Xiaojia Lu

RESOURCE ALLOCATION IN UPLINK COORDINATED MULTICELL MIMO-OFDM SYSTEMS WITH 3D CHANNEL MODELS
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RESOURCE ALLOCATION IN UPLINK COORDINATED MULTICELL MIMO-OFDM SYSTEMS WITH 3D CHANNEL MODELS

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Abstract

Uplink resource allocation strategies in modern cellular networks are studied in this thesis. With the presence of multiple antenna transmission, multiple base station (BS) coordination and multicarrier techniques, the resource allocation problem is reformulated and jointly optimized over a large set of variables. The focus is on the sum power minimization with per user rate constraints.

A centralized multicarrier coordinated cellular network with multiple antennas implemented at the BS side is considered, where BSs can be adaptively clustered to detect signals from one mobile station (MS). The power, subcarrier, beamforming vector and BS cluster (BSC) are the design variables to be jointly optimized to satisfy the rate constraint per user. The first considered scenario is a simple single carrier multicell system. The power control problem with per user rate constraint can be optimally solved by the proposed algorithm, where power vector, BSC and beamforming vectors are separately updated until the sum power converges. The scenario is extended to more complicated multicarrier systems. The resource allocation problem is non-deterministic polynomial-time hard (NP-hard). Suboptimal algorithms are proposed to tackle the problem.

To get more insights to the performance gap between the proposed algorithms and the capacity achieving bound, the scenario is specified to a single cell system with nonlinear receiver so that the calculation of the lower bound is possible. Efficient geometric aided fast converging power minimization algorithms are proposed to calculate the power bound of the multiple access channel (MAC) with per user rate constraint. By comparing the capacity achieving lower bound with the proposed algorithm, the BSW that starts from full rate allocation looks promising to have a good tradeoff between the convergence speed and the sum power consumption.

Besides the resource allocation algorithms in the cellular network, the physical modeling and corresponding design of the network itself are also considered. The radio propagation in the elevation domain is modeled and considered. The diversity gain from the elevation domain is achieved by extra degree of freedom of beamforming in elevation domain. The antenna array can be either a uniform linear array or a uniform planar array with elements placed horizontally. The proposed power control algorithms are simulated in the 3D network scenarios. The effects of antenna array design in different propagation scenarios are compared.

Keywords: 3D channel, MIMO-OFDM, multicell coordination, power control, resource allocation
Lu, Xiaojia, Ylälinkin resurssien kohdentaminen koordinoiduissa MIMO-OFDM matkapuhelinverkoissa kolmiulotteisten kanavamallien kanssa.

Oulun yliopiston tutkijakoulu; Oulun yliopisto, Teknillinen tiedekunta, Tietoliikennetekniikan osasto; Centre for Wireless Communications; Infotech Oulu

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Tiivistelmä

Työssä tutkitaan ylälinkin resurssien kohdentamisstrategioita matkapuhelinverkoissa. Olettaen koordinointi useiden monikantoaaltotekniikoita käyttävien monantennitukiasemien (BS) välillä, resurssien kohdentamisongelma muotoiltaan uudelleen ja optimoidaan yli suuren joukon optimointimuuttujia. Erityisesti keskitytään yhteenlasketun tehon minimointialojaan käyttäjäkohtaisien siirtoneupeusrajoitteiden kanssa.

Työssä oletetaan keskitetty koordinointi useiden monikantoaaltotekniikoita käyttävien monantennitukiasemien välillä, jotka tukiasemat voidaan adaptiivisesti ryhmitellä yhden matkaviestimen signaalin havainnotta varten. Lähetysteho, kantoaaltoallotus, keilanmuodositus ja tukiasemaklusterointi ovat ongelman muuttuja, jotka optimoidaan yhdessä siten, että käyttäjäkohtaiset siirtoneupeusrajoitteet täyttyvät. Ensimmäinen käsittelty tapaus on yksinkertainen, jossa matkaviestin ohjaus on optimoinnissa ylläpitettävää. Keskustelu laajennetaan monimutkaisempaan monikantoaaltojärjestelyyn, jossa käyttäjäkohtainen siirtoneupeustavoite kiinnitetään, joten ongelma voidaan vastaavasti hajottaa osittaisiksi monikantoaaltokohtaisiksi osaongelmiiksi, jossa ongelma voidaan optimoimalla ratkaista. Jos alkikantoaaltojärjestelyä ei ole kiinnitetty, ongelmat yhdellä osatajoella voidaan optimoimalla yhteenlasken tehon minimoinnissa.

Jotta ongelma voidaan analysoida todellisesti suorituskykyerosta ehdotettujen algoritmin ja kapasiteetti-optimointialoisen rajan välillä, vertailu tehdään yhdessä seuran simulointimallissa epälineaarinen vastaanottimen tiedossa. Tässä varten kehitetään tehokas geometriavastauksen tallennus, jotta optimointia mahdollistaa. Työssä oletetaan myös traajien optimointia, jossa eri optimointialoja voidaan optimoida yhdessä. Ongelma optimoidaan yhdessä monivärisesti optimoinnissa, jossa ongelma voidaan optimoida eri monivärisesti optimoinnissa.”

Asiasanat: kolmiulotteiseen verkkoskenaarioihin, MIMO-OFDM, monisolujärjestelmä koordinaatio, resurssien kohdentaminen, tehonsääto
Preface

The research for this thesis was carried out at the Centre for Wireless Communications (CWC) and the Department of Communications Engineering, University of Oulu, Finland. First of all, I want to thank Professor Matti Latva-aho and Dr. Ari Pouttu, the directors of CWC during my stay, for giving me the opportunity to work in the inspiring working environment.

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I am grateful to Dr. Wei Li for advising me and sharing his experience at the beginning phase of the work. It was my honor and pleasure to work with him though the duration of the cooperation was short. I would like to thank Dr. Le-Nam Tran for spending time to read my paper and giving valuable comments. The discussions with him were always fruitful and convincing. I would like to thank Docent Juha Ylitalo for advising me in the first few years at CWC and later being in the technical steering group of my projects. I appreciate Dr. Esa Kunnari, Dr. Jouko Leinonen and Lic. Tech Pirkka Sivola who have helped me at different phases of this journey.

The work presented in this thesis was carried out in the Multicell Scheduling with 3D Antenna Arrays (MuSA), MIMO Techniques for 3G System and Standard Evolution (MITSE) and Baseband and System Technologies for Wireless Evolution (BaSe) projects. I would like to thank the project managers of these projects, Lic. Tech Visa Tapio, Dr. Janne Janhunen and my colleagues in those projects. I would also like to thank the technical steering group
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It was fun to hang out in the spare time with my Chinese friends without whom this journey would be a lot less interesting. I feel lucky to have Miss Jie Ren as my companion at the end. I believe that another extraordinary journey just started. I would like to express my deepest gratitude to my uncle Dr. Weidong Xin, my aunt Dr. Chunguang Wang and their son Guanyu Xin for their close care and support during my stay in Oulu. Finally, I express my warmest gratitude to my parents for their endless love.

Oulu, September 30, 2013

Xiaojia Lu
**Abbreviations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\succeq$</td>
<td>element-wise larger or equal to</td>
</tr>
<tr>
<td>$\preceq$</td>
<td>element-wise smaller or equal to</td>
</tr>
<tr>
<td>$E(X)$</td>
<td>expectation of the matrix</td>
</tr>
<tr>
<td>$(X)^H$</td>
<td>Hermitian of the matrix</td>
</tr>
<tr>
<td>$(X)^T$</td>
<td>transpose of the matrix</td>
</tr>
<tr>
<td>$\text{Tr}(X)$</td>
<td>trace of the matrix</td>
</tr>
<tr>
<td>$(X)^{-1}$</td>
<td>inverse of the matrix</td>
</tr>
<tr>
<td>$</td>
<td>A</td>
</tr>
<tr>
<td>$</td>
<td>\cdot</td>
</tr>
<tr>
<td>$\max(\cdot)$</td>
<td>maximum</td>
</tr>
<tr>
<td>$\min(\cdot)$</td>
<td>minimum</td>
</tr>
<tr>
<td>$\text{conv}(\cdot, \cdot)$</td>
<td>convex hull of the convex arguments</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>complex number</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>real number</td>
</tr>
<tr>
<td>$\log(\cdot)$</td>
<td>logarithm in base 2</td>
</tr>
<tr>
<td>$\log_{10}(\cdot)$</td>
<td>logarithm in base 2</td>
</tr>
<tr>
<td>$\mathbf{I}$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$\mathbf{0}$</td>
<td>zero matrix</td>
</tr>
<tr>
<td>$\mathcal{CN}(\mathbf{0}, \mathbf{C})$</td>
<td>complex circularly symmetric Gaussian vector distribution with mean $\mathbf{0}$ and covariance $\mathbf{C}$</td>
</tr>
<tr>
<td>$A_A$</td>
<td>sector antenna field pattern</td>
</tr>
<tr>
<td>$A_m$</td>
<td>maximum attenuation of sector antenna field pattern</td>
</tr>
<tr>
<td>$B$</td>
<td>total number of base stations in the system</td>
</tr>
<tr>
<td>$C_{\text{MAC}}$</td>
<td>rate region of multiuser MAC</td>
</tr>
<tr>
<td>$C_{\mu,1,2}$</td>
<td>convex rate pentagon formed by two users with weight vector $\mathbf{mu}$</td>
</tr>
<tr>
<td>$C_{\mu,1,2}^\prime$</td>
<td>infeasible rate region that is above the tangent line crossing one of the corner point of two-user rate pentagon with weight vector $\mathbf{\mu}$</td>
</tr>
<tr>
<td>$C_{1,2}^\prime$</td>
<td>infeasible rate regions of two users</td>
</tr>
<tr>
<td>$C_{1,2}^{\prime}[t]$</td>
<td>feasible two-user rate union at time index $t$</td>
</tr>
<tr>
<td>$C_{1,2}^{\star}[t]$</td>
<td>infeasible two-user rate union at time index $t$</td>
</tr>
<tr>
<td>$C^\mu$</td>
<td>multiuser multidimensional rate polygon for given $\mathbf{\mu}$</td>
</tr>
</tbody>
</table>
\(C\) feasible rate set
\(\mathcal{C}\mu'\) the calculated infeasible rate set for given \(\mu\)
\(C'\) infeasible rate set
\(\hat{C}\) set of calculated corner points of multidimensional rate polygons

\(F_{R,u,V}\) field patterns for vertical polarizations of antenna element \(u\)
\(F_{R,u,H}\) field patterns for horizontal polarizations of antenna element \(u\)

\(H\) MIMO channel response matrix
\(h\) channel vector
\(h_{n,k,i}\) the complex channel response from the \(k\)th MS to the \(i\)th BS on subcarrier \(n\)

\(I_k\) interference power that the \(k\)th user experiencing

\(K\) number of mobile stations
\(k\) the \(k\)th mobile station

\(M\) number of base stations in the cluster
\(N\) number of subcarriers
\(N_i\) number of antennas of BS \(i\)
\(N_R\) number of receive antennas
\(N_{RxH}\) number of rows of antenna layout
\(N_{RxV}\) number of columns of antenna layout
\(N_T\) number of transmit antennas
\(n\) the \(n\)th subcarrier
\(\hat{n}^I\) the subcarrier to increase rate target
\(\hat{n}^D\) the subcarrier to decrease rate target

\(p[t]\) power vector at time index \(t\)
\(p^*\) the optimal power vector
\(P_{\text{max}}\) sum power constraint
\(P_{\text{UB}}\) upper bound of the power
\(P_{\text{LB}}\) lower bound of the power

\(P_k\) maximum power constraint on the user \(k\)
\(p_{n,k}\) transmit power of mobile station \(k\) on subcarrier \(n\)

\(p[t]\) the average transmit power per MS at time \(t\)

\(\hat{R}\) the closest point to the time sharing region of the calculated multidimensional rate polygon

\(R_k\) size of preselected subset of BSs to choose the serving cluster
\( R_{\xi_i} \) corner point on the multidimensional rate polygon by decoding order \( \xi_i \)

\( \bar{r}_{\text{Tx},s} \) location vector of elements \( s \) of transmit antenna

\( \bar{r}_{\text{Rx},u} \) location vector of elements \( u \) of receive antenna

\( S_{n,k} \) covariance matrix of the transmitted signal of mobile station \( k \) on subcarrier \( n \)

\( \hat{s}_{n,k} \) estimated signal of MS \( k \) on subcarrier \( n \)

\( v_{n,m} \) Doppler frequency of ray \( n, m \)

\( w_k \) stacked receive beamforming vector of all BS clusters of MS \( k \)

\( w_{k,i} \) receive beamforming vector of MS \( k \) on BS \( i \)

\( x \) transmitted signal

\( x_k \) transmitted signal of mobile station \( k \)

\( x_{n,k} \) transmitted signal of mobile station \( k \) on subcarrier \( n \)

\( \hat{x}_{k} \) estimated signal of MS \( k \)

\( \hat{x}_{n,k} \) estimated signal of MS \( k \) on subcarrier \( n \)

\( \eta \) noise vector

\( y \) received signal

\( y_{n} \) received signal on subcarrier \( n \)

\( \alpha_{n,m}^{\text{VV}} \) complex gains of vertical-to-vertical polarizations of ray \( n, m \)

\( \alpha_{n,m}^{\text{VH}} \) complex gains of horizontal-to-vertical polarizations of ray \( n, m \)

\( \lambda_0 \) wave length of the carrier frequency

\( \bar{\varphi}_{n,m} \) angle of arrival unit vector

\( \tilde{\phi}_{n,m} \) is the angle of departure unit vector

\( \mu_k \) weight of user \( k \)

\( \Xi \) the set of all possible decoding orders using interference cancellation receiver

\( \xi_i \) the \( i \)th decoding order in \( \Xi \)

\( \xi_i(m) \) the \( m \)th user index in \( \xi_i \)

\( \pi_i \) the \( i \)th serving base station cluster

\( \pi_i(m) \) the BS index of the \( m \)th element in the serving base station cluster

\( \Gamma \) rate target vector of all users

\( \Gamma_k \) rate target of user \( k \)

\( \vartheta_{n,m} \) arrival elevation angle of subpath \( n, m \)
$\gamma_{n,k}$ is the signal-to-noise-plus-interference ratio of user $n$ on subcarrier $k$.

$\theta$ arrival angle of the signal

$\theta_{3\text{dB}}$ 3dB beamwidth

$\theta_{n,m}$ arrival azimuth angle of subpath $n,m$

$\eta_i$ AWGN vector at BS $i$

$\eta_{n,i}$ AWGN vector at BS $i$ on subcarrier $n$

$\Delta\psi_{u,n,m}$ phase offset of element $u$ of subpath $n,m$

$\Delta r$ rate increment stepsize

$\epsilon_r$ rate tolerance threshold

$\epsilon_p$ power tolerance threshold

2D two-dimensional

3D three-dimensional

3GPP third generation partnership project

AoA angle of arrival

AoD angle of departure

AWGN additive white Gaussian noise

BC broadcast channel

BSC base station cluster

BS base station

CCI co-channel interferences

CDF cumulated distribution functions

CDMA code division multiple access

CoMP coordinated multi-point transmission

CSI channel information

DDSP dual domain sum power minimization

DSL digital subscriber line

DoF degree of freedom

ER equal rate

GA geometry aided

IGA improved geometry aided

ITU International Telecommunication Union

ISI inter-symbol-interference

LOS line-of-sight

LTE long-term evolution

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<table>
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<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>LTE-A</td>
<td>long-term evolution advanced</td>
</tr>
<tr>
<td>MAC</td>
<td>multiple access channel</td>
</tr>
<tr>
<td>MIMO</td>
<td>multiple-input multiple-output</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean-square error</td>
</tr>
<tr>
<td>MS</td>
<td>mobile station</td>
</tr>
<tr>
<td>MU</td>
<td>multiuser</td>
</tr>
<tr>
<td>OFDM</td>
<td>orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>OFDMA</td>
<td>orthogonal frequency division multiple access</td>
</tr>
<tr>
<td>OWPB</td>
<td>optimal weights and power bisection</td>
</tr>
<tr>
<td>PAPR</td>
<td>peak-to-average power ratio</td>
</tr>
<tr>
<td>RSS</td>
<td>received signal strength</td>
</tr>
<tr>
<td>SCALE</td>
<td>successive convex approximation for low-complexity</td>
</tr>
<tr>
<td>SC-FDMA</td>
<td>single-carrier frequency-domain multiple access</td>
</tr>
<tr>
<td>SIC</td>
<td>successive interference cancellation</td>
</tr>
<tr>
<td>SIMO</td>
<td>single-input multiple-output</td>
</tr>
<tr>
<td>SINR</td>
<td>signal-to-interference-plus-noise ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>single-input single-output</td>
</tr>
<tr>
<td>SPmin</td>
<td>sum power minimization</td>
</tr>
<tr>
<td>TPC</td>
<td>transmit power control</td>
</tr>
<tr>
<td>ULA</td>
<td>uniform linear arrays</td>
</tr>
<tr>
<td>UPA</td>
<td>uniform planar arrays</td>
</tr>
<tr>
<td>WSR</td>
<td>weighted sum rate</td>
</tr>
<tr>
<td>WSRmax</td>
<td>weighted sum rate maximization</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Inter-operability for Microwave Access</td>
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1 Introduction

Recent developments in wireless communications systems has been towards high spectral efficiency and low carbon emissions. On one hand, the popularity of data-oriented communications in the latest commercialized standards, such as third Generation Partnership Project (3GPP) long-term evolution (LTE) and Worldwide Inter-operability for Microwave Access (WiMAX), requires high data rate. On the other hand, due to the global demands of low energy consumption, the mission of wireless system design becomes ever ‘green’. Many new techniques, both at the link level and at the system level, have emerged recently to increase spectral efficiency. The system design should involve as many techniques as possible to achieve large performance gain rather than optimization from a standalone technique.

