MAGNETIC RESONANCE IMAGE DISTORTIONS DUE TO ARTIFICIAL MACROSCOPIC OBJECTS

An example: correction of image distortion caused by an artificial hip prosthesis

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OULU 2002
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An example: correction of image distortion caused by an artificial hip prosthesis

Academic Dissertation to be presented with the assent of the Faculty of Medicine, University of Oulu, for public discussion in the Auditorium 7 of the University Hospital of Oulu, on September 27th, 2002, at 12 noon.

OULUN YLIOPISTO, OULU 2002
Eddy currents and susceptibility differences are the most important sources that interfere with the quality of MR images in the presence of an artificial macroscopic object in the volume to be imaged. In this study, both of these factors have been examined.

The findings show that the RF field is the most important cause of induced eddy currents when gradients with relatively slow slew rates are used. The induced eddy currents amplify or dampen the RF field with the result that the flip angle changes. At the proximal end in the vicinity of the hip prosthesis surface, there have been areas where the flip angle is nearly threefold compared to the reference flip angle. Areas with decreased flip angles have also been found near the surface of the prosthesis top. The incompleteness of the image due to eddy currents manifests as signal loss areas.

Two different methods based on MRI were developed to estimate the susceptibility of a cylindrical object. One of them is based on geometrical distortions in SE magnitude images, while the other takes advantage of phase differences in GRE phase images. The estimate value of the Profile™ test hip prosthesis is \( \chi = (170 \pm 13) \cdot 10^{-6} \).

A remapping method was selected to correct susceptibility image distortions. Correction was accomplished with pixel shifts in the frequency domain. The magnetic field distortions were measured using GRE phase images. The method was tested by simulations and by imaging a hip prosthesis in a water tank and in a human pelvis. The main limitations of the method described here are the loss of a single-valued correction map with higher susceptibility differences and the problems with phase unwrapping in phase images. Modulation transfer functions (MTF) were exploited to assess the effect of correction procedure. The corrected image of a prosthesis in a human hip after total hip arthroplasty appears to be equally sharp or slightly sharper than the corresponding original images.

The computer programs written for this study are presented in an appendix.

**Keywords:** eddy currents, image correction, image distortion, magnetic resonance imaging, susceptibility
To my family
Acknowledgements

This study was carried out at the Department of Diagnostic Radiology, University of Oulu, Finland, during the years 1995-2002.

I wish to express my deepest gratitude to my supervisors, Professor Kalevi Kiviniitty, Ph.D., Medical Physics, and Professor Ilkka Suramo, M.D., Ph.D., Diagnostic Radiology, for their guidance, continuous interest, constructive criticism and support during this work. Professor Kiviniitty first suggested to me the problems at surfaces in magnetic resonance imaging as a topic of study. He has continuously encouraged me to go on with this project. Professor Suramo has never (or very seldom) lost his sincere belief in the success of my work. He has given his warm and firm support to my efforts at all times.

I am very grateful to Docent Osmo Tervonen, M.D., Ph.D., Head of the Department of Radiology, for providing to me good research facilities. He directed my attention to prostheses at the very beginning of the study. The atmosphere at the clinic has always been favourable for scientific work.

Professor Raimo Sepponen, D.Sc. (Tech.), one of the best experts in MRI in Finland, introduced me to the study of the effects of eddy currents. I am deeply grateful for his valuable encouragement and advice during this study.

I want to thank Jukka Jauhiainen, Ph.D., Professor Jukka Jokisaari, Ph.D., and Docent Seppo Koskinen, M.D., Ph.D., as reviewers of this thesis. I greatly appreciate their expert advice and constructive criticism.

I am very grateful to my colleague and friend Kalervo Suominen, Lic.Phil., Medical Physicist, at the Department of Clinical Neurophysiology. He has written part of the computer programs used in this work. We have together studied the secrets of MRI and programming. We have also had many valuable discussion, planned the construction of houses and even built one.

I am indebted to all radiologists for many fruitful and stimulating discussions. Especially I wish to thank Marja Perhomaa, M.D., for her practical help at the final stage of my study.

It has been very pleasant to work with the staff of the MR units, who, even in the midst of the worst rush, have kindly found time for my experiments.
I would also like to present my thanks to the staff at the Library of the Faculty of Medicine for their excellent service during many years. Many thanks to Mrs. Kaisa Punakivi for her friendly and helpful secretarial work, especially on academic topics.

I like to thank Sirkka-Liisa Leinonen, Lic.Phil., for revising the language of the manuscript.

Finally, I wish to express my warmest thanks to my family, my wife Kaarina and all my children, Marja-Kaisa, Antti, Mirja-Liisa, Leena, Ilkka, Matti, Riikka, Ella and Mari, for their love, encouragement and forbearance during these years. Without the great patience of my family, this study would never have been possible.

The financial support given to me by the Science Foundation of Instrumentarium and the Radiological Society of Finland is gratefully acknowledged.

Oulu, September 2002

Antero Koivula
Symbols and abbreviations

\(\hat{A}\) area
\(A\) ampere, unit of electric current
\(\text{AP}\) anterior-posterior
\(\mathbf{B}, B\) magnetic induction or magnetic flux density
\(B_{\text{b}}, B_0\) magnetic flux density of the scanner’s main magnetic field
\(B_1, B_t\) radiofrequency magnetic field
\(\Delta B, \Delta B_{\text{loc}}\) variation of magnetic field, local variation of magnetic field
\(\text{BW}\) bandwidth of pixel
\(\text{CT}\) computed tomography
\(\chi, \chi\) susceptibility
\(dT\) time interval
\(E(r, t)\) relaxation matrix
\(\mathbf{E}, E\) electric field
\(e, e\) electromotive force (emf)
\(\text{EPI}\) echo-planar imaging
\(\text{FID}\) free induction decay
\(\text{FOV}\) field of view
\(\text{FT}\) Fourier transform
\(\text{FFT}\) fast Fourier transform
\(\phi, \varphi\) phase of nuclear spins
\(\Phi\) spatially dependent phase of nuclear spins
\(\mathbf{\Phi}(x, y)\) magnetic flux
\(\mathbf{\Phi}^{-1}(x', y')\) transform to distorted image
\(\mathbf{\Phi}^{-1}(x', y')\) transform to undistorted image (image correction transform)
\(G, G\) field gradient \((G = \partial B / \partial x \cdot i + \partial B / \partial y \cdot j + \partial B / \partial z \cdot k)\)
\(G_n, G_{\phi}, G_s\) frequency-encoding gradient, phase-encoding gradient, slice selection gradient
\(\text{GRE}\) gradient-recalled echo, gradient echo
\(\gamma\) gyromagnetic ratio \((\gamma = 2,67519 \cdot 10^8 \text{ rad s}^{-1} \text{ T}^{-1} \text{ for protons})\)
\(H, H\) magnetic field strength
\(h\) Planck’s constant \((h = 6,6256 \cdot 10^{-34} \text{ Js})\)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$\hbar$</td>
<td>$h/2\pi$</td>
</tr>
<tr>
<td>$I, J$</td>
<td>spin angular momentum, nuclear spin</td>
</tr>
<tr>
<td>$J$</td>
<td>electric current density</td>
</tr>
<tr>
<td>$\mathbf{J}(\frac{x,y}{x',y'})$</td>
<td>Jacobian of the transformation $(x, y) \rightarrow (x', y')$</td>
</tr>
<tr>
<td>$k, \mathbf{k}$</td>
<td>base vector of k-space $(k = k_\mathbf{i} + k_\mathbf{j} + k_\mathbf{k})$</td>
</tr>
<tr>
<td>LSF</td>
<td>line spread function</td>
</tr>
<tr>
<td>$M, \mathbf{M}$</td>
<td>magnetisation</td>
</tr>
<tr>
<td>$M_{xy}, M_{yx}$</td>
<td>transverse magnetisation</td>
</tr>
<tr>
<td>$M_x, M_z$</td>
<td>longitudinal magnetisation</td>
</tr>
<tr>
<td>MR</td>
<td>magnetic resonance</td>
</tr>
<tr>
<td>MRI</td>
<td>magnetic resonance imaging</td>
</tr>
<tr>
<td>MRS</td>
<td>magnetic resonance spectroscopy</td>
</tr>
<tr>
<td>MTF</td>
<td>modulation transfer function</td>
</tr>
<tr>
<td>Mtx</td>
<td>matrix size</td>
</tr>
<tr>
<td>$\mu, \mu_0, \mu_r$</td>
<td>magnetic permeability, $\sim$ in vacuum, relative magnetic permeability</td>
</tr>
<tr>
<td>NMR</td>
<td>nuclear magnetic resonance</td>
</tr>
<tr>
<td>$\nu$</td>
<td>frequency</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>Larmor angular frequency</td>
</tr>
<tr>
<td>pxl</td>
<td>pixel, picture element</td>
</tr>
<tr>
<td>$R$</td>
<td>radius of a cylinder or a prosthesis</td>
</tr>
<tr>
<td>RF</td>
<td>radiofrequency</td>
</tr>
<tr>
<td>ROI</td>
<td>region of interest</td>
</tr>
<tr>
<td>$\rho(\mathbf{r})$</td>
<td>spin density at point $\mathbf{r}$</td>
</tr>
<tr>
<td>$S(t)$</td>
<td>magnetic resonance signal</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>T</td>
<td>Tesla (unit of magnetic induction)</td>
</tr>
<tr>
<td>$T_1$</td>
<td>longitudinal relaxation time</td>
</tr>
<tr>
<td>$T_2$</td>
<td>transverse relaxation time</td>
</tr>
<tr>
<td>TG</td>
<td>transmitter gain</td>
</tr>
<tr>
<td>TE</td>
<td>echo time</td>
</tr>
<tr>
<td>THA</td>
<td>total hip arthroplasty</td>
</tr>
<tr>
<td>TR</td>
<td>repetition time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>flip angle</td>
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1 Introduction

A homogenous magnetic field $B_0$ and linear magnetic gradients are important requirements for high-quality magnetic resonance imaging. Poor construction and shimming of the main magnet or foreign metal objects in the bore of the magnet may impair image quality. Besides these external sources of magnetic field imperfections, there are also internal sources that degrade the homogeneity of the magnetic field. The most important internal sources are susceptibility variations, chemical shifts and induced eddy currents in the volume to be imaged. The biggest susceptibility differences in living objects are between normal soft tissues and air cavities, e.g. in brain imaging, between brain tissue and sinus cavities. When special techniques, e.g. echo-planar imaging (EPI) or water/fat spectroscopic imaging, are used, these susceptibility differences may cause serious image distortions. Many correction methods have been developed for image distortions in EPI or spectroscopic imaging.

Imaging of volumes containing artificial metal objects is increasing, as the number of MRI-guided interventions is rising. Metal objects may, however, cause large image distortions due to their tissue-deviant susceptibility. The strength of distortion depends on the curvature and the orientation of the metal object’s surface relative to the main field $B_0$. The eddy currents induced by alternating gradients and RF magnetic fields are another possible source of image distortions. The importance of eddy currents for image distortions has not been widely studied.

In Finland, about 5000 total hip arthroplasty (THA) operations are performed yearly. One of most typical complications after THA is loosening of the stem or cuff of the prosthesis or both. The early phase of loosening is difficult to detect radiologically.

Magnetic resonance imaging would be very expedient in detecting the loosening of prosthesis. As a result of the loosening process, liquid and soft tissue may develop on the interface between the prosthesis and bone. On the basis of relaxation times, these would be visible in MR images. MR imaging is not, however, widely used in the follow-up of the outcome of THA surgery. The magnetic forces between metal objects and magnetic fields, the thermal effects of induced eddy currents and the image distortions due to metallic objects have been presented as the principal reasons for not using MRI. However, most of the commonly used prostheses are nonferromagnetic, and their magnetic forces disappear. Moreover, the heating of the prosthesis during imaging has
been found to be minimal. However, image distortions may be large, because the deviant susceptibility of metal generates local magnetic field disturbances and off-resonant effects. Another reason for image distortions may be the magnetic fields induced by eddy currents in metal. These eddy currents are induced by alternating gradients and RF fields. An opportunity to correct or eliminate at least part of these image distortions might facilitate the utilisation of MRI in the postoperative follow-up of THA patients and in image-guided interventions. Nowadays, when new MRI scanners with higher magnetic fields are coming into clinical use, the role of image distortions due the susceptibility differences, for example, will be more conspicuous and their elimination more important.
2 Review of the literature

2.1. Image distortions caused by metal objects

2.1.1 MR imaging of volumes containing metal objects

In the early 1980’s, it already became evident that non-ferromagnetic implants were not an obstacle to successful MR imaging (Davis et al. 1981, New et al. 1983), and no magnetic forces affected non-magnetic clips. No significant heating during RF exposure was reported, either (Davis et al. 1981). The first large-scale study of total hip prostheses - 22 Charnley-Müller prostheses - in the 0.5 T field showed that image distortions were larger in sagittal and coronal slices than in axial slices (Lemmens et al. 1986). Moreover, in axial slices, the diagnosis of loosening by detecting demarcation lines at the interfaces was accomplished best in the distal part of the femoral stem prosthesis and least well in the acetabular component and the upper part of the stem prosthesis due to artefacts. MR imaging of the surrounding soft tissues was superior to CT and showed less distortion. Also, MR imaging in the higher 1.5 T field appeared to be safe for patients and, with non-ferromagnetic implants, very useful (Augustiny et al. 1987). The direction of the prosthesis relative to the phase- and frequency-encoding gradients within a static magnetic field and the sequence used affected the image distortions when the best images were obtained with the stem of the prosthesis parallel to the field $B_0$. In that case, distortion was largest in the direction of the frequency-encoding gradient. Image distortions were more marked in the coronal and sagittal slices. Augustiny et al. (1987) pointed out that the image artefacts due to metal implants were manifested as geometrical distortions, regions of signal loss with sharp margins or gradual signal loss towards the implant and linear, sharply demarcated areas of very strong signal. The characteristics and applicability to MRI of more than 250 biomedical implants were presented in a review by Schellock et al. (1991).
2.1.2 Eddy currents in metallic implants

Only very few reports dealing with image distortions due to eddy currents induced in the implants have been published. According to Malko and Nelson (1987), by using an external, movable copper coil, it was possible to detect signal reduction in the vicinity of the coil. They found this signal reduction to arise from the eddy currents induced in the coil, which distorted the excitation field of the MR imager in the neighbourhood of the coil; coils may hence be used to reduce the signal intensity of tissues adjacent to them. Camacho et al. (1995) published a study where they used a copper loop as a current pathway. With this simple geometry, the authors showed that the eddy currents induced in the metal produce a significant local artefact in an image. The artefact was demonstrated experimentally to result from perturbations of the transmission and reception sensitivities of the RF coil. The measurements were done using the methods published by Barker et al. (1993), where the resonance signal intensity is followed by changing simultaneously the transmitter power of the RF field. The intensity of the measured resonance signal depends on the flip angle $\theta$ in the way presented by Hornak et al. (1998). Camacho et al. (1995) showed further that only the RF field induced eddy currents in the loop, while alternating gradient fields were not able to induce eddy currents. On the contrary, Hua and Fox (1996) found that MR images of patients wearing a surgical halo occasionally had signal loss areas, even when the RF field did not penetrate any current loop in the halo. They found that artefacts were substantially reduced if the axis of the halo was adjusted parallel to the phase-encoding direction. Thus, on the basis of published reports, both the RF field and rapidly changing gradients may induce eddy currents in a metal object if a pathway for a current loop exists.

