Carlos Hércules Morais de Lima

OPPORTUNISTIC RESOURCE AND NETWORK MANAGEMENT IN AUTONOMOUS PACKET ACCESS SYSTEMS
CARLOS HÉRACLES MORAIS DE LIMA

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University of Oulu Graduate School; University of Oulu, Faculty of Technology, Department of Communications Engineering; Infotech Oulu

Abstract

This thesis aims to evaluate networking aspects of autonomous packets access systems when dynamically and adaptively performing resource and network management. In this context, Quality of Service (QoS)-aware solutions for resource sharing and control (e.g., channel access, load control, interference management and routing techniques among others) in large-scale wireless networks are envisaged.

We propose and investigate distributed coordination mechanisms for controlling the co-channel interference generated in multi-tier coexistence scenarios consisting of macrocells underlaid with short-range small cells. The rationale behind employing such mechanism is to opportunistically reuse resources without compromising ongoing transmissions on the overlaid macrocells, while still guaranteeing QoS in both tiers. To mitigate the resulting co-channel interference, the underlaid tiers of small cells use distributed mechanism that relies on minimal signaling exchange, e.g., the Time Division Duplexing (TDD)-underlay approach which is based on regular busy tones.

Herein, stochastic geometry is used to model network deployments, while higher-order statistics through the cumulants concept is utilized to characterize the probability distribution of the aggregate interference at the tagged receiver. To conduct our studies, we consider a shadowed fading channel model incorporating log-normal shadowing and Nakagami-$m$ fading. In addition, various network algorithms, such as power control and frequency (re)allocation, are included in the analytical framework. To evaluate the performance of the proposed solutions, we also derive closed-form expressions for the outage probability and average spectral efficiency with respect to the receiver of interest under various channel conditions and network configurations.

Results show that the analytical framework matches well with numerical results obtained from Monte Carlo simulations, and that the coordination mechanisms substantially improve the performance of overlaid macrocell networks, while also benefiting small cells. In contrast to the uncoordinated Frequency Division Duplexing mode, the coordinated TDD-underlay solution shows a reduction in the outage probability, while the average spectral efficiency increases at high loads. Although more elaborated interference control techniques such as, downlink bitmap and distributed antennas systems become needed, when the density of uncoordinated small cells in the underlaid tier gets high.

Keywords: distributed antennas systems, heterogeneous network, interference management, self-organization, stochastic geometry
Tiivistelmä

Tämä väitöskirja pyrkii arvioimaan autonomisia pakettikytentäisiä järjestelmiä verkon näkökulmasta, kun resurssien ja verkon hallinta tapahtuu dynaamisesti ja adaptiivisesti. Tässä yhteydessä suunnitellaan QoS-tietoisia ratkaisuja resurssien jakamiseen ja hallintaan (esim. kanavan allokointi, kuorman hallinta, häiriön käsittely ja reititystekniikat) suurena skaalana langattomiin verkkoihin.

Ehdotamme ja tutkimme hajautettuja koordinointimekanismeja monikanavien häiriöiden hallintaan monitasoisissa skenaarioissa, jotka koostuvat lyhyen kantaman soluista makrosoluissa. Peruste näille mekanismeille on resurssien opportunistinen uudelleenkäyttö tinkimättä käynnissä olevista lähetyksistä suuremmissa makrosoluissa, samalla kun QoS taataan moleman tason lähetyksissä. Pienentääkseen aiheutuvaa monikanavahäiriöitä, alemman tason pienet solut käyttävät hajautettua mekanismia, kuten esimerkiksi säännöllisiin varattu-ääniiin perustuvaa Time Division Duplexing (TDD) -mekanismit, yhällä signaalien vaihdon määrällä.

Stokastista geometriaa käytetään mallintamaan verkkoja, kun taas korkeaman tason tilastollista laskentaa monikanavatien geometríaan, käsittelemme varjostumia ja häipyvää kanavanmallia sisältäen log-normaalisen varjostumisen ja Nakagamim häipyvän. Lisäksi sisällytämme analyysin työhön monenlaisia verkkokoordinoimista tehojärjestelyaan ja taajuuden (uudelleen)allokointiin. Ehdotettujen ratkaisujen tehokkuuden arvioimiseksi johdamme myös suljutut muodot katkosten todennäköisyyksille ja keskimääräisen spektrin käytön tehokkuudelle halun vastaanottimen suhteen monissa kanavatiloissa ja verkon kokonaisuudessa.

Tulokset osoittavat, että analyysin työn tulokset vastaavat hyvin Monte Carlo -simulaatioilla saatujen numeristen tulosten kanssa ja että koordinointimekanismit parantavat makrosoluverkkojen tehokkuutta merkittävästi, samalla kun myös pienet solut hyötyyvät. Toisin kuin koordinoimaton Frequency Division Duplexing-toimintatila, koordinoitu TDD-toimintatila pienentää katkosten todennäköisyyttä samalla kun keskimääräinen spektrin käytön tehokkuus kasvaa suurella kuormalla. Toisaalta kehitettäisiin häiriönhallintateknikat, jotka alalinkki bittikartta sekä hajautetut antennijärjestelmät, tulevat tarpeelliseksi, kun pienten koordinoimattomien solujen tiheys kasvaa alemmalla tasolla.

Asiakirjat: hajautetut antennijärjestelmät, heterogeennon verkko, häiriönhallinta, itse-organisointi, stokastinen geometria
To my Family
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August 2, 2013, Carlos Hércules Morais de Lima
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<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
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<tr>
<td>ABS</td>
<td>Almost Blank Sub-frame</td>
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<td>ADSL</td>
<td>Asymmetric Digital Subscriber Line</td>
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<tr>
<td>BS</td>
<td>Base Station</td>
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<tr>
<td>CAPEX</td>
<td>Capital Expenditure</td>
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<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>CCI</td>
<td>Co-Channel Interference</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CF</td>
<td>Characteristic Function</td>
</tr>
<tr>
<td>CM</td>
<td>Coordination Mechanism</td>
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<tr>
<td>COMP</td>
<td>Coordinated Multi-Point</td>
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<tr>
<td>DAS</td>
<td>Distributed Antenna System</td>
</tr>
<tr>
<td>DER</td>
<td>Dynamic Exclusion Region</td>
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<tr>
<td>DHCP</td>
<td>Dynamic Host Configuration Protocol</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
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<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
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<tr>
<td>ES</td>
<td>Evaluation Scenario</td>
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<tr>
<td>FBS</td>
<td>Femto Base Station</td>
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<tr>
<td>FDD</td>
<td>Frequency Division Duplexing</td>
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<tr>
<td>FU</td>
<td>Femtocell User</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<td>--------------</td>
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<tr>
<td>HII</td>
<td>High Interference Indicator</td>
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<tr>
<td>HetNet</td>
<td>Heterogeneous Network</td>
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<tr>
<td>ICIC</td>
<td>Inter-Cell Interference Coordination</td>
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<tr>
<td>IMT</td>
<td>International Mobile Telecommunications</td>
</tr>
<tr>
<td>IP</td>
<td>Interference Profile</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
</tr>
<tr>
<td>LN</td>
<td>Log-Normal</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>LoS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>MBS</td>
<td>Macro Base Station</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>MPP</td>
<td>Marked Point Process</td>
</tr>
<tr>
<td>MU</td>
<td>Macrocell User</td>
</tr>
<tr>
<td>OPEX</td>
<td>Operating Expenditure</td>
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<tr>
<td>PBS</td>
<td>Pico Base Station</td>
</tr>
<tr>
<td>PC</td>
<td>Power Control</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PPP</td>
<td>Poisson Point Process</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>RAT</td>
<td>Radio Access Technology</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>Research &amp; Development</td>
</tr>
<tr>
<td>REB</td>
<td>Range Expansion Bias</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
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<tr>
<td>RE</td>
<td>Range Expansion</td>
</tr>
<tr>
<td>RNTP</td>
<td>Relative Narrowband Transmit Power</td>
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<tr>
<td>RV</td>
<td>Random Variable</td>
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<tr>
<td>SF</td>
<td>Sub-Frame</td>
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<tr>
<td>SG</td>
<td>Stochastic Geometry</td>
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<tr>
<td>SINR</td>
<td>Signal-to-Interference plus Noise Ratio</td>
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<tr>
<td>SIR</td>
<td>Signal-to-Interference Ratio</td>
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<tr>
<td>SLN</td>
<td>Shifted Log-Normal</td>
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<tr>
<td>SON</td>
<td>Self-Organizing Network</td>
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<tr>
<td>TAS</td>
<td>Transmit Antenna Selection</td>
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<tr>
<td>TCP</td>
<td>Transmission Control Protocol</td>
</tr>
<tr>
<td>TC</td>
<td>Transmission Capacity</td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplexing</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>UL</td>
<td>Uplink</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent and identically distributed</td>
</tr>
</tbody>
</table>
List of symbols

\( \alpha \) Path loss exponent

\( \beta \) Minimum received power at the cell border

\( \Delta p \) Fixed power control step

\( \eta_k \) \( k \)th zero of the Hermite polynomial \( H_K(\eta) \) of degree \( K \)

\( \Gamma \) Random Variable (RV) representing the Signal-to-Interference Ratio (SIR) at the tagged receiver

\( \gamma_{\text{th}} \) SIR detection threshold

\( \kappa_n(\cdot) \) \( n \)th cumulant of a given RV

\( \lambda_k \) Density of small cells in the \( k \)th tier

\( \ln(\cdot) \) Logarithm to the base \( e \)

\( \log_b(\cdot) \) Logarithm to the base \( b \)

\( O \) Observation region

\( \mathcal{R} \) An arbitrary region of area \( A \) in \( \mathbb{R}^2 \)

\( \mathcal{R}_k \) \( k \)th coordination region

\( \mu \) Mean value of the distribution \( \text{Normal}(\mu, \sigma^2) \)

\( \omega_k \) The weight associated to the \( k \)th root of the Hermite polynomial \( H_K(\eta) \)

\( \Omega_p \) Mean squared-envelope

\( \text{E}_n[\cdot] \) \( n \)th partial moment of a given RV

\( \bar{C} \) Location-dependent average channel capacity

\( \Phi \) Homogeneous Point Process

\( \psi(m) \) The Euler psi function
\( \Psi_Z (\omega) \) Characteristic Function (CF) of a RV \( Z \)

\( \rho_{th} \) Predefined coordination threshold

\( \sigma \) Standard deviation of the distribution Normal\((\mu, \sigma^2)\)

\( \mathcal{T}_1 \) The event that interferers detect the victim receiver’s beacon signal \( p_b \) above the predefined coordination threshold \( \rho_{th} \)

\( \varphi \) Random point belonging to the point process \( \Phi \)

\( \tilde{\Phi}_k \) Marked Point Process (MPP) of Base Stations (BSs) in the \( k^{th} \) tier

\( \tilde{\Phi}_t \) Thinned Point Process

\( \zeta (2, m) \) The generalized Riemann zeta function

\( B_c \) Clear part of the spectrum in the partial co-channel configuration

\( B_s \) Shared part of the spectrum in the partial co-channel configuration

\( F_X (x) \) Cumulative Distribution Function (CDF) of a RV \( X \)

\( f_X (x) \) Probability Density Function (PDF) of a RV \( X \)

\( H_K (\cdot) \) Hermite polynomial of order \( K \)

\( I_0 \) The modified Bessel function of first kind

\( j \) The imaginary unity

\( K \) Factor of the Rice distribution

\( l(\cdot) \) Signal strength decay function

\( m \) Shape parameter of the Gamma distribution

\( p \) Transmit power

\( p_{req} \) Transmit power of the requesting signal

\( r \) Separation distance to the tagged receiver

\( R_K \) The remainder value of the Gauss-Hermite quadrature with the polynomial order of \( K \)
\( Y \) Distribution of the power received at the tagged receiver from a random transmitter

\( Z \) Distribution of the aggregate Co-Channel Interference (CCI) at the tagged receiver from random transmitters

\( \frac{\partial^n}{\partial \omega^n} \) \( n \)th partial derivative with respect to the variable \( \omega \)
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1 Introduction

Over the past few years, the intense demand for high data rates and ubiquitous coverage, as well as the introduction of pervasive devices with high computational power have leveraged a significant paradigm shift in wireless communications regarding their design, deployment and operation. The traditional centralized and homogeneous structure of cellular systems has gradually changed to a more dynamic, heterogeneous and infrastructureless configuration in which legacy cellular systems and large scale deployments of low-power short-range access points coexist in an distributed manner [1, 2]. Equally important, as observed by Authors in [3], even simple administrative tasks become expensive or impractical in these highly dynamic deployments, so that new operational methods that minimize the need for (re) configuration with human intervention are of primary interest. As a result of this process, not only the deployment and operation of upcoming networks has evolved, but also the previously established and widely accepted methods to evaluate how these systems perform have to be updated accordingly [2, 4].

The International Telecommunication Union (ITU) radio communication sector issued the International Mobile Telecommunications (IMT) advanced requirements for the next generation of mobile phone and internet access services: downlink data rates of up to 100 Mb/s and 1 Gb/s for mobile and nomadic users, respectively. During the standardization process of Long Term Evolution (LTE)-Advanced (Release 10), the 3rd Generation Partnership Project (3GPP) identified several advanced techniques to meet such stringent requirements [5]. For instance, carrier aggregation, Downlink (DL) spatial multiplexing using up to eight-layer Multiple-Input Multiple-Output (MIMO), DL intracell Coordinated Multi-Point (COMP) transmission and reception (to mitigate outages at cell border), Uplink (UL) Spatial Multiplexing using four-layer MIMO. Unfortunately, the above solutions do not provide significant improvements and may not be always effective, mainly under low Signal-to-Interference plus Noise Ratio (SINR) regime due to the radio channel impairments and Co-Channel Interference (CCI). Thus, Heterogeneous Networks (HetNets) emerged as a viable solution to satisfy the relentless traffic demands of upcoming networks in a cost effective manner [6]. Bearing that in mind, this thesis first defines a comprehensive analytical framework and then assess the feasibility of these self-organizing HetNets in various deployment scenarios.
1.1 Self-organizing networks

The ever increasing popularity of pervasive devices, such as smart phones and tablets, has imposed high traffic demand on current and upcoming wireless systems [7]. Users relentlessly consume cloud-based services (remote applications, data storage and video sharing) as well as expect seamless connection and high-quality experience. To keep up with these stringent requirements, next generation wireless systems need to ensure high spectral efficiency with ubiquitous coverage and fairness at cell border. Unfortunately, legacy cellular systems with their rigid infra-structure and centralized coordination fall short.

Operators have indeed very few options available to meet such requirements: (i) increase the density of macrocell sites, but that hinges on regulatory constraints, the necessary Capital Expenditure (CAPEX), and the rollout time to deploy new sites; (ii) upgrade the Radio Access Technology (RAT) but that need long Research & Development (R&D) cycles and may not fill the capacity gap completely; or (iii) expand the radio frequency resources, though spectrum auctions are definitely very expensive and lingering alternative. In this unfavorable scenario, multi-tier HetNets with very large deployment of small cells emerges as an enabling concept of the future wireless systems.

In this contribution, we investigate the feasibility of deploying such HetNets to meet the strict requirements of the next generation of LTE systems [8]. Herein, HetNets integrate legacy macrocells with underlaid small cells and inherit their complementary features [6, 9]. Based on the 3GPP specifications, we construct HetNet structures by deploying base stations in three distinct levels: femtocells which are characterized by very low transmit power at 24 dBm and unplanned deployment, picocell which are mainly deployed at hot spot areas and operate in the range of 24dBm to 33 dBm, and traditional macrocells which are deployed in regular manner and have large coverage with transmit power up to 46 dBm [8]. In this way, Base Stations (BSs) of various types populate multi-tier hierarchical deployments and share the common air interface (single carrier usage) wherein CCI management becomes critical for reducing the resulting interference across tiers. We consider a single-Radio Access Technology (RAT) multi-tier architecture (based on LTE specifications) wherein BSs use various channel access policies and backhaul architectures to communicate. Fig. 1 illustrates a multi-tier deployment in which macrocells are underlaid with pico- and femtocells [10]. In the scenarios under study, macrocells ensure coverage over large areas, support high mobility users and reduce handover attempts. Picocells are tailored to smaller and high density
areas (hot spot) such as shopping malls, concerts and convention centers. Femtocells are deployed by the end user and depend on user owned backhaul infrastructure such as Digital Subscriber Line (DSL) or cable modems. They serve much restricted areas such as households or small offices and interference becomes an issue.

While cellular systems exhibit a stiff hierarchical structure with centralized control, dependability and scalability; small cells leverage short-range low-power communications so as to improve the quality of radio links, reduce co-channel interference and maximize spectral efficiency. In contrast to macrocells, small cells may be deployed by end-users (low installation complexity) and make use of existing broadband Internet connection (backhaul infrastructure), such as Asymmetric Digital Subscriber Line (ADSL) or optic fiber, to connect to operators core network [1]. By bringing the serving base station closer to the end-user, short-range communications intrinsically provide better link quality which improves Signal-to-Interference Ratio (SIR) and ensures higher data throughput. Picocells are deployed by the operators at building facades or street poles to serve high traffic areas, while femtocells are easily deployed by end-users and have great operational flexibility. Underlaid small cells effectively extend coverage and capacity of overlay networks, mainly at macrocells border where the fairness is an issue. However, the uncoordinated operation and unplanned deployment of small cells cause routine tasks to maintain and operate systems expensive or even unpractical. The distributed nature of small cells introduce new challenges to their deployment and

Fig 1. Illustration of Self-Organizing Networks (SONs).
operation. Several practical issues which are tackled in this works still hinders the viability of HetNet deployments. The harsh CCI that arise from their uncoordinated operation is the main problem to undertake.

To reduce Operating Expenditure (OPEX), self-organization plays a determinant role by reducing human intervention and quickly coordinating inter-cell operation and adapting to network dynamics and channel impairments. Bell Labs found savings of about 20% CAPEX and 50% OPEX by employing SONs [11]. Self-organization plays a determinant role to make HetNets a viable solution, since it provides the foundation to achieve overall system organization, even though constituent parts work independently. In fact, self-organization is a much more pervasive phenomena which appears in natural systems and human interactions. For instance, a school of fishes that swim in a coordinated manner, or a flock of migratory birds that travel together into formation. It is possible to identify a set of desirable properties which become prominent across self-organizing structures. First, the emergent behavior whereby sophisticated organization of the overall system is achieved when participants organize themselves by interacting directly and reacting to changes in their local environments without central coordination [3, 12]. And yet self-organizing systems exhibit adaptability to adapt in response to system and environmental changes; robustness against failure and damage; and, equally important, scalability whereby the systems stays functional by considering a very large number of constituent entities.

Despite these principles being commonplace in several areas, telecommunications did not exploit their potential in depth so far. In computer networks, self-organization principles are used to minimize configuration and human intervention facilitating network operation and maintenance, and allows for totally decentralized operation: dynamic acquisition of Interference Profile (IP) addresses with Dynamic Host Configuration Protocol (DHCP) and autonomous back off with Transmission Control Protocol (TCP). In this contribution self-organization principles are used to established distributed strategies by which network entities coordinate based on local side-information so that the overall network operation and performance are improved. Herein, we devise solutions based on the self-organization concept and on the general guideline that local behavior rules that achieve global properties.
1.2 Related work

In order to tap into the full benefits of large-scale SONs, a number of challenges still need to be tackled, including their deployment, operation, automation and maintenance [3, 12]. The design and implementation of self-organizing functionalities in HetNets is a topic of significant interest as evidenced by the number of recent publications [1, 3, 4, 13–16]. As aforesaid, the CCI generated by standalone small cells can severely compromise the communication of both tiers [17, 18]. A motive is that small cells are often deployed by the end-users in an unplanned manner and, as a result, lack any predefined infrastructure [19]. To further exacerbate this situation, small cells are not necessarily networked and may operate in a totally uncoordinated fashion. Thus, it becomes crucial to tackle the interference problem to make multi-tier networks a viable solution [17, 20].

Since standalone small cells operate independently of the overlaid macrocell network, and of others, conventional (predetermined and centralized) interference management solutions like traditional network planning and optimization, are only partially applicable and do not fully exploit variations over time and space [21]. For instance, the dedicated channel assignment in [22] provides a straightforward approach to cope with this uncoordinated interference by simply dividing the whole spectrum in orthogonal frequency bands and then allocating chunks to each tier such that they do not overlap. Unfortunately, in dense HetNets deployments, each small cell allocates a very limited bandwidth which severely compromises the achievable performance. Although the decentralized allocation of radio resources is identified as a promising solution in several recent publications [23–27], its implementation still constitutes a technical challenge due to its inherent synchronization and signaling exchange issues. Lópes-Pérez et al. in [17] have discussed self-configuration and self-optimization concepts as feasible technologies to allow femtocells to sense changes and adapt their parameters accordingly. Along the same lines, Rangan in [18] proposes a distributed solution in which femtocells select their transmit powers so as to control their aggregate interference at the tagged receiver, while still maximizing their throughput. Actually, the author addresses the interference problem in a way similar to ours, by studying the combined effect of power control and spectrum (re)partitioning for interference avoidance.

In [28], the busy tone concept is used to devise an adaptive technique to mitigate inter-cellular interference in Time Division Duplexing (TDD) networks using Time Division Multiple Access (TDMA) scheme. Furthermore, authors in [29] first identify
the CCI as the key challenge for uncoordinated communication in wireless networks, and then use what is called busy burst to establish exclusion regions around receivers in order to avoid conflicting allocations. As an alternative to the typical spectrum partitioning solutions, the TDD-underlay concept is proposed in [30] to take advantage of the natural traffic asymmetry between the DL and UL, as well as the user spatial diversity. The TDD-underlay concept incorporates a distributed mechanism which dynamically coordinates inter-cell time-slot allocation to avoid strong interference from nearby conflicting transmitters [18, 28]. Time-multiplexed communicating links operating in universal frequency reuse are still exposed to the CCI generated by dominant interferers transmitting in an uncoordinated manner.