This thesis concentrates on resource allocation problems in cellular networks involving the latest transmission and channel modeling schemes. In particular, the target is to develop more power-efficient transmission strategies from the system level point of view to reduce the sum power consumption while maintaining the quality of service. In Section 1.1, an overview of the recent key technologies and resource allocation in wireless systems are presented. Section 1.2 introduces the three-dimensional (3D) network, i.e., 3D channel and antenna array models. Section 1.3 presents the motivations and outlines of this thesis. The author’s contributions to the original publications are summarized in Section 1.4.

1.1 Development of wireless systems

Reliability and throughput are the two most common design criteria for communication systems. To meet the design requirements, advanced techniques, such as multiple-input multiple-output (MIMO) [1] and orthogonal frequency division multiplexing (OFDM) [2] are applied.

The OFDM technique divides the whole system bandwidth into subchannels. The frequency selective channel becomes multiple flat fading subchannels on each subcarrier. In the LTE downlink, orthogonal frequency division multiple access (OFDMA) is used for multiuser access, while in the uplink, single-carrier frequency-domain multiple access (SC-FDMA) is adopted. SC-FDMA has
a lower peak-to-average power ratio (PAPR) than OFDMA. An even better amplifier efficiency can be achieved by block interleaved FDMA [3]. A cyclic prefix whose duration is longer than the longest multipath delay is added to the symbol to eliminate the inter-symbol-interference (ISI). With multiple antennas implemented, the spatial domain degree of freedom (DoF) is utilized to achieve multiplexing gain (high data rates), diversity gain (low error rate) and antenna gain (high signal-to-noise ratio). There is a fundamental trade-off between the multiplexing and diversity gain [4]. Multicell coordinated processing is another technique to exploit the spatial DoF. Multiple base stations (BS) are clustered to form a distributed virtual MIMO system to jointly process signals. [5]

1.1.1 Single-cell multicarrier systems

In MIMO multiple access channels (MACs), the sum power or the weighted sum rate are often relevant optimization objectives. For the general multiple access channel (MAC), the theoretical achievable rate region was addressed in [6] and later extended to the MIMO case by several authors (see [7] and the references therein). The sum power minimization for the MIMO system with optimal non-linear receivers was investigated in [8] and for MIMO-OFDM in [9, 74].

With successive interference cancellation at the receiver, the bound of the MIMO-MAC rate region can be achieved. The shape of the rate region for a fixed weight vector is a convex multidimensional polygon with corner points corresponding to different decoding orders. The whole rate region is a convex hull of all multidimensional polygons with different user weights [7]. The rate bound can be found by solving the weighted sum rate maximization (WSRmax) problems with all user weights. The single-input single-output (SISO) multiuser (MU) capacity region in fading channels is characterized in [10], where the optimal rate and power allocations are obtained. Yu et al. [11] proposed an iterative water filling solution to find the optimal transmitter covariance matrices that achieve the sum capacity of MIMO-MACs.

Unlike the WSRmax with power constraint, the sum power minimization (SPmin) with rate constraints is more involved and cannot be solved with a single shot. For a SISO scenario, Wunder et al. [12] proposed a rate iterative water filling solution for broadcast channels (BCs). It can also be applied to the SPmin problem under rate constraints.
For MIMO broadcast channels (BC), Lee and Jindal [8] proposed a method for finding the symmetric rate vector subject to a sum power constraint. Therein, the achievable rate vector for a given user weights is the solution of a WSRmax problem, which can be solved, for example, by a gradient method [13]. The optimal weights corresponding to the symmetric rate are searched by the ellipsoid method [14]. The algorithm can be used to solve the problem of sum power minimization with symmetric rate constraints with an additional power bisection outer loop. Mohseni et al. [9] characterized the power region of fading MIMO-MACs under various rate constraints. A weighted sum power minimization problem was formulated and the Lagrange dual function was constructed. The minimum sum power was iteratively obtained by solving the Lagrange dual functions for fixed user weights, where the user weights corresponding to the rate target are searched by the ellipsoid method.

The MAC SPmin problem with a linear receiver in single-carrier systems can be addressed under the power control framework [15, 16, 17]. However, in multicarrier systems, the problem is NP-hard. Thus, the optimal solution requires an exhaustive search. Recently in [18, 19, 20], the authors applied the branch-and-bound or linear fractional programming approach to reduce the search space of the optimal solution. In the flat fading case with minimum mean-square error successive interference cancellation (MMSE-SIC) receiver, the SPmin can be solved optimally for a given decoding order by optimizing the receive filter and the power allocation in an iterative manner similarly to the linear case [16]. The optimal decoding order must be exhaustively searched. However, the method does not work if the target is in the time sharing region.

Power control and subcarrier allocation problems have been extensively studied for single-cell SISO OFDM systems, where the users are orthogonally allocated over subcarriers [21, 22, 23, 24, 25, 26, 27, 28]. Typically, time sharing of users on each subcarrier is allowed to make the problem convex [21, 27]. The optimization problem can then be iteratively solved, for example, in its dual domain. For the WSRmax with sum power constraints in the downlink, the optimal strategy is to allocate power to one user per subcarrier [22, 25]. The SPmin problem with per user rate constraint is further considered in [29, 21, 24], but the optimal solution requires high complexity. Suboptimal low-complexity solutions are proposed, for example, in [26, 27, 28], where the subcarrier and power allocations are solved separately.
For the multicarrier digital subscriber line (DSL) or the interference-limited single-cell OFDM radio systems, the power control problem becomes nonconvex due to the presence of interference. In [30, 31], dual methods are used, where Lagrange multipliers are found by the ellipsoid method. Bit loading algorithms are proposed in [32, 33, 34, 35], where bits are loaded based on cost functions, which are normally gradients of power-rate functions. In [35], authors also considered bit subtraction for the WSRmax with sum power constraint. The upper bound of the power spectra is found first, and bits are gradually subtracted from the upper bound solution. A successive convex approximation for a low-complexity (SCALE) algorithm was proposed in [36], where a lower bound and a logarithmic transformation are used to approximate and convexify the original nonconvex WSR function.

1.1.2 Multicell single-carrier systems

Uplink communication in interference-limited cellular systems is challenging and has been studied for decades. In multicell uplink transmission, a mobile station (MS) is not only constrained by its own available resources, but also bears the co-channel interference (CCI) from other interfering MSs. Transmit power control (TPC) is a conventional technique to achieve a desired system performance. For the uplink of code division multiple access (CDMA) systems, TPC was studied in [37, 38, 39, 17] and utilized to combat the near-far problem and satisfy a required carrier-to-interference ratio. Under a fixed MS-to-BS assignment, several centralized or decentralized algorithms were proposed. Given a certain signal-to-interference-plus-noise ratio (SINR) requirement per MS, Yates and Huang reformulated the uplink TPC as an SPmin [40] problem. In their proposal, the BS assignment was considered as another degree of freedom to achieve higher transmit power efficiency. An iterative optimization algorithm was invented therein, which was proven to be able to optimally solve the problem as long as the problem is feasible. This work was extended to three implementation models in [15]. With the extension to the multiple antenna, the SPmin problem was restudied in [16], where both fixed and adaptive BS assignments were considered. An interesting result is that the joint optimization over TPC and the receive beamforming with either fixed or adaptive assignment can still be optimally solved by using the similar iterative approaches as those
in [40, 17], provided it is a feasible problem. The problem was later extended to the multi-antenna-BS systems for both uplink and downlink channels, for examples, in [41, 42, 43, 16, 76, 44]. The downlink-uplink SINR duality and the coordinated transmit beamformer design based on a semi-definite programming were considered. Using the downlink-uplink duality, it was shown that the same SINR can be simultaneously achieved by the downlink and uplink channels, thereby the joint PC and BS assignment problem in the downlink can also be efficiently solved by a similar approach [45, 46, 47].

The recent research has been focusing on the coordination of the multiple BSs to reduce interference and increase cell capacity. For single antenna users, the single-carrier multicell SPmin problem can be transformed into the equivalent form as in [15], which can be optimally solved [44, 76]. The cooperative multi-BS processing has been shown to achieve further spectral efficiency gains in particular for the cell edge users, see, for example, [48, 49, 50]. A central controller was set up to collect all information from its connected BSs. By jointly processing the received signals from different cells, more spatial diversity and larger received signal strength can be obtained and meanwhile less CCI. As all connected BSs work in fact as one distributed MIMO system, the set-up is also called as coordinated multipoint processing (CoMP) in 3GPP long-term evolution (LTE). The term ‘network MIMO processing’ is also used. Theoretically, the more BSs joining the cooperation in a network MIMO system, the better the CCI mitigation capability is. However, it is difficult to perform a large-scale cooperation due to practical limitations. In [51, 52, 53], impact of the error-free but limited backhaul capacity was investigated. Those results were presented based on information theoretic analysis. Other related literature have addressed the crucial problem of the asynchronously arriving signals from multiple transmitters [54]. All those works suggest that for a large-scale network MIMO system, the only feasible solution is partial or local cooperation across a small number of BSs.

In the latest 3GPP LTE Release 10 (LTE-A) standardization process, two local cooperative schemes, i.e., the intra-site and inter-site CoMP, have been introduced. The joint processing of the inter-site and intra-site CoMP is within three sectors belonging to the same BS and the different BSs, respectively. The former requires only physical layer cooperation while the latter one requires crosslayer cooperation. For the inter-site CoMP, fiber connections amongst BSs
are needed for the massive upper-layer signaling exchange. In both schemes, the fixed CoMP based on geographic information was considered.

1.1.3 Multicell multicarrier systems

When jointly considering these techniques in the multicell multicarrier scenario, the dimensions of the design problem become large. It includes power, time, frequency and space. The traditional resource allocation schemes should be redesigned to simultaneously exploit the extra dimensions brought by the emerging techniques. Unfortunately, this is a challenging task since the problem was proved to be NP-hard for most typical performance measures in [55].

In multicell multicarrier systems, the subcarrier is allowed to be reused in different cells causing intercell interference. The resource allocation with linear receiver is NP-hard [55]. Numerous suboptimal solutions have been proposed, and the main direction is to find an efficient algorithm targeted at a local optimal solution. For example, joint proportional fair scheduling, beamforming and power control in MIMO-OFDM systems were considered in [56]. The problem is decoupled into subproblems, the beamforming matrix, user scheduling and power are updated accordingly. In [57, 58, 59], the WSRmax problem for coordinated multi-cell MIMO system with per BS power constraint was investigated. The receiver or transmit filter is optimized separately and iteratively assuming the others are fixed.

1.2 Channel and antenna array models

To achieve high spectral efficiency design targets in the wireless system, the elevation domain effects on the channel capacity calculation were studied in [60, 61]. Meanwhile, physical modeling of the 3D channel is of interests in [62, 63, 64]. Sophisticated antenna applications are implemented at BSs, and the BSs are able to form spatial beams to a desired direction. On the other hand, with the multiple antenna technique, spatial richness of the channel can be exploited. Thus, with a proper antenna array designed for a certain scenario, the system capacity or receive signal quality can be expected to be improved.

Most of the multiple antenna research is based on the simplified two-dimensional (2D) channel model or on the assumption that the majority of the
channel paths are concentrated in a 2D horizontal plane. The simplification and assumption make sense when the radio link is strong line-of-sight (LOS) or the clusters are only horizontally distributed. However, this assumption breaks down if the clusters are also vertically distributed or the incident power is rich in the elevation domain. This is in fact the case and has been shown in recent measurements. In [65], a measurement carried out in Helsinki, Finland, shows that most of the power is concentrated from 0° to 16° above the horizontal level. The measurement in an urban scenario in [66] indicates that if the street is narrow and strong scatterers without LOS are around the mobile station (MS), 65% of the energy is incident with elevation angle larger than 10°. A 20° mean elevation arrival angle with standard deviation of 42° was found in [67]. The elevation dispersion is even larger than in the azimuth domain in scenarios where the height of the building is much larger than the width of the street.

The impacts of elevation propagation on the capacity or spatial correlation have been shown to be significant. In [68], a capacity expression as a function of 3D antenna elements spacing, angular spread and power spectrum is given. In [69, 70], elevation angular spread was shown to have a large impact on the capacity of a uniform planar array (UPA). Shafi [62] analyzed the capacity as a function of the elevation angular spread and suggested a 3D channel model. A closed form upper bound of the MIMO capacity was derived in [71] as a function of statistics of scatterers and antenna element displacement. Based on these measurement results, realistic 3D channel modeling is needed and has drawn attention recently [64]. The recent 3GPP spatial channel model (SCM) [72] considered a cross-polarized two-dimensional (2D) model and has been extended to a 3D model in the European WINNER II project [73].

1.3 Motivations and outline of the thesis

The target of the thesis is to develop practical uplink multiuser multicell MIMO resource allocation algorithms with linear receivers in realistic 3D networks. Because of the nonconvexity of the problem, a global optimal solution is difficult to find. Thus, subproblems, each with a subset of variables and achievable global optimal solutions, are also considered. These subproblems or their variations, such as the SPmin of single-carrier CoMP and SPmin of MIMO-MAC with nonlinear receiver, are investigated as lower or upper bound references of the
final problem, i.e., the UL MIMO-OFDM SPmin with coordinated multi-BS processing.

Chapter 2 [74, 75] investigates the SPmin problem subject to per user rate constraint in the uplink of a MIMO multicarrier system with a nonlinear receiver. Two efficient algorithms are proposed to find the optimal solution to the SPmin problem. Instead of directly solving the SPmin problem, the reverse problems of WSRmax subject the power constraint are iteratively constructed and optimally solved. The first algorithm consists of an outer loop for sum power bisection and an inner loop for adjusting the user weights. By utilizing the geometric properties of the rate region, the full search of user weights corresponding to the scaled target rate vector via WSRmax can be largely avoided in the inner loop of the algorithm. The geometry aided algorithm is further improved by utilizing the power and weight information from the last iteration. The ideas of the algorithms are first illustrated by simple two-MS examples, and later extended to the multiuser scenario. Finally, the convergence behavior of the algorithms is compared numerically with the existing methods in a multiuser setting. It is shown that the computational effort of the improved geometry aided approach is greatly reduced as compared to the existing methods.

Chapter 3 [76, 77, 49] studies the coordinated reception amongst multiple BSs in the uplink of a cellular system, where each MS is served by a BS cluster (BSC). Under the individual SINR constraint per MS, power control and receive beamforming are jointly optimized with adaptive BSC selection to minimize the total transmit power. An iterative optimization algorithm is presented accordingly. The joint optimization problem is non-convex in general, but it can be optimally solved by the proposed algorithm as long as it is feasible. To find the optimum, the proposed algorithm requires an exhaustive search over all BSs per MS at each iteration. To reduce the complexity in a large-scale cellular network, we propose simplified schemes, where a subset of BSs is pre-selected based on the large-scale fading factors. By limiting the search to the subset, the complexity is reduced significantly. Although the obtained power vector with the simplified algorithm is not optimal, a performance close to the macro cellular network full coordination can be achieved by carefully choosing the sizes of the pre-selected BS subset and BSC. The significant advantage is proven by simulations.
Chapter 4 [78, 79] considers joint optimization of transmit power, serving base station cluster (BSC), beamforming and subcarrier allocation for the uplink channel of a multicell multicarrier system. The objective is to minimize the sum transmit power subject to user-specific rate constraints. Because the problem is nonconvex and NP-hard, finding an optimal solution is challenging and not practically appealing. We first propose a bit loading algorithm, in which the rate target is iteratively increased following a greedy manner. Specifically, a power control problem with fixed rate allocation per subcarrier is optimally solved at every iteration. A bit switching (BSW) scheme that starts from full rate allocation is further proposed in order to search for better rate allocations. The performances of the proposed algorithms are compared to the upper and lower bounds achieved by the capacity achieving scheme and equal rate allocation over subcarriers (ER), respectively. Numerical results show that the BSW algorithm initialized with ER (BSW-ER) achieves a better performance and computational complexity trade-off than the bit loading schemes with the BSC size of 1. Typically, more than 5 ∼ 8dB gains can be achieved by BSW-ER with single BS processing.

Chapter 5 [80, 81] considers a 3D multicell multiuser system model along with spatial array processing. Most of the radio propagation in the literature has been studied and modeled on a two dimensional plane. The 3D propagation channel modeling has drawn significant attention recently. This is due to the fact that, in some scenarios, the assumption of small elevation domain angular spread does not hold. With multiple antenna elements displaced in a 3D space, such as uniform linear array (ULA) or uniform planar array (UPA), the BS is capable of forming beams in the vertical domain. This is particularly interesting for interference limited systems, since the 3D array processing provides another degree of freedom to transmit signal and reduce the impact of the interference. The impact of elevation domain angular spread and antenna analog beam pattern on the base station antenna design and power control algorithms that we proposed are studied. Significant gains can be achieved due to the spatial array processing with the extra degree of freedom in vertical domain.

Chapter 6 concludes the thesis and summarizes the main results. The open issues and future research directions are pointed out.
1.4 Author’s contribution

This thesis is in part based on eight publications [81, 77, 76, 80, 78, 74, 79, 75], including two published journal papers, one accepted journal paper and four conference papers. The thesis author played the main role in developing the ideas, analyzing the performance and publishing these papers. The author has written the simulation software codes including the link level simulators and system level simulators and performed all the simulations. The original idea of the first algorithm in [76] is proposed by the second author. The second author also helped implemented the part of the codes in [76]. The original idea of [74, 75] was proposed by the second author. The second and third authors in [74] provided guidance, supervision and criticism during the writing process. The 3D channel model used in the simulation is partly from the WINNER project [63] and was further developed mainly by the second author of [81]. The thesis author was also involved in implementing the 3D channel model.

