Ari Pouttu

PERFORMANCE ANALYSIS OF mMCSK-mMFSK MODULATION VARIANTS WITH COMPARATIVE DISCUSSION
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PERFORMANCE ANALYSIS OF $m$MCSK-$m$MFSK MODULATION VARIANTS WITH COMPARATIVE DISCUSSION

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Abstract

This thesis deals with the fascinating subject of the design of digital communication systems - or more precisely one topic therein, i.e., modulation. Modulation and its counterpart demodulation are the means of making the information ride the beast of a wireless radio channel.

The introduction of $m$MFSK and $am$MFSK generated ideas of applying the approach to other modulation methods. The straightforward extension was to apply the method to CSK modulation to introduce $m$MCSK modulation. The analysis shows that due to the orthogonality of the signaling waveforms of $m$MCSK (with Walsh codes) and $m$MFSK, the same performance is achieved for modulation methods with the same modulation alphabet. But with CSK it is rather easy to construct non-orthogonal signaling waveforms. Hence, the thesis also gives analytical results for non-orthogonal $m$MCSK and especially considers (as an example) the use of Gold and Kasami codes. The results indicate that the best choice of codes for a non-coherent $m$MCSK system is the orthogonal code family, which is a rather intuitive result. However, for a coherent $m$MCSK system, better performance could be achieved with non-orthogonal codes. Given that we can construct a code set where the cross-correlation between the codes in the family is negative ($\rho_{i,j} < 0$), performance improvement compared to the orthogonal code family is achieved. The results show that, for instance, a 0.5 dB performance improvement in the AWGN channel can be achieved by using a specifically constructed set of Kasami codes as the basis functions in the modulation. The thesis also presents the performance analysis results of $m$MCSK in a flat Rayleigh fading channel.

To further enlarge the modulation alphabet sizes (with the price of larger spectrum usage) it was realized that combining the $m$MFSK and $m$MCSK would be an interesting choice. The $m$MCSK-$m$MFSK modulation was hence introduced, which can be viewed as the main contribution of this thesis. A method to analyze this two-component modulation was developed and the performance analyses give results for $m$MCSK-$m$MFSK modulation in AWGN and flat Rayleigh fading channels for both coherent and non-coherent receivers. The performance was also assessed with orthogonal and non-orthogonal code constructs. Moreover, an antipodal extension of the $m$MCSK-$m$MFSK modulation was introduced with the analysis of the performance.

A third contribution of the thesis was to introduce the $m$PPM modulation method following the $m$MFSK principles. The performance results obtained in the $m$MCSK can also be applied to these modulation formats in certain scenarios. PPM modulation has been widely suggested to be used in UWB systems. In UWB systems, the means to adapt the data rate is the use of pulse repetition. Hence, the performance analysis of the $m$PPM modulation methods with pulse repetition and a non-coherent receiver was computed. The performance of pulse repetition in impulsive interference was also computed.

Keywords: $m$MCSK, $m$MFSK, $m$PPM, modulation method, performance analysis, spectrum efficiency
Tiivistelmä

Tässä väitöskirjatyössä tarkastellaan tietoliikennejärjestelmän suunnittelun liittyvää osa-aluetta modulaatiomenetelmiä. Modulaatio ja sen vastinpari demodulaatio ovat mekanismit, joiden avulla siirrettävä tieto saatetaan muotoon, jolla se voidaan siirtää radiokanavassa. mMFSK ja nmMFSK modulaatiomenetelmien kehittäminen loi pohjan, jota lähdeettiin soveltaa myös muille modulaatioteknikoihin. Suoritetut matemaattiset analyysit osoittavat, että samalle modulaatioaakkostoon koolle mMCSK (käyttäen Walsh-koodeja) ja mMFSK saavuttavat saman suorituskyvyn olettaen modulaatioiden olevan ortogonaalisia. CSK-teknikalla on kuitenkin helppo rakentaa koodeja, jotka ovat epäortogonaalisia. Väitöskirjassa analysoidaan myös muutamia tapauksia epaortogonaaliselle mMCSK:lle, jossa esimerkinomaisesti tarkastellaan Gold- ja Kasami-kooodien käyttöä. Tulokset osoittavat, että epäkoherentille mMCSK:lle parantetaan koodeja, joka on myös intuitiivinen. Koherentille mMCSK:lle voidaan kuitenkin rakentaa epäortogonaalisia koodikonstruktoita, joissa koodien välinen ristikorrelaatio on negatiivinen ja tällöin saavutetaan suorituskyvyn paranemista ortogonaaliseen koodiperheeseen. Tulokset Gold- ja Kasami-koodeille AWGN-kanavassa osoittavat, että n. 0.5 dB suorituskyvyn paraneminen on saavutettavissa. Työssä johdataan myös analyysit tulokset vastaaviille tapauksille Rayleigh-häipyvässä kanavassa.

Työssä tutkitaan mMCSK- ja mMFSK-modulaatioita, huomattii, että yhdistämällä nämä kaksi voidaan saavuttaa erittäin suuria modulaatioaakkostoja. Hintana on suurempi spektrin käyttö. mMCSK-mMFSK-modulaation kehittämisistä voidaan pitää tämän työn päätuloksena. Työssä kehitetään liikmääräinen menetelmä tämän kaksi-komponenttisen modulaation suorituskyvyn analysoimiseksi. Tuloksina esitettävän modulaatiomenetelmän suorituskyky sekä AWGN- että Rayleigh-häipyvänä kannassa. Suorituskykytuloksia esitettävän myös epäortogonaaliseen koodikonstruktoille. Lisäksi kehitettiin antipodaalinen laajennus mMCSK-mMFSK-modulaatiosta ja suoritettiin suorituskykyanalyysit AWGN-kanavassa.

Työn kolmantena tuloksena esitetään mPPM-modulaatiomenetelmä hyödyntäen mMCSK-modulaation periaatteita. mPPM-mMCSK-suorituskykyanalyysia voidaan tietyn ehdoin soveltaa myös näihin modulaatiomenetelmiin, joiden käyttöä esitetään usein ultralaajakaistajärjestelmissä (UWB). UWB-järjestelmissä datanopeutta säädetään usein pulssin toistoa käyttämällä. Työssä johdataan suorituskykytulokset epäkoherentille mPPM-vastaanottimelle myös pulssintonoitoon yhteydessä. Lisäksi johdettiin suorituskykyanalyysit ko. modulaatioille impulseissä häiriön läsnäolllessa.

Asiasonat: mMCSK, mMFSK, mPPM, modulaatiomenetelmä, spektritehokkuus, suorituskykyanalyysi
Acknowledgments

The work associated with this thesis has been carried out in the Security and Defence research team at the Telecommunication Laboratory and Centre for Wireless Communications (CWC) in the University of Oulu, Finland, where I was supervised by professor Pentti Leppänen, whom I would like to thank for placing the facilities at my disposal, giving me inspiring scientific challenges, and creating an excellent research atmosphere. I would also like to thank Professor Savo Glisic for pointing out the new trends in multilevel modulation research. I am indebted to Dr. Tech. Harri Saarnisaari for stimulating discussions and brainstorming sessions in which the final scope of this thesis was formulated.

The work toward this thesis was initiated in March 2002 during a traditional Finnish Sauna party that was related to a project we conducted for the Finnish Navy. There Lieutenant Commander Topi Tuukkanen, Dr. Tech. Harri Saarnisaari, Professor Pentti Leppänen and I had a vivid discussion which eventually resulted in establishing the framework for the \( n \)MCSK-\( m \)MFSK modulation. At that time I had already once changed the focus of my thesis and felt somewhat discouraged to do it again. However, the topic stirred a lot of interest in my mind and eventually the second change of thesis topic was a reality.

I am grateful to the reviewers of the thesis, Professor Ramjee Prasad from the Aalborg University and Professor Peter Jung from the University of Duisburg-Essen for providing many beneficial suggestions which significantly improved the quality of this thesis.

I would also like to thank numerous colleagues at the CWC for the challenging environment to pursue research. I am grateful to M. Sc. Sanna Heikkilä who performed simulations for some of the cases. Thanks are due to Harri Saarnisaari, Jari Inatti, Matti Latva-aho and Pentti Leppänen who read and commented on the manuscript. The help in various computer problems provided by M. Sc. Jari Sillanpää has been invaluable.

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As managing first the S&D team and since 2006 the whole CWC operations is a full-time daily job, this thesis was turned into a hobby done mostly in the wee hours of the day – however, it has been the most rewarding and intriguing hobby.
Therefore, I would like to thank my mother-in-law Sirkka for looking after Ida-Maria when needed but especially I would like to dedicate this thesis to my family – my wife Eeva-Kristiina and my daughters Krista and Ida-Maria – for their patience and especially for constantly bringing light into my life, and to my parents – Tuula and Tauno – who urged me to pursue higher education.

Oulu, December 3, 2012

Ari Pouttu
Abbreviations

a  Auxiliary variable
A  Amplitude
A  Forward backward linear prediction matrix
\( \hat{A} \)  Rank reduced matrix from \( A \)
\( A_m \)  Signaling amplitude \( m \)
\( A_{mc} \)  In-phase signaling amplitude \( m \) (QAM modulation)
\( A_{ms} \)  Quadrature signaling amplitude \( m \) (QAM modulation)
b  Auxiliary variable
c  Spreading code
c\(_i\)  Chip \( i \) of the spreading code \( c \)
c\(_i\)\(_t\)  Transmitted spreading code
E  Energy of a signaling waveform
\( E_o \)  Bit energy
\( E_{\text{eff}} \)  Efficiency improvement factor
\( E_i \)  Energy of signaling waveform \( i \)
\( E_j \)  Energy of signaling waveform \( j \)
\( E_m \)  Energy of signaling waveform \( m \)
\( E_s \)  Symbol energy
\( \text{erf}(\cdot) \)  Error function
\( \text{erfc}(\cdot) \)  Complementary error function
f\(_c\)  Center frequency
f\(_i\)  Signaling frequency \( i \)
\( f_j(\cdot) \)  Signaling frequency \( j \)
f\((\cdot) \)  Probability density function
\( F(\cdot) \)  Cumulative distribution function
\( f_r(\cdot) \)  Probability density function of the \( r^{\text{th}} \) largest sample
G\((\cdot) \)  Auxiliary function
i  Index
\( I_0(\cdot) \)  Zero order Modified Bessel function of the first kind
j  Index
k  Index
\( \ell \)  Index for a frequency branch
L  Number of symbol replicas in pulse repetition of \( m \)PPM
m  Number of signaling waveforms transmitted per symbol
\( m_c \)  Number of codes transmitted per symbol
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_c$</td>
<td>Size of the symbol alphabet in $m$MCSK (number of available codes)</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Number of frequencies transmitted per symbol</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Size of the symbol alphabet in $m$MFSK (number of available frequencies)</td>
</tr>
<tr>
<td>$M$</td>
<td>Size of the modulation alphabet</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Size of the modulation alphabet in the new modulation formats</td>
</tr>
<tr>
<td>$N$</td>
<td>Length of a chip sequence</td>
</tr>
<tr>
<td>$N_0$</td>
<td>One-sided noise spectral density</td>
</tr>
<tr>
<td>$n$</td>
<td>Index</td>
</tr>
<tr>
<td>$n(t)$</td>
<td>Noise signal</td>
</tr>
<tr>
<td>$n_b$</td>
<td>Number of bits per symbol</td>
</tr>
<tr>
<td>$N_{tr}$</td>
<td>Gaussian zero mean variable (noise sample)</td>
</tr>
<tr>
<td>$n_m$</td>
<td>Number of bits per symbol in the new modulation formats</td>
</tr>
<tr>
<td>$N_{ref}$</td>
<td>Number of reference symbols</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of symbol intervals</td>
</tr>
<tr>
<td>$P$</td>
<td>Number of complex samples per bit/symbol</td>
</tr>
<tr>
<td>$P_d$</td>
<td>Probability of error for antipodally modulated data on one code</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Bit error probability</td>
</tr>
<tr>
<td>$P_{cc}$</td>
<td>Probability of a correct decision</td>
</tr>
<tr>
<td>$P_{cf}$</td>
<td>Probability of a correct decision in FSK detection</td>
</tr>
<tr>
<td>$P_{comp}(j)$</td>
<td>Probability that $j$ out of $L$ replicas are combined</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Symbol error probability</td>
</tr>
<tr>
<td>$P_{Mc}$</td>
<td>Probability of an error in CSK detection</td>
</tr>
<tr>
<td>$P_{Mf}$</td>
<td>Probability of an error in FSK detection</td>
</tr>
<tr>
<td>$Q(\cdot)$</td>
<td>Q function</td>
</tr>
<tr>
<td>$r(t)$</td>
<td>Received signal</td>
</tr>
<tr>
<td>$R(\cdot)$</td>
<td>Auxiliary function</td>
</tr>
<tr>
<td>$r_{\ell}(t)$</td>
<td>Received signal on frequency branch $\ell$</td>
</tr>
<tr>
<td>$\overline{r}_{lp}$</td>
<td>Received low pass signal in vector format</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Rank of a matrix</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Transmitted data pattern</td>
</tr>
<tr>
<td>$S$</td>
<td>Signal power</td>
</tr>
<tr>
<td>$s_k$</td>
<td>Symbol value at time instant $k$</td>
</tr>
<tr>
<td>$s_m(t)$</td>
<td>Signaling waveform $m$</td>
</tr>
<tr>
<td>$s^2$</td>
<td>Non-centrality parameter on non-central chi-square distribution</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (continuous valued)</td>
</tr>
</tbody>
</table>
$T$  Symbol duration  
$T_c$  Chip duration  
$T_f$  Duration of a frequency hop  
$U$  Decision metric  
$U$  Unitary matrix containing left singular vectors  
$u_i(t)$  Signaling waveform $i$  
$u_j(t)$  Signaling waveform $j$  
$u_{jl}(t)$  Signaling waveform $j$ on frequency branch $l$  
$u_k$  $k$th column of matrix $U$  
$U_k$  Decision variable  
$U_j$  Decision variable  
$u_m(t)$  Signaling waveform $m$  
$U_m$  Decision variable  
$V$  Unitary matrix containing right singular vectors  
$v_k^*$  $k$th column of matrix $V^*$  
$V_n$  Decision metric at time instant $n$  
$V_{n-1}^*$  Complex conjugate of the decision metric at time instant $n - 1$  
$x$  Variable  
$x_n$  Channel input sample at time instant $n$  
$y$  Variable  
$y_n$  Channel output sample at time instant $n$  
$z$  Variable  
$z(t)$  Gaussian noise signal  
$\alpha$  Channel attenuation factor  
$\gamma$  Signal-to-noise ratio for a symbol observed over $L$ channels  
$\gamma_b$  Signal-to-noise ratio for a bit  
$\delta(t)$  Impulse response of a channel  
$\delta_{kn}$  Kronecker delta function  
$\phi$  Phase error  
$\Psi_r$  Rank reduced received samples  
$\xi$  Non-centrality parameter of Rician distribution  
$\xi_{h_{10}}$  Relative interference hit probability  
$\mu_m$  Signal sample $m$  
$\mu_r$  Mean of probability density function $f_r(\cdot)$  
$\mu_z$  Approximate mean of sorted and summed $m$ noise samples  
$\eta$  Noise sample  
$\Delta \omega_d$  Frequency error
\( \omega_n \) Frequency deviation
\( \tilde{\omega}_n^{ref} \) Quasi-coherent reference carrier for symbol frequency \( k \)
\( \rho_{i,j} \) Complex-valued cross-correlation coefficient
\( \rho_i \) Auxiliary variable
\( \rho_2 \) Auxiliary variable
\( \sigma^2 \) Variance of Gaussian noise
\( \sigma^2_{\chi} \) Variance of Rayleigh distribution
\( \sigma_k \) The \( k \)th largest singular value of \( A \)
\( \sigma^2_{f_r} \) Variance of probability density function \( f_r(\cdot) \)
\( \sigma^2_{m} \) Approximate variance of sorted and summed \( m \) noise samples
\( \Sigma \) Diagonal matrix containing singular values of \( A \)
\( \tau \) Delay
\( \theta \) Phase of a signal
\( \theta_m \) Modulation phase in MPSK modulation

AGC Automatic Gain Control
\( amMFSK \) M-ary Frequency Shift Keying with \( m \) simultaneous symbol frequencies transmitted and all with different amplitudes
APSK Amplitude and Phase Shift Keying
ASK Amplitude Shift Keying
AWGN Additive White Gaussian Noise
BCSK Binary Code Shift Keying
BEP Bit Error Probability
BER Bit Error Rate
BFSK Binary Frequency Shift Keying
BPF Band Pass Filter
BS Base Station
B3G Beyond third (3) Generation
B4G Beyond fourth (4) Generation
CDMA Code Division Multiple Access
COTS Commercial Of The Shelf
CMF Chip-Matched Filter
CPM Continuous Phase Modulation
CR Cognitive Radio
CSI  Channel State Information  
CWC  Centre for Wireless Communications  
DAB  Digital Audio Broadcasting  
DAPSK  Differential Amplitude Phase Shift Keying  
DARPA  Defense Advanced Research Projects Agency  
DBPSK  Differential Binary Phase Shift Keying  
DPSK  Differential Phase Shift Keying  
DQPSK  Differential Quaternary Phase Shift Keying  
DRM  Digital Radio Mondiale  
DSM  Dynamic Spectrum Management  
DSP  Digital Signal Processing  
DVB-C  Digital Video Broadcasting - Cable  
DVB-H  Digital Video Broadcasting - Handheld  
DVB-T  Digital Video Broadcasting - Terrestrial  
FBLP  Forward-Backward Linear Prediction  
FCC  Federal Communications Commission  
FH  Frequency Hopping  
GSM  Global System for Mobile Communications  
JSI  Jamming State Information  
IEEE  Institute of Electrical and Electronics Engineers  
IID  Independent Identically Distributed  
IS-95  CDMA-based cellular mobile telephony standard  
ISI  Inter Symbol Interference  
ISM  Industrial, Scientific and Medical  
LOS  Line-Of-Sight  
LP  Low Pass  
LS  Least Squares  
MC  Multi-Carrier  
MCSK  M-ary Code Shift Keying  
MDPSK  M-ary Differential Phase Shift Keying  
MF  Matched Filter  
MIMO  Multiple-Input and Multiple-Output  
mMCSK  M-ary Code Shift Keying with $m$ simultaneous code sequences transmitted  
MFSK  M-ary Frequency Shift Keying  
mMFSK  M-ary Frequency Shift Keying with $m$ simultaneous symbol frequencies transmitted
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPAM</td>
<td>M-ary Pulse Amplitude Modulation</td>
</tr>
<tr>
<td>$m$PPM</td>
<td>M-ary Pulse Position Modulation with $m$ pulses transmitted</td>
</tr>
<tr>
<td>$m$PSM</td>
<td>M-ary Pulse Shape Modulation with $m$ pulses transmitted</td>
</tr>
<tr>
<td>MPSK</td>
<td>M-ary Phase Shift Keying</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>OTM</td>
<td>On-The-Move</td>
</tr>
<tr>
<td>PA</td>
<td>Power Amplifier</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak-to-Average-Power-Ratio</td>
</tr>
<tr>
<td>PPM</td>
<td>Pulse Position Modulation</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>PSM</td>
<td>Pulse Shape Modulation</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>S&amp;D</td>
<td>Security and Defense</td>
</tr>
<tr>
<td>SDR</td>
<td>Software-Defined Radio</td>
</tr>
<tr>
<td>SMF</td>
<td>Sequence Matched Filter</td>
</tr>
<tr>
<td>SMS</td>
<td>Short Message Service</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TV</td>
<td>Television</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>US</td>
<td>United States</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra-Wide Band</td>
</tr>
<tr>
<td>WHT</td>
<td>Walsh-Hadamard Transform</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
<tr>
<td>WRAN</td>
<td>Wireless Regional Area Network</td>
</tr>
<tr>
<td>XG</td>
<td>DARPA Project Name</td>
</tr>
</tbody>
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# Contents

## Abstract

## Tiivistelmä

## Acknowledgments

## Abbreviations

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1 Introduction

This thesis deals with the fascinating subject of the design of digital communication systems - or more precisely one topic therein, i.e., modulation. Modulation and its counterpart demodulation are the means of making the information ride the beast of a wireless radio channel. This introductory chapter will gradually lead the reader to the essence of the topic dealt with in the thesis. The approach in this chapter is to discuss some general trends in modern digital communication, the design procedure of such a system and eventually introduce the state of the art in the field of modulation designs. So let us boldly go where Data has not gone before...

1.1 Digital communications

The era of digital communications was started in the 1830's when Samuel Morse, utilizing the inventions of William Sturgeon and Joseph Henry [1], introduced the method of transmitting data (the first commercial telegraph) by using the code nowadays known as the Morse code. Somebody might claim that this is not an accurate claim, since 'coded' information has been transmitted earlier using smoke signs and fire. This work, however, deals with transmitting data by excitation of electromagnetic fields, and the aim is especially to introduce mechanisms to transmit data using wireless radio systems. Since Morse's time, a lot has changed, as the mobile telephones in almost everyone's pocket demonstrate.

The modern digital radio system design is a function of a multitude of optimization criteria. However, the design paradigm is shifting from a technology oriented approach towards a user or application driven approach. For instance, in the GSM system, the technological solution was well in operation before the bulk of the services (such as SMS) were later invented. As an example of the paradigm shift, the design procedure adopted in the Centre for Wireless Communications (CWC) and especially in the Security and Defence (S&D) research team is depicted in Fig. 1. The user will require or even demand a certain quality of service (QoS) for the services that are expected by the user. In the figure, these are called operational requirements. From the user requirements, technical requirements can be defined or derived, which in turn drive the design of the system. The user may, however, require different services at different times, thus making the starting point for the design even more challenging. A new design philosophy is required to offer the future proof solutions expected by the user.
From the operational and technical requirements, the traffic types and data types expected in the system can be derived, modeled, and afterwards used in the system design. The operating environment has to be assessed as well. In system designs for commercial systems, the usual phenomena to be combatted are the noise and the interference generated either by other systems or more often by the other users of the same system. As an example of this, the suppression of interference caused by other users of the same system is a major design criterion in 3rd generation mobile communication systems such as UMTS [2, 3]. In military and other authority communication, more design criteria related to the operating environment can be identified. In the design of these systems, especially intentional interference (jamming) and the possibility of signal intelligence have
to be accounted for. The most common method in terms of system design is to use spread spectrum technology when countering these challenges. The spectrum spreading readily lends itself to signal processing means in jamming protection, but also to hiding the signal among other signals and noise if signal intelligence is considered. On the other hand, the use of spread spectrum technology is in contradiction to the ever increasing demand of using the available spectrum efficiently.