In [21], the self-organization concept is used to devise cognitive radio resource management schemes to mitigate cross-tier interference and guarantee users Quality of Service (QoS) in distinct heterogeneous deployments scenarios. Traditionally, such cognitive solutions are used to enable unlicensed users (secondary system) to dynamically share the spectrum with the primary system. The motivations are twofold: (i) the unceasing demand for radio spectrum and (ii) the inefficient usage of the licensed bands by primary systems over time and space as evidenced in [31]. In such systems, the accurate detection of free space, i.e., transmission opportunities, is crucial to avoid extra interference to the primary system. In [32], Authors develop a stochastic framework to show that the aggregate interference in spectrum sensing cognitive networks is characterized by the radio channel, as well as the transmit power, sensitivity and density of cognitive radios. The opportunistic spectrum usage is also investigated in [33], wherein it is shown that the harmful interference generated by secondary radios depends on the cognitive solution to accurately detect and rapidly react to varying spectrum usage.

Lately, the small cell concept emerge as promising, cost-effective approach to tackle the stringent requirements of the next generation of wireless systems by providing unprecedented spectrum efficiency and coverage. However, small cells are randomly installed by end-users and lack predefined infrastructure (unplanned deployment) as well as operate in an uncoordinated and decentralized manner. As a result, small cell networks cause severe interference towards the legacy cellular systems (primary) and compromise the overall network sustainability. Then, it comes with no surprise that the concept of cognitive radios is applicable to small cells deployments as well. For instance, Quek et al. propose a cognitive hybrid division duplex scheme whereby the macrocell tier performs Frequency Division Duplexing (FDD) in a pair of frequency
bands, while the underlaid tier of cognitive small cells simultaneously operate in TDD on the same bands [34].

More recently, the Range Expansion Bias (REB) concept is discussed within 3GPP as a baseline solution to boost the offloading potential of heterogeneous deployments. In that regard, Authors in [35] investigate the cell range expansion and interference mitigation in heterogeneous networks. Following the same lines, Güvenç instigates the capacity and fairness of heterogeneous networks with range expansion and interference coordination [13]. In [16], Jo et al. use the Stochastic Geometry (SG) framework to assess how the biased cell association procedure performs in heterogeneous networks by means of the outage probability.

Hereafter, we present our contributions in the perspective of the available solutions. We recall that the (re)use of spectral resources to achieve high data rates brings about harsh CCI, which is doubtlessly a critical source of degradation in interference-limited scenarios and largely depends on the local interactions among communicating nodes and their relative disposition [20]. In such systems, to cope with interference, accurate models are also needed during the several stages of the design, development and deployment of protocols and algorithms. Equally important, new investigative techniques which capture the dynamics and distributed nature of such systems are introduced so as to properly identify problems and provide reliable figures-of-merit of the achievable performance.

1.3 Thesis objectives and outline

The objective in this thesis is to appropriately represent multi-tier HetNets using statistical tools such as stochastic geometry and higher order statistics, as well as effectively devise self-organizing solutions to cope with their unplanned deployment and uncoordinated operation in a distributed manner. The results presented in this thesis are contemplated in three journal papers [36–38] and conference papers [39–42]. The framework introduced herein is also used to analyze the performance of relay selection algorithms and multi-hop network routing in [43–47].

As discussed in [4], computer simulations used to be the typical approach to evaluate the performance of cellular systems owing to their regular infrastructure. Unfortunately, it becomes significantly more complex to model the irregularity of large-scale HetNets and evaluate their performance through simulations [48]. In the heterogeneous scenarios under consideration, legacy cellular systems coexist with
several tiers of small cells which are deployed without any predefine infra structure and exhibit distinct communication capabilities such as transmit power, antenna structure and backhaul capability. Recently, stochastic geometry and the concept of random patterns have been used to represent the spatial interactions between communicating nodes with great success. With this analytical approach, models are surprisingly tractable and closely captures the main performance trends of HetNets [10, 49, 50].

By employing the stochastic geometry principles, we initially construct practical network deployment models and establish an analytical framework in Chapter 2 so as to assess the performance of HetNets consisting of legacy macrocells underlaid with tiers of self-organizing small cells. The Evaluation Scenarios (ESs) are defined by configuring network parameters across tiers, for example density of transmitters, transmit power, scheduling policy and SIR threshold (QoS requirement) among others. This framework consider multi-tier deployments of HetNets wherein femto-, pico- and legacy macrocells coexist in a self-organizing framework. Higher order statistics are used through the cumulants concept to recover the distributions of the power received from the transmitter of interest and aggregate interference. The impact of the Radio Channel impairments on the transmission of the user of interest and interfering links is captured as well. Thereafter, self-organization principles are applied to the HetNets under study in order to devise distributed strategies to partition the meager radio resources across tiers and manage the resulting interference.

In Chapter 3, Coordination Mechanisms (CMs) are implemented to reduce the uncoordinated interference from the underlaid tier of small cells. The CMs are distributed strategies to control the co-channel interference generated by standalone femtocells in two-tier coexistence scenarios consisting of macrocells underlaid with short-range small cells. The rationale behind employing such mechanism is to opportunistically reuse resources without compromising ongoing transmissions on the overlaid macrocell, while still guaranteeing QoS in both tiers. Interference Avoidance Techniques are also used in conjunction with the CMs so as to improve the performance of the scenarios under study [51].

The concept of TDD is investigated in Chapter 4 as an alternative to underlay short-range femtocells on the Uplink of legacy macrocell deployments. The TDD-underlay strategy takes advantage of the natural spatial diversity of users locations as well as the traffic asymmetry between the DL and UL of cellular systems. To mitigate the resulting co-channel interference, the underlaid femtocell tier uses a distributed mechanism which is based on regular busy tones and relies on minimal signaling exchange.
In Chapter 5, Range Expansion Bias is employed to improve spatial reuse and balance load between tiers of the heterogeneous scenarios under study. Almost Blank Sub-frame (ABS) is also considered as the baseline Inter-Cell Interference Coordination (ICIC) technique to cope with the resulting CCI. Thereafter, a soft bitmap indicator, similar to the DL-High Interference Indicator (HII), is used to identify the dominant interferers and improve the SIR at the receiver of interest. We then investigate the concept of virtual Distributed Antenna System (DAS) which is yet another self-organization solution to mitigate interference and improve the received signal at the receiver of interest. In Chapter 6 we draw general conclusions, make our final remarks and provide our perspective about futures directions to enhance the proposed self-organizing solutions.
2 Statistical modeling and evaluation of random networks

Aiming at properly modeling and evaluating the performance of Heterogeneous Networks (HetNets), a mathematical framework based on stochastic geometry is introduced. Traditionally, simulations are the common approach to model and evaluate cellular systems in which homogeneous Base Stations (BSs) are deployed in predefined locations over a regular hexagonal grid. Conversely, multi-tier HetNets exhibit irregular structures with intricate network dynamics so that accurate simulations become a complicated task or even impractical. In that regard, random spatial models offer a tractable approach which closely captures the operational trends and achievable performance of HetNets.

Generally speaking, spatial point processes are a natural approach to model and analyze random point patterns. For instance, point processes are used in statistical ecology to model locations of trees and bird nests, in astrostatistics to study the relative disposition of stars and galaxies, in stereology to provide spatial interpretations of planar sections of materials and tissues, and in wireless systems to account for the effect of the random distribution of communicating nodes in the network performance [52–56].

Lately, stochastic geometry principles have been used to address several open problems in wireless communicating systems. To begin, Authors in [57] address the multihop transmission problem in direct-sequence spread spectrum packet radio networks by modeling interferers as a random Poisson field and then deriving the optimum transmission range as a function of the processing gain, the background noise power spectral density, and the degree of error-correction coding. In similar Code Division Multiple Access (CDMA) networks, stochastic geometry is used in conjunction with the queueing theory to model spatial traffic patterns, recover the distribution of the aggregate interference and evaluate the system performance in terms of the outage probability [58]. The performance trends of spectrum sharing schemes in CDMA systems are investigated in [59] using Poisson field of interferers and recovering the outage probability with respect to the tagged receiver.

In the context of multihop ad-hoc networks, Baccelli et al. use mathematical tools stemming from stochastic geometry to identify a tradeoff between the spatial density of communications and the range of each transmission [60]. The Poisson shot noise field theory is also used in [61] to establish the “Opportunistic Aloha” protocol which
– instead of tossing a coin – evaluates the channel quality of active links and selects the stations with good channels conditions to transmit next. The stochastic theory is also used in the context of cognitive radio networks. For instance, the benefits of employing spectrum sensing are assessed in [32] by modeling the secondary tier as a homogeneous spatial Poisson point process and then characterizing the aggregate interference at the primary user for distinct sensing policies and radio channel conditions.

When modeling random access networks, terminals are typically scattered over the deployment area following a known random process which matches well the physical phenomena and leads to analytically tractable results [62]. In wireless networks wherein no interferer dominates, the central limit theorem applies and the Gaussian random process is commonly used to model the aggregate interference [62]. However, in deployment scenarios with dominant interferers, the central limit theorem is violated and the density function of the aggregate interference exhibits heavy tail. As discussed in [20], if we assume nodes to be distributed in the plane according to a homogeneous Poisson Point Process (PPP) as well as the interference components at the receiver of interest to follow a distance dependent attenuation function and sum incoherently; then, the aggregate interference can be modeled as a shot noise process whose distribution follows a $\alpha$-stable.

In fact, the stable distributions are heavy-tailed and generalize the Gaussian distribution. Symmetric $\alpha$-stable distributions are introduced as a convenient formulation to model the aggregate interference of a large number of terminals distributed in the plane according to a Poisson point process (Non-Gaussian Random Processes) [63]. In [62], Win et. al consider random networks where nodes are scattered following a 2 dimensional Poisson process and the propagation effects (multipath fading and shadowing) follow a spherically symmetric distribution. Therein, it is shown that the distribution of the aggregate interference amplitude has a symmetric stable distribution (skewness parameter equal to zero), while the distribution of the aggregate interference power follows a skewed stable distribution. Similarly, Pinto et al. work with an analytical framework which models the interference distribution generated by a Poisson field of interferers using skewed stable distributions in [64, 65]. In addition, the channel capacity of the tagged link when subject to both network interference and noise is characterized and the concept of spectrum outage probability is introduced to characterize the aggregate interference from combined radio-frequency emissions generated in a wireless network. The opportunistic spectrum access is also studied in [33] where the aggregate interference at the primary user is modeled using the theory of.
truncated-stable distributions which constitute an alternative statistical model to the aggregate interference in more realistic scenarios.

Weber et al. introduced the Transmission Capacity (TC) metric [66] as the product of the maximum spatial density of successful transmissions given an outage constraint, and evaluate the performance of ad hoc networks under various configuration parameters: Power Control (PC) in narrowband fading scenarios [67], fractional PC in decentralized wireless networks [68], and successive interference cancellation [69]. Along the same lines, Authors compare the successive interference cancellation against the joint detection using the TC framework in [70]. In a series of papers Andrews et al. assessed the performance of two-tier HetNets [71] under various network configurations. The Authors develop an interference avoidance strategy for the Uplink (UL) of two-tier femtocell networks and then analyze the resulting system capacity using TC metric [50]. The spectrum sharing problem is revisited in [10, 72]. And yet Distributed Antenna System (DAS) with random layout and distinct antenna selection schemes are addressed in [73], as well as how to improve coverage in multi-antenna two-tier networks [74].

In [75], Stüber emphasizes that the system performance depends on the assumptions made about the receiver front-end and propagation environment. The shadowed fading scenario with multipath fading superimposed on shadowing is typically encountered in dense urban areas (congested downtown) with stationary or slow moving terminals [75, 76]. In these scenarios, it is quite convenient to model the radio channel as a composite distribution of the squared envelope due to Nakagami-\(m\) fading and Log-Normal (LN) shadowing. Indeed, the Nakagami-\(m\) distribution is mathematically tractable and can closely approximate a Rice distribution in Line-of-Sight (LoS) scenarios. Herein, stochastic geometry is used to model coexistence scenarios wherein legacy macrocell sites are underlaid with multiple tiers of self-organizing small cells and incorporating the shadowed fading channel as random marks independent of independent of nodes positions [77]. In preparation for the description and performance analysis of the proposed solutions, we first present our assumptions, make definitions and introduce a benchmark scenario to carry out derivations.

2.1 Preliminaries of Poisson point processes

In this section we introduce mathematical tools from the stochastic geometry theory which are needed to develop our framework in the subsequent sections. To begin with, the spatial Poisson process is defined as [52].
Definition 1. (Spatial Poisson process) A spatial Poisson process with uniform intensity \( \lambda > 0 \), is a point process in \( \mathbb{R}^2 \) such that: (i) for every bounded closet set \( B \), the count \( N(B) = \sum_{\varphi \in \Phi} 1_B(\varphi) \) follows a Poisson distribution with mean given by \( \lambda \lambda_2(B) \), where \( \lambda_2(\cdot) \) yields the Lebesgue measure and the indicator function \( 1_B(\varphi) \) is defined as
\[
1_B(\varphi) = \begin{cases} 
1, & \text{if } \varphi \in B, \\
0, & \text{otherwise}; 
\end{cases}
\]
(1)
and (ii) if \( B_1 \ldots B_k \) are disjoint regions, then \( N(B_1), \ldots, N(B_k) \) are independent.

In this thesis, we are interested in homogeneous Poisson processes where \( \lambda \) is a constant. A homogeneous random process exhibit stochastic properties which are unchanged under translation or rotation of \( S \) [78].

Equally important, the concept of a “typical” node of the process is central to derive our performance metrics and requires calculating event probabilities conditional on having a point of the process at a specific position. Generally speaking, the typical node is selected following a procedure whereby any other point of the process is selected with equal probability. For instance, by employing the concept of a typical node one computes the nearest-neighbor distance distribution which yields the distance from a point of the process \( \Phi \) to the nearest other point of the same process, i.e., \( \Pr[\text{dist}(\varphi, \Phi \setminus \varphi) \leq r | \varphi \in \Phi] \), where \( \setminus \) denotes the set exclusion operator. The Slivnyak’s theorem is actually used to formalize this notion of assuming a point of the process dwelling in an arbitrary location [79].

Theorem 1. Consider that the Poisson point process \( \Phi \) has property \( Y \) and that it contains a point at \( \varphi \); then,
\[
\Pr[\Phi \text{ has property } Y \mid \varphi] = \Pr[\Phi \cup \{\varphi\} \text{ has property } Y \mid \varphi],
\]
(2)

Proof. See [80, Section 4.4]. \( \square \)

Intuitively, (2) suggests that the Palm distribution for a stationary Poisson point process corresponds to that of the original process together with an adjoined point in \( \varphi \). The Palm probability is then used to compute the distribution conditional on the presence of this typical node [80].

In addition, the Campbell theorem allows us to compute summations of the form \( \sum_{\varphi \in \Phi} f(\varphi) \), where \( f(\cdot) \) is a real-valued function on the state space of the arbitrary Poisson process \( \Phi \).
Theorem 2. Consider a spatial Poisson process $\Phi$ on space $S \subset \mathbb{R}^2$ with mean measure $\mu$ and a measurable function $f : S \to \mathbb{R}$. Then, the sum $\sum_{\phi \in \Phi} f(\phi)$ is absolutely convergent with probability if and only if $\int_S \min \{ |f(x)|, 1 \} \mu(dx) < \infty$. And if the above condition holds, we have
\[
E\left[ e^{\theta \sum_{\phi \in \Phi} f(\phi)} \right] = \exp \left( \int_S \left[ e^{\theta f(x)} - 1 \right] \mu(dx) \right),
\]
for any real or complex $\theta$.

Proof. See [78, Section 3.2] \qed

This theorem is useful to compute the mean values of point process characteristics and statistical estimators. From (3), we obtain the characteristic function of the random process $\Phi$ by setting $\theta = -1$ and $f(\phi) \geq 0$ [78],
\[
E \left[ e^{-\sum_{\phi \in \Phi} f(\phi)} \right] = \exp \left( - \int_S \left[ 1 - e^{-f(x)} \right] \mu(dx) \right).
\]

The Marked Poisson point process $\tilde{\Phi}$ is another important concept that needs to be formalized as well.

Definition 2. (Marked point process) A Marked Point Process (MPP) on the space $S \subset \mathbb{R}^2$ with marks on a space $M$ is a point process $\tilde{\Phi}$ on $S \times M$ such that $N_{\tilde{\Phi}}(K \times M) < \infty$ almost surely for all compact set $K \subset S$. In other words, the projected process of points without marks is locally finite.

Generally speaking, it corresponds to labeling every point of the original process with additional information denoted “marks”. Formally, a marked point corresponds to a pair $(\phi, m)$ where $\phi \in \Phi$ and $m$ is its respective mark.

2.2 General definitions

Definition 3. (Tagged receiver) The user of interest who is taken as the reference to compute the aggregate Co-Channel Interference (CCI) and the respective performance metrics. The Slivnyak’s theorem [78, 80] is used to obtain average performance quantities conditional on the presence of the “tagged” receiver at the origin. In other words, Palm distributions and the related Campbell’s theorem are used to characterize a
random pattern with respect to a typical point of the process, so that network-wide performance metrics can be characterized by the average behavior of this tagged node [52, 56].

**Definition 4. (Observation region)** An annular region around the tagged receiver over which we account for the aggregate interference. The observation region is denoted by $O$ and defined by the minimum and maximum radii $R_m$ and $R_M$, respectively.

**Definition 5. (Partial moment of a random variable)** Let $Y$ be a continuous Random Variable (RV) with Probability Density Function (PDF) given by $f_Y(y)$; then, $E_Y[y^m, y^M] = \int_{y^m}^{y^M} y^n f_Y(y) dy$ denotes its $n^{th}$ partial moment with $y_m$ and $y_M$ indicating the lower and upper integration limits, respectively.

After making the above definitions, we can introduce the system models which are used throughout this thesis, namely the propagation radio channel along with the network deployment model.

### 2.3 Propagation channel model

Radio links are degraded by deterministic path loss and shadowed fading, which is assumed to be independent over distinct network entities and positions. The shadowed fading consists of small- and large-scale fading which are mutually independent and multiplicative phenomena of the wireless radio channels [20, 63, 81]. The path loss represents the attenuations suffered by a signal (electromagnetic wave) traveling from the transmitter to the receiver and depends on frequency, antenna height, relative distance between peers and topography. Large-scale fading is a random manifestation cased by prominent obstructions (in comparison to the carrier wavelength) of the terrain such as buildings, hills, walls, towers, billboards, etc.. Experimental observations have shown that this effect follows a LN distribution and make the local mean to vary slowly over distances of several tens of wavelengths. Small-scale fading is caused by the signal propagating through multiple paths. As a result, multiple replicas of the transmitted signal combine either in constructive or destructive manner at the receiver front-end what causing high envelop fluctuations [82, 83].
Here, we describe the path loss attenuation by means of an unbounded path loss model with signal strength decay function $r^{-\alpha}$, where $\alpha$ is the path loss exponent – a minimal separation distance of 1 m between communicating pairs so as to avoid the unbounded path loss model singularity at 0 [84]. The shadowed fading model combines the random effects of the LN shadowing and Nakagami-$m$ fading. The received squared-envelope due to multi-path fading and shadowing is represented by a RV $X \in \mathbb{R}^+$ with Cumulative Distribution Function (CDF) and PDF denoted by $F_X(x)$ and $f_X(x)$, respectively.

An arbitrary interferer disrupts the communication of the tagged receiver with a component given by

$$Y = p \ r^{-\alpha} x,$$

where $p$ yields this interferer transmitted power, $r$ is the separation distance from its position to the tagged receiver, and $x$ yields the corresponding shadowed fading.

We consider LoS scenarios with a dominant multipath component or specular condition. In these scenarios, the magnitude of the received complex envelop at any time has Rician distribution [75] whose PDF includes the modified Bessel function $I_0$ [85]. To model this radio propagation environment, we use the Nakagami-$m$ fading which is mathematically convenient and closely approximate the Rician distribution. In fact, there is a direct relation between the Rice factor $K$ and the Nakagami shape factor $m$.

$$K \approx \sqrt{m^2 - m + m - 1}. \quad (6)$$

The composite distribution of the received squared-envelope due to the shadowed fading has a Gamma-LN distribution with PDF [75],

$$f_X(x) = \int_0^\infty \left(\frac{m}{\omega}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\omega} x\right) \frac{\xi}{\sqrt{2\pi} \sigma \omega} \exp\left[-\frac{\left(\frac{\xi \ln \omega - \mu_{\Omega_p}}{2\sigma_{\Omega_p}^2}\right)^2}{2\sigma_{\Omega_p}^2}\right] d\omega, \quad (7)$$

where $m$ is the shape parameter of the Gamma distribution, $\xi = \ln(10)/10$, $\Omega_p$ is the mean squared-envelope, $\mu_{\Omega_p}$ and $\sigma_{\Omega_p}$ is the mean and standard deviation of $\Omega_p$, respectively.

Ho et al. show in [86] that a composite Gamma–LN distribution can be approximated by a single LN distribution with location and scale parameters (in logarithmic scale) given by

$$\mu_{\text{AB}} = \xi \left[\psi(m) - \ln(m)\right] + \mu_{\Omega_p}, \quad (8)$$
\[ \sigma_{dB}^2 = \xi^2 \zeta(2, m) + \sigma_{\text{i.d.}}^2 , \]  

where \( \psi(m) \) is the Euler psi function and \( \zeta(2, m) \) is the generalized Riemann zeta function [85]. In what follows, we use this single LN approximation to characterize the radio channel attenuations in various evaluation scenarios.

### 2.4 Network deployment model

A HetNet consisting of umbrella Macro Base Stations (MBSs) and underlaid tiers of self-organizing small cells is modeled. We assume that MBSs follow centralized coordination and spectrum allocation such that inter macrocell interference is mitigated, for example, using fractional frequency reuse [15]. In these scenarios, small cells are uniformly scattered over the network area. In every transmission interval, each serving BS schedules a single user terminal. The set of associated user terminals are also uniformly distributed within the transmission range of their serving cells. Nodes communicate using antennas with omni directional radiation pattern and fixed power. The macrocell tier transmits at a maximum power of 46 dBm, picocells use 30 dBm and femtocells at 24 dBm. Fig. 2 shows a realization of the network deployment where macrocells are underlaid with two tiers small cells.