In addition to the publications [81, 77, 76, 80, 78, 74, 79, 75] that comprise the main content of the thesis, the publications [82, 83, 84, 85, 86, 87] by the thesis author are not included in the thesis. The thesis author provided the simulation codes and results that resulted in the publication of [49].

In summary, the main contributions of the thesis include:

– A fast converging algorithm for the calculation of the SPmin problem in MAC with nonlinear receivers (Chapter 2 [74, 75])
– An optimal solution to the SPmin problem in a single-carrier multicell system with allocations on CoMP (Chapter 3 [76, 77])
– Three practical resource allocation algorithms for multicarrier multicell systems (Chapter 4 [78, 79])
– The proposed resource allocation algorithms are compared with the capacity achieving algorithms in a simple scenario to get more insights into how far their performance is from the optimal (Chapter 4 [78])
– Modeling of the 3D multicell multi-user networks and analysis of the performance of two different antenna arrays (Chapter 5 [80, 81])
– The proposed power control and beamforming algorithms are applied in a 3D network scenario and the performance of the algorithms is evaluated (Chapter 5 [80]).
2 Sum power minimization in uplink MIMO multichannel systems

This chapter considers multi-carrier MIMO MAC with a nonlinear receiver. The problem is the SPmin subject to user-specific rate targets. The problem can be solved by the methods introduced in [8] and [9] with slight modifications. In [8], the optimal weights corresponding to the given rate target needs full search over the rate bound, which is computationally exhaustive. The SPmin is solved in the dual domain in [9]. The computational complexity is a major issue in these methods. In this chapter, efficient algorithms are proposed so that the full search of the optimal user weights across the rate bound is avoided.

The main contributions are as follows. Two SPmin algorithms from [8] and [9] are introduced and applied to multi-carrier scenarios as benchmark references. Two algorithms, the geometry aided (GA) algorithm and its improved version, the improved geometry aided (IGA) algorithm, are proposed. It is demonstrated with numerical results that the IGA has significantly faster convergence speed than both the existing algorithms.

The remainder of the chapter is organized as follows. Section 2.1 describes the system model and formulates the problem. In Section 2.2, the algorithms from [8] and [9] and our proposed algorithms are explained, and in Section 2.3, the performance is demonstrated with numerical examples. Finally, the chapter is concluded in Section 2.4.

2.1 System model and problem formulation

A multi-carrier system that consists of $K$ MSs and $N$ subcarriers is considered. All MSs have $N_T$ transmit antennas while the BS has $N_R$ receive antennas. Perfect channel information (CSI) is assumed to be available at the transmitter and receiver sides. The received signal $y_n$ on subcarrier $n$, $1 \leq n \leq N$, is written as

$$y_n = \sum_{k=1}^{K} H_{n,k} x_{n,k} + \eta_n,$$  \hspace{1cm} (1)
where $H_{n,k} \in \mathbb{C}^{N_t \times N_r}$ is the complex channel response from the $k$th MS on subcarrier $n$, $x_{n,k} \in \mathbb{C}^{N_T}$ is the transmitted symbol vector of MS $k$ on subcarrier $n$, $\eta_n \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN) vector on subcarrier $n$. Let the covariance matrix of $x_{n,k}$ be $S_{n,k} = E[x_{n,k}x_{n,k}^H]$. The transmit power of user $k$ on subcarrier $n$ is equal to the trace of $S_{n,k}$, denoted by $p_{n,k} = \text{Tr}(S_{n,k})$.

With the interference cancellation receiver, the achievable rate of MS $\xi_i(m)$ can be calculated as [7]

$$
R_{\xi_i(m)} = \sum_{n=1}^{N} \log \left| I_{N_r} + H_{n,\xi_i(m)} S_{n,\xi_i(m)} H_{n,\xi_i(m)}^H \right| - \left( \sigma^2 I_{N_r} + \sum_{m' < m} H_{n,\xi_i(m')} S_{n,\xi_i(m')} H_{n,\xi_i(m')}^H \right)^{-1} \right|^{-1}
$$

where $\xi_i$ is the $i$th decoding order, $\xi_i \in \Xi$, $1 \leq m \leq K$, is the $m$th MS in the decoding order $\xi$, the $\Xi$ is the set of all possible decoding orders with cardinality $|\Xi| = K!$. Note that a decreasing decoding order is performed, i.e., $\xi_i(K)$ is decoded first and $\xi_i(1)$ is decoded last. The rate region of MU-MIMO-OFDM MAC with a sum power constraint is then the union of multidimensional polygons for all feasible $S_{n,k}$ [7]

$$
C_{\text{MAC}}(H, P_{\text{max}}) = \bigcup_{n=1}^{N} \bigcup_{k=1}^{K} \{ r : \sum_{m=1}^{K} r_{\xi_i(m)} \leq R_{\xi_i(m)} \}
$$

where $H = \{H_{1,1}, \ldots, H_{N,K}\}$.

It is well known that the above rate region is convex. The point on the bound can be achieved by using the successive interference cancellation. In practice, it can be found by solving a WSRmax problem as [7]

$$
\max \mu^T R, \\
\text{subject to } R \in C_{\text{MAC}}(H, P_{\text{max}}),
$$

30
where $\mu = [\mu_1, \mu_2, \ldots, \mu_K]$ is the MS weight vector. Expanding (4), the WSR can be expressed as

$$
\max_{\text{Tr}(S_{n,k})} \sum_{n=1}^{N} \left( \mu_K \log |I_{N_n} + \sum_{k=1}^{K} \frac{1}{N_0} H_{n,k} S_{n,k} H_{n,k}^H| 
+ \sum_{k=1}^{K-1} (\mu_k - \mu_{k+1}) \log |I_{N_n} \sum_{l=1}^{K} \frac{1}{N_0} H_{n,l} S_{n,l} H_{n,l}^H| \right),
$$

where $\mu_k \geq \mu_{k+1}$. The maximum sum rate is achieved when $\mu_1 = \mu_2 = \ldots = \mu_K$. Since (5) is a convex problem, it can be efficiently solved by any standard solver for a given $\mu$. The corresponding user rate of the optimal transmit covariance matrix can then be calculated from (2). The shape of the rate region (5) for a given $\mu$ is a convex multidimensional polygon. Each corner on the outer bound can be achieved by a certain decoding order. By changing the value of $\mu$, all boundary points in the rate region in (3) can be calculated. The gradient of the tangent hyperplane of (3) is defined by the ratios of the user weights.

The problem in this chapter is the SPmin subject to rate constraint which is the reverse problem of (5) and can be formulated as

$$
\min P_{\text{max}} \quad \text{subject to} \quad R \succeq \Gamma, \quad R \in C_{\text{MAC}}(H, P_{\text{max}}),
$$

where $P_{\text{max}} = \sum_{n=1}^{N} \sum_{k=1}^{K} \text{Tr}(S_{n,k})$, $R$ is the rate vector and $\Gamma$ is the rate constraint vector and $\succeq$ is elementary larger or equal to.

### 2.2 SPmin with rate constraints

In this section, four algorithms are presented to solve the SPmin problem (6). The ideas of the first two algorithms [8, 9] are briefly introduced in Sections 2.2.1 and 2.2.2, respectively, and they are applied to the multicarrier scenario. The two algorithms will be used as benchmarks for our proposed algorithms defined in Sections 2.2.3 and 2.2.4. The third algorithm is the proposed geometry aided (GA) algorithm, where the inner loop always starts with $\mu_1 = \mu_2 = \ldots = \mu_K$. The fourth algorithm is the improved geometry aided (IGA) algorithm, where
computation effort is largely reduced further due to the fact that the weights are continuously updated.

### 2.2.1 Optimal weights and power bisection (OWPB) algorithm

The symmetric rate maximization algorithm in [8] is modified to solve the WSRmax problem (4) in a multicarrier scenario. The idea is that given the sum power constraint $P_{\text{max}}$, the optimal weight vector $\mu$ is first iteratively obtained by the modified ellipsoid method, which is listed in Algorithm 11 in Appendix 1.

At each iteration, WSR (4) is calculated with an updated $\mu$. The optimal WSR point $R$ is compared to the rate target $\Gamma$, if $R \succeq \Gamma$, $P_{\text{max}}$ is reduced, otherwise, $P_{\text{max}}$ is increased. The iteration continues until $R$ converges to $\Gamma$. A two-user illustration is plotted in Fig. 1, where the point $(\Gamma_1, \Gamma_2)$ is the target and curves are rate bounds. The algorithm first searches along the rate bound for the point $R$ on the line $r_1 = \frac{r_1}{r_2} r_2$. If $R \succeq \Gamma$, then the power is reduced, otherwise, the power is increased. The algorithm is summarized in Algorithm 1, where $\hat{R}_k, \forall k$, denotes the closest point in the time sharing region (including corners) of the rate bound to the rate target, which will be discussed in Section 2.2.3.

Fig 1. Illustration of optimal weight search and power bisection of OWPB.
Algorithm 1 Optimal Weights and Power Bisection (OWPB) Algorithm

**Input:** $H_{n,k}, \forall n,k$, $P_{\text{max}}, \sigma^2, P_{\text{UB}}, P_{\text{LB}}, \Gamma, \epsilon_r$.

**Initialization:** $\mu_1 = \mu_2 = \ldots = \mu_K = \frac{1}{K}$, $\Delta r = +\infty$, iteration index $t = 0$, $g_1 = \ldots = g_{K-1} = +\infty$, $E = (1 - \frac{1}{K-1})I_{K-1}$.

1: while $\Delta r > \epsilon_r$ do
2: \quad $t \leftarrow t + 1$
3: \quad Solve (5) and find optimal $S_{n,k}, \forall n,k$
4: \quad Calculate $R$ from (2)
5: \quad while $|gEg^H| > \epsilon_r$ do
6: \quad \quad Update $\mu$ using the modified ellipsoid method in Algorithm 11 in Appendix 1
7: \quad end while
8: \quad if $R \succ \Gamma$ then
9: \quad \quad $P_{\text{UB}} \leftarrow P_{\text{max}}$
10: \quad else
11: \quad \quad $P_{\text{LB}} \leftarrow P_{\text{max}}$
12: \quad end if
13: \quad $P_{\text{max}} \leftarrow \frac{1}{2}(P_{\text{UB}} + P_{\text{LB}})$
14: \quad $\Delta p = P_{\text{UB}} - P_{\text{LB}}$, $\Delta r = \sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|$ \quad (7)
15: end while

**Output:** $\mu, P_{\text{max}}$

2.2.2 Dual domain SPmin (DDSP) algorithm

The SPmin can be also solved in the dual domain [9]. The Lagrangian of (6) is

$$\mathcal{L}(S_{n,k}, R_k; \mu) = \sum_{k=1}^{K} \sum_{n=1}^{N} \text{Tr}(S_{n,k}) - \sum_{k=1}^{K} \mu_k(R_k - \Gamma_k).$$ \quad (7)
For a fixed $\mu$, the minimization of (7) is equivalent to solving the dual function $D(\mu)$

$$D(\mu) = \min_{\mu} \sum_{k=1}^{K} \sum_{n=1}^{N} \text{Tr}(S_{n,k}) - \sum_{k=1}^{K} \mu_k R_k$$

subject to $S_{n,k} \succeq 0$, $\forall$ $n,k$.

$$R_k \in C_{\text{MAC}}(H_k, \sum_{k=1}^{K} \sum_{n=1}^{N} \text{Tr}(S_{n,k}))$$

After some manipulations, (8) can be represented in an equivalent convex form and can be solved by any standard solver for a given $\mu$ [9]. The dual variable $\mu$ that makes the rate vector converge to the target is iteratively searched by the ellipsoid method [9, Appendix]. The DDSP algorithm is summarized in Algorithm 2.

**Algorithm 2** Dual Domain SPmin (DDSP) Algorithm

**Input:** $H_{n,k}, \forall n,k$, $\sigma^2$, $\Gamma$, $\epsilon_r$.

**Initialization:** $\mu_1 = \mu_2 = \ldots = \mu_K = \frac{1}{K}$, $\Delta r = +\infty$, iteration index $t = 0$.

1: while $\Delta r > \epsilon_r$ do
2: $t \leftarrow t + 1$
3: Solve (8) and find optimal $S_{n,k}, \forall n,k$ for given $\mu$
4: Calculate $R$ from (2)
5: Update $\mu$ using the ellipsoid method in [9, Appendix]
6: $\Delta r = \sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|$
7: end while

**Output:** $\mu$, $P_{\text{max}}$

### 2.2.3 Geometry aided algorithm

The proposed geometry aided (GA) algorithm is inspired by the approach from [8]. The geometric properties of the rate region are utilized in searching for the rate target. Similar to [8], the GA algorithm consists of two loops. The outer loop of the algorithm bisects the power, while the inner loop searches the user weights corresponding to the rate target.

The convex hull of all WSR points (including the initial sum rate points) and the union of the infeasible half spaces with the given power constraint is
updated at each iteration in the inner loop, i.e., a new vertex of WSR point is included in the convex hull of feasible points and the half space above the tangent plane crossing the WSR point is included in the infeasible set. As long as the feasibility is uncertain, i.e., the rate target is neither in the feasible set nor in the infeasible set, the inner loop is continued with the given power constraint.

For simplicity, the basic idea of the algorithm is first illustrated by an example of two MSs, which means the rate region is on a 2D plane. The algorithm is extended to the general number of users in Section 2.2.3.

Two-user example

Assuming the power constraint is fixed, Fig. 2 illustrates the MAC rate region that is bounded by the curve ‘EBCDF’. The section ‘CD’ is the maximum sum rate that can be achieved by two MSs with $\mu_1 = \mu_2$. The corner points ‘C’ and ‘D’ correspond to the decoding order $\xi_1 : 2 \rightarrow 1$ and $\xi_2 : 1 \rightarrow 2$, respectively. The gradient of the tangent line crossing the point on the curve is equal to $-\frac{\mu_2}{\mu_1}$. The pentagons ‘HCDQ’ and ‘ABJL’ are the sum rate pentagon and weighted sum rate pentagon, respectively. The pentagon can be expressed as

$$\begin{align*}
C_{1,2}^\mu &= \{ r : r_1 \leq R_{\xi_1(2)}, r_2 \leq R_{\xi_2(2)}, \\
r_1 + r_2 \leq R_{\xi_1(1)} + R_{\xi_1(2)} \}, 
\end{align*}$$

(9)

where $R_{\xi_1(1)}$ and $R_{\xi_1(2)}$ are calculated from (2) and the covariance matrix therein is the optimal solution from (5) for a given $\mu$.  

![Fig 2. Illustration of different sets.](image-url)
It is clear that the union of the pentagons, i.e., $\bigcup C_{1,2}^\mu, \forall \mu$, is feasible. Furthermore, the convex hull of the the union, $C_{1,2} = \text{conv}(\bigcup C_{1,2}^\mu, \forall \mu)$, which is the polygon ‘ABCDQ’, is also feasible. Thus, only by connecting ‘BC’, the feasible area is enlarged by a triangle ‘BUC’, which is shaded by cross lines. The computation for searching of the WSR in the area of ‘BUC’ is completely avoided.

Due to the convexity of the rate region, the area above the tangent line crossing any point on the bound ‘EBCDF’ is infeasible. The tangent line crossing point $R$ can be expressed as $\sum_{k=1}^{2} \mu_k(r_k - R_k) = 0$. Because there are two corners of the WSR pentagon, i.e., $R(\mu)_{\xi_1}$ and $R(\mu)_{\xi_2}$, the tangent line determines the infeasible area, which is the outer one of the two planes with the same gradient crossing the two corners. It can be expressed as the intersection of the two half spaces as

$$C_{1,2}^{\mu'} = \bigcap_{\xi_i} \{r : \sum_{k=1}^{2} \mu_{\xi_i}(k)(r_{\xi_i}(k) - R_{\xi_i}(k)) > 0\}, \xi_i \in \Xi. \quad (10)$$

The area above the curve ‘EBCDF’ is infeasible. It is the union of all infeasible sets, i.e., $\bigcup C_{1,2}^{\mu'}, \forall \mu$. In this particular example of two pentagons, the infeasible set is a union of halfspaces above the tangent lines ‘BG’ and ‘CD’.

Take the WSR pentagon ‘ABJL’ as an example. The rate vectors at point ‘B’ and at ‘J’ are $R_{\xi_1}$ and $R_{\xi_2}$. The infeasible set determined by ‘ABJL’ is the intersection of the areas above the lines with gradient $-\mu_2/\mu_1$ crossing ‘B’ and ‘J’, this is actually the area above the line ‘BG’.

The main steps of the algorithm are demonstrated in Fig. 3. The rate pentagons are plotted and labeled with a boxed iteration index, sum power constraint and weights. The red dot at (1,3) is the rate target. The power constraint $P_{\text{max}}$ and $\mu$ is labeled on the edge of the corresponding rate pentagon.

1. At point $[1]$, the algorithm is first initialized with the sum rate pentagon and power constraint $P_{\text{max}}[0]$, where $[0]$ is the iteration index. It is clear that the target is above the tangent plane crossing the sum rate plane, i.e., $\Gamma \in C_{1,2}^{\mu[0]}$, where $\mu_1[0] = \mu_2[0] = 0.5$, thus, the power constraint $P_{\text{max}}$ is increased.

2. Sets $C_{1,2}^{\mu[1]}$ and $C_{1,2}^{\mu[2]}$, where $\mu_1[1] = \mu_2[1]$, are calculated and labeled by $[2]$. Since there is only a sum rate pentagon, $C_{1,2}^{[1]} = C_{1,2}^{\mu[1]}$ and $C_{1,2}^{[2]} = C_{1,2}^{\mu[2]}$. Since the rate target is neither inside the pentagon nor above the tangent
plane, i.e., $\Gamma \notin \{ C_{1,2}^{[1]} \cup C_{1,2}^{[1]} \}$. $\mu$ must be changed and a new WSR point is calculated.

