Zaheer Khan

COORDINATION AND ADAPTATION TECHNIQUES FOR EFFICIENT RESOURCE UTILIZATION IN COGNITIVE RADIO NETWORKS
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Oulu, Finland

Abstract

The aim of this thesis is to devise coordination and adaptation techniques that enable the wireless devices operating in a cognitive network to utilize their available resources efficiently. The first part of this thesis considers the case where multiple autonomous devices sense the frequency channels sequentially in some sensing order for spectrum opportunities. In particular, the first part is interested in the scenario where devices with false alarms autonomously select the sensing orders in which they visit channels, without coordination from a centralized entity. An adaptive persistent sensing order selection strategy that allows autonomous adaptations to collision-free sensing orders is proposed and evaluated. It is shown that the proposed strategy converges and maximizes cognitive network throughput compared to a random selection of sensing orders.

The second part of this thesis considers the case where distributed devices interact with one another to cooperate to fulfill tasks or to improve the efficiency of network resource usage. Tools from coalition formation game theory are adopted to devise dynamic cooperative strategies for distributed devices. Dynamic coalition formation methods, are proposed for two different network scenarios: 1) Distributed devices operating in an interference channel; 2) Distributed devices performing spectrum sensing. It is observed that in distributed spectrum sensing if the devices pursue their goals selfishly then coalition formation may lead to a suboptimal equilibrium where devices, through their interactions, reach an undesirable coalition structure from a network point of view. The proposed selfish model of dynamic coalition formation is then extended to determine whether and how the coalitional behavior of devices will change if coalition formation is "not entirely selfish". It is observed that for the problem of distributed spectrum sensing, average throughput per device is increased when devices cooperate to maximize the overall gains of the group as compared to when they cooperate to increase their individual gains.

Finally, in the last part of the thesis, to reduce spectrum sensing overhead and total energy consumption of a cognitive radio network, the problem of sensor selection is considered. Different techniques for selecting devices with the best detection performance are proposed, and it is shown that the proposed device selection methods are able to offer significant gains in terms of system performance as compared to a random selection of devices.

Keywords: adaptation, autonomous devices, cognitive radio, coordination, game theory


Työn viimeisessä osassa pohditaan sensorien valintaongelmia. Siinä ehdotetaan erilaisia menetelmiä, jotka ovat tavoitteet parhaan suorituskyvyn omat laitteet ja näytetään, että ehdotetut laitteiden valintamenetelmät pystyvät tarjoamaan merkittäviä suorituskykyhakemuksia. Ehdotetun satunnaisen laitteiden valintaan.
For the memory of George Eliot (Mary Anne Evans).

The greatest English novelist of the nineteenth century.

“. . . the pride and paragon of all her sex.”

For my sister, who is an admirable woman full of strength and courage.
"I have no special talents. I am only passionately curious."

Albert Einstein
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The list of original articles presented in this thesis indicates Prof. Luiz A. DaSilva and Dr. Janne Lehtomäki as co-authors, but that doesn’t do them justice. They have been my mentors from the very beginning of PhD thesis. I have really enjoyed writing articles with Prof. Luiz A DaSilva, without his guidance, I might not have published a single article. His picky questions/comments and willingness to speak his mind has been a great influence on my research work and life. I am very grateful of my colleague Dr. Janne Lehtomäki, who offered useful criticisms of my research work, provided useful guidance in simulations and shared a cup of coffee with me when I was working long hours.

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Oulu, October 12, 2011

Zaheer Khan
<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>A set ( A = {1, 2, 3, \ldots, A} ) of actions available to a CR</td>
</tr>
<tr>
<td>( A )</td>
<td>Number of actions available to a CR</td>
</tr>
<tr>
<td>( a_{S_1}(S) )</td>
<td>Altruistic contribution of coalition ( S_1 ) to coalition ( S )</td>
</tr>
<tr>
<td>(  \hat{a}(S) )</td>
<td>Sum of altruistic contributions of two coalitions to coalition ( S )</td>
</tr>
<tr>
<td>( B_N )</td>
<td>The Bell number</td>
</tr>
<tr>
<td>( b )</td>
<td>The binary flag</td>
</tr>
<tr>
<td>( C )</td>
<td>The set of all possible coalition structures</td>
</tr>
<tr>
<td>( \bar{C} )</td>
<td>The set of all integer partition states</td>
</tr>
<tr>
<td>( C(|S|) )</td>
<td>Cost for coalition formation</td>
</tr>
<tr>
<td>( D )</td>
<td>Number of messages per coalition formation proposal</td>
</tr>
<tr>
<td>( d_i )</td>
<td>Distance between the primary user and the ( i )th CR</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>Distance between the transmitter of link ( j ) and the receiver of link ( i )</td>
</tr>
<tr>
<td>( d_i(S) )</td>
<td>Minimum rational payoff of link ( i ) in coalition ( S )</td>
</tr>
<tr>
<td>( E )</td>
<td>The event that, in a given time slot, the CRs autonomously arrive at collision-free sensing orders</td>
</tr>
<tr>
<td>( F )</td>
<td>The fundamental matrix for ( \bar{P} )</td>
</tr>
<tr>
<td>( H )</td>
<td>Link gain matrix</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>Primary user not present</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>Primary user present</td>
</tr>
<tr>
<td>( h_{ji} )</td>
<td>The channel gain between the transmitter of link ( j ) and the receiver of link ( i )</td>
</tr>
<tr>
<td>( I )</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>( i.i.d. )</td>
<td>Independent and identically distributed</td>
</tr>
<tr>
<td>( K )</td>
<td>Number of sub-bands</td>
</tr>
<tr>
<td>( M )</td>
<td>A set ( M = {1, 2, 3, \ldots, M} ) of CRs or links</td>
</tr>
<tr>
<td>( M )</td>
<td>Number of CRs or links in the network</td>
</tr>
<tr>
<td>( M_p )</td>
<td>Total number of coalition formation proposals sent during the entire coalition formation process</td>
</tr>
<tr>
<td>( m_i(S\setminus{i}) )</td>
<td>The marginal contribution of any link ( i \in S ) in the presence of other links in any proposed coalition ( S )</td>
</tr>
<tr>
<td>( N )</td>
<td>A set ( N = {1, 2, 3, \ldots, N} ) channels</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of potentially available channels</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Noise variance</td>
</tr>
<tr>
<td>$Q$</td>
<td>Channel sensing order of a CR</td>
</tr>
<tr>
<td>$O$</td>
<td>The set of available sensing orders</td>
</tr>
<tr>
<td>$O$</td>
<td>Matrix with all zero entries</td>
</tr>
<tr>
<td>$P$</td>
<td>Transmission power</td>
</tr>
<tr>
<td>$\tilde{P}$</td>
<td>Transition matrix of an absorbing Markov chain</td>
</tr>
<tr>
<td>$P_{CS}$</td>
<td>Probability of correct selection</td>
</tr>
<tr>
<td>$P_{C_1C_2}$</td>
<td>The transition probability for the $M$ link coalition game</td>
</tr>
<tr>
<td>$\tilde{P}_d$</td>
<td>Target probability of detection</td>
</tr>
<tr>
<td>$P_{d,i}$</td>
<td>The probability of detection of the $i$th CR</td>
</tr>
<tr>
<td>$\tilde{P}_{d,i}$</td>
<td>Target probability of detection of the $i$th CR</td>
</tr>
<tr>
<td>$\tilde{P}_{w,i}$</td>
<td>Weighted target probability of detection of the $i$th CR</td>
</tr>
<tr>
<td>$P_{d,S}$</td>
<td>The probability of detection of the coalition $S$</td>
</tr>
<tr>
<td>$P_{E}$</td>
<td>The probability that, in a given time slot, the CRs autonomously arrive at collision-free sensing orders</td>
</tr>
<tr>
<td>$P_{E,\text{genie}}$</td>
<td>The probability that the $M$ CRs can arrive at collision-free sensing orders in a time slot using the genie aided scheme</td>
</tr>
<tr>
<td>$\tilde{P}_{fa}$</td>
<td>Target probability of false alarm</td>
</tr>
<tr>
<td>$P_{fa,i}$</td>
<td>The probability of false alarm of the $i$th CR</td>
</tr>
<tr>
<td>$\tilde{P}_{fa,i}$</td>
<td>Target probability of false alarm of the $i$th CR</td>
</tr>
<tr>
<td>$P(D)$</td>
<td>The success probability of an individual CR given that the two CRs select different sensing orders</td>
</tr>
<tr>
<td>$P(K)$</td>
<td>Probability that CR $i$ and one or more other CRs select the same sensing order</td>
</tr>
<tr>
<td>$P(L)$</td>
<td>Probability that CR $i$ selects a sensing order that is not selected by any other CR</td>
</tr>
<tr>
<td>$P[N,M,\theta]$</td>
<td>Probability of success for an individual CR for $N$ potentially available channels, $M$ competing autonomous CRs and probability of a primary user present in a channel $\theta$</td>
</tr>
<tr>
<td>$P_{PU}$</td>
<td>Primary user’s signal power</td>
</tr>
<tr>
<td>$P(S)$</td>
<td>The success probability of an individual CR given that the two CRs select the same sensing order</td>
</tr>
<tr>
<td>$p$</td>
<td>$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>The probability of proposing a coalition structure change</td>
</tr>
</tbody>
</table>
\(p_i\) The probability of choosing the \(i\)th sensing order

\(Q\) The matrix of transition probabilities between the transient states

\(\bar{Q}\) The tail probability for the standard normal distribution.

\(R\) The transmission rate of a CR to its receiver.

\(\bar{R}_i\) Average throughput of the CR \(i\).

\(\bar{\hat{R}}_i\) Average throughput of the CR \(i\) considering the cost in terms of overhead in sensing reports combining within a coalition.

\(R_s\) Radius of a network

\(R_t\) The matrix of transition probabilities from transient to absorbing states

\(S\) Subset of \(\mathbf{M}\)

\(S^c\) Complementary coalition of \(S\)

\(SC\) The success counter

\(s\) A row of a Latin Square

\(T\) Total duration of a time slot

\(\mathcal{T}\) Restart time for coalition formation

\(T_B\) Time-bandwidth product

\(T_p\) Training period

\(T_{\text{sense}}\) The time required to sense a channel

\(V(C_k)\) Worth or value of a coalition structure \(C_k\)

\(\bar{v}(S)\) The value, or worth, of a coalition \(S\)

\(\hat{v}(S)\) The net value, or net worth, of a coalition \(S\) considering the cost for coalition formation

\(v(S;\mathcal{P})\) The value, or worth, of a coalition \(S\) and a partition \(\mathcal{P}\)

\(W\) Bandwidth

\(X_i\) Transmitted signal of the \(i\)th CR

\(X_s\) A random variable representing the number of sensing steps within a time slot until a CR is successful in finding a channel free from PU and other CR activity (given that the CR is successful)

\(x\) Payoff vector

\(Y_E\) The expected number of sensing orders that are not selected by any CR in a given time slot

\(Y_i\) Received signal for \(i\)th CR

\(z\) Noise processes

\(1(V(C_i),V(C_i))\) Indicator function that represents the possible agreement or disagreement among the links participating in the coalition game to form the proposed
coalition $S$

$\alpha_{\delta_M}$ Probability that both the CR $i$ and the fusion center declare the primary user to be present

$\alpha_p$ The path loss exponent

$\alpha_{10,i}$ The transition probability for the $i$th channel from PU-occupied to PU-free

$\beta$ Distance between the primary base station and the center of the CR network

$\beta_{01,i}$ The transition probability for the $i$th channel from PU-free to PU-occupied

$\Gamma'$ One link proposes a coalition structure change

$\Gamma_{NW}$ No one proposes a coalition structure change

$\Gamma_{3W}$ More than one proposes a coalition structure change

$\gamma$ The persistence factor

$\bar{\gamma}$ The average SNR

$\gamma_{r,i}$ Received SNR from the primary user to the $i$th cognitive radio

$\gamma$ The set of stable imputations

$\Delta$ Real state of the primary user

$\delta_i$ Sensing decision of CR $i$

$\eta_i$ The fraction of the band that link $i$ uses

$\Theta$ Circulant matrix

$\theta_{ld}$ The difference between the detection probabilities in the "good set" and the "bad set"

$\theta_i$ The probability of the PU being present in the $i$th channel in a given time slot

$\sigma^2$ Variance of time for the coalition game to reach a stable coalition structure

$\psi$ The path loss constant

$\lambda$ Detection threshold

$\mu$ Mean time for the coalition game to reach a stable coalition structure

$\xi$ Core of the coalition game

$\rho$ A partition of $M$

$\rho$ Real state of the primary user

$\sigma$ Channel error rate

$\sigma^2$ Noise power

16
\( \sigma_0 \) Probability of incorrect collision detection
\( \sigma_1 \) Probability of correct collision detection
\( \varsigma \) CRs with high detection probabilities
\( \tau \) A column vector whose components are the respective state “dwell” times
\( \tau_c \) The time spent on reporting sensing decisions to the coalition head
\( \phi \) A proportionality constant
\( \phi_i(S) \) The average throughput for CR \( i \) in a coalition \( S \)
\( \kappa \) Number of CRs to be selected by the fusion center
\( \Omega \) Set of coalition structure states

ACK Acknowledgement
CD Collision detection
CFF Characteristic function form
ChR Channel release message
CR Cognitive radio
CS Coalition structure
CSCG Circular symmetric complex Gaussian noise
DSA Dynamic spectrum access
ED Energy detector
FC Fusion center
GC Grand coalition
IES Internally and externally stable
IZA Indifference zone approach
LS Latin Square
MAC Medium access control
NTU Non-transferable utility
OSA Opportunistic spectrum access
PAC Partial-agreement counting
PFF Partition function form
PSK Phase Shift Keying
PU Primary user
rand Randomize after every collision strategy
SC Simple counting
SINR Signal-to-interference-plus-noise ratio
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TTC</td>
<td>Time to arrive at collision-free sensing orders</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless sensor networks</td>
</tr>
</tbody>
</table>
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>11</td>
</tr>
<tr>
<td>Tiivistelmä</td>
<td></td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>13</td>
</tr>
<tr>
<td>Abbreviations</td>
<td></td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>23</td>
</tr>
<tr>
<td>1.1 Cognitive radio technology – definitions</td>
<td>24</td>
</tr>
<tr>
<td>1.2 Autonomous devices in a multichannel cognitive radio network</td>
<td>25</td>
</tr>
<tr>
<td>1.2.1 Opportunistic multichannel access for autonomous devices</td>
<td>27</td>
</tr>
<tr>
<td>1.2.2 Challenge: design of efficient opportunistic multichannel access strategies for autonomous devices</td>
<td>28</td>
</tr>
<tr>
<td>1.2.3 A solution: adaptive persistent sensing order selection</td>
<td>29</td>
</tr>
<tr>
<td>1.3 Coalition formation in distributed cognitive networks</td>
<td>30</td>
</tr>
<tr>
<td>1.3.1 Coalition formation for efficient spectrum utilization</td>
<td>31</td>
</tr>
<tr>
<td>1.3.2 Challenge: how to coordinate distributed devices to form coalitions?</td>
<td>31</td>
</tr>
<tr>
<td>1.3.3 A solution: dynamic coalition formation methods</td>
<td>33</td>
</tr>
<tr>
<td>1.4 Cooperative spectrum sensing for opportunistic spectrum access</td>
<td>34</td>
</tr>
<tr>
<td>1.4.1 Challenge: ensuring efficient cooperative sensing</td>
<td>35</td>
</tr>
<tr>
<td>1.4.2 A solution: selection of best detection performance devices</td>
<td>35</td>
</tr>
<tr>
<td>1.5 Contributions of this thesis</td>
<td>36</td>
</tr>
<tr>
<td>1.6 Author’s contribution and thesis structure</td>
<td>38</td>
</tr>
<tr>
<td><strong>2 Efficient channel access for autonomous devices in a multichannel cognitive network</strong></td>
<td>41</td>
</tr>
<tr>
<td>2.1 Background</td>
<td>41</td>
</tr>
<tr>
<td>2.2 Related literature</td>
<td>42</td>
</tr>
<tr>
<td>2.3 System model</td>
<td>43</td>
</tr>
<tr>
<td>2.4 Adaptive persistent sensing order selection</td>
<td>46</td>
</tr>
<tr>
<td>2.4.1 γ-persistent sensing order selection</td>
<td>49</td>
</tr>
<tr>
<td>2.5 Convergence of the adaptive persistent sensing order selection</td>
<td>54</td>
</tr>
<tr>
<td>strategies</td>
<td></td>
</tr>
<tr>
<td>2.6 Randomize after every collision (rand) sensing order selection</td>
<td>57</td>
</tr>
</tbody>
</table>
1 Introduction

The reason there are no humanlike robots is not that the very idea of a mechanical mind is misguided. It is that the engineering problems we humans solve as we see and walk and plan and make it through the day are far more challenging than landing on the moon or sequencing the human genome. Nature, once again, has found ingenious solutions that human engineers cannot yet duplicate.

Steven Pinker: How the Mind Works

The increasing demand for wireless communication and the emergence of numerous smart wireless devices has led to rapid growth of the wireless market [1–3]. According to the recent FCC report on trends in wireless devices, the wireless market had under 100 million subscribers at the dawn of the century, and it has grown to near triple that amount today [4]. However, resource scarcity is a general challenge for wireless communication systems [5–9]. Therefore, to accommodate the increasing demand for wireless communication, design of efficient resource utilization techniques is required.

In the design of wireless communication systems, wireless network engineers face two significant challenges: 1) Communication frequency spectrum is a scarce resource; and 2) Coordination among wireless devices is required to fulfill tasks or to improve the efficiency of spectrum usage. For instance, in ad hoc networks, radios need to interact with one another to establish a network topology, enabling communications between any pair of nodes either through a direct link with limited communication bandwidth or a multihop path with limited bandwidth.

Intelligent radios that can efficiently coordinate and adapt to their environment can increase the network performance. Cognitive radio technology has been envisioned as a chief enabler of intelligence and adaptation in wireless devices.

My thesis is dealing with intelligent adaptation and coordination techniques for wireless communication access that enable the wireless devices to fulfill their tasks effectively and to maximize resource efficiency of their network. Integrating cognition into wireless applications such as dynamic spectrum access and radio resource management enables the devices operating in a wireless network to coordinate and utilize their available resources efficiently.

Some preliminary cognitive radio technologies (e.g., transmit power adaptations and dynamic channel selection in response to changing radio-frequency environments) have
already been used in a few existing wireless networks such as cellular networks and wireless local-area networks (WLANs) [10–14].

The research work presented in this thesis enhances our understanding of cognitive networks by developing and analyzing coordination and adaptation techniques for three different network scenarios. First, an intelligent adaptation scheme for efficient channel access in multichannel cognitive networks is presented, where autonomous cognitive radios have to search multiple potential channels for spectrum opportunities and they face competition from one another to access the channel. Then, this thesis considers the scenario where radios act as independent and autonomous agents, and to maximize their utility they are able to make decisions to either compete or cooperate with one another. Tools from coalition formation game theory are adopted to devise dynamic selfish and altruistic cooperative strategies for distributed radios. It is shown that for some scenarios altruistic cooperation increases the average utility per device as compared to when devices cooperate selfishly. Finally, to reduce sensing overhead and total energy consumption of a cognitive network, the problem of best detection performance device selection for centralized cooperative spectrum sensing is considered.

1.1 Cognitive radio technology – definitions

Ever since Joseph Mitola coined the term “cognitive radio” for the first time in [15], cognitive radio has caught attention of a wider segment of the wireless community and many definitions of a cognitive radio (CR) have emerged in literature [13, 16, 17]. Some widely used definitions are:

– An intelligent wireless communication system that is aware of its environment.
– Radio that interacts with other radios and networks, and adapts its operating parameters to fulfill tasks efficiently.
– Radio that adapts its transmission or reception parameters to avoid interference with licensed or unlicensed users.
– A wireless device that has the ability to sense, detect, adapt and make decisions using feedback.

In this thesis, a cognitive network is defined as a collection of wireless devices that intelligently adapt their operating parameters and efficiently coordinate to achieve their goals effectively. In practice, design of a cognitive network faces certain challenges and complexities that are not found in their noncognitive counterparts [13, 18–20]. Some of
these challenges and complexities include:

- Radios operating in the cognitive network may need to exploit locally vacant or unused radio frequency channels, or ranges of radio spectrum, referred to as spectrum holes or white spaces, to utilize the frequency spectrum more efficiently. This requires that cognitive radios are capable of switching across different frequency channels with minimum delay.
- A CR may need to adapt itself without user intervention to save battery power or to reduce interference to other users. Adaptive decision making may involve the use of the history of past outcomes to find a way to efficiently utilize network resources.
- CRs may need to formulate and issue complex queries, one radio to another and execute commands sent by another radio.

The background to techniques that can be used to overcome some of the challenges and complexities found in cognitive networks is provided in the subsequent sections.

1.2 Autonomous devices in a multichannel cognitive radio network

CR networks are envisioned to utilize the licensed frequency spectrum more efficiently through opportunistic access to (temporarily) unused spectrum bands. Among different opportunistic spectrum access (OSA) schemes, sensing-based OSA is widely investigated because it does not require the licensed (primary) users to alter their existing hardware or behavior [21]. In sensing-based OSA, CRs monitor the environment to reliably detect the primary user signals and operate whenever the band is empty. In practice, the detection of primary users may rely on a combination of sensing and the use of geolocation spectrum occupancy databases [22].

Autonomous wireless devices operating in a multichannel cognitive radio network must cope with limited computational resources, uncertainty and limited knowledge of the environment. Autonomy of devices implies that CRs can perform actions (transmission decisions, etc.) independently in their wireless environment based on spectrum sensing and feedback (occurrence of collisions) information.

Distributed coordination of multiple autonomous devices in multichannel cognitive radio networks has recently attracted strong interest [23–28]. This is mainly due to:

1) The need for developing flexible wireless networks where the independent devices can make autonomous decisions;
2) The need for low complexity methods to find
A widely studied class of problems in multichannel cognitive radio networks are those in which the aim of autonomous devices is to attempt to find one another, such as OSA radio rendezvous problem [29, 30]. In OSA radio rendezvous problem, two or more radios attempt to find one another in a dynamic spectrum access (DSA) environment, where a set of frequency channels is shared opportunistically by CRs with multiple primary users.

In this thesis, a complementary class is considered in which the aim of the CRs is to disperse (avoid one another). The sensing order dispersion problem in multichannel cognitive networks is considered, where autonomous CRs have to search multiple potential channels for spectrum opportunities and they face competition from one another to access the channel. The aim of two or more autonomous CRs is to attempt to
Fig 2. An example of the sensing order dispersion problem.

avoid one another by selecting those sensing orders in which the likelihood of collisions is reduced. In Fig. 1 and Fig. 2, illustrated examples of radio rendezvous and sensing order dispersion problems in multichannel cognitive radio networks are presented.

1.2.1 Opportunistic multichannel access for autonomous devices

In sensing based OSA, the CRs are required to perform periodic spectrum sensing so that when a primary user becomes active in a channel, the CRs can vacate that channel [31]. When multiple frequency channels are available for opportunistic transmissions, time-slotted multiple access is widely considered [23–28]. The first portion of each time slot is used by CRs for spectrum sensing, and the second portion is used to access the free channel, if one is found. In Fig. 3, a taxonomy of periodic spectrum sensing models
Fig 3. Taxonomy of periodic sensing models for opportunistic spectrum access.

Consisting of two branches is presented: periodic sensing for single potential primary user band and periodic sensing for multiple potential primary user bands. Under a single potential primary user band system, CRs are allowed to explore a single licensed spectrum band. The first portion of each time slot is used by CRs for sensing the licensed band, and the second portion is used to access the band, if it is free [32, 33].

In the other branch, periodic sensing for multiple potential primary user bands, CRs are allowed to explore multiple licensed spectrum bands. From here, there are two broad categories of periodic sensing policies: CRs may employ single-channel sensing policy or sequential-channel sensing policy. Under a single-channel sensing policy, in any given time slot a CR first selects a channel to sense and transmits if that channel is free; otherwise, it stays silent for the entire duration of that time slot [26, 34–36]. In an alternative approach, sequential-channel sensing, a CR can sense channels sequentially in some order until it finds a free channel to transmit on, however, if in all sensing steps channels are sensed to be busy then the CR stays silent for the entire duration of that time slot. [24, 25].

1.2.2 Challenge: design of efficient opportunistic multichannel access strategies for autonomous devices

When multiple autonomous CRs have to search multiple channels for spectrum opportunities, these radios face competition from one another to access the channel. For instance, if in a given time slot a particular channel is simultaneously sensed free by two or more autonomous CRs and more than one of them decide to transmit on the channel, then a collision occurs. In this context, the channel sensing order $\emptyset$, i.e., the order in which radios competing for the channels visit those channels, will affect their
probability of successful access. The first part of this thesis investigates how CRs can autonomously select channel sensing orders so as to minimize the likelihood of collisions with other CRs also searching for channels to be utilized opportunistically. This thesis is particularly interested in the case where CRs autonomously choose the order \( \bigcirc \) in which they visit channels, without coordination from a centralized entity.

In a given time slot, CRs searching for a channel face one of the following outcomes: success, collision, or no transmission (when all channels sensed by that CR were found busy). Can CRs then use the history of past outcomes to find a way to autonomously arrive at collision-free sensing orders? If the answer is yes, can we design efficient sensing order selection strategies that maximize the throughput of the distributed CR network?

### 1.2.3 A solution: adaptive persistent sensing order selection

To avoid continuously repeated conflicts among autonomous CRs, some method must be devised using which two or more autonomous CRs attempt to avoid one another by selecting those sensing orders that reduce the likelihood of collisions among one another. In Fig. 4, a taxonomy of autonomous sensing order selection methods consisting of two branches is presented: Adaptive and nonadaptive sensing order selection methods. Under nonadaptive methods, a simple method proposed in the literature is a random selection of sensing orders [25, 37]. In this method, in each time time slot, a sensing order (which may come from either the space of all permutations of \( N \) channel indices,
or some subset thereof) is independently and randomly selected (with equal probability) by each CR and then channels are sensed by each CR according to its sensing order. However, the performance of the random selection method is limited by the collisions among the autonomous CRs [38].

In the other branch, adaptive sensing order selection methods, CRs employ adaptive randomization based on feedback (occurrence of collisions) to minimize the likelihood of collisions with one another [26, 34, 37–39]. Adaptations are in the autonomous choice, by CRs, of the channel sensing order $O_i$. Adaptive sensing order selection methods are of two types: Adaptive persistent sensing order selection and adaptive nonpersistent sensing order selection. Under adaptive nonpersistent sensing order selection, initially each CR independently and randomly (with equal probability) selects a sensing order (which may come from either the space of all permutations of $N$ channel indices, or from a subset thereof). In the next time slots, a CR randomly (with equal probability) selects a new sensing order only if it has experienced a collision in the previous slot; otherwise, it retains the previously selected sensing order. In an alternative approach, adaptive persistent sensing order selection, proposed by us in [37–39], a CR maintains information regarding past successes (and failures) in using a particular sensing order. In Chapter 2 it is shown that the proposed adaptive persistent sensing order selection strategy enable the CRs to reduce the likelihood of collisions with one another, as compared to the other sensing order selection strategies.

1.3 Coalition formation in distributed cognitive networks

In distributed and self-configuring networks, nodes operating in the same spectrum band act as independent and autonomous agents and can use their limited power, spectrum, etc. to either compete or cooperate with one another. Coalition game theory provides useful tools to decide which group of players will cooperate with each other to efficiently achieve their goals [40]. Therefore, tools from coalition formation game theory are adopted to devise dynamic cooperative strategies for distributed devices. In coalition games, a coalition is a set of autonomous agents or players which may cooperate in order to increase their individual gains.

Decision making in a distributed network where the autonomous devices can choose between cooperative and competitive behavior displays some unique characteristics:

- The devices may routinely alter their behavior in response to changes in their radio
environments. As a result, the devices need to employ adaptive decision making strategies.

- The devices may have some behavioral preferences, they may cooperate in order to increase their individual gains, which we call as selfish coalition formation. Or they may cooperate to maximize the overall gains of the group, which we call altruistic coalition formation.

- Coalition formation in wireless networks can create externalities. A coalition game with externalities is a game in which the value that a group of players can achieve through cooperation depends on what other coalitions form. Devices make their decisions to form coalitions independently, but, due to the presence of externalities, their choice may impact all the devices in the network. For instance, positive externalities due to coalition formation may provide an incentive for wireless nodes to free ride, resulting in small stable coalitions. On the other hand, negative externalities may provide an incentive to cooperate, resulting in large stable coalitions.

1.3.1 Coalition formation for efficient spectrum utilization

In this thesis, cooperative interactions among CRs for two different network scenarios are analyzed: 1) Distributed devices operating in an interference channel; 2) Distributed devices performing spectrum sensing. In Fig. 5, coalition formation for two different network scenarios are illustrated.