In this network configuration, positions of small cells in the \( k \)th tier constitute a homogeneous PPP \( \Phi_k \) with density \( \lambda_k \) in \( \mathbb{R}^2 \). The number of small cells belonging to the \( k \)th tier in an arbitrary region \( R \) of area \( A \) is a Poisson RV with parameter \( \lambda_k A \) [78]. Additionally, the fading effect is incorporated into the model as a random mark associated with each point of \( \Phi_k \). By virtue of the Marking theorem [52, 78], the resulting process,

\[ \bar{\Phi}_k = \{ (\varphi_k, x) ; \varphi_k \in \Phi_k \} , \]  

corresponds to a MPP on the product space \( \mathbb{R}^2 \times \mathbb{R}^+ \) with intensity \( \lambda_k g(x) \) [78], whose random points \( \varphi \) represent transmitters locations and belong to the stationary point process \( \Phi_k \). It is worth noticing that a MPP \( \bar{\Phi}_k \) as defined above is also a Poisson process, since each point of the original process \( \Phi_k \) is marked independently of the others. Similarly, Baccelli et al. use a MPP to represent the multidimensional dependence between radio links, medium access protocol, Signal-to-Interference Ratio (SIR) threshold and channel impairments [61, 87]. In the following, we combine the above system models to represent particular network configurations as well as address specific problems regarding their operation and interoperability between tiers.
Fig 2. A illustration of a HetNet realization with random shadow map. A MBS at the center of the grid is represented by a big triangle. Picocells constitute the second tier and are depicted as small triangles. Black shaded squares represent femtocells.

2.5 Characteristic function and cumulants

This analytical framework uses stochastic geometry to model network deployments, and higher order statistics through the cumulants concept to recover both the distributions of the received power $Y$ and the aggregate CCI $Z$ at the tagged receiver. We resort to the Slivnyak’s theorem and its associated Palm probability to derive the aggregate CCI and compute the average performance figures conditional on the location of the tagged receiver [32, 78, 80, 85].

To establish this framework, we begin by applying Campbell’s theorem [78, 80] to the MPP $\tilde{\Omega}$ defined in (10) so as to determine the Characteristic Function (CF) of the
distribution of the aggregate CCI. Hereafter, there is no loss of generality in assuming only one tier of small cells to derive the statistics of the aggregate CCI.

**Definition 6.** Let \( Z = \sum_{(\varphi, x) \in \Phi} Y \) be a RV representing the aggregate CCI generated by the interfering process \( \Phi \), and \( j = \sqrt{-1} \) be the imaginary unity; then, the function \( \Psi : \mathbb{R} \rightarrow \mathbb{C} \) defined as,

\[
\Psi_Z(\omega) = \mathbb{E}\left[e^{j\omega Z}\right],
\]

(11)
is called the CF of \( Z \).

As observed in [88], the CF corresponds to the Fourier transform of the PDF of the RV \( Z \).

As observed by Resnik et al. in [89], the fact that the CF always exist poses as its main advantage. The \( n^{th} \) cumulant is obtained by computing higher order derivatives of (11) as presented in our next proposition [85].

**Proposition 1.** Let \( Z \) be a RV, \( \Psi_Z(\omega) \) its CF, and \( n \in \mathbb{N} \). Provided that the \( n^{th} \) moment exists and is finite, then, \( \Psi_Z(\omega) \) is differentiable \( n \) times and

\[
\kappa_n = \frac{1}{j^n} \left[ \frac{\partial^n}{\partial \omega^n} \ln \Psi_Z(\omega) \right]_{\omega=0},
\]

(12)

**Proof.** See [89, Section 9.4]. \( \square \)

### 2.6 Approximating of the aggregate CCI

The total interference perceived by the tagged receiver is the result of the summation of independent LN RVs, each of which represents a distinct interference contribution from every single interferer currently transmitting [75, 76, 90]. The interference contributions of interferers within a given tier are assumed to be identically distributed. Unfortunately, there is no general closed-form expression for the PDF of the sum of LN RVs, though a plethora of analytical approximations have been suggested in the literature [91–93] and references thereof. In [39] the total interference perceived by the tagged receiver is approximated using a flexible method introduced in [94], which uses the Moment Generating Function (MGF) of a single “equivalent” LN RV to approximate the summation of the individual components. Another option is to approximate its density
function using series expansion in terms of higher order cumulants. For example, in [95] the Gram-Charlier series method is used. In [58], Chan et al. first identifies a noticeable skewness in the distribution of the aggregate interference to the right (positively skewed) and then uses the Edgeworth expansion of the characteristic function utilizing higher order cumulants. The Edgeworth expansion is the Taylor expansion of the characteristic function of the aggregate interference at the user of interest [85]. Note that the third cumulant controls the amount of skewness of the probability distribution. In the context of spectrum-sensing cognitive wireless networks, the Edgeworth expansion of the characteristic function is also used the aggregate interference generated by the secondary nodes [32]. And yet an alternative to approximate the distribution is based on estimating the parameters of an equivalent known distribution by means of the moments of the actual distribution.

Motivated by the fact that the distribution of $Z$ is heavy-tailed and positively skewed [32, 58, 94, 96], we use simple yet accurate LN and Shifted Log-Normal (SLN) approximations whose parameters are estimated using the cumulants of the aggregate CCI as presented next. We first relate the moments of the LN distribution to the cumulants of the aggregate CCI as follows,

\[
\mu = \ln \left( \frac{\kappa_2^2}{\sqrt{\kappa_1^2 + \kappa_2}} \right), \quad \text{and} \quad \sigma^2 = \ln \left( 1 + \frac{\kappa_2^2}{\kappa_1^2} \right),
\]

(13)

where $\mu$ is the mean and $\sigma$ is the standard deviation of the distribution, $\text{Normal}(\mu, \sigma^2)$ in the logarithm scale.

In [97], the SLN distribution is introduced to approximate the distribution of the number of packets that a typical customer need to buy to obtain the complete set – without considering the swapping of cards between friends. Similar to the approach of Ghasemi et al. in [32], we adapt the above approximation to characterize the interference using a SLN RV $Z$, whose PDF with shifting parameter $\delta$ is given by,

\[
f_Z(z) = \frac{1}{\sigma(z - \delta) \sqrt{2\pi}} \exp \left\{ -\frac{[\ln(z - \delta) - \mu]^2}{2\sigma^2} \right\}, \quad z > \delta.
\]

(14)

The parameters of the density function in (14) are computed as follows [97]

\[
\sigma^2 = \ln \tau, \quad \mu = \frac{1}{2} \ln \left( \frac{\kappa_2}{\tau (\tau - 1)} \right), \quad \text{and} \quad \delta = \kappa_1 - \sqrt{\frac{\kappa_2}{\tau - 1}},
\]

(15)
where $\tau = \left(\nu + \sqrt{\nu^2 - 1}\right)^{1/3} + \left(\nu - \sqrt{\nu^2 - 1}\right)^{1/3} - 1$, $\nu = 1 + \frac{\zeta^2}{2}$ and $\zeta = \frac{\kappa_3}{(\kappa_2)^{3/2}}$.

### 2.7 Distribution of the aggregate CCI

In this section, we apply the above framework to recover the CCI distribution experienced by the tagged receiver in the reference scenario. We consider the setup and network operation of Sections 2.3 and 2.4. Fig. 2 illustrates the reference scenario where a macrocell is underlaid with two tiers of randomly deployed small cells. In our investigations, this scenario corresponds to the worst case wherein small cells transmit with fixed power $p$ in a totally uncoordinated way.

**Proposition 2.** Consider the MPP $\tilde{\Phi}$; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged Macrocell User (MU) in $O_{MU}$ is given by,

$$
\kappa_n(\tilde{\Phi}) = \frac{2\pi \lambda p^n}{n\alpha - 2} \left(R_m^{2-\alpha n} - R_M^{2-\alpha n}\right) E_X^n[0, \infty].
$$

**Proof.** We start from (11) to derive the CF of the aggregate CCI perceived by the tagged MU as

$$
\Psi_Z(\omega) = \exp\left\{2\pi \int_0^\infty \int_{R_m}^{R_M} \left[\exp(j\omega p r^{-\alpha} x) - 1\right] \lambda f_X(x) r dr dx\right\}.
$$

By substituting (17) in (12), and after integrating with respect to $r$, we write the $n^{th}$ cumulant of the aggregate CCI $Z$ as

$$
\kappa_n(\tilde{\Phi}) = \frac{2\pi \lambda p^n}{n\alpha - 2} \left(R_m^{2-\alpha n} - R_M^{2-\alpha n}\right) \int_0^\infty x^n f_X(x) dx.
$$

Recalling that $E_X^n[0, \infty] = \int_0^\infty x^n f_X(x) dx$, we turn our attention to the case where transmissions are affected by the shadowed fading, and so from Section 2.3, $E_X^n[0, \infty] = e^{n\mu + \frac{1}{2}n^2\sigma^2}$ which gives (16). □
2.7.1 Distribution of the aggregate CCI in HetNets with multiple tiers of small cells

We now account for the combined effect of interferers in multiple tiers of small cells. For each tier, \( \Phi_k \) represents the corresponding MPP combining the random position of interferers and radio channel effects. It is also assumed that transmitters of distinct tier are independently and randomly scattered over the network deployment area. In this multi-tier configuration, the cumulants of the aggregate CCI perceived by the tagged MU is computed using the following proposition.

**Proposition 3.** Consider a HetNet deployment scenario with \( K \) and that the MPP \( \Phi_k \) represents the random field of interferers of the \( k^{th} \) tier; then, the \( n^{th} \) cumulant of the aggregate CCI perceived by this tagged MU in \( O_{MU} \) is,

\[
\kappa_n(\Phi_1 + \cdots + \Phi_{K-1}) = \kappa_n(\Phi_1) + \cdots + \kappa_n(\Phi_{K-1}).
\]  

(19)

**Proof.** Since transmitters in each such tier are assumed to be independently scattered as well as randomly marked with shadowed fading, the resulting \( K \) MPPs still Poisson and also independent [78]. Thus, we use the cumulants additivity property to obtain (19).

With Proposition 3, we neglect the interference from co-tier transmitters and account for the contribution of interferers in the other \( K - 1 \) interfering tiers. In Chapter 3, we address scenarios where the tagged receiver is also interfered by co-tier transmissions.

2.8 Performance analysis

With regard to the tagged receiver, the performance of the evaluation scenarios is assessed by means of the outage probability and average channel capacity. The scenarios under study are interference limited and hence the thermal noise is negligible in comparison to the resulting CCI [81, 98]. As explained in the following section, by expressing both the desired signal power and interference component using LN RVs, the density function of the ratio is determined in a straightforward manner [99].
2.8.1 SIR and outage probability

The outage probability is given by Pr $[\Gamma < \gamma_{th}]$ where the RV $\Gamma$ represents the SIR distribution of the tagged receiver, and $\gamma_{th}$ is the corresponding SIR detection threshold.

**Theorem 3.** Let $V_0$ and $V$ be Normal RVs (in logarithmic scale) representing the power received from the desired transmitter and the aggregate CCI at the tagged receiver, respectively. Under the assumption of the shadowed fading with composite Gamma-LN distribution, the SIR at the tagged receiver is

$$\Gamma \sim \text{Normal} \left( \mu_{V_0} - \mu_V, \sigma_{V_0}^2 + \sigma_V^2 \right),$$

(20)

and the outage probability is given by

$$\Pr [\Gamma < \gamma_{th}] = Q \left[ (\mu_{\Gamma} - \gamma_{th}) / \sigma_{\Gamma} \right],$$

(21)

where $\mu_{\Gamma} = \mu_{V_0} - \mu_V$ and $\sigma_{\Gamma} = \sqrt{\sigma_{V_0}^2 + \sigma_V^2}$.

**Proof.** The SIR distribution is given by the quotient of two independent LN RVs, namely, $e^{V_0}$ which is the received power from the target transmitter, and $e^{\hat{V}}$ which is an equivalent LN RV approximating the aggregate CCI at the tagged receiver. Hence, the multiplicative reproductive property of LN RVs is applied to obtain the SIR distribution [90].

2.8.2 Average spectral efficiency

We now evaluate how the two-tier coexistence scenarios perform in terms of the location-dependent average channel capacity of the tagged receiver [100]. By using the analytical framework previously established, and assuming that all users are allocated on the same bandwidth $W$, we initially recover the SIR distribution of the tagged receiver, and then compute the corresponding capacity.

**Theorem 4.** Under the assumption of the shadowed fading channel regime, the average channel capacity of the tagged receiver is given as,

$$\bar{C} \approx W \sum_{k=1}^{K} \frac{\omega_k}{\sqrt{\pi}} \log_2 \left[ 1 + \exp \left( \frac{\eta_k \sqrt{2} \sigma + \mu}{\xi} \right) \right].$$

(22)
Proof. To compute the location-dependent average channel capacity,  
\[
C = W \int_{0}^{\infty} \log_2 (1 + \gamma) f_\Gamma(\gamma) \, d\gamma,
\]
we use the PDF of the SIR with respect to the tagged receiver, which is indicated by \( f_\Gamma(\gamma) \). The Gauss-Hermite quadrature [85] with the substitution \( \eta = (\xi \ln \gamma - \mu) / \sqrt{2}\sigma \) are used to obtain (22).

\[
\int_{-\infty}^{+\infty} e^{-\eta^2} f(\eta) \, d\eta = \sum_{k=1}^{K} \omega_k f(\eta_k) + R_K, \tag{24}
\]
where \( \eta_k \) is the \( k \)th zero of the Hermite polynomial \( H_K(\eta) \) of degree \( K \), \( \omega_k \) is the corresponding weight of the function \( f(\cdot) \) at the \( k \)th abscissa, and \( R_K \) is the remainder value. \( K \) corresponds to the number of sample points which are used to approximate \( f(\eta) \) [85, 94].

2.9 Numerical results

In this section, we use this analytical framework to assess the performance of a HetNet in which the umbrella macrocell is underlaid with two tiers of small cells. We assume that MBSs follow centralized coordination and spectrum allocation such that inter macrocell interference is mitigated, for example, using fractional frequency reuse [15]. In this example, the picocells (femtocells) randomly deployed within an annular observation region which is defined by \( R_m = 25 \) m \((R_m = 5 \) m\) and \( R_M = 250 \) m and operate with a fixed power level of 30 dBm (24 dBm). The radio channel is affected by path loss with exponent \( \alpha = 3 \), LN shadowing with standard deviation \( \sigma = 6 \) dB, and Nakagami-\( m \) fading with shape parameter \( m = 16 \) which corresponds to a Rician channel with parameter \( K = 14.8 \) dB. Picocells (femtocells) are uniformly scattered over the network deployment area following a homogeneous PPP with density \( \lambda = 10^{-4} \) PBS/m\(^2\) \((10^{-4} \) PBS/m\(^2\)). The user of interest is connected to the overlay macrocell which transmits at 43 dBm.

After computing the cumulants of the actual distribution of the aggregate CCI using (12), our next step is to recover the distribution of the interference itself. To do that, we use equivalent LN (13) and SLN (15) RVs. By using the analytical framework of Section 2.5, the distributions of the aggregate CCI is calculated for each one the Evaluation
Fig 3. Illustration of the aggregate CCI at the tagged receiver.

Scenarios (ESs). The Complementary Cumulative Distribution Function (CCDF) of the aggregate CCI at the tagged receiver is shown in Fig. 3 for varying density of interfering nodes and variable standard deviation of the LN shadowing. The proposed framework approximates well the power received by the tagged receiver from a random transmitters within its observation region for varying number of interfering scenarios.

To illustrate our mathematical framework, we consider two evaluation scenarios in which the umbrella macrocell is underlaid either (i) with one tier of picocells, or (ii) two tiers of pico and femtocells, respectively. The former is hereafter indicated by Pico Base Station (PBS), while the latter is identified by PBS + Femto Base Station (FBS). Fig. 4 shows the outage probability experienced by the user of interest for two network configurations, while Fig. 5 depicts the respective average channel capacity. The uncoordinated interference generated by the underlaid tiers of smalls cells severely degrades the performance of the tagged receiver. In the subsequent chapters, we propose and evaluate several interference coordination techniques so as to preserve
Fig 4. Outage probability at the tagged receiver using (21).

the communication of the MU, as well as increase the average area spectral efficiency across tiers.
Fig 5. Average channel capacity of the tagged receiver using (22).
2.10 Summary and final remarks

We formalize a stochastic framework to model and evaluate how multi-tier HetNets perform in various network configurations and radio channel propagation environments. The radio channel is represented by an equivalent LN RV which closely approximates the shadowed fading channel with Nakagami-\(m\) fading and LN shadowing. This framework resorts to stochastic geometry to model random network deployments. Higher order statistics are used to capture the effect of radio channel dynamics. Then, the resulting performance is assessed by means of the outage probability and average channel capacity with respect to the tagged receiver. In the following chapters we extend this framework to incorporate the operational effect of self-organizing solutions which coordinate concurrent transmissions and mitigate interference in more elaborated HetNet deployments.
3 Coordination mechanism for interference mitigation in multi-tier HetNets

We propose and investigate distributed coordination mechanisms for controlling the co-channel interference generated by standalone femtocells in two-tier coexistence scenarios consisting of macrocells underlaid with short-range small cells. The rationale behind employing such mechanism is to opportunistically reuse resources without compromising ongoing transmissions on the overlaid macrocell, while still guaranteeing Quality of Service (QoS) in both tiers. Following the framework of Chapter 2, stochastic geometry is used to model network deployments, while higher-order statistics through the cumulants concept is utilized to characterize the probability distribution of the aggregate interference at the tagged receiver. We consider the shadowed fading channel model incorporating Nakagami-$m$ and Log-Normal (LN) shadowing. In addition, the cumulant-based framework is extended to incorporate the effect of various network algorithms, such as power control and frequency (re)allocation. To evaluate how the proposed solutions perform, we also update the expressions of the outage probability and average channel capacity with respect to the tagged receiver. Results show that the coordination mechanisms substantially improve the performance of overlaid macrocell networks, while benefiting femtocells as well.

3.1 Motivations and related work

In order to have better link quality, higher spatial reuse, and greater utilization of spectrum resources, Heterogeneous Networks (HetNets) consisting of macro and femtocell tiers represent an inexpensive alternative for improving the capacity of cellular systems [19, 71]. Unfortunately, the Co-Channel Interference (CCI) generated by standalone femtocells can severely compromise simultaneous transmissions on both tiers [17, 18]. A motive is that Femto Base Stations (FBSs) are randomly deployed by end-users and lack any predefined infrastructure [19]. To further exacerbate the situation, standalone FBSs are not networked and operate in a totally uncoordinated manner. Thus, it becomes crucial to tackle the interference problem to make two-tier networks viable [17, 20].

Since standalone femtocells operate independently of the overlaid macrocell network,
and of other femtocells, conventional (pre-determined and centralized) interference management solutions like traditional network planning and optimization, are only partially applicable and do not fully exploit variations over time and space [22]. For instance, Authors in [23] identify the decentralized allocation of radio resources as a promising solution, but its implementation still constitutes a technical challenge due to its inherent synchronization and signaling exchange issues. Additionally, Lópes-Pérez et al. in [17] discuss self-configuration and self-optimization concepts as feasible technologies to allow femtocells to sense changes and adapt their parameters accordingly. Along the same lines, Rangan et al. in [18] proposes a distributed solution in which femtocells select their transmit powers so as to control their aggregate interference at the tagged receiver, while still maximizing their achievable throughput. Therein, the interference problem is addressed in a way similar to ours, by studying the combined effect of power control and spectrum (re)partitioning for interference avoidance purposes. In [28], the busy tone concept is used to devise an adaptive technique to mitigate inter-cellular interference in Time Division Duplexing (TDD) networks using Time Division Multiple Access (TDMA)-based schemes. Furthermore, Authors in [29] first identify the CCI as the key challenge for uncoordinated communication in wireless networks, and then use what is called busy burst to establish exclusion regions around receivers in order to avoid conflicting allocations.

With the same reasoning, we also understand that adaptive solutions that change the allocation of radio resources in response to local knowledge of network dynamics and channel variations are more advantageous when used in conjunction with self-organizing femtocells [39]. Indeed, we extend previous results [10, 50, 101], where static guard zones are used to avoid interference, by considering dynamic exclusion regions wherein coordinating femtocells employ distinct strategies and algorithms to autonomously (re)use available resources, while still avoiding interference. In this work, we thus emphasize opportunistic strategies aiming at reducing CCI generated by self-organizing femtocells, as well as attaining adequate utilization of Downlink (DL) radio resources in both tiers. Since the excessive signaling overhead could render any distributed solution unfeasible, another important point is that the proposed mechanism relies solely on the feedback from the receiver of interest, and very little if any explicit coordination is required between coordinating femtocells and the overlaid macrocell tier. In other words, our mechanism does not require the X2 interface to operate [102].
3.2 System model

Until further notice, we use here the network deployment and propagation radio channel models of Sections 2.3 and 2.4, respectively.

3.2.1 Two-tier deployment model

We particularize the deployment of Section 2.4 to assess the DL of two-tier HetNets wherein Macro Base Stations (MBSs) are underlaid with standalone FBSs in closed access mode. FBSs are uniformly scattered over the network deployment area, while a single reference MBS models the overlaid tier. Each femtocell schedules a random Femtocell User (FU) in every transmission interval, whereas the serving macrocell schedules a Macrocell User (MU) during the same time – we assume that only one MU is active per macrocell per transmission interval. Using the formulation of Section 2.4, active femtocells constitute a homogeneous Poisson Point Process (PPP) \( \Phi \) with density \( \lambda \) in \( \mathbb{R}^2 \) so that the number of active femtocells in an arbitrary region \( R \) of area \( A \) is a Poisson Random Variable (RV) with parameter \( \lambda A \) [78]. Additionally, the fading effect is associated with each point of \( \Phi \) as a random mark [32]. The resulting process is defined as the Marked Point Process (MPP) \( \tilde{\Phi} \) on the product space \( \mathbb{R}^2 \times \mathbb{R}^+ \) with intensity \( \lambda f_X(x) \).

\[
\tilde{\Phi} = \{ (\varphi, x) ; \varphi \in \Phi \}, \tag{25}
\]

where the points \( \varphi \) represent femtocells locations and belong to the stationary point process \( \Phi \).