In addition to the operating environment effects, the wireless transmission is degraded by the different propagation phenomena introduced by the radio channel. Apart from the rather steady propagation loss which is dependent on antenna heights, the terrain, the link distance and the radio frequency used, the mobility of the desired transmitter and receiver as the main contributors but also the mobility of objects in general introduce the time variant nature of the received signal parameters. The line-of-sight (LOS) and the scattered and reflected signal components together form a sum signal in the receiver antenna, which, due to the previously discussed mobility, can be a constructive or destructive sum of the aforementioned components. The result of this may cause signal-to-noise (SNR) fluctuations in the order of several tens of decibels (dB), thus necessitating methods of compensating the radio channel. The most often used approach is to use a channel matched filter, more commonly known as a RAKE-receiver, which estimates the parameters of the impinging signals and tries to sum them constructively, thus removing the deep degradations from temporal SNR values. This, of course, is dependent on the success and the quality of the estimation process.

The networks in wireless radio systems have mostly been relying on a static or semi-static infrastructure. Examples of this include e.g. GSM and UMTS, where fixed base stations are used as central entities and access points to the wider information infrastructure. In recent years the research in wireless networking has been focused on introducing mobility to the network. The wireless local area network (WLAN) concept already includes some mechanisms to build a network without strict centralized control and it also offers the possibility of using multi-hop communications in coverage extension. Especially in the field of embedded sensor networking and military networking, operations on-the-move (OTM) are emphasized. This has created a high interest in research on ad-hoc networking and semi-ad-hoc networking (clustered ad-hoc). The auto-configuration of the network is the key issue in these networks where centralized control is no longer a viable option and multi-hop communications are the
preferred choice. However, the new trends do not abolish the need for solutions based on a centralized or fixed network infrastructure. It is seen that especially in the civilian marketplace the combination of mobile and fixed network solutions will be used in Beyond 3G (B3G) solutions where the hotspots are covered with fixed solutions and where coverage extensions can be offered by relaying functions either by dedicated relaying stations or by other mobile users of the network.

Nowadays a radio system is normally allocated a given frequency band for its operation. The designer needs to allocate this resource to the users so as to maximize the capacity of the system. One issue in using the frequencies is whether the user is allowed to randomly access the media (either in frequency or in time) or whether one should adhere to the fixed allocation. Another issue is how the multiple users can simultaneously share the same frequency band. The multiple access design comparisons always raise a lot of passionate discussion, for example, on the standardization process for a new system. An example of this was the definition process where TDMA and CDMA based solutions competed in the UMTS standardization process. However, the data types and traffic models in conjunction with the user expectation should drive the selection of the capacity maximizing solution. To make these assessments even harder, the concept of mobile or ad-hoc networks will introduce an almost intractable challenge for future designers unless a new methodology for the process is invented.

Some of the choices in the design are mostly driven by the user requirements. An example would be an ad-hoc network solution, which is basically the only viable option if mobility associated with limited resources is the driver in the design. This kind of situation might arise, for instance, in the design of a radio system for crisis management teams which might have to be deployed in any part of the world. Some of the choices are less obvious, as discussed regarding multiple access schemes. And to further make the life of the designer more challenging, the use of a smart antenna system is becoming more attractive through the increase in the computing capacity of the microchips now available. This will introduce a new set of challenges especially when designing the increasingly mobile networks of tomorrow.

Having completed the general design of the system, the overall transceiver design needs to be performed. These include the architectural design of the hardware, the detailed algorithmic design of the protocol stack and, if applicable, the design of the software architecture and the operating system of the transceiver platform. The latter will be an especially important task in the future radio system
design, given that in the decades to come, the radio platforms will be based on a software defined radio approach.

Until now we have discussed the network design, multiple access design and the importance of user requirements as well as the effects of the operating environment and the radio channel. What remains is to discuss the methods of actually transferring the information through wireless transmission technology, e.g. the solutions on the physical layer. When the user wishes to transmit data, it needs to be somehow modulated on the carrier, i.e., the frequency it is to be transmitted at. One of the hot topics in the research world today is how to modulate the information onto the carrier so as to maximize the frequency utilization in terms of how much information can be conveyed in a given time frame, i.e., efficiency in terms of bits/s/Hz. Sometimes the spectrum efficiency can be traded if some other criterion is the driver in the design. For example, in military communications the spectrum efficiency is traded for interference resistance or for lowering the susceptibility of the transmitted signal to intelligence operations. The most often used method is the spread spectrum technology. This is actually one of the motivating factors of this thesis. How would one construct a method to combine the conflicting concepts of spread spectrum technology and spectral efficiency?

1.2 Spectrum efficiency demand: possible methods

The increasing need for wireless communications resulting from the introduction of new bandwidth consuming services has put a demand on the spectral efficiency of the signal waveforms used in the modern wireless systems. All future wireless systems are expected to convey hundreds of megabits per second (Mbit/s) with the constraint of limited spectrum availability. Hence the spectrum efficiency of the chosen waveforms measured as bits/second/Hertz (bps/s/Hz) should be maximized. There are numerous ways to accomplish spectral efficiency, including space-time coding or MIMO technology, code division multiple access (CDMA), flexible spectrum use, and efficient modulation systems. In this thesis the approach is to develop and analyze new modulation systems yielding improved spectral efficiency. However, the approach is to leave spectrum spreading in place in order to combat multipath propagation and the detrimental effects of interference and possibly jamming. In the next 3 subchapters, a brief discussion of the above mentioned techniques is given.
1.2.1 High order modulation

When one is using a modulation method which has $M$ possible symbols, one can transmit

$$n_b = \log_2 M$$

bits per symbol. Given that the system is not power limited, the symbol alphabet can be increased by increasing the transmit power, and hence the data rate is higher given a fixed symbol time. This is evident e.g. in Figs. 8 and 11 where, for instance, going from 4 bits/symbol to 8 bits/symbol would require 20 dB higher transmit power for MPSK and 10 dB for QAM.

The modulation literature will be discussed in more detail in Chapter 1.3 but here we point out a few high-order modulation techniques suggested/used in real life applications. The most commonly known and used high order modulation technique is Quadrature Amplitude Modulation (QAM). This modulation method is used in or suggested for WLAN technologies (IEEE802.11a, IEEE802.11g, IEEE802.11n), as is indicated in [4] and [5] available in [6]. QAM has also been included in the WiMAX (IEEE802.16) standard [7]. It is also used in Digital Video Broadcasting standards DVB-C, DVB-T, and DVB-H [8], [9], [10]. In audio broadcasting QAM is used in Digital Radio Mondiale (DRM) [11], [12]. In some of these systems, the modulation alphabet suggested uses 256 symbols, thus indicating an 8-fold data rate increase compared to binary modulation methods.

1.2.2 Cognitive radio and dynamic spectrum management

Since the introduction of the term cognitive radio [13], [14] numerous definitions for it have been presented. In this context a cognitive radio is defined as an environment aware, self-reasoning and learning capable radio [15], [101] that can change any of its parameters or protocols based on interaction with the environment in which it operates, depicted in Fig. 2. The figure gives by no means a comprehensive depiction of cognitive radio operation but it captures the essentials of the basic cognitive cycle. Having the great flexibility requires a major paradigm shift for wireless communication systems research: strategies for reasoning and learning must be developed. This requires investigation of artificial intelligence and machine learning techniques, game theory for spectrum and network management, and semantics for ontology representations.
As we have seen, new wireless communication systems and services are introduced with an increasing speed. With the introduction of a new system, the frequency spectrum becomes more and more congested. This in turn creates major problems for user equipment and network design. At the same time, only a few per cent of the average available spectrum is utilized by the current wireless systems [16], [17]. This has given rise to research on more liberal usage of the spectrum. Future systems are depicted to adapt to rapidly changing environmental conditions while ensuring minimal, or at least manageable, interference added to other systems. Such a technology is called a cognitive radio. Enabled by software defined radio (SDR) technology [18], [19], open spectrum allows unlicensed secondary users to share spectrum with primary spectrum users, thereby greatly enhancing the spectral efficiency [20], [21]. Based on agreements and constraints imposed by primary users, secondary users opportunistically utilize unused licensed spectrum on a non-interfering or leasing basis. Open spectrum requires a new line of cognitive radios. The term cognitive refers to a device's ability to sense surrounding environment conditions and adapt its behavior accordingly.

The spectrum management issue has become a critical element for radio system design not only in the military radio systems but also in the civilian marketplace, as the current 4G mobile communications research arena indicates. Two main factors are driving this trend: 1) The often quoted but not so true statement of spectrum scarcity and 2) requirements for seamless operation of user terminals regardless of the geographical area.
The first statement has been verified in recent research, where it has been noted that on average less than 15% of the time on a given frequency or less than 15% of the total frequencies allocated for a system are in use. To use this sporadically available capacity, methods for real-time spectrum monitoring need to be introduced for the capability set of the radio. To make intelligent decisions on how to utilize this sporadically available capacity, intelligent or cognitive decision making and spectrum allocation algorithms need to be devised that operate in a distributed or semi-distributed fashion without causing service interruptions to the primary system for which the capacity is originally allocated. A software adaptable network suggested in [22] is one proposal that also classifies control approaches. Other examples include TV frequencies where the frequency re-use factor prohibits the use of the same frequency in adjacent transmitters, thus leaving large portions of the bandwidth unutilized in large geographical areas. This property is exploited in the IEEE802.22 standard that provides Wireless Regional Area Network technology used for wireless broadband access in rural and remote areas [23].

In the United States, the Defense Advanced Research Projects Agency (DARPA) is funding the XG-program [24], [25] wherein it has been demonstrated by a limited set of field tests that cognitive radio technologies can significantly
improve the spectrum usage [26] and thus the overall spectrum efficiency. The target efficiency improvement in the XG project is to increase 10-fold the spectrum use.

In the IEEE802.11 test case, an up to 200% increase in spectrum utilization was estimated [27].

The second statement (2) was actually emphasized by the US Iraqi operation where it is said that only one Global Hawk UAV could be used at any given time due to spectrum allocation problems. To be able to optimally deploy an UAV system or any system in any operational area requires novel thinking from the system design point of view.

One technological requirement is also implied here. The waveform should also be scalable in terms of its instantaneous bandwidth. The available spectrum hole should be utilized regardless of its bandwidth. The cognitive capability also suggests that with sufficient intelligence the waveform could be refined in an evolutionary fashion where in each operation the most suitable parameters are recorded after having been tested in a real life environment during the operation, thus facilitating the concept of a mission specific waveform.

Dynamic spectrum management (DSM) comprises the identification and characterization of the available spectrum, the duration of the availability of this spectrum, and allocation as well as monitoring of the spectrum, as indicated by Fig. 3. An inherent feature of cognitive radio networks is that there is no or very little centralized control over the system. Hence, spectrum management should be carried out in a non-centralized manner. This calls for so-called spectrum etiquette policies. Co-operative communications in ad-hoc type networks can furthermore negotiate how to use the available spectrum opportunities most efficiently within a sub-net. The target is to improve robustness, channel-capacity usage, and connectivity using DSM. The co-existence between the primary users and opportunistic secondary users must also be guaranteed. Strategies for DSM depend very much on the frequency band being used. On ISM bands the policies are less stringent than on licensed bands. On terrestrial television bands the policies can also be more liberal than on mobile cellular bands, and so on.

The competition between independent radio systems in allocating a common shared radio channel can be modeled as a stage-based game model [28]: players, each representing radio systems or even users within one radio system, interact repeatedly in radio resource sharing games, without direct coordination or information exchange. Solution concepts derived from game theory allow the analysis of such models under the microeconomic aspects of welfare. The
decisions that the players repeatedly have to make are about when and how often to attempt a medium access. In multistage games, players apply strategies in order to maximize their observed utility by summarizing the value for successful supported Quality-of-Service (QoS). Strategies determine whether competing radio networks cooperate or ignore the presence of other radio networks. The requirements of the players determine which strategies guarantee QoS. The application of game theory in spectrum sharing scenarios enables a distributed coordination of multiple cognitive radios sharing the same spectrum opportunity.

Fig. 3. Cognitive radio in conjunction with dynamic spectrum management as indicated by SDRForum. Modified from [29].

1.2.3 MIMO Technology

Multiple-input multiple-output (MIMO) processing [86-89] exploits multiple antenna elements at the transmitting end as well as at the receiving end. The main idea in MIMO systems is space-time signal processing, where time and space domain signals are jointly processed.

In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of all $N_tN_r$ paths between the $N_t$ transmit antennas at the transmitter and $N_r$ receive
antennas at the receiver. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information.

In spatial multiplexing, a high data rate signal is transmitted by using multiple lower data rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the streams can be separated into parallel channels. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher signal-to-noise ratios (SNR). The maximum number of spatial streams is limited by the smaller number of the antennas at the transmitter or receiver, i.e. \( \min\{N_t, N_r\} \).

MIMO technology can be seen as add-on technology, as with proper design it may be used on the top of any modulation technology.

1.3 Literature review on modulation technology

1.3.1 Classification of signals

In general, the signals used in wireless systems can be classified into orthogonal, bi-orthogonal, simplex, antipodal and non-orthogonal signals. Let us denote the low pass (complex) signals used in the general \( M \)-ary transmission as \( u_m(t) \) with \( m = 1, \ldots, M \). The \( M \) signals are characterised individually by their energy defined as

\[
E_m = \int_0^T |u_m(t)|^2 \, dt, \quad m = 1, \ldots, M
\]  (2)

and mutually by their complex-valued cross-correlation coefficients

\[
\rho_{i,j} = \frac{1}{2 \sqrt{E_i E_j}} \int_0^T u_i(t) u^*_j(t) \, dt, \quad i \neq j.
\]  (3)

The signals are said to be orthogonal when the cross-correlation value \( \rho_{i,j} = 0 \), \( i \neq j \) for all \( i \) and \( j \). A set of \( M \) bi-orthogonal signals can be constructed from \( M/2 \) orthogonal signals by including the negatives of the orthogonal signals. Now the cross-correlation between any pair of the waveforms is either 0 or -1.

Simplex signals are generated from \( M \) orthogonal signals by subtracting the geometric mean value of the \( M \) signal waveforms from the orthogonal signal.
waveforms, thus effectively translating the origin of the $M$ signals to the value of the geometric mean. The energy of the waveforms is now $E(1-(1/M))$ and the crosscorrelation value between any pair of the waveforms $\rho_{i,j} = -(1/(M-1)), i \neq j$. Signals are said to be antipodal if the cross-correlation value $\rho_{i,j} = -1, i \neq j$ for all $i$ and $j$. For other values of the cross-correlation value $\rho_{i,j}$ the signals are said to be non-orthogonal.

1.3.2 The scope of the review

This thesis introduces some new modulation formats. The modulation format can be distinguished based on many factors, as shown by Fig. 4. The most obvious classification is to divide the modulation formats with respect to the processing domain: analogue or digital. Numerous text books discuss and distinguish modulation formats using this criterion, e.g. [31], [32]. This thesis will introduce a new concept in the digital modulation domain.

![Fig. 4. Some aspects of modulation classification.](image-url)
The next important factor is related to what property of the signal is excited. These include amplitude, phase, frequency, polarization, time shift, pulse width and pulse shape. All of these will be briefly discussed with the inclusion of the detection method and modulation level. The detection processing, memory of the modulation method and combining coding or antennas with modulation will be discussed briefly in this section, but the review itself will be concentrated on digital modulation with one-shot detection of multilevel modulation methods without memory.

1.3.3 Digital amplitude modulation

\( M \)-ary Pulse amplitude Modulation (\( M \)-PAM) or equivalently Amplitude Shift Keying (ASK) uses signal constellation, depicted in Fig. 5.

![Fig. 5. M-PAM signal constellation for symbol alphabets \( M = 2,4,8 \).](image)

The \( M \)-PAM signal can be expressed as

\[
s_m(t) = A_m \text{Re} \left\{ u(t) e^{2\pi ft} \right\}, \quad m = 1, \ldots, M,
\]

where \( u(t) \) is the pulse shape used, which can be assumed to be rectangular. The signaling amplitude \( A_m \) can take values from the set

\[
A_m = 2m - 1 - M.
\]

If the energy of the pulse shape \( u(t) \) equals \( E \), the symbol energy is then \( E_m = A_m^2 E \). The decision metrics in the receiver are now

\[
U = \text{Re} \left\{ e^{\alpha \int_{0}^{T} [s_m(t) + n(t)] u'(t) dt} \right\} = 2\alpha EA_m + \eta = \mu_m + \eta.
\]
where $\eta$ is zero mean white Gaussian noise with variance $2EN_0$. The performance of M-PAM is obtained by the Gaussian assumption and is given in Table 1 [30].

1.3.4 Digital Phase Modulation

In Fig. 6, a demodulation structure (assuming symbol synchronization) for $M$-ary Phase Shift Keying (MPSK) is presented. The digital phase modulation of the carrier results when the binary digits (bits) from the information source are mapped into a set of discrete phases of the carrier. For a given $M$, $\log_2 M$ bits can be transmitted by mapping them into one of the $M$ corresponding phases $\theta_m = 2\pi(m-1)/M$, $m = 1, \ldots, M$.

![MPSK Demodulator Structure](image)

Fig. 6. The MPSK demodulator structure. Modified from [30].

In the demodulator, the received signal is down-converted into in-phase and quadrature components by multiplying the signal with the cosine and sine components of the local oscillator signal. In phase shift keying, the phase difference between the received signal and the local oscillator signal needs to be kept close to zero in order to be able to perform the demodulation correctly. Hence, the phase synchronizer and frequency synchronizer are important elements of PSK demodulation. The bit error rate performance of a binary PSK as well as the symbol error rate performance of $M$-PSK modulation is given in Table 1. PSK modulation is standard modulation in a number of different applications, including the UMTS system. A comprehensive description of PSK modulation is available in all relevant text books on digital communications, e.g. [30], [31], [32].
In some applications, the phase synchronization will become very difficult or it is not desired for some other reason. One application is frequency hopping in military (or civilian) communication systems. In differential phase shift keying, the demodulation is based on phase differences between two or a few consecutive symbols, and hence the absolute phase values are not required. The only required assumption is that the channel-induced phase rotation between two or a few consecutive symbols is sufficiently small. The synchronization is now required to compensate the frequency difference between the received and local oscillator signals. The demodulator structure for differential quaternary phase shift keying (DQPSK) is presented in Fig. 7. The decision metric for DPSK detection can be written as (with the assumption that the phase difference between consecutive symbols is small)

$$U = V'_nV'_{n-1} = 4\alpha^2 E^2 e^{i(\theta_n - \theta_{n-1})}.$$  \hspace{1cm} (7)

The decision metric verifies that the decision is based on the phase difference of consecutive symbols $\theta_n - \theta_{n-1}$.

The bit error rate performance of binary DPSK as well as the symbol error rate performance of $M$-DPSK modulation is given again in Table 1. A variant of DPSK, $\pi/4$-DQPSK, is part of the digital audio broadcasting (DAB) standard [33].
Table 1. The Bit error rate or symbol error rate for various modulation schemes in an AWGN channel.

<table>
<thead>
<tr>
<th>Modulation method</th>
<th>Bit Error Rate $P_e$ or Symbol Error Rate $P_b$ in AWGN channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$</td>
</tr>
<tr>
<td>MPSK</td>
<td>$P_b = \exp(-\frac{E_b}{N_0}\sin\frac{\pi}{M})$</td>
</tr>
<tr>
<td>DBPSK</td>
<td>$P_e = \frac{1}{2} e^{-\frac{E_b}{N_0}}$</td>
</tr>
<tr>
<td>DQPSK</td>
<td>$P_b = \frac{Q(a,b)}{2} - \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$, where $a = 2\frac{E_b}{N_0}(1 - \frac{1}{\sqrt{2}}), \ b = \frac{2E_b}{N_0}\left(1 + \frac{1}{\sqrt{2}}\right)$</td>
</tr>
<tr>
<td>MFSK</td>
<td>$P_m = \int_{-\pi/2}^{\pi/2} \exp\left(-\frac{B_s}{N_0} M \cos\frac{\pi}{M} \csc\phi\right) d\phi$</td>
</tr>
<tr>
<td>Noncoherent BPSK, BPSK</td>
<td>$P_e = \frac{1}{2} e^{-\frac{E_b}{N_0}}$</td>
</tr>
<tr>
<td>Noncoherent MPSK, MFSK</td>
<td>$P_m = \sum_{m=0}^{\infty} (-1)^m \left(\frac{M-m}{\pi} \frac{1}{m+1} \frac{\pi}{M-M-1}\right)$</td>
</tr>
<tr>
<td>Rectangular QAM</td>
<td>$P_m = 2\left[1 - \frac{1}{M^2} \text{erfc}\left(\frac{3}{2(M-1)k_b}\right)\right] - \frac{1}{2} \left[1 - \frac{1}{\sqrt{M^2-1}} \text{erfc}\left(\frac{3}{\sqrt{M^2-1}}k_b\right)\right]$</td>
</tr>
<tr>
<td>M-PAM</td>
<td>$P_m = \frac{(M-1)}{M} \frac{1}{\sqrt{M^2-1}} \text{erfc}\left(\frac{3}{\sqrt{M^2-1}}k_b\right)$</td>
</tr>
</tbody>
</table>

The symbol error rate performance of MPSK as the function $M$ is plotted in Fig. 8, and it is evident that increasing the size of the symbol alphabet requires a huge increase in the received power, indicating huge transmission powers or very short link distances.
1.3.5 Digital frequency modulation

As digital frequency modulation and especially \( M \)-ary frequency shift keying is in the heart of this thesis, a more comprehensive treatment of MFSK is given in Chapter 2 as a derivative of a generalized \( M \)-ary communication method.