1.3.2 Challenge: how to coordinate distributed devices to form coalitions?

Traditionally, the study of coalition games in wireless networks has focused on cohesive games, [41, 42], i.e., games where the value of the grand coalition formed by the set of all players $M$ is at least as large as the sum of the values of any partition of $M$. The authors in [41, 42] also assume that there is no cost to the coalition formation process. In such coalition games, the coalition structure generation is trivial because the wireless nodes always benefit by forming the grand coalition.

However, many coalition game models of wireless node cooperation are not cohesive (see, for example, [43]), because in wireless networks there is some cost to the coalition formation process itself or because in the grand coalition there is no resource reuse,
which is sometimes beneficial in wireless networks. For instance, coalition formation in wireless networks may require wireless links to activate some coalition message exchange link, which may be costly. Some coalitions, in such scenarios, may gain by merging while the others may not. In such scenarios, it is important to analyze cooperative interactions between small groups of nodes, as well as cooperative interactions between all the network nodes. In other words, we need to analyze the entire coalition formation process until we find the social welfare maximizing coalition structure that satisfies the equilibrium conditions.

Stable coalition structures in coalition games correspond to the equilibrium state in which users do not have incentives to leave the already established coalitions. However, the models proposed for the analysis of stable coalition structures are often static, in the sense that they fail to specify how the players arrive at equilibrium.
Establishing cooperation in a wireless network is a dynamic process and two important questions must be addressed: 1) How are coalitions formed?; and 2) How do players arrive at equilibrium?

1.3.3 A solution: dynamic coalition formation methods

This thesis introduces dynamic models of distributed coalition formation for the efficient spectrum utilization in cognitive networks where multiple users coexist and interact with one another. In the proposed dynamic coalition formation, a time-evolving sequence of steps is used by radios to reach self-organizing stable spectrum sharing/sensing coalition structures. Coalition structures in the dynamic coalition formation game are modeled as a sequence of random variables describing the state of the system, and the mechanism of transitions between coalition structures is represented by a Markov chain. Using Markov chain theory this thesis analyzes the mean $\mu$ and variance $\sigma^2$ of the time for the coalition game to reach a stable coalition structure, i.e., to reach an absorbing state. In Chapters 3 and 4, it is shown that the proposed coalition formation solutions yield significant gains in terms of increased average throughput per CR as compared to the non-cooperative solutions.
Fig 6. Cooperative spectrum sensing cognitive radio network for reliable detection of a primary user. In this example only the sensing decisions of cognitive radio B may be reliable as the other two cognitive radios are shadowed.

1.4 Cooperative spectrum sensing for opportunistic spectrum access

In sensing-based opportunistic spectrum access (OSA), the two types of errors associated with spectrum sensing are false alarms and missed detections. The lower the missed detection probability, the better the primary user is protected. However, to increase the achievable throughput of the CRs, the false alarm probability must also be low [33, 44]. The effects of fading or shadowing may result in unreliable detection of the primary user presence by the single CR user (see Fig. 6). To overcome this problem of unreliable detection, cooperative spectrum sensing has been proposed. Cooperative spectrum sensing has been shown to increase the reliability of sensing [33, 44–47].
1.4.1 Challenge: ensuring efficient cooperative sensing

In cooperative sensing, information from several cognitive radios (CRs) is used for detecting the primary user. In centralized cooperative spectrum sensing, where the sensing process is coordinated from a centralized entity (fusion center), the CRs cannot transmit data until sensing reports are collected and the final combined sensing decision is transmitted to all the cooperating CRs. One simple method of sensing reports collection by the fusion center can be stated as follows. The fusion center grants a contention free channel to individual cognitive radios by polling them (using their identity numbers) for transmitting their local decisions. The fusion center may employ a round-robin scheduler [22, 48, 49], and on being polled, a CR transmits its local decision to the fusion center. In this sensing reports collection method there is cost in terms of time delay in data transmissions of the cooperating CRs due to the overhead in combining sensing reports.

A large number of cooperating CRs typically leads to an increase in total energy consumption and overhead, as more sensing information needs to be reported at the fusion center. To reduce sensing overhead and total energy consumption, it is recommended to cooperate only with the CRs that have the best detection performance. However, the problem is that it is not known \textit{a priori} which of the CRs have the best detection performance.

1.4.2 A solution: selection of best detection performance devices

To reduce sensing overhead and total energy consumption of a cognitive network simple but efficient methods for selecting the best detection performance CRs are proposed in this thesis. The best detection CRs are defined to be those CRs that have the highest probabilities of detection. Clearly, the smaller the set of the best detection performance CRs is used for cooperative sensing, the less the overhead and the total sensing energy is consumed because only the CRs that have the best detection performance are selected for cooperative sensing. The motivation behind this work is the underlying assumption that a selected subset of CRs with the best detection performance will detect better in a subsequent sensing period than a subset of CRs selected purely at random. This assumption is verified with simulation results.
1.5 Contributions of this thesis

The main contributions of this thesis are four-fold:

1) This thesis proposes an adaptive persistent sensing order selection strategy that enables the CRs to reduce the likelihood of collisions with one another, as compared to a random selection of sensing orders. Adaptations are in the autonomous choice, by CRs, of the channel sensing order \( O \). It is observed that the proposed strategy converges to collision-free sensing orders without requiring any coordination among CRs (provided that the number of CRs is less than or equal to the number of potentially available channels). Collision-free sensing orders are those in which two or more CRs never simultaneously sense the same channels and therefore never collide with one another. It is also observed that the adaptive persistent strategy reduces the expected time of arrival at collision-free sensing orders as compared to the \textit{randomize after every collision} strategy, in which a CR, upon colliding, randomly selects a new channel sensing order. The impact of imperfect information, i.e., the effects of sensing and channel errors, on adaptation decisions are also explored. Also, the impact of different primary user (PU) channel occupancy models are investigated and it is observed that the proposed adaptive strategy is not strongly affected by PU behavior. It is shown that, when adaptation is employed, there is an increase in the average number of successful transmissions in the network when the CRs select sensing orders from a predefined Latin Square, as compared to when they select sensing orders from the space of all permutations of \( N \) channels. A Latin Square is an \( N \) by \( N \) matrix of \( N \) channel indices in which every channel index occurs exactly once in each row and column of the matrix [50, 51]. Closed-form expressions are derived for the probability of success (the probability that a given CR finds a channel free) for CRs competing for opportunistic use of channels. These expressions are derived for a general number \( M \) of distributed CRs competing for these channels. Results in the literature typically do not consider distributed decisions without assumptions regarding a priori knowledge of PU activity statistics or knowledge of channel gains. Surprisingly, it is observed that when no adaptation is employed, a non-zero probability of false alarm can actually increase the probability that a CR successfully finds a channel to transmit in. This counter-intuitive result stems from a reduced number of collisions among the autonomous CRs, as further explained in Chapter 2. To validate the closed-form expressions that were derived, these results are compared to results obtained via simulations.

2) The spectrum sharing problem in an interference channel is analyzed as a dynamic
coalition formation game under which distributed wireless links with partial channel knowledge coordinate to form coalitions to maximize their data rates, while also taking into account the overhead in message exchange needed for coalition formation. It is shown that a coalition game in an interference channel is a game that generates positive externalities and has partition function form, then its conversion to characteristic function form is presented. This thesis employs an absorbing Markov chain model to model the equilibrium state of the grand coalition or the equilibrium state of internally and externally stable coalition structures for the coalition game in an interference channel. Using a Markovian model of the coalition game, the dynamics of the coalition formation game and the stability of different coalition structures in an interference channel are analyzed. The mean and variance of the time for the game to reach the stable coalition structures (network partitions) are also studied. Using simulation it is shown that the proposed coalition formation yields significant gains in terms of average rates per link for different network sizes as compared to a noncooperative solution.

3) A coalitional game theoretic framework is applied to the study of stable network partitions for the problem of distributed throughput-efficient sensing, under the constraint of probability of primary user detection, in cognitive radio networks. Distributed cooperative strategies are devised for CRs that are either selfish or altruistic. It is demonstrated that when each CR is assigned the same target detection probability in a coalition then for small to moderate values of sensing reporting time, the altruistic solution yields significant gains in terms of average throughput per CR as compared to the selfish and non-cooperative solutions. A weighted target detection probability for individual CRs in a coalition is then adopted. It is observed that the weighted target detection probability for individual CRs results in higher average throughput as compared to when each CR is assigned the same individual target detection probability in a coalition. It is also observed that when CRs are assigned weighted target detection probabilities in a coalition then the altruistic coalition formation solution significantly increases the average throughput per CR, as compared to a noncooperative solution, selfish coalition formation, and the grand coalition of all CRs.

4) The problem of best detection performance CR selection in cognitive radio networks is considered, where each CR performs spectrum sensing and sends its sensing report to a data collector known as the fusion center. Three methods are proposed for selecting the CRs with the best detection performance, based only on hard (binary) local sensing decisions from the CRs. Simulations are used to evaluate and compare the methods. The results indicate that the proposed CR selection methods are able to offer
significant gains in terms of system performance.

1.6 Author’s contribution and thesis structure

Most of the results in this thesis have been published or are under consideration for publication in the international conferences [37, 52], journals [38, 53–55] and book chapters [56]. The research work published in the listed articles was conducted within three years and the author had the main responsibility in developing the original ideas as well as writing the articles [37, 38, 52–56]. The analytical and simulation results presented in the articles [37, 38, 52–56] have been produced by the author under the supervision of Prof. Luiz A. Dasilva, Dr. Janne Lehtomäki and Prof. Matti Latva-aho.

When designing efficient resource utilization techniques for wireless networks, the following question must be considered: Suppose that two or more devices operating in the same channel simultaneously decide to transmit on the channel, what happens to their transmissions at their receivers? In other words, one must consider how interference from other devices disturbs the reception of a given desired signal (or message). In Chapter 2 of this thesis, the collision model is considered, where two or more simultaneous transmissions in the same channel result in a collision (loss of communications). Chapter 2 discusses adaptive persistent sensing order selection strategies for autonomous devices in a distributed cognitive radio network. It describes the motivation for the design of sensing order selection strategies, their analysis and comparison with related strategies proposed in other works.

In Chapter 3, additive interference model is considered, where two or more simultaneous transmissions in the same channel result in additional interference at the receivers, but not a loss of communications. In Chapters 3, tools from coalition formation game theory are adopted to devise dynamic cooperative strategies for efficient spectrum utilization in distributed networks. This chapter explains the motivation for the design of selfish cooperative strategies for efficient spectrum sharing and also analyzes the performance of the proposed strategies in terms of average throughput per device.

Chapters 4 and 5 consider the problem of cooperative spectrum sensing in cognitive radio networks. In Chapter 4, the formation of selfish and altruistic coalitions for distributed spectrum sensing is explored. After coalitions are formed in Chapter 4, it may be desirable for only a subset of each coalition to actually sense, as in Chapter 5. This would decrease the sensing overhead cost and total energy consumption of large coalitions. In Chapter 5, it is assumed that all nodes are willing to cooperate for
(centralized) cooperative detection, but the goal is to choose the best subset for actual
detection. Finally, Chapter 6 summarizes main conclusions of this thesis and outlines
some directions for future work.


2 Efficient channel access for autonomous devices in a multichannel cognitive network

It is better to be satisfied with probabilities than to demand impossibilities and starve.

Schiller, Friedrich
Quoted in Rudolf Flesch’s
The New Book of Unusual Quotations

In this chapter, a distributed time slotted CR network with multiple potentially available frequency channels is considered. The CRs use the beginning of each time slot to sense the channels sequentially in some order, $\mathcal{O}$, to find a free channel to transmit on, if one exists. To visit the potential channels sequentially, each CR autonomously selects a sensing order, $\mathcal{O}$, from the set of available sensing orders and accesses the first vacant channel it finds, if one is found. When two or more autonomous CRs simultaneously sense the same channel, find it free from primary user activity, and transmit data on the channel at the same time, a collision occurs. In this context, the common goal of the CRs is to autonomously select those sensing orders in which the likelihood of collisions among the CRs is reduced.

2.1 Background

In the classic dispersion problem [57], there are $M$ players that prefer to be more dispersed over $A$ actions. The problem is solved by Alpern in 2002 for $M = A$ under a basic simple strategy, in which, initially, each player independently and randomly selects an action. In the next rounds, a player selects a new action uniformly at random (from the set of those actions where the number of players selecting an action is not one) only if it has experienced a collision (if some other player also choos the same action as that player) in the previous round; otherwise, it retains the previously selected action.

Grenager et al in [58] have extended the basic simple strategy to work in the scenarios where $M \neq A$. They proposed an extended simple strategy, where the player (after experiencing a collision) does not assigns uniform probabilities to all actions where the number of players selecting an action is not one. The works in [57, 58] assume that the players have perfect collision observations and also the proposed strategies require that
each player knows the number of players selecting each action in a given round.

This thesis models and evaluates the opportunistic channel access problem in distributed multichannel cognitive radio networks as a sensing order dispersion problem in which the agents are the autonomous CRs, the possible actions are the selection of the sensing orders, and the equilibrium state of the network is the independent arrival of CRs at collision-free sensing orders. However, the basic simple and extended simple strategies proposed in [57, 58] cannot be applied straightforwardly to the sensing order dispersion problem in distributed CR networks due to the following reasons: 1) These schemes employ adaptive randomization based on occurrence of collisions for the players to maximally dispersed over the finite set of available actions, but do not consider the possibility of errors in collision detection; 2) The proposed strategies require that each player knows the number of players selecting each action in a given round.

In practice, in wireless cognitive radio networks collision detection and sensing errors are inevitable. Moreover, when there are multiple autonomous CRs operating in the network and there is no information exchange among them, the CRs do not have the knowledge of number of players selecting each action in a given round.

2.2 Related literature

Adaptation and learning for efficient channel access in multichannel cognitive radio networks is an active research topic, see, for example, the works published in [26, 34–36]. Different from previous works [26, 34–36, 59, 60], where adaptive channel allocation strategies were proposed for CRs employing a single-channel sensing policy, in this thesis, an adaptive sensing order selection strategy for autonomous CRs employing a sequential-channel sensing policy is proposed. The research presented in this thesis also achieves an improvement on the convergence of adaptation methods proposed in [26, 34, 57, 58] by proposing an adaptive persistent strategy, in which a CR maintains information regarding past successes (and failures) in using a particular sensing order. In addition, unlike [26, 34], this research also explores the impact of imperfect information, i.e., the effects of sensing and channel errors, on the proposed adaptation methods. Moreover, the works in [26, 34, 60] assume that the CRs are able to accurately estimate channel availability statistics. The proposed adaptation methods here do not rely on the CRs being aware of the channel availability statistics, which in practice may be difficult to estimate within the required time scale.

Several optimal policies for the selection of channel sensing orders for the sequential
channel sensing model are proposed in the literature. The works in [27, 61, 62] propose optimal policies for the selection of a channel sensing order $O$ for a single CR with perfect sensing observations. Unlike [27, 61, 62], this research takes into account competition for channels among multiple CRs with imperfect sensing observations. The selection of an optimal sensing order by a coordinator for a two-CR network is the topic of [24]. The coordinator determines the sensing orders to be adopted by the two CRs based on the estimated channel availability statistics and announces these sensing orders to the two CRs. While the two-user CR network with a coordinator is simple to implement, in practice a network comprising a large number of CRs would require significant signaling overhead to coordinate successful channel utilization. Moreover, in some practical scenarios, the CRs may be owned and managed by different service providers, requiring a sensing order selection strategy that does not rely on a common coordinator. A channel sensing order policy for distributed CR networks is proposed in [25]. However, this work assumes that CRs have knowledge of the gains for each channel. Based on this assumption, [25] proposes that each CR should sense channels in descending order of their achievable rates and should transmit in the first channel that is sensed free. Unlike [25], this research does not assume knowledge of the channel gains. Moreover, in contrast with [25], it also proposes and analyzes adaptive sensing order selection strategies to minimize conflicts among CRs when accessing available channels.

The works in [63–65] propose learning-based medium access control (MAC) techniques that discover collision-free schedules. However, since the proposed techniques are designed for traditional Time Division Multiple Access (TDMA) based MACs, these schemes do not consider the possibility that a channel may not be available (due to the presence of a primary user) and that the transmitter must perform sensing before transmission to determine which channels are available. Therefore, the techniques proposed in [63–65] cannot be applied straightforwardly to distributed CR networks.

The radio rendezvous problem studied by [30] is, in a sense, the dual of the problem studied here. While [30] proposes the use of non-orthogonal sequences to increase the probability of rendezvous, in this thesis methods are devised that increase the likelihood that CRs will independently arrive at collision-free sensing orders.

### 2.3 System model

A network of $M$ autonomous CRs and a set $N = \{1, 2, \ldots, N\}$ of channels is considered. A CR is allowed to make use of one of these channels when the channel is not occupied.
by a primary user. The primary users and CRs are both assumed to use a time slotted system, and each primary user is either present for the entire time slot, or absent for the entire time slot [23, 26, 35]. Due to hardware constraints, at any given time each CR can either sense or transmit, but not both. Also, each CR can sense only one channel at a time. To protect transmissions by the incumbent, the probability that a CR can correctly detect the presence of a primary user (PU) must be close to 1 [44]. Hence, the effect of varying probabilities of false alarm is considered, while considering the probability of missed detection to be zero. The probability of a PU being present and the probability of a PU being absent in a given channel are assumed to be unknown to the CRs.

The proposed methods are investigated under two different PU activity models: 1) In each channel, the probability of the PU being present in a given time slot is \( \theta_i, i \in \mathbb{N} \). In this model, for each channel, the PU activity in a time slot is independent of the PU activity in other time slots and is also independent of the PU activity in other channels; this (i.i.d.) model of PU channel occupancy is also adopted by [23, 25, 34]. 2) The second model considers correlation in channel occupancy by a PU in consecutive time slots. In this model, the state of each channel is described by a two-state Markov chain, with \( \alpha_{10,i} \) indicating the transition probability for the \( i \)th channel from PU-occupied to PU-free and \( \beta_{01,i} \) indicating the transition probability from PU-free to PU-occupied. This PU activity model is also adopted in [66].

As illustrated in Fig. 7, the CRs use the beginning of each slot to sense the channels sequentially in some order \( \mathcal{O} \) (based on their sensing order selection strategies, as

![Fig 7. Time slot structure with sensing and data transmission stages. a) If a CR finds a channel free in its \( i \)th sensing step, it transmits in that channel until the end of the slot. b) If in all sensing steps channels are sensed to be busy then the CR stays silent for the entire duration of that time slot.](image-url)
explained in Sections 2.4, 2.6 and 2.7) to find a channel that is free of PU (or other CR) activity. This is referred as the sensing stage. The CR then accesses the first vacant channel it finds. The sensing stage in each slot is divided into a number of sensing steps. Each sensing step is used by a CR to sense a different channel. If a CR finds a channel free in its $i$th sensing step, it transmits in that channel. This is referred as the data transmission stage. However, if in all sensing steps channels are found to be busy, then the CR stays silent for the remaining duration of that time slot (see Fig. 7). When a free channel is found in the $i$th sensing step, the durations of the sensing stage and data transmission stage are $iT_{\text{sense}}$ and $T - iT_{\text{sense}}$, respectively, where $T_{\text{sense}}$ is the time required to sense each channel, $T$ is the total duration of each slot and $T \gg T_{\text{sense}}$. When multiple autonomous CRs search multiple potentially available channels for spectrum opportunities, then from an individual CR perspective one of the following three events will happen in each sensing step: 1) The CR visits a given channel and is the only one to find it free and transmit; the CR then has the channel for itself for the remainder of the time slot; 2) The CR visits a given channel, finds it occupied by the PU or by another CR, then it continues looking in the next sensing step; 3) The CR visits a given channel, finds it free and transmits, but so does at least one other CR; a collision occurs and the CR is not able to transmit until the next time slot, when it again will contend for a channel. Note that a false alarm would have the effect of a CR thinking a channel is busy when it is in fact free of both PU and other CR activity. Fig. 8 illustrates examples of different scenarios for sequential channel sensing using sensing orders.

Let $X_s$ be a random variable representing the number of sensing steps within a time slot until a CR is successful in finding a channel free from PU and other CR activity (given that the CR is successful). With a constant time slot of duration $T$, the duration of successful data transmission in each slot is a function of $X_s$. At the end of each slot, the instantaneous throughput $C(X_s)$ achieved by a CR is given by

$$C(X_s) = \begin{cases} (1 - \frac{X_sT_{\text{sense}}}{T})R, & \text{if the CR successfully transmits} \\ 0, & \text{otherwise,} \end{cases}$$

where $R$ represents the transmission rate of a CR to its receiver. Note that if $N$ is larger than $T/T_{\text{sense}}$, then the CR does not have time to visit all channels within a time slot. However, for simplicity and also for practical reasons, it is assume throughout the paper that $N < T/T_{\text{sense}}$.

In this chapter, we are interested in sensing order selection strategies that maximize the average number of successful transmissions in the distributed CR network under the
means channel occupied by a PU
means two or more CRs find the same channel free
means only one CR finds a channel free
means false alarm is generated by a CR

<table>
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<th>CR 1 sensing order</th>
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<th>CR 3 sensing order</th>
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Scenario a) CR 1 and 2 collide, as they both choose the same sensing order and both find channel 1 free in step 2. CR 3 finds channel 4 free in step 2.

Scenario b) CR 1 and 3 collide, as they choose sensing orders with the same PU-free channel index in step 1 and both find channel 4 free in step 1. CR 2 finds channel 1 free in step 3.

Scenario c) CR 1 and 2 choose the same sensing order but they avoid collision as CR 1 finds channel 1 free in step 1 and CR 2 generates a false alarm in step 1 and finds channel 3 free in step 3. However, CR 3 finds channels 3 and 1 busy in steps 4 and 5, as other CRs found those channels free in the earlier steps.

Scenario d) CR 1 finds channel 1 free in step 1. CR 2 finds channel 4 free in step 2 and CR 3 finds channel 2 free in step 4 after finding channels 1 and 4 busy (which were found free by CR 1 and 2 in earlier steps).

Fig 8. Different scenarios for sequential channel sensing using sensing orders.

PU protection constraint. Note that a successful transmission in a given slot means that a CR finds a channel free from PU activity and is the sole CR to transmit in that channel.

2.4 Adaptive persistent sensing order selection

When autonomous CRs have to search multiple potentially available channels for spectrum opportunities, they face competition from one another to access the channel. The end result of this competition is reduced CR throughput due to collisions among CRs that transmit in the same channel. We are interested in finding a way for distributed CRs to autonomously reach collision-free sensing orders. Collision-free sensing orders are those in which two or more CRs never simultaneously sense the same channels and therefore never collide with one another. In this section, an adaptive persistent sensing
Fig 9. Examples of sensing orders for $N = 4$ potentially available channels. In a), the space of all permutations of $N = 4$ channels is illustrated, while in b), a subset of that space (a Latin Square) is illustrated. A sensing order is a row of a Latin Square or a row of a permutation space of $N$ channel indices.

An order selection strategy is proposed that enables the CRs to reduce the likelihood of collisions. Moreover, the proposed adaptive strategy allow the CRs in a distributed cognitive network to autonomously arrive at collision-free channel sensing orders (provided that the number of CRs is less than or equal to the number of potentially available channels).

Let $O$ denote the set of available sensing orders. Note that the sensing order that a CR employs can either come from the space of all permutations of $N$ channel indices (see Fig. 9), or from some subset thereof. In this section, an approach is proposed in which a sensing order comes from a common pre-defined Latin Square (LS), i.e., an $N$ by $N$ matrix of $N$ channel indices in which every channel index occurs exactly once in each row and column of the matrix [50, 51]. An example of a Latin Square is illustrated in Fig. 9. In Section 2.8, the performance of adaptive and non-adaptive sensing order selection strategies for two different scenarios are compared: 1) Each CR selects its sensing orders from a common predefined Latin Square; 2) Each CR selects its sensing orders from the space of all permutations of $N$ channel indices.
\( \gamma \)-persistent strategy

1) Initialize \( p = \left[ \frac{1}{\gamma}, \frac{1}{\gamma}, \ldots, \frac{1}{\gamma} \right] \) and set the binary flag and the success counter \( b = SC = 0 \).

2) Toss a weighted coin to select a sensing order, with \( p_i \) the probability of choosing sensing order \( i \). Sense the channels sequentially in the order given in the selected sensing order.

3) One of three possibilities occurs:
   a) **Successful transmission:** On a successful transmission using the current sensing order \( i \), the CR updates \( p_i \) and \( p_j \) as \( p_i = 1 \) and \( p_j = 0, \forall j \neq i \), i.e., it utilizes the same sensing order to visit the channels in the next slot. The CR then sets \( SC = SC + 1 \).
   b) **CR finds all channels busy:** On using sensing order \( i \) in the current slot if all the channels visited by the CR are found to be currently occupied by either a PU or another CR, the CR updates \( p_i \) and \( p_j \) as \( p_i = 1 \) and \( p_j = 0, \forall j \neq i \), i.e., it utilizes the same sensing order to visit the channels in the next slot. The CR then sets \( b = 1 \).
   c) **CR collides with another CR:** On experiencing a collision in the current slot using sensing order \( i \), the CR updates \( p_i \) as

   \[
   p_i = \begin{cases} 
   \frac{1}{\gamma} & \text{if } SC = 0 \text{ and } b = 1 \\
   \gamma p_i & \text{otherwise}
   \end{cases}
   \]

   and updates \( p_j \) as

   \[
   p_j = \begin{cases} 
   \frac{1}{\gamma} & \text{if } SC = 0 \text{ and } b = 1 \\
   \gamma p_j + \frac{1-\gamma}{\gamma} & \text{otherwise}
   \end{cases}
   \]

   i.e., on experiencing a collision in the current slot the CR randomly selects a sensing order whenever \( SC = 0 \) and \( b = 1 \); otherwise the CR multiplicatively decreases the probability of picking sensing order \( i \), redistributing the probability evenly across the other sensing orders. The CR then sets \( SC = b = 0 \).

4) Return to 2.

Fig 10. \( \gamma \)-persistent strategy for sensing order selection.
2.4.1 γ-persistent sensing order selection

A γ-persistent strategy is proposed, in which a CR uses a success counter (SC) and a binary flag b that track its successes and failures in using the current sensing order in prior time slots. The proposed strategy has a parameter γ, the persistence factor. Two different approaches for the use of γ are proposed: 1) An autonomous CR employs fixed values of γ ∈ (0, 1); 2) An autonomous CR employs γ = 1 − \( \frac{1}{\text{SC} - \log_2(P_{fa})} \) (which takes into consideration its false alarm probability and success counter (SC)). Note that the latter approach assumes that an autonomous CR can estimate its false alarm probability.

In the following subsections reasonable values for γ will be identified, when fixed values of γ are used, and will also explain the motivation behind the proposed γ as a function of false alarm probability and SC.

Let each CR maintains an \( |O| \)-element probability vector \( \mathbf{p} \) (i.e., all components are non-negative and add to 1) and let \( p_i \) represent the probability of selecting the \( i^{th} \) sensing order (which may come from either the space of all permutations of \( N \) channel indices, or from a Latin Square). The γ-persistent strategy is described in Fig. 10. The detection of a collision is implemented in the following way. A CR infers that a collision has occurred whenever it fails to receive an acknowledgement (ACK) for a transmitted data frame.

In order to understand the motivation for the γ-persistent strategy, let us consider what happens when CRs select sensing orders from a common pre-defined Latin Square. Note that, using a Latin Square, \( |O| = N \) and two or more CRs will collide only if they select the same sensing order.

Rationale of the equations in step 3c for updating \( p_i \) and \( p_j \):

Step 3c) considers the possibility when a CR collides with another CR in the current time slot. On experiencing a collision in the current time slot the success counter SC and the binary flag b of the CR can be in one of the following states:

i) \( SC > 0 \) and \( b = 0 \); ii) \( SC > 0 \) and \( b = 1 \); iii) \( SC = 0 \) and \( b = 0 \); and iv) \( SC = 0 \) and \( b = 1 \)

The rationale for the update of \( p_i \) and \( p_j \) after collision when \( SC > 0 \) and \( b = 0 \) or \( b = 1 \) is as follows:

An autonomous CR cannot determine on its own that the sensing order it has selected was not also selected by any other CR. However, a successful transmission \( SC > 0 \)
indicates a high probability that the CR is the sole user of that sensing order in time slot \( n \). After experiencing a collision, a CR persists with the sensing order \( i \) with probability \( \gamma p_i \), redistributing the probability \((1 - \gamma p_i)\) evenly across the other sensing orders. This persistence improves the speed of convergence to collision-free sensing orders as successful CRs tend to stick with their sensing orders, reducing the number of CRs randomly selecting a sensing order.