2.1.3 Susceptibility difference induced image distortions

Magnetic susceptibility $\chi$ is a well-known and documented physical phenomenon (e.g. Schenck 1996 and references therein). It is a quantitative measure of a material’s tendency to interact with and distort an applied magnetic field. Magnetic susceptibility can be expressed as a relationship between a material’s magnetisation $\mathbf{M}$ and magnetic field strength $\mathbf{H}$. Local variations of susceptibility $\chi$ in the imaged volume generate local variations $\Delta \mathbf{B}$ in the magnetic flux density $\mathbf{B}$. In simple geometries, such as cylindrical rods and spherical balls, image distortions due to susceptibility have been successfully calculated. The corresponding measurements agreed well with the theory (Lüdecke et al. 1985, Bakker et al. 1993). Moreover, the presence of susceptibility yields larger image distortions when gradient echo sequences are used compared to the distortions that arise when spin echo sequences are used (Posse & Aue 1990, Bakker et al. 1993). Particularly strong susceptibility-induced image distortions are present when EPI sequences are used (Jezzard & Balaban 1995).
2.2 Correction methods for image distortions

Eddy currents are another mechanism that produce image distortions in the presence of metal objects. Alternating electromagnetic fields can induce eddy currents, which induce alternating magnetic fields. These fields modify the original magnetic field, and this leads to image distortions. The distortions caused by metal objects in the magnetic field $B_0$ are the other, usually more predominant, effects.

2.2.1 Eddy currents

A few basically different methods have been presented for removing the image artefacts due to eddy currents.

- To cut off the eddy current loop, if possible (Clayman et al. 1990, Camacho et al. 1995)
- To use a gradient-echo sequence instead of a spin-echo sequence, which makes the signal intensity less dependent on the flip angle (Camacho et al. 1995)
- To measure the perturbations of the transmitter and receiver sensitivities of the RF coil, when the intensities of image pixels can be corrected. (Camacho et al. 1995)
- To choose the phase- and frequency-encoding gradients in such a way that the frequency-encoding gradient does not penetrate the loop of eddy currents (Hua & Fox 1996).

2.2.2 Determination of magnetic field distortion due to susceptibility variations

In simple geometrical models, such as a cylinder or a ball, the distortion of the magnetic field has been successfully calculated analytically (Lüdecke et al. 1985, Callaghan 1990, Bakker et al. 1993). Computer programs have also been written to calculate the field distribution in objects of complicated geometries (Bhagwandien 1994, Bhagwandien et al. 1994, Li et al. 1995, de Munck et al. 1996).

The calculation of magnetic field distortion requires exact information of the shape and susceptibility of the foreign object. This can be avoided by using methods where the magnetic field distribution has been determined in the image of object volume. In 1984, two methods were published that could were suitable for determining the distortion of the magnetic field: chemical shift imaging (Sepponen et al. 1984) and “spectroscopic imaging”, which differentiates between the signals of fat and water (Dixon 1984). The latter is known as the two-point Dixon method. In this method, two echoes are registered and two images calculated. The fat and water signal phases of these images are either in phase or in opposite phase ($\Delta \phi = 0$ or $\Delta \phi = \pi$). The method has also been used to shim the main field automatically (Schneider & Glover 1991). The three-point Dixon method was developed for simultaneous shimming and differentiation of water and fat signals (Glover & Schneider 1991). The echo time TE was changed so as to make the phase
differences of the water and fat signals $\Delta \phi = -\pi$, $\Delta \phi = 0$ and $\Delta \phi = \pi$. The simplest way to measure the distribution of a static magnetic field is to use a gradient-echo sequence, when after the echo time TE, the phase difference $\Delta \phi = \gamma \Delta B \cdot TE$ (Jezzard et al. 1995).

The main problem with phase images is the cyclic property of the phase. The phase angles $\phi \pm n \cdot 2\pi$ ($n = 0,1,2...$) cannot be separated from each other. When the phase angle grows and exceeds the value of $2\pi$, the corresponding pixel level in a phase image changes sharply from bright to black. Many methods have been reported to correct or unwrap this aliasing phenomenon: phase image pairs with opposite phases (Borrello 1987), 2-dimensional polynomial surface fitting (Glover & Schneider 1991, Schneider & Glover 1991, Pohjonen & Castren 1991, Liang 1995) and a two-dimensional region-growing approach, which tracks phase evolution and unwraps the phase when a $2\pi$ jump is detected (Hedley & Rosenfeld 1992, Szumowsky et al. 1994).

A double-DANTE tagging method has also been used to indicate and quantitate the distortions of the field (Mosher & Smith 1991, Li et al. 1995). In the tagging method, a sequence of the RF field with many pulses has been used with simultaneous tagging gradients. The image is then tagged with crossing lines, which are curved due to field inhomogeneities.

### 2.2.3 Correction of distortions due to susceptibility differences

The methods of correcting image distortion due to susceptibility differences can be divided into two groups: re-mapping and modified pulse sequence methods (Kim et al. 1995).

In the re-mapping methods, the distortion of the magnetic field, $\Delta B$, at every pixel must be known. The local field distortion term causes a shift in the pixel position in the frequency-encoding direction. Mathematically, the image distortion caused by the field distortion is a map from a true image into a distorted image. With certain conditions, an inverse map also exists, which can be used to correct a distorted image. Inverse mapping can be done either as displacement of pixels in an image in the spatial domain (Weis & Budinsky 1990, Weis et al. 1996, 1997, 1998a, 1998b, Jezzard et al. 1995) or as raw data phase operations in the frequency domain (Weisskoff & Davis 1992), assuming that an unequivocal re-map exists.

Weiss and Budinsky (1990) simulated MR image artefacts caused by a non-perfect static magnetic field and gradient fields and represented an inverse map that can be used to correct image distortions in the spatial domain. The method has been further elaborated and used to correct image distortion effects in NMR microscopy (Weis et al. 1996) and spectroscopic imaging (Weis et al. 1997) and to make images of the head better suitable for stereotaxy (Weis et al. 1998b). A similar method has been used to eliminate the effects of field distortions in spectroscopic imaging, where water and fat signals are separated on the basis of their different resonance frequencies (Glover & Schneider 1991, Schneider & Glover 1991, Szumowsky et al. 1994). The same method has been used to correct susceptibility-induced image distortions in EPI imaging (Weisskoff & Kühne 1992, Jezzard et al. 1995). An inverse map in the frequency domain is obtained during

The image artefacts caused by field distortions can be eliminated with suitable pulse sequences during data collection already (Kim et al. 1995). This method does not require the determination of field distortion, but the signal is registered piecewise by reversing the direction of the read gradient between the registered pieces. The method is based on the previously developed methods used in solid-state MR imaging (Miller & Garroway 1986), in NMR microscopy (Callaghan 1990) and in low-field NMR (Bendel et al. 1985, Wong & Rosenfeld 1987).

### 2.3 Radiological follow-up of total hip arthroplasty

The radiological assessment and follow-up of total hip arthroplasty (THA) patients is based on standard anterior-posterior radiographs taken immediately after surgery. The geometric position and localisation with rotation angles have been measured. The postoperative radiographs are also used as reference for follow-up survey images. New radiographs are taken at the time of the survey. Sinking and calcar resorption are measured and compared to the postoperative reference situation. Density and hypertrophy of cortical bone are evaluated visually, and the thickness of the cortex in the diaphyseal area around the cylindrical part of stem is measured from AP radiographs individually on the medial and lateral sides. Any broken screws are noted, and the radiolucent lines and osteolytic foci are located and expressed in millimetres. Ectopic ossification is assessed, too (Boos et al. 1987, Niinimäki 1995). The shortest distance that can be reliably measured in standard radiographs has turned out to be 2 mm, and movement of 5 mm or more can be accepted as true migration of the acetabulum socket (Sutherland 1982, Niinimäki 1995).

In many studies, the suitability of MRI to image metal prostheses has been evaluated (Lemmens et al. 1986, Augustiny et al. 1987, Ebraheim et al. 1992). The diagnosis of loosening based on detection of demarcation lines at the interfaces has been found to be most reliable in the distal part of the femoral stem prosthesis and poor in the acetabular component and in the upper part of the stem prosthesis due to artefacts. However, imaging of soft tissue has been found to be superior to CT and has showed less distortion (Lemmens et al. 1986, Augustiny et al. 1987).

The possibilities to use MRI in the follow-up of THA patients have also been studied in Oulu University Hospital. We have sought for an optimal sequence to minimise image distortions and have found the usual spin-echo sequence with a small field of view (FOV) and a large matrix (Mtx), which means a steep read-out gradient, to be the best (Törmänen et al. 1996). Further, it has been found that if the stem of the prosthesis is parallel to the main magnetic field, it is possible to assess the contact area between the bone and the stem (Niinimäki 1995).
3 Purpose of this study

This study is part of a series of studies where the suitability of MRI to the follow-up of total hip arthroplasty patients was explored. The aim of this study was to examine the field and image distortions caused by macroscopic metal objects, such as hip prosthesis or intervention needles, to suggest possible ways to correct image distortions and to develop, if possible, a method to correct image distortion due to susceptibility differences. The Profile™ hip prosthesis was used as a test object.

Especially the following tasks were addressed:
1. To assess the image distortion caused by eddy currents
2. To develop a MRI-based method to determine the susceptibility values of cylindrical objects and to test it with the test prosthesis.
3. To study the possibilities and boundary conditions for correcting image distortions caused by susceptibility differences.
4. To develop a correction method for susceptibility-induced image artefacts and to test it with the test prosthesis. To evaluate the limitations of the correction method and to assess the results by using the modulation transfer function (MTF).
5. By writing the necessary computer programs for testing and implementing the ideas, to generate tools for studying image distortions.
4 Magnetic resonance imaging

4.1 Principles of magnetic resonance imaging

4.1.1 Nuclear magnetism and magnetic resonance

The principles of MRI are discussed in detail in many modern textbooks (e.g. Buxton 2002). Some general guidelines are given here. Magnetic resonance imaging (MRI) is based on the magnetic property of atomic nuclei. Every nucleus with a nuclear spin quantum number \( I \neq 0 \) has an intrinsic magnetic moment \( \mu = \gamma I \) associated with the angular momentum of the nucleus, called nuclear spin \( I \), and can be an object in magnetic resonance experiments. In the above relation, \( \gamma \) is the gyromagnetic ratio \( (\gamma = 2.67519 \times 10^8 \text{ rads}^{-1}\text{T}^{-1} \text{ for protons}) \) and \( \hbar = h / 2\pi \), where \( h \) is Planck’s constant \( (h = 6.6256 \times 10^{-34} \text{ Js}) \).

The signal of water protons is used in almost all clinical MR imaging, because biological tissues have high concentrations of water and protons have high gyromagnetic ratios. Hence the magnetic resonance signal of tissue water protons is strong. In this study, only the resonance effects of protons are considered.

In a magnetic field, \( B \) protons precess around the direction of the field with angular frequency

\[
\omega_I = -\gamma B. \tag{4-1}
\]

Any observed magnetic resonance signal always consists of the magnetisation of a huge number of nuclear spins. Nuclear magnetisation is a vector sum of nuclear magnetic moments:

\[
M = \sum_i \mu_i. \tag{4-2}
\]

In classical macroscopic consideration, a small volume element in position \( r_i = (x, y, z) \) to be imaged into one picture element, pixel, has magnetisation \( \mathbf{M}_i \) proportional to the proton density \( \rho(r) \). The magnetic field inside the volume element consists of a homogeneous main field \( B_0 \), a gradient field \( \mathbf{G} = (G_x, G_y, G_z) \) and a local field distortion \( \Delta B(r) \) due to e.g. susceptibility variations:
\[
\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \mathbf{G} \cdot \mathbf{r} + \Delta \mathbf{B}(\mathbf{r})
\]
(4-3)

The resonance frequency of protons
\[
\omega = \frac{\gamma}{2\pi} \mathbf{B}(\mathbf{r}) / 2\pi
\]
(4-4)
is dependent on the local magnetic field \(\mathbf{B}(\mathbf{r})\).

The co-ordinate system generally used in MRI, the magnetic field \(\mathbf{B}_0\) is parallel to the \(z\)-axis, while the \(x\)-axis is a horizontal axis perpendicular to the head-to-feet axis of the person to be imaged. The rotating co-ordinate system \((x', y', z')\) is defined so that the \(z'\)-axis coincides with the \(z\)-axis and the \(x'\)-and \(y'\)-axes are rotating around the \(z\)-axis with the same angular frequency \(\omega_2\) as the RF magnetic field. This co-ordinate system, called rotating frame, is generally used in the examination of magnetic moments’ precession and in considering the effects of radiofrequency fields. Within the rotating frame, the RF field is static and only the effects of slowly varying phenomena, e.g. nuclear magnetic relaxation, on the precessing magnetisation are visible. The only frequency difference arises from variations in the local magnetic field \(\Delta \mathbf{B}_{\text{loc}}\), or from the effects of the field gradients applied to imaging.

### 4.1.2 Magnetic field gradients and k-space

The volume element at position \(\mathbf{r} = x_i + y_j + z_k\) with respect to the isocenter of the scanner system has a spin density \(\rho(\mathbf{r})\). The magnetisation of the volume element along the \(z\)-axis \(\mathbf{M}_z(\mathbf{r})\) is proportional to \(\rho(\mathbf{r})\). A RF pulse rotates this magnetisation at an angle of \(\theta\) and generates a component of magnetisation in the \(xy\)-plane, \(\mathbf{M}_{xy}(\mathbf{r}) = \mathbf{M}_z(\mathbf{r}) \sin \theta\). In the rotating frame, the direction of \(\mathbf{M}_z(\mathbf{r})\) is static, but its magnitude is reduced due to spin-spin relaxation. By applying a linear magnetic field gradient \(\mathbf{G}(t)\), \(\mathbf{M}_{xy}(\mathbf{r})\) can be made to rotate in the rotating frame. At a time \(t\), the phase of magnetisation \(\mathbf{M}_{xy}(\mathbf{r})\), \(\varphi(\mathbf{r}, t)\), is
\[
\varphi(\mathbf{r}, t) = \varphi(\mathbf{r}, 0) + \int_0^t \omega(\mathbf{r}, t') dt'
\]
(4-5)
The vector \(\mathbf{k}\) is defined as the time integral of the field gradient (Ljungren 1983, Twieg 1983, King & Moran 1984):
\[
\mathbf{k}(t) = \frac{\gamma}{2\pi} \int_0^t \mathbf{G}(t') dt'
\]
(4-6)
The phase at location \(\varphi(\mathbf{r})\) can be presented using the \(\mathbf{k}\) vector
\[
\varphi(\mathbf{r}, t) = \varphi(\mathbf{r}, 0) + 2\pi \mathbf{k}(t) \cdot \mathbf{r}.
\]
(4-7)
The measured resonance signal \(S(t)\) is a sum of the signals of all the excited nuclei
\[
S(t) \approx \int \mathbf{E}(\mathbf{r}, t) \mathbf{M}_{xy}(\mathbf{r}) e^{i\varphi(\mathbf{r}, t)} d^3\mathbf{r} \approx \int \mathbf{E}(\mathbf{r}, t) \rho(\mathbf{r}) e^{i\varphi(\mathbf{r}, t)} d^3\mathbf{r},
\]
(4-8)
where \(\mathbf{E}(\mathbf{r}, t)\) is relaxation weight at position \(\mathbf{r}\). Using k-vector presentation and ignoring the effect of relaxation
\[
S(\mathbf{k}(t)) = S(t) \approx \int \rho(\mathbf{r}, t) e^{i2\pi \mathbf{k}(t) \cdot \mathbf{r}} d^3\mathbf{r}.
\]
(4-9)
The signal $S(t)$ is proportional to the Fourier transform (FT) $F(k)$ of the spin density $\rho(r)$. During a MR experiment a discrete set of $F(k)$ values is collected, and by performing an inverse Fourier transform, an image of $\rho(r)$ can be obtained. The calculated image is defined in a real space formed by the spatial vectors $r$ with components $(x, y, z)$. The Fourier transform $F(k)$, however, is defined in the Fourier space, often referred to as the $k$-space, formed by $k$-vectors $(k_x, k_y, k_z)$. The sampling points extracted by a given pulse sequence are determined by the trajectory formed by $k(t)$ in the $k$-space.