1.3.6 Combined modulation techniques: QAM, DAPSK, amMFSK

Quadrature amplitude modulation (QAM)

Quadrature Amplitude Modulation (QAM) has raised a lot of interest and it has been suggested to be used in numerous system proposals. In wireless access systems, QAM is part of the standard for IEEE802.11a [4], IEEE802.11g [5], IEEE802.11n as well as WiMAX, the Worldwide Interoperability for Microwave Access (IEEE802.16) [34], [35]. In broadcasting systems, the Digital Radio Mondiale (DRM) standard uses 4-QAM / 16-QAM / 64-QAM [11], [12], terrestrial digital video broadcasting (DVB-T) and digital video broadcasting for handhelds uses 16QAM and 64 QAM [36], [37], and digital video broadcasting in cable (DVB-C) uses 16, 32, 64, 128, 256 QAM [10]. The satellite service for DVB also uses a form of QAM, namely 16APSK and 32APSK [38].
Xplore there are (Oct2012) 5991 papers (with the keyword QAM) dealing with various aspects of the performance and applications of QAM. There exist several good textbooks on QAM, including [39], [40].

In a Quadrature Amplitude Modulation (QAM) system the transmitted signal can be formulated as [30]

$$s_m(t) = A_{mc} \sqrt{\frac{2E}{T}} \cos 2\pi f_s t + A_{ms} \sqrt{\frac{2E}{T}} \sin 2\pi f_s t,$$

wherein $A_{mc}$ and $A_{ms}$ are the quadrature amplitude components conveying the information. The reliable detection and demodulation of the amplitude values require that the receiver has an automatic gain control (AGC) circuitry to compensate the long term amplitude variations in the radio channel. Moreover, due to the fact that the quadrature axes need to be fixed to obtain absolute values for the amplitudes, the receiver also needs to perform accurate phase estimation and synchronization as in MPSK systems. A QAM receiver/demodulator structure (assuming symbol synchronization) is presented in Fig. 9.

Fig. 9. The QAM demodulator structure. Modified from [30].
The performance of the QAM system depends on the signal constellation used. The most used constellations include circular and rectangular constellations that are represented in Fig. 10. The symbol error rate for rectangular constellation is given in Table 1. The performance of QAM with rectangular constellation as a function of $M$ is plotted in Fig. 11 and it is evident that increasing the size of the symbol alphabet requires an increase in the received power, indicating large transmission powers or short link distances. The performance is, however, better than that achieved by MPSK, as one can see when comparing Figs. 8 and 11.
**Differential amplitude and phase shift keying (DAPSK)**

In Differential Amplitude and Phase Shift Keying (DAPSK), not only phase but also amplitudes are differentially modulated. DAPSK modulation is discussed e.g. in [41-51] where the performance in a few selected cases is presented. DAPSK modulation is also proposed in the papers to be used in HF communications, satellite communications, DRM and digital TV broadcasting.

For example, in 64DAPSK modulation 16DPSK is used to map 4 bits onto differentially encoded phase information and bits 5 and 6 are differentially encoded and mapped onto the amplitude. Fig. 12 presents the signal constellation for 64DAPSK and the differential demodulation of the amplitude information. The 16DPSK demodulation (4 bits) is performed in a classical manner to yield the 6 bits/symbol transmitted by the 64DAPSK symbol.

As indicated earlier, there are performance analyses available only for some selected cases and a general performance analysis is still an open problem.

![Fig. 12. 64DAPSK signal constellation and differential demodulation of amplitude information [46] (© 1998 IEEE).](image-url)
1.4 Contribution of this work

The results described in [52, 61] can be viewed as the starting point for this thesis. The introduction of $m$MFSK and $am$MFSK generated ideas of applying the approach to other modulation methods. The straightforward extension was to apply the method to CSK modulation to introduce $m$MCSK modulation. The analysis shows that due to the orthogonality of the signaling waveforms of $m$MCSK (with Walsh codes) and $m$MFSK, the same performance is achieved for modulation methods with the same modulation alphabet. But with CSK it is rather easy to construct non-orthogonal signaling waveforms. Hence, the thesis also gives analytical results for non-orthogonal $m$MCSK and especially considers (as an example) the use of Gold and Kasami codes. The results indicate that the best choice of codes for a non-coherent $m$MCSK system is the orthogonal code family, which is a rather intuitive result. However, for a coherent $m$MCSK system, better performance could be achieved with non-orthogonal codes. Given that we can construct a code set where the cross-correlation between the codes in the family is negative ($\rho_{ij} < 0$), performance improvement compared to the orthogonal code family is achieved. The results show that for instance a 0.5 dB performance improvement in AWGN channel can be achieved by using a specifically constructed set of Kasami codes as the basis functions in the modulation. The thesis also presents the performance analysis results of $m$MCSK in a flat Rayleigh fading channel.

To further enlarge the modulation alphabet sizes (with the price of larger spectrum usage), it was realized that combining the $m$MFSK and $m$MCSK would be an interesting choice. $m$MCSK-$m$MFSK modulation was hence introduced, which can be viewed as the main contribution of this thesis. A method to analyze this two-component modulation was developed and the performance analyses give results for $m$MCSK-$m$MFSK modulation in AWGN and flat Rayleigh fading channels for both coherent and non-coherent receivers. The performance was also assessed with orthogonal and non-orthogonal code constructs. Moreover, an antipodal extension of $m$MCSK-$m$MFSK modulation was introduced with the analysis of the performance.

The third contribution of the thesis was to introduce the $m$PPM and $m$PSM modulation methods following the $m$MFSK principles. The performance results obtained in $m$MCSK can also be applied for these modulation formats for certain scenarios. PPM and PSM modulation have been widely suggested to be used in UWB systems. In UWB systems, the means to adapt the data rate is the use of
pulse repetition. Hence, the performance analysis of the $m_{PPM}$ and $m_{PSM}$ modulation methods with pulse repetition and a non-coherent receiver was computed. The performance of pulse repetition in impulsive interference was also computed.

A minor contribution to FSK detection was also suggested. In chapter 2, a method to quasi-coherently detect FSK modulation is suggested where the signal used in the demodulation of the FSK symbols was computed from the noisy signal itself.

1.5 Organization of the thesis

This thesis is divided into 8 chapters. Chapter 1 is an introductory chapter wherein the scope of the thesis is stated and a literature review of the relevant modulation technologies is given. The contribution of this work and the author's role in the original papers is stated. In Chapter 2, the model of an $M$-ary communication system is given with the emphasis on a memoryless channel. The chapter also more closely examines the modulation techniques that are later on utilized in the thesis in formulating the combined modulation techniques. Chapter 3 will give the descriptions of the $m_{MFSK}$ and $am_{MFSK}$ modulation techniques. The performance of the methods in Additive white Gaussian noise (AWGN) and flat Rayleigh fading channels are also given. Chapter 4 extends the results in Chapter 3 to MCSK modulation. The results with orthogonal and non-orthogonal code sets are given in AWGN and flat Rayleigh fading channels. Chapter 5 combines the modulation methods described in Chapters 3 and 4 to introduce $m_{MCSK}$-$m_{MFSK}$ modulation. The modulation method is described in detail and the results with orthogonal and non-orthogonal code sets are given in AWGN and flat Rayleigh fading channels. Moreover, an antipodally extended variant is described and analyzed. Chapter 6 applies the approach in Chapters 3 and 4 to introduce the $m_{PSM}$ and $m_{PPM}$ modulation methods. In the simplified radio channels, the performance is similar to the one obtained for $m_{MFSK}$ and $m_{MCSK}$ with the orthogonality assumption. The chapter also gives results for non-coherent combining of pulse repetition, often applied in UWB communications. Diversity combining in impulsive interference with ideal side information is also computed. In Chapter 7 a qualitative discussion of the obtained results is performed and Chapter 8 will summarize the main results and point out some directions for further research in this area.
2 The model of a digital communication system

2.1 The general $M$-ary communication system

In Fig. 13, a block diagram of a digital communication system is presented. The binary source has as its output equiprobable, independent binary symbols.

The encoded binary symbols are spread in time by the interleaver. The role of the interleaver-deinterleaver pair is to produce as memoryless a channel as possible for the encoder-decoder pair. Most error correcting codes perform best on random errors, which suggests a memoryless channel. The binary convolutional codes are examples of such codes.

Fig. 13. A Block diagram of a digital communication system.

The memoryless channel is defined by the probability equation [53]

$$p_s(y|x) = \prod_{n=1}^{X} p(y_{n}|x_{n}).$$

(9)

This means that for a memoryless channel the probability of a channel output sequence $y = (y_1, y_2, ..., y_N)$, given the input sequence $x = (x_1, x_2, ..., x_N)$, is equal to the product of the individual channel use probabilities. From a set of $n_b$ binary encoded and interleaved symbols, the binary-to-$M$-ary mapper produces symbols drawn from the $M$-ary symbol alphabet to be transmitted over a noisy transmission channel by the $M$-ary modulator.

The role of the demodulator is to process the received signal and output an estimate of the transmitted symbol, or sufficient information from which this estimate can be extracted. The demodulated symbols, or rather estimates of the symbols, are converted to the binary form and processed thereafter by the de-
interleaver to produce the original order of the transmitted sequence before the sequence is fed to the decoder. The bits might be assigned a credibility measure, a metric, when decoding the bits. This metric can be calculated from the demodulated sequence, or there might be a separate channel probe, and the network can also provide some form of information on the credibility of the received symbols.

The side info synthesizer uses these sources of information to generate a metric corresponding to the credibility of the received symbols. The decoder uses the side information (JSI or CSI) and the demodulated bits in an attempt to decode the received bits. The bits are thereafter fed to the user or the application layer in general.

In the channel, the $M$-ary symbols are perturbed by a noise process of known statistics, i.e., background noise. Furthermore, intentional jamming signals might be summed to the information signal. Due to multipath propagation, the fading process can also be introduced in the channel. In this thesis only flat Rayleigh fading is considered and other fading processes are left for further studies.

2.2 Modulation formats utilized

In this chapter the descriptions and the performance equations of the existing modulation formats that are utilized are given and the extensions of these known results will be the main focus of this thesis.

2.2.1 Description of MFSK modulation

The $M$-ary modulator transmits one of $M$ equal energy signals corresponding to the received symbols from the binary-to-$M$-ary mapper. The signaling set is denoted by $\{s_0(t), s_1(t), \ldots, s_{M-1}(t)\}$. For a signalling interval $T$, the signal energy $E$ can be written in the form [30]

$$E = \int_0^T s_i^2(t) dt, \ i = 0, \ldots, M - 1.$$  \hspace{1cm} (10)

It is assumed that signals $s_i(t)$ are orthogonal, which can formally be expressed as

$$\int_0^T s_i(t)s_j(t)dt = 0, \ i \neq j.$$  \hspace{1cm} (11)
An example of such a set is the set of $M$ continuous wave carriers wherein the $i^{th}$ signal within the set is defined as

$$s_i(t) = \sqrt{2S}\cos\left(2\pi f_i t + \theta_i\right), \quad 0 \leq t \leq T,$$

where $S$ is the average signal power and $f_i$ is the $i^{th}$ carrier frequency. Such a signal set is used in a $M$-ary frequency shift keying (MFSK) modulation system. The MFSK demodulator is depicted in Fig. 14.

![Fig. 14. A Non-coherent MFSK-demodulator and symbol detector.](image)

The signal power can be expressed in terms of the signal energy as

$$S = \frac{E}{T}. \quad (13)$$

In the non-coherent receiver the signaling frequencies relate to each other, according to

$$|f_i - f_j| = \frac{|k - j|}{T}, \quad (14)$$

whereas in the coherent case the signaling frequencies relate to each other, according to

$$|f_i - f_j| = \frac{|k - j|}{2T}. \quad (15)$$
Non-coherent detection of MFSK in AWGN and Rayleigh fading

Let \( s_i(t) \) be the transmitted waveform. The signal is passed through an AWGN-channel. The received signal can be expressed as

\[
r(t) = \sqrt{2S} \cos(2\pi f_t t + \theta_i) + n(t), \quad 0 \leq t \leq T,
\]

where \( \theta_i \) is the initial phase angle of the received signal. Since we are considering non-coherent demodulation, the receiver computes \( M \) decision metrics [53], which can also be seen in Fig. 15.

\[
y_i = \left\{ \left[ \frac{2}{T} \int_0^T r(t) \cos 2\pi f_t dt \right]^2 + \left[ \frac{2}{T} \int_0^T r(t) \sin 2\pi f_t dt \right]^2 \right\}^{1/2}.
\]

The conditional probability density function of \( y_i \) given that the \( i^{th} \) signal was transmitted can be written as [30]

\[
p\left( y_m | i \right) = \begin{cases} 
y_m e^{-\frac{y_m^2}{2}}, & m \neq i \
y_i e^{\frac{-y_i^2}{2} I_0(\xi y_i)}, & m = i \end{cases},
\]

where \( I_0(\cdot) \) is the zero order modified Bessel function of the first kind and \( \xi \) is the non-centrality parameter of the Rician distribution, which in this case (AWGN) can be expressed as

\[
\xi = \frac{E_s}{2N_0}.
\]
These density functions completely define the $M$-ary channel. The decision metric corresponding to the transmitted signal ($m = i$) has a larger mean than the other metrics. Hence, in one-shot transmission, it suffices to pick the largest of the computed decision metrics to yield the minimum symbol error probability [30].

$$P_s = \sum_{n=1}^{M-1} (-1)^{n+1} \left( \frac{M-1}{n} \right) \frac{1}{n+1} e^{-\frac{A^2}{(n+1)\sigma^2}}.$$  \hfill (20)

The bit error rate of an uncoded system in a Rayleigh-fading channel can be calculated by taking into account the underlying probability density function of the amplitude of the received information signal. The amplitude of the received signal fluctuates randomly, according to Rayleigh distribution

$$p_A(A) = \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}}. \hfill (21)$$

By taking account of the fact that the symbol energy can be expressed as

$$E_s = \frac{1}{2} A^2 T,$$ \hfill (22)

and inserting (21) and (22) into (20), the conditional symbol error probability $P_d(A)$ is obtained. Now the symbol error probability in a Rayleigh fading channel is obtained by calculating

$$P_d = \int_{0}^{A} P_d(A) p_A(A) dA.$$ \hfill (23)
After the integration, the bit error probability in a Rayleigh fading channel can be expressed as [30,54]

\[
P_b = \frac{2^{k-1}}{2^k - 1} \sum_{n=1}^{\infty} (-1)^{n+1} \left( \begin{array}{c} M - 1 \\ n \end{array} \right) \frac{1}{1 + n + n \frac{E_b}{N_0}}. \quad (24)
\]

**Coherent detection of MFSK**

Without going into details about the decision metrics, it is just stated here that the decision metrics are distributed in a similar fashion as was the case for \(m\)MFSK modulation (dealt with in detail in Chapter 3.1.1), and thus we can readily write the symbol error rate performance for the coherent orthogonal MFSK in an AWGN channel as [30]

\[
P_{se} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{y}{\sqrt{2}} \right) \right) \left( 1 - 2 \text{erfc} \left( \frac{y}{\sqrt{2}} \right) \right) e^{-\frac{(y-m)^2}{2}} \, dy. \quad (25)
\]

**2.2.2 A case study: quasi-coherent detection of MFSK**

In a frequency hopping system the center frequency of the carrier is pseudo-randomly picked out of a large pool of frequencies. Each center frequency is maintained only for a brief interval \(T_f\), i.e., the duration of a frequency hop. If the locally generated beat frequencies in the receiver are not perfectly phase synchronized with the received signal, coherent reception is difficult and performance degradation will be introduced. The performance degradation is a function of the difference in phase between the signals, and hence the means to minimize the difference are of importance. In a frequency hopping system the phase error is introduced by the abrupt changes in the carrier frequency. If the number of the transmitted bits in a frequency hop is small, phase synchronization is not an easy task to do. In this section a quasi-coherent receiver, in conjunction with MFSK modulation, is proposed where the carriers for symbol demodulation are obtained from the received signal [55]. The carrier extraction can be based on pilot symbols or the data modulated signal. The performance of the proposed method is compared to that of a coherent and non-coherent MFSK system.

In a MFSK modulated system, demodulation is achieved by multiplying the received signal with carrier signals that represent the possible set of symbol
frequencies. The $M$-ary modulator transmits one of $M$ equal energy signals corresponding to the received symbols from the binary-to-$M$-ary mapper. The signaling set is defined in Eqs. 10-15.

In this, $|f_i - f_j| = \frac{(|i-j|)}{T}$ was assumed for the quasi-coherent system and the extension to $|f_i - f_j| = \frac{(|i-j|)}{2T}$ is left for a further study. The receiver structure is depicted in Fig. 16.

The received signal over $N_s$ symbol intervals, i.e., over the frequency hop, is considered to be

$$r(t) = \sqrt{2SN_s} \sum_{k=1}^{N_s} p(t-kT) \cos \left( \omega_k t + s_k \omega_s + \Delta \omega_d \right) t + \phi + n(t).$$

(26)

Fig. 16. The quasi-coherent frequency hopping MFSK receiver with symbol carrier extraction [55] (© 2000 IEEE).

where $n(t)$ is the background noise of white gaussian distribution and $p(t)$ the bit waveform. The center frequency of the $i$th frequency hop is $\omega_i = 2\pi f_i$. The frequency deviation is $\omega_s = \frac{2\pi}{T}$ for a non-coherent and quasi-coherent system and $\omega_c = \frac{\pi}{T}$ for a coherent system. The symbol $s_k$ on time instant $k$ takes values $1, \ldots, M$. In this section the assumption is made that the channel is one path fixed channel, and hence the impulse response of the channel is $\delta(t)$, i.e., there is no inter symbol interference (ISI), and the phase error $\phi$ is controlled by the use of reference bits as the first bits in each frequency hop. In a coherent system one assumes knowledge of $\phi$, whereas in a non-coherent system the decision rule is devised in a manner that the effect of $\phi$ is removed. The frequency error is defined as $\Delta \omega_d [l] = l2\pi \Delta f_d$, $l = 0, \ldots, N_s - 1$, where $\Delta f_d = f'_l - f'_0$ is the frequency difference between the local oscillator ($f'_l$) and the received signal ($f'_0$). The frequency error presents itself in baseband as rotation of the bit samples.
Assuming that the receiver has a bit waveform matched filter \( p(-t) \) and \( P \) complex samples per bit at the correct timing (time synchronicity, \( \tau = 0 \)), the sampled low-pass equivalent of the received signal is

\[
\mathbf{r}_{\text{LP}}^{\text{rPN}} = \text{diag} \left( \sqrt{2\sigma^2} \left[ e^{i\pi x_0} \mathbf{r}^{\text{rPN}} + \mathbf{h}^{\text{rPN}} \right] \right) + \mathbf{n}^{\text{rPN}},
\]

which is the input to the reference carrier estimator. The superscripts denote the dimensions of a matrix or a vector, and \([\cdot]\) denotes a vector's element-wise exponentiation. The low-pass signal \( \mathbf{r}_{\text{LP}}^{\text{rPN}} \) is fed to the carrier extractor which calculates a noise reduced waveform replica. These calculated first order carriers are filtered to yield a more accurate estimate of the symbol carrier. When extracting the reference carriers from the data modulated signal, the data modulation must first be removed. This can be accomplished with a decision feedback strategy where the symbol decisions are fed to the reference carrier extractor. In such a case the usual problems associated with erroneous symbol decisions are present. In this approach, however, the reference symbol approach is considered.

During the reception of the reference or pilot symbols, one can perform the extraction of the reference carriers since the symbol frequency remains the same, and more importantly known. In the pilot channel case, one uses either an unmodulated carrier or a known data pattern in every frequency hop to send \( N_{\text{ref}} \) reference or pilot symbols. The unmodulated carrier is equivalent to having \( \mathbf{s} = 1 \). Assuming time synchronicity, a known data pattern allows writing \( \mathbf{s} \circ \mathbf{s} = 1 \). Both of the pilot channel approaches allow the reference carriers to be extracted from \( \mathbf{r}_{\text{LP}}^{\text{rPN}} \).

The model (Eq. 27) corresponds to a sinusoidal in noise, and hence the problem can be classified as sinusoidal parameter estimation in noise. However, we do not need the parameters as such but a complex conjugate of a replica of the waveform is sufficient to achieve frequency synchronization. To obtain such a replica of the signal, the following is performed. Assuming that the reference carrier estimation is performed using the low-pass signal \( \mathbf{r}_{\text{LP}}^{\text{rPN}} \), the signal replica can be 'estimated' by firstly arranging the samples of \( \mathbf{r}_{\text{LP}}^{\text{rPN}} \) that represent one reference symbol into a forward backward linear prediction (FBLP) matrix form [56].
where $\overline{r}_{LP}$ means the complex conjugate of $r_{LP}$, $P$ denotes the total number of samples in a symbol, assuming 64 complex samples per symbol interval ($P=64$) in this thesis. For best performance of the FBLP algorithm, Haykin [56] suggests the value

$$M \approx \frac{P}{3},$$

which is used throughout the simulations. Secondly, the SVD of the matrix $A$ can be presented by the product of three matrices:

$$A^{new} = U^{new} \Sigma^{new} V^{H \, new}.$$

The dimensions of each matrix are written in matrix superscripts. The matrices $U$ and $V$ are unitary, containing the left and right singular vectors of $A$, and $\Sigma$ is a diagonal matrix containing the singular values of $A$. The singular values are conventionally in decreasing order. The operator $H$ is used for denoting the complex conjugate transpose (Hermitean) of a matrix. Thirdly, according to the Eckhart-Young Theorem [57], the best $r_s$ rank least-squares estimate $\hat{A}$ (= estimated signal replica) for the given data matrix $A$ is obtained as follows:

$$\hat{A} = \sum_{k=1}^{r_s} \sigma_k u_k v_k^*,$$

where $r_s$ is the rank of matrix $A$, $\sigma_k$ is $k^{th}$ largest diagonal element (singular value) of matrix $\Sigma$, $u_k$ and $v_k^*$ are the $k^{th}$ columns of the matrixes $U$ and $V^*$. The idea is that given signal $\overline{r}_{LP}$ (sinusoidal with cyclostationary properties in noise), it suffices to use

$$r_s = 1$$

(32)
to estimate the reference carrier, i.e., a signal replica. The rank reduced matrix \( \hat{A} \) representing the signal replica and noise is rearranged back to sampled vector form by taking averages of the matrix elements (diagonally) which correspond to the same received sample to obtain a first order reference carrier estimate \( \Psi \).