The rationale for the update of \( p_i \) and \( p_j \) after collision when \( SC = 0 \) and \( b = 0 \) is as follows:

The state \( SC = 0 \) and \( b = 0 \) indicates that, before experiencing a collision in the current time slot, the CR was neither successful nor found all channels busy using the sensing order \( i \). Therefore, the CR decreases the probability of selecting that sensing order (as another CR may have been successful before and may stick with that sensing order) in the next time slot, and redistributing the probability \((1 - \gamma p_i)\) evenly across the other sensing orders.

The rationale for the update of \( p_i \) and \( p_j \) after collision when \( SC = 0 \) and \( b = 1 \) is as follows:

\( SC = 0 \) and \( b = 1 \) means that using a sensing order \( i \) the CR \( k \) found all channels busy in time slot \( n \). Since the CR \( k \) stays quiet, it cannot be sure whether it was the sole user of the sensing order in time slot \( n \) (as it may happen that one or more other autonomous CRs also selected the same sensing order and found all channels busy or they found a channel free and transmitted). Since the CR \( k \) cannot be sure whether it was the sole user of the sensing order in time slot \( n \) (since \( SC = 0 \)), after a collision CR \( k \) will select a sensing order independently and randomly (with uniform probability).

The rationale for setting \( SC = 0 \) and \( b = 0 \) after a collision is as follows:

Let us assume that before experiencing a collision a CR was successful using a sensing order \( i \), i.e., its success counter \( SC \) is set to greater than 0. After experiencing a collision, in the next time slot it may happen that the CR selects with probability \( p_j \) some other sensing order \( j, j \neq i \). In this scenario if \( SC > 0 \) then the CR will incorrectly believe that it was successful using the sensing order \( j \) in the previous time slots. Similarly, if the binary flag \( b \) of the CR was set to 1 before collision and the CR selects with probability \( p_j \) some other sensing order \( j, j \neq i \), then the CR will incorrectly believe that it was unable to find a free channel using the sensing order \( j \) in the previous time slots. To avoid this, the CR after experiencing a collision resets the values of \( SC \) and \( b \) to 0 (see step 3c of the \( \gamma \)-persistent strategy).

Choosing the persistence factor \( \gamma \):
Fig 11. $E[TTC]$ as a function of: a) Number of CRs $M$ (with false alarm probability of each CR set to 0.2); b) False alarm probability (with $M = 10$ CRs). The number of channels ($N$) is 10 and the probability of the PU being present in a given time slot (i.i.d model) is $\theta_i = 0.3$, $\forall i \in N$; in all cases, selected sensing orders come from a Latin Square.
Table 1. E[TTC] for the γ-persistent strategy under i.i.d and Markov PU occupancy models, when \( N = M = 10 \). For the i.i.d PU occupancy model \( \theta_i = 0.3 \), and for the Markov model \( \alpha_{10,j} = 0.3, \beta_{01,j} = 0.12857 \) (see Section 2.3), \( \forall i \in N \).

<table>
<thead>
<tr>
<th>( \gamma )-persistent strategy</th>
<th>( P_{fa} = 0 )</th>
<th>( P_{fa} = 0.1 )</th>
<th>( P_{fa} = 0.2 )</th>
<th>( P_{fa} = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d ( \gamma = 0.5 )</td>
<td>87.5344</td>
<td>91.5180</td>
<td>100.5057</td>
<td>114.8016</td>
</tr>
<tr>
<td>Markov ( \gamma = 0.5 )</td>
<td>93.8481</td>
<td>95.1142</td>
<td>103.3093</td>
<td>113.9485</td>
</tr>
<tr>
<td>i.i.d ( \gamma = 0.6 )</td>
<td>26.8745</td>
<td>37.6155</td>
<td>49.7749</td>
<td>67.0764</td>
</tr>
<tr>
<td>Markov ( \gamma = 0.6 )</td>
<td>29.5679</td>
<td>38.1646</td>
<td>51.5461</td>
<td>68.1112</td>
</tr>
<tr>
<td>i.i.d ( \gamma = 0.9 )</td>
<td>23.1055</td>
<td>35.7700</td>
<td>51.2426</td>
<td>71.5800</td>
</tr>
<tr>
<td>Markov ( \gamma = 0.9 )</td>
<td>25.5543</td>
<td>36.3335</td>
<td>50.1692</td>
<td>72.0425</td>
</tr>
<tr>
<td>i.i.d ( \gamma = 0.95 )</td>
<td>22.2857</td>
<td>37.3654</td>
<td>55.8723</td>
<td>82.7012</td>
</tr>
<tr>
<td>Markov ( \gamma = 0.95 )</td>
<td>24.9660</td>
<td>39.0157</td>
<td>56.4288</td>
<td>80.6743</td>
</tr>
</tbody>
</table>

Fig 12. E[TTC] as a function of varying the probability of the PU being present in a channel in a given time slot (i.i.d model), where \( \theta_i, \forall i \in N \). The number of channels (\( N \)) is 10, the number of CRs is \( M = 10 \) and the false alarm probability of each CR is set to 0.
We propose two different approaches for the use of $\gamma$: 1) An autonomous CR employs fixed values of $\gamma \in (0,1)$; 2) An autonomous CR employs $\gamma = 1 - \left( \frac{1}{\text{SC} - \log_2(P_{fa})} \right)$. Note that by presenting $\gamma$ as a function of the false alarm and the success counter (SC) the latter approach takes into consideration a CR’s false alarm probability and also indirectly (through the SC) the number of active CRs in the distributed network. While we make no claims as to the optimality of the selected persistence factor $\gamma$ for the second approach, we justify it as follows. Using the proposed $\gamma = 1 - \left( \frac{1}{\text{SC} - \log_2(P_{fa})} \right)$, a CR with low false alarm probability and a high number of successful transmissions (using a sensing order $i$), after experiencing a collision persists with that sensing order with high probability. However, if the CR has a high false alarm probability and a low number of successful transmissions using a sensing order $i$, after experiencing a collision it persists with that sensing order with low probability. In Figs. 11(a) and 11(b), we plot the expected time to arrive at collision-free sensing orders ($\mathbb{E}[\text{TTC}]$) of the proposed strategy as a function of the number of CRs $M$ and the false alarm probability. The two figures show that when the proposed strategy employs $\gamma$ as a function of false alarm probability and SC, it performs at least as well or better than the scenarios where fixed values of $\gamma$ are employed.

Table 1 lists the expected TTC for the $\gamma$-persistent strategy (using fixed values of $\gamma$) under the i.i.d and Markov PU occupancy models; in all cases, selected sensing orders come from a Latin Square. The results in Table 1 also indicate that the expected TTC for the proposed adaptive strategy is not strongly affected by different PU occupancy models. Fig. 12 evaluates the effect of varying the values of $\theta_i = \theta$, $\forall i \in \mathbb{N}$, i.e., the probability of the PU being present in a channel, on the performance of the proposed scheme. It can be seen in Fig. 12 that $\mathbb{E}[\text{TTC}]$ increases as the value of $\theta$ increases. This is due to the fact that the proposed scheme utilizes feedback (occurrence of successful transmissions or collisions) to arrive at collision-free sensing orders. High values of $\theta$ reduce the chances of successful transmissions or collisions between CRs (as the number of available free channels is reduced), which in turn require more number of time slots to arrive at collision-free sensing orders.
2.5 Convergence of the adaptive persistent sensing order selection strategies

In this section, the convergence of the proposed $\gamma$-persistent strategy is proved for the scenario when $N = M$ and $0 < P_{fa,i} < 0.5$. Note that when $N > M$ or $P_{fa,i} = 0$, or both, the convergence process is similar to the $N = M$ scenario but faster (see Fig. 11(a) and Fig. 11(b)).

**Theorem 2.5.1.** Under the $\gamma$-persistent strategy, when $N = M$ CRs select sensing orders from a common pre-defined Latin Square, then for any $\gamma \in (0, 1)$, the network converges to collision-free sensing orders.

**Proof.** Let $E$ represent the event that, in a given slot, the CRs autonomously arrive at collision-free sensing orders and $P_E$ the probability of this event. To show that the proposed $\gamma$-persistent strategy converges, a genie-aided modification of autonomous arrival at collision-free sensing orders is considered. In the genie-aided modification each CR employs a common pre-defined sensing order (row) $s$ of the Latin Square for sequential channel sensing. If all the $M$ CRs collide in the first free channel, then the genie instructs the CRs to select the sensing order $s$ with probability $\gamma$ and to select $j$ with probability $(1-\gamma)$, $j \in S, \forall j \neq s$; otherwise, it instructs the CRs to again use the sensing order $s$ in the next time slot. If all $M$ CRs select different sensing orders the genie notifies the CRs that they have arrived at collision-free sensing orders; otherwise, it instructs the CRs to use the sensing order $s$ in the next time slot. The process is repeated until the CRs arrive at collision-free sensing orders. The genie-aided modification is described in Fig. 13.

The probability that the $M$ CRs can arrive at collision-free sensing orders in a time...
slot using the genie aided scheme is given by

\[
P_{E,\text{genie}} = \left[ \sum_{i=1}^{N} \left\{ (1 - \theta_i) \prod_{j=1}^{M} (1 - P_{fa,j}) \prod_{k=1}^{i-1} \theta_k \right\} \right] \times \frac{1}{(N-1)!} \frac{N!}{(N-M)!}.
\]  

(2)

Let us now consider our proposed \(\gamma\)-persistent strategy, without a genie. Using the \(\gamma\)-persistent strategy, initially, \(M\) autonomous CRs select independently and randomly (with equal probability) a row of a common pre-defined Latin Square, and use the selected row as a sensing order. Using the \(\gamma\)-persistent strategy, the probability that the \(M\) autonomous CRs arrive at collision-free sensing orders in the initial time slot is given by:

\[
P_{E,1} = \left( \frac{1}{N} \right)^M \left[ \frac{N!}{(N-M)!} \right] = \frac{N!}{N^M}.
\]

Using the \(\gamma\)-persistent strategy, the probability that the \(M\) CRs arrive at collision-free sensing orders in the initial time slot is greater than \(P_{E,\text{genie}}\), i.e., \(P_{E,1} > P_{E,\text{genie}}\) due to the following reason. Considering \(\theta_i = 0, \forall i \in \mathbb{N}, P_{fa,j} = 0, \forall j \in \mathbb{M}\) and \(N = M\), \(P_{E,\text{genie}}\) is given by:

\[
P_{E,\text{genie}} = \frac{1}{(N-1)!} \frac{N!}{N^N}.
\]  

(3)

It is easy to see that in equation (3), \(P_{E,\text{genie}}\) as a function of \(\gamma\) is a continuous function. By taking the first derivative of equation (3) with respect to \(\gamma\) we have:

\[
\frac{d}{d\gamma} P_{E,\text{genie}} = \frac{N!}{(N-1)^N} \left[ (1 - \gamma)^{N-1} - \gamma(N-1)(1 - \gamma)^{N-2} \right].
\]

Solving \(\frac{d}{d\gamma} P_{E,\text{genie}} = 0\), we get \(\gamma = \frac{1}{N}\). We evaluate \(P_{E,\text{genie}}\) (given in equation 3) at \(\gamma = \frac{1}{N}\), 0 and 1, i.e., at the critical point and the boundary points. We get that \(P_{E,\text{genie}}(1/N) = \frac{N!}{N^N}\) and \(P_{E,\text{genie}}(0) = P_{E,\text{genie}}(1) = 0\). From this, we see that the absolute maximum is obtained at \(\gamma = 1/N\). By substituting \(\gamma = 1/N\) in equation (3), we get \(P_{E,\text{genie}} = P_{E,1}\). It is easy to see that for \(\theta_i > 0, P_{fa,j} > 0\) or both, \(P_{E,1} > P_{E,\text{genie}}\).

The probability that \(M\) CRs autonomously arrive at collision-free sensing orders in the later time slots is always greater than \(P_{E,\text{genie}}\) due to the following reason. The
genie-aided modification corresponds to a worst case situation in which $M$ CRs can only select the sensing orders randomly if they all collide in the first free channel of the sensing order $s$, and after collision, in the next time slot they select the sensing order $s$ with probability $\gamma$ and the sensing order $j$ with probability $\left(\frac{1-\gamma}{N-1}\right)$, $\forall j \in O, \forall j \neq s$. In contrast, according to our proposed strategy when all CRs select the same sensing order, they may collide in any of the $N$ sensing steps, and a colliding CR may select a sensing order either randomly with uniform probability or with probability $\gamma$ (depending on the state of its binary flag and success counter). Moreover, for the $\gamma$-persistent strategy, any CR not experiencing a collision does not select a new sensing order from the Latin Square. Hence, the probability that each CR selects a different sensing order in any time slot is higher than the genie-added scheme, since the CR retains its previous sensing order.

It can be seen from equation (2) that for the genie-aided modification, in any time slot (before convergence) $P_{E,\text{genie}} \in (0, 1)$, and as $n \to \infty$ this implies:

$$\lim_{n \to \infty} [1 - (1 - P_{E,\text{genie}})^n] = 1.$$  

Since in any time slot the probability for $M$ CRs to arrive autonomously at collision-free sensing orders ($P_{E}$) is greater than $P_{E,\text{genie}}$, the network converges to collision-free sensing orders. 

Next, we provide an upper bound on the expected convergence time of the $\gamma$-persistent strategy for the scenario where all $N = M$ channels are free from primary user activity and $M$ autonomous CRs with $P_{fa} = 0$.

**Proposition 2.5.1.** Consider a system where all $N$ channels are free from primary user activity and $M$ autonomous CRs with $P_{fa} = 0$ select sensing orders from an $N \times N$ Latin Square, where $N = M$. Then using the $\gamma$-persistent strategy the expected number of time slots until convergence to collision-free sensing orders ($E[\text{TTC}]$) is $O(N)$.

**Proof.** For the scenario where $P_{fa} = 0$, it is easy to see that $\gamma = 1 - \left(\frac{1}{N-\log_2(P_{fa})}\right)$ takes the value of $\gamma = 1$. This means that when a CR transmits successfully using a sensing order it persists with that sensing order with probability 1. The probability that a particular CR is the only one selecting a given sensing order is $((N-1)/N)^{N-1}$ and therefore the expected number of time slots until the CR is alone is $(N/(N-1))^{N-1}$. By linearity of expectation, the expected number of time slots for all CRs to arrive at collision-free sensing orders must be no more than $N^{N}/(N-1)^{N-1}$, and therefore $E[\text{TTC}]$ is $O(N)$. 

56
In Fig. 14, we plot the theoretical upper bound (presented in Proposition 2.5.1) and simulated E[TTC] as a function of $N$. Unfortunately, the primary user activity in the channels and $P_{fa} > 0$ may slow down the convergence process. However for $0 < \theta_i < 1$, through extensive simulations we find that E[TTC] is increased for the $\gamma$-persistent strategy but is still $O(N)$.

In the next section, the performance of the proposed $\gamma$-persistent strategy in terms of the average number of successful transmissions in the network is compared against: 1) a random sensing order selection strategy; and 2) a randomize after every collision adaptive strategy. These two sensing order selection strategies are explained in the subsequent sections.

### 2.6 Randomize after every collision (rand) sensing order selection

Anandkumar in [26] proposed a learning scheme that employs adaptive randomization based on feedback (occurrence of collisions) for the CRs to arrive at orthogonal channel selections. Similarly, in this section, a randomize after every collision (rand) strategy is analyzed that utilizes adaptive randomization based on feedback for the CRs to arrive...
Table 2. E[TTC] slots for different adaptive strategies. The number of channels \( N = 10 \), false alarm probability of each CR is set to 0.2 and the probability of the PU being present (i.i.d model) is \( \theta_i = 0.3, \forall i \in N \).

<table>
<thead>
<tr>
<th></th>
<th>( M = 4 ) CRs</th>
<th>( M = 6 ) CRs</th>
<th>( M = 10 ) CRs</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand</td>
<td>2.3207</td>
<td>5.8555</td>
<td>499.0901</td>
</tr>
<tr>
<td>( \gamma )-persistent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0.8 )</td>
<td>2.6747</td>
<td>5.9249</td>
<td>51.0014</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>2.3481</td>
<td>5.2689</td>
<td>100.2261</td>
</tr>
</tbody>
</table>

at collision-free sensing orders. In this strategy, initially each CR independently and randomly (with equal probability) selects a sensing order (which may come from either the space of all permutations of \( N \) channel indices, or from a Latin Square). In the next time slots, a CR randomly (with equal probability) selects a new sensing order only if it has experienced a collision in the previous slot; otherwise, it retains the previously selected sensing order. The basic idea is for CRs to randomize their sensing orders in a way that leads them to distributedly arrive at collision-free sensing orders.

In Table 2, the randomize after every collision (rand) strategy is compared with our proposed \( \gamma \)-persistent strategy in terms of E[TTC]; in both cases, selected sensing sequences come from a Latin Square (LS). Table 2 shows that the proposed \( \gamma \)-persistent strategy performs at least as well or significantly better than the rand strategy.

2.7 Random selection of sensing orders

In each time slot, a sensing order (which may come from either the space of all permutations of \( N \) channel indices, or from a Latin Square) is independently and randomly selected (with equal probability) by each CR and then channels are sensed by each CR according to its sensing order.

2.7.1 Probability of successful channel access

To evaluate the performance of the random sensing order selection strategy (using a Latin Square) in terms of the average number of successful transmissions in the distributed CR network, this research first derives the probability of success, i.e., the probability that a given CR finds a channel free from primary user and other CR activity within the time slot. An exact closed-form expression for the probability of success can
be derived for any $N$ when $M = 2$. For $M > 2$, obtaining an exact closed-form expression for the probability of success is challenging due to the combinatorial explosion in the number of ways that $M$ CRs can find channels free from PUs and other CRs. This research provides an approximation for the probability of success for any $N$ and $M > 2$.

a) For $M = 2$ CRs and for any $N$, an exact closed-form expression for the probability of success for an individual CR $i$ (assuming i.i.d PU occupancy model with $\theta_k = \theta$, $\forall k \in N$ and CRs employ a random sensing order selection strategy)

be derived for any $N$ when $M = 2$. For $M > 2$, obtaining an exact closed-form expression for the probability of success is challenging due to the combinatorial explosion in the number of ways that $M$ CRs can find channels free from PUs and other CRs. This research provides an approximation for the probability of success for any $N$ and $M > 2$.

In Fig. 15, the average number of successful transmissions in a given time slot in the network are plotted as a function of the false alarm probability, for $M = 2$ and different
values of $N$. The results given by the closed-form expression that is derived in (4) are compared with the calculated average number of successful transmissions from a Monte Carlo simulation. Observe that the values calculated from Monte-Carlo simulations agree perfectly with those obtained from equation (4).

b) For $M > 2$ CRs, the difficulty in deriving an exact closed-form expression is that, in any sensing step $k$, the number of other competing CRs depends on how many CRs were successful in previous steps (which in turn determines how many channels are available) and how many have collided. Hence, instead of presenting an exact closed-form expression an approximation is presented for the probability of success for an individual CR for any $N$ and $M > 2$. The approximation is given in (7). The derivation of equation (7) is given in Appendix 1.

\[
P(S) = \sum_{k=1}^{N} [P_a + P_b]
\]

\[
= \sum_{k=1}^{N} \left\{ (1 - \theta_k)(1 - P_{fa,i})P_{fa,j} \prod_{l=1}^{k-1} \left( \theta_l + (1 - \theta_l)P_{fa,i}P_{fa,j} \right) \right\}
\]

\[
+ \sum_{l=1}^{N-1} \left( (1 - \theta_l)(1 - P_{fa,i})P_{fa,j} \prod_{l=1}^{N-1} \left( \theta_l + (1 - \theta_l)P_{fa,i}P_{fa,j} \right) \right)
\]

\[
\times \sum_{(m=k+1)}^{N} \left\{ (1 - \theta_m)(1 - P_{fa,i}) \prod_{(n=k+1)}^{m-1} \left( \theta_n + (1 - \theta_n)P_{fa,i} \right) \right\},
\]

where $P_a = (\text{In the first } (k - 1) \text{ steps, the channels visited by CR } i \text{ are either occupied by a PU or CR } i \text{ has false alarmed, and in the } k \text{th step CR } i \text{ visits a PU-free channel and does not collide with competing CR } j)$ and $P_b = (\text{In the first } (k - 1) \text{ steps, the channels visited by CR } i \text{ are either occupied or CR } i \text{ has false alarmed and in the } k \text{th step CR } j \text{ has given up channel search, i.e., it has found a PU-free and CR } i \text{ has false alarmed, and in the } (k + 1) \text{th step CR } i \text{ finds a PU-free channel}).$
\[ P(D) = \frac{1}{(N-1)} \sum_{k=1}^{N} (P_a + P_b + P_c) \]

\[ = \frac{1}{(N-1)} \left[ \sum_{k=1}^{N} \left( \sum_{j=k}^{N} (1-\theta_j)(1-P_{fa,j}) \left( 1-\theta_m \right) \prod_{n=2}^{k-1} (\theta_n + (1-\theta_n)P_{fa,n}) \right)^{\frac{k-2}{m+1}} \right] \sum_{n=2}^{l-1} \left( \prod_{p=m+1}^{l-1} (\theta_p + (1-\theta_p)P_{fa,i}) \right) \left( \theta_1 + (1-\theta_1)P_{fa,i} \right) \]

\[ + \sum_{k=2}^{N} \left( \sum_{j=k}^{N} (1-\theta_j)(1-P_{fa,j}) \left( 1-\theta_m \right) \prod_{n=2}^{k-1} (\theta_n + (1-\theta_n)P_{fa,n}) \right)^{\frac{k-2}{m+1}} \right] \sum_{n=2}^{l-1} \left( \prod_{p=m+1}^{l-1} (\theta_p + (1-\theta_p)P_{fa,i}) \right) \left( \theta_1 + (1-\theta_1)P_{fa,i} \right) \]

(6)

where \( P_a \) = (In the first \((k-1)\) steps, the channels visited by CR \( i \) are either occupied or CR \( i \) has false alarmed and CR \( j \) has given up channel search, i.e., it has already found a PU-free and in the \( k \)th step CR \( i \) visits a PU-free channel.). \( P_b \) = (In the first \((k-1)\) steps, the channels visited by CR \( i \) are either occupied by a PU or CR \( i \) has false alarmed and CR \( j \) has not yet found a PU-free channel in one of these steps, and in the \( k \)th step CR \( i \) visits a PU-free channel.) and \( P_c \) = (In the first \((k-1)\) steps, the channels visited by CR \( i \) are either occupied by a PU or CR \( i \) has false alarmed, and in the \( k \)th step, CR \( i \) finds a PU-free channel (in these steps the probability of success of CR \( i \) is not affected by the competing CR \( j \).)

\[ P[N,M,\theta] \approx \left( \frac{N-1}{N} \right)^{M-1} Y_E \sum_{k=1}^{N} \left( 1-\theta_k \right)(1-P_{fa,j}) \left( 1-\theta_m \right) \prod_{j=1}^{k-1} (\theta_j + (1-\theta_j)P_{fa,i}) \]

\[ + \left( \frac{M-1}{N} \right) \left( \frac{N-1}{N} \right)^{M-2} P(S), \]

(7)

where \( P(S) \) is given by equation (5) and \( Y_E = [(N-1)[1-\frac{1}{(N-1)}]^{M-1}], (N-1)[1-\frac{1}{(N-1)}]^{M-1} \) is the expected number of sensing orders that are not selected by any CR in a given time slot.

In Figs. 16(a) and 16(b), the average number of successful transmissions in a given time slot in the network are plotted as a function of the false alarm probability, for \( M = 4,8 \) and different values of \( N \). The results given by the approximation that is derived in (7) are compared with the calculated average number of successful transmissions from a Monte Carlo simulation. Observe that the values calculated from Monte-Carlo
Fig 16. Analytical and simulation results for the average number of successful transmissions per slot in the $M$ CR network $(M \times P\{N, M, \theta\})$ as a function of false alarm probability for different number of channels $N$. $\theta_k = 0.3$, $\forall k \in N$ and CRs employ a random sensing order selection strategy.
Table 3. Optimal false alarm probability for different values of $\theta_1 = \theta_2$ when $N = M = 2$.

<table>
<thead>
<tr>
<th>$\theta_1 = \theta_2$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{fa}$</td>
<td>0.2884</td>
<td>0.2960</td>
<td>0.3025</td>
<td>0.3081</td>
<td>0.3131</td>
<td>0.3174</td>
<td>0.3213</td>
<td>0.3248</td>
<td>0.3279</td>
<td>0.3307</td>
</tr>
</tbody>
</table>

Simulations are within ±12% of those obtained from equation (7) and the approximation improves for larger values of $N$.

2.7.2 False alarm as p-persistent channel access

As shown in Fig. 11(b), when adaptive strategies are employed for sensing order selection, sensing observations with false alarms lead to slower convergence to collision-free sensing orders, which in turn decreases the average number of successful transmissions in the network. However, when no adaptation is employed, i.e., when CRs use a random sensing order selection strategy, sensing observations with some false alarm increases the average number of successful transmissions in the network, as compared to when sensing observations do not generate false alarms (see Figs. 15, 16(a) and 16(b)). This counter-intuitive result stems from reduced collisions among the autonomous CRs and is further explained as follows. When sensing observations do not generate false alarms, the probability of success for two or more CRs that select the same sensing order is zero, as two or more autonomous CRs that select the same sensing order will find the same channel free (if one exists), transmit in that channel and collision will occur. However, if two or more CRs (with non-zero probabilities of false alarms) select the same sensing order then a CR can be successful in finding a free channel if one of the CRs does not generate a false alarm in a PU-free channel and the other CRs generate false alarms in that channel. Note that this gain in the probability of success of a CR (due to the non-zero probability of false alarm) is only valid for some values of false alarm probability, as for high values of false alarm probability, the likelihood of finding a free channel for the CR is reduced. This observation is formalized for the case of two channels and two CRs, in the claim below.

Claim 2.7.1. Under the random sensing order selection strategy, when two autonomous CRs search two frequency channels sequentially with detection probabilities equal to 1, the success probabilities of both CRs are maximized at values of false alarm.
probabilities that are strictly greater than zero.

**Proof.** Let $\theta_1$ and $\theta_2$ be the probabilities of a PU being present in the two channels, where $\theta_1, \theta_2 < 1$, and $P_{fa,1} = P_{fa,2} = P_{fa}$ be the false alarm probabilities of the two CRs. Note that for the case where $P_{fa,1} \neq P_{fa,2}$, it is trivial to show that the success probability of an individual CR is maximized when its false alarm probability is zero and the false alarm probability of the other CR is one. However, this research considers the case where the success probabilities of both CRs are maximized. Using (5) and (6), the success probability of an individual CR is

$$P\{N = 2, M = 2, (\theta_1, \theta_2)\} = \frac{1}{4} \left[ (2 - \theta_1 - \theta_2)(1 - P_{fa}) + 2\{\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2)\}(1 - P_{fa})P_{fa} + 2(1 - \theta_1)(1 - \theta_2)\{(1 - P_{fa}^2)P_{fa}^2 + P_{fa}(1 - P_{fa})\} \right].$$

(8)

Equation (8) is twice-differentiable, because it is a polynomial. By taking the first and second derivatives of (8) with respect to $P_{fa}$, we have

$$\frac{d}{dP_{fa}} P = \frac{1}{4} \left[ (2 - \theta_1 - \theta_2)(-2P_{fa}) + 2\{\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2)\}(1 - 2P_{fa}) + 2(1 - \theta_1)(1 - \theta_2)\{1 - 2P_{fa} + 3P_{fa}^2 - 4P_{fa}^3\} \right],$$

(9)

and

$$\frac{d^2}{dP_{fa}^2} P = \frac{1}{4} \left[ -4 - 2\theta_1 - 2\theta_2 + 8\theta_1\theta_2 + 2(1 - \theta_1)(1 - \theta_2)\{1 + 6P_{fa} - 12P_{fa}^2\} \right] < 0, \forall P_{fa}$$

(10)

so $P$ is strictly concave $\forall \theta_1, \theta_2 < 1$. The optimal value holds when $\frac{d}{dP_{fa}} P = 0$. Solving $\frac{d}{dP_{fa}} P = 0$, we get $P_{fa}^* > 0$ (see Table 3), $\forall \theta_1, \theta_2 < 1$. Hence the success probabilities of both CRs are maximized at values of false alarm probabilities that are strictly greater than zero.

**2.8 Simulation Results**

We conduct simulations to compare the performance of the proposed $\gamma$-persistent strategy with other sensing order selection strategies in terms of $E[TTC]$ and the average
number of successful transmissions in a given time slot for different scenarios.