### 4.1.3 Pulse sequences

A pulse sequence consists of a suitable train of excitation and gradient pulses that can be used to collect MR imaging signal data $F(k)$ for calculating the image of $\rho(r)$. We can use either spin-echoes (SE) or gradient recalled echoes (GRE) – or for short, gradient-echoes - to produce the signal (Fig. 1). In magnetic resonance spectroscopy (MRS), data points are also detected during free induction decay (FID) signals. The object slice is excited with tailored RF pulses with a narrow excitation bandwidth and simultaneous slice selection gradients. Phase-encoding gradients are used to encode the $k$-space in the orthogonal direction $k_p$ to the read direction. During the data collection, a simultaneous gradient in the read direction (frequency-encoding gradient) encodes spins in the read direction $k_r$ in the $k$-space. Depending on the time parameters in the sequence, the image may be weighted by the proton density and/or by the relaxation times $T_1$ and $T_2$.

![Fig. 1. Spin-echo and gradient-echo sequences.](image)

RF = radiofrequency excitation, $G_s$ = slice selection gradient, $G_p$ = phase-encoding gradient, which varies from excitation to excitation between $-G_p(\text{max})$ to $+G_p(\text{max})$, $G_r$ = frequency-encoding gradient (read-out gradient), and $\omega$ is the phase angle of spins with angular frequencies of $\omega$ and $\omega + \Delta\omega$. 


4.2 MR equipment

Three MR imagers are considered in this study, namely Magnetom SP 42 (Siemens, Erlangen, Germany), Signa Horizon EchoSpeed (GE, Milwaukee, WI, USA) and Proview (Philips Medical Systems MR technologies Finland, Vantaa, Finland).

Table 1. Comparison of MR imagers used in this study.

<table>
<thead>
<tr>
<th>Scanner</th>
<th>$B_0$ [T]</th>
<th>$G_R$ [mT/m]</th>
<th>Gradient slew rate [T m$^{-1}$ s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proview</td>
<td>0.23</td>
<td>16 (x, y), 18 (z)</td>
<td>25 (x, y), 40 (z)</td>
</tr>
<tr>
<td>Magnetom 42 SP</td>
<td>1.0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Signa Echo Speed</td>
<td>1.5</td>
<td>22</td>
<td>120</td>
</tr>
</tbody>
</table>

Most susceptibility images are scanned using Magnetom. Eddy current measurements have been performed with Signa because of the ability to choose easily the transmitter power. Both of these scanners have a superconductive magnet with the main field oriented horizontally, while Proview has an iron core electric magnet producing the main field vertically, perpendicular to the longitudinal axis of a supine patient. In both scanners, Signa and Magnetom, it is possible to place a person so that the stem of the hip prosthesis is parallel to the main field, when the image artefacts are much smaller compared to the situation in the Proview scanner, where the prosthesis is perpendicular to the main field.
5 Eddy currents

5.1 Introduction

A varying magnetic field induces currents, called *eddy currents*, inside metallic objects. During imaging, both the RF field and the varying gradient fields are able to induce eddy currents. Eddy currents induced by gradient fields increase the rising and damping times of the gradient pulses and deform their shapes. In a well-balanced MRI imager, these eddy currents are usually compensated for by overdriving the gradient currents, so that the shape of the gradient pulses remains correct. If an imaged volume contains metallic objects, such as wires, clips or implants, they may contain induced eddy currents during the rise or damping of the gradient. In addition to possible heating, they may also cause artefacts into the images.

Apart from the gradient fields, the high-frequency RF field also induces eddy currents. According to Lenz’s law, a magnetic field induced by eddy currents is opposite in direction to the original RF field. As a result, the RF field decreases inside the metallic object and increases outside it. The flip angle $\theta$ varies locally, and this is visible as variation of signal intensity. When a metal wire loop is imaged, the only important source of eddy currents is the RF field (Camacho et al. 1995).

In this part, the image artefacts caused by eddy currents are studied and their role among the observed artefacts is evaluated.
5.2 Theory

5.2.1 Eddy current in a wire loop

Fig. 2. In a wire loop, an induced eddy current and the corresponding induced magnetic field. The RF field increases (dB/dt) perpendicular to the plane of the wire loop, which induces an electric current $I$ in the loop. This current generates a magnetic field around the loop, being inside the loop opposite to $B_1$.

According to Faraday's induction law, a temporally varying magnet flux $\Phi$ induces an electromotive force (emf) $e$ in a wire loop with an area of $A$ and a normal vector of $\mathbf{n}$:

$$ e = -\frac{d\Phi}{dt} = \int \frac{d\mathbf{B}}{dt} \cdot \mathbf{n} \, dA, $$

(5-1)

where $\mathbf{B}$ is the magnetic flux density. As a result of the electromotive force, a current $I = \sigma e$, where $\sigma$ is the conductivity of the current loop, is induced in the loop. This current is highest when the loop plane is perpendicular to the alternating field and disappears when the plane is parallel to the field.

Biot-Savart's law says that the electric current $I$ in a wire $dl$ in length induces an elementary magnetic field $db$ at a distance of $r$ from the middle point of the loop with a radius of $R$:

$$ db = \frac{\mu I \, dl \times s}{4\pi \, s^3} $$

(5-2)
where $k$ is the unit vector in the $z$-direction.

Using the cosine rule, $s^2 = R^2 + r^2 - 2Rr \cos \phi$:

$$\frac{\mu \cdot I}{4\pi} \frac{rd\phi}{s^2} \left( \frac{1 - r^2 \sin^2 \phi}{s^2} \right) k,$$

where $\mu$ is the magnetic permeability of the material and the notations of Fig. 2 have been used. The magnetic field induced by the current loop at a distance of $r$ from the middle point of the loop is obtained by integrating (5-3) around the loop:

$$B(r) = \frac{\mu \cdot I}{4\pi} \cdot \int_0^{2\pi} \frac{(R^2 - Rr \cos \phi) d\phi}{(R^2 + r^2 - 2Rr \cos \phi)^{3/2}},$$

or

$$B(r) = \frac{\mu \cdot I}{4\pi R} \cdot \int_0^{2\pi} \frac{(1 - \frac{r}{R} \cos \phi) d\phi}{\left(1 + \left( \frac{r}{R} \right)^2 - 2 \frac{r}{R} \cos \phi \right)^{3/2}},$$

if a relative distance scale $r/R$ is used. The direction of the induced magnetic field is perpendicular to the plane of the loop.

Fig. 3. Radial distribution of the magnetic field induced by an eddy current running in the loop. The curve represents the integral part of Eq. 5-5.

In Fig. 3, a graph representing the value of the integral part of Eq. 5-5 is given as a function of the relative radius $r/R$. It is calculated using a trapezoidal numerical integration method. Towards the loop wire, the absolute value of the induced field
increases, and the function has a singularity at \( r/R = 1 \). The magnetic field due to the eddy currents decreases the original RF field inside the loop and increases it outside the loop. The value of the induced field depends on the integral part but also on the coefficient \( \mu I/4\pi R \), where \( \mu \) is the magnetic permeability of the material around the coil, \( I \) is the current in the loop and \( R \) is the radius of the loop.

### 5.2.2 Eddy currents in a circular plate

A conducting plate can be conceived of as many concentric current loops. When the RF field is homogeneous, the magnetic flux

\[
\Phi = \int_I \mathbf{B} \cdot \mathbf{n} \, dA = \pi r^2 B(r,t)
\]

(5-6)

penetrates an area inside a circle with a radius of \( r \). The flux depends on the value of \( r \). According to Maxwell’s III law,

\[
\nabla \times \mathbf{E}(r,t) = -\frac{\partial}{\partial t} \mathbf{B}(r,t),
\]

(5-7)

an alternating magnetic field \( \mathbf{B} \) generates a circular electric field \( \mathbf{E} \) perpendicular to the magnetic field. The electromotive force \( e \) along a circle with a radius \( r \) is now

\[
e = e(r,t) = \mathbf{J} \cdot \mathbf{dr} = 2\pi r E(r,t) = -\pi r^2 \frac{dB}{dt}.
\]

(5-8)

The current density of the induced current is

\[
\mathbf{J}(r,t) = \sigma \mathbf{E}(r,t) = \sigma E(r,t) e_\phi = -\frac{\sigma}{2} \frac{d B}{dt} r e_\phi,
\]

(5-9)

where \( \sigma \) is the conductivity of the metal plate. The current density increases linearly towards the edge of the plate. The term \( \int \mathbf{J} \cdot \mathbf{dl} \) in Eq. 5-2 can now be represented as

\[
I \cdot \mathbf{dl} = \mathbf{J}(\rho,t) \cdot d\rho \cdot d\phi \cdot \rho \mathbf{e}_\phi = \frac{\sigma d}{2 \rho} \frac{dB}{dt} \rho^2 d\rho d\phi \cdot e_\phi = j_o \rho^2 d\rho d\phi \cdot e_\phi,
\]

(5-10)

where \( d \) is the thickness of the plate, \( \rho \) the distance of the current element from the middle point of the plate and \( e_\phi \) the tangential unit vector, and the notations explained in Fig. 4 are used. Now, the magnetic field induced by a current element of \( j_o \rho d\rho d\phi e_\phi \) is considered. It is situated at the radial distance of \( r \) in a metal plate with a radius of \( R \). The distance between the current element and the observation point is \( s \). Introducing these into Eq. (5-2), we get

\[
dl = \frac{\mu j_o \rho d\rho d\phi \cdot \sin \alpha}{4\pi} \frac{\mathbf{k}}{s^2} = \frac{j_o \mu k}{4\pi} \frac{\rho^2 d\rho d\phi (\rho - r \cos \phi)}{(\rho^2 + r^2 - 2r \rho \cos \phi)^{3/2}}.
\]

(5-11)
Fig. 4. Eddy currents in a circular plate. The RF magnetic field $B_1$ increases towards the plane of plate. It induces a current $J\rho d\phi d$ with a current density of $J$ in the elementary ring with a thickness $d$. This induced current generates a magnetic field around the ring element, shown as arrows.

When a relative radius $x = \rho/R$ is used

$$B\left(\frac{r}{R}\right) = \frac{j_o \mu_0 k}{4\pi} \int_0^{2\pi} \int_0^1 x^2 \left( x - \frac{r}{R} \cos \phi \right) d\phi dx$$

(5-12)

Eq. 5-12 represents the magnetic flux density $B$ induced by eddy currents in a metal plate as a function of the relative radial distance $r/R$, when the eddy currents are generated by an alternating magnetic field perpendicular to the plate plane. Fig. 5 represents the dependence of the integral in Eq. 5-12 on the relative distance $r/R$. Multiplication of an integral value by the factor $j_o \mu_0 / 4\pi$ gives the local magnetic flux density.
5.3 Measurement and calculation methods

Measurements of eddy currents were performed using both 1.0 T Magnetom 42 SP (Siemens, Erlangen, Germany) and 1.5 T Signa Echo Speed (GE, Milwaukee, Wisconsin, USA) scanners. The proportions of the eddy currents induced by the gradient pulses or the RF field were qualitatively studied with the Magnetom scanner. The quantitative measurements of eddy currents was done using Signa, because the transmitting power of the Signa scanner is easy to modify by adjusting the transmitter gain (TG) parameter. A water-filled acrylic box (23 x 17 x 12 cm³) was used as an image object, where a copper wire loop (diameter of 70 mm) or a circular copper plate (diameter of 100 mm) had been immersed. The angle between the static magnetic field \( B_0 \) and the plate or loop was adjustable between the measurements. Both spin- and gradient-echo sequences with TR of 690 ms, TE of 10 ms, matrix of 192 x 256, and 2 acquisitions were used in qualitative measurements with the Siemens scanner. The transmitter gain (TG) is an adjustable parameter in the GE Signa scanner. The spin flip angle \( \theta \) after the RF pulses depends logarithmically on the transmitter gain (TG) (Barker et al. 1993)

\[
\theta_A = \theta_0 \cdot 10^{\frac{\text{TG} - \text{TG}_0}{20}},
\]

where the reference angle \( \theta_0 \) is usually \( \pi/2 \). The local signal intensity \( S(r) \) of a spin-echo sequence depends on the flip angle \( \theta \) according to the equation

\[
S(r) = S_0 \cdot \sin^3 \theta(r),
\]
where $S_0 = S(\pi/2)$ (Hornak et al. 1988). Then
\[
S(r) = S_0 \sin^3 \left( \frac{\pi}{2} \frac{\frac{\pi}{2}}{200} \right) (5-15)
\]
The maximal signal intensity is observed when the flip angle is equal to $\pi/2$. A spin-echo sequence with TR of 600 ms and TE of 10 ms was used in the quantitative measurements. The water box was imaged both with and without a copper loop or plate. Copper was chosen as the material, because its susceptibility differs only very slightly from that of water and its electric conductivity is high. A Profile™ prosthesis was also imaged in a water tank with the stem axis parallel to the main magnetic field $B_0$. Both head and body coils were used. When a body coil was used, the value of TG varied from 0 to 200. The copper loop and plate were imaged using a circular polarised head coil to obtain better signal-to-noise (SNR) characteristics. Now, TG ranged between 5 and 140. Corresponding to the maximal signal intensity, the local TG($\pi/2$) values were found by fitting Eq. 5-15 into a series of images with different TGs pixel by pixel. A special computer program was written to perform this fitting procedure, where a Levenberg-Marquard algorithm was used. The MATHCAD program (Mathsoft Inc., Cambridge, Mass., USA) was used in the analysis of the results, too.

5.4 Results

5.4.1 Copper wire loop and plate

The results of the measurements carried out using Magnetom 42 SP are presented in the Figs 6 and 7. In Fig. 6, the plane of the wire loop and the plate is perpendicular to the static magnetic field $B_0$, which is parallel to the RF field, when the RF field does not penetrate the area of the loop or plate. In the images of the top row (a)-(c) a spin-echo (SE) sequence was used, while in the images of the second row a corresponding gradient-echo (GRE) sequence was used. In both cases, only a slice selection gradient penetrates the plate and loop, and no effect of eddy currents is visible. Image artefacts due to local susceptibility variations are more clearly visible in the gradient-echo images.

In Fig. 7, the metal objects are perpendicular to the RF field and penetrated by both the RF field and the slice selection and read-out gradients. Changes in signal intensity caused by eddy currents are evident in both the SE (g)-(i) and the GRE (j)-(l) images. The changes due to eddy currents are stronger in the SE images, while the susceptibility-generated artefacts are bigger in the GRE images.