In [22], the 3rd Generation Partnership Project (3GPP) standardization body defines the partial co-channel configuration to accommodate two-tier networks operating in the Frequency Division Duplexing (FDD) mode whereby the available spectrum is divided into clear and shared parts as illustrated in Fig. 6. By this configuration, the macrocell tier can operate on both parts, whereas the standalone femtocells can only use frequency bands in the shared part only. Herein, we introduce a dynamic implementation of the partial co-channel configuration which equally splits the spectrum into two parts, but assigns distinct priorities to intending transmitters in the clear part instead of preventing femtocells from transmitting altogether, so that the macrocell tier has always precedence over FBSs. In other words, to improve the frequency reuse, while still protecting the macrocell tier, the underlaid femtocells can operate in the clear part provided that no MU
Fig 6. The dynamic partial co-channel arrangement. In the priority part, MUs have precedence over femtocells, whereas in the shared part both tiers transmit with equal priority.

is detected in their vicinity. Hereafter, to better reflect the macrocell precedence over femtocells in the clear part, we rename it as the priority part. In our investigations, the tagged MU operates in this priority part and sends a beacon signal when experiencing high interference [103]. We assume that regular reservation busy tones are used to dynamically allocate one resource block in each consecutive frame as long as the tagged receiver successfully received the previous packet and the intended transmitter has still data to send [28, 30]. During the network setup, it is assumed that FBSs access the priority part with probability $\vartheta$, and the shared part with the complementary probability $1 - \vartheta$.

### 3.3 Coordination mechanisms

The self-organization concept is incorporated into our framework by means of coordination procedures which protect the resource allocation of the tagged receiver through reservation busy tones [28], herein our focus is to stochastically model and analyze such mechanisms. It is worth noticing that in the context of the coordination procedures, the “victim user” corresponds to the tagged user (see Definition 3) which triggers the Coordination Mechanisms (CMs) after perceiving the aggregate CCI from interfering femtocells above a predefined threshold.

We address both the transactions which allow femtocells to coordinate and the
underlying procedures that are used to manage interference. A triggering criterion, which is based on the co-channel carrier received signal strength indicator [22], initializes the mechanism: only if sensing the aggregate CCI above a predefined threshold, the victim MU issues an in-band requesting signal to advertise its presence to surrounding FBSs. The victim user momentarily interrupt its reception and transmit the requesting signal that surrounding interferers detect [18, 29]. Note that alternative performance indicators such as the packet loss and packet delay are equally applicable to trigger the coordination mechanism. By detecting the victim user’s request, interfering FBSs adjust their resource allocation to manage the CCI in a distributed manner. In this contribution, we focus on the interactions that follow the triggering event as described next.

3.3.1 Discovery of victim users

The discovery of victim users plays a crucial role in our coordination procedures, so that interfering FBSs coordinate only if they sense the requesting signal of the victim receiver. For a given coordination threshold, the victim MU controls the power level of the requesting signal to keep the coordination range small and restrict the coordination set to the dominant interferers, while still allowing femtocells that are located farther away to reuse the spectrum. Hence, the victim user intentionally transmits the reference signal with low power (0 dBm in our investigations). Recall that the victim user transmits the requesting signal over the same channel used for payload communication which characterizes in-band signaling. With respect to the tagged MU, consider an arbitrary interferer with the instantaneous shadowed fading \( x \) and separation distance \( r \); then, the event that this interfering femtocell detects the requesting signal above the predefined coordination threshold \( \rho_{th} \) is formulated as follows,

\[
r^{-\alpha} x \geq \varrho_{th},
\]

where \( \varrho_{th} = \rho_{th}/p_{req} \) and \( p_{req} \) is the transmit power of the requesting signal.

3.3.2 Definition of coordination regions

Femtocells are independently split into two distinct groups based on the power level of the requesting signal relative to the coordination threshold. Notice that each such group can take distinct and independent actions to reduce the CCI. Any FBS that has already triggered the coordination procedure ignores further requests that may occur while
transactions related to the first one are still ongoing. There is no loss of generality in assuming the MU that carries out the coordination procedures is in the priority part.

The following indicator function,

\[
1 (r^{-\alpha} x) = \begin{cases} 
1, & \text{if } r^{-\alpha} x \geq \varrho_{th} \\
0, & \text{otherwise},
\end{cases}
\]  

(27)
defines the first coordination region, which is denoted by \( R_1 \), and is composed of FBSs that do detect the victim receiver in their vicinity. In accordance with the notation of Section 3.2.1, femtocells within this region constitute a MPP denoted by \( \Phi_{p,1} = \{(\varphi, x) \in \Phi_p \mid r^{-\alpha} x \geq \varrho_{th}\} \). Similarly, femtocells in \( R_2 \), which do not detect the victim MU, form the process \( \Phi_{p,2} = \{(\varphi, x) \in \Phi_p \mid r^{-\alpha} x < \varrho_{th}\} \). Notice that the coordination regions \( R_1 \) and \( R_2 \) are disjoint and statistically independent by construction, therefore it follows immediately from the Superposition theorem [78] that \( \Phi_p = \Phi_{p,1} \cup \Phi_{p,2} \).

To conduct our investigations in a more intelligible way, we separate the coexistence scenarios into two categories which depend on the interaction levels among FBSs, namely, uncoordinated and coordinated. In the former standalone FBSs work independently and do not cooperate, whereas in the latter case, self-organizing FBSs coordinate their operation to better manage the aggregate interference. We now have enough information to properly characterize the interference scenarios encountered throughout this work by defining the Interference Profiles (IPs) in Table 1. Each IP is described by its field of interferers, frequency group and victim user.

In this chapter, we define three uncoordinated evaluation scenarios which depend on the tagged receiver type and the partial co-channel arrangement. When the user of interest connects to the reference MBS, its transmissions are disrupted by the IP1 composed of non-coordinating femtocells within the observation region \( O_{MU} \) and belonging to the point process \( \Phi_p \). Fig. 7 illustrates this reference scenario and shows a realization of the CM with the resulting sets of interfering FBSs within the independent regions \( R_1 \) and \( R_2 \). Similarly, two uncoordinated IPs are established with respect to the tagged femtocell user and the respective frequency part selected for transmission. Since the tagged FU uses the priority part in the IP2, it is interfered by other femtocells of the process \( \Phi_{s} \), while the reference MBS as well. In the IP3, the tagged FU uses the shared part and is interfered by co-channel femtocells in \( \Phi_s \). When the femtocells operate with the CMs, we define four distinct evaluation scenarios. With respect to the tagged MU, the IP4 is composed of the detecting femtocells of the process \( \Phi_{p,1} \), while
### Table 1. Description of the interference profiles.

<table>
<thead>
<tr>
<th>Victim user</th>
<th>Priority Part</th>
<th>Shared Part</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uncoordinated Scenarios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MU</td>
<td>IP1: accounts for the interference that is caused by femtocells within $\Omega_{MU}$ and belonging to the point process $\Phi_p$ with intensity $\theta \lambda f_X(x)$.</td>
<td>Not considered.</td>
</tr>
<tr>
<td>FU</td>
<td>IP2: is jointly caused by other femtocells dwelling in $\Omega_{FU}$ that are in $\Phi_p$, and also by the serving MBS in $\Phi_m$.</td>
<td>IP3: is generated by femtocells within $\Omega_{FU}$ that are members of the point process $\Phi_v$ with density $(1 - \theta) \lambda f_X(x)$.</td>
</tr>
<tr>
<td><strong>Coordinated Scenarios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MU</td>
<td>IP4: is generated by femtocells that after coordinating are in $R_1$ of $\Omega_{MU}$, and then constitute a point process $\Phi_{p,1}$. IP5: is caused by femtocells within $\Omega_{MU}$ that do not detect the victim MU, and constitute the point process $\Phi_{p,2}$.</td>
<td>Not considered.</td>
</tr>
<tr>
<td>FU</td>
<td>IP6: is jointly generated by the femtocells in the process $\Phi_{p,1}$ that remain in the same frequency allocation of the tagged MU after coordinating, and by the serving MBS in $\Phi_m$.</td>
<td>IP7: generated by femtocells that switch to the shared part $\Phi_{p,2}$ after coordination, and by the femtocells originally in that frequency group and in the point process $\Phi_s$.</td>
</tr>
</tbody>
</table>
Fig 7. Illustration of the coordination regions in the priority part. The shaded circles represent interferers inside $R_1$, while the unshaded circles identify interferers within $R_2$. The black shaded square identifies the tagged MU at the origin, and the unshaded one identifies the tagged FU. The dashed circle with the tagged FU at the center depicts the femtocell observation region with radius $R_0$ and located $r_0$ meters away from the tagged MU.

the IP5 is characterized by the femtocells which do not detect the victim user. When evaluating the tagged FU, we consider either the IP6 in which the reference MBS and co-channel femtocells in the priority part interferer with the user of interest, or the IP7 which is generated by the femtocells within the shared part after the coordination.
3.4 Macro-to-femtocell coexistence scenarios

The framework of Section 2.5 is now used to assess the CMs in practical deployments regarding various channels, and also the set of algorithms that femtocells employ to manage their utilization of resources. We investigate the CCI experienced by the victim MU in the priority part for two reasons: first, given the restriction of space, we focus on the most challenging scenario; second, it is reasonable to assume that under high interference MUs migrate to the priority part [22]. Afterward, we analyze the impact of the CMs on the underlaid tier with regard to both parts.

3.4.1 Full interference scenario

The IP1 is equivalent to the reference scenario of Section 2.7; however, we repeat the cumulants derivation for sake of completeness and to maintain our notation consistent. IP1 is an uncoordinated scenario and corresponds to the worst case in our investigations. In what follows, we consider the setup of Section 3.2, the process $\Phi_p$, and that all femtocells transmit with fixed power $p$.

Proposition 4. Consider the IP1; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU in $G_{MU}$ with respect to $\Phi_p$ is given by,

$$\kappa_n(\Phi_p) = \frac{2\pi \theta \lambda p^n}{n\alpha - 2} \left( R_m^{2^{-\alpha n}} - R_M^{2^{-\alpha n}} \right) \mathbb{E}[\chi^n[0, \infty]].$$ (28)

Proof. We start from (11) to derive the Characteristic Function (CF) of the aggregate CCI perceived by the tagged MU as

$$\Psi_{Z_p}(\omega) = \exp \left\{ 2\pi \int_0^R \int_{R_m} \exp \left( j\omega pr^{-\alpha} x \right) - 1 \right\} \theta \lambda f_X(x) r dr dx, \quad (29)$$

where $Z_p = \sum_{(\varphi, x) \in \Phi_p} Y$. After substituting (29) in (12) and integrating with respect to $r$, we write the $n^{th}$ cumulant as

$$\kappa_n(\Phi_p) = \frac{2\pi \theta \lambda p^n}{n\alpha - 2} \left( R_m^{2^{-\alpha n}} - R_M^{2^{-\alpha n}} \right) \int_0^\infty x^n f_X(x) dx. \quad (30)$$

Recalling that $\mathbb{E}[\chi^n[0, \infty]] = \int_0^\infty x^n f_X(x) dx$, we turn our attention to the case where transmissions are affected by the shadowed fading, and so from (13), $\mathbb{E}[\chi^n[0, \infty]] = e^{n\mu + \frac{1}{2} n^2 \sigma^2}$ which gives (28). \hfill \Box
3.4.2 Full interference with power control

This is also an uncoordinated scenario, but now femtocells carry out the standard Power Control (PC) algorithm to fully compensate for channel attenuations of their desired users [104]. We assume that femtocells schedule a random user in every transmission interval, FUs are uniformly distributed within the transmission range of serving cells, and the transmit power is set as a function of the distance between transmitter and receiver pairs, but independently of the interference at that receiver.

Lemma 1. Under the assumptions given above, the $n^{th}$ partial moment of the distribution of the femtocells transmit power is,

$$
E^n_P[p_m, p_M] = \frac{2\beta^n}{d_M^2 - d_m^2} \times \frac{d_M^{2+n\alpha} - d_m^{2+n\alpha}}{2 + n\alpha},
$$

(31)

where $d_M = (p_M / \beta)^{1/\alpha}$ is the radio range of FBSs, such that the received power at the desired user is $\beta$, $p_m$ and $p_M$ are the minimum and maximum FBSs transmit powers, respectively, and $d_m = 1$ m is the minimum distance between an FU and its serving FBS.

Proof. See Appendix 1.

Next, we extend Proposition 4 to incorporate the PC algorithm into our analytical framework.

Proposition 5. Under the assumptions of Proposition 4, and with femtocells using the standard PC algorithm, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU in $O_{MU}$ with respect to $\Phi_p$ becomes

$$
\kappa_n(\Phi_p) = \frac{2\pi \theta \lambda E^n_P[p_m, p_M]}{n\alpha - 2} \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) E^n_X[0, \infty].
$$

(32)

Proof. In this case, the CF assumes the following form

$$
\Psi_{Z_p}(\omega) = \exp \left\{ 2\pi \int_0^\infty \int_{R_m}^{R_M} \int \left[ \exp (jwpr^{-\alpha} x) - 1 \right] \theta \lambda f_X(x) f_p(p) rdr dp dx \right\},
$$

(33)

where $f_p(p)$ is the Probability Density Function (PDF) of the distribution of interfering femtocells transmit power given in Lemma 1. By repeating the mathematical treatment to get (30), and using (31), the resulting $n^{th}$ cumulant in (32) is obtained.
3.4.3 Opportunistic power control with discrete levels

In this coordinated scenario, FBSs do not fully compensate for the channel attenuations of their desired users (as in the previous case), but alternatively adjust their transmit power using a fixed step $\Delta p$ depending on their relative distance to the tagged receiver which is estimated from the requesting signal strength. Coordinating femtocells then reduce the aggregate CCI by simply maintaining their transmit power levels below a predefined upper limit. In line with this model, 3GPP suggests halting the FBSs pilot channel temporarily to avoid interfering with a nearby MU [22].

To simplify our formulation, we use arbitrary power levels, which are not optimized to specific channel and network dynamics, but still provide valuable insight into the achievable gains of such adaptive interference avoidance technique. Femtocells need less signaling exchange to implement this solution, though link quality of already connected FUs may be degraded. From Fig. 7, one can observe that FBSs in $R_1$ are most likely to be the closest interferers, and because of that, they reduce their transmit power to a level $p' < p$. Femtocells in $R_2$, which have not sensed the victim user, maintain their same power level $p$. After including the concepts of coordination regions, and discrete power levels in our mathematical framework, we derive the CF for each one of the coordination regions in the following.

First coordination region

As femtocells in this region do detect the victim user, they decrease their transmit power by a predefined value $\Delta p$ to reduce the interference. In our investigations, we consider fixed $\Delta p$ equal to $-3$ and $-6$ dB.

**Proposition 6.** Consider the IP, then, the $n$th cumulant of the aggregate CCI perceived by the tagged MU in $O_{MU}$ with respect to $\Phi_{p,1}$ has the following form.

$$
\kappa_n(\Phi_{p,1}) = \frac{2\pi \theta \lambda (p')^n}{na - 2} \left( (R_m^{2-an} - R_M^{2-an}) E_X[\varrho_m, \infty] - \varrho_{th}^{-\frac{\alpha}{2}} E_X[\varrho_m, \varrho_M] + R_m^{2-an} E_X[\varrho_m, \varrho_M] \right),
$$

(34)

where $p' = p + \Delta p$, $\varrho_m = \varrho_{th} R_m^\alpha$ and $\varrho_M = \varrho_{th} R_M^\alpha$.

**Proof.** See Appendix 2.

\[ \Box \]
Second coordination region

FBSs in this region do not detect a request from the victim user, and hence keep transmitting at the standard power level ($p$ dBm).

**Proposition 7.** Consider the IP5; then, the $n$th cumulant of the aggregate CCI perceived by the tagged MU in $O_{MU}$ with respect to $\Phi_{p,2}$ has the following form.

$$
\kappa_n(\tilde{\Phi}_{p,2}) = \frac{2\pi \theta \lambda p^n}{n\alpha - 2} \left\{ \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) E_X^2([-\infty, \varrho_m]) + \varrho_0^n - \frac{2}{\pi} \int_{\varrho_m}^{\varrho_M} E_X^2 \right\}.
$$

(35)

**Proof.** See Appendix 3. □

Aggregate CCI at the tagged MU

To compute the total CCI perceived by the tagged MU, we need to account for the combined effect from both interfering regions, $R_1$ and $R_2$ as follows.

**Theorem 5.** Consider the IPs4, 5, and the MPP $\tilde{\Phi}_p$ on the priority part; then, the $n$th cumulant of the aggregate CCI perceived by the tagged MU in $O_{MU}$ is,

$$
\kappa_n(\tilde{\Phi}_{p,1} + \tilde{\Phi}_{p,2}) = \kappa_n(\tilde{\Phi}_{p,1}) + \kappa_n(\tilde{\Phi}_{p,2}).
$$

(36)

**Proof.** Since the coordination regions are independently marked, the resulting process in each region is also independent, and still Poisson [78]. Therefore, we can use the additivity property of cumulants to readily obtain (36). □

3.4.4 Dynamic Exclusion Regions (DERs)

Femtocells dynamically establish exclusion regions based on the power strength of the requesting signal from the victim user and on the coordination threshold. After coordinating, FBSs employ spectrum (re)allocation to reduce interference and vacate the priority part altogether. Next, we characterize the interference from each coordination region and the aggregate interference that both regions together impose on the tagged receiver.
First coordination region

Whenever the victim user is detected, femtocells leave all frequency bands in the priority part currently allocated to that receiver. During the next frame, femtocells in $\mathcal{R}_1$ do not interfere with the tagged receiver in the reserved resource block and, as a result, constitute a dynamic exclusion region around the victim user. Thus, the aggregate interference at the tagged receiver is generated only by the remaining femtocells that do not listen to the requesting signal, i.e., $\kappa_n(\Phi_{p,1}) = 0$.

Second coordination region

Femtocells dwelling in $\mathcal{R}_2$ do not detect the presence of the victim user. The $n^{th}$ cumulant of the interference contribution of femtocells in this region is computed using Proposition 7.

Aggregate CCI at the tagged MU

For the reason that interfering femtocells dwelling in $\mathcal{R}_1$ switch to non-overlapping allocations, the aggregate CCI includes only the interference components from femtocells in $\mathcal{R}_2$. That fact is reflected in the following corollary that disregards the contribution of interferers in $\mathcal{R}_1$ on the computation of the resulting $n^{th}$ cumulant of the aggregate CCI.

Corollary 1. When the DER strategy is used, $\kappa_n(\Phi_{p,1}) = 0$ and, as a result, (36) in Theorem 5 readily simplifies to

$$\kappa_n(\Phi_{p,1} + \Phi_{p,2}) = \kappa_n(\Phi_{p,2}).$$

(37)

3.4.5 DERs with PC

This solution combines the benefits of the PC algorithm with that of the interference avoidance provided by DER. Similar to the default implementation of the DER mechanism, femtocells located inside the exclusion region, which corresponds to the $\mathcal{R}_1$, switch to non-overlapping frequency allocations. Interferers in $\mathcal{R}_2$ which do not detect the victim user’s beacon do not change their transmission scheme and continue to use the standard PC algorithm to compensate for the channel attenuations of their desired
First coordination region

As previously said, this strategy relies on DERs to coordinate, and so femtocells in $R_1$ do not contribute to the aggregate interference, meaning that $\kappa_n(\Phi_{p,1}) = 0$ as well.

Second coordination region

Femtocells in this region do not detect the victim user, but use PC to manage their interference contribution. By combining Propositions 5 and 7, we obtain the next corollary as an immediate result.

**Corollary 2.** Under the assumptions of Propositions 7, and with femtocells employing the standard PC algorithm, the $n$th cumulant of the total interference caused by the femtocells in $R_2$ under a shadowed fading channel regime is given as

$$
\kappa_n(\Phi_{p,2}) = \frac{2\pi \theta \lambda}{\alpha - 2} \left\{ \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) E_{X}[\rho_m, \rho_M] \right\}^{\frac{1}{n}} \left\{ \left[ \int_{-\infty}^{\rho_m} + \rho_{\text{th}}^{\frac{1}{\alpha}} E_{X}[\rho_m, \rho_M] \right] \right\}^{\frac{1}{n}}.
$$

Aggregate CCI at the tagged MU

Similar to the result of Section 3.4.4, Corollary 1 yields the $n$th cumulant of the aggregate CCI perceived by the tagged MU when DER is used in conjunction with the PC algorithm.

3.4.6 Approximating the aggregate CCI at the tagged MU

Figs. 8 and 9 compare the Complementary Cumulative Distribution Function (CCDF) of the aggregate CCI from Monte Carlo simulations with those from the LN and Shifted Log-Normal (SLN) approximations. In this example, the macrocell observation region is determined with $R_m = 1$ m and $R_M = 100$ m. Additionally, we set a high density of interferers with $\lambda = 0.1$ FBS/m² and $\theta = 50\%$ as the probability that femtocells choose the priority part. The radio channel is affected by path loss with exponent $\alpha = 3$, LN shadowing with $\sigma_{dB} = 6$ dB, as well as Nakagami fading with shape factor of $m = 2$ in
Fig. 8. CCDF of the aggregate CCI at the tagged MU under shadowed fading with $\sigma_{dB} = 6\,\text{dB}$ and $m = 2$ (corresponds to a Rician factor $K = 3.8\,\text{dB}$).

Fig. 8 and $m = 16$ shown in Fig. 9. In the full interference case, all FBSs transmit with a fixed power level $p = 20\,\text{dBm}$ and the serving MBS transmit with $43\,\text{dBm}$. In the PC with fixed levels, femtocells dwelling in the first coordination region transmit with $p' = p - 6\,\text{dB}$. When using PC with full compensation, femtocells control their transmit power to fully compensate for the average value of the desired receivers’ large-scale fading.

As can be seen from Figs. 8 and 9, both the LN and SLN approximations match well with the Monte Carlo simulation results, though the proposed approximations work slightly better with lower fading variance ($m = 16$). In the uncoordinated scenarios, a high fading variance ($m = 2$) worsens the interference perceived by the tagged receiver. However, in the coordinated case, a high fading variance causes more femtocells to detect the requesting signal, and consequently reduces the aggregate CCI at the tagged receiver. PC indeed provides gains, because less power is radiated, even in the
uncoordinated scenarios where femtocells do not cooperate with each other. By using PC with fixed power levels, the dominant interferers lower their radiated power, which further reduces the aggregate interference. In the coordinated scenarios, a coordination threshold of $\rho_{th} = -40$ dBm and an MU requesting power of $p_{req} = 0$ dBm are used. Comparing the aggregate CCI of uncoordinated scenarios with that of the coordinated ones, we observe that the two-tier networks under study benefit most from avoiding dominant interferers through the CMs. Like the uncoordinated scenarios, it is also possible to achieve even greater gains by employing PC in conjunction with DERs in the coordinated deployments.