Figs. 17 and 18 evaluate the effect of varying the number of sensing steps on the performance of the proposed scheme in terms of $E[\text{TTC}]$ and the average number of successful transmissions ($M \times P\{N,M,\theta\}$). All simulated scenarios in Fig. 17 consider the case $NT_{\text{sensing}} > T$, i.e., CRs cannot sense all $N$ channels within the duration of a slot. In Fig. 18, except from one scenario where number of steps is 10, all scenarios consider the case $NT_{\text{sensing}} > T$. It can be seen in Figs. 17 and 18 that $E[\text{TTC}]$ decreases and the average number of successful transmissions increases as the number of sensing steps are increased. However, it can also be seen in Figs. 12 and 13 that for the considered scenarios there is little or no gain when more than 4 sensing steps are utilized by an autonomous CR.

In Fig. 19 we evaluate our proposed strategy for the scenario where autonomous CRs are able to sense only one channel in a given time slot. It can be seen that our proposed adaptive persistent strategy significantly outperforms the randomize after every collision (rand) adaptive strategy.

In Figs. 20(a) and 20(b), for $N = 16$ channels, the average number of successful transmissions achieved by the different strategies under two different scenarios, when
Fig 18. Simulation results for the average number of successful transmissions for the $\gamma$-persistent strategy ($\gamma = 1 - \left( \frac{1}{S_{\text{en}} + \log_2(P_{\text{fa}})} \right)$) in a given time slot for different strategies when $\theta_i = 0.3$, $\forall i \in N$, false alarm probability of each CR is set to 0.2, $N = 10$, and $M = 5$.

$M < N$ and when $M = N$, are compared. From the two figures it can be seen that the adaptive sensing order selection strategies achieve the highest average number of successful transmissions. The two figures (Figs. 20(a) and 20(b)) show that the proposed $\gamma$-persistent strategy performs at least as well or significantly better than all other sensing order selection strategies evaluated. Particularly for the scenario where $M = N$, the $\gamma$-persistent strategy significantly outperforms all the other strategies evaluated (see Fig. 20(b)). The two figures (20(a) and 20(b)) also compare the performance of the random sensing order selection strategy for two different scenarios, when selected sensing orders come from the space of all permutations of $N$ channels (RPS strategy), and when selected orders come from a Latin Square (LS strategy). It can be seen from the two figures (20(a) and 20(b)) that the average number of successful transmissions is greater for the RPS strategy than for the LS strategy. This is due to the fact that the RPS strategy increases the number of ways to visit $N$ channels and therefore reduces the chances of collisions between CRs. However, when adaptation is employed then the scenario where CRs select sensing orders from a common pre-defined Latin Square, i.e., randomize after every collision using a Latin Square (rand-LS) strategy, results in increased number of successful transmissions, as compared to the case where they select
Fig. 19. $E\{TTC\}$ as a function of varying the number of channels $N$ for the scenarios where $N = M$, $\theta_i = \theta = 0.5$, $\forall i \in N$, false alarm probability of each CR set to 0.2 and number of sensing steps $= 1$.

sensing orders from the space of all permutations of $N$ channels, i.e., randomize after every collision using all permutations (rand-AP) strategy, (see Fig. 20(a)).

In Fig. 21, for $N = 10$ channels, this research compares the average number of successful transmissions achieved in time slot, $n = 200$, as a function of number of CRs for different adaptive strategies. It can be seen from Fig. 21 that when $N = 10$ channels are available for opportunistic transmissions then for difficult scenarios, i.e., $N \leq M$, the proposed $\gamma$-persistent strategy maximizes the average number of successful transmissions as compared to the randomize after every collision (rand-LS) strategy; in all cases, selected sensing orders come from a Latin Square.

When $N < M$ it is not possible for all CRs to arrive at collision-free sensing orders. However, Fig. 21 shows that for $N < M$ our proposed strategy enables the CRs to reduce the likelihood of collisions. Fig. 21 also shows that, for the proposed $\gamma$-persistent strategy, when $N < M$, high values of $\gamma$, i.e., $\gamma = 0.9$ (high persistence) performs slightly better, as compared to when $\gamma$ is employed as a function of false alarm probability and SC. This is due to the fact that for $N < M$ there are more chances of collisions and high persistence enables successful CRs to stick with their sensing orders with high probability, which increases the probability of arriving at collision free sensing orders.

The presented adaptive sensing order selection strategies use transmission failure
Fig 20. Simulation results for the average number of successful transmissions in a given time slot \( M \times P[N, M, \theta] \) for different strategies when \( \theta_k = 0.3, \forall k \in \mathbb{N} \) and false alarm probability of each CR set to 0.2.
Fig 21. Simulation results for the average number of successful transmissions achieved in time slot, \( n = 200 \), as a function of number of CRs for different strategies when \( \theta_k = 0.3 \), \( \forall k \in \mathbb{N} \) and false alarm probability of each CR set to 0.2. The number of channels \( N \) is 10. Calculations are performed by Monte Carlo method using 30000 Monte Carlo runs for opportunistic transmissions in 200 time slots using different strategies.

as an indication that a collision has occurred, i.e., the selected sensing order is also used by another CR. In practice, transmission failures can also be due to channel errors introduced by fading. In Fig. 22, the performance of the adaptive sensing order selection strategies with and without channel errors are compared. This research utilizes a simple model where channel errors are introduced at a particular rate \( \sigma \). It can be seen in Fig. 22 that the performance of the proposed adaptive strategies is degraded due to channel errors (\( \sigma = 20\% \)). However, it can also be seen in Fig. 22 that for different values of \( \gamma \), the proposed \( \gamma \)-persistent strategy outperforms the \textit{randomize after every collision} (rand-LS) adaptive strategy; in all cases, selected sensing orders come from a Latin Square. It is observed in Fig. 22 that high persistence \( \gamma = 0.9 \) is more robust to the presence of channel errors. This is due to the fact that high persistence allows a successful CR to stick with its sensing order (after experiencing a collision) with high probability even after few consecutive transmission failures.
Fig 22. Simulation results for the average number of successful transmissions in a given time slot \((M \times P[N, M, \theta])\) for different strategies when \(\theta_k = 0.3, \forall k \in N\) and false alarm probability of each CR is set to 0.2. The number of channels \(N\) is 16 and the number of CRs \(M\) is 16.

### 2.9 Summary

In distributed cognitive radio networks, devices may have to search multiple potential channels for spectrum opportunities. These devices face competition from one another to access the channel. In this chapter, the problem of efficient sensing order selection strategies to maximize cognitive network throughput is considered. Efficient sensing order selection can be achieved by carrying out negotiation among distributed cognitive radios via a coordinator (base station) or with the help of a common control channel. While the use of a coordinator or a common control channel simplifies the problem, it may create significant signaling overhead or potential contention under heavy load.

Distributed sensing order selection strategies that can be pursued by autonomous cognitive radios have been discussed. An adaptive persistent sensing order selection strategy is proposed and it is shown that this strategy converges and reduces the likelihood of collisions among the autonomous CRs as compared to a random selection of sensing orders.
3 Dynamic coalition formation method for spectrum sharing in an interference channel

... for he who seeks for methods without having a definite problem in mind seeks for the most part in vain.

Bulletin of the American Mathematical Society
Hilbert: Mathematical Problems
Volume 8, July 1902 (p. 444)

In this chapter, a dynamic coalition formation game based on a Markovian model is formulated to analyze the spectrum sharing problem among $M$ wireless links (transmitter/receiver pairs), co-existing in an interference channel of bandwidth $W$. Our model is dynamic in the sense that distributed transmitter/receiver pairs, with partial channel knowledge, reach stable coalition structures (CSs) through a time-evolving sequence of steps. The transmitters on each of the $M$ links have the same power constraint $P$. The strategic decision for any link consists of an allocation of power $P$ across the available bandwidth to maximize the individual data rate. Coalitions are formed if some rational demands of wireless links are satisfied, with coalition participants agreeing to share (allocate power over) bandwidth $W$ according to certain ratios; otherwise, each link will form a singleton coalition (individual player) by allocating power $P$ over the entire spectrum band. A key question this chapter tries to address, is how to coordinate distributed wireless links with partial channel knowledge to form coalitions to maximize their data rates, while also taking into account the overhead in message exchange needed for coalition formation.

3.1 Background

The next generation of wireless networks will increasingly rely on distributed and self-configuring architectures. The deployment of such networks raises new research challenges related to the adaptation of the participating links to the changing network environment. Cooperation among distributed and self-configuring wireless network links may lead to the efficient use of network resources. To model cooperative scenarios, where distributed radios interact with one another to share spectrum or to fulfill the
task of spectrum sensing, this research considers the cooperative network as a dynamic coalition game, using concepts from coalition game theory.

Using the standard framework of coalition game theory, let $M$ denote the set of players playing the coalition game, $M = \{1, 2, 3, ..., M\}$. A coalition, $S$, is defined as a subset of $M$, $S \subseteq M$. An individual player is called a singleton coalition and the set $M$ is also a coalition, called the grand coalition (GC), where all players cooperate [40].

### 3.1.1 Coalition forms of $M$-person games

To understand the dynamic coalition formation methods presented in this thesis, it is useful to characterize two possible representations or forms of coalition games:

1. Characteristic function form (CFF); and
2. Partition function form (PFF);

The most common form of a coalition game is the characteristic function form. In the characteristic function form (CFF) of coalition games, utilities achieved by the players in a coalition are unaffected by those outside it.

**Definition 1.** The $M$-player coalitional game in characteristic function form (CFF) is given by the pair $\langle M, v \rangle$, where $M$ is the set of players and $v$ is a real-valued function, called the value of the game, defined on the subsets of $M$, with $v(\emptyset) = 0$.

The quantity $v(S)$ in the CFF game is a real number associated with each subset of $S \subset M$, which may be considered as the value, or worth, of a coalition when its members group together as a unit. Similarly, the quantity $v_{i \in S}(S)$ in the CFF game is a real number for each player $i \in S$, which represents the payoff obtained by link $i$ from coalition $S$. To model all $M$ player coalitions, this research defines coalition structures as follows:

**Definition 2.** A coalition structure (CS) is a partition of $M$ into exhaustive and disjoint subsets, where each subset is a coalition. The set of all possible CSs is denoted as $C$, where $C = \{C_1, C_2, ..., C_{|C|}\}$.

The cardinality of $C$, i.e., the number of all possible CSs (excluding empty coalitions) for $M$ players is given by the Bell number

$$B_0 = 1$$

$$B_M = \sum_{k=0}^{M-1} \binom{M-1}{k} B_k, \text{ for } M \geq 1. \quad (11)$$

72
For instance, $\mathbf{C}$ for $M = 3$ players is given as:

$$\mathbf{C} = \left\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\right\}.$$ 

Worth or value $V$ of a CS is defined as the sum of values of coalitions that are elements of that CS, i.e., $V(C_j) = \sum_{S_i \in C_j} v(S_i)$, for $j = 1, 2, ..., |C|.$

**Characteristic function form (CFF)**

The characteristic function assigns to every coalition a value which is the aggregate payoff that a coalition can secure for its members, irrespective of the behavior of players outside this coalition. An example inspired by [67] and modified to illustrate the concept of CFF of coalition games is next presented.

**Example 3.1.1.** Consider a three-player buyer-seller game in which player 1 (a seller) has a car which is worthless to her (unless she can sell it). Players 2 and 3 (buyers) value the car at $\$4000$ and $\$5000$, respectively.

If player 1 sells the car to player 2 at a price of $x$, she will effectively make a profit of $x$, while player 2’s profit is $4000 - x$. The total value (profit) of the coalition $\{1, 2\}$ is $v(\{1, 2\}) = 4000$. Similarly, $v(\{1, 3\}) = 5000$. However, a single player, or the two buyers together can obtain no profit. Thus, $v(\{1\}) = v(\{2, 3\}) = 0$. If some side payments which do not change the total amount of utility are allowed, the coalition of all three players, i.e., the grand coalition can do no better than $\{1, 3\}$. Thus, $v(\{1, 2, 3\}) = 5000$.

**Partition function form (PFF)**

A coalition game with externalities is a game in which the value that a group of players can achieve through cooperation depends on what other coalitions form [68]. Coalition games with externalities are described using the PFF. In partition function games, the real-valued function $v$ is a function of a coalition $S$ and a partition $\rho$. In other words, in partition function games, any coalition $S \subset M$ generates a value $v(S; \rho)$ where $\rho$ is a partition of $M$, with $S \in \rho$. This research defines $v(\emptyset; \rho) = 0$ for all partitions $\rho$ of $M$.

In partition function games, the worth of a coalition depends on the entire coalition structure, i.e., the partition of players inside and outside a coalition. One of the strengths of partition function games is that they take into account the impact of any positive or negative externality present in the network environment on coalition formation.
Fig 23. Different wireless users coexisting in the same area and interfering with each other.
In what follows, an example of wireless network environment is provided in which there are externalities from coalition formation. A scenario is modeled in which different wireless systems operating in the same band (for example, IEEE 802.11 [69] and Bluetooth systems [70]), each formed by a single transmitter-receiver pair, coexist in the same area. The received signal for any user $i$ in a Gaussian interference channel is given by:

$$Y_i = \sum_{j=1}^{M} h_{ji} X_j + z_i, \quad i \in M,$$

where $X_j$ and $Y_i$ are input and output signals, respectively. The noise processes are independent and identically distributed (i.i.d.), zero mean and unit variance Gaussian random variables, i.e., $z_i \sim \mathcal{N}(0, N_0)$, with $N_0 = 1$. $h_{ji}$ is the channel gain between the transmitter of user $j$ and the receiver of user $i$. The channel from each transmitter to each receiver is assumed to be flat fading. The user $i$ is assumed to have an average power constraint $P_i$. The transmission strategy for each user is the way that it allocates power in the given bandwidth. The user can either spread the power over the available bandwidth $W$ or it can allocate the same power in a segment of $W$. It also assumed that each wireless user treats multiuser interference as noise and no interference cancelation techniques are employed. In many spectrum sharing problems the issue of fair and efficient solutions arises due to asymmetries between the systems. Fig. 23 illustrates an example in which users 1 and 2 operate in symmetric situations while user 3 operates in an asymmetric situation. In Fig. 23, users 1 and 2, i.e., Tx1/Rx1 and Tx2/Rx2 with same power capabilities (for example, two IEEE 802.11 systems) share the same spectrum band and, due to the locations of the transmitters and receivers, both 1 and 2 strongly interfere with each other [71]. There is also a third user with a low power capability (for example, a Bluetooth system) sharing the same band with the two other users. Authors in [41] have shown that, for arbitrary SNR values, the users in the stable coalitions benefit from the exclusion of the weak interferer. In Fig. 23 all the channel gains are comparable, so intuitively if the three users decide to play a coalition game then the users 1 and 2 will prefer to form a coalition, i.e., share the spectrum with each other, rather than to form a coalition with the weak user. Assuming, the users 1 and 2, i.e., Tx1/Rx1 and Tx2/Rx2 decide to form a coalition $S = \{1, 2\}$ and share the spectrum bandwidth $W$, the value of $S$ can be expressed as:

$$v(S) = \sum_{i \in S} R_i = \sum_{i \in S} W \eta \log_2 \left( 1 + \frac{P_i}{\eta h_{ii}^2} \right), \quad S \subset M,$$

where $X_j$ and $Y_i$ are input and output signals, respectively. The noise processes are independent and identically distributed (i.i.d.), zero mean and unit variance Gaussian random variables, i.e., $z_i \sim \mathcal{N}(0, N_0)$, with $N_0 = 1$. $h_{ji}$ is the channel gain between the transmitter of user $j$ and the receiver of user $i$. The channel from each transmitter to each receiver is assumed to be flat fading. The user $i$ is assumed to have an average power constraint $P_i$. The transmission strategy for each user is the way that it allocates power in the given bandwidth. The user can either spread the power over the available bandwidth $W$ or it can allocate the same power in a segment of $W$. It also assumed that each wireless user treats multiuser interference as noise and no interference cancelation techniques are employed. In many spectrum sharing problems the issue of fair and efficient solutions arises due to asymmetries between the systems. Fig. 23 illustrates an example in which users 1 and 2 operate in symmetric situations while user 3 operates in an asymmetric situation. In Fig. 23, users 1 and 2, i.e., Tx1/Rx1 and Tx2/Rx2 with same power capabilities (for example, two IEEE 802.11 systems) share the same spectrum band and, due to the locations of the transmitters and receivers, both 1 and 2 strongly interfere with each other [71]. There is also a third user with a low power capability (for example, a Bluetooth system) sharing the same band with the two other users. Authors in [41] have shown that, for arbitrary SNR values, the users in the stable coalitions benefit from the exclusion of the weak interferer. In Fig. 23 all the channel gains are comparable, so intuitively if the three users decide to play a coalition game then the users 1 and 2 will prefer to form a coalition, i.e., share the spectrum with each other, rather than to form a coalition with the weak user. Assuming, the users 1 and 2, i.e., Tx1/Rx1 and Tx2/Rx2 decide to form a coalition $S = \{1, 2\}$ and share the spectrum bandwidth $W$, the value of $S$ can be expressed as:

$$v(S) = \sum_{i \in S} R_i = \sum_{i \in S} W \eta \log_2 \left( 1 + \frac{P_i}{\eta h_{ii}^2} \right), \quad S \subset M,$$
where $R_i$ is the data rate for user $i$, $h_{ii} = 1$, $M = \{1,2,3\}$, $h_{ij}^2$ represents the interference channel power gain to the members of coalition $S$, $0 \leq \eta_i \leq 1$ is the fraction of the band that user $i$ uses, and $\sum_{i \in S} \eta_i \leq 1$.

To illustrate the above situation, an example for the three users of Fig. 23 can be constructed.

**Example 3.1.2.** Consider for example a three-user scenario with bandwidth $W = 2$, $N_0 = 1$, link gain matrix $H = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$, given as: $H = \begin{pmatrix} 1 & 7 & 3 \\ 6 & 0 & 1 \\ 0 & 6 & 1 \end{pmatrix}$. $P_1 = P_2 = P$, with $P > N_0$ and $P_3 = P/5$. If all three users spread their powers through the entire band and prefer to form singleton coalitions then they obtain coalition values $v(\{1\}) = v(\{2\}) = \log(1 + \frac{P}{1+0.02P})$ and $v(\{3\}) = \log(1 + \frac{2P}{5(1+0.02P)})$. However, if the users 1 and 2 form a coalition $S$ and share the same spectrum band in a certain ratio $\sum_{i \in S} \eta_i \leq 1$ (since $h_{12} = h_{21} = .7$ it is natural to assume that both users share the band in equal ratios, i.e., $\eta_1 = \eta_2 = 0.5$), the value of coalition $S$ is: $v(S) = \sum_{i \in S} R_i = \log(1 + \frac{P}{1+0.02P})$ and $v(\{3\}) = \log(1 + \frac{5P}{3(1+0.02P)})$. In this scenario, the coalition value $v(\{3\})$ is not affected by the formation of new coalition $S$.

Consider now the scenario in which $H = \begin{pmatrix} 1 & 6 & 3 \\ 6 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$. If users 1 and 2 share the same spectrum band then it is natural to assume that the two share the spectrum in a certain ratio $\eta_2 > \eta_1$, as $h_{21} > h_{12}$. The value of coalition $S$ is: $v(S) = \eta_1 \log(1 + \frac{P}{1+0.02P}) + \eta_2 \log(1 + \frac{P}{1+0.02P})$ and $v(\{3\}) = \eta_1 \log(1 + \frac{P}{5(1+0.02P)}) + \eta_2 \log(1 + \frac{2P}{5(1+0.02P)})$. In this scenario, the coalition value $v(\{3\})$ is a function of $\eta_i$, i.e., the coalition value of the third system depends on the way users 1 and 2 allocate the spectrum, although the third user is not participating in the coalition formation.

In other words, the coalition formation game process in an interference channel generates externalities and the value of coalitions in an interference channel depends on a partition $\rho$ of $M$.

Broadly speaking, the presence of positive externalities, i.e., if players outside any coalition $S$ either do not lose or they gain by any player $i$ joining $S$, may provide an incentive for wireless nodes to free ride, resulting in small stable coalitions. On the other hand, negative externalities, i.e., if players outside any coalition $S$ lose by any player $i$ joining $S$, may provide an incentive to cooperate, resulting in large stable coalitions. Unfortunately, coalition games with externalities have PFF and such partition functions are difficult to analyze [41].
3.1.2 The core and the grand coalition

Most solution concepts related to coalition games analyze stable coalition structures. Stable coalition structures correspond to the equilibrium state in which players do not have incentives to leave the existing coalitions. In analyzing stable coalition structures for networks, it is essentially required to determine if the grand coalition (GC) is stable, and this can be done using the concept of core.

In the study of coalition games, a payoff vector $\mathbf{x}$ is said to be in the core if it satisfies the following two properties: 1) $\mathbf{x}$ is an imputation; 2) imputation $\mathbf{x}$ is stable [72].

**Definition 3.** An imputation $\mathbf{x}$ is a payoff vector that satisfies the following two conditions: 1) $\sum_{i \in M} x_i = v(M)$; 2) $x_i \geq v(\{i\}) \forall i$.

**Definition 4.** An imputation $\mathbf{x}$ is unstable if there is a coalition $S \subset M$ such that $\sum_{i \in S} x_i < v(S)$. Otherwise, $\mathbf{x}$ is said to be stable.

**Definition 5.** The set, $\gamma$, of stable imputations is called the core

$$\gamma = \{ \mathbf{x} : \sum_{i \in M} x_i = v(M) \text{ and } \sum_{i \in S} x_i \geq v(S), \text{ for all } S \subset M \}. \quad (14)$$

As already stated, in certain scenarios, self-interested players may prefer to cooperate with only a selected subset of users to achieve maximum gains. In such scenarios, the GC will not be formed. In such network scenarios, the network welfare maximizing coalition structure varies. The aim in such scenarios is to maximize the social welfare of $M$ by finding a coalition structure [73]:

$$C_j^* = \arg \max_{C_j \in \mathcal{C}} V(C_j). \quad (15)$$

For such scenarios, it can be said that any coalition has reached individual stability or equilibrium if it is internally and externally stable (IES).

**Definition 6.** Internal stability means that no player has an incentive to leave its coalition to become a singleton (individual non-cooperative player), i.e., $v_{i} \in S_1(S_1) \geq v(\{i\})$, $\forall i \in S_1$, and external stability means that no other coalition has an incentive to join coalition $S_1$, i.e., $v(S_2) > v(S_1 \cup S_2) - v(S_1), \forall S_2 \subseteq S_{1}^{c}$, where $S_{1}^{c}$ represents the complement of $S_1$.

If each coalition in a CS is IES, then the CS is called IES stable; this is also known as a multi-coalition equilibrium. For instance, if $M = 4$ players are participating in a
coalition game, then the CS \( \{\{1,2\},\{3\},\{4\}\} \) is IES stable CS if all the coalitions, i.e., \( \{1,2\},\{3\} \) and \( \{4\} \), satisfy the IES requirements. The CS that consists of all singleton coalitions, i.e., \( \{\{1\}\},\{\{2\}\},\{\{3\}\},...,\{\{M\}\}\) is stable if each singleton coalition satisfies \( v(\{i\}) > v_i, \forall i, \forall S = \{i\} \cup \{j\}, i \neq j \). This is known as the Nash Equilibrium with no cooperation. This research defines minimum rational payoff for an individual player in the coalition formation game as:

**Definition 7.** A payoff is a minimum rational payoff for player \( i \) in the coalition \( S \), if a payoff partition gives \( i \) at least as much value as it receives before forming the coalition \( S \). The minimum rational payoff for player \( i \) in coalition \( S \) is denoted by \( d_i(S), i \in S \).

### 3.2 Related literature

The autonomous formation of coalitions is an important form of interaction in multi-agent systems, such as ad hoc wireless networks and networks of autonomous robots, because many applications require agents to cooperate to fulfill tasks or to improve the efficiency of network resource usage. In ad hoc wireless networks, distributed nodes operating in the same spectrum band act as independent and autonomous agents and can use their limited power and spectrum to either compete or cooperate with one another [71, 74]. In these networks, radios interact with one another to establish a network topology, enabling communications between any pair of nodes either through a direct link or a multi-hop path [75, 76].

In certain scenarios network nodes can optimize their achievable data rates through mutual cooperation [71, 77]. Cooperative game theory has been used to analyze fair and efficient sharing of available spectrum in [78]. A resource allocation scheme for delay-sensitive services based on a Nash bargaining model is analyzed in [79]. In [41], it is shown that in a wireless multiuser network, when nodes play rationally, i.e., to maximize their payoffs, and are allowed to cooperate, cooperation among all nodes is desirable only when all nodes experience similar signal-to-noise ratio (SNR). Nodes with arbitrary SNR values may achieve maximum gains by cooperating only with the nodes that are causing high interference to them. It is shown in [41, 80–82] that it is useful to model these scenarios as a game in which the possibility of cooperative interactions between subsets of players, i.e., coalitions, is analyzed. While strategies based on non-cooperative game theory are mainly concerned with individual utility, coalition game theory deals with maximizing the entire system payoff while satisfying...
individual rational demands of the network nodes [41, 54, 56, 80–83]. These features make coalition game theory suitable for analyzing cooperative interactions among wireless nodes in distributed scenarios. In [41], coalition game theory was used to analyze the stability of the grand coalition in a cooperative wireless network. A coalition game approach to provide incentives to selfish nodes in wireless packet-forwarding networks was utilized in [81]. In [84], the problem of task allocation among a number of autonomous agents such as Unmanned Aerial Vehicles (UAVs) is modeled as a coalition formation game. Coalition game theory has also been applied to the field of Distributed Artificial Intelligence, in most cases to find efficient solutions to the allocation of tasks to group of agents [85].

Most traditional approaches to coalition formation analyze stable coalition structures. Stable coalition structures correspond to the equilibrium state in which players do not have incentives to leave already established coalitions. To determine stable coalition structures in a coalition game one must in general take into account whether the gain achieved by a coalition is also influenced by any externality. However, coalition game models for wireless networks are often analyzed with an assumption that the gain of any coalition is independent of co-existing coalitions in the network [41, 83, 86]. Thus the possibility of interaction between coalitions is ruled out while analyzing the wireless network.

Establishing cooperation in a wireless network is a dynamic process and three important questions must be addressed: 1) How are the coalitions formed?; 2) How do players arrive at equilibrium?; and 3) What is the long term behavior of the coalition formation process? As cooperation among nodes may involve some short term costs, long term behavior can provide more detailed insight into the true benefits of coalition formation.

The dynamics of coalition formation for economic behavior and artificial intelligence is the topic of [87, 88]. Individual adaptation rules are defined as a finite Markov chain in [7] to analyze the dynamic model of coalition formation for trade and finance. The main difference between that model and ours is that [87] assumes that each player can observe all the other players’ demands at any time t, while the players in our model learn the demands of the other players through coalitional interactions. The authors in [87] also assume that no single player can do better on her own than as a member of any coalition; our model does not make such an assumption. Moreover, in contrast with [87], this research analyzes its Markovian model to evaluate how long it will take the players to establish stable coalition structures. Coalition structures are modeled as a sequence
of random variables describing the state of the system, and the transition mechanism between coalition structures is modeled as a Markov chain.

A Bayesian reinforcement learning model is presented in [88] to analyze agent interaction in uncertain environments and to evaluate the long term impact of agent coalition technologies on e-commerce. In [86], the dynamics of coalition formation for networks of autonomic agents are presented using graph theory. While [86] investigates network formation under the fundamental assumption that nodes in networks always want to cooperate with one another, our model does not make such an assumption.

3.3 System setup

A network consisting of $M$ transmitter/receiver pairs is considered. Each transmitter/receiver pair is referred to as a link. Links operate in a wireless channel with bandwidth $W$. It is assumed that each wireless link treats multiuser interference as noise and no interference cancelation techniques are employed. The transmission strategy for each link is simply its power allocation. Links are aware of one another’s presence, for instance by overhearing one another’s transmissions. The received signal for link $i$ is given by Eq. 12, where $X_j$ and $Y_i$ are input and output signals, respectively. The noise processes are independent and identically distributed (i.i.d.). $h_{ji} = |c_{ji}|^2$ is the channel gain between the transmitter of link $j$ and the receiver of link $i$. It is assumed that in a given scenario the direct channel gains, i.e., when $j = i$ ($h_{ii} = |c_{ii}|^2$), are the same for all the links and when $j \neq i$ then $h_{ji} = \psi/d_{ji}^{\alpha_p}$ with $\psi$ being the path loss constant, $\alpha_p$ the path loss exponent and $d_{ji}$ the distance between the transmitter of link $j$ and the receiver of link $i$. The transmitter of any link $i$ has an average power constraint $P_i$. It is also assumed that at the start of any coalition formation, each link knows its respective direct channel gains and the aggregate interference caused by all others but has no knowledge of the level of interference it inflicts on the other receivers in the network. A link will form a singleton coalition by allocating power uniformly over the entire band $W$.