This signal is related to \( \hat{\omega}_r \). These first order reference carriers are filtered over the \( N_{\text{ref}} \) symbols to obtain a better estimate by

\[
\hat{\omega}^{\text{ref}}_r(l) = \frac{1}{N_{\text{ref}}} \sum_{i=1}^{N_{\text{ref}}} \hat{\Psi}^{(i)}(l), \quad l = 1, \ldots, P. \tag{33}
\]

The reference symbols were all assumed to be one, and hence \( \hat{\omega}^{\text{ref}}_r = \hat{\omega}^{\text{ref}}_r \) can be directly used to demodulate symbol 1. The reference carrier \( \hat{\omega}^{\text{ref}}_r \) for symbol 2 can be obtained by taking every second sample of the vector containing two replicas of \( \hat{\omega}^{\text{ref}}_r \), i.e., \( \hat{\omega}^{\text{ref}}_r \). For the \( k^{\text{th}} \) carrier \( \hat{\omega}^{\text{ref}}_r \) one takes every \( k \)th sample of a vector containing \( k \) replicas of \( \hat{\omega}^{\text{ref}}_r \).

Given the signals \( \hat{\omega}^{\text{ref}}_r, \> 1, \ldots, M \), the decision variables for the symbol decisions are obtained as

\[
y_k = \sum_{j=1}^{P} \hat{T}_{LP} \odot \hat{\omega}^{\text{ref}}_r, \quad k = 1, \ldots, M. \tag{34}
\]

The operator \( \odot \) represents element-wise multiplication of vector elements. The algorithm described in Eqs. 28-34 can be considered the best signal correlator in LS-sense, i.e., the singular vectors \( U \) and \( V^* \) span the signal subspace.

**Some numerical results**

Since the phase estimation problem is especially severe in a frequency hopping system, such a system was chosen for the simulations. One assumption was made which is hard to achieve in practice, namely that the frequency settling time of the FH-dehopper was set to zero. The number of transmitted bits in a frequency hop \( N \) was set to 50, which would suggest a low frequency hopping rate or a high data rate. The number of signaling frequencies was chosen as 4, i.e., 4FSK was the modulation method as in Fig. 16. The symbol carriers were constructed to be phase continuous. The performance of the proposed receiver structure was assessed in the one-path AWGN channel. The received signal had a random initial phase for each frequency hop. For the quasi-coherent system, 1, 3, 5 or 10 reference bits/ frequency hop were assumed. In the simulations the receiver took 64 samples of the received signal in a symbol interval \( T_s \) to obtain a proper
symbol carrier for all the symbols. The results of the simulations in an AWGN channel assuming perfect time and frequency synchronization are presented in Fig. 17. One can see that using only one reference bit results in unacceptable performance, but with as few as 3 reference bits the performance is very good and for $E_b/N_0 > 7\text{dB}$ the quasi-coherent structure outperforms the non-coherent structure. With 10 reference bits the performance is rather close to that of the coherent system.

![Fig. 17. The simulated bit error rate performance of the quasi-coherent MFSK in an AWGN channel with the number of the reference symbols as a parameter [55] (© 2000 IEEE).](image)

The purpose of the quasi-coherent structure is to achieve operability with any initial phase of the received signal symbol carrier, similarly to the non-coherent structure. This property was tested by introducing any initial phase to the symbol carrier in a simulation for each frequency hop. The results of this simulation are presented in Fig. 18 with $E_b/N_0 = 7\text{dB}$. One has to point out that the results obtained for the coherent structure are somewhat unfair since no phase estimator in that case was assumed. The results demonstrate that a quasi-coherent structure achieves the desired phase resistance, and the next question was whether the quasi-coherent structure was more susceptible to the frequency error, such as Doppler, than the coherent or non-coherent structure.

Fig. 19 presents the bit error rate as a function of the normalized frequency error $\Delta f = f_d / R$. The curves obtained show that the performance degradation in
terms of the normalized frequency error with the quasi-coherent structure is similar to that of the coherent or non-coherent structure, somewhat outperforming the traditional detection methods with higher frequency errors.

Fig. 18. The simulated bit error rate performances of coherent, non-coherent and quasi-coherent MFSK with the initial phase error $\phi \in [0, (\pi / 2)]$, $N_{\text{it}} = 5$. [55] (© 2000 IEEE).
Fig. 19. The simulated bit error rate performances of coherent, non-coherent and quasi-coherent MFSK in a Doppler environment, $N_{ref} = 5$. [55] (© 2000 IEEE).

### 2.2.3 Description of MCSK modulation

In IS-95 [58], [59], [60] uplink uses MCSK modulation where one out of 64 codes of 64 chips (orthogonal Walsh functions) is transmitted. The system transmits 6 bits/symbol, i.e., per code transmission. To enhance the spectral efficiency of the MCSK signaling set, multi-code shall be considered in Chapter 3. A standard MCSK modulation uses one out of $M$ codes each $T_s$ seconds to transmit a block of $n_b = \log_2 M$ bits. The optimum coherent receiver has a bank of $M$ sequence (code) matched filters (SMF), and every $T_s$ seconds it makes a decision based on the largest output filter sample. The non-coherent receiver uses $M$ envelope detectors (after MF) to produce the decision.
Let us assume that the system is using orthogonal codes. Let us assume that the spreading codes transmitted are denoted by
\[ c_{li}(t) = \sum_{i} c[i] p(t - iT_c), \]
where \( c[i] \) is the sequence of chips of length \( N \), \( p(t) \) is the chip waveform and \( T_c \) is the duration of one chip. The transmitted signals can be represented as
\[
\begin{align*}
    u_k(t) &= Ac_k(t) = Au_{k0}(t) \\
    s_k(t) &= \text{Re}\{u_k(t)e^{j2\pi ft}\} = A\text{Re}\{u_{k0}(t)e^{j2\pi ft}\} \\
    &= As_{k0}(t), \quad k = 1, 2, \ldots, M
\end{align*}
\]
with (for orthogonal configuration)
\[
\int_{0}^{T} u_k(t)u_{k0}(t)dt = \delta_{kk},
\]
where \( u_k(t) \) is the complex envelope of the signal and \( \delta_{kk} \) is the Kronecker delta function. The energy of these signals is
\[
E_s = \frac{1}{2} \int_{0}^{T} |u_k(t)|^2 dt = A^2 E_k, \quad k = 1, \ldots, M.
\]
After frequency down-conversion the received signal envelope is
\[
r(t) = \alpha e^{-j\phi}u_k(t) + z(t), \quad 0 \leq t \leq T,
\]
where \( \alpha \) is due to the channel attenuation, \( \phi \) is the phase difference between the input and local signal, and \( z(t) \) is Gaussian noise. For simplicity, and without loss of generality, let us assume that the signal in the first code is transmitted, i.e., \( s_1(t), l = 1 \). The received low-pass equivalent of the signal becomes
\[
r(t) = \alpha e^{-j\phi}u_1(t) + z(t), \quad 0 \leq t \leq T.
\]
Without going into details about the decision metrics (which will be done in detail in Chapter 4), it is just stated here that the decision metrics are distributed in a similar fashion as was the case for MFSK (and \(m\text{-MFSK}\) modulation, and thus we can readily write the symbol error rate performance for the coherent orthogonal MCSK in an AWGN channel as

\[
P_{se} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ 1 - \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right) \right) e^{-\left( r - \sqrt{\gamma} \right)^2 / 2} \right] dy ,
\]

(40)

and similarly for the non-coherent detection of MCSK in an AWGN channel,

\[
P_{se} = \sum_{s=1}^{M-1} (-1)^{s+1} \binom{M - 1}{n} \frac{1}{n+1} e^{-\frac{\text{erfc}(s+1) \nu}{\gamma}} .
\]

(41)
3 Description of \( m \)MFSK and \( am \)MFSK modulation

\( m \)MFSK and \( am \)MFSK are modifications of the well-known MFSK modulation described in Section 2.2.1. One can use a standard MFSK detector in signal detection of \( m \)MFSK and \( am \)MFSK [52, 61]. The difference when compared to MFSK is in the signal processing rules after the detection has occurred. One can see an example in Fig. 14 where the block – rules for symbol detection – will be different for \( m \)MFSK and \( am \)MFSK. To enhance the spectral efficiency of the signaling set, multi-tone and multi-amplitude signals are considered. Standard MFSK modulation uses one out of \( M \) frequencies each \( T_s \) seconds to transmit a block of \( n = \log_2 M \) bits. The optimum coherent receiver has a bank of \( M \) matched filters and every \( T_s \) second it makes a decision based on the largest output filter sample. The non-coherent receiver uses envelope detection, as depicted in Fig. 15.

Let us suppose that instead of sending one out of \( M \) frequencies, the transmitter sends \( m \) out of \( M \) equal amplitude frequencies simultaneously. The optimum receiver will now have to find the \( m \) largest signals at the output of the \( M \) detectors. Given that the amplitudes are the same, one can form

\[
M_m = \binom{M}{m}
\] (42)

different combinations. The number of transmitted bits per \( T_s \) is now increased to

\[
n_m = \log_2 \binom{M}{m}
\] (43)

bits. This modulation format is denoted as \( m \)MFSK modulation. In Table 2, some example values have been calculated. By observing Table 2, one notices that with the constraint \( m \leq \lfloor M / 2 \rfloor \), where \( \lfloor \cdot \rfloor \) denotes the integer part of the argument, one achieves all the possible combinations.
Table 2. The Number of possible signaling combinations for \(m\)MFSK with \(m\) and \(M\) as parameters.

<table>
<thead>
<tr>
<th>(m)</th>
<th>(M)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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If the amplitudes of the transmitted tones are all different, the number of the possible signaling combinations is further increased to

\[
M_n(a) = M(M - 1)(M - 2)\ldots(M - m + 1),
\]

or equivalently

\[
\binom{M}{m} = \frac{M!}{(M-m)!},
\]

and the number of transmitted bits is

\[
n_n(a) = \sum_{i=0}^{m-1} \log_2 (M - i),
\]

or equivalently

\[
n_n(a) = \log_2 \left( \frac{M!}{(M-m)!} \right).
\]

The name for this type of modulation is \(am\)MFSK, where \(M\) is the number of frequencies available, \(m\) the number of simultaneously transmitted tones and the letter \(a\) is to point out that all the tones are of different amplitudes. In Table 3, some example values have been calculated. By observing Table 3, one notices that with constraint \(m < M - 1\), one achieves all the possible combinations.
Table 3. The Number of possible signaling combinations for \(amMFSK\) with \(m\) and \(M\) as parameters.

<table>
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<tr>
<th>(m)</th>
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<td>360</td>
<td>840</td>
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<td>120</td>
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<td>720</td>
<td>5040</td>
<td>20160</td>
<td>60480</td>
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In these cases the signal energy is divided into \(m\) separate tones, making it more vulnerable to noise and fading, but the overall flow of useful information will still be increased under a large range of signal, channel and interference parameters, thus offering better spectral efficiency. To support this statement, the bit error probability results for \(mMFSK\) are presented in the following sections. The results for \(amMFSK\) with \(m = 2\) are available in [52] but the generalization of the \(amMFSK\) analysis is left for further study.

### 3.1 Bit error probability results for \(mMFSK\) in an AWGN channel

In this chapter the bit/symbol error probability results for coherent and non-coherent reception of \(mMFSK\) and \(amMFSK\) signals are presented. The results have been first presented in [52, 61]. The channel is the additive white Gaussian noise (AWGN) channel.

#### 3.1.1 Error probability for a coherent \(mMFSK\) receiver

The transmitted signals can be represented as

\[
\begin{align*}
    u_k(t) &= A e^{j2\pi f t} = A u_{\text{re}}(t) \\
    s_k(t) &= \text{Re}[u_k(t) e^{j2\pi f t}] 
\end{align*}
\]  

with (for orthogonal configuration)
where \( u_k(t) \) is the complex envelope of the signal and \( \delta_{\text{low}} \) is the Kronecker delta function. The energy of these signals is

\[
E_k = \int_0^T |u_k(t)|^2 dt = \int_0^T |u_0(t)|^2 dt
\]

\[= A^2 E_0 = E, \quad k = 1, \ldots, M. \] (48)

After frequency down-conversion the received signal complex envelope is

\[
r(t) = \alpha e^{-j\phi} u_k(t) + z(t), \] (49)

where \( \alpha \) is the channel attenuation coefficient, \( \phi \) the phase difference between the input and local signal, and \( z(t) \) is Gaussian noise. For simplicity, and without loss of generality, let us assume that the first \( m \) frequencies are transmitted, i.e., \( s_l(t), \quad l = 1, 2, \ldots, m \). The received low-pass equivalent of the signal becomes

\[
r(t) = \alpha e^{-j\phi}[u_1(t) + \ldots + u_m(t)] + z(t), \quad 0 \leq t \leq T. \] (50)

The decision variables are now given as

\[
U_k = \text{Re} \left\{ e^{j\phi} \int_0^T r(t) u_k^*(t) dt \right\}, \quad k = 1, \ldots, M. \] (51)

An optimum receiver will choose the \( m \) largest ones. Parameter \( U_k \) can be represented as

\[
U_l = 2\alpha E + N_{\omega_l}, \quad l = 1, 2, \ldots, m \]

\[
U_p = N_{\omega_p}, \quad p = m + 1, \ldots, M, \] (52)

where \( N_{\omega_l} \) are Gaussian zero mean variables with variance \( \sigma^2 = 2EN_0 \), and \( N_0 \) is the one-sided noise spectral density. The probability density functions (pdf’s) for \( U_k \) can be represented as

\[
p(U_k) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{(U_k - 2\alpha E)^2}{2\sigma^2}} \] (53)

\[
p(U_p) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{U_p^2}{2\sigma^2}} \]
A correct decision is reached if and only if \( U_i > U_p \) with all combinations of \( l \) and \( p \). The probability of a correct decision is

\[
P_c = P(U_i > U_{m+1}, \ldots, U_i > U_M) \cdot P(U_m > U_{m+1}, \ldots, U_m > U_M).
\]

(54)

Given the fact that all the \( U_i \) are mutually identically distributed and that all the \( U_p \) are mutually identically distributed, we can write

\[
P_c = [P(U_i > U_{m+1}, U_i > U_{m+2}, \ldots, U > U_M)]^m.
\]

(55)

The probability of a correct decision can be then formulated as

\[
P_c = \left[ \frac{1}{2^{M-\alpha}} \int_{\infty}^{\infty} e^{-x^2} \left( 1 + \text{erf} \left( x + \sqrt{\frac{\alpha^2 E}{N_0}} \right) \right)^{M-\alpha} dx \right]^m.
\]

(56)

Given the probability of a correct decision \( P_c \), the probability of a symbol error \( P_m \) can be written as a complement of the probability of a correct decision to yield

\[
P_m = 1 - P_c.
\]

(57)

This performance equation should collapse to that of Eq. (25) when \( m = 1 \). This can be easily verified.

**3.1.2 Error probability for a non-coherent mMFSK receiver**

Observing Fig. 15, one notices that the decision variables in the detection of FSK are given for the non-coherent receiver as

\[
U_i = \int_0^\tau r(t) u_k(t) dt, \quad k = 1, \ldots, M_e,
\]

(58)

and the receiver then chooses the \( m \) largest ones. Parameter \( U_k \) can be represented as

\[
U_i = 2\alpha E + N_i, \quad i = 1, \ldots, m_e
\]

\[
U_p = N_p, \quad p = m_e + 1, \ldots, M_e,
\]

where \( N_i \) are Gaussian zero mean variables with variance \( \sigma^2 = 2EN_0 \), and \( N_0 \) is the noise spectral density. One can show that pdf's for \( U_i \) can be expressed as
\[ p(U_i) = \frac{U_i}{\sigma^2} e^{-\frac{U_i^2}{2\sigma^2}} \text{I}_0 \left( \frac{aU_i}{N_0} \right), \quad (60) \]

when \( l = 1, \ldots, m \), where \( \text{I}_0(\cdot) \) is the zero order modified Bessel function of the first kind. For \( p = m + 1, \ldots, M \) the pdf is

\[ p(U_p) = \frac{U_p}{\sigma^2} e^{-\frac{U_p^2}{2\sigma^2}}. \quad (61) \]

The probability of a correct decision is

\[ P_c | U_1, \ldots, U_m = P(U_1 > U_{m+1}, \ldots, U_l > U_M) \ldots P(U_m > U_{m+1}, \ldots, U_m > U_M) \quad (62) \]

Because \( P(U_{x > m+1}, \ldots, U_x > U_M), \ x \leq m \) are the same, the probability of a correct decision assumes the form

\[ P_c = [P(U_1 > U_{m+1}, U_1 > U_{m+2}, \ldots, U_1 > U_M)]^m, \quad (63) \]

which after some calculus becomes [52, 61]

\[ P_c = \sum_{n=0}^{m} (-1)^n \left( \begin{array}{c} M - m \\ n \end{array} \right) \frac{1}{n+1} e^{\frac{-a^2}{(m+1)N_0}}, \quad (64) \]

and thus the symbol error probability is \( P_m = 1 - P_c \).

### 3.2 Bit error probability results for mMFSK in a frequency non-selective Rayleigh fading channel

#### 3.2.1 Error probability for a coherent mMFSK receiver

Let us assume that the receiver has perfect estimates of complex valued channel coefficients \( a e^{-j\phi} \) which imply that the channel is slowly fading. Let us make the notation

\[ G(z) = 1 - \frac{1}{2^{M-m} \sqrt{\pi}} \int_0^\infty \left[ 1 + \text{erf}(x + \sqrt{z}) \right]^{M-m} dx \quad (65) \]
to simplify the following derivations. $G(z)$ is the symbol error rate of coherent \textit{mMCSK} in a Gaussian channel with $z = \alpha^2 E / N_0$. In a Rayleigh fading channel the amplitude of the received signal varies according to the Rayleigh distribution,

$$ p(a) = \frac{a}{\sigma_c^2} e^{-\frac{a^2}{2\sigma_c^2}}. \quad (66) $$

The function $G(z)$ can be modified to show the amplitude information by noting that $\alpha^2 E / N_0 = a^2 / \sigma^2$. Now one can derive the probability of a symbol error as

$$ P_m = \int_0^\infty G \left( \frac{a^2}{\sigma^2} \right) \frac{a}{\sigma_c^2} e^{-\frac{a^2}{2\sigma_c^2}} da. \quad (67) $$

Making a change of variables, $a^2 / \sigma^2 = x^2 \rightarrow da = xdx$, $a = \sigma x$ allows writing

$$ P_m = \int_0^\infty G \left( x^2 \frac{\sigma^2}{\sigma_c^2} \right) x e^{-\frac{x^2}{2}} dx, \quad (68) $$

where signal-to-noise ratio is defined by $\sigma^2 = 2EN_0$.

\textbf{3.2.2 Error probability for a non-coherent mMFSK receiver}

Let us consider a square law detection of the received signals. The $m_f$ outputs containing the signal are

$$ U_l = |2E \alpha e^{-j\phi} + N_{l,p}|^2, \quad l = 1, \ldots, m_f \quad (69) $$

and the remaining $M_f-m_f$ outputs not containing the signal are

$$ U_p = |N_{p}|^2, \quad p = m_f + 1, \ldots, M_f. \quad (70) $$

The probability of error is the complement of the case that $U_l$ exceeds all the $U_p$ for $l=1,\ldots,m_f$ and $p=m_f+1,\ldots,M_f$. Since the signals are orthogonal and the additive noise processes are mutually statistically independent, the random variables $U_1, U_2, \ldots, U_{M_f}$ are also mutually statistically independent. The complex-valued random variables $N_{l,p}$ and $N_{p}$ are zero-mean Gaussian-distributed. Hence it follows that the decision variables $U_l$ and $U_p$ are distributed according to a chi-square probability distribution with 2 degrees of freedom. The distributions are
\[ p(U_l) = \frac{1}{2\sigma^2} e^{-\frac{U_l}{2\sigma^2}}, \quad l = 1, \ldots, m_f \]  
(71)

and

\[ p(U_p) = \frac{1}{2\sigma^2} e^{-\frac{U_p}{2\sigma^2}}, \quad p = m_f + 1, \ldots, M_f \]  
(72)

with

\[ \sigma^2 = 2EN_0. \]  
(73)

A correct decision is reached if and only if \( U_l > U_p \) with all combinations of \( l \) and \( p \). The probability of a correct decision is

\[
P_f | U_1, \ldots, U_{m_f} = P(U_1 > U_{m_f + 1}, \ldots, U_1 > U_{M_f}) \ldots.
\]
\[
P(U_{m_f} > U_{m_f + 1}, \ldots, U_{m_f} > U_{M_f}).
\]  
(74)

Given the fact that all the \( U_l \) are mutually identically distributed and that all the \( U_l \) are mutually identically distributed allows to write

\[
P_f | U_1, \ldots, U_{m_f} = [P(U_1 > U_{m_f + 1}, \ldots, U_1 > U_{M_f})]^m, \quad k = 1, \ldots, M_f.
\]  
(75)

With \( U_l \) fixed, the joint probabilities \( P(U_1 > U_{m_f + 1}, U_1 > U_{m_f + 2}, \ldots, U_1 > U_M) \) are equal to \( P(U_1 > U_{m_f + 1}) \) raised to the \( M_f - m_f \) power, yielding

\[
P_f = \left[ P(U_1 > U_{m_f + 1})^{M_f - m_f} \right]^m.
\]  
(76)

Now the probability

\[
P(U_l > U_{m_f + 1}) = \int_0^U p(U_p) dU_p = 1 - e^{-\frac{U_l}{2\sigma^2}}.
\]  
(77)

The \( M_f - m_f \) power of this probability is then averaged over the probability density function of \( U_l \) to yield the probability of a correct decision

\[
P_f = \left[ \int_0^U \left( 1 - e^{-\frac{U_l}{2\sigma^2}} \right)^{M_f - m_f} dU_l \right]^m.
\]  
(78)

After some calculus [30, 54], it follows that the probability of a symbol error is
\[ P_e = 1 - \left[ \sum_{n=2}^{M_f-m_f} (-1)^n \binom{M_f-m_f}{n} \frac{1}{1+n+n\frac{E}{N_0}} \right]^{m_f}. \quad (79) \]

### 3.3 Some Numerical Results

To assess the performance of \( m \)MFSK, some simulations were performed. The simulations and analysis agree. As is expected, the performance is significantly worse than in an AWGN channel and the linear drop of the symbol error probability is observed. In general, the coherent version outperforms the non-coherent one by approximately 2 dB.