3. At step 3, new $c_{1,2}^{[2]}$ and $c_{1,2}^{[2]}$, $\mu_1^{[2]} \neq \mu_2^{[2]}$, are calculated.

4. Both $C_{1,2}$ and $C_{1,2}^{'}$ are updated at step 4. $C_{1,2}^{[2]} = \text{conv}\{ c_{1,2}^{[1]}, c_{1,2}^{[2]} \}$ and $C_{1,2}^{[2]} = \cup\{ c_{1,2}^{[1]}, c_{1,2}^{[2]} \}$. Now, $\Gamma$ is above the tangent line with gradient $-\frac{\mu_2^{[2]}}{\mu_1^{[2]}}$ crossing the upper corner of the pentagon 3, which means that $\Gamma \in C_{1,2}^{[2]}$. Thus, $P_{\text{max}}$ is increased.

5. At 5, the new sum rate pentagon, $C_{1,2}^{[3]}$ and $C_{1,2}^{[3]}$ is initialized. $C_{1,2}^{[3]}$ and $C_{1,2}^{[3]}$ are updated accordingly. Similarly as in 2, $\Gamma \notin \{ C_{1,2}^{[3]} \cup C_{1,2}^{[3]} \}$, the target is neither feasible nor infeasible.

6. A new WSR pentagon with a new $\mu$ is calculated at 6. The target is inside the pentagon, thus, $P_{\text{max}}$ is decreased. The iteration continues until the rate target is found.

The algorithm can be easily generalized to a multiuser case with little modifications.
General case

For a multiuser case, the algorithm presented in the previous section does not change much but only works in a larger dimension. The multidimensional rate polygon for a given $\mu$, the feasible set of the calculated rate points, the infeasible set for a given $\mu$ and the infeasible set become

$$C^\mu = \left( \cup_{\xi_i \in \Xi} \{ r : r_{\xi_i(k)} \leq R_{\xi_i(k)}, \forall k \} \right) \cap \left( \cup_{\xi_i \in \Xi} \{ r : \sum_{k=1}^{K} r_{\xi_i(k)} \leq \sum_{k=1}^{K} R_{\xi_i(k)} \} \right),$$

$$C = \text{conv}\{C^\mu, \forall \mu\},$$

$$C'^\mu = \cap_{\xi_i \in \Xi} \{ r : \sum_{k=1}^{K} \mu_{\xi_i(k)}(r_{\xi_i(k)} - R_{\xi_i(k)}) > 0 \},$$

$$C' = \bigcup C'^\mu, \forall \mu,$$

where $R_{\xi_i(k)}, \forall k$, is the optimal rate allocation from (5) and (2) for a given $\mu$.

The detailed algorithm is listed in Algorithm 3. It is initialized by calculating the sum rate with $P_{\text{max}} = \frac{1}{2}(P_{\text{UB}} + P_{\text{LB}})$, where $P_{\text{UB}}$ and $P_{\text{LB}}$ is the upper and lower bound of the power constraint, respectively. The sum rate region is always calculated once the power constraint is changed. The inner loop is continued until the feasibility of the current power constraint is certain. The weights are changed to make the WSR point move towards the target point. Thus, the sizes of both $C'$ and $C'$ are getting larger and the infeasible set is getting smaller. As long as the rate target is found inside $C$ or $C'$, the inner loop stops. The computation for optimal weights is avoided. After the convergence criterion $\Delta r < \epsilon_r$ is met, $P_{\text{UB}}$ is chosen as the output to guarantee the feasibility. Different from the two-user case, where weights can be searched by bisection, the multiuser weights are calculated by the modified ellipsoid method listed in Algorithm 11 in Appendix 1.

Time sharing region

If for any two users $k$ and $k'$, $\mu_k \neq \mu_{k'}$, the maximum WSR point $R$ is one out of $K!$ vertices of the multidimensional rate polygon that is touching the rate bound and can be calculated by (2). However, if $\exists k \neq k'$, such that $\mu_k = \mu_{k'}$, the maximum sum rate of user $k$ and $k'$ at $\mu$ can only be achieved by time sharing between user $k$ and $k'$, e.g., the section ‘CD’ in Fig. 2. It is possible
Algorithm 3 Geometry aided algorithm

**Input:** \( H_{n,k} \forall n,k, \sigma^2, P_{UB}, P_{LB}, \Gamma, \epsilon_r. \)

**Initialization:** \( \mu_1 = \mu_2 = \ldots = \mu_K = \frac{1}{K}, P_{\text{max}} = \frac{1}{2}(P_{UB} + P_{LB}), \Delta p = +\infty, \Delta r = +\infty, \) iteration index \( t = 0, C[t] = \emptyset, C'[t] = \emptyset, E = (1 - \frac{1}{K})I_{K-1}. \)

1: while \( \Delta r > \epsilon_r \) do
2: \( t \leftarrow t + 1 \)
3: Solve (5) and find optimal \( S_{n,k} \forall n,k \)
4: Calculate the multidimensional rate polygon \( R \) from (2)
5: Calculate \( C[\mu], C'[\mu], C \) and \( C' \) from (11)
6: \( C[t] \leftarrow \text{conv}(C[t-1], C[\mu]), C'[t] \leftarrow C'[t-1] \cup C[\mu]' \)
7: if \( \Gamma \in C[t] \cup C'[t] \) then
8: \( \) if \( \Gamma \in C[t] \) {decrease the power} then
9: \( P_{LB} \leftarrow P_{\text{max}}, \)
10: else if \( \Gamma \in C'[t] \) {increase the power} then
11: \( P_{UB} \leftarrow P_{\text{max}} \)
12: end if
13: \( P_{\text{max}} \leftarrow \frac{1}{2}(P_{UB} + P_{LB}) \)
14: \( C[t] \leftarrow \emptyset \)
15: \( E = (1 - \frac{1}{K})I_{K-1}, \mu = [\frac{1}{K}, \ldots, \frac{1}{K}]^T \)
16: else
17: Update \( \mu \) using the modified ellipsoid method in Algorithm 11
18: end if
19: \( \Delta p = P_{UB} - P_{LB}, \Delta r = \sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|. \)
20: end while

**Output:** \( \mu, P_{\text{max}} \)

that the target point is closer to the time sharing hyperplane than to any corner of the multidimensional rate polygon. Thus, the stopping criterion is the shortest distance between the target and the time sharing region instead of the WSR corner points. Recall from (2), that \( R_\xi \) is one corner point on the multidimensional rate polygon.

Let \( \tilde{C}, 1 \leq |\tilde{C}| \leq K! \), denote a set of corner points of the calculated multidimensional rate polygons touching the rate boundary for a given \( \mu \). The time sharing region is the convex hull \( \text{conv}(\tilde{C}) \) of the corner points that maximize the
WSR. If $|\tilde{C}| = 1$, then there is no time sharing, meaning only one WSR point is touching the rate region. If two users are time sharing, then $\text{conv}(\tilde{C})$ is a section connecting two WSR points. The closest point to the time sharing region $\text{conv}(\tilde{C})$ can be found by calculating the minimum Euclidian distance

$$\hat{R} = \arg\min_{R \in \text{conv}(\tilde{C})} \sqrt{\|R - \Gamma\|^2}. \quad (12)$$

Because $\text{conv}(\tilde{C})$ consists of infinite number of points, simple linear sampling can be used to find $\hat{R}$ from a discrete subset of $\text{conv}(\tilde{C})$. As a stopping criterion in Algorithm 3, $\sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|$ is checked instead of the the corner point $R$ of the multidimensional rate polygon.

**Initial power constraint**

Because the GA algorithm requires power bisection, the starting point has impacts on the performance of the algorithm. An impractically large upper bound of $P_{\text{max}}$ will increase the number of iterations in the power bisection. Too small upper bound of $P_{\text{max}}$ may cause the problem to become infeasible in the first place. A good estimate of the initial $P_{\text{UB}}$ can be calculated by assuming a successive cancellation receiver with a fixed decoding order. The user-specific power is calculated in an iterative water-filling manner with rate constraint.

### 2.2.4 Improved geometry aided algorithm

The GA algorithm avoids full search of the rate bound with the given power constraint. After locating the target in the feasible or infeasible sets, the inner loop terminates. The power constraint is bisected in the outer loop and the search always starts from the equal weight point. If the optimal WSR point at the previous iteration is already close to the target, a similar pattern of WSR points will be repeated with a new power constraint. Thus, intuitively, some modifications could be made to avoid the unnecessary repetitive pattern. One idea is that, at the stage of power bisection, $\mu$ and geometry information $E$ are not erased at step 15 in Algorithm 3, i.e., step 15 is removed. Once the target is found to be inside the feasible or infeasible region, the power is bisected and a new WSR point is calculated using $\mu$ from the last iteration. The physical
meaning of such approach is that the search is kept in the area that is close to the target point rather than going back to the sum rate point.

The idea can be further improved by utilizing the past geometry information. The search of the target can be thought of in two-dimensional space with an axis of power and weights. Until now, the search at each iteration only happens in 1-D, i.e., either in power or in weights, but not both. If the search happens both in power and weights at the same time, the number of iterations can be further reduced.

The modification of the idea of the 2-D search results in the improved GA algorithm (IGA) and can be made as follows. The initial point is the sum rate point. At every iteration, both the weight is calculated and the power is bisected with the same criterion as in the GA algorithm. Thus, the past geometry information of both power and weights is always utilized.

The IGA algorithm needs little modification from the GA algorithm. The sum rate point is calculated only once at the first iteration. The ellipsoid is always updated based on the previous iteration and not erased. Furthermore, the matrix $E$ containing the direction and step size information is not erased. The calculated WSR point is set to be at the center of the new ellipsoid, and the search direction and step size is saved to update the ellipsoid. The algorithm is summarized in Algorithm 4.

To illustrate the benefit of heritance of geometry information from the last point and 2-D search, the power bisection and weight search process are plotted in Fig. 4 under the same channel realization as in Fig. 3. It can be seen that after iteration 2, the target point is confirmed to be outside of the rate region. Power is increased and $\mu$ is updated. The pentagon moves to $\mathcal{P}$. It can be noted from the figure that the calculated pentagon is generally converging to the target, while in Fig. 3, the search always goes back to the sum rate pentagon after power bisection.

2.3 Numerical example

The numerical simulation is carried out for 3 MSs and 8 subcarriers in i.i.d channels with $1 \times 4$, $2 \times 2$ and $4 \times 4$ antenna setups. The tolerance threshold for rate discrepancy $\sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|$ is $\epsilon_r = 0.1$ bits/s/Hz. The reason that power
Algorithm 4 Improved geometry aided (IGA) algorithm

Input: $H_{n,k}, \forall n,k, \sigma^2, P^\text{UB}, P^\text{LB}, \Gamma, \epsilon_r$.

Initialization: $\mu_1 = \mu_2 = \ldots = \mu_K = \frac{1}{K}, P_{\text{max}} = \frac{1}{2}(P^\text{UB} + P^\text{LB}), \Delta r = +\infty$, iteration index $t = 0$, $C^{[t]} = \emptyset$, $C'^{[t]} = \emptyset$, $E = (1 - \frac{1}{K})I_{K-1}$.

1: while $\Delta r > \epsilon_r$ do
2: $t \leftarrow t + 1$
3: Solve (5) and find optimal $S_{n,k}, \forall n,k$
4: Calculate the multidimensional rate polygon $R$ from (2)
5: Calculate $C^\mu, C'^\mu, C$ and $C'$ from (11)
6: $C^{[t]} \leftarrow \text{conv}(C^{[t-1]}, C^\mu), C'^{[t]} \leftarrow C'^{[t-1]} \cup C'^\mu$
7: if $\Gamma \in C^{[t]} \cup C'^{[t]}$ then
8: $C^{[t]} \leftarrow \emptyset$
9: end if
10: if $\Gamma \in C^{[t]} \{\text{decrease the power}\}$ then
11: $P^\text{LB} \leftarrow P_{\text{max}}$
12: else if $\Gamma \in C'^{[t]} \{\text{increase the power}\}$ then
13: $P^\text{UB} \leftarrow P_{\text{max}}$
14: end if
15: $P_{\text{max}} \leftarrow \frac{1}{2}(P^\text{UB} + P^\text{LB})$
16: Update $\mu$ and $E$ using the modified ellipsoid method in Algorithm 11
17: $\Delta p = P^\text{UB} - P^\text{LB}, \Delta r = \sum_{k=1}^{K} |\hat{R}_k - \Gamma_k|$.
18: end while

Output: $\mu, P_{\text{max}}$

Convergence is not considered in the stopping criterion is due to the fact that the DDSP algorithm has no power bisection. The power of DDSP is not bounded, which makes measuring the convergence of power difficult. The complexity of the algorithms is mainly due to solving (4). Because at each iteration, the complexity caused by execution of (4) of the four algorithms is practically equal, the total complexity of the algorithms can be compared based on the number of times that (4) is executed.

Note that when the number of MSs becomes large, the search for the optimal $\mu$ becomes more difficult and demands more iterations. In addition, the calculation
of the WSR convex hull becomes more difficult because the number of vertices is large. There are a total of $K!$ corners on the WSR hyperplane of $K$ MSs. The algorithm for the calculation of the WSR convex hull is from [88].

The proposed GA and IGA algorithms with the OWBP and DDSP algorithms modified from [8] and [9], respectively, are compared in Fig. 5 with a $2 \times 2$ MIMO setup, where the rate target is $\mathbf{r} = [1 \ 2 \ 3]$. The upper and lower bounds of the power constraint are plotted on a linear scale, where the solid lines correspond to the geometry aided algorithm and dashed lines to the algorithm in [8]. Since DDSP does not need power bisection, only the sum power is plotted. It is clear that the proposed algorithm needs significantly less iterations than that in [8] for convergence. It can be noticed that after about 25 iterations, the power of IGA has already converged. The GA algorithm needs 35 iterations to converge in power. The power of OWBP has not converged yet before it meets the stopping criterion of the rate. The power convergence behavior of DDSP is clearly worse than that of the GA and IGA algorithms.

The rate convergence behavior, i.e., $\sum_{k=1}^{K} |\hat{R}_k - R_k|$, is plotted in Fig. 6, where MSs converge to the non-symmetric rate target vector $[1 \ 2 \ 3]$ bits/s/Hz at the
end. The IGA needs only 30 iterations to converge to the target, while the GA and DDSP need 55. The OWPB has the worse performance. From the figure, it can be seen that the OWPB algorithm has similar repetition patterns between the iteration interval 1 to 45, 46 and 85, 86 and 120, etc. It is because the OWPB needs to find the optimal weights in the inner loop before the power bisection. The search in the outer loop starts form the sum rate point. Thus, a nearly repeated pattern of WSR points are calculated again like before the power bisection. The repetition characteristic of the rate pattern is greatly reduced in the GA algorithm. It is also clear that the IGA has even less redundant recitation pattern.

The comparisons under the symmetric rate target $\mathbf{\Gamma} = [2 \ 2 \ 2]$ bits/s/Hz for $4 \times 4$ MIMO are plotted in Figs. 7 and 8, respectively. It shows that the proposed GA and IGA algorithms outperform the existing methods in power convergence and IGA has the best performance in rate convergence.

![Fig 5. Comparison of power convergence of presented algorithms of a 3 MSs and 8 subcarrier example. $N_R = 2, N_T = 2$. The rate target vector is $\mathbf{\Gamma} = [1 \ 2 \ 3]$ bits/s/Hz.](image-url)
Fig 6. Rate of MSs vs. number of iterations of a 3 MSs and 8 subcarrier example. $N_R = 2, N_T = 2$. The rate target vector is $\Gamma = [1 \ 2 \ 3]$ bits/s/Hz.

Fig 7. Comparison of power convergence of presented algorithms of a 3 MSs and 8 subcarrier example. $N_R = 4, N_T = 4$. The rate target vector is $\Gamma = [2 \ 2 \ 2]$ bits/s/Hz.
Fig 8. Rate of MSs vs. number of iterations of a 3 MSs and 8 subcarrier example. $N_R = 4, N_T = 4$. The rate target vector is $\Gamma = [2 \ 2 \ 2]$ bits/s/Hz.

Two hundred channel realizations are simulated and the average numbers of iterations are listed in Table 1. IGA clearly outperforms the existing algorithms. The IGA is about $50 - 100\%$ and $300 - 450\%$ more efficient than the DDSP and OWPB algorithms, respectively, depending on the scenario. OWPB has the worst performance in all scenarios.

Table 1. Average number of iterations.

<table>
<thead>
<tr>
<th>Antenna configuration &amp; rate target (bits/s/Hz)</th>
<th>OWPB</th>
<th>DDSP</th>
<th>GA</th>
<th>IGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 4, \ [2 \ 2 \ 2]$</td>
<td>129.64</td>
<td>49.78</td>
<td>58.34</td>
<td>29.20</td>
</tr>
<tr>
<td>$2 \times 2, \ [1 \ 2 \ 3]$</td>
<td>136.22</td>
<td>47.60</td>
<td>76.02</td>
<td>26.55</td>
</tr>
<tr>
<td>$2 \times 2, \ [2 \ 2 \ 2]$</td>
<td>157.37</td>
<td>49.31</td>
<td>92.43</td>
<td>33.19</td>
</tr>
<tr>
<td>$4 \times 4, \ [1 \ 2 \ 3]$</td>
<td>107.48</td>
<td>48.15</td>
<td>85.77</td>
<td>25.15</td>
</tr>
<tr>
<td>$4 \times 4, \ [2 \ 2 \ 2]$</td>
<td>166.72</td>
<td>48.84</td>
<td>70.87</td>
<td>30.22</td>
</tr>
</tbody>
</table>

2.4 Summary and discussions

Four sum power minimization algorithms are presented for multiuser uplink MIMO multicarrier systems. The first two were modified versions of existing algorithms and the latter two are the proposed geometry aided algorithms. The
The geometric properties of the rate region are utilized in the proposed methods. The proposed algorithms do not need to find the optimal weight vector before adjusting the power constraint. Instead, at each iteration, the feasible and infeasible sets are updated. The power bisection is performed as long as the feasibility is certain, otherwise, the weight adjustment is performed. The iteration continues until the stopping criterion is met. The GA algorithm is further improved by searching in power and weights at the same time. The search pattern converges towards the target point both in power and in weight dimensions at each iteration. The numerical results show that IGA has significant faster average convergence speed than the existing methods in various multicarrier MIMO scenarios.
3 Adaptive coordinated reception for multicell MIMO uplink

This chapter concentrates on the single-carrier coordinated multicell MIMO uplink. The SPmin problem subject to user-specific rate targets is studied. In the formulated problem, the transmit power, serving BSC and receive beamforming are jointly optimized. The diversity reception model introduced in [15] can be regarded as one of the simplest examples of the model, where each BS has only one receive antenna. With adaptive BSC selection per MS, an SPmin problem is formulated and an iterative optimization algorithm is presented to solve the problem accordingly. The SPmin problem is non-convex, but it can still be optimally solved by the proposed algorithm.