The payoff obtained by player $i$ is quantified as the Shannon capacity of the $i$th link. Assuming uniform power allocations over the entire band $W$ from the other players, the value $v(\{i\})$ of a singleton coalition, i.e., the rate payoff achieved by link $i$ when acting alone, can be expressed as:

$$v(\{i\}) = R_i = \frac{W}{2} \log_2 \left( 1 + \frac{h_{ii}P_i}{WN_0 + \sum_{j \neq i} h_{ji}P_j} \right),$$

Eq. 16 can be rewritten in terms of SNR of link $i$ as:
\[ v([i]) = R_i = \frac{W}{2} \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{1 + \sum_{j \neq i} h_j \text{SNR}_j} \right), \]  
(17)

where \( \text{SNR}_i = \frac{P_i W}{N_0} \) [89]. The value \( v(S) \) of a coalition \( S \), i.e., the rate payoff achieved by all links in \( S \), can be expressed as:

\[ v(S) = \sum_{i \in S} R_i = \sum_{i \in S} \frac{W}{2} \eta_i \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{1 + \sum_{j \in S} h_j \text{SNR}_j} \right), \quad S \subset M, \]  
(18)

where \( S^c \) is the complement of \( S \) in \( M \), \( 0 \leq \eta_i \leq 1 \) is the fraction of the band that link \( i \) uses and \( \sum_{i \in S} \eta_i \leq 1 \). Finally, the value \( v(M) \) of a grand coalition is:

\[ v(M) = \sum_{i \in M} R_i = \sum_{i \in M} \frac{W}{2} \eta_i \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{\eta_i} \right), \]  
(19)

with \( \sum_{i \in M} \eta_i \leq 1 \). We now consider the cases of a 2-player game and an \( M \)-player game.

**Case 1: \( M = 2 \) links**

When two links transmit with an average power constraint \( P \) in the spectrum band \( W \), it is a good strategy to form the grand coalition whenever:

\[ \frac{W}{2} \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{1 + h_j \text{SNR}_j} \right) \leq \frac{W}{2} \eta_i \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{\eta_i} \right) = \frac{W}{2} \log_2 \left( 1 + \frac{h_i \text{SNR}_i}{\eta_i} \right) \eta_i, \]  
(20)

or

\[ h_j \text{SNR}_j \geq \frac{h_i \text{SNR}_i}{(1 + \frac{h_i \text{SNR}_i}{\eta_i})^{\eta_i} - 1}, \quad \text{where } i \neq j. \]  
(21)

**Case 2: \( M \) links**

For \( M \) links there are \( 2^M \) possible coalitions. As compared to the singleton coalitions, links will benefit from the formation of the grand coalition as long as:

\[ \sum_{j \neq i} h_j \text{SNR}_j \geq \frac{h_i \text{SNR}_i}{(1 + \frac{h_i \text{SNR}_i}{\eta_i})^{\eta_i} - 1}, \quad i, j \in M. \]  
(22)
It is also possible that some links form coalitions \( S \subset M \). Links will benefit from the formation of the coalition \( S \) as long as:

\[
\sum_{j, j \neq i} h_{ji} \text{SNR}_j \geq \frac{h_{ii} \text{SNR}_i}{(1 + \left( \frac{h_{ii} \text{SNR}_i}{\sum_{k \in S} h_{ik} \text{SNR}_k} \right)^\eta)} - 1, \quad i, j \in S, \tag{23}
\]

where \( \sum_{k \in S} h_{ik} \text{SNR}_k \) is the perceived interference from the links in \( S' \).

### 3.3.1 Interference channel with positive externalities

**Definition 8.** A coalition game is said to have positive externalities if for any mutually disjoint coalitions \( S_1, S_2, S_3 \subset N \):

\[
\nu(S_3; \{S_1 \cup S_2, S_3\}) \geq \nu(S_3; \{S_1, S_2, S_3\}). \tag{24}
\]

In other words, the coalition value \( \nu(S_3) \) either remains the same or increases whenever \( S_1 \cup S_2 \) is formed as compared to when \( S_1 \) and \( S_2 \) prefer to remain apart.

**Theorem 3.3.1.** The proposed coalition formation game in an interference channel is a game with positive externalities.
Proof. Consider any three singleton coalitions \( \{i\}, \{j\} \) and \( \{k\} \) such that \( \{i\} \cap \{j\} \cap \{k\} = \emptyset \). To maximize their rate payoffs, each singleton coalition either allocates its constrained power over the entire band \( W \), or two of them decide to form \( \{i\} \cup \{j\} \) by allocating the same power across disjoint sub bands in certain ratios \( \eta \) and \( (1 - \eta) \), of the spectrum band \( W \), subject to the minimum rational payoff condition. Then, using the Shannon capacity equation for an interference channel, coalitions values of \( \{k\} \) are:

\[
v(\{k\}; \{\{i\} \cup \{j\}, \{k\}\}) = \frac{W}{2} \eta \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + h_{ik} \text{SNR}_k}) + \frac{W}{2} (1 - \eta) \log_2(1 + \frac{h_{jk} \text{SNR}_k}{1 + h_{ik} (1 - \eta) \text{SNR}_k}),
\]

(25)

\[
v(\{k\}; \{i, \{j\}, \{k\}\}) = \frac{W}{2} \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + h_{jk} \text{SNR}_j + h_{ik} \text{SNR}_i}).
\]

(26)

By using notation \( e = h_{ik} \text{SNR}_i \), \( f = h_{jk} \text{SNR}_j \), for inequality (24) to hold, it is required to prove that:

\[
\eta \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + \frac{e}{\eta}}) + (1 - \eta) \log_2(1 + \frac{h_{jk} \text{SNR}_k}{1 + \frac{1}{(1 - \eta)}})
\]

\[
\geq \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + (e + f)}).
\]

(27)

It can be seen that inequality (27) becomes equality whenever \( \{i\} \cup \{j\} \) share the spectrum in the ratio \( \eta = \frac{e}{e + f} \) and \( 1 - \eta = \frac{f}{e + f} \). To prove inequality (27), it is required to show that the point \( \eta = \frac{e}{e + f} \) is the minimum point for the left hand side of inequality (27). If \( 0 \leq \eta \leq 1 \) in the left hand side of inequality (27) is varied, with \( \text{SNR}_i = \text{SNR}_j \), keeping the direct channel gain \( h_{ik} \) normalized to one and cross channel gains \( h_{jk} \) and \( h_{jk} \) taking values such that \( 0 \leq \sum_{m} h_{mk} \leq 1, m \in \{i, j\} \), it can be seen from Fig. 24 that the left hand side of inequality (27) is convex with respect to \( \eta \). By taking the first and second derivatives it can be verified that:

\[
\frac{d}{d\eta} \left[ \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + \frac{e}{\eta}}) + (1 - \eta) \log_2(1 + \frac{h_{jk} \text{SNR}_k}{1 + \frac{1}{(1 - \eta)}}) \right]_{\eta = \frac{e}{e + f}} = 0
\]

(28)

and

\[
\frac{d^2}{d\eta^2} \left[ \log_2(1 + \frac{h_{ik} \text{SNR}_k}{1 + \frac{e}{\eta}}) + (1 - \eta) \log_2(1 + \frac{h_{jk} \text{SNR}_k}{1 + \frac{1}{(1 - \eta)}}) \right]_{\eta = \frac{e}{e + f}} > 0.
\]

(29)

Eq. (28) and inequality (29) show that \( \eta = \frac{e}{e + f} \) corresponds to the minimum point for the left hand side of inequality (27). Hence, whenever the coalition \( \{i\} \cup \{j\} \) is formed,
the outside coalition is either indifferent to it or it gains from the coalition formation. Therefore, the game has positive externalities.

Coalition games with externalities have PFF and are difficult to analyze [41]. To overcome this problem and to avoid any impact of externalities on the coalition formation, a characteristic function is assigned to the proposed coalition formation game. The usual way to assign a characteristic function is to define $v(S)$ for each $S \subset M$ in a way that assumes that the players within each coalition $S$ cooperate to act together as a unit and the players in $S^c$ allocate their power over the entire band to compete with $S$ [40]. The value $v(S)$ may be called the safety level of coalition $S$. It represents the total payoff that coalition $S$ can guarantee for itself, even if the only objective of the members of $S^c$ is to keep the sum of the payoffs to players in $S$ at a minimum. The value $v(S)$ is a lower bound to the payoff that $S$ should receive because it assumes that the players in $S^c$ ignore what possible payoffs they might receive as a result of their actions. For a coalition game in an interference channel, this characteristic function form conversion means that each coalition $S$ determines its value $v(S) = \sum_{i \in S} R_i$, where $S \subset M$, by using Eq. 18.

It is assumed that links are rational and myopic, i.e., links are maximizing their payoffs, conditional on feasibility, and care only about their current payoffs. The decisions by the links to form coalitions are made unanimously, i.e., a coalition is formed only if it is acceptable to everyone involved. Hence, CS changes are decided by the members of a coalition acting together as a unit. Any singleton coalition can decide on its own, as it is an individual coalition of a single link.

A group of links or a coalition can deviate only if all links within the coalition are at least as well off as a result of the proposed deviation. In other words, once links decide to form a coalition they enter a binding agreement and hence cannot unilaterally deviate on their own [40].

### 3.3.2 Value function

The value $v(S)$ of the coalition formation game is the sum of the rates achieved by the member links in the coalition $S$ (as explained in the previous section). Coalition formation in wireless networks requires a decentralized negotiation process, which in general is costly. This cost represents the overhead in message exchange needed for coalition formation, and it can be modeled as directly proportional to coalition size. In this sense, there is no cost to “form” the singleton coalition. A cost function satisfying
the above assumption is given as:

\[ C(|S|) = \phi \cdot (|S| - 1), \]  
\[ \text{where } \phi \text{ is a proportionality constant.} \]

Therefore, the net value of a coalition \( S \) considering the cost for coalition formation is given as:

\[ \hat{v}(S) = v(S) - (\phi \cdot (|S| - 1)). \]  
\[ \text{(31)} \]

3.4 A Markovian model of coalition formation

In this section, a dynamic model of distributed coalition formation is introduced for the spectrum sharing problem in an interference channel where multiple users coexist and interfere with one another. In the proposed dynamic coalition formation a time-evolving sequence of steps is used by links to reach self-organizing stable spectrum sharing CSs. CSs in the dynamic coalition formation are modeled as a sequence of random variables describing the state of the system, and the mechanism of transitions between CSs is represented by a Markov chain. To incorporate slow changes in the interference environment, the coalition formation after reaching equilibrium restarts after some time \( \mathcal{T} \). This time \( \mathcal{T} \) may be assigned according to variations in the interference environment. It is assumed that during one round of coalition formation the interference environment does not change.

3.4.1 Steps in coalition formation

The coalition formation involves four steps. At the beginning of each round of the coalition formation, the network is composed of all singleton coalitions, i.e., non-cooperative links. The four steps of the coalition formation game are summarized as follows:

1) Initialization: Each individual link computes the aggregate interference caused by all other links.

2) Coalition formation proposal: a) At each time slot, each coalition, with probability \( \mathcal{p} \), proposes a new CS. In this process, one of the links in the coalition acts on behalf of the coalition. (In the case of singleton coalitions, each singleton coalition, i.e., link, individually proposes a new CS with some probability \( \mathcal{p} \); when two or more links form a coalition \( S_1 \), then any link within \( S_1 \) is selected as a coalition head to propose a new CS
with some probability \( p \) on behalf of that coalition.) b) The evolution from one CS to the next can only occur through the merging of two existing coalitions. For instance, any coalition head of any existing coalition \( S_1 \) may propose to merge with another coalition \( S_2 \), forming \( S_1 \cup S_2 = S \).

3) **Coalition formation decision**: Coalition formation only occurs when all links in the new coalition are at least as well off through the merge as they were before it, i.e., the new coalition satisfies the **minimum rational payoff** condition. Due to this coalition formation condition, whenever links agree to form \( S \) the new coalition is internally stable, i.e., no link has an incentive to become a singleton (an individual non-cooperative link).

4) **Payoff distribution**: The total value of the newly formed coalition is arbitrarily partitioned among the links by dividing the spectrum in \( \eta_1, \eta_2, \ldots, \eta_{|S|} \), ratios of the spectrum band \( W \), subject to the **minimum rational payoff** condition, where \( |S| \) is the cardinality of set \( S \), with \( \sum_{k=1}^{|S|} \eta_k = 1 \). The total value may also be partitioned according to some fairness criteria, for instance proportional fairness [43]. The study of the distribution of these payoffs is beyond the scope of this thesis. Note that for generating the numerical results, the total value of the newly formed coalition is partitioned among the links by first dividing the spectrum in ratios that satisfy the minimum rational payoff condition for each link. Then any remaining spectrum is partitioned equal among the members of the newly formed coalition.

The four steps of the coalition formation are repeated until all the coalitions have made their coalition formation decisions, resulting in a final CS. The negotiation process described above can be achieved using a common control channel where links can exchange coalition formation messages to perform the proposed distributed coalition formation.

### 3.4.2 Dynamics of coalition formation

In our dynamic game model, a finite set \( C \) of all possible CSs for \( M \) links forms the state space of the coalition formation game. Let:

\[
C = \{ C_1, C_2, \ldots, C_{|C|} \},
\]

(32)

where each element of \( C \) is a state representing a CS. The initial state of the game process is the set of singleton coalitions. If the Markov chain is currently in state \( C_k \), it moves to state \( C_l \) at the next step with a transition probability denoted by \( P_{C_kC_l} \).
It is said that the coalition game process moves forward when a CS changes due to the merging of two coalitions that results in an internally stable new coalition, and it stays in the same state if no new coalition is formed. For example, transition from the CS state of all singleton coalitions, i.e., $C_1$ given as \{1\}, \{2\}, ..., \{M\} to $C_2$ or $C_3$ given as \{(1, 2), \{3\}, ..., \{M\}\} and \{(1), \{2, 3\}, \{4\}...., \{M\}\} respectively, is possible, because both $C_2$ and $C_3$ result from the merger of two singleton coalitions. However, the transition from $C_2$ to $C_3$ is not possible because it implies that link 2 will break away unilaterally from the coalition \{1, 2\} and merge with \{3\} to form \{2, 3\} at the same time [40]. In other words, state transitions will occur only when two coalitions decide to merge.

Each of $n$ coalitions with some probability $p$ proposes a new CS. The probabilities that a single coalition proposes a CS change, no one proposes a CS change, and more than one coalition proposes a CS change are:

$$\Gamma_{NW} = (1 - p)^n, \text{ no one proposes a CS change};$$ (33)

$$\Gamma = n p (1 - p)^{n - 1}, \text{ one proposes a CS change};$$ and

$$\Gamma_{SW} = 1 - \Gamma_{NW} - \Gamma, \text{ more than one proposes a CS change}.\quad (35)$$

Any coalition may propose a CS change to another coalition in the current state $C_k$, and if all the links in the proposed coalition are at least as well off as before the merge, the game moves to $C_l$. The transition probabilities for the $M$ link coalition game with $B_M$ (see Eq. 11) possible CSs as the state space $C$ are given as:

$$P_{C_kC_l} = \frac{2p(1 - p)^{|C_k| - 1}}{|C_k| - 1} 1(V(C_l), V(C_k)),$$ $\quad P_{C_kC_k} = 1 - \sum_{C_l \in \Omega_l} P_{C_kC_l},\quad (36)$$

where

$$1(V(C_l), V(C_k)) = \begin{cases} 1 & \text{when } V(C_l) \geq V(C_k). \\ 0 & \text{otherwise} \end{cases}$$

$V(C_k)$ and $V(C_l)$ are the values of CS states $C_k$ and $C_l$, respectively, $|C_k|$ represents the number of coalitions in the present CS state $C_k$, $C_l$ represents any one of the new possible CS states to which coalitions can transit from $C_k$, and $\Omega_l$ represents the set of all new possible CS states to which coalitions can transit from $C_k$. The set $\Omega_l$ is given as: $\Omega_l = \left\{ \{S_1 \cup S_2, S_3, ...., S_{|C_k|}\}, \{S_1, S_2 \cup S_3, ...., S_{|C_k|}\}, ...., \{S_1, S_2, ...., S_{|C_k| - 1} \cup S_{|C_k|}\} \right\}$. Given
If \( |C_k| > 1 \) coalitions in the present state \( C_k \), it is possible to transition from \( C_k \) to one of the \( |\Omega_l| \) possible states, where \( |\Omega_l| \) can be calculated as:

\[
|\Omega_l| = \left( \frac{|C_k|}{2} \right).
\]

For instance, if the number of coalitions \( |C_1| = 4 \), then it is possible to move from the state \( C_1 \) to one of \( |\Omega_l| = 6 \) different states (besides itself), provided that the coalition formation condition is satisfied. As explained previously, two coalitions can merge only if all links within the proposed coalition \( S \) are at least as well off as before the merge. The indicator function \( 1(V(C_l), V(C_k)) \) in Eq. 36 represents the possible agreement or disagreement among the links participating in the coalition game to form the proposed coalition \( S \). As the transition probability at any present state \( C_k \) does not depend upon the prior states of the CSs, the Markov property holds.

### 3.5 Dynamic coalition formation with reduced complexity

From the above specified transition model for the proposed coalition formation game, the transition matrix \( \bar{P} \) of the Markov chain can be constructed. The dimension of the transition matrix is \( B_M \times B_M \), where \( M \) is the number of links playing the coalition formation game and \( B_M \) is the Bell number given by Eq. 11. It can be seen from Eq. 11 that \( B_M \) grows super exponentially. For large \( M \) coalitional interaction becomes an issue, leading to a combinatorial explosion. Moreover, if the number of coalitions playing the coalition formation game is large then there is an increased coalition message collision probability. For large values of \( |C_k| \), the number of links attempting to play the coalition formation game with probability of success \( \bar{p} \) on behalf of their coalitions grows, and there is an increased probability of more than one coalition attempting to transmit coalition messages at the same time, resulting in coalition packet collisions. Imposing some hierarchy, through clustering, becomes the practical solution to this problem in large scale wireless networks, with links interacting with a small subset of peers. This can be modeled as a network of wireless links only interacting with a limited subset of links known to them to form stable CSs. One simple and practical way of reducing the complexity of the proposed coalition formation model for the large number of links can be explained as follows. Typically, for many applications each link in the wireless network is required to satisfy a target signal-to-interference-plus-noise ratio (SINR). The presence of such an additional constraint will motivate only those links that have SINR below the target value to participate. This may help in reducing the
Fig 25. Transition probability graph of dynamic coalition formation with GC representing the grand coalition, when the number of states in state space follows the Bell number with $1_{|\mathcal{Y}|}$ representing the indicator function as explained in Eq. 36.
number of participating links and hence reduce the size of the Markov chain. Moreover, in this context, during coalition formation, once a coalition achieves the target SINR, it will have no further incentives to keep participating in the coalition formation as further increasing the size of coalition incurs further increase in communication cost. The exclusion of the links satisfying the target SINR from the coalition formation may help in reducing the size of the Markov chain and may allow the coalition formation to converge quickly (as the game is only played between the left over links).

The explanation and analysis of the Markov chain model that allows links to leave the coalition formation process is a topic for future study and is beyond the scope of this thesis.

In Figs. 25(a) and 25(b), examples of the proposed $M$-link dynamic coalition formation game, using $M = 3$ and $M = 4$ links, are presented. It can be seen from Figs. 25(a) and 25(b) that the size of the state space comprising all possible CSs, i.e., the cardinality of $C$, grows considerably even for a small increase in the number of links from $M = 3$ to $M = 4$.

To reduce the super exponential growth of the state space of the Markov chain, a reduced complexity model for the analysis of stable CSs is introduced. In this model the state space of the Markov chain represents subspaces of CSs containing coalitions of particular sizes. For example, the state $(2, 1, 1)$ for $M = 4$ links represents the subspace of all CSs containing three coalitions with one coalition of size two and two singleton coalitions of size one, i.e., $\{\{1, 2\}, \{3\}, \{4\}\}$, $\{\{1, 3\}, \{2\}, \{4\}\}$, $\{\{1, 4\}, \{2\}, \{3\}\}$, $\{\{1\}, \{2, 3\}, \{4\}\}$, $\{\{1\}, \{2, 4\}, \{3\}\}$ and $\{\{1\}, \{2\}, \{3, 4\}\}$. The state space $\bar{C}$ for this reduced model is equal to all the integer partitions that can be generated for $M$ links. For example, the state space $\bar{C}$ for $M = 4$ links is the integer partition of $M = 4$, given as: $\bar{C} = (1, 1, 1, 1), (2, 1, 1), (3, 1), (2, 2), (4)$. In this model we are not interested in stable CSs of any particular link, but we are only interested in analyzing stable grouping of the CSs according to the sizes of coalitions they contain. The growth rate of the state space of the Markov chain for this model follows an integer partition function, and the growth rate of this function is known to be $\Theta(\frac{e^{\sqrt{2M/3}}}{M})$, where $M$ represent the number of links [90]. This growth is much slower with respect to $M$ than the super exponential growth of the state space $C$. The proposed reduction in complexity for Markov model is particularly relevant to symmetric network scenarios. Fig. 26 presents the state transition graph for $M = 8$ links, using integer partition as the state space. The number of integer partition states for this game is only 22, as compared to 4140 states.
Fig 26. Transition graph of dynamic coalition formation game for $M = 8$ links, when the number of states in the state space follows the integer partition function.
when all possible CSs are used to represent the state space of the Markov process. The transition probability from the present state $\hat{C}_k$ to any next state $\bar{C}_l$ for $M$ links using this model can be calculated as:

$$P_{\hat{C}_k \bar{C}_l} = \sum_{r=1}^{s(\hat{C}_k, \bar{C}_l)} \frac{2p(1-p)^{|\hat{C}_k|-1}}{|\bar{C}_l| - 1} 1_r(V(\hat{C}_l), V(\bar{C}_k)),$$

(38)

where

$$1_r(V(\hat{C}_l), V(\bar{C}_k)) = \begin{cases} 1 & \text{when } V(\hat{C}_l) \geq V(\bar{C}_k). \\ 0 & \text{otherwise} \end{cases}$$

$V(\hat{C}_l) = \sum_{S_i \in \hat{C}_l} v(S_i)$, with $l = 1, 2, ..., |\hat{C}|$, and $s(\hat{C}_k, \bar{C}_l)$ is a function that represents the number of ways coalitions can combine in $\hat{C}_k$ to reach a particular integer partition state $\bar{C}_l$. For instance, for $M = 8$ links if the coalition game is at present in state $(2, 1, 1, 1, 1, 1, 1)$ then it can be seen from Fig. 26 that the merger of coalitions can result in a transition to either integer partition state $(3, 1, 1, 1, 1, 1)$ or to the integer partition state $(2, 2, 1, 1, 1, 1)$. If the coalition formation process moves from $(2, 1, 1, 1, 1, 1, 1)$ to $(3, 1, 1, 1, 1, 1)$, then to calculate the transition probability, there are $n = |\hat{C}_k| = 7$ coalitions in the present integer partition state $\hat{C}_k$ and $s(\hat{C}_k, \bar{C}_l) = 6$, as a coalition of size two within $\hat{C}_k$ can combine in six different ways with the other singleton coalitions to reach $\bar{C}_l = (3, 1, 1, 1, 1, 1)$. However, if the coalition formation process moves from $(2, 1, 1, 1, 1, 1, 1)$ to $(2, 2, 1, 1, 1, 1)$, we have $s(\hat{C}_k, \bar{C}_l) = 15$ as singleton coalitions within $\hat{C}_k$ can combine in fifteen different ways to reach $\bar{C}_l = (2, 2, 1, 1, 1, 1)$.

The indicator function $1_r(V(\hat{C}_l), V(\bar{C}_k))$ in Eq. 38 represents the possibility of agreement or disagreement among the links participating in the coalition game to form a new CS. From Eqs. 36 and 38 and also from Figs. 25(a), 25(b) and Fig. 26 it can be seen that the grand coalition state can be reached in different ways depending on the state transitions. Whenever the grand coalition is formed each link is promised at least the payoff $v_{i,j} \in M(M)$. Using simulation results in Section 3.8 we will quantify that in the proposed coalition formation game, rational payoffs and coalition state transitions depend on the interference perceived by the links.

### 3.6 Analysis of the dynamic coalition formation

Using the standard framework of coalitional game theory, we analyze the stable CSs for the proposed dynamic model of the coalition game for an interference channel. In
analyzing the stable CSs, we need to determine whether the grand coalition is stable, and this can be done using the concept of core.

Due to the myopic nature of links in the coalition formation process, it is possible that for certain rate allocations the core of the game is not empty but links cannot form the grand coalition. In other words, sometimes, even though the grand coalition is advantageous to all links, there is no path to reaching it because the intermediate coalitions are not sustainable. Any utilization of core allocations among wireless links is possible only if the grand coalition is formed. On the other hand, there may be circumstances where there is no path to the grand coalition, even if it ultimately would yield better value, i.e., the core is not empty but the grand coalition cannot be formed.

To illustrate the above situation, an example for $M = 3$ links can be constructed, where the core is non-empty but the grand coalition cannot be formed even if the cost for cooperation is zero.

**Example 3.6.1.** Let $H = \left( \begin{array}{ccc} 1 & 7 & 3 \\ 3 & 1 & 7 \\ 7 & 3 & 1 \end{array} \right)$ be the link gain matrix for $M = 3$ links, with $W = 3$ and $\text{SNR} = 10\text{dB}$ for all links. In this three-link game, due to perceived interference at the myopic links, link 1 wants to cooperate with link 2 but not with 3, link 2 wants to cooperate with 3 but not with 1 and link 3 wants to cooperate with link 1 but not with link 2. Transitions from state $C_1 = \{\{1\}, \{2\}, \{3\}\}$ to any next state $C_l$ are not possible. Calculating coalition values using Eqs. (16)-(18) and rules explained in Sections 3.3 and 3.4, we obtain: $v(\{1\}) = v(\{2\}) = v(\{3\}) = 1.9573$, $v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 3.6306$, $v(\{1, 2, 3\}) = 7.4313$.

To show that the core of this game is not empty, suppose that the links form the grand coalition and divide $v(\{1, 2, 3\})$. Suppose some payoff vector $x$ is proposed as a division of $v(N) = v(\{1, 2, 3\})$. The solution of the core for the three-link game in example 3.6.1 consists of all vectors $(x_1, x_2, x_3)$ satisfying
\begin{align*}
    x_1 &\geq v(\{1\}), \quad x_1 + x_2 \geq v(\{1, 2\}), \\
    x_2 &\geq v(\{2\}), \quad x_2 + x_3 \geq v(\{2, 3\}), \quad x_1 + x_2 + x_3 = v(\{1, 2, 3\}), \\
    x_3 &\geq v(\{3\}), \quad x_1 + x_3 \geq v(\{1, 3\}).
\end{align*}
Solving (39), it can be seen that the core for the three-link game is not empty and is given by
\begin{equation}
    \gamma = \{x : x_1 + x_2 + x_3 = 7.4313, x_1 \geq 1.9573, x_2 \geq 1.9573, x_3 \geq 1.9573\}.
\end{equation}
The three links are treated symmetrically in this game by the core and all Pareto-efficient payoff allocations satisfying $x_i \geq v(\{i\})$ are included in it. However, using coalition
values of the three-link game in example 3.6.1, it can be seen that \( v(\{i, j\}) - v(\{i\}) < v(\{j\}) \), \( \forall i, j \). Therefore, any proposed \( v(\{i, j\}) \) in the CS state \( C_k = \{\{1\}, \{2\}, \{3\}\} \) cannot guarantee that all links within the proposed coalition are at least as well off as without the proposed coalition.

Now that it is shown that a non-empty core does not necessarily lead to the formation of the grand coalition, this research will next present a condition for the formation of a stable grand coalition for the proposed dynamic coalition game.

**Definition 9.** The marginal contribution of any link \( i \in S \) in the presence of other links in any proposed coalition \( S \) is defined as: \( m_i(S \{i\}) = v(S) - v(S \{i\}) \).

**Theorem 3.6.1.** In the proposed dynamic coalition game a stable grand coalition is formed if in every CS state (or, correspondingly, in every integer partition state) there is at least one proposed coalition \( S \) that satisfies: \( m_i(S \{i\}) \geq d_i(S), \forall i \in S \), for \( |S| > 1 \).