In quantitative measurements, a water box with either a copper wire loop or a circular copper plate was lying perpendicular to the RF field. A slice in the plane of the metal object was imaged. The images of a loop and plate obtained with two transmitter gain values, TG of 20 and TG of 130, are presented in Fig. 8. The lower TG of 20 corresponds to a flip angle of $\theta < \pi/2$ and the higher TG of 130 produces a flip angle of $\theta > \pi/2$ without any effect of eddy currents.
Fig. 6. A copper wire loop and a circular copper plate in a water box. The loop and the plate are parallel to the RF field when the RF field does not penetrate the plane of the objects. Figs. (a)-(c) were imaged using a SE sequence and (d)-(f) using a GRE sequence. The images in (a) and (d) are coronal slices, while the others are sagittal slices. The loop was imaged in the Figs. (b) and (e), and the plate in (c) and (f).

Fig. 7. The box has been turned 90°, and the Cu wire loop and plate are perpendicular to the RF field and the maximal penetration of RF samples is achieved. The imaging parameters and slices are the same as in Fig. 6.
With a low TG, the areas outside both the plate and the loop are bright. This is an area where the eddy current field increases the RF field. When a high TG value is used, the image pixels inside and near the loop or the edge of the plate are bright. In those areas, the eddy current fields decrease the RF field.

The same effect is visible in Fig. 9, where the typical pixel intensity variations as a function of transmitter gain (TG) and the corresponding fits of Eq. 5-15 to the pixel intensity are shown. The TG value that gives a maximal pixel intensity corresponds to the flip angle $\theta = \pi/2$ in this pixel position. Only the TG values from zero to the first minimum are included in the fit.

Now, an image (Fig. 10) with a pixel intensity corresponding to the local values of $\text{TG}\pi/2$ can be calculated by fitting Eq. 5-15 pixel by pixel to a series of images with different TGs in the same way as presented in Fig. 9. The pixel intensities are calibrated in such a way that black (pixel value = 0) corresponds to a transmitter gain TG = 0, while bright (pixel value = 255) corresponds to TG = 150. Some example regions where the fit procedure failed are marked with an arrow. In the vicinity of the Cu wire loop, the

---

**Fig. 8.** Cu wire loop (a) and (b) and Cu plate (c) and (d) in a water box imaged with a SE sequence with TR = 500 ms, TE = 10 ms, and Acq = 1. In (a) and (c), transmitter gain TG = 20, and in (b) and (d) TG = 130. The average TG of $\pi/2$ pulses was 76.
changes in eddy current fields are so large that they cannot be measured with the methods available here because of the limited adjustable region of TG.

Fig. 9. Intensity variations of an image pixel according to the different transmitter gains (TG), when a circular Cu plate (a) or a Cu loop in a water box (b) was imaged. The Cu objects are perpendicular to the RF field, and the image slices are in the plane of the plate and the loop. The corresponding background images without a plate or a loop are registered in the same planes compared to the metal objects. Inside the plate edge and the loop, the intensity maximum demands a higher transmitter gain compared to the corresponding background image. On the other hand, outside the plate edge and the loop, maximal intensity can be obtained with lower TG compared to the corresponding background image.
Fig. 10. Transmitter gain (TG) used to obtain the maximal image intensity presented pixel by pixel. The grey levels are defined so that the black pixels correspond to transmitter gain TG = 0 and the bright pixels to TG = 150. The fit has failed in the regions marked with arrows. (a) Circular copper plate, (b) copper wire loop.

Fig. 11. Measured and calculated radial profiles of relative TG\(_{g/2}\) in (a) Cu plate and (b) Cu ring.

In Fig. 11, a radial profile of the relative flip angle is shown as calculated as a mean of two horizontal and two vertical radii. The measured flip angles are scaled against the background ones, which have a value of 1. The disturbances due to the curvature of the plate, which are visible in the plate area as extra peaks, are shown in Fig. 10 (a). These peaks reduce the quality of fit. In the immediate vicinity of the Cu loop, the eddy currents in the loop change the RF field so strongly that the adjustment range of TG is
insufficient to balance the power of the RF field correspondingly. This can be seen in Fig. 10 (b), where the fit is incomplete close to the wire.

5.4.2 Measurements with an artificial hip prosthesis

A prosthesis lying in a water tank with its stem parallel to the main magnetic field $B_0$, was imaged using a normal SE sequence: TR=300 ms, TE=10 ms and Acq=1. A body coil was used as a transmitter and a receiver coil. Transmitter gain TG was varied from 10 to 200 in steps of 10. The image series was analysed as described above for a Cu loop and a Cu plate.

In Fig. 12, the TG values corresponding to maximal signal intensity are represented as an image. The positions of the axial slices are marked in a coronal image 12(a). Close to the distal top of the prosthesis, dark points or regions are visible, which are shown in more detail in the Figs. 13 (c) and (d). A circular region can be seen at the distal top of the prosthesis, where the field induced by an eddy current enhances the RF field to be nearly 1.5-fold compared to the reference. In the direction outside the prosthesis, there is a region where the RF field has decreased due to an eddy current, the flip angle corresponding to the $\pi/2$ pulse being only about $60^\circ$.

The changes in the RF field at the proximal top of the prosthesis are visible in the Figs. 12 (b) and (f) and 14 (a) and (b). Both in the sagittal and in the axial slices 14 (a) and (b), respectively, a region can be seen where the RF field is nearly threefold in height compared to a situation without eddy currents.
Fig. 12. Transmitter gain (TG) corresponding to a flip angle of $\pi/2$ presented as grey-scale images of a Profile™ prosthesis in different slice orientations. The regions with a high eddy current induced field are visible as dark regions in the Figs (a), (c) and (f).
Fig. 13. The changes in the eddy current induced field in sagittal (a) and (c) and axial slices (b) and (d) at the distal top of the prosthesis.
5.5 Discussion

It is evident that rapidly changing magnetic fields, such as gradient fields and RF fields, are able to induce eddy currents in metal objects present in the imaged volume, and these eddy currents may cause image artefacts (Malko & Nelson 1987, Camacho et al. 1995, Hua & Fox 1996). On the other hand, the relative role and importance of gradient fields compared to RF fields have been reported controversially (Camacho et al. 1995, Hua & Fox 1996). In this study, a copper wire loop and a circular copper plate were used to evaluate the relative importance of eddy currents induced by the gradient and RF fields. Copper was used as conductor, because the susceptibility of Cu is very close to that of water, and susceptibility distortions were hence negligible. When relatively slowly changing gradient fields (maximal 10 T m⁻¹ s⁻¹) are applied, the image artefacts caused by eddy currents induced by the gradient fields are very small compared to the artefacts due to induced eddy currents induced by the RF fields. This is in good accordance with the results of Camacho et al. (1995). However, the eddy currents
induced by the gradient field may play a more important role in image distortions when strong and very rapidly switching gradients are used, as in EPI.

The intensity of the RF field varies in the vicinity of a Cu object, depending on the increasing or decreasing effect of the eddy current induced magnetic field. The results of the present measurements with a Cu loop and a Cu plate were in satisfactory agreement with the theory.

The results of the measurements done with a prosthesis (Profile\textsuperscript{TM}) showed that the induced eddy currents affect the image artefacts in images including a prosthesis. Artefacts are seen especially at the distal and proximal tops of the prosthesis, where the magnetic flux penetrates the surface of the metal object and the curvature of the surface is conspicuous. The image artefacts are shown as signal loss areas due to the exceptional flip angles consequent to RF pulses. According to the present results, the flip angle may be 2.5-fold at the proximal end of the prosthesis and about 1.5-fold at the distal top compared to a normal value. At the middle stem, no image artefacts due to the eddy current are visible. Depending on the signal loss regions due to eddy currents at both ends of the prosthesis and on the marked image distortions caused by susceptibility differences in the same regions, distortionless imaging of these regions is extremely difficult, or even impossible.

The signal intensity obtained by using a GRE sequence $S = S_x \sin \theta$, while the intensity obtained by using a SE sequence is $S = S_x \sin^3 \theta$, where $\theta$ is the flip angle of magnetisation. The dependence of signal intensity on the flip angle is smaller in the GRE sequence. However, the possibility to avoid eddy current artefacts by using a GRE sequence is very limited, because GRE sequences will increase more the dominant susceptibility artefacts.
6 Image distortions induced by susceptibility differences

6.1 Introduction

6.1.1 Magnetic susceptibility

The properties of magnetic fields are described in terms of quantities: magnetic field strength \( H \) [A/m], magnetic flux density or magnetic induction \( B \) [T] and magnetisation \( M \) [A/m]. The coefficients between these quantities are magnetic permeability \( \mu \) and magnetic susceptibility \( \chi \). Magnetic susceptibility tensor \( \chi_{ij} \) measures magnetisation \( M \) inside a piece material located in a magnetic field \( H \). In liquids and soft tissues, susceptibility is isotropic, i.e. a scalar \( \chi = \chi_{ii} \). Magnetic field strength \( H \), magnetic flux density \( B \) and magnetisation \( M \) are vectors. They are related according to the equations

\[
B = \mu H = \mu_0 \mu H, \tag{6-1}
\]

\[
M = \chi H, \tag{6-2}
\]

where \( \mu_0 \) is the magnetic permeability of vacuum and \( \mu \) is the relative permeability of corresponding material. Let \( B_0 \) be the magnetic flux density in vacuum and \( B \) the magnetic flux density in material, then

\[
B = \mu_0 (H + M) = \mu_0 (1 + \chi) H = B_0 + \chi B_0 = (1 + \chi) B_0. \tag{6-3}
\]

The magnetic flux density inside material depends on magnetic susceptibility. Hence, susceptibility differences affect the homogeneity of the magnetic field in the imaged volume.

Susceptibility variations can be manifested in two different ways: microscopic, if the variation is across an object volume element, and macroscopic, if the susceptibility and the field strength inside the different volume elements differ from one another. Susceptibility variation inside the volume elements affects relaxivity and signal intensity, while macroscopic variations are visible as image distortions. In this study, only macroscopic effects are considered.
Body tissues are typically weakly diamagnetic when their susceptibility is negative ($\chi \approx -7.0 - -11.0 \cdot 10^{-6}$), and the magnetisation of tissue is opposite in direction to the external magnetic field. The weakly paramagnetic air cavities have the most exceptional susceptibility values inside a body ($\chi = 0.36 \cdot 10^{-6}$). Deoxyhemoglobin, which contains iron, also has elevated susceptibility. However, the normal susceptibility differences within a human body are so small that they only cause very small image distortions if spin-echo sequences are used. Air cavities cause the most serious image distortions when fast gradient-echo sequences, such as EPI, are used. Examples of the susceptibility values of different materials are given in Table 2.

Some metals, e.g. bismuth, gold, mercury, lead and copper, are diamagnetic. In diamagnetic materials, the direction of magnetisation is opposite to the magnetic field, and flux density is lower inside than outside diamagnetic material. Many metals, such as magnesium, aluminium, tungsten, titanium, nickel, vanadium, and iron and ferrous alloys, are paramagnetic or ferromagnetic. Apart from copper, the susceptibility of metals differs significantly from that of water and tissues. Magnetisation in para- and ferromagnetic materials is parallel to the polarising magnetic field, and the field inside the material increases. The magnetic flux density in metal objects is so strong that $B$ also changes in the vicinity of a metal object.

Table 2. Susceptibility $\chi$ of some materials (Schenck 1996).

<table>
<thead>
<tr>
<th>Material</th>
<th>Susceptibility $[10^{-6}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi</td>
<td>-164</td>
</tr>
<tr>
<td>Au</td>
<td>-34</td>
</tr>
<tr>
<td>Hg</td>
<td>-28</td>
</tr>
<tr>
<td>Ag</td>
<td>-24</td>
</tr>
<tr>
<td>Pb</td>
<td>-15.8</td>
</tr>
<tr>
<td>Zn</td>
<td>-15.7</td>
</tr>
<tr>
<td>Cu</td>
<td>-9.63</td>
</tr>
<tr>
<td>Water (+37 °C)</td>
<td>-9.05</td>
</tr>
<tr>
<td>Human tissue</td>
<td>$\approx (-1.0 \ldots -7.0)$</td>
</tr>
<tr>
<td>ZrO$_2$</td>
<td>-8.3</td>
</tr>
<tr>
<td>Whole blood (deoxy)</td>
<td>-7.90</td>
</tr>
<tr>
<td>Red cells (deoxy)</td>
<td>-6.52</td>
</tr>
<tr>
<td>Air (NTP)</td>
<td>0.36</td>
</tr>
<tr>
<td>Mg</td>
<td>11.7</td>
</tr>
<tr>
<td>Al</td>
<td>20.7</td>
</tr>
<tr>
<td>W</td>
<td>77.2</td>
</tr>
<tr>
<td>Mb</td>
<td>123</td>
</tr>
<tr>
<td>Ti</td>
<td>182</td>
</tr>
<tr>
<td>Co</td>
<td>250</td>
</tr>
<tr>
<td>Pt</td>
<td>279</td>
</tr>
<tr>
<td>Cr</td>
<td>320</td>
</tr>
<tr>
<td>V</td>
<td>384</td>
</tr>
<tr>
<td>Ni</td>
<td>600</td>
</tr>
<tr>
<td>Stainless steel (nonmagnetic)</td>
<td>3520 - 6700</td>
</tr>
</tbody>
</table>
6.1.2 Susceptibility-induced distortion in MRI

The measured signal $S(t)$ in magnetic resonance imaging consists of a sum over the whole volume interacting with the radiofrequency field and the receiver coil:

$$S(t) = \iiint \rho(\mathbf{r}) e^{i\varphi(\mathbf{r}, t)} d^3 \mathbf{r},$$

(6-4)

where $\rho(\mathbf{r})$ is the spin density at point $\mathbf{r}$ and $\varphi(\mathbf{r}, t)$ is the phase angle at point $\mathbf{r}$ at time $t$. With a well-compensated 2D spin-echo sequence, the digitalised signal $S_{k,i}$ is

$$S_{k,i} = \frac{1}{N} \sum_{x_u} \sum_{y_v} M_0(x_u, y_v) e^{-i\varphi(x_u, y_v)} e^{i[\varphi(y_1, y_v, t_1 + \Delta B_{xy})]},$$

(6-5)

where $N$ is the number of pixels, $M_0(x_u, y_v)$ is the equilibrium magnetisation in the image object plane at point $(x_u, y_v)$, $G_i$ is amplitude and $T$ is the duration of the phase-encoding gradient, $\Delta B_{xy}$ is the static magnetic field distortion at point $(x_u, y_v)$ and $t_i$ is the time interval of the signal point $i$ from the spin-echo centre. In a well-balanced spin-echo sequence, the refocusing $\pi$-pulse re-phases all the spins at the middle of the spin echo independently of their resonance frequency.

Fig. 15. A typical spin-echo sequence, where all the spins are re-phased at the middle of the spin echo. RF is a radiofrequency field. $G_s$, $G_p$ and $G_r$ are slice selection, phase-encoding and frequency-encoding gradients, respectively, and $\Phi$ is the phase angle of the spins.
Figure 15 shows a typical spin-echo sequence, where one group of spins (dashed line) has a higher resonance frequency than the others due to their higher susceptibility. The phase angle $\Phi$ of these spins grows faster. After a $\pi$-pulse, the phase angle of this faster resonating group is more negative compared with the other group. At the echo-time, all the spins have again the same zero phase angle, but the faster resonating group has a higher frequency $\omega = d\Phi/dt$.

The field distortion term $\Delta B$ causes a shift in pixel positions in the frequency-encoding direction ($x$-axis):

$$x' = x + \frac{\Delta B(x, y)}{G_x} = x + \Delta x.$$  

(6-6)

This shift deforms the image, as shown in Fig. 16.

Fig. 16. Original cylindrical object and corresponding distorted image.