Fig 9. CCDF of the aggregate CCI at the tagged MU under shadowed fading with $\sigma_{dB} = 6$ dB and $m = 16$ (corresponds to a Rician factor $K = 14.8$ dB).
3.5 Femto-to-macrocell coexistence scenarios

To evaluate how the CMs impact the underlaid femtocell tier, we assess the femtocell observation region (see Definition 4), which is illustrated in Fig. 7. The tagged MU still triggers the coordination procedure, but now the performance is evaluated with respect to the tagged FU. We approximate the CCI at the tagged FU by first computing the coordination probability of femtocells inside $O_{FU}$, and thereafter by using that probability to perform independent $p$-thinning [52]. Generally speaking, a thinning operation establishes a deletion rule by which points of an original process $\Phi$ are either deleted or retained thus yielding a thinned point process such that $\Phi_t \subset \Phi$. According to an independent $p$-thinning each point of $\Phi$ is deleted with probability $1 - p$ and its deletion is independent of the locations and possible deletions of any other point of the original point process [80]. As an outcome of the coordination procedure, the serving femtocell of the tagged FU, hereafter denoted by the tagged femtocell, may end up in two distinct spectrum allocations: if in IP6, it remains in the same spectrum allocation of the victim MU, or if in IP7, it switches to a non-overlapping resource allocation in the shared part.

Coordination probability

Since interfering femtocells coordinate with respect to the origin of the coordinate system, where the tagged MU is positioned, it becomes difficult to evaluate a simple expression for the aggregate CCI by translating the reference to the tagged FU. Due to that, we propose a simplification that is motivated by the fact that, in this work, transceivers have antennas with omni directional radiation pattern, and that for computing the interference only the relative distances matter. To use this simplification, we begin with the coordination probability of femtocells inside the observation region, and use that probability to independently thin the MPP.

Lemma 2. Consider the MPP $\Phi$ in $\mathbb{R}^2$ with intensity $\lambda f_X(x)$, and the set of points $\varphi$ uniformly distributed in a bounded region $W \subset \mathbb{R}^2$. Let $C$ be a bounded set, which corresponds to the dynamic coordination region, and the event $r^{-\alpha} x \geq \varphi$ means that a coordinating femtocell is inside $C \subseteq W$, then the probability of this event is,

$$\Pr \{ \varphi \in C \} = \frac{\lambda_2(C \cap W)}{\lambda_2(W)},$$

(39)

where $\lambda_2(\cdot)$ is the Lebesgue measure of the enclosed region [80].
Proof. From geometric probability [105], we know that if the geometric size of the whole domain equals, for instance, $\lambda_2(W)$, the size of a partition of it equals $\lambda_2(C \cap W)$, and a favorable event means hitting $C$, then the probability of this event is defined to be (39). □

The total area of $O_{FU}$ is simply $\pi R_0^2$, where $R_0$ is the radius of this observation region and is equal to the separation distance between the origin (where the tagged MU is located) and the tagged femtocell receiver. The region encompassing the subset of points of the MPP $\tilde{\Phi}$ that are inside $O_{FU}$ and that detect the requesting signal from the victim MU is

$$\lambda_2(C \cap W) = \int_{r_m}^{r_M} \int_{\theta_m}^{\theta_M} \int_{r^a_{\theta h}}^\infty f_X(x) \ r \ dr \ d\theta \ dx = \int_{r_m}^{r_M} \int_{\theta_m}^{\theta_M} [1 - F_X(r^a_{\theta h})] \ r \ dr \ d\theta,$$

(40)

where $r_0$ is the distance from the center of $O_{FU}$ to the tagged MU, $r_m = r_0 \cos \theta - \sqrt{R_0^2 - r_0^2 \sin^2 \theta}$, $r_M = r_0 \cos \theta + \sqrt{R_0^2 - r_0^2 \sin^2 \theta}$, $\theta_m = -\sin^{-1} \left(\frac{R_0}{r_0}\right)$ and $\theta_M = \sin^{-1} \left(\frac{R_0}{r_0}\right)$. Notice that the distance from the origin to a victim FU should be greater than or equal to the radius of the femtocell observation region $R_0$.

CCI approximation

To compute the $n^{th}$ cumulant of the aggregate CCI at the tagged FU, we perform an independent $p$-thinning of $\tilde{\Phi}$ within $O_{FU}$ by which each point $\phi$ has a probability $Pr\{\phi \in C\}$ of suffering deletion [80]. In the sequel, we show how to compute the $n^{th}$ cumulant of the interference perceived by the tagged FU depending on the outcome of the coordination procedures.

1. The tagged femtocell remains in the priority part: the tagged FU is exposed to the IP6, as defined in Table 1. To account for the serving MBS interference, we model a process $\Phi_m$ with a single point, and relate its $n^{th}$ cumulant to the moments of a LN distribution through the logarithm generating function [90]. It is worthy mentioning that for conducting this study, we neglect other-macrocell interference, and only consider the serving MBS interference. The interfering femtocells are assumed in a
process with intensity $\Pr \{ \varphi \in C \} \vartheta \lambda f_X (x)$, hence we can employ Proposition 4 to approximate the corresponding $n^{\text{th}}$ cumulant.

2. The tagged femtocell switches to the shared part: the tagged FU is exposed to the IP7. The process of switching femtocells has intensity $(1 - \Pr \{ \varphi \in C \}) \vartheta \lambda f_X (x)$, and we approximate its $n^{\text{th}}$ cumulant as in the previous case.

Fig. 10 shows the CCDF of the aggregate CCI at the tagged FU for the frequency groups with and without priority. In the priority part, the serving MBS dominates the interference, so that the DL performance of those femtocells located too close to the macrocell antenna is severely deteriorated. When this interference condition occur, femtocells can use interference cancellation techniques, such as successive interference cancellation [69], or smart antennas [106], to suppress macrocell interference. Although the distance to the serving MBS is what matters for those femtocells which remain in the priority part, the separation distance to tagged MU becomes determinant for the femtocells that migrate to the shared frequencies, since more or less interfering femtocells coordinate depending on the distance to the victim user.

### 3.6 Numerical Results

The numerical results in this section are generated with the same assumptions and configuration parameters of Section 3.4. In addition, high densities of interfering FBS are considered to highlight gains that are achieved with the proposed CMs. We are aware that in actual deployments a victim receiver may be subject to less severe interference (lower number of surrounding femtocells), and that such density scales down the performance results.

The analytical framework previously established in Chapter 2 is used to assess the benefits of coordinating standalone femtocells in the two. In our investigations, the performance of the multi-tier coexistence scenarios scenarios is evaluated in terms of the outage probability and average spectral efficiency of the tagged link as defined in Section 2.8. We evaluate the DL performance with regard to the tagged receiver in terms of the outage probability and average channel capacity, when both are functions of the density of interfering femtocells and the desired signal strength.

Fig. 11 shows the outage probability for increasing $d$ – separation distance between the tagged MU receiver and its serving MBS. The QoS, in terms of outage probability, experienced by a tagged MU significantly degrades by considering its serving MBS.
Fig 10. CCDF of the aggregate CCI at the tagged FU on the priority and shared parts under shadowed fading with $\sigma_{\text{dB}} = 6 \text{dB}$ and $m = 16$. When the tagged FU is in the priority part, $d_{\text{MBS-FU}}$ indicates the distance from the serving MBS to that tagged receiver. Similarly, for the shared part, $d_{\text{MU-FU}}$ indicates the distance from the tagged MU to the tagged FU.

Increasingly farther. In the uncoordinated scenarios, PC has a pivotal role in maintaining interference at tractable levels even in high density deployments. For instance, we observe that a tagged user located 50 m away from its serving MBS undergoes an improvement of nearly 40% on its outage probability when comparing the default scenario with that where interfering FBSs employ PC with full compensation. By employing coordination procedures even greater gains are attained, though for the discrete PC strategy one has to still set a optimum transmit power so as to reach a trade-off between system wide spectrum efficiency, and remaining interfering femtocell in the priority part [18]. For example, discrete PC with $-6 \text{dB}$ outperforms the standard PC with full compensation, whereas with $-3 \text{dB}$ reduction it does not. An MU receiver
Fig 11. Outage probability experienced by the tagged MU receiver for increasing separation distances to the serving MBS ($\gamma_{th} = 0$dB).

benefits most from coordinating through DER, because dominant interferers that detected a requesting message switch to non-overlapping resource allocations. DER with PC further improves QoS of the victim user, since the aggregated contribution of femtocells in $\mathcal{R}_2$ is reduced.

Fig. 12 presents the outage probability experienced by the tagged MU, but now regarding an increasing density of interfering femtocells. As a consequence of increasing the density of interferers, the aggregate CCI at the victim user worsens, which confirms the cumulant formulation in Section 2.5 in which the $n^{th}$ cumulant and density of interferers vary directly. On the other hand, owing to the guard zones that are dynamically established around the victim user, DER-based solutions are less sensitive to higher densities of interfering femtocells, and renders greater gains. Fig. 13 shows the location-dependent average channel capacity experienced by the tagged MU receiver. As expected,
independent of the coordination procedure that is employed, the system capacity reduces with increasing density of interfering femtocells. However, by employing the CMs, the tagged user may achieve higher rates owing to the reduction of the aggregate CCI.

For the next set of results, we compare the evident benefits of the CMs for the macrocell tier with their impact on the underlaid femtocell tier using outage probability and average channel capacity at the tagged FU as figures of merit. To perform that comparisons we consider all situations where serving femtocell may end up, namely, in either shared or priority parts. First, we compare the outage probability between tiers in Fig. 14. From this figure, one can see the effect of the FU proximity to the serving MBS, which dominates the interference in the priority part: even though the tagged receiver in the priority part enjoys the benefits of the CM, the closer that FU is to the serving MBS antenna the worst performance it achieves in terms of outage probability.
Fig 13. Average spectral efficiency experienced by the tagged MU distant $d = 10$ m from the serving MBS, and for increasing density of interfering femtocells.

There is a transition point that depends on the density of femtocells where even if the FU is fairly close the macrocell antenna the combined effect of interferers in both tiers does not destroy its transmission. Although, when the density goes too high the outage probability becomes worse than the one that would be experienced if the user is instead in the shared part. In Fig. 15, we show how the average channel capacity varies with increasing values of density of femtocells. When the serving femtocell of the tagged FU remains in the priority part, the FU performance strongly depends on the distance to the serving MBS which dominates the CCI; conversely, femtocells in the shared part use orthogonal spectrum allocation and are not affected by that serving MBS. It is also possible to identify a crossing point where the capacity in the shared part becomes worse than in the priority part. And if the tagged FU is far from the MBS, 50 m for instance, it can take better advantage of the coordination performed by the MU.
Fig 14. Outage probability experienced by a tagged FU for increasing density of interfering femtocells located at the $(d_{FU} = 3 \text{ m}, \gamma_{th} = 0 \text{ dB})$. 
Fig 15. Average channel capacity experienced by the tagged FU for increasing density of interfering femtocells ($d_{FU} = 3$ m, $\gamma_{th} = 0$ dB).
3.7 Summary and final remarks

In this chapter, Coordination Mechanisms are assessed in the DL of macrocell-to-femto coexistence scenarios. We investigate self-organizing deployments where standalone femtocells operate autonomously. The analytical framework of Chapter 2 is used to approximate the aggregate interference at the tagged receiver. Numerical results obtained with Monte Carlo simulations corroborate the accuracy of the proposed framework. We consider two evaluation scenarios, namely, the coordinated and uncoordinated cases, which depend on the coordination levels of femtocells. The coordinated scenarios employing CMs substantially outperform the operation of an uncoordinated deployment of femtocells. Results show that the CMs are indeed a promising strategy to cope with interference and opportunistically reuse radio resources on both tiers. As a result of using such distributed strategies, spectral efficiency is substantially increased in the hierarchical scenarios under study. The concept of extending the range of cooperation by repeating a victim user’s reference message among femtocells that have not detected it initially is considered for further study [32]. We consider a PC algorithm which fully compensates for the desired user’s channel attenuation, but the idea of fractional PC can be also used to further reduce femtocell’s transmit power. In addition, the optimization of the investigated solutions constitute an interesting aspect of the problem under study. For instance, one can combine stochastic geometry with a distributed optimization framework, such as game theory in a way similar to [107] where authors study the mobile association problem through spatial SINR games.
4 Coordinated TDD-underlay for self-organizing femtocells in two-tier coexistence scenarios

The Time Division Duplexing (TDD) concept is investigated as an alternative to underlay short-range small cells on the Uplink of legacy macrocells. To cope with the resulting co-channel interference across tiers, small cells use a distributed mechanism which is based on regular busy tones and requires minimal signaling exchange. The framework of Chapter 2 is adapted to the TDD-underlay scenarios so as to properly capture network dynamics as well as reflect channel variations on the achievable performance. In that regard, we investigate how the fading correlation between data and signaling channels affect the proposed coordination strategies. After recovering the distribution of the co-channel interference using the Log-Normal (LN) approximation, we evaluate the system performance in terms of the outage probability and average channel capacity with respect to the receiver of interest. Monte Carlo simulations are used to corroborate the analytical results as well.

4.1 Motivations and related work

Lately, the concept of small cells emerged as a promising solution to achieve the stringent requirements of the next generation of cellular systems owing to the intrinsic better link quality of short range communications. In this context, small cells constitute an inexpensive alternative to provide better indoor coverage, fairness at cell border and offloading of the overlaid macrocells [19, 71]. However, the unplanned deployment and uncoordinated operation of such small cells leads to harsh Co-Channel Interference (CCI) across tiers [36].

To deal with the interference problem in two-tier Heterogeneous Networks (HetNets), various solutions have been proposed. For instance, López–Pérez et al. first characterize the cross-tier interference problem and then provide a comprehensive summary of candidate solutions wherein the use of dynamic spectrum allocation in conjunction with self-configuration and optimization of femtocells play a determinant role [17]. Similarly, a sub-band scheduling and interference cancellation mechanism is proposed.
in [18] by which the macrocell bandwidth is partitioned and femtocells use load-spillage to control their power across the sub-bands. As an alternative to the typical spectrum partitioning solutions, the TDD-underlay concept is proposed in [30] to take advantage of the natural traffic asymmetry between the Downlink (DL) and Uplink (UL), as well as the natural user spatial diversity. Unfortunately, time-multiplexed links operating in universal frequency reuse are still exposed to the CCI generated by dominant interferers transmitting in an uncoordinated manner. Thus, we augment the TDD-underlay concept by incorporating a distributed mechanism which dynamically coordinates inter-cell time-slot allocation to avoid strong interference from nearby conflicting transmitters [18, 28].

Hence, we propose coordination strategies which consist of a decision criterion (whether to coordinate or not) and an interference avoidance algorithm. According to the adopted policy, short-range femtocells then coordinate their transmissions with co-channel interferers by means of regular busy tones. Additionally, we evaluate the two-tier coexistence scenarios considering a practical radio channel propagation model which includes path loss attenuation, shadowing and multi-path fading.

4.2 System model

To properly characterize the TDD-underlay evaluation scenarios, we update the propagation radio channel and deployment models of Sections 2.3 and 2.4 as presented next.

4.2.1 Network deployment model

Using the framework introduced in Chapter 3, we model two-tier HetNets with small cells underlaid in the UL of a reference Macro Base Station (MBS) through the TDD mode [77]. The locations of femtocells in the underlaid tier (unplanned deployment) constitute the homogeneous Poisson Point Process (PPP) $\Phi$ with density $\lambda$ (Femto Base Stations (FBSs)/m$^2$) in $\mathbb{R}^2$ at any given time instant. From Section 2.4, recall that a Marked Point Process (MPP) on the product space $\mathbb{R}^2 \times \mathbb{R}^+$ with intensity $\lambda f_X(x)$ is obtained by associating the fading effect with each point of $\Phi$ [32] as

$$\tilde{\Phi} = \{(\varphi, x) : \varphi \in \Phi\}, \quad (41)$$
where $\varphi$ represents the point location (an element of the original PPP $\Phi$) and $x$ is the shadowed fading mark attached to it.

4.3 **Coordinated TDD-underlay and network operation**

Macro and femtocells communicate following two transmission schemes: either (i) they operate in the Frequency Division Duplexing (FDD) mode and interfere with each other on the DL and UL frequency bands [22]; or (ii) femtocell transmissions are time multiplexed in the UL of the macrocell tier which works in the FDD mode. In either scheme, serving cells schedule only one of their associated users per frequency-time resource block allocation (transmission interval). Furthermore, simultaneous transmissions are assumed to be synchronized and communicating nodes use omnidirectional antennas. We also consider that all deployments operate under the partial co-channel configuration wherein the available spectrum is split into clear and shared parts [22] which are denoted $B_c$ and $B_s$, respectively. From Fig. 16, the clear part is allocated such that the macrocell traffic requirements are guaranteed, while active femtocells in the shared part compete for the remaining spectrum and are exposed to the uncoordinated CCI. Without loss of generality, we consider that the macrocell operate in the clear part only, and the transmission frames are composed of two subframes. Thereby, the Macrocell User (MU) does not interfere with the tagged receiver but opportunistic strategies which allow femtocells to dynamically coordinate their transmissions so as to reduce the CCI are still needed. Next, we introduce such a strategy which is based on regular busy tones [108] and enables nearby femtocells to coordinate their simultaneous transmissions. In what follows, the scenarios with the coordination mechanisms are denoted as coordinated, whereas the uncoordinated term is used to designate deployments that do not use such collaborative procedures.

4.3.1 **Coordination mechanisms**

Inspired by the concept of reservation busy tones which are used in [29, 108, 109] to effectively mitigate interference, we define a new strategy to control the CCI in the uncoordinated scenarios under study. Following our approach, the tagged Femtocell User (FU) triggers the coordination of surrounding FBSs by issuing an in-band requesting signal to advertise its presence. To achieve that, the victim user momentarily suspend its reception and transmit a requesting signal that surrounding interferers detect [18, 29].
Different from the original busy burst solution whereby detecting transmitters estimate their interference parcel from the received feedback; herein, surrounding femtocells only use the busy tone as an indication of the nearby victim receiver presence. To reduce the likelihood that multiple simultaneous requests trigger the coordination procedure, an interference margin can be introduced in order to reduce the sensitivity of potential interferers to the triggering criterion. The beacon power is maintained low so as to ideally restrict the group of detecting femtocells to the dominant set of interferers. It is also worth noting that any FBS that has already triggered the coordination procedure ignores further requests that may occur while transactions related to the first one are still afoot.

Transmitters use distinct Coordination Mechanisms (CMs) to control their CCI parcel which is inflicted on the macrocell tier and themselves. The tagged femtocell receiver initiates the coordination procedure by experiencing the aggregate CCI above a predefined triggering threshold [103]. The network performance is assessed provided that the triggering criterion has been already satisfied. Potential interferers use the tagged receiver requesting beacon to decide about their participation in the upcoming coordination procedure.

In a distributed manner, interferers employ two distinct decision criteria, namely, CM1 and CM2. By following the former, potential interferers coordinate based solely on the received signal strength of the coordination beacon. With the latter, potential interferers alternatively use the received beacon to estimate their channel gain to the tagged receiver and then use their intending transmit power to compute the interference
they would cause on that receiver similar to the busy burst solution introduced in [28]. In fact, surrounding interferers do not coordinate only by detecting the victim receiver, but use the received beacon to estimate if their interference component is above the coordination threshold $\rho_{th}$. Notice that the channel gain between each such interferer and the tagged receiver is assumed to be perfectly estimated (this is important to guarantee that the coordination criterion of CM2 encompasses the one of CM1).

In the CM1, the event that surrounding interferers detect the coordination beacon of the user of interest $p_b$ above the predefined coordination threshold $\rho_{th}$ is denoted by

$$\Upsilon_1 = \{p_b R^{-\alpha} X \geq \rho_{th}\}. \tag{42}$$

where $R$ represents the distribution of the distance separating interferers from the tagged receiver and $X$ is the distribution of the shadowed fading. From the event in (42), we introduce the following indicator function,

$$1_{\Upsilon_1}(p_b r^{-\alpha} x) = \begin{cases} 1, & \text{if } p_b r^{-\alpha} x \in \Upsilon_1 \\ 0, & \text{otherwise} \end{cases}, \tag{43}$$

which defines the first coordination region denoted by $\mathcal{R}_1$, and is composed of FBSs that do detect the victim receiver in their vicinity (under the assumptions of $\Upsilon_1$). In accordance with the formulation of Section 4.2.1, femtocells within this region constitute a MPP denoted by $\tilde{\Phi}_1 = \{(\varphi, x) \in \tilde{\Phi} | p_b r^{-\alpha} x \geq \rho_{th}\}$. Similarly, femtocells which do not detect the victim MU form a process $\tilde{\Phi}_2 = \{(\varphi, x) \in \tilde{\Phi} | p_b r^{-\alpha} x < \rho_{th}\}$ in $\mathcal{R}_2$. The coordination regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are disjoint and statistically independent by construction, therefore it follows immediately from the Superposition theorem [78] that $\tilde{\Phi} = \tilde{\Phi}_1 \cup \tilde{\Phi}_2$.

Similarly, the detection event of the strategy CM2 is defined as

$$\Upsilon_2 = \{PR^{-\alpha} X \geq \rho_{th}, P_b R^{-\alpha} X \geq \rho_{th}\}. \tag{44}$$

where $P$ yields the potential interferer’s transmit power Random Variable (RV). Finally, we specify the coordination regions for the event $\Upsilon_2$ using (44) as in the previous case. Considering two distinct realizations of the HetNet under study, Fig. 17 compares the random sets of non-coordinating small cells (in $\mathcal{R}_2$) which are obtained using each coordination criteria at a time. It is worth noticing that after coordinating, the tagged receiver is only interfered by active transmitters in $\mathcal{R}_2$, since nodes in $\mathcal{R}_1$ switch to non-conflicting resource allocations.
Fig 17. Illustration of the non-coordinating small cells in $R_2$. Circles identify the random set of interfering User Equipments (UEs) with the CM1, while crosses represent remaining interferers with the CM2. The black shaded square identifies the tagged receiver at the origin.