**Proof.** First, any link leaving the coalition in any CS state or in any integer partition state could form a singleton coalition and would get the payoff \( v(\{i\}) \leq v_i, i \in S(S) \), so no link has an incentive to leave. If in every CS state (or correspondingly in every integer partition state) of the dynamic coalition game there is at least one proposed coalition \( S \) in which the marginal contributions of all links are at least equal to their individual minimum rational payoffs \( d_i(S) \), it means that with the merger of coalitions the payoff values of each link participating in the merger of coalitions is non-decreasing. So the rate \( R_i \) of each link will increase monotonically with the merging of coalitions. As a result, the wireless links forming coalitions have an incentive to allow merging of coalitions at every state. Hence, a stable grand coalition is formed if in every CS state or in every integer partition state there is at least one proposed coalition \( S \) that satisfies: \( m_i(S \{i\}) \geq d_i(S), \forall i \in S \), for \( |S| > 1 \).

3.7 Dynamic coalition formation transients

Using standard theory of absorbing Markov chains one can calculate the mean \( \mu \) and variance \( \sigma^2 \) of the time for the dynamic coalition game starting from the initial state of all singleton coalitions to reach stable CSs. Let \( \bar{P} \) represent the state transition probability matrix of an absorbing Markov chain in canonical form [91–93]:

\[
\bar{P} = \begin{pmatrix}
I & O \\
R & Q
\end{pmatrix},
\]
where $I$ is an identity matrix, $O$ is a matrix with all zero entries, $R_t$ is the matrix of transition probabilities from transient to absorbing states and $Q$ is the matrix of transition probabilities between the transient states. The matrix $F = (I - Q)^{-1}$ is called the fundamental matrix for $\tilde{P}$. Using $F$, one can calculate the mean $\mu$ and variance $\vartheta^2$ of the time for the game before the coalition game process converges to the absorbing state [91]:

$$
\mu = F \tau, \quad \vartheta^2 = (2F - I)\mu - \mu_{sq},
$$

where $\mu$ is a column vector whose $i$th entry $\mu_i$ is the expected number of discrete steps or time intervals before the process reaches a stable CS, i.e., an absorbing state of the Markov chain, given that the process starts in state $C_i$ (or correspondingly $\tilde{C}_i$). $\mu_{sq}$ is a column vector whose entries are the square of the entries of $\mu$ and $\tau$ is a column vector whose components are the respective state “dwell” times, i.e., the time it takes to make a coalition formation decision.

### 3.8 Simulation results

An ad hoc network comprising $M$ links randomly deployed in a $150m \times 150m$ area is simulated. The noise variance is set to $N_0 = -40dBm$ and $\psi = 1$. The spectrum band parameter $W$ is set to 5. The coalition formation overhead $\phi$ in Eq. (30) is set to 0.2.

Transmission power is chosen to be $P = 100mW$ (as it is a typical transmission power value for ad hoc networks [94]). Path loss value ($\alpha_p$) in wireless communications is typically between 2 to 4 [95, 96]. If $\alpha_p = 2$ is chosen as a parameter (that is the value for free space propagation which can model a channel with few obstructions, such as encountered in rural areas) then there will be little signal propagation loss over distance and as a result links will interfere more with each other and will have greater incentives to form coalitions of large sizes. If $\alpha_p = 4$ is chosen as a parameter (that is the value for urban channel modeling) then there will be high signal propagation loss over distance and as a result links will interfere less with each other and will have incentives to form coalitions of small sizes. $\alpha_p$ is set to 3 for our simulation purposes which would represent the channel model for suburban area.

For a fixed area, increasing the number of links $M$ will lead to more interference and decreasing the number of links will lead to less interference. To analyze the coalition
formation process under varying interference conditions, the number of links $M$ is varied (between 1 to 20) for a fixed network area of $150m \times 150m$ and evaluate the average rate per link and average maximum coalition sizes (see Fig. 27 and Fig. 28(b)).

Fig. 27 shows the average rate per link for different network sizes, when all links act independently, when links can form coalitions but there is no cost to coalition formation, and when links can form coalitions but incur some cost in the coalition formation process. It can be seen in Fig. 27 that the distributed process of coalition formation yields an improvement in the average link rates as compared to the non-cooperative strategy. Fig. 28(a) compares the gain in average rate per link due to coalition formation with the non-cooperative strategy for different network sizes. In this figure, it is shown show that although coalition formation yields significant average rate gains when compared with the non-cooperative strategy, these gains are reduced as the distance between the transmitter and its own receiver is decreased. Since the performance of the coalition formation solution is dependent on the distance between the transmitters and their own receivers, average maximum coalition sizes for different distances are shown in Fig. 28(b). It can be seen from Fig. 28(b) that the coalition formation solution results in the formation of the grand coalition (for network sizes of 15 and 20 links) whenever the distances between the transmitters and their own receivers is greater than $40m$ for the given simulation scenario. Therefore, the terminal state grand coalition of the
(a) Average rate gain per link due to coalition formation with no cost as compared with the non-cooperative strategy for different network sizes. $d$ represents the distance between a transmitter and its own receiver.

(b) Average maximum coalition size for different distances between a transmitter and its own receiver with network sizes of $M = 15$ and 20 links.

Fig 28. ([55] [©IEEE 2010]).
finite Markov chain of the proposed dynamic coalition game is an absorbing state and, in game theoretic terms, a Nash equilibrium for the distances between transmitters and their own receivers greater than 40m. However, if there is no path to the grand coalition state, i.e., the stable grand coalition cannot be formed, then any state of the proposed dynamic coalition game that satisfies the coalition structure internal and external stability condition is an absorbing state, as links have no incentive to leave that internal and external stable state. This is verified in Fig. 28(b): as the distance between the transmitters and their own receivers decreases, the average maximum coalition size, i.e., the number of links in a coalition also decreases. In other words, as the distance between the transmitters and their own receivers decreases, the network for such scenarios is composed of independent disjoint coalitions (internally and externally stable coalitions) of smaller sizes. It can also be seen from Fig. 28(b) that as the distance between the transmitters and their own receivers is reduced to 12m then the coalition formation solution results in a network structure mostly composed of individual non-cooperative links, i.e., all singleton coalitions. In other words for the distances of less than 12m between a transmitter and its own receiver the coalition formation solution mostly coincides with the non-cooperative solution.

Definition 10. A coalition game is superadditive if for every pair of coalitions \( S_i \) and \( S_j \),

\[
v(S_i) + v(S_j) \leq v(S_i \cup S_j), \text{ if } S_i \cap S_j = \emptyset.
\]  

(42)

Superadditivity in coalition games implies that forming the grand coalition is efficient. To show that superadditivity is a more restrictive requirement to form the grand coalition, as compared to the formulated condition in Theorem 3.6.1, the following simulation set up is considered: \( M = 3 \) links are deployed with direct channel gain \( h_{ii} \) between the transmitter and its own receiver normalized to one and \( SNR = \frac{P}{N_0 W} = 10dB \). The coalition formation game always starts from the initial state of all singleton coalitions. To analyze the coalition formation game at a given SNR for the various interference environments, the asymmetry between the three links is controlled by varying the cross gains \( h_{ji} \) each time the coalition game is played, where \( 0 < h_{ji} \leq 1, j \neq i \) and \( i, j \in M \). It can be seen from Fig. 29 that the proposed coalition game in an interference channel is superadditive for the perceived high symmetric interference gains at the receivers. In Fig. 29, the interference gains of links 1 and 2 are varied while for the third link interference gain is kept fixed at \( \sum_{j: j \neq i} h_{ji} = 0.85 \). It can be seen from Fig. 29 that inequality (42), i.e., superadditivity, is a more restrictive condition for forming the stable grand coalition.
Fig 29. Light grey and dark grey region represents where GC can form for our formulated condition and only dark grey region represents where GC can form for superadditive condition. Regions with percent coalition gains are shown for various interference channel gains of links 1 and 2 interference varied while for the third link $\sum_{j \neq i} h_{ji} = 0.85$. For instance percent gain for GC values are calculated as: $\frac{v(M) - \sum_{\{i\}} v(\{i\})}{\sum_{\{i\}} v(\{i\})} \times 100\%$, ([55] [©IEEE 2010]).

The time required to transmit/receive and decode a message may range from few micro seconds to few milliseconds. This research takes 1 millisecond dwell time as an example. All the links are assumed to be enduring moderate to high interference. Using the Markov chain model in which the state space follows an integer partition function, Fig. 30(a) and 30(b) illustrate the mean and variance of the time for the coalition game to reach the absorbing state of grand coalition from initial state $\bar{C}_i$ for $M = 3, 4, 5$ and 6 links. In Fig. 30(a) and 30(b), simulation results are generated by assuming that all the links are enduring moderate to high interference [97], probability of coalition formation proposal is $p = 0.3$, the direct channel gain $h_{ii}$ between the transmitter and its own receiver is normalized to one, and $SNR = \frac{P}{N_0W} = 10dB$. It can be seen from Fig. 30(a) and 30(b) that the mean and variance of the time to converge to the absorbing state of the grand coalition decreases with the number of coalitions participating in the coalition game. For instance in state $\bar{C}_1$, i.e., the state of all the singleton coalitions, the number of coalitions playing the game is highest so it takes longer to converge to the grand
Fig 30. Mean time and its variance for the coalition game to form the GC from different transient states. All the links are assumed to be enduring moderate to high interference [97]. The dwell time in each state is set to 1ms , ([55] [©IEEE 2010]).
Fig 31. Mean time and its variance for the coalition game to form the GC from the transient state of all singleton coalitions for different $\bar{\rho}$. All $M = 6$ links are assumed to be enduring moderate to high interference [97]. The dwell time in each state is set to 1msec, ([55] [ ©IEEE 2010 ]).
coalition from $\mathcal{C}_1$ as compared to the other states. The mean and variance of the time to converge to the grand coalition from states 3 and 4 are same in Fig. 30(a) and 30(b), as the number of coalitions participating in the two states are the same.

The two figures in Fig. 31 illustrate the mean and variance of the time for the coalition game to converge to grand coalition with different probabilities of coalition formation proposal. Fig. 31(a) shows that for small $\bar{p}$, $\mu$ is high because the mean time between coalition formation messages is too long. If $\bar{p}$ is high then the mean time between coalition formation messages is shorter but the number of coalition message collisions is higher, resulting in a longer time between CS changes. This suggests that depending on the number of players in the game there is an optimum value for $\bar{p}$. The optimum value for $M = 6$ links is illustrated in Fig. 31.

### 3.9 Summary

In this chapter, the spectrum sharing problem as a dynamic coalition formation game is considered, where links self-organize to reach stable coalition structures through a time-evolving sequence of steps. The proposed game accounts for the case of partial channel knowledge, where a specific link is aware of the aggregate strength of interfering signals but has no knowledge of the level of interference it inflicts on the other receivers in the network. Stable coalition structures (stable network partitions) are modeled as the absorbing states of a Markov process. Depending on the interference environment, the proposed coalition formation model either converges to the absorbing state of the grand coalition, or it converges to the absorbing state of any coalition structure satisfying the condition of internal and external stability (IES). Simulations are used to analyze the average data rate per link for different network sizes, when all links act independently, when links can form coalitions but there is no cost to coalition formation, and when links can form coalitions but incur some cost in the coalition formation process. Results show that the proposed coalition formation solution yields significant gains in terms of average data rates per link for different network sizes.
4 Dynamic coalition formation for throughput-efficient spectrum sensing

Trying to be number one and trying to do a task well are two different things.

Alfie Kohn
No Contest: The Case Against Competition

In this chapter, the problem of distributed throughput-efficient sensing in cognitive radio networks is formulated as a dynamic coalition formation game based on a Markovian model. A key question this chapter tries to address is how to coordinate distributed cognitive radios to perform cooperative sensing to minimize their false alarm probabilities and therefore increase their achievable throughput, under a probability of detection constraint. Our model also takes into account the overhead in combining sensing reports.

4.1 Related literature

It is shown in [45] that cooperation in sensing can reduce the false alarm probability of the cooperating CRs. In cooperative spectrum sensing, each CR performs spectrum sensing and sends its sensing report to a data collector known as the fusion center. The report may be binary (hard decision), consisting of zeros (primary user not present) and ones (primary user present). Hard decisions may be combined at the fusion center using, for example, “OR”, “AND” and “MAJORITY” rules [33, 44]. However, in distributed CR networks, individual CRs need to interact with each other without a centralized fusion center.

Two different game theoretic analysis of distributed cooperative spectrum sensing are presented in [84] and [98]. In [98], an evolutionary game theoretic framework is used to analyze the interactions among distributed selfish CRs in cooperative sensing. It is assumed in [98] that the selfish CRs overhear the detection results from the other CRs and can free ride by refusing to take part in spectrum sensing. Hence [98] models the spectrum sensing problem as a non-cooperative game. Distributed coalition formation for cooperative spectrum sensing CRs is the topic of [84]. Using a merge-and-split based coalition formation game model, the authors in [84] analyze the average missed detection
probability per CR. However, unlike [84], a value function is proposed in this research that encourages collaborating CRs to minimize their false alarm probabilities for a given target primary user detection probability ($\tilde{P}_d$). This is an important requirement for coexistence with primary users; otherwise, CRs will not be allowed to operate in the primary user band [99]. Moreover, in contrast with [84], the coalition formation model proposed in this paper also takes into account the overhead in combining sensing reports within a coalition. In [55], a dynamic coalition formation game based on a Markovian model is used by us to analyze the *selfish* interactions among distributed nodes for spectrum sharing in an interference channel. Unlike [55], this research analyzes the interactions among CRs for the problem of distributed throughput-efficient sensing in CR networks. Moreover, *selfish* coalition formation proposed in [55] for distributed networks may lead to a suboptimal equilibrium where nodes, through their interactions, reach an undesirable coalition structure from a network point of view. In this chapter and in [52], the dynamic model of coalition formation proposed in [55] is extended to determine whether and how the coalitional behavior of CRs will change if coalition formation is “not entirely selfish”.

### 4.2 System model

The system setup used in this chapter includes a primary user transmitter and a distributed CR network of $M$ active CRs (transmitter/receiver pairs). The CRs are uniformly and independently distributed in a circle with radius $R_s$ and centered at the coordinates $(\beta, 0)$. The PU (primary user transmitter) is at coordinates $(0, 0)$ as shown in Fig. 32. This corresponds to, for example, using only downlink frequencies for CR access. The primary user and CRs are both assumed to use a time slotted system, and one
transmission by primary user corresponds to one time slot, as in [100]. In this approach, the primary user is either present for the whole time slot, or absent for the whole time slot. The CRs use the beginning of each slot for sensing. It is assumed that $M$ cognitive radios employ energy detection to make primary user detection observations in the frequency band they are monitoring. The probability that a primary user is present is denoted by $\theta$. In order to detect the primary user, each CR can either sense the spectrum on its own (non-cooperative strategy) or it can perform cooperative sensing by forming coalitions with other CRs (cooperative strategy).

Let us represent the received signal-to-noise ratio (SNR) from the primary user to the $i$th CR by:

$$\gamma_{r,i} = \frac{P_i}{\sigma^2},$$

(43)

where $\sigma^2$ represents noise power and $P_i = \frac{P_{PU}}{d_i^{\alpha_p}}$ is the signal power received by CR $i$. $P_{PU}$ is the primary user’s signal power, $d_i$ is the distance between the primary user and the $i$th CR, $\alpha_p$ is the path-loss exponent and $\psi$ is a scalar. A complex-valued PSK primary user signal and circular symmetric complex Gaussian (CSCG) noise is assumed. For the CSCG noise case the probability of false alarm of CR $i$ for a chosen detection threshold $\lambda_i$ is given by [101, 102]:

$$P_{f,i}(\lambda_i) = Q\left(\frac{\lambda_i}{\sigma^2} - 1\right)\sqrt{T_B},$$

(44)

where $Q(.)$ is the tail probability for the standard normal distribution and $T_B$ represents the time-bandwidth product, given as $T_B = \tau_s W$, where $\tau_s$ is the sensing duration and $W$ is the measurement bandwidth. For a chosen threshold $\lambda_i$, the probability of detection of CR $i$ is approximated by [101, 102]:

$$P_{d,i}(\lambda_i, \gamma_{r,i}) = Q\left(\frac{\lambda_i}{\sigma^2} - \gamma_{r,i} - 1\right)\sqrt{T_B},$$

(45)

To protect the primary user against harmful interference from the CRs, the detection probability is fixed at a desired target value, $\tilde{P}_d$. In practice, $\tilde{P}_d$ is required to be close to 1 [44]. The probability of false alarm of each CR $i$ for the targeted $\tilde{P}_d$ can be rewritten as:

$$P_{f,i}(\tilde{P}_d, \gamma_{r,i}) = Q\left(\sqrt{2\gamma_{r,i} + 1}Q^{-1}(\tilde{P}_d) + \sqrt{T_B}\gamma_{r,i}\right).$$

(46)

It may be seen from (46) that a high detection probability requirement may lead to a high false alarm probability for an individual CR if its $\gamma_{r,i}$ is low, thus reducing the
achievable throughput of that CR. In this case, the individual CRs may interact to form coalitions to help decrease the false alarm probability. Within a coalition, sensing decisions by individual CRs are transmitted over the narrowband common control channel to a CR selected as a coalition head. Although the optimal decision fusion rule is the Chair-Varshney rule [103], for simplicity of implementation it is assumed that an OR fusion rule is used by the coalition head to combine the individual CR sensing decisions within a coalition. The OR rule is a simple decision rule explained as follows: if one out of $|S|$ CRs in a coalition detects the primary user, the final decision for the coalition declares a primary user is present, where $|S|$ represents the number of CRs in a coalition $S$. CRs in any coalition $S$ decide to transmit or not, based on the final combined sensing decision of the coalition head. Therefore, the probabilities of detection and false alarm of a coalition head are also the probabilities of detection and false alarm of each CR $i$ that is the member of $S$.

When the observations of CRs are not conditionally independent, i.e., they are correlated, such as spatially correlated. Selecting independent CRs for cooperation can improve the robustness of sensing results. However, additional information of location estimates is required at the coalition head to perform selection of spatially independent CRs. In this thesis, we do not assume that any information about CR positions are known at the coalition head, so it is not possible to assess the correlation between observations made by the various CRs. Therefore, the coalition formation methods treat the measurements of CRs as conditionally independent.

Assuming that all decisions are conditionally independent (this means that the sensing measurements performed by CRs are independent, but that for each CR the same hypotheses $\{H_0, H_1\}$, $H_0 =$ PU not present and $H_1 =$ PU present, apply) then using the OR rule, the detection probability of the coalition $S$ is given as:

$$P_{d,S} = 1 - \prod_{i=1}^{S} (1 - P_{d,i}).$$

(47)

For a given $\tilde{P}_d$, the individual CR’s target probability of detection in a coalition using OR fusion rule is written as (assuming same target probability of detection for every CR, as in [98, 101]):

$$\tilde{P}_{d,i} = 1 - (1 - \tilde{P}_d)^{\frac{1}{|S|}}.$$  

(48)

The probability of false alarm of each CR $i$ for the $\tilde{P}_{d,i}$ can be written as:

$$P_{f,i}(\tilde{P}_{d,i}, \gamma_i) = Q\left(\sqrt{2\gamma_i + \frac{1}{\gamma_i}}(\tilde{P}_{d,i}) + \sqrt{\gamma_i} \right).$$

(49)
Fig 33. Probability of false alarm for different detection probabilities and for different different values of SNR, $T_B = 6000$.

However, it may happen that some of the CRs have better received SNR from the PU than the others. To gain from this SNR diversity, a weighted target probability of detection is adopted for any CR $i \in S$. The weighted target probability of detection modifies $\tilde{P}_{d,i}$ in (48) and takes a sensing CR’s received SNR $\gamma_{r,i}$ into consideration. The proposed weighted target probability of detection for CR $i$ is given by:

$$\tilde{P}_{w,d,i} = 1 - (1 - \tilde{P}_{d,i}) \sum_{j \in S} \gamma_{r,j}.$$  

(50)

While this research makes no claims as to the optimality of the selected weighting method, these probabilities preserve the relationship in (47) and assign a higher expectation on detection accuracy to the CRs that experience higher SNR. The use of $\tilde{P}_{w,d,i}$ is justified as follows. For a given SNR $\gamma_{r,i}$ and $T_B$, false alarm probability of CR $i$ is a function of the target detection probability for that CR (see equation (49) and Fig. 33). It can be seen from Fig. 33 that the impact of target detection probability on the false alarm probability of individual CRs varies for the different values of $\gamma_{r,i}$. If a CR has high $\gamma_{r,i}$, for instance $\gamma_{r,i} = 0.05$, then the target detection probability has little impact on its false alarm probability. However, if a CR has low $\gamma_{r,i}$, for instance $\gamma_{r,i} = 0.005$, then the target detection probability has strong impact on its false alarm probability.

In this approach, the CR with high $\gamma_{r,i}$ is assigned relatively high target detection probability. However, due to high $\gamma_{r,i}$, the assignment of high target detection probability
has little impact on the false alarm probability of that CR. The CR with low $\gamma_r$ is assigned relatively low target detection probability which results in low false alarm probability for that CR (see Fig. 33), as compared to when each CR is assigned the same target detection probability in a coalition.

The probability of false alarm of each CR $i$ for the $P_{wd,i}$ is written as:

$$P_{f,i}(\tilde{P}_{wd,i}, \gamma_r,i) = Q\left(\sqrt{2\gamma_{rd} + 1}Q^{-1}(\tilde{P}_{wd,i}) + \sqrt{T_{Bd}\gamma_{rd}}\right).$$  \hspace{1cm} (51)

Using the OR rule, the false alarm probability $P_{f,S}$ of the coalition $S$ is given as:

$$P_{f,S} = 1 - \prod_{j=1}^{\lvert S \rvert} (1 - P_{f,j}).$$  \hspace{1cm} (52)

It may be seen from (49) and (51) that, for a given $P_d$ and $T_B$, the CRs with low values of $\gamma_r$ have incentives to form coalitions as it helps to decrease $P_{f,i}$ (due to the increase in $Q^{-1}(\tilde{P}_{wd,i})$ or $Q^{-1}(\tilde{P}_{wd,i})$ term which in turn decreases $P_{f,i}$). However, for a given $P_{f,i}$ the coalitional false alarm probability given by (52) is an increasing function of coalition size $\lvert S \rvert$.

4.3 Distributed spectrum sensing as coalition formation

Coalition game theory provides useful tools to decide which group of players will cooperate with each other to efficiently achieve their goals [40]. Therefore, to analyze cooperative interactions among CRs performing distributed spectrum sensing, this research models the problem as a coalition formation game.

Let $M = \{1, 2, ..., M\}$ denote the set of players (CRs) playing the coalition game. A coalition, $S$, is a subset of $M$, $S \subseteq M$. The utility of a coalition in a coalition game is called the coalition value, and is denoted by $v$. Coalitions are assumed to be non-overlapping, i.e., CRs are members of at most one coalition.

For a non-cooperative CR network with periodic spectrum sensing, each slot consists of sensing duration $\tau_s$ and data transmission duration $T - \tau_s$. To focus on the coalition formation model for the sensing-throughput tradeoff problem in distributed CR networks; it is assumed that the entire primary user band is divided into $K$ sub-bands and, when the primary user is absent each CR operates exclusively in one of the $K$ sub-bands. Note that scheduling policies are not analyzed in this research and it assumes a very simple and predetermined orthogonal sub-band allocation policy for the CRs when the primary
user band is free for access. Chapter 2 of this thesis addresses competition among CRs for multiple potentially available channels. This assumption is in line with the other models in the literature [98, 104]. For the non-cooperative case, the average throughput of the CR \( i \) is approximated by [33, 98]:

\[
\bar{R}_i = (1 - \theta)(1 - \frac{\tau_s}{T})(1 - P_{f,i})R_i,
\]

where \( 1 - \theta \) is the probability of primary user absent, \( T \) represents the total frame length and \( R_i \) represents the transmission rate of the CR \( i \) to its receiver when the primary user is absent. In distributed CR networks when CRs decide to perform cooperative sensing by forming coalitions with one another they may incur overhead costs in terms of time delay due to the process of coalition formation and due to combining sensing reports within a formed coalition. Note that the time delay due to the process of coalition formation depends on how frequently the coalition process is initiated which in turn depends on changes in the network configuration (e.g. CR mobility). However, the time delay in data transmissions of a coalition due to the overhead in combining sensing reports is periodic (since this overhead is incurred in every time slot). This research analyzes the impact of overhead costs in terms of time spent during combining sensing reports on the throughput of distributed CRs. It ignores the overhead in terms of time delay due to the process of coalition formation. One of the extensions that is envisioned for this research is the consideration of the effects due to overhead in terms of spending time and resources for coalition formation and also the effects due to overhead in terms of energy consumed by a coalition head to collect and combine sensing reports.

When the CRs decide to form a coalition then the entire coalition cannot transmit data until sensing reports are collected and the final combined sensing decision is transmitted to all the coalition members (see Fig. 34). One simple method of sensing reports collection by the coalition head can be stated as follows: The coalition head grants a contention free channel to individual cognitive radios by polling them (using their identity numbers) for transmitting their local decisions. The coalition head may
employ a round-robin scheduler [48, 49], and on being polled, a CR transmits its local decision to the coalition head. In this sensing reports collection method there is cost in terms of time delay in data transmissions of a coalition due to the overhead in combining sensing reports. This cost generally increases with the number of coalition forming CRs as more decisions need to be reported to the coalition head. The average throughput of the CR $i$ considering the cost in terms of overhead in sensing reports combining within a coalition is approximated as:

$$\hat{R}_i = (1 - \theta)(1 - \frac{\tau_s}{T} - \frac{\tau_c}{T}(|S| - 1))(1 - P_{f,s})R_i,$$

where $\tau_c$ is the time spent on reporting a sensing decision to the coalition head.

For a target detection probability, $\tilde{P}_d$, the CRs may form coalitions to reduce their false alarm probability and therefore increase their average throughput given by (54). The coalition forming CRs may also increase their average throughput by reducing their sensing time $\tau_s$ via joint coalitional sensing. However, we note that for the distributed spectrum sensing problem when the coalition forming CRs are allowed to vary sensing time $\tau_s$, it generates significant uncertainties in the coalition values. For instance, if two or more CRs reduce their sensing time $\tau_s$ (or in other words increase their data transmission time) via forming the coalition $S$ then it may happen that the CRs in $S$ start transmitting data while some CRs outside $S$ may still be sensing. This may lead to a change in the coalition value (in terms of false alarm probabilities) of the CRs outside the coalition $S$. To avoid this uncertainty in the coalition value due to CR transmissions, the sensing duration $\tau_s$ is fixed for each individual CR.

### 4.3.1 Value function

**Definition 11.** A non-transferable utility (NTU) game is a coalition game in CFF, in which the value $v(S)$ of a coalition $S$ cannot be arbitrarily divided among the coalition’s players. In such games, each player will have its own value within a coalition $S$. The value function $\varphi_i(S)$ represents the value of player $i$ that belongs to a coalition $S$.

For a target detection probability, $\tilde{P}_d$, and the fixed sensing duration $\tau_s$, the coalition value $v(S)$ must characterize the incentives to form coalitions in terms of the decreased false alarm probability $P_{f,S}$ of the coalition. Moreover, the coalition value must also take into account the cost in terms of delay in data transmissions of a coalition due to the overhead in combining sensing reports. A suitable coalition value that satisfies the
above requirements is given by:

\[ v(S) = \left( \phi_1(S), \phi_2(S), \ldots, \phi_{|S|}(S) \right) = (\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_{|S|}), \]  

(55)

where \( \phi_i(S) \) denotes the average throughput for CR \( i \) given by (54), for \( i = 1, 2, \ldots, |S| \). In the proposed coalition formation game, each CR has its own value within a coalition and its non-transferable due to the following reasons.

1) \textit{Indivisible false alarm probability:} The probability of false alarm \( P_{f_i} \) of each CR \( i \), where \( i \in S \), is also given by the probability of false alarm of the coalition \( S \), i.e., \( P_{f_S} \) (as explained in Section 4.2) and it cannot be divided among the radios.

2) \textit{Indivisible cost:} Each CR incurs the same cost in terms of overhead in combining sensing reports, i.e., \( |S| - 1 \), and this cost cannot be divided among the CRs.

3) \textit{Indivisible bandwidth:} Finally, each CR operates exclusively in one of the \( K \) sub-bands. Therefore, they cannot arbitrarily divide the spectrum among themselves.

Therefore, each CR will have its own value within the coalition \( S \) and hence, the proposed game is an NTU game.

When two or more CRs form a coalition \( S \), then any CR within \( S \) is selected as a coalition head to combine the individual CR sensing decisions within the coalition. The decisions to form coalitions by the CRs are based on consensus, i.e., a coalition is formed only if it is acceptable to everyone involved. It is also assumed that CRs are myopic, i.e., CRs care only about their current payoffs.