In a real 3D imaging procedure, objects with abnormal susceptibility compared to the surrounding tissues and water bend the image plane and distort the image in the frequency-encoding direction.

### 6.2 Assessment of the susceptibility of a prosthesis

#### 6.2.1 Introduction

Field distortion in the neighbourhood of a macroscopic object is affected by the angle between the tangent of the surface of the macroscopic object and the direction of the main magnetic field $B_0$, the curvature of the surface of the object and the susceptibility difference between the object and the surrounding medium. The extent of field distortion can be estimated if the susceptibility of the object is known. Field distortion can be calculated with high precision if the 3D shape and susceptibility of the object are known.
(Bhagwandien et al. 1994). In order to use correct susceptibility values in simulation, it is necessary to estimate the susceptibility of the prosthesis is used as a test object in this study.

### 6.2.2 Methods

In this work, two basically different methods have been used. The first method – the geometrical method - is based on geometrical image distortion. The second method – the phase difference method – utilises the phase distortion caused by a field susceptibility difference induced field modification.

#### 6.2.2.1. Estimating the susceptibility of a metal cylinder on the basis of geometrical image distortion

According to Schenck (1993), a cylinder with a radius of \( R \) oriented transversely to the main magnetic field \( B_0 \) causes a distortion in the spin-echo images, which can be analysed. The co-ordinates are chosen so that the field \( B_0 \) and the read gradient \( G_r \) are in the \( z \)-direction and phase encoding is in the \( x \)-direction. The distances are made dimensionless by dividing by \( R \). The following equations describe this situation:

\[
\Delta B_y = \begin{cases} 
\frac{\Delta \chi \cdot B_0 \cdot R^2}{2} \left( \frac{z_i^2 - x_j^2}{(z_i^2 + x_j^2)^2} \right), & \text{if } \left( \frac{z_i^2 + x_j^2}{R^2} \right) > 1 \\
\frac{\Delta \chi \cdot B_0}{2}, & \text{otherwise}
\end{cases}
\]

\[
z' = f(z, x) = z + c \cdot \frac{z^2 - x^2}{(z^2 + x^2)^2}; \quad c = \frac{\Delta \chi B_0}{2 G_r \cdot R^3},
\]

where \( \Delta B(z, x) \) is field distortion, \( \Delta \chi \) the susceptibility difference between the cylinder material and the surrounding path medium (water), \( f(z, x) \) is co-ordinate transformation in the distorted image and \( G_r \) is the strength of the read-out gradient.

Schenck (1993) has shown that the anterior intensity maximum is located near \( x = x' = 0, z = (2c)^{1/3} = 1.25992 \, c^{1/3} \) and \( z' = 3(c/4)^{1/3} = 1.8898 \, c^{1/3} \). The posterior maxima are near the cusps at \( x = x' = \pm 1.1337 \, c^{1/3}, z = -0.46959 \, c^{1/3} \), and \( z' = -0.939186 \, c^{1/3} \). The bright intensity cusps and the measured distances \( L_x \) and \( L_y \) are visible in Fig. 4. Now

\[
L_x = 2 \cdot 1.1337 \, c^{1/3} \equiv k_x \, c^{1/3},
\]

\[
L_z = \Delta z' = (1.8898 - (-0.939186)) \, c^{1/3} \equiv k_z \, c^{1/3},
\]

where

\[
k_x = 2.2674 \quad \text{and} \quad k_z = 2.828986.
\]

Finally, when relative distances \( \Delta q/R \) are used
where \( q = x, z \)

\[
\Delta \chi = \frac{2G_r \cdot R}{k_q^3 \cdot B_0} (\Delta q)^3, \quad (6-11)
\]

Fig. 17. Sagittal image of a cylindrical metal rod lying perpendicular to the main field. The field perturbation leads to three image highlights – one located anteriorly (leading edge) and two posteriorly (trailing edge). The distances between the highlights are marked as \( L_x \) and \( L_z \).

Measurements were performed using a Siemens Magnetom SP 42 scanner (Erlangen, Germany). A spin-echo sequence with TR of 600 ms and TE of 10 ms (FOV 300 mm) or 15 ms (FOV 200 mm) was used to produce images. The distances of the high-intensity cusps were measured using both ImagePro 4.0 and home-written KS_Image programs.

6.2.2.2 Assessment of susceptibility on the basis of the measured phase difference around metal cylinder

The method is also based on the field distortion of a cylindrical object located perpendicular to the main magnetic field \( B_0 \) when the local field is modified according to Eq. 6-11. Now, the phase angle distortion at the echo time TE is

\[
\Delta \phi_{ij} = \gamma \cdot \Delta B_{ij} \cdot TE. \tag{6-12}
\]

According to Eq. 6-12, the maximal field and phase distortions occur on the surface of the prosthesis:

\[
\Delta B_{\text{max}} = \frac{\Delta \chi \cdot B_0}{2}, \quad \text{at } z_i = \pm R \text{ and } y_j = 0, \tag{6-13}
\]

and the susceptibility difference \( \Delta \chi \):

\[
\Delta \chi = \frac{2 \cdot \Delta B_{\text{max}}}{B_0} = \frac{2 \Delta \phi_{\text{max}}}{\gamma B_0 \cdot TE}. \tag{6-14}
\]
Several technical methods involving phase unwrapping and fitting phase data to mathematical field models were tested to obtain the true distribution of $\Delta \Phi_y$. Aluminium and copper rods (length 100 mm, diameter 10 mm) were used to determine the susceptibility difference between a metal rod and surrounding water. These rods were in a water container perpendicular to the main magnetic field, and they were imaged with a gradient echo sequence (FOV=300mm, TR=690 ms, BW=130Hz/pxl, TE = 6, 7, 10 ms). The most reliable way to determine the field distortion due to the rod was to measure the distances between the first phase wraps in the horizontal direction (Fig. 18) and to calculate the maximal field distortion on the surface of the rod according to Eq. 6-14. The measurements were repeated many times using the different values of TE.

![Fig. 18. Phase images of Al and Cu rods (TE=6 ms). The arrows show the positions of the first phase wraps where $\Delta \Phi = \pm \pi$.](image)

A slightly modified method was used to assess the susceptibility of the prosthesis. The distances of the first to the fourth wraparounds were measured, the dimensionless distance $\rho$ was calculated from $\rho = \Delta \chi / 2R$, where $\Delta \chi$ is the distance between the wraparounds and $R$ is the radius of the prosthesis. Angular frequencies $\omega = \Delta \Phi / TE$ were calculated at the wraparounds. The problem was then linearized using $x = 1 / \rho^2$, and a line $\omega = \alpha(x)$ was fitted, as shown in Fig. 19. The susceptibility for the prosthesis was then calculated according to Eq. 6-14.

### 6.2.3 Results

#### 6.2.3.1 Results of the geometrical method

The prescribed method was tested with an aluminium rod in a water basin. The results are given in table 3. The estimated susceptibility difference between aluminium and
water is \((35.3 \pm 14.9) \cdot 10^{-6}\) measured in the \(x\)-direction and \((36.6 \pm 8.3) \cdot 10^{-6}\) in the \(z\)-direction. These are higher than the corresponding reference value of \(29.75 \cdot 10^{-6}\) reported in the literature (e.g. Schenck 1996). The reference value is, however, within the mean ± standard deviation of the measurements.

Table 3. Estimated susceptibility of an Al rod.

<table>
<thead>
<tr>
<th>Case #</th>
<th>(L_x) [g215]</th>
<th>(L_z) [g215]</th>
<th>(\Delta \chi_x [10^{-6}])</th>
<th>(\Delta \chi_z [10^{-6}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.37</td>
<td>14.06</td>
<td>22.84</td>
<td>39.74</td>
</tr>
<tr>
<td>2</td>
<td>9.38</td>
<td>15.63</td>
<td>22.88</td>
<td>54.59</td>
</tr>
<tr>
<td>3</td>
<td>8.59</td>
<td>10.16</td>
<td>25.21</td>
<td>29.96</td>
</tr>
<tr>
<td>4</td>
<td>10.16</td>
<td>10.16</td>
<td>58.19</td>
<td>29.96</td>
</tr>
<tr>
<td>5</td>
<td>7.03</td>
<td>10.94</td>
<td>19.30</td>
<td>37.44</td>
</tr>
<tr>
<td>6</td>
<td>9.38</td>
<td>10.55</td>
<td>45.77</td>
<td>33.55</td>
</tr>
<tr>
<td>7</td>
<td>7.81</td>
<td>10.94</td>
<td>26.46</td>
<td>37.44</td>
</tr>
<tr>
<td>8</td>
<td>9.77</td>
<td>10.16</td>
<td>51.80</td>
<td>29.96</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td>35.31\cdot 10^{-6}</td>
<td>36.58\cdot 10^{-6}</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td>14.88\cdot 10^{-6}</td>
<td>8.25\cdot 10^{-6}</td>
</tr>
</tbody>
</table>

The detailed results of measurements with a Profile™ prosthesis are given in table 4.

Table 4. Estimated susceptibility of Profile™ prosthesis.

<table>
<thead>
<tr>
<th>Case #</th>
<th>(L_x) [mm]</th>
<th>(L_z) [mm]</th>
<th>(R) [mm]</th>
<th>GR [10^{-6} T/m]</th>
<th>(\Delta \chi_x [10^{-6}])</th>
<th>(\Delta \chi_z [10^{-6}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.13</td>
<td>37.50</td>
<td>8.15</td>
<td>2.606</td>
<td>175.3</td>
<td>189.2</td>
</tr>
<tr>
<td>2</td>
<td>26.95</td>
<td>36.33</td>
<td>7.90</td>
<td>2.606</td>
<td>145.2</td>
<td>183.1</td>
</tr>
<tr>
<td>3</td>
<td>20.31</td>
<td>26.17</td>
<td>8.15</td>
<td>7.817</td>
<td>175.1</td>
<td>179.7</td>
</tr>
<tr>
<td>4</td>
<td>19.53</td>
<td>25.78</td>
<td>7.90</td>
<td>7.817</td>
<td>165.7</td>
<td>196.3</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>165.3 \cdot 10^{-6}</td>
<td>187.1 \cdot 10^{-6}</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.1 \cdot 10^{-6}</td>
<td>7.3 \cdot 10^{-6}</td>
</tr>
</tbody>
</table>

The geometrical method gives \(\Delta \chi = 165 \pm 14\) in the \(x\)-direction and \(\Delta \chi = 187 \pm 7\) in the \(z\)-direction (direction of the read-out gradient).

6.2.3.2 Results of the phase difference method

The results of the Al and Cu rod measurements are shown in tables 5a and 5b. The mean and standard deviation for the susceptibility difference between water and aluminium is \(\Delta \chi_{Al} = (29.9 \pm 2.8) \cdot 10^{-6}\), which is in good agreement with \(29.75 \cdot 10^{-6}\), which has been reported in the literature (Schenck 1996).
Table 5a. Results of the susceptibility determinations for Al rod.

<table>
<thead>
<tr>
<th>Case #</th>
<th>TE (ms)</th>
<th>Δx (mm)</th>
<th>ΔΦ at R (rad)</th>
<th>ΔB [10^{-6} T]</th>
<th>Δχ [10^{-6}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>25.8</td>
<td>20.90</td>
<td>7.81</td>
<td>16.2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>23.5</td>
<td>17.28</td>
<td>10.8</td>
<td>22.3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>23.5</td>
<td>17.28</td>
<td>10.8</td>
<td>22.3</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>24.6</td>
<td>19.04</td>
<td>10.2</td>
<td>21.1</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>25.8</td>
<td>20.90</td>
<td>7.81</td>
<td>16.2</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>22.3</td>
<td>15.58</td>
<td>9.71</td>
<td>20.1</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td>9.50 \cdot 10^{-6}</td>
<td>29.9 \cdot 10^{-6}</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td></td>
<td>1.37 \cdot 10^{-6}</td>
<td>2.8 \cdot 10^{-6}</td>
</tr>
</tbody>
</table>

Table 5b. Results of the susceptibility determinations for Cu rod.

<table>
<thead>
<tr>
<th>Case #</th>
<th>TE (ms)</th>
<th>Δx (mm)</th>
<th>ΔΦ at R (rad)</th>
<th>ΔB [10^{-6} T]</th>
<th>Δχ [10^{-6}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>21.1</td>
<td>13.99</td>
<td>8.71</td>
<td>18.0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>22.3</td>
<td>15.58</td>
<td>8.32</td>
<td>17.2</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>25.8</td>
<td>20.90</td>
<td>7.81</td>
<td>16.2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>17.6</td>
<td>9.71</td>
<td>6.05</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>17.6</td>
<td>9.71</td>
<td>5.18</td>
<td>10.7</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>17.6</td>
<td>9.71</td>
<td>3.63</td>
<td>7.52</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>14.1</td>
<td>6.22</td>
<td>3.87</td>
<td>8.02</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>17.6</td>
<td>9.71</td>
<td>6.05</td>
<td>12.5</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td>6.20 \cdot 10^{-6}</td>
<td>12.8 \cdot 10^{-6}</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td></td>
<td>1.95 \cdot 10^{-6}</td>
<td>4.0 \cdot 10^{-6}</td>
</tr>
</tbody>
</table>

On the contrary, the susceptibility value derived for Cu (Δχ_{Cu} = (12.8 \pm 4.0) \cdot 10^{-6} has less good agreement with the corresponding value of \(-0.58 \cdot 10^{-6}\) reported in the literature (Schenck 1996).

6.2.3.3 Prosthesis

The detailed results of measurements are presented in table 6. The fit of angular frequency $\omega$ versus inverse square of distance $1/r^2$ is shown in Fig. 6, where Fig. 6a represents the fit in the region of the measurement points, while in Fig. 6b the fit is extrapolated to the surface of the prosthesis, where $1/r^2 = l$. A logarithmic scale is used in the extrapolated fit for clarity.

Extrapolation to the surface of the prosthesis gives an angular frequency on the surface: $\omega = \Delta \phi_{\text{max}}/TE = 0.230 \cdot 10^3$ rad/s (0.189 – 0.232 \cdot 10^3 rad/s with 95% confidence limits), which gives for the susceptibility difference between the prosthesis and water $\Delta \chi = 178.2 \cdot 10^{-6}$ (146 – 211 \cdot 10^{-6} with 95% confidence).
Table 6. Detailed results of susceptibility measurements for a Profile™ prosthesis.

<table>
<thead>
<tr>
<th>$\Delta \Phi$ [rad]</th>
<th>Image 1</th>
<th>Image 2</th>
<th>Image 3</th>
<th>Image 4</th>
<th>Image 5</th>
<th>Image 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/2$</td>
<td>109</td>
<td>99</td>
<td>119</td>
<td>106</td>
<td>152</td>
<td>87</td>
</tr>
<tr>
<td>$3\pi/2$</td>
<td>79</td>
<td>74</td>
<td>87</td>
<td>78</td>
<td>98</td>
<td>64</td>
</tr>
<tr>
<td>$5\pi/2$</td>
<td>65</td>
<td>61</td>
<td>70</td>
<td>66</td>
<td>73</td>
<td>53</td>
</tr>
<tr>
<td>$7\pi/2$</td>
<td>56</td>
<td>53</td>
<td>67</td>
<td>57</td>
<td>61</td>
<td>48</td>
</tr>
<tr>
<td>R [mm]</td>
<td>7.90</td>
<td>8.15</td>
<td>7.90</td>
<td>8.15</td>
<td>7.80</td>
<td>8.12</td>
</tr>
<tr>
<td>TE [ms]</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. 19. Measured dependence of angular frequency on the distance from the prosthetic surface. A line has been fitted to the measured points (a) and extrapolated to the prosthetic surface (b). The confidence limits of 95% of the line are also shown on the logarithmic scale.
The final estimate of the magnetic susceptibility of the prosthesis is

\[ \chi_{\text{prosthesis}} = (170 \pm 13) \cdot 10^{-6}, \]  

(6–15)

which is given as a weighted mean and standard error of the mean of the three estimates given above. The inverses of SD were used as weighting factors.