The remainder of this chapter is organized as follows. In Section 3.1, the system model and the problem formulation for minimum power design with adaptive BSC selection are introduced. The algorithm derivation is presented with the convergence and complexity analysis in Section 3.2. The simulation results are given in Section 3.3 and the conclusion in Section 3.4.

3.1 System model and problem formulation

3.1.1 The Multicell multiuser system

The considered cellular system consists of $K$ MSs and $B$ BSs in flat fading channels. Assuming that all MSs have one transmit antenna while BS $i$ has $N_i$ receive antennas, the received signal $y_i$ at the $i$th BS is written as

$$y_i = \sum_{k=1}^{K} \sqrt{p_k} h_{k,i} x_k + \eta_i,$$

where $p_k$ is the transmit power of MS $k$, $h_{k,i} \in \mathbb{C}^{N_i \times 1}$ is the complex channel response from the $k$th MS to the $i$th BS, $x_k$ is the transmitted symbol of MS $k$ with average power normalized to 1, and $\eta_i \sim \mathcal{CN}(0, \sigma^2 I)$ is the AWGN vector at BS $i$ on subcarrier $n$. 

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Assuming MS \( k \) is simultaneously served by a set of chosen BSs, a set of linear beamformers can be utilized to extract its signal from those BSs, i.e.,

\[
\hat{x}_k = \sum_{i \in \pi_k} w_{k,i}^H y_i = \sqrt{p_k} w_{k,i}^H h_{k,i} x_k + \sum_{i \in \pi_k} \left( \sum_{k'=1 \atop k' \neq k}^K \sqrt{p_k'} w_{k',i}^H h_{k',i} x_{k'} + w_{k,i}^H \eta_i \right),
\]

where \( w_{k,i} \in \mathbb{C}^{N_i \times 1} \) is the receive beamforming vector for MS \( k \)'s received signal at BS \( i \), \((\cdot)^H\) denotes the Hermitian operator and \( \pi_k \) is the BSC serving MS \( k \). The elements in \( \pi_k = \{ \pi_k(1), \pi_k(2), \ldots, \pi_k(|\pi_k|) \} \) are the indices of the BSs jointly providing service to MS \( k \), where \(|\pi_k|\) denotes cardinality of \( \pi_k \).

The interference-plus-noise that MS \( k \) experiences can be calculated as

\[
I_k(p, \pi_k, w_k) = \sum_{k'=1 \atop k' \neq k}^K \left| \sum_{i \in \pi_k} w_{k,i}^H h_{k',i} \right|^2 p_{k'} + \sum_{i \in \pi_k} \|w_{k,i}\|^2 \sigma^2,
\]

where \(|\cdot|\) denotes the absolute value and \(\|\cdot\|\) the standard Euclidean norm. Let \( p = [p_1 \ p_2 \ \cdots \ p_K]^T \) be the stacked vector including all MSs transmit powers, \( w_k \) is the stacked beamformer of MS \( k \), i.e., \( [w_{k,\pi_k(1)}^T w_{k,\pi_k(2)}^T \cdots w_{k,\pi_k(|\pi_k|)}^T]^T \).

### 3.1.2 Problem formulation

Using (15), the effective SINR of MS \( k \) can be written as

\[
\gamma_{k,\pi_k,w_k} = \frac{\left| \sum_{i \in \pi_k} w_{k,i}^H h_{k,i} \right|^2 p_k}{I_k(p, \pi_k, w_k)}.
\]

The considered SPmin problem becomes

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^K p_k \\
\text{subject to} & \quad \log (1 + \gamma_{k,\pi_k,w_k}) = \Gamma_k, \forall k \\
& \quad p_k \leq P_k
\end{align*}
\]

where \( \Gamma_k \) is the minimum SINR requirement of MS \( k \) and \( P_k \) is the maximum transmit power of MS \( k \). The optimization problem (17) is a non-convex problem.
in general. When \(|\pi_k| = 1\), it becomes exactly the same as the one in [16]. On the other hand, when all \(|\pi_k| = B\), it is reduced to coherent beamforming across all BSs [15].

By defining a mapping function \(m(p) = [m_1(p) \ m_2(p) \ \cdots \ m_K(p)]^T\) where

\[
m_k(p) = \min_{\pi_k, w_k} \frac{I_k(p, \pi_k, w_k)}{\sum_{i \in \pi_k} w_{k,i} h_{k,i}^H} \gamma_k,
\]

the following result holds.

**Lemma 1** \(m(p)\) satisfies three properties of a standard interference function.

1) **Positivity**: \(m(p) \succ 0\), where \(\succ\) denotes element-wise larger.
2) **Monotonicity**: if \(p \succeq p'\), then \(m(p) \succeq m(p')\), where \(\succeq\) is element-wise no smaller.
3) **Scalability**: For any \(\alpha > 1\), \(\alpha m(p) \succ m(\alpha p)\).

**Proof** See Appendix 2.

### 3.2 Minimum power beamforming with adaptive BSC selection

#### 3.2.1 Algorithm derivation

The minimum transmit power of a MS, jointly optimized over beamforming and BSC in (18), is shown to be a standard interference function. This allows us to follow the standard power control approach [40]. The SPmin problem with adaptive BSC selection can also be solved by using the iterative optimization algorithm, where at each iteration the power vector is updated by

\[
p^{[t+1]} = m(p^{[t]}).
\]

Recall that \(m(p^{[t]})\), which can be calculated from (18), is the minimum power vector that is optimized over the BSC and beamforming space to meet the SINR target. The optimal receive beamforming matrix for multiuser joint power control and BS assignment is known to be minimum mean-square error filter [47], which is given by

\[
w_{k, \pi_k} = \left(\sum_{k' \neq k} p_{n,k'} h_{k', \pi_k} h_{k', \pi_k}^H + \sigma^2 I\right)^{-1} h_{k, \pi_k}
\]

(20)
where

\[ w_{k,\pi^k} = \left[ w_{k,\pi^k(1)}^T, \ldots, w_{k,\pi^k(|\pi^k|)}^T \right]^T \]

and

\[ h_{k,\pi^k} = \left[ h_{k,\pi^k(1)}^T, \ldots, h_{k,\pi^k(|\pi^k|)}^T \right]^T. \]

Substituting \( m(p^{|n|}) \) by (18) and \( w_k \) by (20), a detailed description of the iterative joint algorithm is given in Algorithm 5.

**Algorithm 5** Joint PC, beamforming and BSC selection with fixed rate target.

**Input:** \( h_k, \forall k; \sigma^2, \epsilon_p \) and \( \Gamma \)

**Initialization:** \( \tilde{p}_k^{[t]} = 0, \forall k. \)

1. \( t \leftarrow 0 \)
2. while \( \sum_{k=1}^{K} \tilde{p}_k^{[t]} - \sum_{k=1}^{K} \tilde{p}_{m,k}^{[t-1]} < \epsilon_p \) do
3. \( t \leftarrow t + 1 \)
4. Calculate \( w_k^{[t]}, \forall \pi_k^{[t]} \in \Pi_k \) from (20)
5. Calculate \( p_{k,\pi_k^{[t]}}^{[t]} \) by

\[
p_{k,\pi_k^{[t]}}^{[t]} = \frac{1}{\sum_{i \in \pi_k^{[t]}} \left( \sum_{k' \neq k} \left| \sum_{i \in \pi_k^{[t]}} w_{k,i}^{[t]} h_{k',i} \right|^2 \tilde{p}_{k'}^{[t-1]} + \sum_{i \in \pi_k^{[t]}} \left| w_{k,i}^{[t]} \right|^2 \sigma^2 \right)}.
\]

6. \( \tilde{\pi}_k^{[t]} \leftarrow \arg \min_{\pi_k^{[t]} \in \Pi_k, |\pi_k^{[t]}| = M} p_{k,\pi_k^{[t]}}^{[t]} \)
7. \( \tilde{w}_k^{[t]} \leftarrow w_{k,\tilde{\pi}_k^{[t]}}^{[t]} \)
8. \( \tilde{p}_k^{[t]} \leftarrow p_{k,\tilde{\pi}_k^{[t]}}^{[t]} \)
9. end while

**Output:** \( \tilde{p}_k^{[t]}, \tilde{w}_k^{[t]}, \tilde{\pi}_k^{[t]} \)

### 3.2.2 Optimality and convergence analysis in a feasible case

The optimization problem has been formulated in (17) and a power control approach has been given in the last subsection. In this subsection, the global optimality and convergence are studied. First, we give the following theorem.
Theorem 1 If (17) is feasible, then the minimum power vector $p^*$ must be a fixed point of the mapping function $m(p)$ and the fixed point is unique.

Proof See Appendix 3

To prove the convergence, we first define $p_t$ as the power vector produced by Algorithm 5 at iteration $t$ with the initial power vector $p'$. Then, we have the following theorems.

Theorem 2 If (17) is feasible and the initial power vector $p'$ is a feasible solution, the output power vector of Algorithm 5 must monotonically decrease after every iteration and will converge to the optimal solution $p^*$.

Proof See Appendix 4

Theorem 3 Algorithm 5, with an initial power vector $p' = 0$, produces a monotonic increasing power vector sequence. Moreover, if (17) is feasible, the sequence must converge to the optimal solution $p^*$.

Proof See Appendix 5

Theorem 4 With any initial power vector $p'$, despite whether $p'$ is feasible or not, the produced power sequence from Algorithm 5 converges to the optimal solution $p^*$, as long as (17) is feasible.

Proof See Appendix 6

3.2.3 Discussion on convergence

When the problem is feasible, we proved the convergence of Algorithm 5 in the previous subsection. In practical systems, the SINR constraint is not always achievable due to large amount of interference or the time selectivity of the wireless channels. In this case, we have the following corollary.

Corollary 1 For any initial power vector $p'$, the power vector sequence produced by Algorithm 5 approaches infinity, i.e., $\sum_{k=1}^{K} p_k \to +\infty$, if (17) is infeasible.

Proof The proof is straightforward from the previous theorems. First, Theorem 3 points out that the algorithm always generates an increasing sequence when using $p' = 0$. As the problem is not feasible and the produced sequence is not upper bounded, $\sum_{k=1}^{K} p_k \to +\infty$. Then using the proof of Theorem 4, we know that the produced sequence by Algorithm 5 with arbitrary $p'$ is element-wise no smaller than the one with $p' = 0$. As a result, it must also approach infinity. This completes the proof.
Although Algorithm 5 may not always converge to a fixed solution, it can be terminated by a cutoff threshold, for example, the maximum transmit power constraint in practical systems. If Algorithm 5 is initialized with zero power vector, the value of the generated sequence is always increasing. The value of the updated power vector will either increase to the optimal solution or reach/exceed the cutoff value $P_k$. The latter case denotes the infeasibility of the problem.

### 3.2.4 Complexity discussion and algorithm simplification

Given $|\pi_k|$ fixed, Algorithm 5 can optimally solve the problem, but its complexity is still high due to the exhaustive search over all BSs combination set to find the optimal BSC for each MS. At each iteration, the number of candidates in the exhaustive search is given by $K \prod_{l=1}^{B} l \prod_{m=1}^{\pi_k} m \prod_{n=1}^{B-|\pi_k|} n$, which dominates the total computational complexity if the network size $B$ is large. On the other hand, in order to achieve larger spatial diversity gain, a larger size of BSC, such as $|\pi_k| = 3$, is required, which also increases complexity. To achieve a better tradeoff between performance and complexity, it is proposed to use BS pre-selection to limit the number of the candidate BSs to be searched per MS. Practically, a central controller can pre-select $R_k$ BSs for MS $k$ based on measured received signal strengths (RSS) at each BS, where $|\pi_k| \leq R_k \leq B$. Then, the central controller in the network runs Algorithm 5 but limits the adaptive BSC selection over those $R_k$ BSs instead of all BSs. By doing so, the number of iterations of the exhaustive search for MS $k$ reduces to $\sum_{k=1}^{K} \prod_{l=1}^{R_k} l \prod_{m=1}^{\pi_k} m \prod_{n=1}^{R_k-|\pi_k|} n$ and we are able to flexibly balance complexity and performance.

The optimal and simplified schemes are summarized as follows.

**Full set selection (FS):** $R_k = B$, which means that an exhaustive search over all BSs is carried out to find the optimal $\pi_k$ for MS $k$.

**Single element selection (SE):** $R_k = |\pi_k|$, which means that the selected $|\pi_k|$ BSs are those which provide $|\pi_k|$ largest RSS values for MS $k$. Compared to FS, SE does not need exhaustive search, and is thereby remarkably simpler.

**Partial set selection (PS):** Consider the case where $|\pi_k| < R_k < B$ and $R_1 = R_2 = \cdots = R_K$. This scheme is significantly computationally more efficient.
than FS when $R_k$ is far smaller than $B$, but it is still more complex than SE. However, it will show performance close to that of FS in the simulations.

Adaptive-sized partial set selection (APS). In this scheme, $R_k$ is also dynamically changed according to the BSs’ RSS values. To do so, a threshold $P_L$ is set for the BS pre-selection. Only those BSs whose RSS differences to the strongest BS are less than $P_L$ are taken into account. In this case, the number of exhaustive search of each MS is not fixed and could be different from time to time.

3.3 Simulation results

The proposed schemes with different sizes of $\pi_k$ are evaluated by system-level simulations. The uplink intra/inter-site CoMP are also compared with the proposed algorithms. Considering a cellular system containing 19 BSs with 3 sectors each, adding up to 57 sectors in total. As shown in Fig. 9, the central 57 sectors are the original sectors, while the outer sectors are the copies of the central sectors. The edge effect is then eliminated by wrapping around the network [89]. Other simulation parameters are listed in Table 2. For convenience, it is assumed that $|\pi_1| = |\pi_2| = \cdots = |\pi_K|$ in all the simulations. The SINR constraints are set to 0 dB and 8 dB in order to simulate voice and data oriented systems.
Fig 9. The considered network layout in the simulation.

Table 2. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layout</td>
<td>19 cells, 3 sectors/cell, wrap around</td>
</tr>
<tr>
<td>Propagation scenario</td>
<td>Base coverage urban</td>
</tr>
<tr>
<td>Cell radius</td>
<td>1000m</td>
</tr>
<tr>
<td>Maximum MS transmit power</td>
<td>24dBm</td>
</tr>
<tr>
<td>Maximum antenna gain</td>
<td>17dBi</td>
</tr>
<tr>
<td>Scheduling interval</td>
<td>10 transmission time interval</td>
</tr>
<tr>
<td>1 transmission time interval</td>
<td>1 ms</td>
</tr>
<tr>
<td>Thermal noise density</td>
<td>$-174$ dBm/Hz</td>
</tr>
<tr>
<td>Number of users</td>
<td>30 in 19 cells</td>
</tr>
<tr>
<td>BS receiver antenna array</td>
<td>ULA</td>
</tr>
<tr>
<td>BS receiver antenna elements</td>
<td>2</td>
</tr>
<tr>
<td>UE antenna</td>
<td>1</td>
</tr>
<tr>
<td>Number of BSs for coordinate reception</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>SINR constraint per MS</td>
<td>0, 4, 8dB</td>
</tr>
<tr>
<td>MS speed</td>
<td>3km/h</td>
</tr>
<tr>
<td>Shadow fading</td>
<td>Log-Normal, 8dB standard deviation</td>
</tr>
<tr>
<td>Shadowing correlation</td>
<td>independent</td>
</tr>
<tr>
<td>Down tilt angle</td>
<td>8 degrees</td>
</tr>
</tbody>
</table>

The large-scale fading parameters, including path loss, shadow fading and antenna beam pattern gain are from International Telecommunication Union.
For simplicity, the BS and link index are dropped. The general sector antenna field pattern can be modeled as

$$A_A(\theta) = -\min \left[ 12\left(\frac{\theta}{\theta_{3dB}}\right), A_m \right], \quad -180^\circ \leq \theta \leq 180^\circ,$$

\[ (22) \]

where \( \theta \) is the arrival angle, \( \theta_{3dB} \) is the 3dB beamwidth which is 70° for 3-sector cell and \( A_m = 20 \text{dB} \) is the maximum attenuation. The general path loss model is given by

$$P_{\text{pathloss}} = (44.9 - 6.55 \log_{10}(h_{\text{BS}})) \log_{10}(d_{\text{BS-MS}}) + 34.46 + 5.83 \log_{10}(h_{\text{BS}}) + C_{\text{pathloss}} \log_{10}(f_c/5),$$

\[ (23) \]

where \( h_{\text{BS}} \) is the BS antenna height, \( d_{\text{BS-MS}} \) is the distance between the considered BS and MS, \( C_{\text{pathloss}} \) is the frequency dependence factor, which is 23 in urban macro non-line-of-sight scenario and \( f_c \) is the carrier frequency. Finally, the shadow fading is log-normal distributed with standard deviation of 8 (dB). The MS antenna is assumed to be placed at 1.5m above the ground. [90]

### 3.3.1 Performance analysis of the proposed algorithm

First, the feasibilities against the number of MSs of the alternative schemes are compared in Fig. 10, where the SINR constraint per MS is 0dB. As an important benchmark, the performance of the full cooperative scheme is given, where all BSs jointly process their received signals as a huge virtual MIMO system. This gives us the theoretical upper bound of the system performance, which is shown by the black solid line in the figure. Obviously, FS with \( |\pi_k| = 3 \) can support more MSs than other partially cooperative schemes as it achieves the highest feasibility. Compared with the fully cooperative network MIMO, it still has a perceptible gap when the number of MSs increases up to 100 or higher. From the complexity viewpoint, SE is the simplest scheme. However, it always has the worst performance amongst all compared schemes with the same number of \( |\pi_k| \). When \( |\pi_k| \) is increased, the performance gap reduces. By setting \( R_1 = R_2 = \cdots = R_K = 5 \), the performances of PS with different \( |\pi_k| \) values are obtained. Among all simplified schemes, it is noticed that PS always achieves higher feasibilities than others schemes with the same \( |\pi_k| \). Another interesting result is that PS shows a little better performance than APS, where
$R_k$ is dynamically changed based on the measured RSS values of MSs. The reason is that the threshold $P_L$ is not large, thus the number of BSs involved in the joint receiving is less than 5 as in PS. By setting $P_L$ to a larger value, a better performance from APS can be expected, besides a higher complexity. A similar behavior could also be found in Fig. 11 where the SINR constraint per MS is set at 8dB.