\subsection*{4.3.2 Selfish coalition formation}

It is assumed that CRs are individually rational, i.e., CRs seek to maximize their payoffs, conditional on feasibility. Therefore, for selfish coalition formation, the merge of two coalitions only occurs when all CRs in the new coalition \( S \) are at least as well off through the merge as they were before it. Mathematically speaking, coalitions \( S_1 \) and \( S_2 \) will merge to form \( S \) only: if \( \forall i, j, \) where \( i \in S_1 \) and \( j \in S_2 \), \( (\phi_i(S) - \phi_i(S_1)) \geq 0 \) and also \( (\phi_j(S) - \phi_j(S_2)) \geq 0 \). Due to this coalition formation condition, whenever CRs agree to form \( S \) the new coalition is internally stable, i.e., no CR has an incentive to become a singleton (an individual non-cooperative CR).
4.3.3 **Altruistic coalition formation**

The model of selfish coalition formation discussed above is in line with much of the coalition formation literature, which assumes that users form coalitions to maximize their individual payoffs. However, it is equally interesting to investigate the question of whether and how the achievable throughput of CRs will change if CRs are assumed to be “not entirely selfish”. Intuitively, we want to model the case when two or more CRs propose to form a new coalition $S$ by taking into account one another’s welfare. Before studying altruistic coalition formation, this research first defines the concept of altruistic contribution of a coalition as [105, 106]:

**Definition 12.** Let $S_1$ and $S_2$ be two disjoint coalitions. The altruistic contribution of $S_1$ to $S$, where $S = S_1 \cup S_2$, is $a_{S_1}(S) = \sum_{i \in S_2} (\phi_i(S) - \phi_i(S_2))$; the altruistic contribution of $S_2$ to $S$ is $a_{S_2}(S) = \sum_{i \in S_1} (\phi_i(S) - \phi_i(S_1))$; and the sum of altruistic contributions of $S_1$ and $S_2$ is $\hat{a}(S) = a_{S_1}(S) + a_{S_2}(S)$.

In simple words, the altruistic contribution of coalition $S_1$ to $S$ represents the change in the value of the CRs in $S_2$, when the CRs in $S_1$ are added to the coalition $S$. It can be easily seen that if any proposed coalition $S$ has $\hat{a}(S_j \cup S_k) > 0$, then the merger of $S_1$ and $S_2$ to form $S$ would do more good than harm to the overall value of the coalition $S$. Formally, it is assumed that to maximize the achievable throughput of the proposed coalition $S$, an altruistic coalition decides to form the coalition $S$ whenever

$$\hat{a}(S) > 0.$$  \hfill (56)

### 4.4 Dynamic coalition formation model

The dynamic coalition process can be modeled using a Markov chain, with each state representing a different coalition structure (as explained in the previous chapter). A finite set $C = \{C_1, C_2, ..., C_M\}$ of all possible coalition structures for $M$ CRs forms the state space of the coalition formation game.

To incorporate slow changes in the network configuration (for e.g. due to CR mobility), the initial round of coalition formation game restarts some time $T$ after reaching equilibrium. This time $T$ may be assigned according to variations in the network configuration. It is assumed that during one round of coalition formation the received primary user’s SNR, i.e., $\gamma_{r,i}$, and the transmission rate $R_i$ of each CR $i$ does not change. At the very beginning of each round of the coalition formation game, the
distributed CR network is composed of all singleton coalitions, i.e., non-cooperative CRs.

### 4.4.1 Selfish model

Any coalition may propose a coalition structure change to another coalition in the current state $C_k$, and if all the CRs in the proposed coalition are at least as well off as before the merge, the game moves to $C_l$. When each of $n$ prevailing coalitions with some probability $p$ proposes a new coalition structure then the transition probabilities for the $M$ CR selfish coalition game with coalition structures as state space $C$ are given as:

$$P_{C_k C_l} = \frac{2p(1-p)^{|C_k|-1}}{|C_k| - 1}, \forall i, j,$$  \hspace{1cm} (57)

where $1[Y]$ is an indicator function equal to 1 if condition $Y$ is satisfied, and zero otherwise, where $Y$ represents $[(v_i(S) - v_i(S_1)) \geq 0$ and $(v_j(S) - v_j(S_2)) \geq 0]$, $v(S_1)$ and $v(S_2)$ are the values of the two coalitions participating in the coalition formation to form the proposed coalition $S$, $v(S)$ is the value of the proposed coalition $S$ and $\Omega_l$ represents the set of all new possible coalition structure states to which coalitions can transit from $C_k$. The set $\Omega_l$ is given as:

$$\Omega_l = \left\{ \{S_1 \cup S_2, S_3, \ldots, S_{|C_k|}\}, \{S_1, S_2 \cup S_3, \ldots, S_{|C_k|}\}, \ldots, \{S_1, S_2, \ldots, S_{|C_k|-1}, S_{|C_k|}\} \right\}.$$  \hspace{1cm} (58)

Given $|C_k| > 1$ coalitions in the present state $C_k$, it is possible to transition from $C_k$ to one of the $|\Omega_l|$ possible states. As explained previously, two coalitions can merge only if all CRs within the proposed coalition $S$ are at least as well off as before the merge. The indicator function $1[Y]$ in (57) represents the possible agreement or disagreement among the CRs participating in the coalition game to form the proposed coalition $S$. As the transition probability at any present state $C_k$ does not depend upon the prior states of the coalition structures, the Markov property holds.

### 4.4.2 Altruistic model

For the altruistic-cooperation case, the condition $Y$ (coalition formation decision) in (57) is changed from $[(v_i(S) - v_i(S_1)) \geq 0$ and $(v_j(S) - v_j(S_2)) \geq 0]$ to $[\delta(S) > 0]$. It simply
means that each CR now decides to merge if the merger of two coalitions would do more good than harm to the overall value of the merged coalition $S$.

Using simulation results in Section 4.5 the performance of the coalition formation based on altruistic decision will be compared with the selfish decision in terms of average throughput per CR, for different network sizes.

### 4.4.3 Steps in coalition formation

The coalition formation game involves five steps. The five steps of the proposed model are summarized as follows:

1. **Node discovery**: Discover the CRs within the network.
2. **Initialization**: Each individual CR computes its received SNR $\gamma_i$ from the PU.
3. **Coalition formation proposal**:
   a) At each time slot, each coalition, with probability $\pi$, proposes a new coalition structure. In this process, one of the CRs in the coalition acts on its behalf. (In the case of singleton coalitions, each singleton coalition, i.e., each CR, individually proposes a new coalition structure with some probability $\pi$; when two or more CRs form a coalition $S$, then any CR within $S$ is selected as a coalition head to propose a new coalition structure with some probability $\pi$ on behalf of that coalition.)
   b) The evolution from one coalition structure to the next can only occur through the merging of two existing coalitions. For instance, any coalition head of any existing coalition $S_1$ may propose to merge with another coalition $S_2$, forming $S_1 \cup S_2 = S$.
4. **Coalition formation decision**:
   a) When the CRs are assumed to be selfish then they form a coalition if $\forall i, j$, where $i \in S_1$ and $j \in S_2$, $(\phi_i(S) - \phi_i(S_1)) \geq 0$ and also $(\phi_j(S) - \phi_j(S_2)) \geq 0$.
   b) When the CRs are assumed to be altruistic then they form a coalition if $\hat{a}(S) > 0$. (see Section 4.4.1 and 4.4.2 for the details).

The steps 3 and 4 of the coalition formation are repeated until all the coalitions have made their coalition formation decisions, resulting in a final stable coalition structure $C_F$.

5. **Coalitional spectrum sensing**: Each CR within a coalition computes its local sensing
decision and transmits it to the coalition head over the common control channel. The coalition head combines the local sensing decisions (including its own sensing decision) using an OR decision fusion rule.

The proposed distributed coalition formation can be performed by coalition formation message exchanges between CRs over a common control channel. In practice, if all CRs use the same control channel to report sensing decisions to their respective coalition heads then the reporting time may increase. To decrease the reporting time coalitions may use different control channels. However, since the number of available channels in general is limited and the number of coalitions may be large, control channels must be spatially reused.

Using standard theory of absorbing Markov chains one can calculate (as presented in the previous chapter) how long will it take for the process to reach stable coalition structures, i.e., one can calculate the mean time $\mu$ and its variance $\vartheta^2$ for the dynamic coalition game starting from the initial state of all singleton coalitions to reach stable coalition structures (where no two coalitions have an incentive to merge) [55].

### 4.4.4 Analysis of the dynamic coalition formation

To perform the coalition formation, CRs need to exchange their received primary user’s signal-to-noise ratio (SNR). The amount of information exchange necessary to reach stable coalition structures can be measured by the total number of coalition formation proposals sent by the $M$ CRs during the coalition formation process. For instance if $m$ number of coalition formation proposals are sent during the entire coalition formation process and each proposal requires the exchange of $D$ messages for coalition heads to take a coalition formation decision then to reach equilibrium $m \times D$ messages need to be exchanged among the CRs. When the loss of coalition formation proposals due to collisions among distributed uncoordinated CRs is not taken into account then in the worst case, i.e., where almost all proposals are rejected, the number of proposals is only $\binom{M}{2} + \sum_{i=1}^{M-2} i$, where $\binom{M}{2} + \sum_{i=1}^{M-2} i = M^2 - 2M + 1$. Thus for the worst case, coalition formation proposals can be said to be order of $O(M^2)$. In the best case, i.e., where all proposals are accepted and the coalition formation leads to the formation of the grand coalition, the number of coalition formation proposals necessary is only $M$. Thus for the best case, coalition formation proposals can be said to be order of $O(M)$. In practical scenarios the complexity is between these two extremes.

115
This reduction in the information exchange is due to the reason that when coalitions are formed then instead of all the CRs only the coalition heads exchange proposals on behalf of their respective coalition members.

4.4.5 Stable coalition structures

As CRs form self-organizing spectrum sensing coalition structures, this research analyzes under what conditions the coalition formation process will reach a stable coalition structure (where no two coalitions have an incentive to merge anymore).

1. Selfish Coalition Formation: For the proposed selfish coalition formation, a coalition structure state $C^*$ is an equilibrium state if it satisfies the following condition: $\forall S_j, S_k, k \neq j \in C^*$, for some $i \in S_j \left( \phi_i(S_j \cup S_k) - \phi_i(S_j) \right) < 0$, or for some $i \in S_k \left( \phi_i(S_k \cup S_j) - \phi_i(S_k) \right) < 0$.

The above-stated condition ensures that no two coalitions in the prevailing coalition structure $C^*$ have an incentive to merge anymore.

The following simple fact proves that the selfish coalition formation process converges to an equilibrium state: In the proposed selfish coalition formation, if a certain coalition structure is not an equilibrium state, there must exist at least two coalitions that can decide to merge to improve their value functions. As long as such two coalitions exist, the coalition structure changes to another coalition structure, until an equilibrium state is reached.

2. Altruistic Coalition Formation: For the proposed altruistic coalition formation, a coalition structure $C^*$ is an equilibrium state if it satisfies the following condition: $\forall S_j, S_k, k \neq j \in C^*$, if $\hat{a}(S) \leq 0$.

Following the same reasoning as we did for the selfish coalition formation process, it is easy to see that the altruistic coalition formation process converges to an equilibrium.

4.4.6 Coalition sizes

For the proposed selfish coalition formation where CRs seek to maximize their payoffs, conditional on feasibility, it is interesting to note that the higher the $\gamma_i$ (the signal power received by CR $i$), the lower the $P_{f,i}$ given by (49). CRs with high values of $\gamma_i$ may have either less or no incentive to cooperate with CRs with low values of $\gamma_i$ (or with high $P_{f,i}$). Hence, cooperation among all CRs is desirable only when all CRs experience
Fig 35. Average throughput per CR for different network sizes and for different scenarios.
similar $\gamma_{ri}$. Therefore, in the proposed coalition formation, the grand coalition of all the CRs may not always form.

For the proposed altruistic coalition formation where CRs seek to merge if the merger of two coalitions would do more good than harm to the overall value of the merged coalition, it is interesting to note that the CRs with high $\gamma_{ri}$ values may form coalitions with the CRs having low $\gamma_{ri}$ values as long as the overall value of the newly formed coalition $S$ is increased.

Using simulation results in the next section average maximum number of CRs per formed coalition for both selfish and altruistic coalition formation will be presented.

4.5 Simulation results

Using simulations the aim of this research is to compare the performance (e.g. in terms of average throughput per CR) of the coalition structure that emerges as the outcome of the proposed coalition formation solutions to a non-cooperative solution and to the grand coalition.

For simulation illustrations, the following distributed CR network is set up: $M$ CRs are uniformly and independently distributed in a circle with radius $R_s = 1000\text{m}$ and centered at the coordinates $(\beta,0)$. The PU transmitter is at coordinates $(0,0)$ as
shown in Fig. 32. The sensing time \( \tau_s = 1\, \text{ms} \), the time-bandwidth product is set as \( T_B = 6000 \), and the frame duration is set to be \( T = 100\, \text{ms} \). The path loss exponent \( \alpha_p \) is set to 3. The PU power \( P_{PU} \), scalar \( \psi \) and noise power \( \sigma_i^2 \) are set at a value such that \( \gamma_{ri} \) (PU’s SNR at CR \( i \)) at the coordinates \((\beta, 0) = (2000, 0)\) is -15dB. The probability of primary user present is assumed to be \( \theta = 0.2 \). To keep our simulation analysis simple, it is assumed that all the CRs have the same transmission rate, i.e., \( R_i = R = \log(1 + \text{SNR}^r) = 3.4594\, \text{bits/sec/Hz} \) in (54), where \( \text{SNR}^r \) is signal-to-noise ratio from a CR to its receiver. Simulations were performed by “dropping” the CRs randomly around the coordinates \((\beta, 0)\).

For the target detection probability \( \hat{P}_d = 0.9 \) and \( \hat{P}_d = 0.99 \). Figs. 35(a) and 35(b) show the average (averaged over the simulation runs) throughput per CR for different network sizes, when all CRs sense independently (non-cooperative strategy), and when CRs can form coalitions (selfish and altruistic). It can be seen from Fig. 35(a) and 35(b) that the proposed coalition formation (both selfish and altruistic) yields an improvement in the average throughput as compared to the non-cooperative solution. However, the selfish coalition formation solution leads to a loss in average throughput as compared to the altruistic coalition formation solution. Fig. 36 compares the performance of altruistic and selfish coalition formation solutions with the non-cooperative solution in terms of average false alarm probability per CR for different network sizes. It can be seen that the altruistic coalition formation solution significantly reduces the average false alarm per CR, as compared to both selfish coalition formation and non-cooperative solutions. It can also be seen from Figs. 35 and 36 that the weighted target detection probability for individual CRs in a coalition results in better average throughput and reduced average false alarm per CR as compared to when each CR is assigned the same target detection probability.

When the overhead cost, i.e., the cost in terms of collecting and combining sensing reports at the coalition head is not taken into account then the altruistic coalition formation solution yields the same results as if all the CRs perform cooperative sensing, i.e., form the grand coalition, see Fig. 37. This is because, for no overhead cost, with increasing the number of cooperating CRs, a target detection probability may be achieved by having low detection probability at the individual CRs. The low detection probability at the individual CRs is translated to a low false alarm probability and therefore increase in throughput. However, in practice the reporting of the local CR sensing decisions to the report combining entity (coalition head, fusion center, etc.) incurs overhead in the sense that the entire reporting group cannot transmit until all
the sensing reports are collected and combined by that entity. In the literature (also in IEEE 802.22), polling of CRs by the report combining entity is suggested for the collection of sensing reports [22, 45]. This method has communication overhead that increases linearly with the number of cooperating CRs. When the overhead cost of collecting and combining sensing reports is taken into account (for instance, $\tau_c$ in (54) is set to 1ms) then it can be seen in Fig. 37 that the performance of the grand coalition degrades significantly as the number of CR increases, as compared to the altruistic coalition formation solution. Fig. 38(a) compares the performance of altruistic and selfish coalition formation solutions in terms of average throughput per CR for a network size of 20 CRs. In this figure, it is shown that for small $\tau_c$, the altruistic coalition formation (same $P_{d,i}$) solution yields significant average throughput gains when compared with the selfish coalition formation solution (same $P_{d,i}$). However, when $\tau_c$ is large then the selfish coalition formation solution (same $P_{d,i}$) either outperforms or at least matches altruistic solution (same $P_{d,i}$) with respect to average throughput per CR. This is because the average coalition size for the altruistic coalition formation solution (same $P_{d,i}$) is large as compared to the selfish coalition formation solution (same $P_{d,i}$), see Fig. 38(b), and for large values of $\tau_c$, the sensing reporting overhead becomes significant. Due to this reason, the large coalitions can be more costly as compared to the small ones.
(a) Average throughput per CR for selfish and altruistic coalition formation solutions, when the values of $\tau_c$ (the time spent for reporting a sensing decision to the coalition head) are varied between 0.001ms to 0.017s, $\hat{P}_d = 0.99$ and $\beta = 2000m$.

(b) Average maximum coalition size for selfish and altruistic coalition formation solutions, when the values of $\tau_c$ (the time spent for reporting a sensing decision to the coalition head) are varied between 0.001ms to 0.017s, $\hat{P}_d = 0.99$ and $\beta = 2000m$.

Fig 38. Average throughput per CR and average maximum coalition size for different network sizes and for different scenarios.
To illustrate the above situation, an example can be constructed for $M = 3$ CRs, where the altruistic coalition formation solution (same $P_{d,i}$) outperforms selfish solution (same $P_{d,i}$) when $\tau_c$ is small. However, for the same scenario, when $\tau_c$ is set to be large then selfish coalition solution (same $P_{d,i}$) may outperform altruistic solution (same $P_{d,i}$).

**Example 4.5.1.** Let $\gamma_{c,1} = 0.0263$, $\gamma_{c,2} = 0.0148$ and $\gamma_{c,3} = 0.0233$ be the received primary user’s SNRs at $M = 3$ CRs. In this three-CR game, when $\tau_c = 0.001 ms$ and assuming $R_1 = R_2 = R_3 = 3.4594$ bits/sec/Hz then calculating coalition values using (55) and rules explained in Sections 4.2 and Section 4.3 we obtain: $v(\{1\}) = 0.9968$, $v(\{2\}) = 0.3076$, $v(\{3\}) = 0.7718$, $v(\{1,2\}) = (0.9198, 0.9198)$, $v(\{1,3\}) = (1.4433, 1.4433)$, $v(\{2,3\}) = (0.8272, 0.8272)$, $v(\{1,2,3\}) = (1.3036, 1.3036, 1.3036)$. It may be seen that the altruistic coalition solution (same $P_{d,i}$) for this scenario results in the formation of grand coalition $\{(1,2,3)\}$ of all CRs. However, the selfish coalition solution (same $P_{d,i}$) results in the formation of either $\{1,3\}$ or $\{2,3\}$. Since, $v(\{1,2,3\})$ is greater than $v(\{1,3\}) + v(\{2\})$ and $v(\{2,3\}) + v(\{1\})$, therefore, the altruistic solution (same $P_{d,i}$) outperforms the selfish solution (same $P_{d,i}$) for this scenario. However, when $\tau_c = 15 ms$ then we obtain: $v(\{1\}) = 0.9968$, $v(\{2\}) = 0.3076$, $v(\{3\}) = 0.7718$, $v(\{1,2\}) = (0.7805, 0.7805)$, $v(\{1,3\}) = (1.2247, 1.2247)$, $v(\{2,3\}) = (0.7019, 0.7019)$, and $v(\{1,2,3\}) = (0.9086, 0.9086, 0.9086)$. It may be seen that the selfish coalition solution (same $P_{d,i}$) for this scenario results in the formation of $\{1,3\}$ or $\{2,3\}$. However, the altruistic coalition solution (same $P_{d,i}$) results in the formation of either $\{1,2,3\}$ or $\{1,3\}$ or $\{2,3\}$. Since, $v(\{1,3\}) + v(\{2\})$ is greater than $v(\{1,2,3\})$, therefore, the selfish solution (same $P_{d,i}$) either outperforms or at least matches altruistic solution (same $P_{d,i}$) for this scenario.

It can also be seen from Fig. 38(a) that for very large values of $\tau_c$, the altruistic solution (same $P_{d,i}$) matches the selfish solution (same $P_{d,i}$) in terms of average throughput per CR. This is because for very large values of $\tau_c$ there are either very little or no gains in terms of average throughput per CR for coalition formation, and network structure is mostly composed of individual non-cooperative CRs (see Fig. 38(b)).

In Fig. 39, the parameter $\beta$ (distance between the primary user base station and the center of the distributed CR network, see Fig. 32) is varied, and average maximum coalition sizes for the selfish and altruistic coalition formation solutions are shown. It can be seen from Fig. 39 that for large $\beta$, the network for such scenarios is composed of coalitions of large sizes. In this figure, it also shown that for the given simulation scenario the altruistic coalition formation solution results in the formation of the grand
coalition of all CRs whenever $\beta \geq 2000$ m. As the value of parameter $\beta$ is decreased, the network for such scenarios is composed of independent disjoint coalitions of smaller sizes. It can also be seen from Fig. 39 that as the value of the parameter $\beta$ is reduced to 500 m then the coalition formation solution results in a network structure mostly composed of individual non-cooperative CRs, i.e., all singleton coalitions.

4.6 Summary

In this chapter, using a coalition game-theoretic framework distributed cooperative strategies for CRs that are either selfish or altruistic are devised. A coalition formation model is proposed for these CRs to utilize primary user spectrum efficiently, under the constraint of a target probability of detection. The proposed model takes into account the cost of distributed cooperative sensing in terms of overhead in combining sensing reports within a coalition. The impact of this cost on the distributed cooperative strategies of CRs is also evaluated. Given a target detection probability for a coalition, a weighted target detection probability is adopted for individual CRs in a coalition and its impact on the average throughput per CR is analyzed. Using simulations the performance of
the proposed altruistic and selfish coalition formation solutions in terms of gains in increased average throughput per CR are evaluated. These results are compared to the throughput achieved by a non-cooperative strategy and by the grand coalition (when all the CRs cooperate).
5  Selection of best detection performance devices for cooperative sensing

Everyone is constantly faced with the problem of choosing among several alternatives. This choice is a decision about which alternative is best, in some well-defined sense of best. Some decisions are of relatively little important in the long run, such as when an individual is selecting a pair of shoes to purchase. Other decisions are of greater significance to the individual, such as the choice of which college to attend or of whom (or whether) to marry.

In this chapter, the problem of best detection performance CR selection for cognitive radio networks is studied, where the spectrum allocation and access procedures are controlled by the fusion center and each CR performs spectrum sensing and sends its sensing report to the fusion center (data collector). Cognitive radios in such networks cannot transmit data until sensing reports are collected and the final combined sensing decision is transmitted to all the cooperating CRs in the network (as explained in the previous chapter). In sensing reports collection there is cost in terms of time delay in data transmissions of a network due to the overhead in combining sensing reports. This cost generally increases with the number of cooperating CRs, as more decisions need to be reported to the fusion center [22, 48, 49]. Moreover, a large number of CRs performing cooperative sensing typically leads to an increase in total energy consumption of the network. To reduce the sensing overhead and total energy consumption of a cognitive radio network three methods are proposed for the selection of the best detection performance CRs. The best detection CRs are defined to be those CRs that have the highest probabilities of detection. The proposed methods perform this challenging task by using binary decisions sent by each CR to the fusion center.

5.1 Related literature

It is shown in [101] that optimum (best) detection performance is usually achieved by cooperating only with the CRs that have the highest primary-user-signal-to-noise ratio (SNR) values. However, in general, the SNRs of different CRs are not known, so that it
is not known \textit{a priori} which CRs have the best detection performance.

The received SNR can be estimated, for example, using soft information as in [107]. However, sending soft information to the fusion center requires estimation of SNR. Sensor selection based on the knowledge of sensor positions is presented in [108]. In [103, 109, 110], iterative algorithms to estimate the probabilities of detection and false alarm of sensing nodes are proposed. However, these papers derive their results based on the assumption that if the local decision is the same as the final fused decision, it is the correct decision. Also in [109, 111], weights are assigned to each CR’s local sensing decision. These weights are computed by estimating the probabilities of detection and false alarm of CRs in the network. However, it is important to emphasize that the proposed methods in [109] do not select the best CRs, but only assign weights to their decisions. Moreover, it is also noteworthy that the estimation of detection and false alarm probabilities would require a large number of samples. The problem of making a prediction about binary events based on local decision makers (experts) with unknown performances has been studied extensively in machine learning (see [8] and references therein). However, in most of these works it is assumed that either there is at least one expert that makes always perfect decisions or the actual value of the binary outcome is known after the data collector makes the final decision. In [112], symmetric error probabilities are assumed and a method is proposed for estimating the competence of the local decision makers without the need to know the actual value of the estimated binary event.

It is important to note that the sensor selection schemes proposed in the existing literature for wireless sensor networks (WSN) are mostly formulated to guarantee the prolonged unattended deployment of the WSN for long periods of time, such as months or even years [113]. However, CRs in CR networks do not require such prolonged unattended deployment. Hence, new methods are required for intelligent selection of the CRs. Our work in [53] considered the usage of hard (binary) decision information from CRs at the decision-making process to select the best detection performance CR.

5.2 System model

The system setup used in this section consists of a set \( M = \{1, 2, \ldots, M\} \) of CRs. The number of CRs selected by the proposed methods is \( \kappa \). The total number of best detection performance CRs in the network is denoted by \( \varsigma \). There is one primary user (PU) (denoted as a base station (BS) in Fig. 40), at the origin. This corresponds to, for
example, using only downlink frequencies for the secondary access. The fusion center (FC) is at coordinates $(\beta, 0)$. The CRs are uniformly and independently distributed in a circle centered at the FC with radius $R_S$, as shown in 40. The PU and CRs are both assumed to use time slotted system [100]. The probability of PU present, $\theta$, and the probability of PU absent, $1 - \theta$, in a given time slot are assumed to be unknown. One transmission by PU corresponds to one time slot. The CRs use the beginning of each slot for sensing. The sensing process at each CR is characterized with detection probability $P_{d,i}$ and false alarm probability $P_{f,i}$. For energy detector (ED) in Rayleigh fading channel the detection and false alarm probabilities for CR $i$ are [44, 114]

$$P_{d,i} = e^{-\lambda} \frac{T_B}{\pi} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{\pi} \right)^n \gamma_i^n$$

$$\times \left[ e^{-\frac{\lambda}{2(1+\gamma_i)}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{2(1+\gamma_i)} \right)^n \right]$$

$$P_{f,i} = \tilde{P}_f = \frac{\Gamma(T_B, \frac{\lambda}{2})}{\Gamma(T_B)}$$

where $T_B$ is the time-bandwidth product of the energy detector, $\gamma_i$ is the average SNR (from the PU), $\lambda$ is the ED detection threshold and $\tilde{P}_f$ is the desired false alarm probability. For simplicity and also because adapting threshold before sensor selection can be difficult, it is assumed that each CR uses the same threshold $\lambda$ so that the individual false alarm probability is equal to $\tilde{P}_f$ [33, 101]. The average SNR for CR $i$ using path-loss model can be found with $\bar{\gamma}_i = \frac{\pi}{d_i^{\alpha_p}}$, where $d_i$ is the distance of the CR $i$ to the PU, $\alpha_p$ is the path-loss exponent, and $\pi$ is a constant depending on the transmit power, noise level, $T_B$, etc., see [83] for details. To focus on the performance evaluation of the proposed selection methods, it is assumed that an error-free separate control
channel is used for sending the local CR decisions to the FC.

A training period of $T_p$ slots is assumed. In each slot CR $i$ performs sensing and produces local hard decisions. The local decisions are collected by the FC. At the end of $T_p$ slots, based on the local hard decisions ($T_p$ decisions from each CR), the FC selects for cooperation $\kappa$ CRs using the proposed selection methods.

At the start of each cognitive radio selection round, i.e., at the very start of training period, the FC assigns an identity number to each cognitive radio. The FC grants a contention free channel to individual cognitive radios by polling them (using their identity numbers) for transmitting their local decisions. The FC may employ the round-robin scheduler \cite{48}, and on being polled, a CR transmits its local decision to the FC. At the end of the selection period, only the cognitive radios with the best detection performance are polled.

It is also possible to communicate (transmit) during the training period as we can combine all the local decisions which are reported to the FC anyways. This means that there is no more delay than usual in these kinds of systems \cite{19}.

Mobility of CRs causes changes to the probabilities of detection. Slow mobility can be taken into account by periodically performing the CR selection process. This research does not assumes that any information about CR positions are known at the FC, so it is not possible to assess the correlation between observations made by the various CRs. Therefore, the methods treat the measurements of CRs as conditionally independent, which means that the measurements of CRs are independent, but that for each CR the same hypothesis $\{H_0, H_1\}$, $H_0 =$PU not present and $H_1 =$PU present, applies. This assumption is in line with the other works \cite{33, 44, 115}.