### 6.2.4 Discussion of the estimation of susceptibility

#### 6.2.4.1 Geometrical method

The presented methods for estimating the susceptibility of cylindrical metal rods seem to be reliable for aluminium but not for copper rods. The susceptibility difference between copper and water is very low and cannot have caused the image distortions visible in Fig. 6b. It was impossible to determine the susceptibility of a Cu rod with the geometrical method, because the geometrical distortion of the image was negligible. Further, copper has higher electrical conductivity (59.59 \cdot 10^6 \ \Omega^{-1} \text{m}^{-1}) than aluminium (37.66 \cdot 10^6 \ \Omega^{-1} \text{m}^{-1}) and titanium (2.381 \cdot 10^6 \ \Omega^{-1} \text{m}^{-1}). Thus, eddy currents rather than a susceptibility difference are probably the main reason for the image distortions of the copper rod. The geometrical method is more reliable if the susceptibility difference is higher than that between Cu and water, as in the case of Al and Ti.

The statistical deviation SD=42 \% is nearly twofold in the phase-encoding \( x \)-direction compared to the frequency-encoding \( z \)-direction, where SD=23 \%. Hence, the \( z \)-direction gives much more reliable estimates of susceptibility. The difference between the susceptibilities of aluminium and water is relatively small, and the image deformation with high intensity points is not sharply visible in the images. This causes large deviations in the determined susceptibility of aluminium. The deviation figures are much lower with the prosthesis: 8.5 \% in the phase-encoding direction and 3.9 \% in the frequency-encoding direction. The most reliable results can be obtained using a small FOV and a large matrix. In images with FOV=300 and matrix size of 256x256, an error of one pixel in the measured distances leads to an error of 13 \% in \( \Delta \chi_x \) and 10 \% in \( \Delta \chi_z \), while in images with FOV=200; Mtx = 512x512, the error of \( \Delta \chi_x \) is 6 \% and that of \( \Delta \chi_z \) 4.5 \%.

This tendency to marked deviation is in accordance with Schenck (1993), who concluded that the method is suitable for quick estimations of susceptibility values.

#### 6.2.4.2 Phase difference method

The estimated susceptibility of aluminium 20.8 \cdot 10^{-6} is in good agreement with the value reported in the literature, 20.7 \cdot 10^{-6} (e.g. Schenck 1996), while the estimation of the susceptibility of copper failed (3.8 \cdot 10^{-6} compared with the literature value of –9.63 \cdot 10^{-6}).
The greater susceptibility difference between the prosthesis and water enabled the use of many wraparound positions in the assessment of the susceptibility of the prosthetic material. The relative accuracy of the method is lower than that of the geometrical method. In the case of a prosthesis, the inaccuracy of the phase difference method seems to be slightly greater compared to the geometrical method. The confidence limits of 95 % are 17 % for \( \Delta \chi \), and 7.8 % for \( \Delta \chi \), estimated with the geometrical method and 18.3 % for \( \Delta \chi \) estimated with the phase difference method. The final estimate for the prosthetic material \( \chi_{\text{prosthesis}} = 170 \cdot 10^{-6} \) is slightly lower than the susceptibility of titanium \( \chi = 182 \cdot 10^{-6} \), but the latter is within the standard error limits of estimated susceptibility.

### 6.3 Methods of correcting image distortion

#### 6.3.1 Determination of magnetic field distortion

Because pixel shifts depend on the field distortion \( \Delta B \) at the pixel position \((x, y)\) (Eq. (6-6)), the first step in many image distortion correction procedures is to determine the magnetic field distortion. Only the methods based on distortion-correcting pulse sequences or exclusively on the features of the image do not require information about field distortion. There are, in principle, two different methods that can be used to determine the shape and size of the magnetic field distortion due to the susceptibility difference. The distortion can be calculated on the basis of the shape and susceptibility of the distorting object (Bhagwandien 1994), or it can be measured using field-mapping methods (e.g. Glover & Schneider 1991). The first method requires exact data about the susceptibility and the shape of the object. Heavy computing is also needed to calculate the field deformation. On the contrary, no \textit{a priori} knowledge about the susceptibility and shape of the object is necessary if field mapping is used. On the basis of a phase field map, it is possible to calculate the shape and size of field distortion.

By recording a shifted phase image \( \Delta \Phi(u, v) \), a map of magnetic field distortions can be produced. A major problem with phase images is that the phase wraps, resulting in wraparounds. The phase image is calculated from real and imaginary images \( \text{Re}(x, y) \) and \( \text{Im}(x, y) \):

\[
\arg (x, y) = \arctan \left( \frac{\text{Im}(x, y)}{\text{Re}(x, y)} \right),
\]

which has singularities at \(-\pi/2\) and \(\pi/2\). If the phase angle exceeds a singularity, e.g. \(\pi/2\), the phase changes from \(-\infty\) to \(+\infty\) and the intensity level in the phase image turns suddenly from black to white, as shown in Fig. 20.
Fig. 20. Wraparound zones are visible in a simulated phase image of a rod perpendicular to the main magnetic field $B_0$. The susceptibility difference between the rod and the surrounding material is $\Delta \chi = 200 \times 10^{-6}$, and a temporal shift of 5 ms for the $\pi$-pulse in the SE sequence was used in the simulation.

Two basically different field-mapping methods can be used to determine field distortion $\Delta B$. The field map can be generated by using either the shifted spin-echo (SE) or the gradient-echo (GRE) method. In a shifted spin-echo sequence, the inverting $\pi$-pulse is temporally shifted by time $dT$ for the second acquisition (see Fig. 21a). The local field distortion $\Delta B$ causes a phase shift $\Delta \Phi$ relative to $B$:

$$\Delta \Phi = \Delta B \cdot \gamma \cdot dT \Leftrightarrow \Delta B = \frac{\Delta \Phi}{\gamma \cdot dT}, \quad (6-17)$$

where $\gamma$ is the gyromagnetic ratio.

A field map can also be generated from a single GRE image or a pair of such images. The phase shift in a single GRE image depends upon the field distortion $\Delta B$ and the echo time $TE$ according to the equation

$$\Delta B = \frac{\Delta \Phi}{\gamma \cdot TE}, \quad (6-18)$$

which is presented in Fig. 21b. A limitation of this method is that, during the echo time $TE$, the phase difference $\Delta \Phi$ increases if the field distortion $\Delta B$ is high and many phase wraps appear.
Fig. 21. Spin-echo and gradient-echo sequences for recording a field map. In a shifted spin-echo sequence (a), during the second acquisition the refocusing $\pi$-pulse is temporally $dT$ shifted (the lowest row in (a)), when the phase shift is relative to the field distortion. In a gradient-echo sequence (b), the phase difference is proportional to the field distortion $\Delta B$ and the echo-time $TE$.

The pixel intensities of a pair of GRE images can be presented as $I_1 = \rho_1 \cdot \exp(i\phi_1)$ and $I_2 = \rho_2 \cdot \exp(i\phi_2)$, where $\rho_1$ and $\rho_2$ are magnitudes and, correspondingly, $\phi_1$ and $\phi_2$ are phases of the images $I_1$ and $I_2$, so that $\arg(I_k) = \phi_k$. Hence

$$\arg\left(\frac{I_1}{I_2}\right) = \arg\left(\frac{\rho_1}{\rho_2} \cdot e^{i(\phi_1 - \phi_2)}\right) = \phi_1 - \phi_2 = \Delta \Phi.$$  \hspace{1cm} (6-19)

The phase difference $\Delta \Phi$ can be calculated as a phase image from the ratio of two GRE images with different $TE$, if no phase wraps exist. This method allows the use of phase differences $\Delta \Phi$ generated during a shorter time interval $\Delta TE$, but now the phase wraps in two images must be corrected at first. An inverted pulse shift in SE sequences is not a routine tool of sequence modification, and pulse sequence programming is needed to use them. However, it is possible to use time intervals $dT$ shorter than the shortest echo times in GRE sequences, which causes less problems with phase wraps. In this work, only single GRE phase images are used for field maps.
6.3.2 Methods for phase unwrapping

Several methods were tested as ways to analyse phase unwrapping. The basic task is to compare the difference between the intensity of one pixel and that of its neighbours. If the difference between two pixels exceeds an appointed limit $\Delta \phi_{\text{limit}}$, phase wrapping exists between these pixels.

In many phase images coded with 12 bits, the intensity variation is from 0 to 4095. When the phase difference $\Delta \phi$ grows from $-\pi$ to $\pi$, the pixel value changes from 0 to 4095, respectively. The pixel value drops to zero when the phase difference exceeds $\pi$. If the change in the phase difference is very steep, the phase wraps are close to each other and difficult to distinguish from statistical noise with computational methods.

![A phase image of a prosthesis parallel to $B_0$ in a water tank. A GRE sequence with TR of 700 ms and TE of 6 ms was used. A horizontal profile line shows the variation of pixel intensity. The arrows indicate the positions where the determination of phase wraps is difficult because of a small maximum-to-minimum difference.](image)

A phase image of a prosthesis in a water tank lying parallel to the main magnetic field $B_0$ is shown in figure 22. A gradient echo sequence with TR of 700 ms and TE of 6 ms was used to obtain this image. The arrows show the positions where the gradient of $\Delta \phi$ is very steep, the phase wraps are close to each other and the intensity difference between
the maximum and minimum is much lower than 4095. These examples demonstrate the practical difficulties of finding phase wrap points.

Fig. 23. Block diagram of the phase unwrap procedure.
A block diagram of the developed wraparound method is shown in Fig. 23. First, the black low-signal areas outside the imaged object and at the position of the prosthesis are removed using a logical AND operation between the phase image and the corresponding magnitude image, which has been interactively thresholded. If the prosthesis is along the main magnetic field, as in Fig. 22, the field distortion near the prosthesis stem has a circular symmetry. Then, a co-ordinate transformation from Cartesian to polar co-ordinates can be interactively chosen and used, when most phase wraps occur in the radial direction. According to the cylindrical model, the phase difference $\Delta \Phi$ should vanish proportional to $1/r^2$. This relationship was applied to extrapolate the phase difference to the surface of the prosthesis with poor results. Cubic splines are used to minimise statistical noise. Laplace filtration is used to find the wraparound positions, too. The unwrapping limit, i.e. the phase difference between two adjacent pixels, $\Delta \Phi_{\text{limit}}$, is an adjustable parameter. The unwrapped phase in polar co-ordinates has been transformed back to Cartesian co-ordinates, to obtain the final unwrapped phase image.

### 6.3.3 Re-mapping in the spatial domain

The following consideration assumes that 2D spin-echo imaging is used and that the gradients are linear. Then, the images are distorted in such a way that the image co-ordinates $(x', y')$ are altered from the true image co-ordinates $(x, y)$ in accordance with the transformation $\Phi(x, y)$ (Fig. 24)

$$
\begin{align*}
x' &= x + \frac{\Delta B(x, y)}{G_x} , \\
y' &= y .
\end{align*}
$$

(6-20a) (6-20b)

Then, the inverse transformation $\Phi^{-1}(x, y)$ is:

$$
\begin{align*}
x &= x' - \frac{\Delta B(x, y)}{G_x} , \\
y &= y' .
\end{align*}
$$

(6-21a) (6-21b)

The observed signal in SE imaging

$$
S_{k,j} = \frac{1}{N} \sum_{u} \rho(x_u, y_v) \cdot e^{-i\gamma [G_{y_1} y_1 + (G_{x_1} x_1 + \Delta B_{x_1}) (T-\tau)]}
$$

(6-22)

can now be transformed into

$$
S_{k,j} = \frac{1}{N} \sum_{v} \rho'(x'_u, y'_v) \cdot e^{-i\gamma [G_{y_1} y_1 + (G_{x_1} x_1 + \Delta B_{x_1}) (T-\tau)]}
$$

(6-23)

where

$$
\rho'(x', y') = \frac{\rho(\Phi^{-1}(x', y'))}{j \left| \frac{x'}{x}, \frac{y'}{y} \right|},
$$

(6-24)

and
\[
J(x', y'; x, y) = \begin{vmatrix}
\frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\
\frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y}
\end{vmatrix} = 1 + \frac{\delta(AB(x, y))}{G_x \cdot \partial x} \cdot \frac{\delta(AB(x, y))}{G_y \cdot \partial y} = 1 + \frac{\delta(AB(x, y))}{G_x \cdot \partial x}
\]

(6-25)

is the Jacobian of transformation (6-20).

Fig. 24. Transformations between an object and an image plane. The main magnetic field is perpendicular to the image plane, and the read-out gradient \(G_x\) lies along the \(x\)-axis.

In the presence of susceptibility differences, distorted images \(\rho'(x', y')\) are recorded instead of true images \(\rho(x, y)\), and true images can be obtained with inverse transformation of (6-24):

\[
\rho(x, y) = \rho(\Phi^{-1}(x', y')) = \rho'(x', y') \cdot J\left(\frac{x', y'}{x, y}\right).
\]

(6-26)

Weis and Budinsky (1990) were the first to discuss the presented method.

### 6.3.4 Correction in the frequency domain

The method refers to the Fourier-shift theorem in the time domain, using a distorted image \(S_{k,i}\) (6-5) and the field map to produce the time signal data that would have been created with the opposite frequency shifts (Weisskoff 1992). Let \(\rho'(x', y')\) be the intensity of pixel\((u, v)\) in the distorted image and \(\Delta x_u\) the shift of pixel\((u, v)\), then the corrected time signal is

\[
S_{k,q} = \frac{1}{N} \sum_m \rho'(x'_m, y'_k) \cdot e^{i2\pi\Delta x_u/N} \cdot e^{-i2\pi q/N}.
\]

(6-27)
Fourier transformation of these synthetic time data produces an image with reduced distortion.

According to Weisskoff and Davis (1992), the Fourier correction produces more homogenous results for a larger scale of susceptibility differences compared to spatial correction. Therefore, the frequency domain correction was chosen to be used here. A block diagram of a correction method in the frequency domain is presented in Fig. 25. At first, the original spin-echo image, which is to be corrected, and the corresponding gradient-echo phase image with the shortest possible echo time are imaged. Then, the phase image is unwrapped to obtain the local field distortion \( \Delta B \). The corrected raw image of the SE image \( S_{c,i,j} \) is calculated using a Fourier transform (FFT) with Fourier shift (Eq 6-28).

\[
S_{c,i,j} = \frac{1}{N^2} \sum_u \sum_v (-1)^{uv} \cdot I_{u,v} \cdot \hat{e}^{\left(\frac{2\pi i v \Delta x_{u,v}}{N} + \frac{2\pi i u \Delta y_{u,v}}{N}\right)},
\]  

(6-28)

where the co-ordinate shift

\[
\Delta z_{u,v} = \frac{\Delta B_{u,v}}{G_r},
\]

(6-29)

and \( G_r \) is the strength of the read-out gradient. The final corrected spin-echo image \( I_{c,u,v} \) is calculated using an inverse Fourier transform of \( S_{c,i,j} \) according to

\[
I_{c,u,v} = \sum_i \sum_j S_{c,i,j} \cdot \hat{e}^{\left(\frac{2\pi i v \Delta x_{u,v}}{N} + \frac{2\pi i u \Delta y_{u,v}}{M}\right)}.
\]

(6-30)

In order to perform the described correction procedure, computer programs must be written. The block diagrams and more detailed descriptions of the used computer programs are given in Appendix A.
Fig. 25. A block diagram of the image correction method in the frequency domain.
6.4 Results of susceptibility correction

6.4.1 Simulation

Figure 26 shows two series of simulated images with different susceptibility differences between a cylindrical object and the surrounding medium.