4.4 Evaluation scenarios

The Evaluation Scenarios (ESs) are identified by the transmission mode (either FDD or TDD) and by the coordination mechanism used by nodes to autonomously avoid interference. In the UL of such scenarios, we initially characterize CCI distribution at the tagged receiver. Campbell’s theorem is employed to determine the Characteristic Function (CF) of the aggregate interference generated by the Poisson field of transmitters. Thereafter, Proposition 1 is used to compute the cumulants of the actual distribution of the interference perceived by the tagged receiver. Then, (13) is used to estimate the parameters of the LN approximation from the respective cumulants.
4.4.1 Uncoordinated scenarios

Co-channel transmitters use $B_s$ without exchanging any information about their intending transmissions. We first address the UL performance of the two-tier HetNet in the FDD configuration, and the TDD-underlay mode afterward.

FDD mode

We name this uncoordinated deployment as ES1. FUs transmit in the FDD mode lacking any sort of coordination with other co-channel transmitters in surrounding femtocells. As a consequence, communicating links are exposed to the highest interference levels. The resulting interference caused by interferers within $\mathcal{O}$ and belonging to the point process $\tilde{\Phi}$ as defined by (41). The respective cumulants are derived by following a mathematical treatment similar to the full interference case in Section 3.4.1 (on page 61). Our next corollary gives the $n$th cumulant of the aggregate CCI at the tagged receiver in the configuration of the ES1.

**Corollary 3.** Consider the ES1; then, the $n$th cumulant of the aggregate CCI perceived by the tagged receiver within $\mathcal{O}$ and with respect to $\tilde{\Phi}$ is given by,

$$
\kappa_n(\tilde{\Phi}) = \frac{2\pi \lambda p^n}{n\alpha - 2} \left( R_m 2^{-\alpha n} - R_M 2^{-\alpha n} \right) E_X^n[0, \infty]. 
$$

TDD-underlay mode

This is an uncoordinated deployment as well, though FUs are now time multiplexed in the UL of the macrocell tier (see Fig. 16). The tagged receiver operates in the first of the two slots which compose the UL frame structure [30]. This scenarios is designated by ES2 and the aggregate interference is caused by transmitters which operate in the first time slot within $\mathcal{O}$ and belong to the point process $\Phi$ with intensity $\vartheta \lambda f_X(x)$. Hence, by observing that transmitters independently access either slot with equal probability ($\vartheta = 50\%$) and that all interfering nodes communicate with the fixed transmit power $p$, we extend the result of (45) to derive the cumulants for the uncoordinated TDD-underlay case as follows.

**Corollary 4.** Under the assumption of the ES2, the $n$th cumulant of the aggregate CCI
perceived by the tagged receiver within $O$ and with respect to $\Phi$ is given by,

$$\kappa_n(\Phi) = \frac{2\pi \vartheta \lambda p^\alpha}{n\alpha - 2} \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) E_X^n[0, \infty].$$  \hfill (46)

4.4.2 Coordinated scenarios

We discuss coordinated scenarios wherein self-organizing femtocells coordinate with nearby potential interferers so as to reduce the overall CCI. When operating in the FDD mode, femtocells use the strategy CM1 only, while in the TDD-underlay configuration the femtocell tier benefits from both criteria as described in Section 4.3.

FDD mode and CM1

In this scenario, which is denoted by ES3, femtocells coordinate using the CM1 given that $\Upsilon_1$ is satisfied. In the FDD mode, UL and DL operate over distinct frequency bands which gives rise to uncorrelated fading between beacon and data (signaling and communication) channels. Data and signaling channels have shadowed fading with composite Gamma and LN distributions (see Section 2.3) and are represented by the independent and identically distributed (i.i.d.) RVs $X$ and $Y$, respectively. The remaining CCI at the tagged receiver is characterized by and its $n^{th}$ cumulant is,

**Proposition 8.** Consider the ES3 with FDD mode and CM1 as described above; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged receiver in $O$ with respect to $\Phi_2$ has the following form,

$$\kappa_n(\Phi_2) = \frac{2\pi \vartheta \lambda p^\alpha}{n\alpha - 2} \left( \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) F_Y(R_m \varrho_{\text{th}}) \right) E_X^n[0, \infty] \left\{ \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) F_Y(R_m \varrho_{\text{th}}) \right\} \\
+ \varrho_{\text{th}}^{\alpha - \frac{2}{n}} E_Y^{\frac{2}{n} - \alpha} \left[ R_m^\alpha \varrho_{\text{th}}, R_M^\alpha \varrho_{\text{th}} \right] \left[ F_Y(R_m^\alpha \varrho_{\text{th}}) - F_Y(R_M^\alpha \varrho_{\text{th}}) \right].$$  \hfill (47)

where $\varrho_{\text{th}} = \frac{\varrho_{\text{th}}}{p^\alpha}$ is the normalized coordination threshold.

**Proof.** See Appendix 4. \hfill $\Box$

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TDD-underlay and CM

We characterize the ES4 where femtocells still follow the coordination criterion \( \Upsilon_1 \) as shown in (42). However, recall that the coordination is carried out through in-band signaling exchange so that data and signaling channels are fully correlated (channel reciprocity) in the TDD configuration. As described in Section 4.3, non-detecting femtocells of \( \tilde{\Phi}_2 \) in \( R_2 \) constitute the remaining set of interferers.

**Proposition 9.** Consider the Interference Profile (IP) \( \text{IP}_A \), then the \( n \)th cumulant of the aggregate CCI perceived by the tagged receiver in \( \tilde{O} \) with respect to \( \tilde{\Phi}_2 \) has the following form,

\[
\kappa_n(\tilde{\Phi}_2) = \frac{2\pi \theta \lambda p^n}{n\alpha - 2} \left\{ (R_m^2 - \alpha n) E_X^\alpha [-\infty, \tilde{\varrho}_m] + \frac{\varrho_m^{n-2}}{\pi} E_X^{\frac{\varrho_m^{n-2}}{\alpha}} [\tilde{\varrho}_m, \tilde{\varrho}_M] - R_M^{2-\alpha n} E_X^n [\tilde{\varrho}_m, \tilde{\varrho}_M] \right\}.
\]  

(48)

**Proof.** See Appendix 5. \( \square \)

TDD-underlay and CM

This scenario is represented by the ES5 wherein transmitters decide whether or not to coordinate based on their interference contribution to the aggregate CCI at the tagged receiver [28]. Conditional \( \Upsilon_2 \), the coordination procedure is carried out in the two following steps. After detecting the tagged receiver, intending transmitters rely on the channel reciprocity and use the received beacon strength to estimate their channel attenuation to that victim receiver. Thereafter, transmitters adapt their link to meet the minimum requirement of their desired receivers and still being able to transmit together with the tagged link. The standard Power Control (PC) algorithm is used by FUs to adjust their power so as to compensate for the desired receiver channel attenuations [104]. We assume that FUs are uniformly distributed within the transmission range of serving cells which schedule a random user in every transmission interval. The transmit power is set as a function of the distance between transmitter and receiver peers, but independently of the interference at the latter. Using our previous result in Lemma 1, the \( n \)th cumulant of the resulting aggregate CCI is written as follows,

**Proposition 10.** Consider the IPS, then the \( n \)th cumulant of the aggregate CCI perceived
by the tagged receiver in $O$ with respect to $\tilde{\Phi}_2$ has the following form,

$$
\kappa_n(\tilde{\Phi}_2) = \frac{2\pi \theta \lambda}{n \alpha} - \frac{2}{2} [T_1 + T_2 - T_3]
$$

where $T_1$, $T_2$ and $T_3$ are derived in Appendix 6.

Proof. See Appendix 6. $\square$

### 4.4.3 Approximating the aggregate CCI

For each one the evaluation scenarios, the analytical framework of Section 2.5 is used to recover the respective aggregate CCI. In this example, the observation region is determined with $R_m = 1 \text{ m}$ and $R_M = 100 \text{ m}$. The density of interferers is $\lambda = 0.05 \text{ FBS/m}^2$. In the TDD-underlay mode, femtocells choose either slot with equal probability $\theta = 50\%$. The radio channel is affected by path loss with exponent $\alpha = 3$, LN shadowing with $\sigma_{\text{dB}} = 6 \text{ dB}$, and Nakagami fading with shape factor either $m = 16$. FUs transmit at fixed power level of $p = 20 \text{ dBm}$. With the standard PC algorithm, transmitters control their power to fully compensate for the average value of the desired receivers’ large-scale fading.

Fig. 18 compares the Complementary Cumulative Distribution Function (CCDF) of the aggregate CCI from Monte Carlo simulations with those from the LN approximation. As can be seen from these figures, the LN approximation matches well with the simulation results. In the coordinated scenarios, a coordination threshold of $\rho_{\text{th}} = -40 \text{ dBm}$ and requesting power of $p_b = 0 \text{ dBm}$ are used. Comparing the aggregate CCI of uncoordinated scenarios with that of the coordinated ones, we observe that the two-tier networks under study benefit most from avoiding dominant interferers through the coordination strategies. In fact, the coordinated cases provide gains due to the formation of dynamic exclusion regions around the tagged receiver.

### 4.5 Numerical results

Using the same assumptions and configuration parameters given above in Section 4.4.3, we assess how the CM perform in terms of outage probability and average channel capacity. In Fig. 19, we use Theorem 3 to plot the outage probability for an increasing density of FUs. The uncoordinated scenarios present the poorest performance, while in the time-multiplexing scenario the tagged receiver experiences slightly better
Fig 18. CCDF of the aggregate CCI at the tagged FBS under shadowed fading with $\sigma_{dB} = 6$ dB and $m = 16$ (corresponds to a Rician factor $K = 14.8$ dB).

performance since the CCI is reduced by a factor that is proportional to the number of time-slots available in a UL frame. By avoiding the dominant interferers through the coordination procedure, the tagged receiver performance significantly improves, whereas the extent of that gain depend on the correlation between beacon and data channels. When nodes coordinate by $\Upsilon_1$, the time-multiplexed scenarios outperform the typical FDD transmission mode. However, by comparing to the coordination criteria in the TDD-underlay case, we observe an interesting trade-off between individual link quality and overall spectrum efficiency. In fact, when nodes coordinate following the criterion given by $\Upsilon_1$, the tagged receiver experiences better link quality since less interferers are active. When nodes follow $\Upsilon_2$, the overall utilization of radio resources is increased since more simultaneous transmissions are allowed at the expense of slightly worse outage figures for the tagged link.

We use Theorem 4 to present the average channel capacity of the tagged link for an
increasing density of interfering nodes in Fig. 20. As previously observed with the outage probability, the coordinated scenarios present better performance overall. The uncoordinated TDD-underlay case presents ergodic channel capacity comparable to the FDD deployment with the CM1. Moreover, by assessing the configuration with TDD-underlay and CM2, it becomes clear that the tagged link experiences lower average channel capacity owing to the higher number of simultaneous transmission which are allowed in this case.
Fig 20. Average channel capacity at the tagged receiver for increasing density of interfering FUs. We consider a Nakagami shape parameter $m = 16$ (that corresponds to the Rice parameters $K = 14.8$ dBm) and network radius of 100 m.

### 4.6 Summary and final remarks

As opposed to the typical spectrum partitioning approaches, the TDD concept is assessed as an alternative to underlay short-range small cells on the Uplink of legacy macrocell deployments. The associated CCI problem which results from the uncoordinated operation and unplanned deployment of small cells is addressed as well. Herein, we consider distributed mechanisms based on the regular busy tones and that rely on minimal signaling exchange to coordinate the underlaid femtocell tier to reduce the co-channel interference. We use an analytical framework based on stochastic geometry and cumulants concept to recover the distribution of the co-channel interference and evaluate the system performance in terms of the outage probability and average spectral efficiency with respect to the tagged link. Our analytical model matches well with
numerical results obtained using Monte Carlo simulations. When compared to the uncoordinated Frequency Division Duplexing deployment, the outage probability of the evaluation scenarios with the coordinated TDD-underlay solution is reduced by nearly 80\%, while the average spectral efficiency increases by approximately 90\% at high loads.
5 Self-organization in Long Term Evolution (LTE) systems and advanced techniques

We assess the self-organizing Heterogeneous Networks (HetNets) composed of legacy macrocells and self-organizing picocells. Aiming to improve coverage, cell-edge throughput and overall system capacity, self-organizing solutions, such as range expansion bias, almost blank subframe and distributed antenna systems are considered. We use stochastic geometry to model network deployments and the cumulants concept to characterize the probability distribution of the received power and aggregate interference at the user of interest. A shadowed fading channel model incorporating log-normal shadowing and Nakagami-$m$ fading is used. The formulation of Section 2.8 is then used to evaluate the performance of such self-organizing networks in terms of their outage probability and average channel capacity with respect to the tagged receiver. Monte Carlo simulations are also used to validate analytical results.

5.1 Motivation and related work

Targeting at upcoming releases, the 3rd Generation Partnership Project (3GPP) standardization body has focused on enhancing the end-user satisfaction and performance of LTE systems by adopting new deployments strategies and concepts such as HetNets and self-organization. In fact, legacy cellular systems with predefined structure and centralized coordination cannot keep up with the stringent requirements of next generation wireless systems, which demand high spectral efficiency and ubiquitous coverage with fairness at cell border. For instance, LTE-Advanced aims at peak data rates up to 1 Gbps which contrasts with current LTE systems which deliver at most 100 Mbps or even Asymmetric Digital Subscriber Line (ADSL) technology over cooper landlines that can transmit at 24 Mbps only. Operators have indeed very few options available to meet such requirements: increase the density of macrocell sites, but that hinges on regulatory studies and approval; upgrade Radio Access Technology (RAT) which takes time and do not fill the capacity gap completely; or expand the radio spectrum resource, but that is definitely a very expensive and lingering alternative.
In this context, heterogeneous deployments which underlay legacy macrocells with low-cost, -power and -complexity small cells emerge as a promising and inexpensive alternative to meet these strict requirements. Future networks indeed benefit from self-organization in several situations, for example, to cope with the uncertainties of random networks wherein moving nodes need to communicate over volatile wireless channels; and to dynamically reconfigure and maintain infrastructureless deployments of small cells with large amount of nodes in which traditional and centralized methods become costly or even unfeasible. In order to tap into the full benefits of large-scale Self-Organizing Networks (SONs), a number of challenges still need to be tackled, including their deployment, operation, automation and maintenance [3, 12].

The design and implementation of self-organizing functionalities in HetNets is a topic of significant interest as evidenced by the number of recent publications [1, 3, 4, 13–16]. For instance, the self-organization concept is used to devise cognitive radio resource management schemes to mitigate cross-tier interference and guarantee users Quality of Service (QoS) in distinct heterogeneous deployments scenarios [21]. More recently, the Range Expansion Bias (REB) concept is discussed within 3GPP as a baseline solution to boost the offloading potential of heterogeneous deployments. In that regard, Authors in [35] investigate the cell range expansion and interference mitigation in heterogeneous networks. Following the same lines, Güvenç instigates the capacity and fairness of heterogeneous networks with range expansion and interference coordination [13]. In [16], Jo et al. use the Stochastic Geometry (SG) framework to assess how the biased cell association procedure performs in heterogeneous networks by means of the outage probability. Moreover, in [110] Hosseini et al. introduce the reversed Time Division Duplexing (TDD) scheme and compare the trade off between deploying massive Multiple-Input Multiple-Output (MIMO) or dense deployment of small cells in a time based network architecture. Authors evaluate the above system in terms of the achievable throughout considering the needed overhead for channel training and estimation of the interference covariance matrix for downlink precoding.

5.2 System model

In preparation for the description of the evaluation scenarios and their performance analysis, we first introduce our system models and configuration parameters.
5.2.1 Network deployment model

In this section, the Downlink (DL) of a heterogeneous networks consisting of an umbrella Macro Base Station (MBS) and an underlaid tier of self-organizing small cells is modeled. We assume that MBSs follow centralized coordination and spectrum allocation such that the inter macrocell interference is mitigated, for example, using fractional frequency reuse [15]. In these scenarios, picocells are uniformly scattered over the network area, while both tiers operate in TDD mode and share the whole spectrum. In every Sub-Frame (SF), each serving Base Station (BS) schedules a single user terminal and interference coordination are implemented in the time-domain. The set of associated user terminals are also uniformly distributed within the transmission range of their serving cells. Nodes communicate using antennas with omni directional radiation pattern and fixed power. The macrocell tier transmits at a maximum power of 46 dBm and picocells use 30 dBm.

From the framer work of Chapter 2, we know that active picocells constitute a homogeneous Poisson Point Process (PPP) \( \Phi \) with density \( \lambda \) in \( \mathbb{R}^2 \). The number of picocells in an arbitrary region \( R \) of area \( A \) is a Poisson Random Variable (RV) with parameter \( \lambda A \) [78]. Additionally, we assume the fading effect as a random mark associated with each point of \( \Phi \) so that the resulting process,

\[
\tilde{\Phi} = \{(\varphi, x) : \varphi \in \Phi\},
\]

corresponds to a Marked Point Process (MPP) on the product space \( \mathbb{R}^2 \times \mathbb{R}^+ \), whose random points \( \varphi \) belong to the stationary point process \( \Phi \) and denote transmitters locations.

5.3 Biased cell association and handover probability

Following the standard handover procedure [111], the tagged Macrocell User (MU) is transferred to the underlaid picocell tier only if the pilot signal of the target Pico Base Station (PBS) is strictly higher than the umbrella MBS\(^1\) as follows,

\[
Y^P > Y^M + \Omega,
\]

where the RV \( Y^P \) refers to the power received from the target PBS, \( Y^M \) yields the power received from the umbrella MBS and \( \Omega \) is the handover hysteresis to avoid the ping-pong effect.\(^1\)

\(^1\)We consider that migrating MUs are not affected by the ping-pong effect and that the predefine triggering time has already elapsed [112].
However, in most circumstances, the umbrella MBS overpowers the underlaid tier which shrinks the coverage of the small cells and compromises the expected gains of spatial and frequency reuse [13, 35]. In such large-scale heterogeneous deployments, transceivers have various communication capabilities and the restrictive nature of the typical handover procedure worsen the load unbalance problem across tiers. To alleviate this problem, 3GPP suggests adding a positive bias \( \Delta \text{REB} \) to the picocells received power so that the rate of MUs handovers to the underlaid tier increases [35] as given next

\[ Y^P + \Delta \text{REB} > Y^M + \Omega. \]  \( (52) \)

Indeed, the REB prompt the macrocell offloading and improves the spectral efficiency by relaxing the standard association criteria used by MUs. Unfortunately, by doing so, MUs within the expanded region of picocells do not actually connect to the strongest BSs and are exposed to high interference levels from the macrocell tier. Fig. 21 illustrates the operation of the REB concept in heterogeneous scenarios composed of an umbrella macrocell and underlaid picocells. The coverage area of the target picocell is artificially increased by the positive bias \( \Delta \text{REB} \) as indicated by the handover criterion in (52). As a result, MUs are offloaded to the picocell tier more often and unburden the umbrella macrocell.

From (52), we derive the probability that the tagged MU within the coverage of the umbrella MBS is offloaded to the target PBS. The Log-Normal (LN) approximation in (13) is used here to recover the distribution of the received power at the tagged receiver.

**Proposition 11.** Consider the observation region \( O \) centered at the tagged receiver and the biased cell association as described above; then, the probability that the tagged receiver connects to the target PBS is given by,

\[ \Pr \left[ Y^M < Y^P + \delta \right] \sim \sum_{k=1}^{K} \frac{\omega_k}{2\sqrt{\pi}} g(\eta_k), \]  \( (53) \)

where \( \delta = \Delta \text{REB} - \Omega \), \( \eta_k \) is the \( k \)th zero of the Hermite polynomial \( H_K(\eta) \) of degree \( K \), \( \omega_k \) is the corresponding weight of the function \( g(\cdot) \) at the \( k \)th abscissa and \( g(\eta) = 1 + \text{Erf} \left( \frac{-\mu_M + \mu_P + \eta \sigma_P}{\sqrt{2} \sigma_M} \right) \).

**Proof.** See Appendix 7. \( \Box \)
Fig 21. Illustration of the REB concept. Circles indicate MUs, the shaded triangle depicts the umbrella MBS and the shaded square depicts the target picocell.

When using the standard procedure in (51), one needs to make the substitution $\delta = -\Omega$ in (53) to derive the handover probability.

Fig. 22 shows the handover probability for distinct network configurations. To generate this plot, we consider that the tagged receiver is randomly placed around the umbrella MBS in an annular region with inner radius equal to 25 m and outer radius of either 250 m or 500 m. Notice that this region actually defines the minimum and maximum distances between the tagged receiver and the umbrella MBS. In addition, the distance from the tagged receiver to its serving picocell varies within the set $\{15, 30, 45\}$ m. When the MU is near to the target picocell, the REB does not affect the
handover probability so significantly. However, the REB effect becomes pronounced when the user is located farther away from the picocell of interest. For sake of illustration, we consider the tagged user located 45 m away from the target picocell and bias of $\Delta REB = 5$ dB. In contrast to the standard approach in (51), the handover probability increases from 52% to 74% (dashed line with up-triangles).

![Handover Probability](image)

**Fig 22. Handover probability as a function of increasing $\Delta REB$ values.**

### 5.4 Network operation

In the coexistence scenarios under study, self-organizing PBSs employ distributed strategies to control their cross-tier interference [3]. We describe these solutions and translate their operation to our mathematical framework so as to identify their impact on the overall system performance two tier HetNets under consideration.
5.4.1 Almost blank sub-frame

By observing Fig. 21 and recalling our results in Fig. 22, it is clear that within the range expanded region the received power of the target picocell with REB is weaker than the umbrella MBS. To cope with this problem in SONs, the Almost Blank Sub-frame (ABS) is considered as a baseline strategy to implement interference control. The ABS is a time-domain resource partitioning strategy whereby MUs in the expanded region of picocells only transmit within the reserved slots. During these reserved slots, the umbrella MBS either implements soft ABS by transmitting with less power; or does not transmit at all what characterizes the zero-power ABS [113]. In Fig. 23, the aggressor MBS does not transmit during the reserved slots so as to protect vulnerable users in the range expanded region of the target picocell. We follow [35] in assuming that only cells with REB are allowed to transmit within the reserved subframes. The downside of the ABS strategy is that non-REB cells undergo capacity loss since reserved subframes are left idle. Different from the typical approach and depending on the network configuration, we consider that ABS applies to both macro and picocells. There is no loss of generality in assuming that the umbrella MBS reserves 1/2 of the frame for the ABS allocation [15] when operating with REB.