![Fig. 21. The symbol error probability as a function of the average signal-to-noise ratio of coherent \( m \)MFSK in a frequency non-selective Rayleigh fading channel with \( M = 8 \) and \( m = 1,2,3 \).](image)
Fig. 22. The symbol error probability as a function of the average signal-to-noise ratio of non-coherent \( m \)MFSK in a frequency non-selective Rayleigh fading channel with \( M_f = 8 \) and \( m_f = 1, 2, 3 \).
4 \textit{mMCSK modulation}

4.1 Description of \textit{mMCSK} modulation

To enhance the spectral efficiency of the MCSK signaling set, multi-code signals are considered [62]. Standard MCSK modulation uses one out of $M_c$ codes each $T_s$ seconds to transmit a block of $n = \log_2 M_c$ bits. The optimum coherent receiver has a bank of $M_c$ matched filters (MF) and every $T_s$ seconds it makes a decision based on the largest output filter sample. The non-coherent receiver uses $M_c$ envelope detectors (after MF) to produce the decision.

Let us assume that the system is using orthogonal codes. Let us assume that the spreading codes transmitted on each frequency are denoted by $c_i(t) = \sum_{n=1}^{N_c} c_{i}[n] p(t-nT_c)$, where $c_i[i]$ is the sequence of chips of the length $N_c$, $p(.)$ is the chip waveform and $T_c$ is the duration of one chip. Assuming perfect frequency and code time synchronization, the receiver can be depicted as in Fig 23.

![Fig. 23. An \textit{mMCSK} receiver [62] (© 2002 IEEE).](image)

The transmitted signals can be represented as

$$u_k(t) = AC_k(t) = Au_{in}(t)$$

$$s_k(t) = Re[u_k(t)e^{j2\pi B_k T_c}] = ARe[u_{in}(t)e^{j2\pi B_k T_c}]$$

$$= As_{in}(t), k = 1, 2, ..., M$$

with (for orthogonal configuration)

$$\int_0^T u_k(t)u^*_k(t)dt = \delta_{kk}.$$  \hfill (81)

where $u_k(t)$ is the complex envelope of the signal and $\delta_{kk}$ is the Kronecker delta function. The energy of these signals is
Hence the symbol energy is defined as \( E_s = m_i E \). After frequency down-conversion, the received signal envelope is

\[
r(t) = a e^{-j\phi} u_i(t) + z(t), \quad 0 \leq t \leq T,
\]

where \( a \) is due to channel attenuation, \( \phi \) is the phase difference between the input and local signal, and \( z(t) \) is Gaussian noise. For simplicity, and without loss of generality, let us assume that signals in the first \( m_c \) codes are transmitted, i.e., \( s_n(t), n = 1, 2, ..., m_c \). The received low-pass equivalent of the signal becomes

\[
r(t) = a e^{-j\phi} \left( u_1(t) + u_2(t) + ... + u_{m_c}(t) \right) + z(t), \quad 0 \leq t \leq T.
\]

### 4.2 Performance analysis of \( m \)MCSK modulation in an AWGN channel

#### 4.2.1 Error probability for coherent \( m \)MCSK with an orthogonal code set

In this chapter the coherent receiver is considered, and hence the receiver has knowledge of the phase difference \( \phi \).

Observing Fig. 23 one notices that the decision variables in the detection of CSK are given for the coherent receiver as

\[
U_k = R e \left\{ e^{j\phi} \int_0^T r(t) u_i(t) dt \right\}, \quad k = 1, ..., M_c.
\]

An optimum receiver will choose the largest \( m_c \). Parameter \( U_k \) can be represented as

\[
U_l = 2aE + N_{0}, \quad l = 1, 2, ..., m_c, \\
U_p = N_{0}, \quad p = m_c + 1, ..., M_c,
\]

where \( N_{0} \) are Gaussian zero-mean variables with variance \( \sigma^2 = 2EN_0 \), and \( N_0 \) is the noise spectral density. Probability density functions (pdf) for \( U_k \) can be represented as
\[ p(U_l) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(U_l - 2\alpha E)^2}{2\sigma^2}}, \quad l = 1, 2, \ldots, m_c \]  

\[ p(U_p) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{U_p^2}{2\sigma^2}}, \quad p = m_c + 1, 2, \ldots, M_c. \]  

(87)

The probability of a correct decision is

\[ P_c | U_1, \ldots, U_{m_c} = P(U_1 > U_{m_c+1}, \ldots, U_1 > U_{M_c}). \]  

(88)

Because \( P(U_i > U_{m_c+1}, \ldots, U_i > U_{M_c}) \), \( x \leq m_c \) are the same, the probability of a correct decision assumes the form

\[ P_c = \left[ P(U_1 > U_{m_c+1}, U_1 > U_{m_c+2}, \ldots, U_1 > U_{M_c}) \right]^{m_c}, \]  

(89)

which after some calculus becomes [62]

\[ P_c = \frac{1}{2^{M_c - m_c}} \int_{-\infty}^{\infty} e^{-x^2 - \sqrt{\pi} x} \left[ 1 + \text{erf} \left( x + \sqrt{\frac{\alpha^2 E}{N_0}} \right) \right]^{m_c} dx. \]  

(90)

### 4.2.2 Error probability for non-coherent mMCSK with an orthogonal code set

Observing Fig. 23 one notices that the decision variables in the detection of the CSK component are given for the non-coherent receiver as

\[ U_k = \int_0^1 r(t) u_k(t) dt, \quad k = 1, 2, \ldots, M_c \]  

(91)

and it chooses the \( m_c \) largest ones. Parameter \( U_k \) can be represented as

\[ U_i = 2\alpha E + N_0, \quad l = 1, 2, \ldots, m_c \]  

\[ U_p = N_0, \quad p = m_c + 1, 2, \ldots, M_c, \]  

(92)

where \( N_0 \) are Gaussian zero-mean variables with variance \( \sigma^2 = 2EN_0 \), and \( N_0 \) is the noise spectral density. One can show that pdf's for \( U_k \) can be expressed as:

\[ p(U_i) = \frac{U_i^{\frac{1}{2} + 4\alpha^2 E}}{\sigma^2} I_0 \left( \frac{\alpha U_i}{N_0} \right), \]  

(93)
when \( l = 1, \ldots, m_c \) where \( I_0(\cdot) \) is the zero order modified Bessel function of the first kind. For \( p = m_c + 1, \ldots, M_c \) the pdf is

\[
p(U_p) = \frac{U_p}{\sigma^2} e^{-\frac{U_p^2}{2\sigma^2}}.
\]  

(94)

The probability of a correct decision is

\[
P_c \mid U_1, \ldots, U_{m_c} = P(U_1 > U_{m_c+1}, \ldots, U_{l} > U_{M_c}) \ldots.
\]

(95)

\[
P(U_{m_c} > U_{m_c+1}, \ldots, U_{l} > U_{M_c}).
\]

Because \( P(U_x > U_{m_c+1}, \ldots, U_x > U_{M_c}) \), \( x \leq m_c \) are the same, the probability of a correct decision assumes the form

\[
P_c = \left[ P(U_1 > U_{m_1+1}, U_1 > U_{m_2+2}, \ldots, U_1 > U_{M_c}) \right]^{m_c},
\]  

(96)

which after some calculus becomes [62, 52, 61]

\[
P_c = \left[ \sum_{n=0}^{M-c} (-1)^n \left( \frac{M_c - m_c}{n} \right) \frac{1}{n+1} e^{-\frac{m_c^2}{(n+1)\sigma^2}} \right]^{m_c}.
\]  

(97)

4.2.3 Error probability for coherent mMCSK with a non-orthogonal code set

The decision variables are given for the coherent receiver as

\[
U_k = \text{Re}\left\{ e^{j\phi} \int_{r(t)u_l(t)dt} \right\}, k = 1, \ldots, M_c.
\]  

(98)

Taking into account that non-orthogonal codes are used, the parameter \( U_k \) can be now be represented as [62]

\[
U_l = 2aE\sqrt{\rho_l} + N_{l}, \quad l = 1, 2, \ldots, m_c,
\]

\[
U_p = 2aE\sqrt{\rho_l} + N_{p}, \quad p = m_c + 1, \ldots, M_c,
\]  

(99)

where \( N_{l} \) are Gaussian zero-mean variables with variance \( \sigma^2 = 2EN_0 \) and \( N_0 \) is the noise spectral density. The probability density functions (pdf) for \( U_k \) can be represented as

70
where $\rho_i = m_i, \rho_{ij}$ and $\rho_f = 1 + (m_i - 1)\rho_{ij}$, and $\rho_{ij}$ ($i \neq j$) is the cross-correlation value at the correct time instant between a code pair within the code family used in the mMCSK alphabet, and $N$ is the code length. The probability of a correct decision after some calculus becomes [62]

$$P_{eq} = \left[ \frac{1}{2^{M_c-M}} \int_{-\infty}^{+\infty} e^{\frac{(x-2\alpha\sqrt{\rho_f}E)}{2\sigma^2}} \left[ 1 + \text{erf} \left( \frac{x-2\alpha\sqrt{\rho_f}E}{\sqrt{2\sigma^2}} \right) \right]^{M_c-M} dx \right].$$

(101)

This analysis applies to cases where $\rho_{ij}$ is the same between any code pair within the family. Such codes are, for instance, Gold codes and a restricted set of Kasami codes.

### 4.2.4 Error probability for non-coherent mMCSK with a non-orthogonal code set

Given that we have a non-orthogonal code set as our basis functions, cross-correlation between the basis functions is observed. This cross-correlation might produce performance degradation but will, on the other hand, allow for free code set selection. The transmitted signal, its energies and the equivalent baseband form of the received signal are given again by Eqs. (80)-(84).

The receiver will create decision variables

$$U_k = \int_0^{\infty} r(t)\nu_k(t)dt, k = 1, \ldots, M_c$$

(102)

and choose the $m_c$ largest ones. One can show that pdf's for $U_k$ can be expressed as

$$p(U_i) = \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-\frac{(U_i-2\alpha\sqrt{\rho_f})^2}{2\sigma_i^2}}, i = 1, \ldots, m_c$$

(103)
when \( l = 1, \ldots, m \) and for \( p = m_t + 1, \ldots, M_c \)

\[
p(U_p) = \frac{U_p}{\sigma_j} e^{\frac{\nu_j + \alpha \sqrt{j_k \beta}}{2\sigma_j^2}} \left( \frac{\alpha \rho \sigma_j}{N_0} \right), \quad p = m_t + 1, \ldots, M_c, \tag{104}
\]

where \( I_0(\cdot) \) is the zero order modified Bessel function of the first kind and \( \rho_i = m \left| \rho_{ij} \right| \) and \( \rho_j = 1 - (m - 1) \left| \rho_{ij} \right| \), and with variance \( \sigma_j^2 = 2EN_0 + \sigma_i^2 \). The extra noise term \( \sigma_i^2 \) is due to the fact that the noise in the decision variables is correlated and can be approximated (Gaussian approximation) by \( \sigma_i^2 = \frac{P_i}{N} 2EN_0 \), where \( \rho_{ij} \) \((i \neq j)\) is the cross-correlation value at the correct time instant between a code pair within the code family used in the mMCSK alphabet, and \( N \) is the code length. The probability of a correct decision is defined in general form by Eq. (95) and (96) it and now becomes \( \left( \beta_1 = 4\alpha^2 \rho_i^2 E^2, \beta_2 = 4\alpha^2 \rho_j^2 E^2 \right) \) [62]

\[
P_{cc} = \left[ \int_0^{\alpha \rho \sigma_j} \int_0^{\alpha \rho \sigma_j} e^{\frac{\nu_j + \alpha \sqrt{j_k \beta}}{2\sigma_j^2}} \left( \frac{\alpha \rho \sigma_j}{N_0} \right) dy \right]^{M_c-m_t} \left[ \int_0^{\alpha \rho \sigma_j} \int_0^{\alpha \rho \sigma_j} e^{\frac{\nu_j + \alpha \sqrt{j_k \beta}}{2\sigma_j^2}} \left( \frac{\alpha \rho \sigma_j}{N_0} \right) dx \right]^{M_c-m_t}. \tag{105}
\]

### 4.3 Numerical results

A system employing either coherent or non-coherent mMCSK modulation was simulated. For the orthogonal code set case we used \( N = 64 \) chip Walsh functions and for the non-orthogonal code set case we used \( N = 63 \) chip small set Kasami codes and Gold codes. The number of codes used in the receiver was set at \( M_c = 8 \), whereas the number of transmitted codes \( m_t \) was varied between 1 and 4. Figs. 24, 26 present the simulated and theoretical bit error rate results for coherent and non-coherent mMCSK modulation in an AWGN channel. To show that the analytical results agree with reality, we simulated both coherent and non-coherent mMCSK-modulated systems with \( M_c = 8 \) and \( m_t = 1, 2, 3 \). Fig. 26 shows the results obtained for the non-coherent case and Fig. 24 shows the results for the coherent detection case. From the depicted results, one can easily see that the analytical results agree well with the simulated results.
Fig. 24. The Bit Error rate performance of coherent $m$MCSK in an AWGN channel [62] (© 2002 IEEE).

Fig. 25. The Performance of coherent $m$MFSK with Walsh functions and Kasami codes for $m = 2, 3$ and $M_c = 8$ [62] (© 2002 IEEE).
In Fig 25 the error rate performance of a non-orthogonal code set for coherent detection is considered. We compare the performance of Kasami codes and Walsh functions. The cross-correlation between the individual Kasami codes was $-9/63$, Gold codes $-1/63$, whereas Walsh functions were naturally orthogonal.
The results indicate that with coherent reception and Kasami codes, one achieves approximately ½ dB better performance compared to the orthogonal case. The performance of the non-coherent receiver with Gold codes and Kasami-codes is presented in Fig. 27. The results indicate that the performance deteriorates with increasing cross-correlation value in the code set and with increasing $m_c$. Hence, for non-coherent $m$MCSK one would choose a code family with very low cross-correlation value.

To assess the efficiency of the modulation method against standard MCSK modulation efficiency, the improvement factor metric defined in [63] was used,

$$E_{\text{eff}} = \frac{(1-P_e)n}{(1-P_0)n_0},$$

where $P_e$ is the probability of a symbol error and $n_0$ the number of bits/symbol for baseline MCSK modulation.

It is observed that for higher SNR, efficiency improvement is observed and it is saturated to maximum (e.g. 2.05 for $m = 4$) with SNR values of less than 20 dB.
Fig. 29. Efficiency improvement for coherent mMCSK with Kasami codes and \( m = 2, 3, 4 \) \((M_c = 8)\). MCSK uses orthogonal codes. [62] © 2002 IEEE.
5 Combined $m$MCSK-$m$MFSK modulation

This chapter contains the bulk of the new modulation ideas and the derived results of this thesis. First the new modulation method will be introduced and thereafter the performance analysis will be computed.

5.1 Description of combined $m$MCSK-$m$MFSK modulation

Consider a system transmitting on $m_f$ out of $M_f$ frequencies (such as an $m$MFSK system). To further enhance the system spectral efficiency, consider the system to transmit $m_c$ signals on each $m_f$ frequencies (such as spreading codes) out of a set of $M_c$ possible signals. The proposed system in a coherent configuration is depicted in Fig. 30 [64].

Fig. 30. A $m$MCSK-$m$MFSK receiver [64] (© 2004 IEEE).
The possible \( n \)MCSK signal set on each frequency is the same. The symbol alphabet for the proposed system is enlarged to

\[
M_m = \begin{pmatrix} M_f \\ m_f \\ \vdots \\ M_c \end{pmatrix}^{m_m}.
\]

(107)

Let us assume that the system is using orthogonal codes. Let us assume that the spreading codes transmitted on each frequency are denoted by

\[
c_i(t) = \sum c_i[t] p(t - iT_c),
\]

where \( c_i[t] \) is the sequence of chips of the length \( N_c \), \( p(\cdot) \) is the chip waveform and \( T_c \) is the duration of one chip. Assuming perfect frequency and code time synchronization and that the frequencies are separated so that the signals on different frequencies are orthogonal, the receiver in frequency branch \( \ell \) can be depicted as in Fig. 31.

\[
\text{Fig. 31. The receiver branch on frequency } \ell.
\]

The transmitted signals can be represented as

\[
u_{k,j}(t) = A c_i(t) = A u_{kn}(t)
\]

\[
s_{k,j}(t) = \text{Re}[u_{k,j}(t)e^{j2\pi ft}] = A \text{Re}[u_{k,j}(t)e^{j2\pi ft}]
\]

\[
= A s_{kn}(t), \quad k = 1,2,\ldots,M
\]

(108)

with (for orthogonal configuration)

\[
\int_0^T u_{k,j}(t)u_{n,j}(t)dt = \delta_{kn},
\]

(109)

where \( u_{k,j}(t) \) is the complex envelope of the signal and \( \delta_{kn} \) is the Kronecker delta function. The energy of these signals is
Hence the symbol energy is defined as \( E_k = m_m E \). After frequency down-conversion, the received signal on frequency \( \ell \) is

\[
r(\ell)(t) = \alpha e^{-j\varphi} u_{\ell,1}(t) + z(t), 0 \leq t \leq T.
\]

where \( \alpha \) is due to channel attenuation, \( \varphi \) is the phase difference between the input and local signal, and \( z(t) \) is Gaussian noise. For simplicity, and without loss of generality, let us assume that signals in the first \( m_c \) codes are transmitted, i.e., \( s_{\ell,n}(t), n = 1, 2, \ldots, m_c \). The received low-pass equivalent of the signal becomes [64]

\[
r(t) = \alpha e^{-j\varphi} \left( u_{\ell,1}(t) + u_{\ell,2}(t) + \ldots + u_{\ell,m_c}(t) \right) + z(t), 0 \leq t \leq T.
\]

The detection is performed in two steps. First the frequencies where the transmission is assumed to reside are found by energy comparison. Thereafter the codes within the found frequencies are searched.

### 5.2 Performance analysis of \( m \)MCSK-\( m \)MFSK modulation in an AWGN channel

This section will present the results of the analysis and the simulations performed for \( m \)MCSK-\( m \)MFSK modulation. The performance is assessed in AWGN and flat Rayleigh fading (in Chap. 5.3) channels. For the AWGN channel, both orthogonal and non-orthogonal code sets are studied as well as an antipodal extension of the modulation.

#### 5.2.1 Coherent \( m \)MCSK-\( m \)MFSK with orthogonal code sets

In this chapter the coherent receiver is considered, and hence the receiver has knowledge of the phase difference \( \varphi \). Let \( P_{\ell} \) be the probability of detecting correctly the transmission frequencies and \( P_{\ell,1} \) the probability of detecting correctly the transmitted codes within the frequency. In the sequel we derive expressions for \( P_{\ell,1} \) and \( P_{\ell} \) and assess the performance of the modulation using these results.
Error performance of the CSK component

Observing Fig. 31 one notices that the decision variables in the detection of the CSK component on frequency $\ell$ are given for the coherent receiver as

$$U_k = \text{Re}\left\{ e^{j\pi} \int_0^T r(t) u_{k,\ell}(t) dt \right\}, \quad k = 1, \ldots, M_c.$$  \hfill (113)

An optimum receiver will choose the $m_c$ largest. Parameter $U_k$ can be represented as

$$U_l = 2\alpha E + N_p, \quad l = 1, 2, \ldots, m_c$$
$$U_p = N_p, \quad p = m_c + 1, \ldots, M_c,$$  \hfill (114)

where $N_p$ are Gaussian zero-mean variables with variance $\sigma^2 = 2EN_0$, and $N_0$ is the noise spectral density. The probability density functions (pdf) for $U_k$ can be represented as

$$p(U_l) = \frac{1}{\sqrt{2\pi}\sigma^r} e^{-\frac{(U_l - 2\alpha E)^2}{2\sigma^r}}, \quad l = 1, 2, \ldots, m_c$$
$$p(U_p) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{U_p^2}{2\sigma^2}}, \quad p = m_c + 1, \ldots, M_c.$$  \hfill (115)

The probability of a correct decision is

$$P_c \left| U_1, \ldots, U_m = P(U_1 > U_{m+1}, \ldots, U_1 > U_{M_c}) \ldots \right.$$  
$$P(U_m > U_{m+1}, \ldots, U_m > U_{M_c}).$$  \hfill (116)

Because $P(U_x > U_{m+1}, \ldots, U_x > U_{M_c})$, $x \leq m_c$ are the same, the probability of a correct decision assumes the form

$$P_c = \left[ P(U_1 > U_{m+1}, U_1 > U_{m+2}, \ldots, U_1 > U_{M_c}) \right]^{m_c},$$  \hfill (117)

which after some calculus becomes [64, 52]

$$P_c = \left[ \frac{1}{2^{M_c-m_c}\sqrt{\pi}e^{-\frac{\xi^2}{2}}} \left[ 1 + \text{erf} \left( \frac{x + \sqrt{\frac{\alpha^2 E}{N_0}}}{} \right) \right] \right]^{m_c}.$$  \hfill (118)
Error performance of the FSK component

In CSK detection, the \( M_c \) decision variables on each frequency were sorted in descending order. For FSK detection, one chooses the largest \( m_c \) on each frequency. Before the sorting operation, these \( M_c \) individual decision variables on each frequency are distributed according to Eq. 115.