5.3 The proposed methods

Three methods for selecting the best detection performance CRs are proposed: Simple Counting (SC), Partial-Agreement Counting (PAC), and Learning from Collisions (LC).

5.3.1 Simple counting (SC)

Given that each CR has the same target false alarm rate, it is intuitive to count the number of ones (PU present decisions) during the training period and select for cooperation those CRs that have the highest counts. In case of ties (in terms of counts of ones), randomized selection is assumed. The presented approach can be justified as follows. In
a given slot, let us denote the sensing decision of CR \( i \) by \( \delta_i \in \{0, 1\} \), and the real state of the PU by \( \rho \in \{0, 1\} \). The probability that the CR \( i \) declares the PU to be present (either due to correct detection or false alarm) is given as

\[
\Pr(\delta_i = 1) = \Pr(\rho = 0) \Pr(\delta_i = 1 \mid \rho = 0) + \Pr(\rho = 1) \Pr(\delta_i = 1 \mid \rho = 1) = (1 - \theta)\tilde{P}_f + \theta P_{d,i}.
\]

Since, in (61), the first term and \( \theta \) are the same for every CR, the CRs with the best detection performance are simply those CRs that have the highest probability of deciding that the PU is present, \( \Pr(\delta_i = 1) \). In practice, \( \Pr(\delta_i = 1) \) is not known at the FC. Instead, the above mentioned counting based approach can be applied. It is assumed that every CR has the same target false alarm rate \( \tilde{P}_f \) and that there is only negligible amount of noise-uncertainty. In practice, the noise level at each CR is not known perfectly and, thus, the actual false alarm rates will be somewhat different from the desired target value. Note, that this method is vulnerable to malicious or malfunctioning nodes always sending ones.

### 5.3.2 Partial-agreement counting (PAC)

In this method, it is assumed that the \( M \)th CR is the fusion center that performs local sensing as well as collects information from the other CRs. Assume that the FC (at fixed location) has reasonably high \( P_{d,M} \), due to, for example, a high performance antenna at a good location. Also, for this approach the primary BS should be at fixed location so that the detection performance of the FC does not vary too much. With these assumptions, it is reasonable to count how many times each CR agrees with the FC (they both say one), and select for cooperation those CRs that have the highest agreement with the FC (which is here always included for cooperation). This approach can be motivated as follows. Let \( \alpha_{iM} \) represent the probability that both the CR \( i \) and the FC declare the PU to be present (either due to correct detection or false alarm). This probability can be expressed as

\[
\alpha_{iM} = (1 - \theta)\tilde{P}_f + \theta P_{d,i} P_{d,M}.
\]

Since, in (62), the first term, \( P_{d,M} \), and \( \theta \) are the same for every CR, the CRs with the best detection performance are simply those CRs that have the highest \( \alpha_{iM} \). Results in Section 5.4 show, this approach can be used even when the FC is not always correct.

In practice, \( \alpha_{iM} \) is not known at the FC. Instead, we can use the above mentioned PAC method for the selection.
5.3.3 Learning from collisions (LC)

The (LC) method is different than previous ones because its performance rely on the specific decision fusion rule employed at the FC to make global detection decisions during the training period. In this method, the FC calculates the correctness count for every CR \( i \). In other words, the FC calculates how many times the CR \( i \) correctly detected the presence of the PU, when the global (false) decision was that the PU is not present, i.e., there was a missed detection by the global decision. At the end of \( T_p \) slots, the FC selects the CRs with the highest correctness counts. In this method, the FC learns from collisions so that the CRs can better avoid them after the training period.

The (simplified) collision detection can be achieved as follows. Assume that the global decision is that the PU is not present in the studied slot. Then the FC transmits channel release (ChR) message to the CRs on the PU channel, indicating that the channel is available for message exchange. The CRs, upon receiving the ChR successfully, reply with an acknowledgement (ACK). The FC upon unsuccessful ChR-ACK exchange concludes that a missed detection error has occurred. In practice, an unsuccessful ChR-ACK exchange may also be caused by channel fading or interference from the other users [116]. It is also interesting to note that due to the capture effect, ChR-ACK message exchange may be successful even if collision with the PU occurs. If the probability of correct collision detection is denoted by \( \sigma_1 \) and that of false collision detection by \( \sigma_0 \), the two probabilities can be expressed as follows [116]

\[
\begin{align*}
\sigma_1 &= \Pr[\text{Collision detected}|\text{Collision with PU}] \\
\sigma_0 &= \Pr[\text{Collision detected}|\text{No Collision with PU}]
\end{align*}
\]

5.4 Results

5.4.1 Indifference zone approach

Let us apply the indifference zone approach (IZA) [117] and assume \( \varsigma \) CRs with high detection probabilities ("good set"). In the IZA, the distance measure \( \theta_d \) is a measure of the differences between the population we want to identify and the remaining populations. In terms of CR selection, this research defines the \( \theta_d \) as the difference between the smallest probability of detection among the \( \varsigma \) best CRs and the largest probability of detection among the \( M - \varsigma \) of the remaining CRs.

Using the IZA [117], it is generally assumed that CRs in the good set are equal (here
Table 4. Required number of samples for \( P_{CS} = 0.9 \), #1 refers to situations with \( \tilde{P}_f = 0.01 \) and #2 to situations with \( \tilde{P}_f = 0.1 \), ([53] [©IEEE 2010]).

<table>
<thead>
<tr>
<th>( \theta_d )</th>
<th>SC#1</th>
<th>SC#2</th>
<th>PAC#1</th>
<th>PAC#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>&gt;500</td>
<td>&gt;500</td>
<td>&gt;500</td>
<td>&gt;500</td>
</tr>
<tr>
<td>0.2</td>
<td>292</td>
<td>&gt;500</td>
<td>241</td>
<td>300</td>
</tr>
<tr>
<td>0.3</td>
<td>148</td>
<td>370</td>
<td>128</td>
<td>151</td>
</tr>
<tr>
<td>0.4</td>
<td>95</td>
<td>218</td>
<td>86</td>
<td>97</td>
</tr>
</tbody>
</table>

Table 5. Required number of samples for \( P_{CS} = 0.9 \), #3 refers to situations with "MAJORITY" rule, #4 and #5 to "AND" rule (with perfect and imperfect collision detection, \( \sigma_0 = 0.1 \) and \( \sigma_1 = 0.9 \), respectively) employed at the fusion center, ([53] [©IEEE 2010]).

<table>
<thead>
<tr>
<th>( \theta_d )</th>
<th>LC#3</th>
<th>LC#4</th>
<th>LC#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>&gt;500</td>
<td>&gt;500</td>
<td>&gt;500</td>
</tr>
<tr>
<td>0.2</td>
<td>&gt;500</td>
<td>229</td>
<td>318</td>
</tr>
<tr>
<td>0.3</td>
<td>&gt;500</td>
<td>124</td>
<td>167</td>
</tr>
<tr>
<td>0.4</td>
<td>&gt;500</td>
<td>85</td>
<td>100</td>
</tr>
</tbody>
</table>

in terms of the detection probability) and that CRs in the bad set are also equal. It is important to emphasize that this assumption gives worst configuration for any given distance measure \( \theta_d \). In a sense \( \theta_d \) measures the minimum separation of the probability of detection between the two groups composed respectively of the \( \varsigma \) best CRs and the \( M - \varsigma \) worst CRs.

For all studied methods, this research utilizes the probability of making a correct selection for a given distance measure \( \theta_d \) as the basis for comparison between the proposed methods. Let us denote the probability of correct selection by \( P_{CS} \). The probability of correct selection (\( P_{CS} \)) is the probability that all members of the "good set" have been selected for cooperation.

Computer simulations were performed using the indifference zone approach with various values of \( \theta_d \). Tables 4 and 5 summarize the results which present (approximately)
the required number of samples, i.e., the length of the training period, $T_p$, for the proposed methods to obtain $P_{CS} = 0.9$ with $\theta = 0.2$ (not known to the system), $\kappa = \varsigma = 4$, and $M = 9$.

It can be seen from Table 4 that the SC method returns good selection with $\hat{P}_f = 0.01$ (SC#1). However, its selections performance is reduced when $\hat{P}_f = 0.1$ (SC#2). The PAC method returns a good selection both with $\hat{P}_f = 0.01$ and $\hat{P}_f = 0.1$ (PAC#1 and PAC#2).

Results for the LC method are shown in the Table 5. It can be seen that the LC method has poor performance when using the “MAJORITY” rule. This is because with the assumed parameters, “MAJORITY” rule is correct most of the time. This means few collisions, which leads to a poor selection performance. With “AND” rule, there are more collisions during the training period, leading to improved performance. Thus, there is a trade-off between collisions during the (rather short) training period and better selection performance (which can lead to less collisions in the longer term). From Table 5, it can also be seen that with “AND” rule, the LC method with the perfect collision detection assumption requires only 124 samples to have $P_{CS} = 0.9$ for $\theta_d = 0.3$. However, if the probabilities of correct collision detection and false collision detection
Missed detection probability with the “MAJORITY” rule, \( M = 9 \) CRs, \( P_{fS} = 0.1 \), \( T_b = 5 \), \( \theta = 0.2 \), \( T_p = 300 \), \( \beta = 2 \) km, and \( R_S = 1 \) km, ([53] [©IEEE 2010]).

Given by (63) are set to \( \sigma_1 = 0.9 \) and \( \sigma_0 = 0.1 \), the performance of the LC method degrades and now it requires 167 samples to have \( P_{CS} = 0.9 \) for the same \( \theta_d = 0.3 \). Thus imperfect collision detection somewhat reduces the performance of the proposed selection method. When \( \sigma_1 \) is only 0.5, more than 300 samples are required to have \( P_{CS} = 0.9 \) for \( \theta_d = 0.3 \).

### 5.4.2 System performance

Assume a path-loss exponent \( \alpha_p = 3 \) and the time-bandwidth product \( T_b = 5 \). Also, it is assumed for illustration purposes that \( \pi \) equals 110 dB [83] (distances are in meters). The “MAJORITY” rule is used for global detection decisions after the selection process has been performed. Note that since the same threshold for all CRs is assumed [44], the fusion rule thus used is not optimal. The false alarm probability per CR was selected so as to achieve the system false alarm probability of \( \bar{P}_{fS} = 0.1 \). Simulations were performed by “dropping” the CRs randomly around the FC. Fig. 41 shows the average (averaged over the drops) missed detection probability for \( \kappa = 1, 2, 3, 4 \) CRs when the SC method is used for CR selection. It can be seen from Fig. 41 that after \( T = 300 \)
samples, there are no further significant gains. Fig. 42 shows the average missed detection probability for random selection and counting based selection (using the SC method) for $\kappa = 1, 2, \ldots, 9$ with $T_p = 300$ samples. The difference between random selection and counting based selection decreases as $\kappa$ increases. It can be seen from Fig. 42 that to achieve system detection probability of 0.95, the SC method requires only $\kappa = 2$ CRs, whereas the random selection needs two times more CRs ($\kappa = 4$).

5.5 Summary

In cooperative spectrum sensing, information from several cognitive radios (CRs) is used for detecting the primary user. To reduce sensing overhead and total energy consumption, selection of a few CRs for cooperation that have better detection performance than others is recommended. However, it is not known a priori which of the CRs have the best detection performance. Methods are proposed for the fusion center (data collector) to select the CRs with the best detection performance based only on hard (binary) decisions from the CRs. Simulations are used to evaluate and compare the methods. The results indicate that the proposed CR selection methods are able to offer significant gains in terms of system performance.
6 Conclusions and future directions

The imagination must feed on the past in creating the future.

Cognitive wireless devices are envisioned to make better use of available network resources through learning from and adaptation to their environment. The work described in this thesis designs adaptation and coordination techniques for wireless communication access that enable the wireless devices to fulfill their tasks effectively and maximize network resource efficiency.

For autonomous cognitive radios operating in a distributed multichannel network, in Chapter 2, design of efficient adaptive persistent sensing order selection strategies is considered. It is observed that the performance that results from a random selection of sensing orders is limited by the collisions among the autonomous CRs. A $\gamma$-persistent strategy that enables the CRs to reduce the likelihood of collisions with one another is proposed and evaluated. It is shown that the $\gamma$-persistent strategy converges to collision-free channel sensing orders. It is observed that when adaptation is employed, there is an increase in the average number of successful transmissions in the network when the CRs select sensing orders from a predefined Latin Square, as compared to when they select sensing orders from the space of all permutations of $N$ channels. The effects of false alarm and channel errors on adaptation decisions are also explored. It is shown that even in the presence of false alarm and channel errors, the proposed $\gamma$-persistent strategy increases the average number of successful transmissions in the network.

For the spectrum sharing problem in an interference channel, in Chapter 3, a dynamic coalition formation game is analyzed, where transmitter/receiver pairs self-organize to reach stable coalition structures through a time-evolving sequence of steps. It is shown that a coalition game in an interference channel is a game that generates positive externalities and has partition function form, and its conversion to characteristic function form is presented. An absorbing Markov chain model is employed to model the equilibrium state of the grand coalition or the equilibrium state of internally and externally stable coalition structures (CSs) for the coalition game in an interference channel.
channel. Using a Markovian model of the coalition game, the dynamics of the coalition formation game and the stability of different CSs in an interference channel are analyzed. When a coalition game for various interference environments is analyzed in terms of CSs, the number of the states in the state space of the Markov chain follows the Bell number. When the state space of the Markov chain is generated by grouping the CSs according to the sizes of coalitions they contain, the number of states follows the integer partition function. It is shown that, given sufficient time, the coalition process converges either to the grand coalition or to internally and externally stable CS, depending on the interference perceived by the links. The mean and variance of the time for the game to reach the stable CSs is studied. It is demonstrated that for certain interference channel gains, although the grand coalition can yield optimal payoffs, due to the myopic nature of wireless links the grand coalition cannot be formed and links cannot obtain the optimal core payoffs. A condition for the formation of the stable grand coalition is formulated. It is shown that the restrictive condition of superadditivity for the stable grand coalition formation reduces the region of stable coalition structures, as compared to the condition formulated in this thesis.

In chapter 4, the problem of distributed throughput-efficient sensing in cognitive radio (CR) networks is formulated as a dynamic coalition formation game based on a Markovian model. The proposed coalition formation enables the CRs to increase their achievable throughput, under the detection probability constraint, while also taking into account the overhead in sensing reports combining. The dynamic model of coalition formation is used to express and model the behavior of the coalition forming CRs over time. In the proposed model, CRs formed coalitions either to increase their individual gains (selfish coalition formation) or to maximize the overall gains of the group (altruistic coalition formation). It is shown that the proposed coalition formation solutions yield significant gains in terms of reduced average false alarm probability and increased average throughput per CR as compared to the non-cooperative solutions. Given a target detection probability for a coalition, a weighted target detection probability for individual CRs in a coalition is adopted. It is observed that the weighted target detection probability for individual CRs results in increased average throughput per CR as compared to when each CR is assigned the same target detection probability in a coalition.

In chapter 5, the problem of best detection performance CR selection is studied. Three different selection methods for this purpose are proposed and evaluated. Simulation results show that the Partial-Agreement Counting method and the Collision
Detection method employing “AND rule” require less number of sensing samples to select the subset of best detection performance CRs with high probability, as compared to the Collision Detection employing “MAJORITY” rule. Also, the Simple Counting method performs well, at least when the false alarm probability of each CR is small.

There are several directions for future research relating to opportunistic multichannel access in distributed cognitive radio networks (presented in Chapter 2). The research presented in this thesis analyzes adaptive strategies for autonomous CRs with symmetric interference links. One possible extension of this research work is to explore the impact of asymmetric interference links, i.e., when CR \( i \) interferes with CR \( j \) but not vice-versa, on adaptation decisions. Moreover, it may happen that the set of available channels differs from one CR to another. In the future, the proposed adaptive strategies can be evaluated for the scenarios where the set of available channels vary from one CR to another.

The research presented in chapters 3 and 4 can be extended in several directions, for example, there are many open problems unresolved regarding the presence of externalities in coalition forming wireless networks. For instance, one issue of concern is the design of incentives for nodes to form coalitions in games with positive externalities. Furthermore, the explanation and analysis of the Markov chain model that allows links to leave the coalition formation process is also a possible topic for future research. Also, one of the extensions we envision for our work in chapter 4 is the consideration of the effects due to overhead in terms of spending time and resources due to the process of coalition formation and also the effects due to overhead in terms of energy consumed by a coalition head to collect and combine sensing reports.

The future directions of the work presented in Chapter 5 could be, for example, to investigate the problem of sensor selection using optimum subset size of CRs performing cooperative spectrum sensing. Further, the incorporation of correlation between observations made by the various CRs in the proposed methods would also be an interesting problem to address.
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Appendix 1 Derivations of the equations for probability of success

This appendix contains the derivations of equations (5), (6) and (7) in Chapter 2.

For a given number of channels $N$ there can be many Latin Squares. For instance the number of all possible Latin Squares for $N = 6$ is $812851200$. To select a sensing order from a common predefined Latin Square, CRs can employ any of the many Latin Squares. However, to explain the derivation of equations (5) and (6), for simplicity, this research utilizes a circulant matrix, which is an example of a Latin Square. A circulant matrix associated to $N$ (set of channels) is defined as the $|N| \times |N|$ matrix whose rows are given by the iterations of the shift operator acting on $N$ [51]. For example, with $|N| = 5$, the circulant matrix $\Theta$ is given as:

$$
\Theta = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
5 & 1 & 2 & 3 & 4 \\
4 & 5 & 1 & 2 & 3 \\
3 & 4 & 5 & 1 & 2 \\
2 & 3 & 4 & 5 & 1
\end{pmatrix}
$$

To validate the closed-form expressions that are derived in this appendix, these results are compared to results obtained via simulations in Fig. 15 (see Chapter 2). The simulation results are obtained when CRs employ a common predefined Latin Square that is randomly selected out of many Latin Squares. It can be seen that the values calculated from Monte-Carlo simulations agree perfectly with those obtained from closed-form equations, i.e., the probability of success is not affected by the choice of a Latin Square.

Derivations of equations (5) and (6):

In a given time slot, when CR $i$ and $j$ randomly select one of the rows (sensing orders) of a Latin Square then one of the following two events occurs. 1) CR $i$ and $j$ select the same sensing order. This is represented as event $S$. 2) CR $i$ and $j$ select different sensing orders. This is represented as event $D$. In any time slot, given that CR $i$ and $j$ select the same sensing order, the CR $i$ can obtain a channel in the $k$th step if one of the following two events happens. a) In the first $(k - 1)$ steps, the channels visited by CR $i$ are either occupied by a PU or CR $i$ has false alarmed, and in the $k$th step CR $i$ visits
a PU-free channel and does not collide with competing CR \( j \). This is represented as event \( A \) and the probability that this event will happen in the \( k \)th step is \( P_k(A \mid S) \). b) In the first \((k-1)\) steps, the channels visited by CR \( i \) are either occupied or CR \( i \) has false alarmed and in the \( k \)th step CR \( j \) has given up channel search, i.e., it has found a PU-free channel and in the \((k+1)\)th step CR \( i \) finds a PU-free channel. This is represented as event \( B \) and the probability that this event will happen in the \( k \)th step is \( P_k(B \mid S) \). The probability that CR \( i \) can obtain a free channel given that the two CRs select the same row is given by the first term in equation (4), i.e., \( P(S) = \sum_{k=1}^{N} P_k(A \mid S) + \sum_{d=1}^{N} P_k(B \mid S) \), and is derived as follows.

CR \( i \) can obtain a channel in its first step when the channel is free of primary user, no false alarm is generated by CR \( i \) and CR \( j \) generates a false alarm. So the probability

![Fig A-1. Two levels of a tree diagram for \( P(S) \) (CR \( i \) perspective).](image-url)
of success in the first step for CR $i$ is given by:

$$P_1(S) = P_1(A | S) = (1 - \theta_1)(1 - P_{fa,i})P_{fa,j} \tag{A-1}$$

If in the first step, a channel was occupied by a PU, or it was PU-free but the CR $i$ generated a false alarm, then CR $i$ can obtain a channel in the second step if it visits a PU-free channel and CR $j$ generates a false alarm in the second step (event $A$) or CR $j$ has already found a PU-free channel in the first step and has given up channel search (event $B$).

$$P_2(S) = P_2(A | S) + P_2(B | S)$$

$$= \left( (1 - \theta_1)P_{fa,i}P_{fa,j}(1 - \theta_2)P_{fa,j}(1 - P_{fa,i}) + \theta_1(1 - \theta_2)P_{fa,j}(1 - P_{fa,i}) \right)$$

$$+ \left( (1 - \theta_1)P_{fa,i}(1 - P_{fa,j})(1 - \theta_2)(1 - P_{fa,i}) \right) \tag{A-2}$$

Using a probability tree diagram $P(S)$ for $N$ sensing steps are generalized. This generalization is given by equation (5). Fig. A-1 presents the probability tree diagram for calculating $P(S)$ that represents the probabilities associated with a two-stage experiment (first and second sensing steps). However, a CR $i$ can also obtain a channel if event $D$ happens. Given that CRs $i$ and $j$ select the different rows in a given time slot, the CR $i$ can obtain a channel in the $k$th step if one of the following three events happen. a) In the first $(k - 1)$ steps, the channels visited by CR $i$ are either occupied or CR $i$ has false alarmed and CR $j$ has given up channel search, i.e., it has already found a PU-free channel and in the $k$th step CR $i$ finds a PU-free channel. This is represented as event $F$, and the probability that this event will happen in the $k$th step is represented as $P_k(F | D)$. b) In the first $(k - 1)$ steps, the channels visited by CR $i$ are either occupied or CR $i$ has false alarmed, and in the $k$th step CR $i$ visits a PU-free channel and competing CR $j$ is still searching for a PU-free channel. This is represented as event $C$, and the probability that this event will happen in the $k$th step is represented as $P_k(C | D)$. Events $F$ and $C$ can happen in those sensing steps where probability of success of CR $i$ is affected by the competing CR $j$. c) In the first $(k - 1)$ steps, the channels visited by CR $i$ are either occupied by a PU or CR $i$ has false alarmed, and in the $k$th step CR $i$ visits a PU-free channel. Event $E$ can happen in those steps where probability of success of CR $i$ is not affected by the competing CR $j$. This is represented as event $E$ and the probability that this event will happen in the $k$th step as $P_k(E | D)$. The probability that CR $i$ can obtain a free channel given that the two CRs select different rows is given by the second term.
in equation (4), i.e., \( P(D) = \sum_{k=1}^{N} P_k(F \mid D) + \sum_{k=1}^{N} P_k(C \mid D) + \sum_{k=1}^{N} P_k(E \mid D) \), and is derived as follows.

When the two CRs select different sensing orders, the probability of success of CR \( i \) depends on the sensing order selected by the other competing CR \( j \). Given that CR \( i \) selects any one of \( N \) rows, then each row in a total of \( N - 1 \) other rows is equally likely to be selected by CR \( j \), and the probability of selection is \( 1/(N - 1) \) for each sensing order. It can be easily seen that for the first sensing step the probabilities of event \( F \) and event \( C \) is zero, i.e., in the first step probability of success of CR \( i \) is not affected by the competing CR \( j \). Hence, CR \( i \) can obtain a channel in its first step when the channel is free of primary user and no false alarm is generated by CR \( i \). So the probability of success in first step for CR \( i \) is:

\[
P_1(D) = P_1(E \mid D) = (1 - \theta_1)(1 - P_{fa,i}). \tag{A-3}
\]

If in the first step the channel was occupied by a PU, or it was PU-free but the CR \( i \) generated a false alarm, then CR \( i \) can obtain a channel in the second step if it visits a PU-free channel and does not generate a false alarm (it visits one of those channels where its success is not affected by the competing CR \( j \), i.e., event \( E \)) or it visits a PU-free channel and CR \( j \) generated a false alarm while visiting that channel in the first step (event \( C \)). Considering a circulant matrix, which is an example of a Latin Square, in the second sensing step there is one sensing order where event \( C \) can happen and \( N - 2 \) sensing orders where event \( E \) can happen. The probability of success in second step \( P_2(D) \) for CR \( i \) is given by:

\[
P_2(D) = P_2(C \mid D) + P_2(E \mid D)
= \left( \frac{1}{N - 1} \right) \left( (1 - \theta_2)(1 - P_{fa,i})P_{fa,j} (\theta_1 + (1 - \theta_1)P_{fa,i}) \right)
+ \left( \frac{N - 2}{N - 1} \right) \left( (1 - \theta_2)(1 - P_{fa,i}) (\theta_1 + (1 - \theta_1)P_{fa,i}) \right).
\tag{A-4}
\]

If in the first and second steps channels were occupied or they were free but either the CR \( i \) generated a false alarm in both steps or CR \( i \) generated a false alarm in the first step and found the channel busy in the second step (as the channel is occupied by competing CR \( j \), which has already found a PU-free channel and has given up channel search), then it can obtain a channel in the third step if it visits a PU-free channel and does not generate a false alarm (it visits one of those channels where its success is not affected by the competing CR \( j \), i.e., event \( E \)) or it visits a PU-free channel and CR \( j \) generated a
false alarm while visiting that channel in the first or second steps (event $C$) or it visits a PU-free channel and CR $j$ has already found a PU-free channel in the first step and has given up its channel search (event $F$). In step three, there are two sensing orders where event $C$ can happen, one sensing order where event $F$ can happen and $N-3$ sensing orders where event $E$ can happen. The probability of success in the third step, $P_3(D)$, for CR $i$ is given by:

$$P_3(D) = P_3(C | D) + P_3(F | D) + P_3(E | D)$$

$$= \left( \frac{1}{N-1} \right) \left( (1 - \theta_1)(1 - P_{fa,i})P_{fa,j} \left( \theta_2 + (1 - \theta_2)P_{fa,j} \right) \right) \times \left( \theta_1 + (1 - \theta_1)P_{fa,i} \right) + \left( \frac{1}{N-1} \right) \left( (1 - \theta_3)(1 - P_{fa,i})(1 - \theta_2)(1 - P_{fa,j}) \right) \times \left( \theta_1 + (1 - \theta_1)P_{fa,i} \right) + \left( \frac{N-3}{N-1} \right) \left( (1 - \theta_3)(1 - P_{fa,i}) (\theta_1 + (1 - \theta_1)P_{fa,j}) \right) \times \left( \theta_2 + (1 - \theta_2)P_{fa,j} \right) .$$

(A-5)

Using a probability tree diagram $P(D)$ for $N$ sensing steps is generalized. This generalization is given by equation (6).

**Derivation of equation (7):**

For $N$ channels there are $N$ sensing orders (rows of the Latin Square) that a CR can select from. When $M$ autonomous CRs (uniformly) randomly select from these sensing orders then one of two events occurs. 1) CR $i$ and one or more other CRs select the same sensing order. This is represented as event $K$ and the probability that this event will happen is $P(K)$. 2) CR $i$ selects a sensing order that is not selected by any other CR. This is represented as event $L$ and the probability that this event will happen is $P(L)$. The probability of success for an individual CR for any $N$ and $M > 2$ is given by $P(N,M,\theta) = P(K) + P(L)$. To present an exact closed form expression for probability of success of an individual CR when $M > 2$ is challenging due to combinatorial explosion in the number of ways that $(M-1)$ other CRs can influence the success probability of a particular CR. An approximation for probability of success is derived as follows. When CR $i$ and one or more other CRs select the same sensing order this research considers a pessimistic scenario that the CR $i$ can only be successful in finding a free channel when only one other CR selects the same sensing order as selected by CR $i$, i.e., $P(K) \approx \left( \frac{M-1}{N-1} \right)^{M-2} P(S)$, (where $P(S)$ is given by equation 5).

When the CR $i$ selects a sensing order which is not selected by any other CR (event $L$), the probability of success of CR $i$ depends on which sensing orders are selected by
the other competing CRs. Given that CR $i$ selects any one of $N$ sensing orders, then each sensing order in a total of $(N-1)$ other rows is equally likely to be independently selected by others CRs. To arrive at an approximation for the probability of success, the following assumptions are made: 1) In a given time slot, the number of sensing orders that are not selected by any CR is exactly $Y_E$ (see equation 7); 2) The $M-1$ other CRs that select one of remaining $N-Y_E-1$ sensing orders always find a channel free in their first sensing step (yielding $Y_E + 1$ possibilities for CR $i$ where it can win a free channel). Under this assumption the probability of success when event $L$ happens is given by

$$P(L) = \left( \frac{N-1}{N} \right)^{M-1} Y_E + 1 \sum_{k=1}^{Y_E} \left[ (1 - \theta_k)(1 - P_{fa,i}) \prod_{j=1}^{k-1} \left( \theta_j + (1 - \theta_j)P_{fa,i} \right) \right]. \quad (A-6)$$

By adding $P(K)$ and $P(L)$ the approximation for $P\{N, M, \theta\}$ given in equation (7) can be obtained.
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