![Graph of probabilities of feasibility vs. number of MSs with the SINR constraint of 0 dB.](image)

**Fig 10. Comparison of probabilities of feasibility vs. the number of MSs with the SINR constraint of 0 dB ([76], published by permission of EURASIP).**

In Fig. 12, the cumulated distribution functions (CDF) of transmit power per MS of alternative schemes are plotted. In this comparison, 30 MSs are uniformly distributed over the whole interested area and the SINR constraint is also set at as 0 dB at first. In this comparison, only the powers obtained under the feasible channel realizations are counted. As can be expected, FS with $|\pi_k| = 3$ achieves a very close performance to the upper bound in this comparison, where the difference is less than 0.8dB at most. Naturally, its performance is getting worse when $|\pi_k|$ is reduced to 1. Same as in the feasibility comparison, SE still performs the worst with the same $|\pi_k|$ while PS shows very good performance. Compared with SE, the performance gap between PS and FS is almost negligible. Moreover, PS shows slightly better performance than APS, as in the previous comparison. Similar findings can also be found in Fig. 13, where the SINR constraint is 8dB per MS.

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Fig 11. Comparison of probabilities of feasibility vs. the number of MSs with the SINR constraint of 8 dB ([76], published by permission of EURASIP).

Fig 12. Comparison of CDFs of transmit power of different sizes of joint processing BSC with the SINR constraint of 0 dB ([76], published by permission of EURASIP).
Fig 13. Comparison of CDFs of transmit power of different sizes of joint processing BSC with the SINR constraint of 8 dB ([76], published by permission of EURASIP).

3.3.2 Comparison of the proposed algorithm with the LTE inter/intra-site CoMP

The LTE inter-site and intra-site CoMP [91], are in fact two special cases of the full set selection, i.e., FS, with $R_k = 3$ and $|\pi_k| = 3$. In the inter-site and intra-site CoMP, BSs have been grouped based on the geographic information. The receive strategies of the two schemes are also chosen from a single-element set. The cell combination set of the CoMP is fixed, while the single-element set has an adaptive combination. With minor modifications, our proposed TPC algorithm can be easily applied to the inter-site and intra-site CoMP. An illustration of the inter/intra-site cooperation is included in Fig. 14.
Fig 14. An illustration of inter-site and intra-site reception schemes.

Fig 15. Comparison of probabilities of feasibility of intra-site, inter-site cooperation and proposed adaptive reception schemes. Again SINR constraint is 0dB ([76], published by permission of EURASIP).

The feasibility comparisons are shown first in Figs. 15 and 16, where the SINR constraint per MS is 0dB and 8dB respectively. The adaptive scheme shows obvious advantage over the conventional schemes in both figures. It is also
observed that the inter-site cooperation achieves higher probability of feasibility than the intra-site cooperation.

The CDF curves of the average transmit power are plotted in Fig. 17 and Fig. 18 with the same SINR constraint settings as the feasibility comparisons. When the transmit power is small, we see that the intra-site cooperation performs a little better than the inter-site cooperation. As the required transmit power is increased, the inter-site cooperation starts to outperform the intra-site cooperation. The reason is that the small and large transmit powers usually indicate that the MS is located at the cell-center and cell-edge, respectively. The sector-beam attenuation, rather than the path loss, dominates the channel gains in the cell-center case, while in the cell-edge case, the path loss becomes more significant. From the CDF results, the obvious advantage of the adaptive scheme is still observed. PS with $|\pi_k| = 1$ sometimes outperforms 3-sector CoMP because PS is near optimal and the best sector is chosen based on small-scale fading out of 5 candidate sectors, while the fixed-cluster CoMP is only chosen based on large-scale fading parameters.

### 3.3.3 Complexity analysis

The complexity of Algorithm 5 is composed of two parts including the convergence speed, i.e., the number of iterations it takes to converge, and the number of candidates in the exhaustive search set at each iteration. For convenience, the complexity of APS in this comparison is not considered, as it does not show better performance than PS and its complexity of the second part can not be directly calculated.

In order to see the convergence behavior of the alternative schemes, the maximum number of the iterations is fixed to be 100 and stop criteria in Algorithm 5 is skipped. At each iteration $t$, Algorithm 5 calculates the average transmit power per MS, i.e. $\bar{p}_t = \frac{\sum_{k=1}^{K} p_k[t]}{K}$. Later, the difference between two consecutive iterations is computed as $\epsilon_p = \bar{p}_t[t] - \bar{p}_t[t+1]$. After averaging over 1000 channel realizations, $\epsilon_p$ is illustrated in Fig. 19. As the iteration index $t$ is increased, FS shows the fastest convergence speed with the same value of $|\pi_k|$. For example, when setting $|\pi_k| = 3$ and $\epsilon_p = 5e-3$, FS takes about 1.8 iterations.
Fig 16. Comparison of probabilities of feasibility of intra-site, inter-site cooperation and proposed adaptive reception schemes. Again SINR constraint is 8dB ([76], published by permission of EURASIP).

Fig 17. Comparison of CDFs of transmit power of inter-site, intra-site and adaptive reception schemes with the SINR constraint of 0dB ([76], published by permission of EURASIP).

to converge, while SE and PS take 4 and 1.85 iterations, respectively. In addition, the larger $|\pi_k|$ values, the faster convergence speed for all the schemes.

Then by setting $B = 57$, $R_k = 5$ and $\epsilon_p = 5e-3$, Algorithm 5 can calculate the product of the number of iterations and the number of the candidates in the set to be exhaustively searched, which, to some extent, represents the total complexity of the algorithm. The obtained results of the alternative schemes
with different values of $|\pi_k|$ are shown in Fig. 20. Although FS shows the fastest convergence speed in the previous comparison, it still has significantly higher complexity than SE and PS do. When $|\pi_k| = 1$, SE and PS have about the same complexities. But when $|\pi_k|$ is increased to 2 and 3, the complexity of SE is reduced significantly while that of PS does not change so much. As the total complexity depends on both convergence speed and number exhaustive searches, thus, PS almost has the similar complexity for different $|\pi_k|$.

### 3.4 Summary and discussion

The joint TPC and receive beamforming with adaptive BSC selection for uplink communications is studied. To minimize the total transmit power, an algorithm that optimally solves the problem is presented accordingly. As the optimal BSC selection involves exhaustive search per MS over all BSs, several simplified schemes with different BS pre-selection schemes are presented. The proposed algorithm with both optimal and simplified BSC selection is evaluated through system level simulations. The results show that using the simplified scheme, better tradeoffs between complexity and performance can be achieved. The proposed adaptive scheme is also compared to the conventional fixed inter-cell
Fig 19. Comparison of convergence speed of different algorithms ([76], published by permission of EURASIP).

Fig 20. Comparison of the total number of iterations for different algorithms and $\pi_k$ ([76], published by permission of EURASIP).

and intra-site CoMP, where the obvious advantages of the adaptive scheme are observed.
This chapter considers multicell multicarrier multiantenna systems. The SPmin problem that involves a large set of variables, i.e., power, BSC, beamforming vector and subcarrier allocation is studied. The problem considered in this chapter differs from the existing literature that generally consider simpler problems with only a subset of the aforementioned variables. In particular, all these parameters are jointly considered to minimize the sum power subject to per user rate constraints. Two bit loading algorithms are proposed, in which the rate target is iteratively increased following a greedy manner. Specifically, a power control problem with fixed rate allocation per subcarrier is optimally solved at every iteration. The SPmin problem is then split into two subproblems, i.e., the optimal power control problem with fixed rate assignment and the rate assignment problem with fixed other-variables. A BSW scheme based on full rate allocation is further proposed in order to search for better rate allocations.

The structure of the rest of the chapter is organized as follows. In Section 4.1, the system model and problem are presented. In Section 4.2, the proposed bit loading algorithms are explained. In Section 4.3, the bit switching algorithm is explained. The numerical results are presented in Section 4.4, and, finally, the chapter is concluded in Section 4.5.

4.1 System model and problem formulation

4.1.1 System model

Consider a system that consists of $K$ MSs, $B$ BSs and $N$ subcarriers. All MSs have a single transmit antenna while BS $i$ has $N_i$ receive antennas, the received signal $y_{n,i}$ at the $i$th BS on subcarrier $n$ is written as

$$y_{n,i} = \sum_{k=1}^{K} \sqrt{p_{n,k}} h_{n,k,i} x_{n,k} + \eta_{n,i} \quad (24)$$

where $p_{n,k}$ is the transmit power of MS $k$ on subcarrier $n$, denoted as link $(n,k)$, $h_{n,k,i} \in \mathbb{C}^{N_i}$ is the complex channel response from the $k$th MS to the $i$th BS.
on subcarrier $n$, $x_{n,k}$ is the transmitted symbol of MS $k$ on subcarrier $n$ with average power normalized to 1, and $\eta_{n,k} \sim \mathcal{CN}(0, \sigma^2 I)$ is the AWGN vector at BS $i$ on subcarrier $n$.

The joint processing among BSs to serve a particular MS is considered. More explicitly, it is assumed that $M$ out of $B$ BSs cooperate to detect signals from one MS. This gives rise to the problem of selecting a set of $M$ BSs that offers the best performance. For mathematical exposition, let us denote by $\Pi_{n,k}$ the set of all possible combinations of $M$ serving BSs on link $(n,k)$. The cardinality of $\Pi_{n,k}$ is thus equal to $|\Pi_{n,k}| = \binom{B}{M}$. In this way, each $\pi_{n,k} \in \Pi_{n,k}$ represents a cluster of multiple BSs simultaneously serving MS $k$ on subcarrier $n$. In this chapter, the cluster size $M$ is a fixed design parameter that is assumed to be equal for all links, though varying sizes of BSC on different links does not affect our problem. The impact of the cluster size is evaluated numerically in the numerical section.

A set of linear receive beamformers formed by $\pi_{n,k}$ can be utilized to extract the signal of MS $k$, i.e.,

$$\hat{x}_{n,k} = \sum_{i \in \pi_{n,k}} w_{n,k,i}^H y_{n,i}$$

$$= \sum_{i \in \pi_{n,k}} \sqrt{p_{n,k}} w_{n,k,i}^H h_{n,k,i} x_{n,k}$$

$$+ \sum_{i \in \pi_{n,k}} \left( \sum_{k'=1, k' \neq k}^K \sqrt{p_{n,k'}} w_{n,k',i}^H h_{n,k',i} s_{n,k'} + w_{n,k,i}^H \eta_{n,i} \right)$$

(25)

where $w_{n,k,i} \in \mathbb{C}^{N_i}$ is the receive beamforming vector for MS $k$ at $i$th BS. The stacked vector of the optimal linear receiver of the $k$th user and $n$th subcarrier for the BSC $\pi_{n,k}$, is the minimum mean-square error (MMSE) filter which can be expressed as

$$w_{n,k,\pi_{n,k}} = \left( \sum_{k' \neq k} p_{n,k'} h_{n,k',\pi_{n,k}} h_{n,k',\pi_{n,k}}^H + \sigma^2 I \right)^{-1} h_{n,k,\pi_{n,k}}$$

(26)

where

$$w_{n,k,\pi_{n,k}} = \left[ w_{n,k,\pi_{n,k}(1)}, \ldots, w_{n,k,\pi_{n,k}(|\pi_{n,k}|)} \right]^T$$

$$h_{n,k,\pi_{n,k}} = \left[ h_{n,k,\pi_{n,k}(1)}, \ldots, h_{n,k,\pi_{n,k}(|\pi_{n,k}|)} \right]^T$$

(27)
The signal-to-interference-plus-noise ratio (SINR) of MS $k$ on the $n$th subcarrier can be calculated as
\[
\gamma_{n,k} = \frac{\left| \sum_{i \in \pi_{n,k}} w_{n,k,i}^H h_{n,k,i} \right|^2 p_{n,k}}{\sum_{k' \neq k} \left| \sum_{i \in \pi_{n,k}} w_{n,k,i}^H h_{n,k',i} \right|^2 p_{n,k'} + \sum_{i \in \pi_{n,k}} \| w_{n,k,i} \|^2 \sigma^2} \tag{28}
\]
where $\pi_{n,k} \in \Pi_{n,k}$ is the set of indices of $M$ BSs as mentioned above.

### 4.1.2 Problem formulation of power control in coordinated multicell multicarrier systems

The uplink joint PC, subcarrier allocation, BSC selection and beamforming problem that is considered can be formulated as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{n \in \mathcal{N}_k} p_{n,k} \\
\text{subject to} & \quad \sum_{n \in \mathcal{N}_k} \log (1 + \gamma_{n,k}) \geq \Gamma_k, \forall k
\end{align*}
\tag{29}
\]

where $\gamma_{n,k}$ is the SINR defined in (28), $\Gamma_k$ is the rate target of MS $k$ and $\mathcal{N}_k$ is the set of subcarriers allocated to the $k$th MS with $1 \leq |\mathcal{N}_k| \leq N$. It is easy to see that for the SPmin problem in (29), the inequality holds with equality at optimal point.

The transmit power $p_{n,k}$ on link $(n,k)$ can be calculated as
\[
p_{n,k} = \frac{(2^{\log(1+\gamma_{n,k})} - 1) I_{n,k}(p)}{\left| \sum_{i \in \pi_{n,k}} w_{n,k,i}^H h_{n,k,i} \right|^2} \tag{30}
\]
where
\[
I_{n,k}(p) = \sum_{k' \neq k} \left| \sum_{i \in \pi_{n,k}} w_{n,k,i}^H h_{n,k',i} \right|^2 p_{n,k'} + \sum_{i \in \pi_{n,k}} \| w_{n,k,i} \|^2 \sigma^2 \tag{31}
\]
is the total interference that MS $k$ is experiencing on subcarrier $n$. 

The problem (29) can be relaxed into the following equivalent form if per link rate target $\Gamma_{n,k}$ is included in the original problem as a variable

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{n \in N_k} p_{n,k} \\
\text{subject to} & \quad \log (1 + \gamma_{n,k}) \geq \Gamma_{n,k}, \forall n, k \\
& \quad \sum_{n \in N_k} \Gamma_{n,k} \geq \Gamma_k, \forall k
\end{align*}
\]

(32)

where the inequalities holds with equalities at optimal point.

In the following subsection, the optimal solution with any fixed $\Gamma \in \mathbb{R}^{N \times K}$ in (32) is presented, where $\Gamma$ is the matrix of $\Gamma_{n,k}$. With fixed $\Gamma$, (32) is equivalent to a power control, beamforming and BSC selection problem without considering subcarrier allocation. Later in Sections 4.2 and 4.3, how to find a good $\Gamma$ efficiently is discussed.

### 4.1.3 Optimal solution for the fixed rate per subcarrier

As mentioned earlier, (32) is nonconvex and NP-hard, and finding the optimal solution to it efficiently is an open problem. However, if the rate target $\Gamma$ is fixed, the optimal solution of (32) can be found jointly over power, BSC and beamforming vector. Because given rate target vectors over subcarriers, (32) can be transformed into $N$ parallel single-carrier power control subproblems, which can be efficiently solved by [76] on the subcarrier $n$ as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} p_{n,k} \\
\text{subject to} & \quad \log (1 + \gamma_{n,k}) \geq \Gamma_{n,k}
\end{align*}
\]

(33)

The algorithm to solve the multicarrier fixed rate target problem is listed in Algorithm 6. The BSC $\pi_{n,k}$ is adaptively chosen at every iteration. The sum power is monotonically decreasing until converge. The output of the algorithm, i.e., power, BSC and beamforming vector are globally optimal for fixed $\Gamma$ [76].
Algorithm 6 Joint PC, beamforming and BSC selection with fixed rate target.

Input: $h_{n,k}, \forall n,k, \sigma^2$, set of available subcarriers $\mathcal{N}$ and $\Gamma$

Initialization: $\tilde{p}_{n,k}^t = 0$, $\forall n,k$.