Symbol on frequency

In this case, \( M_c - m_c \) SMFs output Gaussian noise only and \( m_c \) SMFs output signal+noise. Since we are searching for the probability of a correct decision, we can assume that the detector correctly identifies the \( m_c \) signals present. The pdf of the \( m_c \) variables is of the form

\[
p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{119}
\]

The decision variable for FSK detection is the sum of these \( m_c \) decision variables. The pdf of the sum of Gaussian variables is also Gaussian with mean \( \mu = \sum \mu_i \) and variance \( \sigma^2 = \sum \sigma_i^2 \), resulting in our case in a pdf of the form

\[
p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{120}
\]

Symbol not on frequency

In this case all the \( M_c \) SMFs output Gaussian noise only. After the sorting operation, by using order statistics, one can find the distributions of the elements of the sorted ensemble. Let us denote the pdf before sorting as \( f(x) \) and the corresponding cumulative distribution function (cdf) as \( F(x) \), i.e., [64]

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}}
\]

\[
F(x) = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{x}{\sqrt{2\sigma}}\right) \tag{121}
\]

with \( \sigma^2 = 2EN_0 \). Now the pdfs of the sorted ensemble can be expressed as [65]
What we have is $m_r$ decision variables which are distributed according to $f_r(x)$ with $r \in [M_r - m_r + 1, ..., M_r]$. Prior to FSK detection, we sum these $m_r$ decision variables, and what we need in the analysis is the pdf of this sum. The convolution of pdfs cannot be used because the $f_r(x)$'s are dependent. The exact pdf proves to be rather elusive, but we can approximate it with a Gaussian pdf.

First we need the mean of the desired pdf. This can be obtained in the following fashion. The mean of each of the $m_r$ decision variables can be obtained by calculating the factor (using normal distribution $x \sim N(0,1)$)

$$\mu_r = \int_{-\infty}^{\infty} x f_r(x) dx.$$  \hspace{1cm} (123)

The mean of the pdf of the decision variable in FSK detection can now be expressed as [64]

$$\mu_s = \sigma \sum_{r=1}^{M} \mu_r.$$  \hspace{1cm} (124)

After an exhaustive search, a reasonable rule to calculate the variance of the sum was found. The variance of each of the $m_r$ decision variables can be obtained by calculating

$$\sigma_r^2 = \sigma^2 \int_{-\infty}^{\infty} (x - \mu_r)^2 f_r(x) dx.$$  \hspace{1cm} (125)

The variance of the pdf of the decision variable in FSK detection can now be expressed as [64]

$$\sigma_s^2 = \sum_{k=1}^{m} \sum_{r=1}^{M} \sigma_r^2.$$  \hspace{1cm} (126)

The pdfs for symbol on frequency and not on frequency can hence be expressed as
with \( p = 1, \ldots, m_f \) and \( n = m_f + 1, \ldots, M_f \). Having obtained the pdfs for symbol on frequency and not on frequency, the probability of a correct decision can be expressed as in Eqs. (116) and (117). This can be further expressed as

\[
P_{\text{correct}} = \left[ \int_{-\infty}^{\infty} \left( P \left( U_{m_f+i} < U_i | U_i = u_i \right) \right)^{M_f-m_f} p(u_i) du_i \right]^{m_f}. \tag{128}
\]

where

\[
P \left( U_{m_f+i} < U_i | U_i = u_i \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy.
\tag{129}
\]

The probability of a correct decision now assumes the form \([64]\)

\[
P_{\text{correct}} = \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{y - \mu_{m_f,i}}{\sqrt{2}\sigma_z} \right) \right]^{M_f-m_f} \frac{1}{\sqrt{2\pi m_f^2}} e^{\frac{(y-2\alpha m_f, k)^2}{2m_f^2}} \]. \tag{130}
\]

Since the decision variables in different frequencies are independent, the probability of a correct symbol decision is

\[
P_c = P_{\text{correct}} \prod_{i=1}^{m_f} P_{c_i}, \tag{131}
\]

which in a case where random variables in different frequencies are identically distributed is simplified to

\[
P_c = P_{\text{correct}} P_{c_1} \prod_{i=1}^{m_f} P_{c_i}. \tag{132}
\]

The probability of a symbol error is now simply obtained as the complement of the probability of a correct symbol decision, i.e., \( P_m = 1 - P_c \).

The analyzed coherent \( m \)MCSK-\( m \)MFSK modulation is verified with computer simulations. To get some insight into the performance of the proposed system, we chose a few cases to be presented in \( E_s / N_0 \) vs. \( P_m \).
The cases are introduced in Table 5, and by using the procedure given in Eqs. (121)-(126), the values for $\mu_\Sigma$ and $\sigma_\Sigma$ are tabulated in Table 4. In Table 5 the notation under the column 'modulation' denotes the number of the frequencies and codes used in the following fashion: $\left( n_f m_f \right)$. 

**Table 4. The Precomputed values for $\mu_\Sigma$ and $\sigma_\Sigma$ for coherent mMCSK-mMFSK modulation for the assessed cases. [64] (© 2004 IEEE).**

<table>
<thead>
<tr>
<th>$\left( n_f m_f \right)$</th>
<th>$\mu_\Sigma$</th>
<th>$\sigma_\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( 8 2 \right)$</td>
<td>2.2758</td>
<td>0.9852</td>
</tr>
<tr>
<td>$\left( 8 3 \right)$</td>
<td>2.7486</td>
<td>1.7982</td>
</tr>
<tr>
<td>$\left( 8 4 \right)$</td>
<td>2.9012</td>
<td>2.7985</td>
</tr>
<tr>
<td>$\left( 8 8 \right)$</td>
<td>3.0507</td>
<td>0.7644</td>
</tr>
<tr>
<td>$\left( 8 16 \right)$</td>
<td>4.8042</td>
<td>2.0938</td>
</tr>
</tbody>
</table>

**Table 5. Definition of the assessed modulation types. [64] (© 2004 IEEE).**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>M</th>
<th>Bits/Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( 8 2 \right)$</td>
<td>21952</td>
<td>14.4</td>
</tr>
<tr>
<td>$\left( 8 3 \right)$</td>
<td>9 834496</td>
<td>23.2</td>
</tr>
<tr>
<td>$\left( 8 4 \right)$</td>
<td>1 680 700000</td>
<td>30.6</td>
</tr>
<tr>
<td>$\left( 8 8 \right)$</td>
<td>94080</td>
<td>16.5</td>
</tr>
<tr>
<td>$\left( 8 16 \right)$</td>
<td>43 698 200000</td>
<td>35.3</td>
</tr>
<tr>
<td>$\left( 8 16 \right)$</td>
<td>403200</td>
<td>18.8</td>
</tr>
<tr>
<td>$\left( 8 16 \right)$</td>
<td>7.680395632·10¹⁴</td>
<td>49.4</td>
</tr>
</tbody>
</table>
Fig. 32 presents results in a case where the system bandwidth is maintained as the same between modulation approaches. What we see is that in this case we can double the data rate with losing 6 dB in the $E_s/N_0$-domain.

![Graph showing performance of different modulation schemes in an AWGN channel.](image)

**Fig. 32. Performance of mMCSK-mMFSK in an AWGN channel [64] (© 2004 IEEE).**

Fig. 33, on the other hand, demonstrates that if one can afford a fixed number of matched filters, it is more advantageous to use a smaller number of frequencies and a higher number of matched filters per frequency since the performance degradation is marginal but the data rate is increased. This is evident by comparing the two pairs of curves in Fig. 33.
In this chapter, the derivation of the symbol error rate results has been performed for a modulation scheme known as $m$MCSK-$m$MFSK in a coherent configuration. The results show that with $m$MCSK-$m$MFSK one can easily construct practical modulation alternatives with a huge alphabet with excellent performance and hence achieve good spectral efficiency.

### 5.2.2 Coherent $m$MCSK-$m$MFSK with non-orthogonal code sets

**Error performance of the CSK component**

The decision variables are given for the coherent receiver as

$$U_k = \text{Re}\left\{e^{j\phi} \int r(t) u_k(t) dt\right\}, \quad k = 1, \ldots, M_{t} \ .$$

(133)

Taking into account that non-orthogonal codes are used, the parameter $U_k$ can be now be represented as [62]
\[ U_l = 2\alpha E \sqrt{\rho_l} + N_{\nu_l}, \quad l = 1, 2, \ldots, m_x \]
\[ U_p = 2\alpha E \sqrt{\rho_p} + N_{\nu_p}, \quad p = m_x + 1, \ldots, M_x, \]

where \( N_{\nu_l} \) are Gaussian zero-mean variables with variance \( \sigma^2 = 2EN_0 \) and \( N_0 \) is the noise spectral density. The probability density functions (pdf) for \( U_l \) can be represented as

\[
p(U_l) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(U_l - \alpha \sqrt{\rho_l})^2}{2\sigma^2}}, \quad l = 1, 2, \ldots, m_x
\]

\[
p(U_p) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(U_p - \alpha \sqrt{\rho_p})^2}{2\sigma^2}}, \quad p = m_x + 1, \ldots, M_x,
\]

where \( \rho_l = \rho_{ij_l} \) and \( \rho^2 = 1 + (m_x - 1)\rho_{ij_l} \) and \( \rho_{ij} \) (i \( \neq \) j) is the cross-correlation value at the correct time instant between a code pair within the code family used in the \( m \)MCSK alphabet, and \( N \) is the code length. The probability of a correct decision after some calculus becomes [62]

\[
P_{cc} = \left[ \frac{1}{2^{M_x - m_x} \sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x - 2\alpha \sqrt{\rho_l}E)^2}{2\sigma^2}} \left[ 1 + \text{erf} \left( \frac{x - 2\alpha \sqrt{\rho_l}E}{\sqrt{2\sigma}} \right) \right] \right]^{M_x - m_x}. \quad (136)
\]

This analysis applies to cases where \( \rho_{ij_l} \) is the same between any code pair within the family. Such codes are, for instance, Gold codes and a restricted set of Kasami codes.

**Error performance of the FSK component**

In CSK detection, the \( m \) decision variables on each frequency were sorted in descending order. For FSK detection, one chooses the largest \( m_x \) on each frequency. Before the sorting operation, these \( m_x \) individual decision variables on each frequency are distributed according to Eq. 135.

**Symbol on frequency**

In \( m \)MFSK detection, \( m_x \) out of \( M_x \) frequencies contain signal. On these \( m_x \) frequencies, \( M_x - m_x \) SMFs output Gaussian noise only and \( m_x \) SMFs output
signal and noise. Since we are searching for the probability of a correct decision, we can assume that the detector correctly identifies the $m_s$ signals present. Given that we have the cross-correlation effects due to the non-orthogonal codes used, the pdf of the $m_s$ variables is of the form

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\|x - \mu\|_2^2}{2\sigma^2}}. \quad (137)$$

The decision variable for FSK detection is the sum of these $m_s$ decision variables. The pdf of the sum of Gaussian variables is also Gaussian with mean $\mu = \sum \mu_i$ and variance $\sigma^2 = \sum \sigma_i^2$, resulting in our case in a pdf of the form [62, 64]

$$p(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{\|x - \mu\|_2^2}{2\sigma^2}}. \quad (138)$$

**Symbol not on frequency**

In mMFSK detection, $M_f - m_f$ out of $M_f$ frequencies contain noise only. In these cases, all the $M_f$ SMFs output Gaussian noise only. After the sorting operation, by using order statistics, one can find the distributions of the elements of the sorted ensemble.

Now, following the procedure described in Eqs. (121)-(126), the values for $\mu_z$ and $\sigma_z$ are obtained. Now the pdfs for symbol on frequency and not on frequency can be expressed as

$$p(u_r) = \frac{1}{\sqrt{2\pi \sigma_r^2}} e^{-\frac{(u_r - \mu_r)^2}{2\sigma_r^2}} \quad (139)$$

$$p(u_n) = \frac{1}{\sqrt{2\pi \sigma_n^2}} e^{-\frac{(u_n - \mu_n)^2}{2\sigma_n^2}}$$

with $p = 1, \ldots, m_f$ and $n = m_f + 1, \ldots, M_f$. Having obtained the pdfs for symbol on frequency and not on frequency, the probability of a correct decision can be derived in the following manner:

$$P_{sf} = \left[ \int_{\infty}^{\infty} P(U_{m_f+1} < U_1 = u_1)^{M_f-m_f} p(u_1) du_1 \right]^{m_f}, \quad (140)$$
where

\[ P\left(U_{m\left(\omega\right)} < U_{1} | U_{1} = u_{1}\right) = \int_{-\infty}^{u_{1}} p(u) du = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{u - \mu_{m}}{\sqrt{\sigma_{m}^{2}}}} e^{-x^{2}} dx. \tag{141} \]

The probability of a correct decision now assumes the form [62, 64]

\[ P_{e} = \left[ \int_{-\infty}^{\infty} \left( 1 - \frac{1}{2} \text{erfc}\left( \frac{y - \mu_{m}}{\sqrt{2}\sigma_{m}} \right) \right)^{M_{f}} \frac{1}{\sqrt{2\pi m_{\sigma}}} e^{\frac{(y-2am_{\sigma}\sqrt{2\pi})^{2}}{2am_{m}^{2}}} dy \right]^{m_{j}}. \tag{142} \]

Since the decision variables in different frequencies are independent, the probability of a correct symbol decision is

\[ P_{e} = P_{e} \prod_{j=1}^{M_{f}} P_{e}^{m_{j}}, \tag{143} \]

which in a case where random variables in different frequencies are identically distributed is simplified to

\[ P_{e} = P_{e} \left[ P_{e} \right]^{m_{j}}. \tag{144} \]

The probability of a symbol error is now simply obtained as the complement of the probability of a correct symbol decision, i.e., \( P_{e} = 1 - P_{e} \).
Fig. 34. Performance of \( \left( \begin{array}{c} 3 \\ 3 \end{array} \right) \) and \( \left( \begin{array}{c} 4 \\ 4 \end{array} \right) \) mMCSK-mMFSK in an AWGN channel with orthogonal Walsh-functions, Gold codes and Kasami codes (© 2004 IEEE).

5.2.3 Non-coherent mMCSK-mMFSK with orthogonal code sets

The detection is again performed in two steps. First the frequencies where the transmission is assumed to reside are found by energy comparison. Thereafter the codes within the found frequencies are searched. In this chapter we consider the non-coherent receiver, and hence the receiver has no knowledge of the phase difference \( \phi \). The receiver compensates the phase by taking the absolute values of the complex-valued decision variables. Let \( P_{cf} \) be the probability of detecting correctly the transmission frequencies and \( P_{cc} \) the probability of detecting correctly the transmitted codes within a used frequency. In the sequel we derive expressions for \( P_{cf} \) and \( P_{cc} \) and assess the performance of the modulation using these results.

Error performance of the CSK component

Observing Fig. 31 one notices that the decision variables in the detection of the CSK component are given for the non-coherent receiver as
and the receiver chooses the \( m_c \) largest ones. Parameter \( U_k \) can be represented as

\[
U_k = \left[ \int_0^\infty r(t) u_{i_k}(t) \, dt \right] , \quad k = 1, \ldots, M_c ,
\]  

where \( N_\nu \) are Gaussian zero-mean variables with variance \( \sigma^2 = 2EN_0 \), and \( N_0 \) is the noise spectral density. One can show that pdf's for \( U_k \) can be expressed as [66]:

\[
p(U_l) = \frac{U_l}{\bar{\sigma}^2} e^{-\frac{(U_l + \mu l)^2}{2\sigma^2}} I_0 \left( \frac{\alpha U_l}{N_0} \right),
\]

when \( l = 1, \ldots, m_c \) where \( I_0(\cdot) \) is the zero order modified Bessel function of the first kind. For \( p = m_c + 1, \ldots, M_c \) the pdf is

\[
p(U_p) = \frac{U_p}{\bar{\sigma}^2} e^{-\frac{U_p^2}{2\sigma^2}} .
\]

The probability of a correct decision is

\[
P_{cc} = \prod_{x=1}^{M_c} P(U_{x-1} > U_{x+1}, \ldots, U_{x} > U_{x+1}, \ldots).
\]

Because \( P(U_x > U_{x+1}, \ldots, U_x > U_{M_c}) \), \( x \leq m_c \) are the same, the probability of a correct decision assumes the form

\[
P_{cc} = \left[ P(U_1 > U_{m_c+1}, U_1 > U_{m_c+2}, \ldots, U_1 > U_{M_c}) \right]^{M_c} ,
\]

which after some calculus becomes [66, 52]

\[
P_{cc} = \left[ \sum_{n=0}^{M_c-m_c} (-1)^n \binom{M_c-m_c}{n} \frac{1}{n+1} e^{-\frac{\mu l^2}{(n+1)\sigma^2}} \right]^{M_c} .
\]

**Error performance of the FSK component**

In CSK detection, the \( M_c \) decision variables on each frequency were sorted in descending order. For the FSK-detection one chooses the \( m_c \) largest on each
frequency. Before the sorting operation, these \( M \) individual decision variables on each frequency are distributed according to Eqs. (147) and (148).

**Symbol on frequency**

In this case \( M - m \) SMFs output Gaussian noise only and \( m \) SMFs output signal+noise. Since we are searching for the probability of a correct decision, we can assume that the detector correctly identifies the \( m \) signals present. The CSK detection has produced squared decision variables out of Gaussian variables, and hence the pdf of the sum of \( m \) variables is a non-central chi-square distribution of the form [66]

\[
p(x) = \frac{x^{m-1}}{\sigma^{m}} e^{-\frac{x^2}{2\sigma^2}} I_{m-1} \left( \frac{\alpha x}{\sigma} \right)
\]

with a non-centrality parameter \( s^2 \):

\[
s^2 = \sum_{i=1}^{2m} E^2[U_i] = 2mE^2 + 2m\sigma^2.
\]

**Symbol not on frequency**

In this case all the \( M \) SMFs output Gaussian noise only. Thereafter the modulo operation will produce Rayleigh distributed random variables. After the sorting operation, by using order statistics, one can find the distributions of the elements of the sorted ensemble. Let us denote the pdf before sorting as \( f(x) \) and the corresponding cumulative distribution function (cdf) as \( F(x) \), i.e.,

\[
f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}
\]

\[
F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}
\]

with \( \sigma^2 = 2EN_0 \). Now the pdfs of the sorted ensemble can be expressed as [65]

\[
f_s(x) = \frac{M!}{(r-1)!(M_r-r)!} F(x)^{r-1} \left[ 1 - F(x) \right]^{M_r-r} f(x).
\]

What we have is \( m \) decision variables which are distributed according to \( f_s(x) \) with \( r \in [M_r - m + 1, \ldots, M_r] \). Prior to FSK detection we sum these \( m \) decision
variables, and what we need in the analysis is the pdf of this sum. This can be obtained in the following fashion. The mean of each $m_r$ decision variables can be obtained by calculating the factor

$$\mu_r = \int_{-\infty}^{\infty} x f_r(x) \, dx.$$ \hspace{1cm} (156)

The mean of the pdf of the decision variable in FSK detection can now be expressed as [66]

$$\mu_z = \sigma \sum_{r=M_r-M_r+1}^{M_r} \mu_r.$$ \hspace{1cm} (157)

The variance of each $m_r$ decision variables can be obtained by calculating

$$\sigma_r^2 = \sigma^2 \int_{-\infty}^{\infty} (x - \mu_r)^2 f_r(x) \, dx.$$ \hspace{1cm} (158)

The variance of the pdf of the decision variable in FSK detection can now be expressed as [66]

$$\sigma_z^2 = \sum_{k=1}^{M_r} \sum_{n=M_r-M_r+1}^{M_r} \sigma_k^2.$$ \hspace{1cm} (159)

The pdfs for symbol on frequency and not on frequency can hence be expressed as

$$p(u_p) = \frac{1}{\sqrt{2\pi m_r \sigma}} e^{-\frac{(u_p-\mu_{m_r})^2}{2m_r \sigma^2}}$$

$$p(u_n) = \frac{1}{\sqrt{2\pi \sigma_z}} e^{-\frac{(u_n-\mu_{z})^2}{2\sigma_z^2}}$$ \hspace{1cm} (160)

with $p = 1, \ldots, m_r$ and $n = m_r + 1, \ldots, M_f$. Having obtained the pdfs for symbol on frequency and not on frequency, the probability of a correct decision can be expressed as in Eq (160). The probability of a correct decision now assumes the form [66]

$$P_{sf} = \left[ \sum_{m_{sf}} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{y - \mu_{n}}{\sqrt{2\sigma_z}} \right) \right]^{M_f-m_{sf}} \frac{y^n}{\sigma^n} e^{\frac{y^2}{2\sigma^2}} I_{m_{sf}} \left( \frac{\alpha y s}{\sigma} \right) dy \right]^{m_{sf}}.$$ \hspace{1cm} (161)
Since the decision variables in different frequencies are independent, the probability of a correct symbol decision is

$$P_c = P_m \prod_{f} P_{m}^f,$$  \hspace{1cm} (162)

which in a case where random variables in different frequencies are identically distributed is simplified to

$$P_c = P_m \left[P_m^f \right]^{m_f}.$$  \hspace{1cm} (163)

The probability of a symbol error is now simply obtained as the complement of the probability of a correct symbol decision, i.e., $P_e = 1 - P_c$.

The analyzed non-coherent $m$MCSK-$m$MFSK modulation is verified with computer simulations. To get some insight into the performance of the proposed system, we chose a few cases to be presented in $E_s / N_0$ vs. $P_m$. The cases are introduced in Tables 6 and 7.

**Table 6. Pre-computed values for $\mu_e$ and $\sigma_e$ for non-coherent $m$MCSK-$m$MFSK modulation for the assessed cases [66] (© 2004 IEEE).**

<table>
<thead>
<tr>
<th>$(\mu_e, \sigma_e)$</th>
<th>$\mu_e$</th>
<th>$\sigma_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(3, 7)$</td>
<td>3.2631</td>
<td>0.4194</td>
</tr>
<tr>
<td>$(4, 9)$</td>
<td>4.4800</td>
<td>0.7451</td>
</tr>
<tr>
<td>$(5, 7)$</td>
<td>5.5138</td>
<td>1.1284</td>
</tr>
<tr>
<td>$(16, 2)$</td>
<td>3.7610</td>
<td>0.3478</td>
</tr>
<tr>
<td>$(16, 4)$</td>
<td>6.8781</td>
<td>0.9238</td>
</tr>
</tbody>
</table>
Table 7. Definition of the assessed modulation types [66] © 2004 IEEE.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>M</th>
<th>Bits/Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu\nu$</td>
<td>21952</td>
<td>14.4</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>9,834,496</td>
<td>23.2</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>1,680,700,000</td>
<td>30.6</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>94,080</td>
<td>16.5</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>43,698,200,000</td>
<td>35.3</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>403,200</td>
<td>18.6</td>
</tr>
<tr>
<td>$\nu\nu$</td>
<td>$7.680395632 \times 10^4$</td>
<td>49.4</td>
</tr>
</tbody>
</table>

In Table 6, by using the procedure given in Eqs. (154)-(159), the values for $\mu_z$ and $\sigma_z$ are tabulated. In Table 7 the notation under the column 'modulation' denotes the number of the frequencies and codes used in the following fashion:

$$
\begin{pmatrix}
M_f \\
m_f
\end{pmatrix} \begin{pmatrix}
M_z \\
m_z
\end{pmatrix}.
$$

Fig 35 presents the results in a case where the system bandwidth is maintained as the same between modulation approaches. What we see is that in this case we can double the data rate with losing 6 dB in the $E_s / N_0$ domain.
Fig. 35. Performance of the non-coherent and MFSK in an AWGN channel [66] (© 2004 IEEE).