![Fig 23. Illustration of the ABS strategy with rate of 1/2. The umbrella MBS leaves every second subframe (reserved slot) empty so that cell edge MUs which were reassigned to the picocell tier experience less interference.](image-url)
5.4.2 **Downlink–high interference indicator**

To avoid the inherent capacity loss of the ABS strategy, we also investigate distributed strategies that rely on the autonomous coordination of nearby BSs. Inspired by the busy tones concept [28] and the interference mitigation technique in [114], we consider the utilization of DL channel measurements for the coordination of interfering picocells. A bitmap indicator which is similar to the Relative Narrowband Transmit Power (RNTP) indicator in Release 8 is used to identify dominant interferers [115]. The tagged MU identifies potential interferers by monitoring their pilot signal and reporting the measurements to its serving BS. After acquiring this measurement report, the serving BS then coordinates by exchanging the interference bitmap with the surrounding picocells via the X2 interface. The updating period of the DL-High Interference Indicator (HII) messages is a configurable parameter which is comparable to the handover procedure [116].

Within our mathematical framework, the tagged receiver uses the interference threshold $\rho_{th}$ to identify potential interferers in its vicinity. Since our network operates in TDD mode, we assume that the channels for measurements and data transmissions are fully correlated. Furthermore, the channel gain between active interferers and the tagged receiver are assumed to be perfectly estimated by the receiver of interest.

Under the above assumptions, the set of dominant interferers is identified by the following indicator function,

$$\mathbb{1}_{\Phi} (p_{br^{-\alpha}x}) = \begin{cases} 1, & \text{if } p_{br^{-\alpha}x} \geq \rho_{th} \\ 0, & \text{otherwise} \end{cases}$$

which defines the first coordination region denoted by $\mathcal{R}_1$, and where $p_{br}$ is the transmit power of the reference signal of surrounding picocells.

In accordance with the formulation of Section 5.2.1, picocells within this region constitute a MPP which is denoted by $\tilde{\Phi}_1 = \{ (\phi, x) \in \tilde{\Phi} | p_{br^{-\alpha}x} \geq \rho_{th} \}$. Similarly, picocells in $\mathcal{R}_2$, which are not detected by the MU of interest, form the process $\Phi_2 = \tilde{\Phi}\setminus\Phi_1$. Recall that the coordination regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are disjoint and statistically independent by construction, therefore it follows immediately from the Superposition theorem [78] that $\Phi = \Phi_1 \cup \Phi_2$. Fig. 24 illustrates the resulting coordination regions by following the criterion in (54). It is worth noticing that after coordinating, the tagged receiver is only interfered by active transmitters in $\mathcal{R}_2$, since nodes in $\mathcal{R}_1$ switch to non-conflicting resource allocation.

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Fig 24. Illustration of the resulting interfering regions by using (54). Unshaded circles identify dominant interferers within $R_1$ whose received power is above the threshold predefined $\rho_{\text{th}}$. Shaded circles identify remaining interferers within $R_2$.

5.4.3 Virtual distributed antenna system

By following the standard REB strategy, a user is served by a BS which does not actually provide the strongest received power which brings about the side effect of exposing the user of interest to high interference levels. With the virtual Distributed Antenna System (DAS) strategy, we intend to further exploit the HII bitmap information so that surrounding picocells coordinate over the X2 interface and establish virtual DAS with random antenna layout [117]. As discussed in [118], such techniques constitute a work item in the 3GPP standardization where large scale remote radio heads are seen as a promising solution to meet the requirements of release 11. Note that with this solution we do not consider Transmit Antenna Selection (TAS) or precoding and the received
the serving DAS coherently add at the user of interest. Instead of considering the REB, the tagged MU uses the aggregate received power from the serving group which belongs to $\tilde{\Phi}_1$. Similar to the coordinated multipoints strategy, this solution requires the user data to be available at all coordinating picocells which in its turn requires extra singling exchange and more elaborated backhaul infrastructure [119]. Our strategy is similar to the maximum ratio transmission [73] by which all antenna elements transmit the same information and the aggregate received power is

$$\sum_{(\varphi, x) \in \tilde{\Phi}_1} Y^p(\varphi, x),$$

where $Y^p(\varphi, x)$ yields the received power from the picocell at $\varphi$ with shadowed fading $x$.

In what follows, we initially extend the analytical framework presented in [36] to evaluate how these distributed strategies perform in heterogeneous networks composed of self-organizing small cells and legacy macrocells. Thereafter, the performance of the SON is evaluated in terms of Signal-to-Interference Ratio (SIR), outage probability and average spectral efficiency as shown in Section 2.8.

### 5.5 Interference model

Under the assumptions of Section 5.2 and with respect to the tagged receiver, we now use our analytical framework to derive the probability distributions of the desired signal and the resulting aggregate Co-Channel Interference (CCI) for each one of the Evaluation Scenarios (ESs) described next. Each ES characterizes a particular network configuration, in which macro and picocells employ distributed strategies to mitigate the cross-tier interference. The following list summarizes the evaluation scenarios under consideration.

- **ES 1**: the tagged MU connects to the umbrella MBS and experiences full interference from the underlaid picocell tier.
- **ES 2**: the tagged MU connects to the target PBS with REB, but no Inter-Cell Interference Coordination (ICIC) strategy, such as ABS, is carried out. As a result, the user of interest which dwells in the Range Expansion (RE) region of the serving picocell is subject to high interference levels from the umbrella macrocell.
- **ES 3**: the tagged MU connects to the target PBS with REB, and the umbrella MBS implements the ABS scheme with rate $1/2$.
- **ES 4**: the tagged MU connects to the target PBS, and the surrounding picocells
coordinate based on the DL-HII bitmap [116], although the umbrella MBS still interferes. This configuration is particularly relevant when the density of small cells nearby the tagged receiver is high, or there are multiple tiers of interfering small cells, such as femtocells.

- ES5: based on the DL-HII bitmap, the strongest picocells coordinate so as to implement a virtual DAS. However, the umbrella MBS and small cells in $R_2$ still interfere with the user of interest. It is assumed that the picocells coordinate through the X2 interface. However, picocells can also coordinate over the air interface for example using the coordination mechanism introduced in [36].

5.5.1 Received power from the umbrella MBS

In this section, we initially derive the Characteristic Function (CF) [32, 85, 90] of the RV which describes the power received from a random transmitter within $O$ and thereafter particularize it to the umbrella MBS case. By considering the communication model of Section 5.2.1, we write the CF of the power received at the tagged MU from a random transmitter within its observation region as follows.

**Proposition 12.** Let $Y = R^{-\alpha} X$ be a RV describing the power received at the tagged receiver from a random transmitter in $O$ with $R$ varying from $R_m$ to $R_M$ and $X$ following the Gamma-LN distribution as described in Section 5.2. Then, the CF of $Y$ is

$$\Psi_Y(\omega) = \frac{2}{R_M^2 - R_m^2} E_X[R(\omega)],$$

(56)

where $R(\omega) = \int_{R_m}^{R_M} \exp(j\omega pr^{-\alpha}) r dr$ and $E_X[\cdot]$ yields the expectation of the enclosed expression over the RV $X$.

**Proof.** See Appendix 8. □

It is worthy noting that (56) is a general formulation which characterizes the distribution of any random transmitter within the reception range of the tagged receiver including the umbrella MBSs and small cells in the underlaid tier.

Thereafter, by taking the $n^{\text{th}}$ derivative of the CF as given in (12), the corresponding cumulant $\kappa_n$ is obtained.

**Proposition 13.** Consider the CF of the power received from a transmitter randomly
deployed within the observation region $O$; then, the $n^{th}$ cumulant of $Y$ is given by

$$\kappa_n = \frac{1}{j^n} \sum_{k=0}^{n} g^{(k)}(\beta_0) \cdot B_{n,k} [\beta_1, \beta_2, \ldots, \beta_{(n-k+1)}],$$  \tag{57}$$

where $g(u) = \ln(u)$. $B_{n,k} [\beta_1, \beta_2, \ldots, \beta_{(n-k+1)}]$ is the partial Bell polynomial \[120\] and $\beta_n = j^n p^n \times \frac{R_m^{2-\alpha n} - R_M^{2-\alpha n}}{n\alpha - 2} E_X [x^n]$.

Proof. See Appendix 9. \qed

5.5.2 Aggregate CCI from the underlaid tier of small cells

This scenario represents our default configuration in which the umbrella MBS serves the tagged receiver, whereas the underlaid picocell tier is the only source of interference. A careful observation reveals that this scenario is equivalent to the Full Interference scenario described in Section 3.4.1. Therefore, we can apply Proposition 4 to derive the $n^{th}$ cumulant of the distribution of the aggregate CCI at the tagged receiver with respect to the MPP $\bar{\Phi}$ \[32, 36\] as given next.

Corollary 5. Consider the ES1; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU within $O$ and with respect to $\bar{\Phi}$ is given by,

$$\kappa_n(\bar{\Phi}) = \frac{2\pi \lambda p^n}{n\alpha - 2} \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) E_X^n [0, \infty].$$ \tag{58}$$

The aggregate CCI from the underlaid tier of picocells is computed with respect to a limited region of the total field of interfering nodes (from $R_m$ to $R_M$). To account for the neglected interference parcel beyond $R_M$, one needs to change in (32) the upper limit of integration with respect to $r$ until $\infty$. For instance, considering $\alpha = 3$, $R_m = 5$ m and $R_M = 250$ m, the aggregate interference from the region beyond $R_M = 250$ m (towards $\infty$) represents only 2% of the aggregate interference, whereas for an observation region within $R_m = 25$ m and $R_M = 500$ m the neglected region contributes with 5% of the total interference.

5.5.3 Aggregate CCI from multiple tiers

the additivity property of cumulants is used to compute the aggregate CCI in HetNets with multiple tiers \[121\]. In order to apply this property, we observe that the interference
components from distinct tiers are assumed to be independent. Then, Proposition 3 (in page 45) is applied as follows.

**Corollary 6.** Consider the two tier deployment scenario where an umbrella MBS is underlaid with self-organizing small cells; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU in $O$ is,

$$\kappa_n = \kappa_n^M + \kappa_n^P.$$  

(59)

### 5.5.4 Aggregate CCI with inter-cell interference coordination

As shown in Section 5.4.2, picocells use the DL-HII bitmap to self-organize into two coordination regions $R_1$ and $R_2$. Herein, we derive the cumulants of the aggregate CCI generated by each such region with respect to the tagged receiver. Small cells that are detected by the tagged receiver within $R_1$ decrease their transmit power by a predefined value, i.e., $p' = p + \Delta p$ so as to reduce their interference towards the user of interest. In the following we use Proposition 6 to compute the cumulants of the dominant interfering picocells belonging to MPP $\Phi_1$ are identified.

**Corollary 7.** Consider the network operation of Section 5.4.2; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU in $O$ with respect to $\Phi_1$ is written as,

$$\kappa_n(\Phi_1) = \frac{2\pi \lambda (p')^n}{n \alpha - 2} \left\{ (R_m^2 - R_M^2 - R_m^{2-\alpha n}) E_X^n [\varrho_M, \varrho] - \frac{n-2}{2} E_X^n [\varrho_m, \varrho] + R_m^2 - R_m^{2-\alpha n} E_X^n [\varrho_m, \varrho_M] \right\}. \quad (60)$$

During the coordination mechanism, the tagged receiver does not detect the picocells within $R_2$ which contribute to the aggregate interference with transmit power $p$ dBm. Therefore, the distribution of the remaining interference is similar to the case 7 and the corresponding the respective cumulants are characterized using the following formulation.

**Corollary 8.** Consider the network operation of Section 5.4.2; then, the $n^{th}$ cumulant of the aggregate CCI perceived by the tagged MU in $O$ with respect to $\Phi_2$ has the following
5.6 Numerical results

By using the analytical framework of Section 2.5, the distributions of the received power, aggregate CCI and the resulting SIR are calculated for each one the ESs. The outage probability and average channel capacity are also used to evaluate how the system performs with biased cell association and interference coordination techniques.

Fig. 25 compares the Cumulative Distribution Function (CDF) of the picocell received power at the tagged receiver \( Y_p \) from Monte Carlo simulations with those obtained using the LN approximation. In this example, the annular observation region is defined by \( R_m = 5 \) m and \( R_M = 75 \) m and picocells operate with a fixed power level of 30 dBm. The radio channel is affected by path loss with exponent \( \alpha = 3 \), LN shadowing with standard deviation \( \sigma = 6, 8, 10 \) and \( 12 \) dB, and Nakagami fading with shape parameter \( m = 16 \) which corresponds to a Rician channel with parameter \( K = 14.8 \) dB (see relation (6)). The proposed framework approximates well the power received by the tagged receiver from a random picocell within its observation region for varying number of interfering scenarios.

Similarly, Fig. 26 shows the CDF of the distribution of the power received from the umbrella macrocell at the tagged receiver \( Y_M \). The umbrella MBS transmits at 43 dBm. As can be seen, our LN model matches well the simulation results in the macrocell configuration. In addition, the LN approximation is tighter for larger \( \sigma \) (standard deviation), when the resulting shadowing dominates the variation of the received power distribution.

Fig. 27 compares the Complementary Cumulative Distribution Function (CCDF) of the aggregate CCI from Monte Carlo simulations with those from the proposed LN approximation. Our approximation matches well with the simulation results for the evaluation scenarios under study. An annular observation region with \( R_m = 25 \) m and \( R_M = 250 \) m is considered. Picocells operate with a fixed power level of 30 dBm and constitute a Poisson field of interferers with intensity \( \lambda = 10^{-5} \) PBS/m\(^2\) (about...
2 picocells on average). By comparing the scenario where PBSs are the only source of interference with that in which PBSs and the umbrella MBS jointly interferer, it is possible to identify the harmful impact of the macrocell component at the tagged receiver (about 12 dB). With that effect in mind, the benefits of using ABS to avoid the interference from the umbrella macrocell altogether becomes evident. Afterwards, we increase the density of picocells to $\lambda = 10^{-4}$ PBS/m$^2$ (about 20 cells on average) and allow surrounding picocells to use the downlink HII in order to coordinate with the serving BS. As a result, the interference experienced by the tagged receiver is further reduced. An interesting observation is that depending on the density of picocells and their relative distance to the tagged receiver, the underlay picocell tier dominates the aggregate interference.

![CDF of the power received by the tagged receiver from a random pico-cell within its observation region under shadowed fading with $\sigma = \{6, 8, 10, 12\}$ dB and shape parameter $m = 16$ (corresponds to a Rician factor $K = 14.8$ dB). PBSs transmit with constant power equal to 30 dBm.](image)

**Fig 25.** CDF of the power received by the tagged receiver from a random pico-cell within its observation region under shadowed fading with $\sigma = \{6, 8, 10, 12\}$ dB and shape parameter $m = 16$ (corresponds to a Rician factor $K = 14.8$ dB). PBSs transmit with constant power equal to 30 dBm.
Fig. 28 shows the outage probability for distinct evaluation scenarios and increasing density of PBSs. The expressions (20) and (21) in Theorem 3 are used to generate the numerical results shown in this figure. By comparing the outage probability of the tagged receiver in ES2 and ES3, one observes a performance improvement by avoiding the interference from the umbrella MBSs, which is the dominant interferer. However, the underlaid tier of picocells dominates the resulting interference as the density of picocells increases – The ABS gains are not so evident for a density $\lambda$ higher than $7 \times 10^{-5}$ PBS/m$^2$. Hence, the coordination mechanisms are considered in this work to further reduce the interference levels at the tagged receiver. When interfering picocells coordinate their transmissions by fulfilling the criterion $\tilde{\Phi}(p_b r^{-\alpha})$ in scenarios ES4 and ES5, the network operation outperforms the standard configuration wherein BSs do not coordinate.

Fig 26. CDF of the power received by the tagged receiver from a random transmitter within its observation region under shadowed fading with $\sigma = \{6, 8, 10, 12\}$ dB and shape parameter $m = 16$ (corresponds to a Rician factor $K = 14.8$ dB). The umbrella MBSs transmit with constant power equal to 43 dBm, respectively.
not self-organize. In fact, when nodes coordinate following the criterion given in (54),
the tagged receiver experiences much better link quality since less interferers are active
in its reserved subframes.

In Fig. 29, we use (22) (see Theorem 4) to compute the average channel capacity
of the tagged link for an increasing density of interfering picocells. The performance
of the tagged receiver is severely degraded by the umbrella MBS which corroborates
our observation in the previous outage probability results. By employing interference
avoidance techniques in scenarios ES4 and ES5, the channel capacity of the tagged
receiver link improves significantly. In addition, the tagged receiver benefits mostly from
the coordination of surrounding picocells by means of the DL–HII which corresponds
to ES4 and ES5. For instance, the tagged receiver attains at most 1bps/Hz in ES3, while
an average channel capacity of about 2.5bps/Hz is achieved in ES5.
Fig 28. Outage probability at the tagged receiver for increasing density of interfering picocells.
Fig 29. Average channel capacity at the tagged receiver for increasing density of interfering picocells.
5.7 Summary and final remarks

In this paper, we investigate the problem of co-channel interference in heterogeneous networks composed of self-organizing small cells and legacy macrocells. An analytical framework which resorts to stochastic geometry and higher-order statistics through the concept of cumulants is introduced in order to characterize network dynamics and channel variations. We use this framework to recover the distribution of the CCI and to evaluate the system performance in terms of outage probability and average spectral efficiency of the tagged link. For the scenarios under study, results show that our analytical model matches well with numerical results obtained using Monte Carlo simulations. Aiming to reduce the co-channel interference generated at the underlaid tier, picocells coordinate their transmissions using the DL-HII incurring minimum overhead. Finally, by employing the concept of virtual DASs, the user of interest not only benefits from reduced interference, but also from the maximum ratio transmission among the serving picocells. We also observed that by simply using almost blank subframes the aggregate interference at the tagged receiver is reduced by about 12dB. Although more elaborated interference control techniques such as, downlink bitmap and distributed antennas systems become needed, when the density of picocells in the underlaid tier gets high.
6 Conclusions and final remarks

In this thesis, the feasibility of self-organizing Heterogeneous Networks (HetNets) under various network configurations and radio channel regimes has been considered. In view of our observations, HetNets constitute an inexpensive alternative to reduce Capital Expenditure (CAPEX), while still meeting the stringent requirements of upcoming systems, improving cell border user experience, increasing spectral efficiency and filling in coverage holes. Equally important, self-organizing solutions reduce Operating Expenditure (OPEX) by limiting human intervention and allowing small cells to autonomously coordinate their transmissions across tiers and control the resulting inter-tier Co-Channel Interference (CCI) in a distributed way.

It is also observed that the traditional evaluation methods based solely on computer simulations with Base Stations (BSs) scattered in a regular structure over hexagonal grid does not fully capture the dynamics of Self-Organizing Networks (SONs) [122]. Moreover, the unplanned deployment and uncoordinated operation of small cells make the performance evaluation of such HetNets a challenge undertaking. Thus, we initially introduced an analytical framework which resorts to the stochastic geometry to model random network deployments and capture the spatial interaction between communicating nodes. It is worth noting that this framework is used to capture the interworking over distinct tiers and assess their performance as well. Using the concept of Marked Point Process (MPP), multidimensional random patterns which combine spatial disposition of nodes with radio channel impairments are generated. A shadowed fading channel model which combines Nakagami-$m$ fading with Log-Normal (LN) shadowing was used to model channel impairments in our studies. Using this framework we are able to model the operation of network algorithms and capture their impact on the performance of nodes communicating over the air interface. In that regard, various network algorithms were incorporated into the model such as power control, frequency partitioning, antennas structures with random layout; as well as the effect of the correlation between data and control channels on the Coordination Mechanisms (CMs). We then characterized the scenarios under study in terms of the received power of the desired transmitter, resulting aggregate CCI, outage probability and average channel capacity conditional on the tagged receiver.

Thereafter, the performance of the HetNets with distinct network configurations and
transmission modes were investigated. We initially established the CMs which allow small cells to coordinate their transmissions either over the air interface (in Chapters 3 and 4) or through a wired backhaul (in Chapter 5). In a totally distributed way, the CMs enable nearby small cells to become aware of each other and dynamically coordinate their transmissions so as to reduce the CCI. After incorporating the operation of standard network algorithms into our framework, we introduced new self organizing solutions which significantly improve the performance of two-tier networks, namely, opportunistic Power Control (PC) with discrete power levels and Dynamic Exclusion Region (DER). Results showed that the CMs are indeed a promising strategy to cope with interference and opportunistically reuse radio resources on both tiers. Hence, the spectral efficiency is substantially increased in the heterogeneous scenarios under study.

Subsequently, the feasibility of using the Time Division Duplexing (TDD) mode to underlay tiers of small cells on the Uplink (UL) of legacy macrocells was investigated. In this study, two coordination criteria were considered: a small cell coordinates if (Υ₁) the received power level of the beacon from the tagged receiver is above the coordination threshold; or (Υ₂) conditional on the previous criterion, the interference contribution of a given BS on the tagged receiver needs to be above that threshold as well. When compared to the uncoordinated Frequency Division Duplexing deployment, results show that the outage probability of the evaluation scenarios with the coordinated TDD-underlay solution is reduced by nearly 80 %, while the average spectral efficiency increases by approximately 90 % at high loads.

Thereafter, advanced solutions were considered to enable SONs to offload the umbrella macrocell and to improve the spectral efficiency on both tiers were considered. The biased cell association was initially assessed through the corresponding handover probability. To cope with the resulting interference, standard Inter-Cell Interference Coordination (ICIC) network algorithms were modeled and their performance was assessed: Almost Blank Sub-frame (ABS), binary bitmap indicator, and spectrum repartitioning. We then introduced the Downlink (DL)-High Interference Indicator (HII) which was implemented through a soft map (reflecting the interferers power) and virtual Distributed Antenna System (DAS) solution. It was observed that by simply using almost blank subframes the aggregate interference at the tagged receiver is reduced by about 12 dB. Although more elaborated interference control techniques such as, downlink bitmap and distributed antennas systems become needed, when the density of picocells in the underlaid tier gets high.
6.1 Future work

An interesting alternative to further develop the CMs study is to incorporate the concept of the cooperative range expansion [32] by allowing small cells to forward the victim user’s coordination request among other cells that have not detected it initially. In addition, the optimization of the CM parameters in order to improve the achievable performance also constitute a compelling aspect of the problem under study. For instance, by combining stochastic framework with distributed optimization methods, such as game theory it becomes possible to investigate the benefits of strategic decision making. As an example, authors study the mobile association problem through spatial Signal-to-Interference plus Noise Ratio (SINR) games in [107].