1: for $n \in \mathcal{N}$ do
2:     $t \leftarrow 0$
3:     while $\sum_{k=1}^{K} \tilde{p}_{n,k}^t - \sum_{k=1}^{K} \tilde{p}_{n,k}^{t-1} < \epsilon$ do
4:         $t \leftarrow t + 1$
5:         Calculate $w_{n,k,\pi_{n,k}^t}$, $\forall \pi_{n,k}^t \in \Pi_{n,k}$ from (26)
6:         Calculate $p_{n,k,\pi_{n,k}^t}^t$ from (30)
7:         $\hat{\pi}_{n,k}^t \leftarrow \underset{\pi_{n,k}^t \in \Pi_{n,k}, |\pi_{n,k}^t|=M}{\arg \min} p_{n,k,\pi_{n,k}^t}^t$
8:         $\hat{w}_{n,k,\hat{\pi}_{n,k}^t}^t \leftarrow w_{n,k,\hat{\pi}_{n,k}^t}^t$
9:         $\tilde{p}_{n,k}^t \leftarrow p_{n,k,\hat{\pi}_{n,k}^t}^t$
10:    end while
11: end for

Output: $\tilde{p}_{n,k}^t, \hat{w}_{n,k,\hat{\pi}_{n,k}^t}^t, \hat{\pi}_{n,k}^t$

4.1.4 Discrete form of problem

A straightforward way to achieve the global optimal solution to (32) is to carry out Algorithm 6 for all possible rate allocation $\Gamma_{n,k}, \forall n,k$, such that $\Gamma_k = \sum_{n\in\mathcal{N}_k} \Gamma_{n,k}, \forall k$, by Algorithm 6, and choose the one that minimizes the sum power. This is obviously computationally impractical since the rate is a continuous function which will result in infinite number of allocation alternatives.

One step towards tractability is to discretize the rate target $\Gamma_{n,k}$ with granularity $\Delta r$, such that $\Gamma_{n,k} = t_{n,k}\Delta r$, where $t_{n,k}$ is a nonnegative integer. Thus, (32) is
transformed into an equivalent discrete form

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{n \in \mathbb{N}_k} p_{n,k} \\
\text{subject to} & \quad \log (1 + \gamma_{n,k}) \geq t_{n,k} \Delta r, \forall n, k \\
& \quad \sum_{n \in \mathbb{N}_k} t_{n,k} \Delta r \geq \Gamma_k, \forall k
\end{align*}
\]

(34)

Again, for fixed \( t_{n,k}, \forall n, k \), the problem can be optimally solved by Algorithm 6. However, the optimal solution for the discretized version (34) requires an exhaustive search over all possible \( t_{n,k}, \forall n, k \) that satisfy the second constraint in (34). The complexity of the exhaustive search depends on the granularity \( \Delta r \) and also grows exponentially with the number of users and subcarriers. Thus, finding good rate allocations \( t_{n,k}, \forall n, k \) efficiently is necessary. The motivation of introducing the problem in (34) is that its discrete form and the variables \( t_{n,k} \) are suitable for presenting the bit loading algorithms in the following section.

### 4.2 Multicarrier bit loading algorithms

As stated in the previous section, optimally solving problem (34) needs exhaustive search, which has exponential complexity. Thus, our focus now is to find a good rate allocation efficiently, so that the exhaustive search can be avoided. In this section, multicarrier bit loading algorithms are proposed, which are based on a proper combination of Algorithm 6 and the idea in orthogonal subcarrier allocation [21] and the thesis comes up with a multicarrier bit loading algorithm which continuously increase the rate target iteratively until the rate constraint is reached.

#### 4.2.1 Greedy algorithm

The idea of the greedy bit loading algorithm is that at each iteration, \( \Delta r \) bits are added to the optimal link while the rates of interfering users are kept unchanged, so that the total power increment in the system is minimum. The iteration continues until all MSs reach their rate targets. At every iteration, the considered problem is a fixed-rate problem that can be optimally solved by Algorithm 6, as discussed in Section 4.1.3.
The iteration of the algorithm is initialized from zero allocation \( \Gamma^{[0]} = 0 \). At iteration \( t \), the algorithm tests with an allocation such that rate on link \((n, k)\) has \( \Delta r \) increment and others are kept fixed, i.e.,

\[
\begin{align*}
\Gamma_{n,k}^{[t]} &= \Gamma_{n,k}^{[t-1]} + \Delta r, \\
\Gamma_{n',k'}^{[t]} &= \Gamma_{n',k'}^{[t-1]}, \forall (n', k') \neq (n, k)
\end{align*}
\]

(35)

Obviously there are \( NK \) such candidate allocations. To obtain the current best link, Algorithm 6 is executed \( NK \) times each for one of the candidates. The sum power of rate target (35) is denoted by \( \Phi_{n,k} \). The optimal link \((\hat{n}, \hat{k})\) is the one that causes the minimum sum power increment from the last iteration, i.e.,

\[
(\hat{n}, \hat{k}) = \arg \min_{n, k} (\Phi_{n,k} - \Phi_{\hat{n},\hat{k}}^{[t]}), \forall n, k
\]

(36)

where \( \Phi_{\hat{n},\hat{k}}^{[t]} = N \sum_{n=1}^{N} \sum_{k=1}^{K} p_{\hat{n},\hat{k}}^{[t]} \) is the sum power of the optimal rate target \( \Gamma^{[t]} \) from the last iteration. Finally, \( \Delta r \) bits are allocated on link \((\hat{n}, \hat{k})\). The sum power \( \Phi(\hat{n}, \hat{k})^{[t+1]} \) and rate target

\[
\begin{align*}
\Gamma_{n,k}^{[t+1]} &= \begin{cases} \\
\Gamma_{\hat{n},\hat{k}}^{[t]} + \Delta r, & (n, k) = (\hat{n}, \hat{k}) \\
\Gamma_{n,k}^{[t]}, & \forall (n, k) \neq (\hat{n}, \hat{k})
\end{cases}
\end{align*}
\]

(37)

are updated. The power allocation, beamforming vector and BSC are global optimal for the given rate target \( \Gamma^{[t+1]} \).

The operations in (35) and (36) are carried out only for the set of MSs that have not reached the rate constraint. This set is updated at the end of every iteration and denoted by \( K^{[t]} \). The MS that reaches the rate target will be excluded from \( K^{[t]} \). The iteration continues until \( K^{[t]} = \emptyset \), i.e., all MSs reach their rate requirements. The greedy bit loading algorithm is outlined in Algorithm 7. Note that given a fixed rate target per subcarrier, the sum power on one subcarrier is independent from the other subcarriers. When calculating the total sum power increment, only the set of subcarriers that has user rate increment is need to be recalculated.

### 4.2.2 Gradient based algorithm

Even though the proposed greedy bit loading algorithm in the previous subsection has linear complexity, it needs to compute the optimal beamforming
Algorithm 7 Greedy bit loading algorithm.

**Input:** \( h_{n,k}, \forall n,k, P_{\text{max}}, \) step size of rate increment \( \Delta r, \sigma^2 \) and \( \Gamma \).

**Initialization:** \( t = 0, P^{[t]} = 0, \Gamma^{[t]} = 0 \), MS set that does not satisfies the rate constraint \( \mathcal{K}^{[t]} = \{1, \ldots, K\} \), MS set that satisfy rate constraint \( \tilde{\mathcal{K}}^{[t]} = \emptyset \).

1: while \( \mathcal{K}^{[t]} \neq \emptyset \) do
2: \( t \leftarrow t + 1 \)
3: for \( k \in \mathcal{K}^{[t]} \) do
4: for \( n \in \mathcal{N}_k \) do
5: \( \Gamma_{n,k} \leftarrow \Gamma_{n,k}^{[t-1]} + \Delta r \)
6: \( \Gamma_{n',k'} \leftarrow \Gamma_{n',k'}^{[t-1]}, \forall n' \neq n, k' \neq k \)
7: Calculate corresponding \( P(n,k), W(n,k) \) and \( \pi(n,k) \) from Algorithm 6 given \( \Gamma \) and \( \mathcal{N} = \{n\} \)
8: end for
9: end for
10: Find the optimal link \( (\hat{n}, \hat{k}) \) by (36)
11: \( \tilde{P}^{[t]} \leftarrow P(\hat{n}, \hat{k}), \tilde{W}^{[t]} \leftarrow W(\hat{n}, \hat{k}), \tilde{\pi}^{[t]} \leftarrow \pi(\hat{n}, \hat{k}) \)
12: \( \Gamma_{n,k} \leftarrow \Gamma_{n,k}^{[t-1]} + \Delta r \)
13: \( \tilde{\mathcal{K}}^{[t]} \leftarrow \tilde{\mathcal{K}}^{[t-1]} \cup \{ \arg \sum_{n \in \mathcal{N}_k} \Gamma_{n,k}^{[t]} = R_{\text{min}} \} \)
14: \( \mathcal{K}^{[t]} = \mathcal{K}^{[t-1]} \setminus \tilde{\mathcal{K}}^{[t]} \)
15: end while

**Output:** \( \tilde{P}^{[t]}, \tilde{W}^{[t]}, \tilde{\pi}^{[t]} \)

vector, power vector and BSC for each of \( NK \) candidate rate assignments, which is computationally intense. With the motivation of further reducing the computational effort and avoiding extensively executing Algorithm 6, a gradient based power updating algorithm is proposed.

The gradient method is similar to Algorithm 7 in the sense that every link is tested with \( \Delta r \) bits increment while keeping the rates of all other links unchanged. Since the optimal link is found by calculating the sum rate power gradient, Algorithm 6 is only executed once at each iteration instead of \( NK \) times in Algorithm 7. To reach the final rate targets of all MSs as soon as possible, the
optimal link is chosen such that it has the maximum gradient of the sum rate power function. The gradient measures how efficiently power increases on one link to reach the final rate targets. The gradient is calculated as

\[
g_{n,k} = \frac{d}{dp_{n,k}} \sum_{k=1}^{K} \sum_{n \in \mathcal{N}_k} \log (1 + \gamma_{n,k}) \]

\[
= \frac{d}{dp_{n,k}} \sum_{n=1}^{N} \sum_{n \in \mathcal{N}_k} \log \left(1 + \left| \sum_{i \in \mathcal{P}_{n,k}} w_{n,k,i}^H h_{n,k,i} \right|^2 p_{n,k} \right)

= \frac{1}{\sum_{k'=1,k' \neq k}^{K} I_{n,k'}(p) + \sum_{i \in \mathcal{P}_{n,k}} \left| w_{n,k,i}^H h_{n,k,i} \right|^2} \sum_{k'=1,k' \neq k}^{K} \frac{I_{n,k'}(p)}{n,k'}(p) + \sum_{i \in \mathcal{P}_{n,k}} \left| w_{n,k,i}^H h_{n,k,i} \right|^2.
\]

(38)

The optimal link is then chosen by

\[
\{\hat{n}, \hat{k}\} = \arg \max_{n,k} \{g_{1,1}, \ldots, g_{N,K}\}.
\]

(39)

Once the optimal link is found, Algorithm 6 is performed to update the power, beamforming vectors and BSC.

For initialization, in order to get the initial gradients, \(\Delta \mathbf{r}\) is assumed to be allocated on link \((n, k)\)

\[
\begin{align*}
\Gamma_{n,k} &= \Delta \mathbf{r} \\
\Gamma_{n',k'} &= 0, \forall (n', k') \neq (n, k)
\end{align*}
\]

(40)

Algorithm 6 is executed for all links and the calculated power, beamforming vector and BSC are saved. It is in fact equivalent to select the optimal BSC for \(\Delta \mathbf{r}\) allocation without interference.
The difference between the two link selection schemes (39) of gradient algorithm and (36) of greedy algorithm is that, the execution of Algorithm 6 is only once per iteration in the gradient algorithm, while in the greedy algorithm it is \( NK \) per iteration. The algorithm runs until rate constraints of all MSs are fulfilled. The gradient based algorithm is summarized in Algorithm 8.

**Algorithm 8** Gradient based algorithm.

**Input:** \( h_{n,k}, \forall n, k, \) step size of rate increment \( \Delta r, \sigma^2 \) and \( \Gamma \).

**Initialization:** \( t = 0, \) \( P[t] = 0, \) \( \Gamma[t] = 0 \), MS set that does not satisfy the rate constraint \( K[t] = \{1, \ldots, K\} \), MS set that satisfy rate constraint \( \tilde{K}[t] = \emptyset \).

Calculate \( P(n,k), W(n,k), \pi(n,k), \forall n,k \) by Algorithm 6 assuming rate allocation is (40).

1. while \( K[t] \neq \emptyset \) do
2. \( t \leftarrow t + 1 \)
3. for all \( k \in K[t], n \in N_k \) do
4. Calculate gradients by (38)
5. end for
6. Find the optimal link \((\hat{n}, \hat{k})\) by (36)
7. \( \Gamma_{\hat{n},\hat{k}} \leftarrow \Gamma_{\hat{n},\hat{k}}^{[t-1]} + \Delta r \)
8. \( \Gamma_{n,k} \leftarrow \Gamma_{n,k}^{[t-1]}, \forall (n,k) \neq (\hat{n},\hat{k}) \)
9. Calculate \( P(\hat{n},\hat{k}), W(\hat{n},\hat{k}) \) and \( \pi(\hat{n},\hat{k}) \) by Algorithm 6 given \( \Gamma \) and \( N = \{\hat{n}\} \)
10. \( \hat{P}[t] \leftarrow P(\hat{n},\hat{k}), \hat{W}[t] \leftarrow W(\hat{n},\hat{k}), \hat{\pi}[t] \leftarrow \pi(\hat{n},\hat{k}) \)
11. \( \hat{K}[t] \leftarrow \hat{K}^{[t-1]} \cup (\arg_k \sum_{n \in N_k} \Gamma_{n,k}^{[t]} = R_{\min}) \)
12. \( K[t] = K^{[t-1]} \setminus \hat{K}[t] \)
13. end while

**Output:** \( \hat{P}[t], \hat{W}[t], \hat{\pi}[t] \)

4.3 Bit switching algorithm

The bit loading algorithm proposed in the previous section can find the optimal solution for a fixed rate allocation. However, it starts from zero allocation state
and requires large computational effort to achieve the final full rate allocation state. Meanwhile the final rate allocation is likely to be non-optimal due to the greedy nature of the algorithm. To achieve the global optimality, an exhaustive search is required, which is of course not practical for a large-scale system.

In this section, a so-called bit switching algorithm is considered that can find more efficient power allocation for the target $\Gamma$. The idea is that, instead of increasing rates iteratively from zero, it starts with the full rate allocation, i.e., $\Gamma_k = R_{\text{min}}, \forall k$. The initial rate allocation $\Gamma$, could be any feasible solution. If $\Gamma$ is not optimal, there must exist a better allocation $\Gamma'$ with $\Gamma_{n,k} - \Delta r$ on subcarrier $k$ and $\Gamma_{n,k'} + \Delta r$ on subcarrier $k' \neq k$. It is in fact switching $\Delta r$ bits from a subcarrier to a more suitable subcarrier of one MS. Since the rate targets are already fulfilled, i.e., $\Gamma_k = R_{\text{min}}, \forall k$, the switching is only among subcarriers of one MS. The criterion of bit switching is such that the overall sum power should be reduced. The required transmit power on link $(n,k)$ can be calculated by (30).

For the current rate allocation, it first calculates and saves the cost/bonus of increment/decrement of $\Delta r$ on subcarrier $n$ and on subcarrier $n' \neq n, \forall n$. If the bonus is larger than the cost, $\Delta r$ bits are switched. In another words, keeping the sum rate of MS $k$ unchanged, $\Delta r$ bits are switched between a subcarrier pair $\{n, n'\}$, if the sum power is to be decreased. The searching for the subcarrier switching pairs goes through all MSs. There may be multiple pairs of switching, out of which the one with the largest sum power decrement is chosen. The iteration runs until any further bits switching will cause the sum power to be increased. The optimal subcarrier pair $\hat{n}^i, \hat{n}^D$ and MS $\hat{k}$ to perform the switching can be expressed as

$$\{\hat{n}^i, \hat{n}^D, \hat{k}\} = \arg \max_{n^i, n^D, n^i \neq n^D, k} \left( \Phi_{n^i,k} - \sum_{m=1}^{K} p_{n^i,m} - \Psi_{n^D,k} \right),$$

(41)

where $\Phi_{n^i,k}$ is the sum power of rate target of $\Delta r$ increment on link $(n^i, k)$. Similarly, $\Psi_{n^D,k}$ is the sum power of rate target of $\Delta r$ decrement on link $(n^D, k)$.

The calculation for the power vector of the new rate constraint is an iterative search like Algorithm 6. The difference is that the BSC and beamforming vectors are kept fixed and only powers are updated. By doing so, the computation effort is dramatically decreased. The generated power vector is a monotonically decreasing sequence and will converge to a stationary point. The description
of this scheme is summarized in Algorithm 9. However, since the optimal beamforming vector is not calculated, the final power vector is probably not globally optimal. Once the subcarrier pair is found, Algorithm 6 is performed on both subcarriers, the new power and beamforming vectors are calculated and saved. The iterative procedure keeps going until the power converges.

Algorithm 9 Power updating with fixed beamforming vectors and BSCs.

Input: \( h_{n,k}, I_{n,k}, \forall n,k, P_{\text{max}}, \Delta r, p_n \)

Initialization: \( t \leftarrow 0, \tilde{p}_n^{[t]} \leftarrow p_n \)

1: while \( \sum_{n=1}^{N} \tilde{p}_n^{[t]} - \sum_{n=1}^{N} \tilde{p}_n^{[t-1]} < \epsilon \) do
2: \( t \leftarrow t + 1 \)
3: Calculate \( \tilde{I}_{n,k}(\tilde{p}_n^{[t]}) \) with fixed \( I_{n,k}(\tilde{p}_n^{[t]}) \)
4: Calculate \( I_{n',k}(\tilde{p}_n^{[t]}), \forall n' \neq n \) by (31)
5: Calculate \( \tilde{p}_{n',k}^{[t]}, \forall n' \neq n \) by (30)
6: end while

Output: \( \tilde{p}_n^{[t]} \)

The switching algorithm searches for a locally optimal solution which depends on the starting point. The starting point can be any full-rate allocation. It uses the output of the proposed bit loading algorithm or an equal rate allocation as the initialization point. The algorithm is summarized in Algorithm 10. Note that better performance can be achieved if steps 7 and 14 in Algorithm 10 are replaced by Algorithm 6. However, this performance gain comes at the expense of larger computational effort.