Fig. 36, on the other hand, demonstrates that if one can afford a fixed number of matched filters, it is more advantageous to use a smaller number of frequencies and a higher number of matched filters per frequency since the performance degradation is marginal but the data rate is increased. This is evident by comparing the two pairs of curves in Fig. 36.

Comparing the results described in the previous sections, one can conclude that the performance difference between the coherent and non-coherent versions of the modulation is approximately 1 dB in favor of the coherent configuration. The advantage that we get with the non-coherent structure is that phase estimation circuitry is not needed.

The results show that with \( m \text{-MCSK}-m \text{MFSK} \) one can easily construct practical modulation alternatives with a huge alphabet with excellent performance and hence achieve good spectral efficiency.
5.3 Performance analysis of \(m\)MCSK-\(m\)MFSK modulation in a Rayleigh fading channel

This section considers the performance of \(m\)MCSK-\(m\)MFSK in a Rayleigh fading channel. The performance will be analyzed in a case where the whole signal experiences flat Rayleigh fading.

5.3.1 Coherent \(m\)MCSK-\(m\)MFSK modulation

Again the approach is to calculate the probability of error for CSK and FSK transmission independently and to use these probabilities for assessing the performance of the combined modulation.

**Error performance of the CSK component**

Let us assume that the receiver has perfect estimates of complex-valued channel coefficients \(ae^{-i\phi}\), which imply that the channel is slowly fading. Let us make the notation [67]
to simplify the following derivations. \( G(z) \) is the symbol error rate of coherent \( m \)MCSK in a Gaussian channel with \( z = \alpha^2 E / N_0 \). In a Rayleigh fading channel the amplitude of the received signal varies according to the Rayleigh distribution:

\[
G(z) = 1 - \frac{1}{2^{M-\alpha} \sqrt{\pi} \sigma_e} \int e^{-x^2} \left[ 1 + \text{erf} \left( x + \sqrt{z} \right) \right]^{M-\alpha} dx
\]  

(164)

The function \( G(z) \) can be modified to show the amplitude information by noting that \( \alpha^2 E / N_0 = a^2 / \sigma_a^2 \). Now one can derive the probability of a symbol error as

\[
P_e = \int G \left( \frac{a^2}{\sigma_a^2} \right) \frac{a}{\sigma_a} e^{-\frac{a^2}{2\sigma_a^2}} da.
\]  

(166)

Making a change of variables, \( a^2 / \sigma_a^2 = x^2 \rightarrow da = xdx, a = \sigma_a x \) allows to write [67]

\[
P_e = \int G \left( \frac{x^2}{\sigma_a^2} \right) xe^{-\frac{x^2}{2\sigma_a^2}} dx,
\]  

(167)

where signal-to-noise ratio is defined by \( \sigma_a^2 = 2EN_0 \).

**Error performance of the FSK component**

Let us start to calculate the performance of the FSK component by making the notation [67]

\[
R(z) = \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{y - \mu_y}{\sqrt{2} \sigma_y} \right) \right]^{N_f-\alpha} \frac{1}{\sqrt{2\pi} m_c \sigma} e^{-\frac{(y-2m_c \sqrt{\sigma} e)^2}{2m_c^2 \sigma}} dy
\]  

(168)

By noting that \( \sigma_a^2 = 2EN_0 \) and that \( \alpha^2 E / N_0 = a^2 / \sigma_a^2 \), one can formulate the probability of error in FSK detection as

\[
P_{ef} = \int_0^\infty R(a) \frac{a}{\sigma_a} e^{-\frac{a^2}{2\sigma_a^2}} da,
\]  

(169)

and again the symbol error probability can be expressed as
The numerical results are presented in Fig. 37. The simulated and analytical results agree and the expected linear symbol error rate performance with respect to the average SNR is observed.

\[ P_m = 1 - \left( 1 - P_{m_{\text{theory}}} \right) \left[ 1 - P_{m_{\text{sim}}} \right]^{n_Y}. \]  

(170)

5.3.2 Non-coherent mMCSK-mMFSK modulation

Error performance of the CSK component

Let us consider a square law detection of the received signals. The \( m_c \) outputs on frequency containing the signal are \[ U_l = |2E_a e^{-j\theta} + N_{r_{\text{p}}}|^2, \ l = 1, \ldots, m_c \]  

(171)

and the remaining \( M_c - m_c \) outputs not containing the signal are \[ U_p = |N_{r_{\text{p}}}|^2, \ p = m_c + 1, \ldots, M_c. \]  

(172)

The probability of error is the complement of the case that \( U_l \) exceeds all the \( U_p \) for \( l = 1, \ldots, m_c \) and \( p = m_c + 1, \ldots, M_c \). Since the signals are orthogonal and the
additive noise processes are mutually statistically independent, the random variables $U_1, U_2, \ldots, U_M$ are also mutually statistically independent. The complex-valued random variables $N_o$ and $N_p$ are zero-mean Gaussian-distributed. Hence it follows that the decision variables $U_i$ and $U_p$ are distributed according to a chi-square probability distribution with 2 degrees of freedom. The distributions are

$$p(U_i) = \frac{1}{2\sigma^2} e^{-\frac{U_i}{2\sigma^2}}, \quad i = 1, \ldots, m_c$$

(173)

and

$$p(U_p) = \frac{1}{2\sigma^2} e^{-\frac{U_p}{2\sigma^2}}, \quad p = m_c + 1, \ldots, M_c$$

(174)

with

$$\sigma^2 = 2E_{N_0}.$$ 

(175)

Noting that the formulation is the same as in Chapter 3.2.2 allows to write (with $E_{s} = m,m,M$)

$$P_M = 1 - \sum_{n=1}^{m-m_c} (-1)^{n} \left( \frac{M_c - m_c}{n} \right) \frac{1}{1 + n + \frac{E_s}{m,m,m_{E_{N_0}}}}.$$ 

(176)

**Error performance of the FSK component**

Let us start to calculate the performance of the FSK component in a Rayleigh fading channel by making the notation (following from Eq. 161)

$$R(z) = \left[ \int_{-\infty}^{\infty} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{y - \mu_y}{\sqrt{2} \sigma_y} \right) \right)^{m_c-m} \frac{1}{\sigma^2 s(z)^{m_c}} e^{-\frac{y^2}{2\sigma^2}} I_{m_c-1} \left( \frac{\alpha y s(z)}{\sigma^2} \right) dy \right]^{-m_c}.$$ 

(177)

wherein

$$s(z) = 2m_c z^2 + 2m_c \sigma^2.$$ 

(178)

By integrating Eq. 177 over the Rayleigh distribution, we reach the result for the probability of a correct decision as
\[ P_{sf} = \int_{0}^{\infty} R(z) \frac{z}{\sigma^2} e^{-\frac{z^2}{2 \sigma^2}} dz. \]  

(179)

With the assumption that the decision within FSK and CSK detection is approximately independent, the symbol error probability for the \( m \)MCSK-\( m \)MFSK modulation is therefore

\[ P_{se} = 1 - P_{sf} \left[ P_{se}^{\alpha} \right]. \]  

(180)

5.4 Performance analysis of antipodally extended \( m \)MCSK-\( m \)MFSK modulation in an AWGN channel

Consider a system transmitting on \( m_f \) out of \( M_f \) frequencies (such as a \( m \)MFSK system), and transmitting on each \( m_f \) frequencies \( m_c \) signals (such as spreading codes) out of a set \( M_c \) possible signals. The proposed system with an antipodal configuration is depicted in Fig. 38.
Fig. 38. A coherent mMCSK-mMFSK receiver with antipodal extension [69]. (© 2004 WPMC).

The detection is performed in three steps. First the frequencies where the transmission is assumed to reside are found by energy comparison. Thereafter the codes within the found frequencies are searched. The last stage is to determine whether the phase of the code is 0 or $\pi$. In this derivation we consider the 'coherent' receiver, and hence the receiver has knowledge of the phase $\phi$.

**Error performance of the CSK component**

Observing Fig. 38 and by realizing that the antipodal data modulation in CSK detection has to be removed by squaring, one notices that the decision variables in the detection of the CSK component are given to the 'coherent' receiver as [69]
\[ U_k = e^{\mu_k} \int_0^{\tau_k} r_i(t) u_{k,i}(t) \, dt, \quad k = 1, \ldots, M. \]  

An optimum receiver will choose the largest \( m_c \). Parameter \( U_i \) can be represented as

\[ U_i = 2\alpha E + N_p, \quad i = 1, 2, \ldots, m_c \]
\[ U_p = N_p, \quad p = m_c + 1, \ldots, M_c \]  

where \( N_p \) are Gaussian zero-mean variables with variance \( \sigma^2 \), and \( N_0 \) is the noise spectral density. One can show that pdf for \( U_i \) can be expressed as noncentral chi-square distribution

\[ p(U_i) = \frac{1}{\sqrt{2\pi U_i \sigma}} e^{-\frac{U_i \sigma^2}{2}} \cosh \left( \frac{U_i}{2\sigma} \right), \quad i = 1, \ldots, m_c, \]  

and for \( p = m_c + 1, \ldots, M_c \) with central chi-square distribution

\[ p(U_p) = \frac{1}{\sqrt{2\pi U_p \sigma}} e^{-\frac{U_p \sigma^2}{2}}. \]  

The probability of a correct decision is

\[ P_{cc} = P(U_1 > U_{m+1}, \ldots, U_i > U_{m_c}) \ldots \]
\[ P(U_{m_c} > U_{m+1}, \ldots, U_{m_c} > U_{m}). \]  

Because \( P(U_i > U_{m+i}, \ldots, U_i > U_{m_c}) \), \( x \leq m_c \) are the same, the probability of a correct decision assumes the form

\[ P_{cc} = \left[ P(U_i > U_{m+1}, U_i > U_{m+c}, \ldots, U_i > U_{m_c}) \right]^{m_c}. \]  

This can be further expressed as

\[ P_{cc} = \left[ \int_{-\infty}^{\infty} P(U_{m+1} < U_i = u_i)^{M_c - m_c} p(u_i) \, du_i \right]^{m_c}, \]  

where

\[ P(U_{m+1} < U_i = u_i) = \int_{-\infty}^{u_i} p(u_i) \, du_i = \text{erf} \left( \frac{\sqrt{U_i}}{\sqrt{2\sigma}} \right), \]  

which then becomes [69]
\[ P_x = \int_0^\infty \frac{1}{\sqrt{2\pi s\sigma}} \left[ \text{erf}\left(\frac{\sqrt{x}/\sqrt{2\sigma}}{2\sigma}\right) e^{-\frac{x+C^2}{2\sigma^2}} \cosh\left(\frac{\sqrt{x}}{\sigma}\right) dx \right]^m. \]  

(189)

**Error performance of the FSK component**

In CSK detection, the \( M \) decision variables on each frequency were sorted in descending order. For FSK detection one chooses the largest \( m \) on each frequency.

**Symbol on Frequency**

In this case, \( M - m \) SMFs output only Gaussian noise and \( m \) SMFs output signal+noise. Since we are searching for the probability of a correct decision, we can assume that the detector correctly identifies the \( m \) signals present. The CSK detection has produced squared decision variables out of Gaussian variables, and hence the pdf of the sum of \( m \) variables is a non-central chi-square distribution of the form

\[ p(x) = \frac{1}{2\sigma^2} \left( \frac{x}{s^2} \right)^{m-2} e^{-\frac{x+C^2}{2\sigma^2}} I_{\frac{s^2}{2\sigma^2}} \left( \frac{\sqrt{x}}{\sigma} \right) \]  

(190)

with \( I_{\alpha}(x) \) being the \( \alpha \)th order modified Bessel function of the first kind and the non-centrality parameter \( s^2 \)

\[ s^2 = \sum_{i=1}^{2m} E^2(U_i) = mE^2 + m\sigma^2. \]  

(191)

**Symbol not on Frequency**

In this case, all the \( M \) SMFs output only Gaussian noise. Because the noise in the quadrature branch can be eliminated due to the phase coherence of the receiver, the modulo operation will produce generalised Rayleigh distributed rv’s. After the sorting operation, by using order statistics, one can find the distributions of the elements of the sorted ensemble. Let us denote the generalized Rayleigh (with 1 degree of freedom) pdf before sorting as \( f(x) \) and the corresponding cumulative distribution function (cdf) as \( F(x) \), i.e., [69]
\[ f(x) = \frac{\sqrt{2}}{\sqrt{\pi \sigma}} e^{-\frac{x^2}{\sigma^2}} \]
\[ F(x) = \text{erf}\left(\frac{x}{\sqrt{2\sigma}}\right) \] (192)

with \( \sigma^2 = 2EN_0 \). Now the pdfs of the sorted ensemble can be expressed as [65]
\[ f_r(x) = \frac{M_r!}{(r-1)!(M_r - r)!} F(x)^{-1} \left[1 - F(x)\right]^{M_r - r} f(x). \] (193)

What we have is \( m_c \) decision variables which are distributed according to \( f_r(x) \) with \( r \in [M_c - m_l + 1, ..., M_c] \). Prior to FSK detection, we sum these \( m_c \) decision variables, and what we need in the analysis is the pdf of this sum. This can be obtained in the following fashion. The mean of each \( m_c \) decision variables can be obtained by calculating the factor
\[ \mu_r = \int_{-\infty}^{\infty} x f_r(x) dx. \] (194)

The mean of the pdf of the decision variable in FSK detection can now be expressed as
\[ \mu_r = \sigma \sum_{r-M_c-m_l+1}^{M_c} \mu_r. \] (195)

The variance of each \( m_c \) decision variables can be obtained by calculating
\[ \sigma_r^2 = \int_{-\infty}^{\infty} (x - \mu_r)^2 f_r(x) dx. \] (196)

The variance of the pdf of the decision variable in FSK detection can now be expressed as
\[ \sigma_r^2 = \sigma^2 \left( \sum_{k=1}^{m_c} \sum_{l=k+M_c-m_l+1}^{M_c} \sigma_z^2 \right)^2. \] (197)

The pdfs for symbol on frequency and not on frequency can hence be expressed as
\[ p(u_p) = \frac{1}{2\sigma^2} \left( \frac{u_p}{s^2} \right)^{\frac{n-2}{4}} e^{-\frac{u_p}{\sigma^2}} I_{\frac{n-1}{2}} \left( \sqrt{\frac{u_p}{\sigma^2}} \right) \]
\[ p(u_n) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{(u_n-m)^2}{2\sigma_c^2}} \]

with \( p = 1, ..., m_f \) and \( n = m_f + 1, ..., M_f \). Having obtained the pdfs for symbol on frequency and not on frequency, the probability of a correct decision can be expressed as in Eqs. (140)-(141). The probability of a correct decision now assumes the form [69]
\[ P_f = \left[ \int_{\infty}^{\infty} \frac{1}{2\sigma^2} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{y - \mu_c}{\sqrt{2\sigma_c^2}} \right) \right)^{M_f-M_f'} \left( \frac{y}{s'} \right)^{\frac{n-2}{4}} e^{-\frac{y}{\sigma_c^2}} I_{\frac{n-1}{2}} \left( \sqrt{\frac{y}{\sigma_c^2}} \right) dy \right]^{M_f'} . \]

Since the decision variables in different frequencies are independent, the probability of a correct symbol decision is
\[ P_c = P_f \prod_{i=1}^{M_f'} P_{\omega i} , \]
which in a case where random variables in different frequencies are identically distributed is simplified to
\[ P_c = P_f [ P_{\omega} ]^{M_f'} . \]

The probability of a symbol error is now simply obtained as the complement of the probability of a correct symbol decision, i.e., \( P_m = 1 - P_c \).

**Error Rate for the Antipodal Data**

The data that is modulated on each of the transmitted \( m, m_f \) codes can be considered to be BPSK modulated after the matched filtering. Hence it is easy to write the error performance of the data as [69]
\[ P_y = \frac{1}{2} \text{erfc} \left( \frac{E_s}{\sqrt{m, m, N_0}} \right) . \]

This is of course subject to the fact that in \( m \)MCSK-\( m \)MFSK modulation both the detection of transmitted frequencies as well as the transmitted codes has occurred.
correctly. Hence we can approximate the total error performance of the antipodal data as

\[ P_M = \frac{1}{2} (1 - P_{\text{off}} P_f (1 - P_f)). \]  

(203)

To get some insight into the performance of the proposed system, we chose a few cases to be presented in $E_r / N_0$ vs $P_M$. The cases are introduced in Table 8 and the pre-computed values for $\mu_x$ and $\sigma_x$ in Table 9.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>M</th>
<th>Bits/Symbol</th>
<th>antipodal ext. (bits/sym)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>21952</td>
<td>14.4</td>
<td>18.4</td>
</tr>
<tr>
<td>1/2</td>
<td>9 834496</td>
<td>23.2</td>
<td>29.2</td>
</tr>
<tr>
<td>1/2</td>
<td>1 680 700000</td>
<td>30.6</td>
<td>46.6</td>
</tr>
<tr>
<td>1/2</td>
<td>94080</td>
<td>16.5</td>
<td>20.5</td>
</tr>
<tr>
<td>1/2</td>
<td>43 698 200000</td>
<td>35.3</td>
<td>51.3</td>
</tr>
<tr>
<td>1/2</td>
<td>403200</td>
<td>18.6</td>
<td>22.6</td>
</tr>
<tr>
<td>1/2</td>
<td>7.680395632 $\times 10^4$</td>
<td>49.4</td>
<td>65.4</td>
</tr>
</tbody>
</table>
Table 9. The Pre-computed values for $\mu_\Sigma$ and $\sigma_\Sigma$ for antipodally extended $m$MCSK-$m$MFSK modulation for the assessed cases. [69] (© 2004 WPMC).

<table>
<thead>
<tr>
<th>$\mu_\Sigma$</th>
<th>$\sigma_\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.32</td>
<td>9.38</td>
</tr>
<tr>
<td>4.1083</td>
<td>1.2391</td>
</tr>
<tr>
<td>7.1949</td>
<td>13.1617</td>
</tr>
<tr>
<td>7.4188</td>
<td>6.3012</td>
</tr>
<tr>
<td>11.0801</td>
<td>7.5719</td>
</tr>
</tbody>
</table>

Fig. 39 presents the results in a case where the system bandwidth is maintained as the same between modulation approaches. What we see is that in this case we can double the data rate with losing 6 dB in the $E_b / N_0$ domain.

Fig. 40, on the other hand, demonstrates that if one can afford a fixed number of matched filters, it is more advantageous to use a smaller number of frequencies and a higher number of matched filters per frequency since the performance degradation is marginal but the data rate is increased. This is evident by comparing the two pairs of curves in Fig. 40.

Comparing the results in [64] and [66], one can conclude that the performance difference between the coherent and non-coherent versions of the modulation is approximately 1 dB in favor of the coherent configuration, and the performance of this version falls between the two.

Fig. 41 presents the performance achieved for the antipodally modulated bits. The error performance is reasonable, considering the fact that it is obtained free on top of the $m$MCSK-$m$MFSK modulation.
Fig. 39. Performance of the antipodally extended \( (\frac{8}{2}, \frac{8}{2}) \), \( (\frac{8}{3}, \frac{8}{3}) \), and \( (\frac{8}{4}, \frac{8}{4}) \) mMCSK-
mMFSK in an AWGN channel [69]. © 2004 WPMC.

Fig. 40. Performance of the antipodally extended \( (\frac{16}{2}, \frac{8}{2}) \), \( (\frac{8}{2}, \frac{16}{2}) \), \( (\frac{8}{4}, \frac{16}{4}) \), and \( (\frac{16}{4}, \frac{16}{4}) \) mMCSK-
mMFSK in an AWGN channel [69]. © 2004 WPMC.
Fig. 41. Performance of the antipodally modulated bits in $m$MCSK-$m$MFSK with $\left(\frac{8}{2}, \frac{4}{4}\right)$, $\left(\frac{8}{3}, \frac{4}{4}\right)$, and $\left(\frac{8}{3}, \frac{3}{3}\right)$ (AWGN channel) [69]. © 2004 WPMC.
6 mPPM modulation

6.1 Introduction

In [70] the ultra-wideband (UWB) signaling method was accepted by the Federal Communications Commission (FCC) to be used in different applications. In UWB [79, 80, 81, 82] and especially in optical communications, pulse position modulation (PPM) has often been considered as the data modulation format. In this chapter we examine a spectrally efficient mPPM modulation format which is a modification of the well-known PPM principle. Different versions of the PPM modulation format, also including differential and combinatorial PPM, are described e.g. in [71], [72], [73]. mPPM was introduced for use in optical communications in [74] under the name MPMM. MPMM has been considered e.g. in [75], [76], [77], [78]. In this chapter it is considered for UWB-communications and it gives exact bit error rate (BER) results for one-shot transmission. One can use a standard PPM detector in signal detection of mPPM, and hence the receivers are practically of the same complexity as PPM receivers. The difference when compared to PPM is in the signal processing rules after the detection has occurred. To enhance the spectral efficiency of the signaling set, multi-slot signals are considered. A standard PPM modulation uses one out of M timeslots each \( \frac{T_s}{M} \) seconds to transmit a block of \( n = \log_2 M \) bits in a time window of \( T_s \). The optimum coherent receiver has a bank of \( M \) matched filters or correlator integrator processors and every \( T_s \) second it makes a decision based on the largest output filter sample. The coherent and non-coherent orthogonal PPM in an AWGN channel has the symbol error rate performance [30]

\[
P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{y}{\sqrt{2}} \right) \right] e^{-\frac{[y - \sqrt{2}]}{2}} \, dy
\]

(204)

and

\[
P_e = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} e^{-\frac{2\sigma^2 \gamma^2}{n+1}}.
\]

(205)
6.1.1 System model

In Fig. 42, a coherent \( m \)PPM receiver is depicted. The receiver has switches \( S_1, \ldots, S_M \) which pass one out of \( M \) timeslots to subsequent correlator-integrator processing. The switch is open in each receiver branch only for the duration of the time slot being processed by the given receiver branch. It is easily seen that using this type of a receiver, the orthogonality between the time slots is maintained, given correct synchronization. This receiver is a generalization of a PPM-receiver. In Fig. 42, one notices that the waveforms \( u_{1,t}(t), \ldots, u_{M,t}(t) \) in each receiver branch can be different. The more often used approach, however, is to use only one signaling (pulse) waveform since the timeslot structure (given synchronization) makes orthogonality possible.