![Simulated images of a cylinder with different susceptibilities. The cylinder is perpendicular to the magnetic field. The distorted images are shown on the left side and the corresponding corrected ones on the right side.](image)

Fig. 26. Simulated images of a cylinder with different susceptibilities. The cylinder is perpendicular to the magnetic field. The distorted images are shown on the left side and the corresponding corrected ones on the right side.
The following parameters were used in simulations: radius of the cylinder \( R = 8.53 \) mm, strength of the reading gradient \( G_r = 10 \) mT/m and magnetic flux density of the main field \( B_0 = 0.975833 \) T.

The corrected images in Fig. 26 measure the ability of the Fourier shift method to correct distorted images. The exact shape and the magnitude of the magnetic field distortion are known in simulations. With a small susceptibility difference \( \Delta \chi \), the correction succeeds well. The correction is not perfect with higher values of \( \Delta \chi \), but in the corrected image the distortion is now symmetric in the frequency-encoding direction and the intensity distortions at the edge of rod are much smaller than in the uncorrected images.

### 6.4.2 Prosthesis in a water tank

A Profile\textsuperscript{TM} prosthesis in a water tank parallel to the main magnetic field \( B_0 \) was imaged using the SE and GRE sequences. The coronal magnitude and phase images in Fig. 27 show the locations of the axial slides to be corrected.

![Fig. 27. A Profile\textsuperscript{TM} prosthesis in a water tank is parallel to the main magnetic field. The axial slide locations have been marked by 1-5 in the magnitude image (a) and the phase image (b).](image)

The magnitude image (Fig. 27a) was obtained using a SE sequence with TR = 700 ms and TE = 10 ms. Slice thickness was 8 mm. A gradient-echo sequence with the same parameters, except TR = 6 ms, was used for the phase image (Fig. 27b).

The corresponding axial images are presented in Fig. 28. The spin-echo images with TR = 700 ms, TR = 10 ms, two acquisitions and FOV = 350 mm are presented in the left column. In the right column, corresponding phase images obtained with the same parameters as the magnitude images, with the exception that TR = 6 ms, are shown. The corrected images are shown in the middle column. The correction procedure has been described above. Median filtration with a 3x3 mask was added to remove pixel noise from final corrected images.
Fig. 28. A Profile™ prosthesis in a water tank parallel to the main magnetic field. The left column shows the original images without any correction. The middle column shows the corrected images and the right column the corresponding phase images, where phase distortion is relative to the distortion of the magnetic field.

The three topmost images (level 1-3) are from the distal end of the prosthesis stem, where the magnetic field lines penetrate the surface of the prosthesis and the field is distorted. The outcome of the correction procedure is not perfect. The high-intensity area is visible on the opposite site of the prosthesis, but the area is smaller and its intensity is not so bright as in the uncorrected image. At the level of the third row distortion is small, and between the slides 3 and 4 it almost vanishes. The direction of the distortion field is opposite in the slides 1 and 2. This distortion at the distal end of the prosthesis stem impairs the visibility of the prosthesis stem top. At the level of the slides 4 and 5, the shape of the prosthesis deforms, affecting distortion in the field.
6.4.3 Correction of patient images

As an example, the correction procedure was tested with images of a prosthesis installed during a total hip arthroplasty operation. The model of a titanium hip prosthesis (Bi-Metric, Harris-Galante) with a cylindrical stem was imaged. Fig. 29 shows coronal SE and GRE magnitude and phase images obtained at the prosthesis level. The sequence parameters were FOV of 300 mm, one acquisition, matrix of 256 x 256, slice thickness of 8 mm, TR of 500 ms and TR of 10 ms in SE or TE of 6 ms in the GRE sequence. A comparison of the SE and GRE images makes it evident that the distorted area is larger in the GRE image (Fig. 33b) than in the corresponding SE image (Fig. 29a). The locations of the axial images presented in Fig. 30 are also shown in Fig. 29.

![Fig. 29. Coronal spin-echo (a), and gradient-echo magnitude (b) and phase (c) images of an artificial prosthesis in a human hip. The locations of axial slices are visible and marked with 1-4.](image)

Fig. 30 shows the original SE and GRE images, the corresponding corrected images, the difference images between the original and corrected images, the GRE phase images and the unwrapped phase images relative to the field distortion. The first level is at the distal top of the prosthesis stem. The next two levels are in the region of the prosthesis stem, where the magnetic field distortion is relative small. The fourth level involves larger distortion, especially in the GRE images. The magnetic field distortion is small at the first three locations, and correction has a very small effect on the images. The possible lack of sharpness in the corrected images is due to median filtration. At the fourth location, the phase image shows the shape of the field distortion to be irregular. The image plane has been curved so much as to prevent a successful unwrapping procedure. The dark region in the GRE image has not been eliminated.
Fig. 30. Axial images obtained at four different levels in the region of a hip prosthesis stem. The top row at each location shows the original SE image, the corresponding corrected one, the difference image between the original and corrected images and the unwrapped phase image showing the field inhomogeneity. The lower row shows the corresponding GRE images at the same locations.
6.5 Modulation transfer function (MTF) in original and corrected images

6.5.1 Calculation of MTF

Fig. 31. Determination of MTF in a prosthesis image. (a) First, a profile parallel to the frequency-encoding gradient was selected. (b) A line spread function (LSF) was calculated as a derivative of the profile. A part of derivative curve on the surface of the prosthesis between the zero crossing points can be selected for the calculation of MTF. (c) The modulation transfer function (MTF) is a Fourier transform of LSF.
The determination of the modulation transfer function (MTF) is presented in detail in Fig. 31. First, a line profile along the reading gradient was selected. Then, the line spread function (LSF) was calculated at the edge of the prosthesis as the first derivative of the profile curve. The derivative was calculated using the equation

\[
\frac{dpixels(x_i)}{dx} = -pixels(x_{i-2}) - 2 pixels(x_{i-1}) + 2 pixels(x_{i+1}) + pixels(x_{i+2}),
\]

which also includes a minor smoothing operation. A part of the derivative curve corresponding to the image intensity change on the surface of the prosthesis or at the bone-tissue interface was then selected interactively. The selection was standardised to be between the zero crossing points closest to the top of the derivative curve. A zero filling operation was then used at both ends of the derivative curve, so that the number of points was the same in the original profile and in the final line spread function. The modulation transfer function was calculated from LSF by a Fourier transform.

6.5.2 Prosthesis in a water tank

A comparison of the original and corrected images is shown in Fig. 32. The topmost row shows the images at location 1, which is at the distal top of the prosthesis stem. The corresponding phase image in Fig. 28 shows relatively high field distortion, which is also visible as high-intensity distortion in the magnitude image. On the basis of the calculated MTF curves, the intensity variation at the prosthesis edge in the corrected image is sharper than in the original, which is also clear in a visually comparison of the image pair.

At location 2, the direction of the distorted field is opposite to that at location 1. The strength of distortion is relative high. In the original image, the left edge of the prosthesis is especially deformed, with a high-intensity region outside the prosthesis changing gradually into dark in the region of the prosthesis. After the correction, the high-intensity region is smaller, the intensity lower and the intensity change from bright to dark steeper. The intensity alterations due to the correction at the right edge of the prosthesis are much smaller. Also, the corresponding MTF curves show that the visible line density in the corrected image is much higher at the left edge, while no clear change is visible at the right edge.

Location 3 is in the part of the prosthesis stem where field distortion is minimal. The left edge is sharper after the correction, but the intensity at the right edge in the corrected image varies more gradually compared to the original image. However, the differences between the images are small.

At location 4, the shape of the prosthesis is changing slowly and the field distortion is growing compared to location 3. The right edge looks sharper after the correction, but the left edge is less sharp. The differences are small, which can be seen by comparing visually the original and corrected images.

At location 5, field distortion is rapidly growing towards the proximal end of the prosthesis. A visual comparison of the original and corrected image pairs gives an impression that the original image is better. MTF analysis shows, however, that the
corrected image repeats the higher line densities at the right edge of the prosthesis better than the original image. At the left edge, sharpness is poorer in the corrected image.

Fig. 32. Original and corrected axial images of a prosthesis in a water tank. The stem of the prosthesis is along the field B₀. The image locations are given in Fig. 27. Modulation transfer functions have been calculated on the basis of the horizontal profile intensity variations on the surface of the prosthesis. The profiles are marked with white lines in the images. The following annotations have been used:

- Left: original
- Right: original
- Left: corrected
- Right: corrected
6.5.3 Prosthesis installed in a total hip arthroplasty (THA) operation

The method used to analyse a prosthesis in a water tank was applied to the comparison of the original and the corrected images of a prosthesis implanted in a human hip during total hip arthroplasty. The interface between the prosthesis stem and bone is invisible in most images. Therefore, the modulation transfer functions have been determined using intensity variations at the interface between bone and surrounding soft tissue.

![Fig. 33. Original and corrected SE image pairs showing the prosthesis stem after a THA operation at different locations presented in Fig. 29. Modulation transfer functions have been calculated on the basis of intensity variation at the interface between bone and soft tissue. The profile lines are marked with white lines. Annotations as used in MTF curves.](image_url)
### Table

<table>
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<tr>
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<th>Corrected</th>
<th>MTF</th>
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<td><img src="image8.png" alt="Image 8" /></td>
<td><img src="mtf4.png" alt="MTF 4" /></td>
</tr>
</tbody>
</table>

**Fig. 34.** Original and corrected GRE image pairs showing a prosthesis stem implanted in a THA operation at different locations presented in Fig. 29. Modulation transfer functions have been calculated on the basis of intensity variation at the interface between bone and muscle tissue. White lines indicate the profiles used in MTF calculations. Annotations as used in MTF curves.
The results are shown in Fig. 33 for spin-echo images and in Fig. 34 for gradient-echo images. In the spin-echo images, the MTF of corrected images is on the left side of the prosthesis lower than or equal to the original image. On the right side, except at location 1, the MTF of the corrected image is slightly higher than that of the original image. Distortion in the spin-echo images is generally small at all of the locations studied, which is why the original images can be used for diagnostic purposes without any corrections.

In gradient-echo images, the calculated modulation transfer function of the corrected image is nearly the same on the left side in all the corrected images except at location 4, where the MTF of the original image is higher. On the right side, the MTF of the corrected image is slightly higher at the locations 1-3. The distortions are also small in the gradient-echo images, except at location 4, where the tissue on the right side of the prosthesis displays decreased intensity. It was difficult at this location to find any intensity edge for measuring LSF. The very small increase of sharpness in the corrected images can be explained by the high quality and sharpness of the original images together with a smoothing property of the correction procedure.

6.6 Discussion of the correction of susceptibility-induced image distortion

6.6.1 Boundary conditions for the feasibility of single-valued correction

The transformation pair $\Phi(x, y)$ (6-20) and $\Phi^{-1}(x, y)$ (6-21) connected to susceptibility-related image distortions represents transformations which map a true image into a distorted image or a distorted image back into a true image.

A cylinder perpendicular to the main magnetic field $B_0$ can be considered as an example. Co-ordinates are chosen so that the field $B_0$ and the read-out gradient $G_R$ are in the $z$-direction and the phase-encoding gradient is in the $x$-direction. Distances are made dimensionless by dividing by the radius of the prosthesis $R$. A notation

$$c = \frac{\Delta y B_0}{2G_R R}$$

has been used, too.

Now, the co-ordinate shift due to the susceptibility difference is

$$z' = \begin{cases} 
    z + c \cdot \left( \frac{z^2 - x^2}{z^2 + x^2} \right) & \text{(outside the cylinder)} \\
    z + c & \text{(inside the cylinder)}
\end{cases}$$

and the Jacobian of transformation
\[
J \left( \frac{x', z'}{x, z} \right) = \begin{cases} 
1 + 2c \cdot \frac{z}{(x^2 + z^2)^2} & \text{outside the cylinder} \\
1 - 4c \cdot \frac{(z^2 - x^2)}{(x^2 + z^2)^3} & \text{inside the cylinder}
\end{cases} 
\] (6-34)

The values used in the calculation correspond to those of the Magnetom scanner and the Profile™ prosthesis \( B_0 = 0.9658 \) T, \( G_r = 7.82 \cdot 10^{-3} \) T/m, \( R = 0.075 \) m. A graphical representation of the transformation (6-33) is shown in Fig. 35. It is a surface curved by susceptibility distortion. In the ideal case without any susceptibility distortions, the transformation is a plane with a unit gradient along the read-out direction. It has a single-value inverse transformation if the gradient of the transformation along the read-out direction is systematically positive.

![Mapping from true to distorted image](image)

**Fig. 35.** Transformation from a true image to a distorted image, which takes place during imaging. The positions of central and tangential lines have been marked in the figure.
Fig. 36. Lines of transformation marked in Fig. 35. The regions where transformation is not single-valued have been marked. For example, the points (a), (b) and (c) are mapped to a single image point (d).

Fig. 37. Regions with a negative Jacobian of transformation in the frequency-encoding direction.

Both the distorting transformation and its derivative depend on the magnetic flux density $B_0$ and the strength of the read-out gradient $G_r$. Schenck (1993) has shown that the depth of distortion with a cylindrical object depends on the parameter $c$ in Eq. 6-31. If $c \leq 0.5$, the distortion transformation is single-valued. This is equivalent to

$$\Delta x \leq \frac{B_0}{G_r R}.$$  \hspace{1cm} (6-35)
Fig. 38. Maximal susceptibility differences at different read-out gradient $G_r$ and main field $B_0$ values. The position of the MRI scanners at Oulu University Hospital has been marked in the figure, too.

Figure 38 shows the maximum value of susceptibility with different values of the magnetic field $B_0$ and the read-out gradient $G_r$ according to Eq 6-35. The same cylinder radius of the prosthesis (7.5 mm) has been used.

Table 7 shows the maximal susceptibility differences for three MR scanners in Oulu University Hospital in a case where a cylinder ($R = 7.5$ mm) is perpendicular to the main magnetic field.

Table 7. Maximal susceptibility values of a cylinder perpendicular to the main magnetic field. The radius of the cylinder is 7.5 mm.

<table>
<thead>
<tr>
<th>Scanner</th>
<th>$B_0$ [T]</th>
<th>$G_r$ [mT/m]</th>
<th>$\Delta \chi$ [$10^6$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proview</td>
<td>0.23</td>
<td>16</td>
<td>522</td>
</tr>
<tr>
<td>Magnetom 42 SP</td>
<td>0.97</td>
<td>10</td>
<td>78</td>
</tr>
<tr>
<td>Signa Echo Speed</td>
<td>1.5</td>
<td>22</td>
<td>110</td>
</tr>
</tbody>
</table>

The susceptibility difference between the Profile™ prosthesis and water is about $180 \cdot 10^{-6}$. The distorted image of a prosthesis perpendicular to the main field can only be corrected when a Proview scanner is used. On the contrary, when a Magnetom or Signa scanner is used, the patient with a prosthesis can be placed into the imager in such a way that the stem of the prosthesis is parallel to the main field. If the stem has a constant shape along some part of its length, this region does not have any field and image distortions, or the distortions are small and correctable.