In our investigations we considered fully loaded heterogeneous systems where small cells always schedule a random user within their communication range in every transmission interval. However, partially loaded systems where the number of scheduled users is a Random Variable (RV) which also depend on the traffic distribution create new opportunities to study heterogeneous systems with variable service class requirements and distinct link adaptation algorithms [20].

With regard to the TDD-underlay concept, we initially developed interference avoidance techniques using the distinct CM criteria. Using that criteria, more elaborated transmission strategies, for example, cooperative diversity techniques are also applicable so that nearby small cells which are tagged as potential interferers can also cooperate through distributed antenna structures to improve overall network capacity. In that respect, distributed antennas systems were evaluated on the Frequency Division Duplexing (FDD) mode and significant results were obtained. However, only maximum transmission ratio is employed in the transmitter side. On the receiver side, diversity combining techniques – such as selection, equal gain or maximal ratio combining – which are triggered by victim users and rely on the CMs introduced here are possible. Moreover, precoding which exploit multiple antenna systems on both transmitter and receiver ends are applicable as well.

Another plausible extension is to take into effect the temporal and spatial correlations on the outage probability and achievable capacity so that distinct dispositions of communicating nodes over time and space effect the overall network performance. Hence, the analysis of the aggregate interference (incorporating multipath fading and correlated LN RVs) generated by the multi-tier HetNet becomes a more evolved problem. In that regard, geometry-inclusive models which combine the stochastic
profile of radio channels with the variations due to the geometry of the problem (spatial distribution of points) are particularly interesting. Moreover, both directional antennas and precoding techniques for multi-stream transmissions constitute appealing directions for future development as well. In addition, the interference modeling for a finite number of interferers, non-homogeneous Poisson fields of interferers, and 3 dimensional space become relevant within specific contexts which establish reasonable grounds to investigate more practical constraints on the deployment of self-organizing HetNets.
References


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Appendix 1 Proof of Lemma 1

In our studies the PC fully compensates for the desired channel attenuations (channel inversion), so \( P = \beta R^\alpha \), where \( P \) and \( R \) are RVs representing the Femto Base Stations (FBSs) transmit power and the distance to a random user within range, respectively. The density function of \( R \) is \( f_R(r) = \frac{2r}{(r^2_M - r^2_m)} \), where the radii \( r_m \) and \( r_M \) define an annular region centered at a serving femtocell. Thereby, after using the standard procedure of the change of variates in probability theory, we have \( f_P(p) = \frac{1}{\alpha \beta^{1/\alpha}} p^{-1+1/\alpha} f_R \left[ \frac{p}{\beta} \right]^{1/\alpha} \), which yields the distribution of FBSs transmit power. After computing \( E \left[ p_m, p_M \right] = \int_{p_m}^{p_M} p^n f_P(p) \, dp \), we obtain the \( n^{th} \) partial moment of the distribution of the femtocells transmit power in (31).
Appendix 2 Proof of Proposition 6

By using the indicator function in (27), we write the Characteristic Function (CF) of the aggregate CCI for the $R_{1}$ as,

$$
\Psi_{Z_{n,1}}(\omega) = \exp \left\{ 2\pi \int_{0}^{\infty} \int_{R_{m}} \left[ \exp(jwp'r^{-\alpha}x) - 1 \right] \vartheta \lambda f_{X}(x) \mathbb{1}(r^{-\alpha}x \geq \varrho_{th}) dr \, dx \right\}.
$$

(62)

And from (12) the $n^{th}$ cumulant is,

$$
\kappa_{n} = 2\pi \vartheta \lambda \int_{0}^{\infty} \min_{R_{M}, \varrho_{M}} \frac{(p')^{n}r^{-n\alpha}x^{n}f_{X}(x)dr \, dx}{\varrho_{M} R_{m}}.
$$

(63)

where $\varrho_{m} = \varrho_{th} R_{m}^{2\alpha}$ and $\varrho_{M} = \varrho_{th} R_{M}^{2\alpha}$. By integrating (63) with respect to $r$, we obtain

$$
\kappa_{n} = \frac{2\pi \vartheta \lambda}{n\alpha - 2} \left\{ \left( R_{m}^{2\alpha n} - R_{M}^{2\alpha n} \right) \int_{\varrho_{M}}^{\infty} x^{n}f_{X}(x)dx + \int_{\varrho_{m}}^{\varrho_{M}} \frac{\varrho_{M}^{n}}{\varrho_{m}^{n}} \left[ x^{n} R_{m}^{-n\alpha} - x^{\frac{n\alpha}{2}} \varrho_{th} \right] f_{X}(x)dx \right\}.
$$

(64)

Finally, we compute the partial moments of the approximating LN RV $X$ by repeatedly applying Definition 5, and by using the change of variable $X = e^{\mu+\sigma Z}$, where $Z \sim \text{Normal}(0, 1)$, along with the substitutions $\tilde{\varrho}_{M} = \frac{ln \varrho_{M} - \mu}{\sigma}$ and $\tilde{\varrho}_{m} = \frac{ln \varrho_{m} - \mu}{\sigma}$.

$$
\text{E}_{X} \left[ \varrho_{M}, \infty, n \right] = e^{n\mu + \frac{n^{2}\alpha^{2}}{2}} Q \left[ \tilde{\varrho}_{M} - n\sigma \right],
$$

(65)

$$
\text{E}_{X} \left[ \varrho_{m}, \varrho_{M}, 2 \right] = e^{\frac{2\mu + \frac{2n\alpha^{2}}{2}}{2\sigma}} \left( Q \left[ \tilde{\varrho}_{m} - 2\sigma \right] - Q \left[ \tilde{\varrho}_{M} - 2\sigma \right] \right),
$$

(66)

$$
\text{E}_{X} \left[ \varrho_{m}, \varrho_{M}, n \right] = e^{n\mu + \frac{n^{2}\alpha^{2}}{2}} \left( Q \left[ \tilde{\varrho}_{m} - n\sigma \right] - Q \left[ \tilde{\varrho}_{M} - n\sigma \right] \right),
$$

(67)

where $Q[.] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{v^{2}}{2}} dv$. And by replacing the above expressions in (63), (34) results.
Appendix 3 Proof of Proposition 7

For computing the $n^{th}$ cumulant of the aggregate interference for interfering femtocells in $R_2$, we, once again, begin formulating the corresponding CF as

$$
\Psi_{Z_{p,2}}(\omega) = \exp \left\{ 2\pi \int_0^\infty \int_{R_m}^R \left[ \exp \left( j\omega p r^{-\alpha} x \right) - 1 \right] \theta \lambda f_X(x) \mathbb{1}(r^{-\alpha} x < \varrho_{th}) r dr dx \right\}.
$$

(68)

The $n^{th}$ cumulant is then given by

$$
\kappa_n = 2\pi \theta \lambda \int_0^\infty \int_{R_m}^R p^n r^{1-n\alpha} x^n f_X(x) dr dx
$$

$$
= 2\pi \theta \lambda \int_0^{\varrho_m} \int_{R_m}^R p^n r^{1-n\alpha} x^n f_X(x) dr dx + \int_{\varrho_m}^{\varrho_M} \int_0^{R_m} p^n r^{1-n\alpha} x^n f_X(x) dr dx.
$$

(69)

Similar to the derivation of (64), we first integrate with respect to $r$ and obtain

$$
\kappa_n = \frac{2\pi \theta \lambda}{n\alpha - 2} \left\{ (R_m^{2-\alpha n} - R_M^{2-\alpha n}) \int_{-\infty}^{\varrho_m} x^n f_X(x) dx + \int_{\varrho_m}^{\varrho_M} \left[ \frac{2}{x^{\alpha}} \frac{\varrho_{th}^{n-2}}{\varrho_{th}^{n-2}} - x^n R_M^{2-n\alpha} \right] f_X(x) dx \right\}.
$$

(70)

And after computing the following partial moment, we obtain (35).

$$
E_X^n \left[ -\infty, \varrho_m \right] = e^{n\mu + \frac{n\sigma^2}{2}} \left( 1 - Q \left[ \frac{\varrho_m - n\sigma}{\sigma} \right] \right).
$$

(71)
Appendix 4 Proof of Proposition 8

Considering interfering Femtocell Users (FUs) in $R_2$, the $n$th cumulant of the aggregate interference is determined from the CF as follows

$$
\Psi_{Z_2}(\omega) = \exp\left\{ 2 \pi \int_0^\infty \int_0^{R_M} \int_{\min(r,\frac{\rho_{th}}{\rho_{th}})} \left[ \exp(j\omega p_{th} r^{-\alpha} x) - 1 \right] \lambda f_X(x) f_Y(y) 1_{1-Y_1}(r^{-\alpha} y) r dr dy dx \right\},
$$

(72)

where $Z_2 = \sum_{(x,y) \in \tilde{\Phi}_2} Y$ and $Y_1 = 1 - Y_1$ corresponds to the event of not detecting a victim receiver.

The $n$th cumulant is then given by

$$
\kappa_n = 2 \pi \lambda \int_0^\infty \int_0^{R_M} p_n r^{1-n\alpha} x^n f_X(x) f_Y(y) dr dy\,
$$

$$
= 2 \pi \lambda p_n \int_0^\infty f_X(x) dx \left[ \int_0^{\varrho_m} \int_{R_m} r^{1-n\alpha} f_Y(y) dr dy + \int_{\varrho_m}^{\varrho_M} \int_{\varrho_m}^{R_M} r^{1-n\alpha} f_Y(y) dr dy \right].
$$

(73)

where $\varrho_{th} = \rho_{th}/p_{th}$, $\varrho_m = \rho_{th} R_m$, and $\varrho_M = \rho_{th} R_M$. By integrating (73) with respect to $r$, we obtain

$$
\kappa_n = \frac{2 \pi \lambda p_n}{n \alpha - 2} \left\{ (R_m^{2-n\alpha} - R_M^{2-n\alpha}) \int_{-\infty}^{\varrho_m} f_Y(y) dy + \int_{\varrho_m}^{\varrho_M} \left[ y \frac{1}{\varrho_{th}} \frac{1}{\varrho_{th}} - R_M^{2-n\alpha} \right] f_Y(y) dy \right\}.
$$

(74)

Finally, we obtain (47) by computing the partial moments of the LN approximations of the RVs $X$ and $Y$, repeatedly applying the Proposition 1, as well as using the change of variable $Y = e^{\mu + \sigma Z}$, where $Z \sim \text{Normal}(0, 1)$, along with the substitutions $\tilde{\varrho}_M = \frac{\ln \varrho_{th}^{\frac{\mu}{\sigma}} - \mu}{\sigma}$ and $\tilde{\varrho}_m = \frac{\ln \varrho_{th}^{\frac{\mu}{\sigma}}}{\sigma}$.
Appendix 5 Proof of Proposition 9

By considering the indicator function in (43), we write the CF of the aggregate CCI for the $R_2$ as,

$$
\Psi_{Z_2}(\omega) = \exp \left\{ 2\pi \int_0^\infty \int_{R_m} \exp \left( j\omega pr^{-\alpha} \right) - 1 \right\} \theta \lambda f_X(x) \mathbb{1}_{Y_1}(r^{-\alpha} x) r dr dx \right\}, \quad (75)
$$

where $Z_2 = \sum_{(\varphi, x) \in \tilde{\Phi}_2} Y$. And from (12), the $n^{th}$ cumulant is

$$
\kappa_n = 2\pi \theta \lambda \int_0^\infty \int_{R_m} p^n \rho^{1-\alpha} x^n f_X(x) dr dx
= 2\pi \theta \lambda \left[ \int_0^\infty \int_{R_m} p^n \rho^{1-\alpha} x^n f_X(x) dr dx + \int_{\rho_m}^{\rho_M} \int_{R_m} p^n \rho^{1-\alpha} x^n f_X(x) dr dx \right]. \quad (76)
$$

Similar to the derivation of (74), we first integrate with respect to $r$ and obtain

$$
\kappa_n = \frac{2\pi \theta \lambda}{\rho M} \int_0^\infty p^n \rho^{1-\alpha} x^n f_X(x) dr dx \left[ R_m^{2-\alpha n} - R_M^{2-\alpha n} \right] + \int_{\rho_m}^{\rho_M} \int_{R_m} p^n \rho^{1-\alpha} x^n f_X(x) dr dx \left[ x^n R^{-\alpha n}_m - x^n R^{-\alpha n}_M \right]
$$

(77)

Likewise the previous case, after computing the following partial moments with respect to $X$, and using the change of variable $X = e^{\mu+\sigma Z}$, we obtain the expression (48).

$$
E^n_X \left[ -\infty, \rho_n \right] = e^{\mu + \frac{n^2\sigma^2}{\alpha}} \left( 1 - Q \left[ \tilde{\rho}_n - n\sigma \right] \right) \quad (78)
$$

$$
E^\frac{1}{2}_X \left[ \rho_n, \rho_M \right] = e^{\frac{\mu}{\alpha} + \frac{2\sigma^2}{\alpha}} \left( Q \left[ \tilde{\rho}_n - \frac{2\sigma}{\alpha} \right] - Q \left[ \tilde{\rho}_M - \frac{2\sigma}{\alpha} \right] \right) \quad (79)
$$

$$
E^n_X \left[ \rho_n, \rho_M \right] = e^{\mu + \frac{n^2\sigma^2}{\alpha}} \left( Q \left[ \tilde{\rho}_n - n\sigma \right] - Q \left[ \tilde{\rho}_M - n\sigma \right] \right). \quad (80)
$$

where $Q[u] = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{v^2}{2}} dv$. 

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Appendix 6 Proof of Proposition 10

By using the indicator function of (44), we write the CF of the aggregate CCI for the resulting $\mathcal{R}_2$ as,

$$
\Psi_I(\omega) = \exp \left\{ 2\pi \int_{p_m}^{p_M} \int_0^{R_M} \int \exp \left( j \omega pr - \alpha \right) \rho \lambda f_X(x) f_P(p) 1_{\mathcal{C}_2}(pr - \alpha) r dr dx dp \right\}.
$$

(81)

where $Z_2 = \sum_{(\varphi, x) \in \tilde{\Phi}_2} Y$. And from (12), the corresponding $n^{th}$ cumulant becomes

$$
\kappa_n = 2\pi \theta \lambda \int_0^{\max\{R_m, (xp/\rho_th)^{1/\alpha}\}} \int_{p_m}^{p_M} \int_{R_m}^{R_M} p^n r^{1-n\alpha} x^n f_X(x) f_P(p) dx dp
$$

$$
= 2\pi \theta \lambda \int_{p_m}^{p_M} \int_{R_m/R_M}^{R_M} \int_{p_m}^{p_M} \int_{R_m/R_M}^{R_M} p^n r^{1-n\alpha} x^n f_X(x) dx dp
$$

$$
+ \int_{p_m}^{p_M} \int_{R_m/R_M}^{R_M} \int_{p_m}^{p_M} \int_{R_m/R_M}^{R_M} p^n r^{1-n\alpha} x^n f_X(x) f_P(p) dr dx dp
$$

(82)

where $\rho_m = \rho_{th} R_m^{\alpha}$ and $\rho_M = \rho_{th} R_M^{\alpha}$. After integrating (82) with respect to $r$, we obtain

$$
\kappa_n = \frac{2\pi \theta \lambda}{n\alpha} \left( R_m^{2-\alpha n} - R_M^{2-\alpha n} \right) \int_{p_m}^{p_M} \int p^n x^n f_X(x) f_P(p) dx dp
$$

$$
+ \int_{p_m}^{p_M} \int \left( p^n x^n R_m^{2-\alpha n} - p^n x^n R_M^{2-\alpha n} \right) f_X(x) f_P(p) dx dp
$$

(83)

Using the change of variate from Appendix 4 we can rewrite the cumulant expression as,

$$
\kappa_n = \frac{2\pi \theta \lambda}{n\alpha} \left[ T_1 + T_2 - T_3 \right]
$$

(84)
where $\hat{\rho}_M = \frac{\ln(p_M/p) - \mu}{\sigma}$, $\hat{\rho}_m = \frac{\ln(p_m/p) - \mu}{\sigma}$, and

$$T_1 = \left( R_m^{2-an} - R_M^{2-an} \right) \int_{p_m}^{p_M} p^n e^{n\mu + \frac{n^2 \sigma^2}{2}} \left( 1 - Q [\hat{\rho}_m - n\sigma] \right) f_P(p) \, dp,$$

$$T_2 = \int_{p_m}^{p_M} p^n R_m^{1-an} e^{\frac{3n \mu}{\alpha^2}} \left( Q [\hat{\rho}_m - \frac{2n\sigma}{\alpha}] - Q [\hat{\rho}_M - \frac{2n\sigma}{\alpha}] \right) f_P(p) \, dp,$$

$$T_1 = \int_{p_m}^{p_M} \frac{2}{\pi} \rho_{nm}^{n^{\alpha} + \frac{n^2 \sigma^2}{2}} \left( Q [\hat{\rho}_m - n\sigma] - Q [\hat{\rho}_M - n\sigma] \right) f_P(p) \, dp. \quad (85)$$

The cumulant in (84) can be computed in a closed form expression, but unfortunately it is final format is overly cumbersome. Thereby, we decide to simply indicate the corresponding operation in (49).
Appendix 7 Proof of Proposition 11

From (13), we know that $Y_M$ and $Y_P$ follow LN distribution with parameters $(\mu_M, \sigma_M)$ and $(\mu_P, \sigma_P)$, respectively. Thus, the handover probability is given by

$$\Pr \left[ Y_M < Y_P + \delta \right] = \int_0^{\infty} \int_0^{y_P + \delta} f_{Y_M}(y_M) f_{Y_P}(y_P) \, dy_M \, dy_P.$$  \hfill (86)

where $f_{Y_M}(y_M)$ and $f_{Y_P}(y_P)$ yield the probability density function of the umbrella Macro Base Station (MBS) and target picocell, respectively. After evaluating the inner-most integral, we obtain

$$\Pr \left[ Y_M < Y_P + \delta \right] = \int_0^{\infty} \frac{1}{2} \text{Erfc} \left( \frac{\mu_M - \log \left( c + y_P \right)}{\sqrt{2}\sigma_M} \right) f_{Y_P}(y_P) \, dy_P$$  \hfill (87)

After making the change of variate $\eta = \frac{-\mu_P + \log(y_P)}{\sqrt{2}\sigma_P}$ in (87), we obtain

$$\Pr \left[ Y_M < Y_P + \delta \right] = \int_{-\infty}^{\infty} e^{-\eta^2} \text{Erfc} \left( \frac{\mu_M - \log \left( c + e^{\mu_P + \sqrt{2}\sigma_P} \right)}{\sqrt{2}\sigma_M} \right) \frac{d\eta}{2\sqrt{\pi}}$$  \hfill (88)

To evaluate $\Pr \left[ Y_M < Y_P + \delta \right]$ in (88), we then use the Gauss-Hermite quadrature [85],

$$\int_{-\infty}^{\infty} e^{-\eta^2} f(\eta) \, d\eta = \sum_{k=1}^{K} \omega_k f(\eta_k) + R_K,$$  \hfill (89)

where $\eta_k$ is the $k^{th}$ zero of the Hermite polynomial $H_K(\eta)$ of degree $K$, $\omega_k$ is the corresponding weight of the function $f(\cdot)$ at the $k^{th}$ abscissa, and $R_K$ is the remainder value. Finally, we obtain (53) by performing the substitutions indicated above.
Appendix 8 Proof of Proposition 12

From (5), the CF of the representative interference component of a random transmitter within the observation region $O$ is written as,

$$\Psi_Y(\omega) = E[e^{j\omega Y}]$$

$$= \int_0^{R_M} \int_{R_m}^{R_M} e^{j\omega pr^{-\alpha} X} f_{R,X}(r, x) \, dr \, dx.$$ \hspace{1cm} (90)

where $f_{R,X}(r, x)$ is the joint density function of the separation distance between interferers and the tagged receiver, and the shadowed fading. Recalling that the finite field of interferers is within an observation region which is delimited by $R_m$ and $R_M$, the Probability Density Function (PDF) of the distances from random points uniformly scattered within $O$ to the tagged receiver is,

$$f_R(r) = \frac{2r}{R_M^2 - R_m^2}.$$ \hspace{1cm} (91)

By substituting (91) in (90), we obtain

$$\Psi_Z(\omega) = \frac{2}{R_M^2 - R_m^2} \int_0^{R_M} \int_{R_m}^{R_M} \exp(j\omega pr^{-\alpha} X) f_X(x) \, r \, dr \, dx.$$ \hspace{1cm} (92)

After manipulating the above expression by performing substitutions and simplifications as indicated in Proposition 12, we obtain (56).
Appendix 9 Proof of Proposition 13

Consider the auxiliary functions $f(\omega) = \int_0^\infty \int_{R_m}^{R_M} \exp(j\omega pr^{-\alpha}x)f_X(x)rdrdx$ and $(g \circ f)(\omega) = \ln[f(\omega)]$. Now, using the Faà di Bruno’s formula [85] which generalizes the chain rule to compute higher order derivatives of the composition of two functions $(g \circ f)(\omega)$, we have

$$\frac{\partial^n}{\partial \omega^n} (g \circ f)(\omega) = \sum_{i=0}^{n} g^{(i)}[f(\omega)] \cdot B_{n,i}[f'(\omega), f''(\omega), \ldots, f^{(n-i+1)}(\omega)],$$

(93)

where $B_{n,i}[f'(0), f''(0), \ldots, f^{(n-i+1)}(0)]$ is the partial Bell polynomial [120]. After evaluating (93) at $\omega = 0$ and using the definition of cumulants from (12), we obtain the following result

$$\kappa_n = \frac{1}{j^n} \sum_{i=0}^{n} g^{(i)}[f(0)] \cdot B_{n,i}[f'(0), f''(0), \ldots, f^{(n-i+1)}(0)].$$

(94)

The derivatives of the auxiliary function $f(\omega)$ at zero are given by,

$$\beta_n = \left. \frac{\partial^n f(\omega)}{\partial \omega^n} \right|_{\omega=0} = j^n p^n \int_0^{\infty} x^nf_X(x)dx \int_{R_m}^{R_M} r^{1-n\alpha}dr.$$  

(95)

By substituting (95) into (94), the final expression for the $n$th cumulant of the aggregate CCI in (57) results.
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