Fig. 42. A \( m \)PPM receiver in a coherent configuration [83] (© 2003 IEEE).
Fig. 43. A mPPM receiver in a non-coherent configuration [83] (© 2003 IEEE).

Fig. 43 depicts the non-coherent mPPM receiver. As previously, the orthogonality of the decision variables is assured by allowing only one timeslot out of $M$ to be processed in one receiver branch through the use of switches $S_1,...,S_M$. The signals used in Figs. 42 and 43 are presented in the next chapter.

6.2 Error probability of mPPM systems

In this chapter the bit error probability (BEP) results for coherent and non-coherent mPPM are presented. The channel is the additive white Gaussian noise (AWGN) channel. The assumption made in this analysis is that the time slots in the PPM window are orthogonal (this is sometimes hard to achieve due to pulse ringing introduced by the derivative nature of the antenna). Orthogonality is thus not always necessarily the case but the performance with non-orthogonal time slot assignment is left for further study.

6.2.1 Error probability for coherent mPPM

The transmitted signals are represented as $u_k(t-kT)$, $k=1,2,...,M$ with (for orthogonal configuration)

$$\int_0^T u_k(t)u_m(t)dt = \delta_{km},$$

(206)

where $u_k(t)$ is the complex envelope of the signal and $\delta_{km}$ is the Kronecker delta function. The energy of these signals is
The received signal complex envelope is
\[ r(t) = \alpha e^{-j\varphi} s(t) + z(t), \] (208)

where \( \alpha \) is the channel attenuation coefficient, \( \varphi \) the phase difference between the input and local signal, and \( z(t) \) is Gaussian noise. For simplicity, and without loss of generality, let us assume that in the first \( m \) timeslots are used for transmission, i.e., \( s_l(t), l = 1,2,...,m \). The received low-pass equivalent of the signal becomes
\[ r(t) = \alpha e^{-j\varphi} \left[u_1(t-T) + \ldots + u_m(t-mT)\right] + z(t), 0 \leq t \leq T. \] (209)

The decision variables are now given as
\[ U_k = \text{Re} \left\{ e^{j\varphi} \int_{0}^{T} r(t) u_k^*(t-kT) dt \right\}, \quad k = 1,\ldots,M. \] (210)

An optimum receiver will choose the \( m \) largest ones. Parameter \( U_k \) can be represented as
\[ U_l = 2\alpha E + N_l, \quad l = 1,2,...,m \]
\[ U_p = N_p, \quad p = m+1,...,M, \] (211)

where \( N_l \) are Gaussian zero-mean variables with variance \( \sigma_l^2 = 2EN_l \) and \( N_p \) is the one-sided noise spectral density. The probability density functions (pdf’s) for \( U_k \) can be represented as [30, 83]
\[ p(U_l) = \frac{1}{\sqrt{2\pi\sigma_l^2}} e^{\frac{(U_l-2\alpha E)^2}{2\sigma_l^2}} \]
\[ p(U_p) = \frac{1}{\sqrt{2\pi\sigma_p^2}} e^{\frac{U_p^2}{2\sigma_p^2}}. \] (212)

Noting that this is equivalent to the case of orthogonal \( m \)MFSK, we can directly write the expression for the probability of a correct decision as [83]
\[ P_c = \left[ \frac{1}{2^{M-m}} \int_{0}^{\gamma} e^{-x^2} \left[ 1 + \text{erf}\left( x + \sqrt{\frac{\alpha^2 E}{N_0}} \right) \right]^{M-m} dx \right]^M. \] (213)
Given the probability of a correct decision \( P_c \), the probability of a symbol error \( P_M \) can be written as a complement of the probability of a correct decision to yield

\[
P_M = 1 - P_c. \tag{214}
\]

The bit error probability \( P_b \) presented as a function of \( \frac{E_b}{N_0} \) in the numerical results is obtained as

\[
P_b = \frac{2^{\log_2 M}}{2^{\log_2 M} - 1} P_M \tag{215}
\]

with \( \alpha^2 E / N_0 \) in Eq. 213 replaced by \( \alpha^2 E / N_0 = 2^{\log_2 M} \alpha^2 E / N_0 \).

### 6.2.2 Error Probability for Non-Coherent mPPM

Using the same reasoning as for the coherent mPPM case with respect to the obtained decision metrics and remembering that the orthogonality assumption holds, we can again readily write an expression of a symbol error for non-coherent mPPM as \[83\]

\[
P_M = 1 - \left[ \sum_{n=0}^{M-1} (-1)^n \binom{M - m}{n} \frac{1}{n+1} e^{-\frac{\alpha^2 E}{N_0} (n+1)} \right]^m. \tag{216}
\]

### 6.3 Pulse integration with mPPM (multichannel signaling with mMFSK)

In UWB-applications it is customary to send several pulses (up to thousands of PPM symbols) per data symbol. In a coherent configuration, MRC combining is available, and hence (ideally) no losses are incurred in pulse repetition. The pulse duration in practical applications, however, is often so short that coherent reception is not feasible and one resorts to non-coherent receivers. In a non-coherent configuration, however, non-coherent combining loss is observed. Given that we send \( L \) replicas of the mPPM symbol to transmit one data symbol and that we have a square law detector, the pdfs of the decision variables are \((l = 1, ..., m \text{ and } k = m+1, ..., M) \ [83, 84, 30]\)
\[
p(u_i) = \frac{1}{4EN_0} \left( \frac{u_i}{s^2} \right)^{\frac{L+1}{2}} e^{-\frac{\sqrt{u_i}}{4EN_0} I_{L-1} \left( \frac{s^2}{2EN_0} \right)}
\]

(217)

\[
p(u_i) = \frac{1}{(4EN_0)^L(L-1)!} u_i^{L-1} e^{-\frac{u_i}{4EN_0}}
\]

(218)

with the non-centrality parameter \( s^2 = 4E^2 \sum L \alpha_i^2 \). The conditional probability that the transmitted timeslot has higher energy than an empty timeslot is

\[
p(U_{m+1} < u_i | U_i = u_i) = 1 - e^{-\left( \frac{\sum_{i=1}^{L-1} \alpha_i^2}{4EN_0} \right) L^{-1}}.
\]

(219)

The probability of a correct symbol decision is

\[
P_e = \left[ \int_0^\infty \left[ P(U_{m+1} < u_i | U_i = u_i) \right]^{M-m} p(u_i) du_i \right]^m,
\]

(220)

which after some mathematical manipulations gives an expression for symbol error probability as

\[
P_\varepsilon = 1 - \left[ e^{-\frac{1}{4EN_0} \left( \frac{X}{\gamma} \right)^{L-1}} \sum_{k=0}^{M-m} \frac{1}{k!} e^{(\gamma+1)} I_{L-1} \left( 2\sqrt{\gamma} X \right) \right]^{m},
\]

(221)

where \( \gamma = E / N_0 \) is the SNR per symbol observed over \( L \) channels.

### 6.4 Diversity combining in impulsive interference with mPPM

Let us assume that \( \xi_{\text{hit}} \) represents the relative interference hit probability, i.e., \( \xi_{\text{hit}} = 0.5 \) denotes that 50\% of the diversity replicas are interfered on average. Without considering the practical algorithms, let us assume that we can identify the interfered diversity replicas reliably and discard them (i.e., the receiver has perfect side information). In addition, assuming that the hit probability on each diversity replica is uniformly distributed, we can find an expression for the probability that \( j \) out of \( L \) diversity branches are combined

\[
P_{\text{comb}}(j) = \binom{L}{j} \left( 1 - \xi_{\text{hit}} \right)^{L-j}.
\]
Moreover, given that we combine \( j \) out of \( L \) diversity branches \( (j > 1) \), the symbol error probability is

\[
P_e(j) = 1 - \left[ \frac{1}{\Gamma(j+1)} \left( \frac{1}{\gamma} \right)^j \int_0^{\infty} \frac{1}{4EN_0} \frac{x^{j-1}}{\Gamma(j)} e^{-\frac{x}{\gamma}} I_{j-1} \left( 2\sqrt{\gamma x} \right) dx \right]^m.
\]  

For \( j = 0 \) we get \( P_{comb}(0) = \left[ \left( \frac{m}{m} \right)^{-1} \right] \) and for \( P_M(1) \) we use equation (216). In the above equations, \( \gamma = (N_s / (LN_0)) \) is the SNR per symbol observed over \( j \) combined channels. The final symbol error rate is of the following form [85]:

\[
P_s = \sum_{j=0}^L P_{comb}(j) P_M(j).
\]

### 6.5 Some numerical results

To show that the analytical results agree with reality, both coherent and non-coherent \( m \)PPM-modulated systems were simulated with \( M = 8 \) and \( m = 1, 2 \) and 3. Figs. 44 and 43 show that the analytical results agree well with the simulations. One also observes that the performance degradation is not severe with increasing \( m \), suggesting higher spectral efficiency.

In Fig. 46 the performance of non-coherent \( m \)PPM with pulse repetition is presented as a function of the symbol error rate and \( E_s / N_0 \) (\( E_s \) is the symbol energy). The results indicate that the theoretical curves agree well with the simulated ones. The non-coherent combining loss is 2–3 dB with \( L = 10 \) and 5–7 dB for \( L = 100 \), depending on the SNR.

Fig. 47 presents the result in impulsive interference as a function of diversity order \( L \). The results show that the analysis is useful in obtaining the optimal diversity order, given that we have somehow acquired knowledge of the interference hit probability.
Fig. 44. The Bit error rate performance of the non-coherent mPPM scheme in an AWGN channel with $m$ as a parameter [83] (© 2003 IEEE).

Fig. 45. The Bit error rate performance of the coherent mPPM scheme in an AWGN channel with $m$ as a parameter [83] (© 2003 IEEE).
Fig. 46. The Symbol error rate performance of the non-coherent mPPM scheme in an AWGN channel with number pulse repetition ($L$) as a parameter [83] (© 2003 IEEE).

Fig. 47. The Symbol error rate performance of the non-coherent mPPM scheme in an AWGN channel with pulse repetition order and impulsive interference ($\xi_{in} = 0.3, 0.5$ or $0.7$), markers simulated. [83] (© 2003 IEEE).
7 Qualitative discussion of the proposed methods and further directions

This thesis introduces new combinatorial modulation techniques and the approach in the analysis is similar to that of many textbooks [30-32], i.e., to derive symbol error results under idealized conditions. The approach is to use the assumption of ideal synchronization and simple channel models, including an AWGN channel and a flat Rayleigh fading channel. And as the system parameters (such as center frequency, data rates, mobility parameters…) are not fixed, the results are thus very general. The results in an AWGN channel give us the best possible performance and the results in Rayleigh fading give us a good indication of the performance in bad channel conditions. Most of the real life radio channel conditions fall between the two.

7.1 mMFSK and mMCSK

For mMFSK and mMCSK modulation, statements on improving spectral efficiency can be made if we set the performance achieved by standard MFSK or MCSK modulation as the baseline. For high SNR, one can reconsider Eq. (106) by noting that approximately $P_m \to 0$ and $P_0 \to 0$, thus allowing to rewrite it as

$$E_{eff} = \frac{n}{n_0} = \frac{\log_2 \left( \frac{M}{m} \right)}{\log_2 M}. \quad (225)$$

Considering only one link (or one user in a system) and applying this to a system where $M = 64$ (such as an IS-95 system), one can plot maximal efficiency improvement for the link with respect to increasing $m$, as is depicted in Fig. 48. A ten-fold efficiency improvement ($M = 64$) can be observed in ideal conditions with $m = 32$. In real life and with the practical values of SNR (the acceptable bit error rate region), one cannot assume that $P_m \to 0$ and $P_0 \to 0$, and thus it is expected that lower values for efficiency improvement would be observed. The SNR value used in a real life system is a design parameter that can be used to fine-tune system performance.
Fig. 48. Efficiency improvement (for high SNR) as a function of $m$ for $M=64$ of a $m$MFSK or $m$MCSK system when compared to a standard MFSK or MCSK system, respectively.

As was indicated in Chapter 3, all possible symbol combinations can be achieved with the constraint $m \leq \lfloor (M / 2) \rfloor$. Thus, the maximum efficiency improvement as a function of $M$ can be given as

$$E_{\text{eff, max}} = \frac{\log_2 \left( \frac{M}{M / 2} \right)}{\log_2 M}$$

(226)

with $M$ being even. The maximum efficiency improvement for $m$MFSK and $m$MCSK as a function of $M$ is depicted in Fig. 49.
Another appealing property of \( m \text{MFSK} \) and \( m \text{MCSK} \) (again comparing to standard MFSK and MCSK) is the fact that the complexity increase is minimal. In practice, the only change that is required is in the rules for the symbol decision after symbol detection has occurred. All other signal processing elements remain the same. Thus, the proposed modulation formats may be readily used in systems employing standard MFSK or MCSK with a minimal software change.

One notable difference in complexity, however, is observed for non-orthogonal code constructs for \( m \text{MCSK} \). For Walsh functions (orthogonal \( m \text{MCSK} \)), fast Walsh-Hadamard Transforms (WHT) are available [90-94], which significantly ease the computational burden compared to the matched filter approach. For non-orthogonal code family constructs, these transforms are not readily available and the matched filter approach is required.
7.2 \( m \text{MCSK-}m \text{MFSK} \)

Combining \( m \text{MCSK} \) and \( m \text{MFSK} \) gives us an opportunity to create huge symbol alphabets with the price of increased spectrum usage. This spectrum spreading may be useful in systems targeted to be interference resistant, such as military communication systems. The complexity of the system is naturally increased compared to MFSK or MCSK. Roughly speaking, the complexity increase with respect to MFSK modulation is of the order \( NM_jM_f \), and with respect to MCSK, it is of the order \( M_f \).

To get an idea of the efficiency of the modulation, a metric (bandwidth efficiency) is used in which the number of bits/symbol is normalized with the spectrum usage.

\[
E_{\text{max}} = \frac{\log_2 \left( \frac{M_f}{m_f} \right) M_c}{M_j M_c}. \tag{227}
\]

This has four parameters. Recalling that for \( m \text{MFSK} \) and \( m \text{MCSK} \), all possible symbol combinations can be achieved with constraint \( m \leq \left\lfloor \frac{M}{2} \right\rfloor \), and by making the assumption that \( M_f = M_c = M \), we can rewrite (227) into

\[
E_{\text{max}} = \frac{(M/2 + 1) \log_2 \left( \frac{M}{M/2} \right)}{M^2} \approx \frac{1}{2} \tag{228}
\]

for all \( M \). This means that the ultimate limit (\( \frac{1}{2} \) for BFSK and 4FSK) for FSK modulated signals can be now reached with \( m \text{MCSK-}m \text{MFSK} \) for all \( M \), as is indicated in Fig. 50. In Fig 50, the same efficiency metric for MFSK is \( E_{\text{max}} = \log_2 M / M \).

In addition to the complexity issues discussed earlier, another topic that needs to be assessed is the PAPR for this modulation. As we are transmitting \( m_jm_c \) signals, the PAPR issue is even more pronounced than with \( m \text{MFSK} \) and \( m \text{MCSK} \), and thus needs attention in further studies.
Without going into details, it can be stated that the \( m \)PPM approach has similar advantages as was discussed in conjunction with \( m \)MFSK and \( m \)MCSK modulation. The efficiency improvement is the same as was given in Eqs. (225) and (226), and it can thus be depicted as in Figs. 48 and 49. The assumption is that the time slots are orthogonal. Systems that use standard PPM may also readily use the \( m \)PPM approach by simple software changes, given that the pulse generator can produce pulses in adjacent time slots.

In real life systems, however, pulse ringing is observed due to transmit antenna dispersion \([97-99]\), which spreads the pulses to adjacent time slots. This effect should be analyzed in conjunction with \( m \)PPM if it is to be used in practical systems.

The approach can also be used with pulse shape modulation PSM \([100]\), given that the pulses transmitted are orthogonal. One can transmit \( m \) simultaneous orthogonal pulses to achieve similar performance for \( m \)PSM as with \( m \)PPM.
7.4 Further directions

As indicated at the beginning of this chapter, the performance analysis was performed under the assumption of an ideal system. Therefore, a plethora of open issues arise when one applies the proposed modulation techniques in real life systems. The \( m \text{MFSK} \), \( m \text{MCSK} \) and \( m \text{PPM} \) modulations may be readily used with systems that use standard MFSK, MCK or PPM modulation. \( m \text{MCSK-} m \text{MFSK} \) modulation, on the other hand, could be used in systems where spectrum spreading is desired (due to jamming/interference resistance), such as military communication systems. Given the modulation method, the performance of the system is dependent on the real life radio channel. For mobile communication systems, the radio channel is dependent on the center frequency of the transmission, the bandwidth of the system (data rate), the mobility of the devices, terrain, and antenna heights to name a few. Therefore, if one wants to apply the proposed modulation methods in practical systems, a re-iteration on the performance analysis needs to be performed where the statistical properties of multipath signals, fast and slow fading and frequency errors need to accounted for. Moreover, synchronization mechanisms and the related estimation algorithms need to be proposed and the performance of these assessed. For military communications, the jamming resistance could be evaluated as well as the performance of the methods with interference suppression algorithms.

The peak-to-average-power-ratio will also become an issue as \( m_f \) and/or \( m_c \) are increased, setting higher requirements for the transmitter power amplifiers (PA) similarly to multi-carrier communication systems. This needs to be accounted for in PA-design/selection and/or research can be performed on PAPR reduction methods to assess the performance degradation of such methods.

Further spectrum efficiency may be obtained if the \( m \text{MFSK} \) signaling frequencies could be packed closer together. An interesting research problem is whether a methodology used in multi-tone code division multiple access MT-CDMA and described in [96, 95] could be applied to \( m \text{MCSK} \) and \( m \text{MCSK-} m \text{MFSK} \) in such a fashion that the FSK frequencies relate to each other with respect to \( 1/T_r \) rather than \( 1/cT \), which is the case studied in this thesis. If the MT-CDMA approach could be applied (with a spectrum efficiency increase of \( \approx T_r / T \)), it would be interesting to find out under which constraints the method could be used.
8 Summary

The introduction of $m$MFSK and $am$MFSK generated ideas of applying the approach to other modulation methods. The straightforward extension was to apply the method to CSK modulation to introduce $m$MCSK modulation. The analysis shows that due to the orthogonality of the signaling waveforms of $m$MCSK (with Walsh codes) and $m$MFSK, the same performance is achieved for the modulation methods with the same modulation alphabet.

The thesis also gives analytical results for non-orthogonal $m$MCSK and especially considers (as an example) the use of Gold and Kasami codes. The results indicate that the best choice of codes for a non-coherent $m$MCSK system is the orthogonal code family, which is a rather intuitive result. However, for a coherent $m$MCSK system, better performance could be achieved with non-orthogonal codes. Given that we can construct a code set where the cross-correlation between the codes in the family is negative ($\rho_{i,j} < 0$), performance improvement compared to the orthogonal code family is achieved. The results show that, for instance, a 0.5 dB performance improvement in an AWGN channel can be achieved by using a specifically constructed set of Kasami codes as the basis functions in the modulation. The thesis also presents the performance analysis results of $m$MCSK in a flat Rayleigh fading channel.

To further enlarge the modulation alphabet sizes (with the price of larger spectrum usage), it was realized that combining the $m$MFSK and $m$MCSK would be an interesting choice. $m$MCSK-$m$MFSK modulation was hence introduced, which can be viewed as the main contribution of this thesis. A method to analyze this two-component modulation was developed and the performance analyses give results for $m$MCSK-$m$MFSK modulation in AWGN and flat Rayleigh fading channels for both coherent and non-coherent receivers. The performance was also assessed with orthogonal and non-orthogonal code constructs. Moreover, an antipodal extension of the $m$MCSK-$m$MFSK modulation was introduced with the analysis of the performance.

The third contribution of the thesis was to introduce the $m$PPM and $m$PSM modulation methods following the $m$MFSK principles. The performance results obtained in $m$MCSK can also be applied to these modulation formats in certain scenarios. The PPM and PSM modulation have been widely suggested to be used in UWB systems. In UWB systems, the means to adapt the data rate is the use of pulse repetition. Hence, the performance analysis of the $m$PPM and $m$PSM modulation methods with pulse repetition and a non-coherent receiver was
computed. The performance of pulse repetition in impulsive interference was also computed.

The thesis also discusses the merits of the proposed solutions. For $m$MFSK, $m$MCSK and $m$PPM, the only change that is required is in the rules for the symbol decision after symbol detection has occurred. All other signal processing elements remain the same. Thus, the proposed modulation formats may be readily used in systems employing standard MFSK, MCSK or PPM with a minimal software change. Moreover, the maximal efficiency improvement achievable is discussed. Especially for $m$MCSK-$m$MFSK, it is shown that the fundamental spectrum efficiency limit $\frac{1}{2}$ for systems employing FSK is reachable for all $M$.

This thesis introduces new combinatorial modulation techniques, and the approach in the analysis is to derive symbol error results under idealized conditions. If the methods are to be used in real systems, several issues need to be solved, including synchronization mechanisms and estimation algorithms in real mobile radio channels. PAPR performance should be assessed, as well as interference rejection performance in a multi-signal environment.
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