6.6.2 Critical role of the field map in the correction procedure

Good correction requires exact information about the magnetic field distortion. The unwrapping procedure turned out to be most critical for the correction procedure. When it was possible to use a precisely known magnetic field distortion to correct the calculation, as in case of the simulation in Fig. 26, the correction succeeded at low susceptibility differences $\Delta \chi$. The limit for the single-valued inverse transformation in simulations presented in Fig. 26 is about $\Delta \chi \leq 88 \cdot 10^{-6}$ according to Eq. (6-35). If the susceptibility difference $0 \leq \Delta \chi \leq 50 \cdot 10^{-6}$, the simulated correction procedure works well, but with $\Delta \chi \geq 100 \cdot 10^{-6}$, the correction is incomplete.
7 General discussion

MRI is sensitive to various disturbances that distort images. Inhomogeneity of the main magnet field $B_0$, nonlinearity of the gradients and imperfections of the shape of the gradient and RF pulses are the most common deficiencies in MR scanning. Moreover, many factors inside the object volume impair the quality of the image. The chemical shift between fat and other tissues, the flow of liquids, e.g. blood, microscopic and macroscopic susceptibility differences and motion inside the object volume may give rise to image distortions. In this study, attention was drawn to image distortions due to non-moving macroscopic artificial objects, such as a metal prosthesis inside a human body. In this case, induced eddy currents in metal and the susceptibility difference between metal and the surrounding tissue are the mechanisms most likely to distort images.

Only some studies can be found in the literature concerning image distortions due to eddy currents in metal objects (Malko & Nelson 1987, Malko & Hoffman 1989, Camacho et al. 1995, Hua & Fox 1996). Image distortions caused by eddy currents around a metal prosthesis have not been reported earlier. Qualitative measurements using a Magnetom scanner showed that the eddy currents are mainly induced by the RF field, when the gradients have a relative low slew rate. This study also shows that, in the vicinity of both ends of the prosthesis, there are signal loss regions due to eddy currents induced in the prosthesis. In these regions, the flip angle $\theta$ can be estimated to be 1.5-fold compared to the undisturbed value at the distal end and over 2.5-fold compared to the reference value at the proximal end of the prosthesis.

The stem of a Profile™ hip prosthesis was used as a test object in the measurements. This prosthesis is not one of the most commonly used prostheses in Finland or in our catchment area. In 1997, only 20 Profile™ prostheses were installed in Finland, while the most common model, Lubinus™ SP II, was installed into 958 hips (Nevalainen 1998). The reason for choosing this prosthesis model was that it has been an object in many earlier studies at our department. (Niinimäki 1995, Törmänen et al. 1996). The shape of the middle stem is nearly a regular cylinder, which enables comparisons of the results to the theoretical calculations based on a cylindrical model. The results of this study can be generalised to the non-ferromagnetic prosthesis models.

Two different methods have been developed and used to estimate the susceptibility of cylindrical metal objects. One is based on the geometrical deformation of the image and
the other on the phase differences of phase images. The estimated susceptibility value of
the Profile™ prosthesis is $\chi = (170 \pm 13) \times 10^{-6}$. This value is slightly lower than the
susceptibility of pure titanium ($\chi = 182 \times 10^{-6}$).

Two basically different methods can be used to correct susceptibility-induced image
distortions. Using properly modified pulse sequences, it is possible to eliminate the effect
of susceptibility variations during data collection (e.g. Kim et al. 1995). The other way is
to use re-mapping, in which case routine pulse sequences can be used to collect image
data, but information about the field distortion is needed. This information can be
calculated on the basis of the shape and susceptibility of the artificial object. These
calculations set demands on the performance of computers. The field distortion can also
be measured as a phase image using a normal GRE or a SE sequence with a shifted echo.
Now, a problem arises from the phase wrapping in the phase images. An unwrapping
algorithm must be used to obtain field distortion data for the re-mapping procedure. In
this study, cubic spines were exploited to smooth statistical noise and to extrapolate over
wraparound positions, which were sought using Laplace filtration and comparison of the
intensity differences between the adjacent pixels of the phase image. The re-mapping
operation can be performed in either a spatial or a frequency domain. The re-mapping
method has been developed on the basis of routine pulse sequences by avoiding serious
and heavy field distortion calculations. Field distortion has been measured using an extra
GRE sequence with a short TE. Re-mapping has been calculated in the frequency
domain, because it has been shown to give better results than re-mapping in the spatial
domain (Weisskoff & Davis 1992). Using the methods described, a correction procedure
has been built and programmed into a computer. The ideas and methods have first been
tested by simulations.

Using these methods, the corrected images of a prosthesis appear to be equally good as
or slightly better than the original images when compared to calculated MTFs. When the
images of a prosthesis in a water tank are corrected, the increase of sharpness in the
corrected images appears to be biggest in the images of the distal end of the prosthesis. In
the middle part of the prosthesis stem, where image distortions are small, the effect
of correction is negligible. However, the image quality achieved here without any correction
is sufficient to visualise the surface of the prosthesis stem. Although, in images of the
human hip, another type of titanium prosthesis Bi-Metric, Harris-Galante) has been used,
the results are similar to the Profile™ prosthesis in a water tank. The quality of
uncorrected SE images is relatively good when the effect of correction is small. In GRE
images the effect is bigger. In the most proximal images, the phase unwrapping procedure
did not work well due to distortion of the GRE image plane. However, image quality is
not markedly improved by the correction procedure. When exact field distortion was
usable, as in the case of simulation, correction was possible at low susceptibility
differences $\Delta \Phi$ according to the theory.

High-quality estimation of the field distortion is most critical for successful correction.
The shortest possible TE of 6 ms was used for a minimally small phase difference $\Delta \Phi$.
However, the distances between the wraparounds near the surface of the prosthesis might
be only some pixels. Hence, precise quantification of field distortion on the surface of the
prosthesis was unreliable or downright impossible. A spin-echo sequence with a shifted
echo may offer shorter time intervals for $\Delta \Phi$. This requires, however, the pulse sequences
to be programmable. Several unwrapping methods have been developed and tested. The
methods based on testing pixel pairs (e.g. Hedley & Rosenfeld 1992, Szumowski et al. 1994) turned out to be unreliable with metallic implants, because the magnetic gradient field was very steep near the implant surface, the distances between the wraps around were some pixels, and the intensity steps at the wraps around were occasionally much smaller than the difference between the minimal and maximal intensities of the image.

A theoretical consideration of a cylinder perpendicular to the main magnet field shows that the feasibility of single-valued remapping depends linearly on the magnetic induction $B_0$ and inversely on the strength of the read-out gradient and the curvature radius of the cylinder surface. These dependencies can be generalised to objects of different shapes and orientations in a magnetic field. If a prosthesis radius is used ($R = 7.5$ mm), the estimated maximum $\Delta\chi = 522 \cdot 10^{-6}$ for $B_0 = 0.23$ T and $G_r = 16$ mT/m (Proview), $\Delta\chi = 88 \cdot 10^{-6}$ for $B_0 = 0.97$ T and $G_r = 10$ mT/m (Magnetom 42 SP) and $\Delta\chi = 110 \cdot 10^{-6}$ for $B_0 = 1.5$ T and $G_r = 22$ mT/m.

Single-value inverse mapping is not possible at higher susceptibility differences. Then, the correction does not succeed in the frequency domain, but must be performed in the spatial domain. This warrants further investigation. In the future elaboration of the presented correction method, one of most important tasks will be to develop a robust and functional phase unwrapping procedure. Also, correction methods which do not require information about field distortion may be useful (Kim et al. 1995, Bui et al. 2000).

To obtain high-quality images in the neighbourhood of artificial macroscopic objects with small image distortions, a spin-echo or fast spin-echo sequence with the highest possible value of read-out gradient must be used. This means the smallest possible field of view (FOV) and the biggest matrix size (Mtx) with the chosen bandwidth of pixel (BW). This agrees with our earlier results (Törmänen et al. 1996).
8 Conclusions

On the basis of results of this study, the following general conclusions can be made:

1. Eddy currents cause visible signal loss regions in images if the volume to be imaged contains metal objects. The eddy current may increase the flip angle manyfold compared to the reference value or reduce it to a fraction of the reference.

2. Two methods have been developed to assess the susceptibility of cylindrical objects. The susceptibility of a Profile™ prosthesis has been estimated to be \((170 \pm 13) \cdot 10^{-6}\).

3. Susceptibility distortion can be corrected if the susceptibility difference between the artificial object and the surrounding medium is not too great. The limit for a feasible single-valued correction transform depends linearly on the magnetic flux density of \(B_0\) and inversely on the curvature radius of the object and the strength of the frequency-encoding gradient.

4. Many correction methods can be used to eliminate image distortions due to susceptibility differences. The method developed in this study does not require \textit{in priori} knowledge about the shape and susceptibility of the implant. The necessary image data can be collected using routine pulse sequences. The method uses correction re-mapping in the frequency domain, and the field distortion can be measured using GRE phase images with short TE. Comparison using modulation transfer functions shows slight improvement or no changes in the sharpness of the corrected images. The main weakness in the selected method is its poor ability to assess field distortion from phase images due to problems in phase unwrapping.

5. Two different computer programs have been written and described in this study. One of them simulates image formation. Field imperfections can be entered to produce image distortions. The other program visualises and processes images, including filtration and arithmetic operations. The correction procedures have also been incorporated in this program.
9 References


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Appendix A: Computing methods and programs

A-1 Introduction

Some commercial computer programs, such as MathCad (MathSoft Engineering & Education, Inc, Cambridge, MA; USA) and Image-Pro®Plus (Media Cybernetics, L.P., Silver Spring, MD, USA), were used in image processing and data analysis during this study. However, the testing of many ideas and the algorithms for image processing, data analysis and artefact corrections required self-written programs. A short description of these programs is given in this chapter. The Visual Basic, Visual C++ and MacroAssembler languages (Microsoft Corporation, USA) have been used as programming tools, and the applications run under the MS Windows operating system.

A-2 Simulation of MR image formation

For testing the effects of different aspects of the object to be imaged and parameters of the imaging procedure on the generated MR images, a computer simulation program was written. Figure A-1 shows a block diagram of this program. The theory used in simulation has been presented in detail by K. Suominen (1983a). He has also written the kernel of the program.

Four matrices - spin density $\rho$, distributions of relaxation times $T_1$ and $T_2$, and distortion of the main field as an angular frequency difference of resonance frequency $\delta\omega$ - are given pixel by pixel as input information about the object plane. In the simulation process, at every sample moment $t_k$, the components of the magnetisation vector of every object pixel $M_q(x, y)$ have been summed into the total magnetisation vector $M$, which induces the measurable signal. The effects of relaxation times and resonance frequency offsets on all the object points $(x, y)$ have been taken into account. As a result, we have obtained the time-dependent components of magnetisation $M_x(t_k)$, $M_y(t_k)$ and $M_z(t_k)$. In image calculation, the $y$-component $M_y(t_k)$ and the $x$-component $M_x(t_k)$ are the real and the
imaginary parts of the input data for a complex Fourier transform, which gives the final complex image data. Magnitude and phase images are then calculated and saved in the internal format of the KS_Image program. An example of simulated images is shown in Fig. A-2.

Fig. A-1. Block diagram of the MRI simulation program WinSimu.
Fig. A-2. An example of simulated images. The simulation was done using a spin-echo sequence with TR=500 ms, TE=15 ms, the susceptibility of a cylinder, and an $\pi$ pulse shifted by 2 ms in order to get a map of field distortion. (a) A metal cylinder with $\chi=175\cdot10^{-6}$ parallel to the main field. (b) Magnet field variation due to a susceptibility difference between the cylinder and surrounding water. (c) Simulated image. Typical high signal intensity and signal void areas are visible. (d) The corresponding phase image shows field distortion as a phase difference.

The presented simulation method is almost the same as that presented by Bittoun et al. (1984) and by Summers et al. (1985), but differs from the formalism of Petersson (Petersson et al. 1993, Petersson 1998), who uses the spin partition concept in simulation.

A-3 Image-processing software (KS_Image)

The need of a tailored image-processing software application arose before the introduction of the DICOM standard, when different MRI and CT scanners used their own image formats. Our own project of building a MRI scanner (Jämsä 1983, Suominen 1983b) also required software to visualise images. K.Suominen wrote at that time the program kernel, including the user environment and the basic measuring and image-processing tools. These parts are included in the modules IMAGE and image.dll presented in Fig. A-3. Visual Basic is used in the user environment for menus, image frames, command buttons, etc. Routines in image.dll containing operations for display,
profile, icon, ROI, and measurements of angles and distances as well as some pixel operations have been coded using MacroAssembler.

Fig. A-3. Block diagram of the functionality of the KS_IMAGE program.

Especially the module SPECIAL and the modules included in it were used in this study. The modules FILTER, OPER, FFT, filter.dll and afft.dll were also important in this study. All of them have been written by the present author using Visual Basic in the modules and Visual C++ in the dynamic linked libraries pxl.dll, filter.dll and afft.dll.
Pxl.dll includes operations of image data buffers, such as copying images, reading and writing from and into files, converting pixels from 8 to 16 bits and vice versa, reading and writing single pixels from image buffers into program variables and vice versa.

Filter.dll consists of image filtration routines, such as average, median, Gauss, gradients and Laplace. Common methods are used in them (e.g. Gonzales & Woods 1992). Transformations from Cartesian to polar co-ordinates and vice versa (Alakuijala 1992) are also included in this module. In the FILTER module, which uses filter.dll, a user can define the image or images and the filtration operation.

Arithmetic and logical operations, such as add, subtraction, and, or, etc., can be performed between pixels of two different images. Also, the user can define a cut-off level for the posterization of an image.

The dynamic linked library afft.dll contains 1-D and 2-D fast Fourier transform routines, including the Fourier shift operation needed for the calculation of image correction. The basic algebra of complex variables is included in this library, too. These routines are needed in the FFT and SPECTRO modules. Image calculation from raw data produced by simulation or obtained from a scanner has been programmed in the FFT module. Transformation of an image from the spatial domain to the frequency domain and vice versa is possible in this module. The SPECTRO module contains routines for magnetic resonance spectrum analysis.

Especially in this study, some necessary routines have been integrated into the module of SPECIAL and its submodules. The calculation of integrals in the Eqs. (5-5) and (5-12) has been coded in the EDDY module. The code for the fitting procedure according to Eq. (5-15) is also in the same module. Levenberg-Marquard fitting written using Visual Basic according to the algorithm presented in Numerical Recipes in C (Press 1997) has been used in this procedure. This is located in the LEV_MRQT module.

The routines used in image correction calculation are shown in Fig. A-3 as a block diagram. First, the image to be corrected and the corresponding field echo phase image are read into the program. At the second step, a poster copy image is generated by a cut-off operation. The cut-off level can be chosen either automatically or interactively. The prosthetic area is black and the rest of the image is white. When the AND operation between this image and the phase image has been performed, a phase image with the area of the prosthesis cut off is obtained as a result. When the unwrapping method and the parameters have been selected, the unwrapping of the phase image with the prosthesis cut off is performed and the distortion of the main magnetic field $\Delta B$ is obtained. Using fast Fourier transform with Fourier shift, a corrected raw image is calculated. The final corrected image has been obtained as a result of an inverse fast Fourier transform of the corrected raw image, and it has been shown using routines of the IMAGE module.
Fig. A-4. Schematic diagram of the IMAGE program modules needed in image correction calculations.