4.4 Simulation results

Consider a cellular system containing 7 BSs, each with 3 sectors, or 21 sectors in total. As shown in Fig. 21, the wrap-around model contains central 21 sectors as the original sectors and the outer sectors as the copies of the central sectors. The circles indicate BSs and the triangles indicate the MSs. The MSs are uniformly distributed in the network. The units of the x and y axes are meters. The edge effect is eliminated by wrapping around the network. Other simulation parameters can be found in Table 2.
Algorithm 10: Bit switching algorithm.

Input: $h_{n,k}$, $\forall n,k$, $P_{\text{max}}$, step size of rate increment $\Delta r$, $\sigma^2$, $\Gamma$, $P$, $W$ and $\pi$.

Initialization: $t \leftarrow 0$, $\Gamma_n^{[t]} \leftarrow \Gamma_n$, $p_n^{[t]} = p_n$, $w_{n,k}^{[t]} = w_{n,k}, \forall k,n$, $\Delta p = +\infty$.

1. while $\Delta p > 0$ do
   2. $t \leftarrow t + 1$
   3. for all $k \in K^{[t]}$, $n \in N_k$ do
      4. {Calculate costs}
      5. $\Gamma_n,k \leftarrow \Gamma_n^{[t]} + \Delta r$
      6. $\Gamma_{n',k'} \leftarrow \Gamma_{n',k'}, \forall n', k' \neq n, k$
      7. Calculate $\tilde{p}_n$ by Algorithm 9 (alternatively by Algorithm 6 for better performance)
      8. $\Phi_{n,k} \leftarrow \sum_{m=1}^{K} \tilde{p}_{n,m}$
   9. end for
   10. for all $k \in K^{[t]}$, $n \in N_k$ do
       11. {Calculate bonus}
       12. $\Gamma_n,k \leftarrow \Gamma_n^{[t]} - \Delta r$
       13. $\Gamma_{n',k'} \leftarrow \Gamma_{n',k'}, \forall n', k' \neq n, k$
       14. Calculate $\tilde{p}_n$ by Algorithm 9 (alternatively by Algorithm 6 for better performance)
       15. $\Psi_{n,k} \leftarrow \sum_{m=1}^{K} \tilde{p}_{n,m}$
   16. end for
   17. Find the links $\hat{n}^I, \hat{n}^D, \hat{k}$ to switch bits by (41)
   18. $\Gamma_{\hat{n}^I,k} \leftarrow \Gamma_{\hat{n}^I,k}^{[t-1]} + \Delta r$
   19. $\Gamma_{\hat{n}^D,k} \leftarrow \Gamma_{\hat{n}^D,k}^{[t-1]} - \Delta r$
   20. $\Delta p \leftarrow \left(\sum_{m=1}^{K} p_{n^D,m} - \Psi_{n^D,k} - (\Phi_{n^I,k} - \sum_{m=1}^{K} p_{n^I,m})\right)$
   21. Calculate $\tilde{P}^{[t]}, \tilde{W}^{[t]}, \tilde{\pi}^{[t]}$ by Algorithm 6 given $\Gamma^{[t]}$ and $N = \{n^D, n^I\}$
   22. end while

Output: $\tilde{P}^{[t]}, \tilde{W}^{[t]}, \tilde{\pi}^{[t]}$
4.4.1 Comparisons of the proposed schemes

The CDF of the individual transmitted power are plotted. The compared algorithms include greedy bit loading (Algorithm 7), gradient based bit loading (Algorithm 8), and bit switching (Algorithm 10) that uses the output of the greedy bit loading and equal rate allocation as the initialization. The solid curve marked with a circle is the solution of Algorithm 6 with equal rate (ER) allocation over subcarrier, which serves as the reference.

In Fig. 22, the CDFs of the average power of 10 MSs are plotted. The blue curves are $M = 1$ and green curves are $M = 3$. With $M = 1$, the gradient based algorithm (Algorithm 8) is about 1.5dB better than the ER scheme at CDF of 0.5. The greedy bit loading algorithm (Algorithm 7) is about 3.5dB better than the gradient based algorithm. The BSW algorithm initialized with ER is 0.5dB worse than that initialized by the greedy algorithm. If $M = 3$, more spatial subchannels are involved in beamforming and the frequency selectivity is less than that of $M = 1$. From Fig. 22, it is clear that the performance gains of BSW of $M = 3$ become less than the $M = 1$ case. The greedy algorithm is about 1dB better than the ER scheme. The BSW-greedy performs the best. It is about 2.3dB better than the ER. The gap between the BSW-greedy and BSW-ER is only 0.2dB at CDF of 0.5.
Fig 22. Comparison of CDF of transmit powers. Number of coordinated receiving BS is 1 or 3. 10 MSs, $R_{\text{min}} = 6$ bits, 6 subcarriers, $\Delta R = 1$ bit, cell radius is 1 km ([78], published by permission of IEEE).

Similarly, the performance of 20 MSs are plotted in Fig. 23. The system becomes more interference limited. The proposed algorithms are able to utilize the multiuser diversity gain. It can be observed from the figure that all proposed algorithms have larger gains than the 10 MSs case in Fig. 22. With $M = 3$, the BSW-greedy is 1dB better than the BSW-ER and the latter one is 3.5dB better than the ER allocation.

Performance comparison of 30 MSs is shown in Fig. 24. Because of the larger number of MSs, it is difficult to provide a feasible solution with $M = 1$. Thus, only curves of $M = 3$ is plotted. Compared to the result in Fig. 23, it is observed that the performance gain becomes larger as the number of MSs is growing. This is due to the large diversity gain in dense networks. The BSW-greedy is 6dB better than ER. The performance gap between BSW-greedy and BSW-ER is 0.7dB at CDF of 0.5. It can be observed that the BSW algorithm has better performance in $M = 1$ than that of $M = 3$. Because when $M$ is large, the transmit power is much smaller than that of $M = 1$. The sum rate-power function in $M = 3$ case has much deeper gradients than $M = 1$, i.e., $M = 3$ needs much less power than $M = 1$ to increase $\Delta r$ on top of the current rate.
target. Thus, when $M = 1$, power is less sensitive to the rate change and loading $\Delta r$ bits at each iteration becomes relatively inefficient to search for the optimal point, because at every iteration it only moves a relatively small step forward comparing to the total power needed. As a result, there is a large room left for the BSW algorithm in the $M = 1$ case. Larger numbers of bits can be switched, and possible switching options are much more numerous than in the $M = 3$ case.

The performance of 15 MSs in a smaller cell is plotted in Fig. 25, where the cell radius is reduced to 0.5km. Similar performance can be found as in 1 km scenarios, only that less power is needed.

4.4.2 Complexity analysis

The complexity of the algorithm is mainly caused by the execution of Algorithm 6 to calculate the optimal power, BSCs and beamforming vectors. The average numbers of executions of Algorithm 6 of the proposed algorithm and the

Fig 23. Comparison of CDF of transmit powers. Number of coordinated receiving BS is 1 or 3. 20 MSs, $R_{\text{min}} = 6$ bits, 6 subcarriers, $\Delta R = 1$ bit, cell radius is 1 km ([78], published by permission of IEEE).
Fig 24. Comparison of CDF of transmit powers. Number of coordinated receiving BS is 1 or 3. 30 MSs, $R_{\min} = 6$ bits, 6 subcarriers, $\Delta R = 1$ bit, cell radius is 1 km ([78], published by permission of IEEE).

exhaustive search are plotted in Figs. 26 and 27 for $M = 1$ and $M = 3$, respectively. The results are averaged out over 1000 channel realizations.
Fig 25. Comparison of CDF of transmit powers. Number of coordinated receiving BS is 1 or 3. 15 MSs, $R_{min} = 6$ bits, 6 subcarriers, $\Delta R = 1$ bit, cell radius is 0.5 km ([78], published by permission of IEEE).

Fig 26. Average numbers (in logarithmic scale) of execution of Algorithm 6 for $M = 1$. 
Surprisingly, $M = 3$ needs less iterations than that of $M = 1$. When $K = 20$, BSW-ER needs 58.5 and 34.7 iterations for single BS and 3 BSs reception, respectively. The BSW-ER starting from equal rate allocation needs 10 to 35 iterations to converge, where as the greedy algorithm needs $2.5 \times 10^2$ to $3.9 \times 10^3$. The complexity of greedy and BSW-greedy are almost the same with BSW-greedy slightly larger. It is because the BSW procedure converges very quickly given any full rate allocation.

## 4.4.3 Comparison with the capacity achieving scheme

It is known that the sum power minimization with per MS rate constraint problem in single-cell multicarrier systems has an optimal solution with successive interference cancellation (SIC) receivers [8]. This capacity achieving solution clearly outperforms the exhaustive search of optimal rate allocations and computationally expensive branch-and-bound method [18, 92, 20] with linear receivers. To have more insights into the performance gap between the proposed schemes and the capacity achieving solution, the proposed algorithms with the absolute upper bound [8] are compared in a single-cell scenario. The sum power
minimization in the nonlinear receiver case is a reverse problem of capacity maximization. Thus, the capacity achieving bound is provided by the power minimization in [8], rather than [20], which uses linear receivers. Due to the high computational complexities with the nonlinear receivers [8], only a small number of MSs, i.e., 4, is simulated. The comparison of CDF of average transmit power are plotted in Fig. 28. It can be seen that there is about 5 dB gap between the performance of SIC and ER. The two bit loading algorithms have almost the same performances which are about 1.3 dB better than the ER. The BSW algorithm is 1 dB better than the bit loading algorithms and about 1.5 dB worse than the SIC upper bound.

\[ R_{\text{Min}} = 4, 4 \text{ subcarriers, } R_{\text{Cell}} = 700\text{m, NumMS} = 4, \text{Nr} = 8, |\pi| = 1. \]

Fig 28. Comparison of proposed algorithm and optimal SIC receiver ([78], published by permission of IEEE).

4.5 Summary and discussions

A greedy bit loading algorithm was proposed for the resource allocation problem in multicell multicarrier systems. The algorithm iteratively loads bits to the best candidate link. It requires \( NK \) times execution of fixed rate allocation algorithm for the optimal power, BSCs and beamforming vectors with the given rate target. To reduce the complexity, a gradient based algorithm is proposed so
that the explicit calculation of power, beamforming vectors and BSCs is avoided at each iteration. Finally, to revise the final non-optimal rate allocations caused by the greedy nature of the bit loading schemes, a BSW algorithm that starts from any full rate allocation is proposed. The advantage to start from the full rate allocation is that update procedure of power, beamforming vectors and BSCs from the zero rate state to the full rate state, is avoided. This results in a dramatic computation efforts reduction over the bit loading schemes. Moreover, it will reach a locally optimal rate allocation state. It is shown by simulations, that the proposed algorithms have better performance than the schemes that do not consider subcarrier allocation, i.e., equal rate allocation over subcarriers. The BSW is observed to perform $5 \sim 8$ dB better than the equal rate allocation with single BS reception. The performance gain becomes larger as number of MSs grows because of the better exploration of the multiuser diversity gain. Thus, the BSW that starts from full rate allocation, e.g., equal rate allocation, is promising in the single BS processing setup for its good performance and fast convergence speed. When the size of the BSC becomes large, the spectrum of the equivalent channel at the output of the receive filter becomes less frequency selective. The greedy and the gradient algorithms become better than the BSW-ER in the large BSC case.
5 Comparison of antenna arrays in a 3-D multicell network

The purpose of this chapter is two-fold. The first aim is to get insights into the system level performance of the proposed resource allocation algorithms with a realistic multicell 3D channel and antenna array model. The second objective is to find out which type of spatial antenna array is the most suitable one with the extra vertical dimension of propagation. The focus is mainly on uplink transmission in scenarios specified in LTE Release 9 (LTE-A) [93], where multiple BS coordination reception and power control are included.

This chapter is structured as follows. In Section 5.1, the multicell uplink model, spatial channel model and antenna array model are presented. Section 5.2 applies the power control and BS cooperation. The simulation results will be given in Section 5.3, and, finally, in Section 5.4, the chapter is concluded.

5.1 System and channel model

5.1.1 Multicell system

The considered cellular system consists of $K$ MSs and $B$ BSs in flat fading channels. Assuming that all MSs have one transmit antenna while BS $i$ has $N_i$ receive antennas, the received signal $y_i$ at the $i$th BS is written as

$$y_i = \sum_{k=1}^{K} \sqrt{p_k} h_{k,i} x_k + \eta_i,$$

(42)

where $p_k$ is the transmit power of MS $k$, $h_{ij} \in \mathbb{C}^{N_i \times 1}$ is the complex channel response from the $k$th MS to the $i$th BS, $x_k$ is the transmitted symbol of MS $k$ with average power normalized to 1 and $\eta_i$ is the AWGN vector at BS $i$ with variance $\sigma^2$ for each receive antenna.

For multi-BS cooperation reception and beamforming, MS $k$ is simultaneously received and processed by a set of chosen BSs $\pi_k$, a set of linear beamformers
can be utilized to extract its signal from those BSs, i.e.,

\[
\hat{x}_k = \sum_{i \in \pi_k} w_{k,i}^H y_i = \sqrt{p_k} \sum_{i \in \pi_k} w_{k,i}^H h_{k,i} x_k + \sum_{i \in \pi_k} \left( \sum_{k' = 1, k' \neq k}^K \sqrt{p_{k'}} w_{k',i}^H h_{k',i} x_{k'} + w_{k,i}^H \eta_i \right)
\]

(43)

where \((\cdot)^H\) means Hermitian of the matrix and \(w_{k,i} \in \mathbb{C}^{N_{Rx} \times 1}\) is the receiver beamformer gain vector for MS \(k\)'s received signal at BS \(i\) and \(\pi_k\) is the multi-BS set serving MS \(k\). Its elements are the indices of BSs jointly providing service to MS \(k\) which are denoted by \(\pi_k(\cdot)\).

### 5.1.2 3D spatial channel

The 3D channel model is based on the WIM II channel model, which is a geometry based stochastic model [63]. A more general form of a MIMO channel matrix than (43) is given by

\[
H(t, \tau) = \sum_{n=1}^{N} H_n(t; \tau),
\]

(44)

where \(n\) is the path index, \(N\) is the total number of paths, \(\tau\) is the delay time and \(t\) is the time index. \(H_n(t; \tau)\) is the channel matrix for cluster \(n\), which is expressed as [63]

\[
H_n(t; \tau) = \int F_R(\phi)H(t; \tau, \phi, \varphi)F_T^\dagger(\varphi)d\phi d\varphi,
\]

(45)

where \(F_R(\phi)\) and \(F_T(\varphi)\) are the beam gain matrix for the received antenna (Rx) and transmit antenna (Tx) in direction \(\phi\) and \(\varphi\) respectively. \(H(t; \tau, \phi, \varphi)\) is the dual-polarized channel response matrix. The channel coefficient from the Tx element \(s\) to Rx element \(u\) for cluster \(n\) is given as follows [63]

\[
h_{u,s,n}(t; \tau) = \sum_{m=1}^{M} \left[ \begin{array}{c} \alpha_{VV,n,m}^+ \\ \alpha_{VH,n,m}^+ \\ \alpha_{HV,n,m}^+ \\ \alpha_{HH,n,m}^+ \end{array} \right] \left[ \begin{array}{cccc} F_{R,u,V}(\varphi_{n,m}) & F_{R,u,H}(\varphi_{n,m}) \\ \bar{F}_{R,u,V}(\varphi_{n,m}) & \bar{F}_{R,u,H}(\varphi_{n,m}) \end{array} \right] \left[ \begin{array}{cccc} \alpha_{VV,n,m}^- & \alpha_{VH,n,m}^- & \alpha_{HV,n,m}^- & \alpha_{HH,n,m}^- \end{array} \right] \\
\times \exp(j2\pi \lambda_0^{-1}(\varphi_{n,m} \cdot \bar{r}_{R,u})) \exp(j2\pi \lambda_0^{-1}(\varphi_{n,m} \cdot \bar{r}_{T,u})) \exp(j2\pi v_{n,m} t) \delta(\tau - \tau_{u,m}),
\]

(46)
where $F_{R,u,V}$ and $F_{R,u,H}$ are the field patterns for vertical and horizontal polarizations of antenna element $u$ respectively, $\alpha_{n,m}^{VV}$ and $\alpha_{n,m}^{VH}$ are the complex gains of vertical-to-vertical and horizontal-to-vertical polarizations of ray $n$, $m$ respectively. Parameter $\lambda_0$ is the wave length of the carrier frequency, $\varphi_{n,m}$ is the angle-of-arrival (AoA) unit vector, $\hat{\varphi}_{n,m}$ is the angle-of-departure (AoD) unit vector, $\bar{r}_{T,s}$ and $\bar{r}_{R,u}$ are the location vectors of elements $s$ and $u$ respectively, and $v_{n,m}$ is the Doppler frequency of ray $n$, $m$. If polarization is not considered, the central matrix in the second line of (3) is replaced by scalar $\alpha_{n,m}$ and only the vertically polarized field pattern is considered. More detailed description of the channel model can be found in [63].

The single-element field pattern is adopted from [90] as

$$A_A(\theta) = -\min \left[12\left(\frac{\theta}{\theta_{3dB}}\right), A_m\right], -180^\circ \leq \theta \leq 180^\circ, \quad (47)$$

where $\theta_{3dB}$ is the 3dB beamwidth, which is 70° for 3-sector cell and $A_m = 20$dB is the maximum attenuation.

### 5.1.3 Antenna array

The spatial displacement of the receive and transmit antenna elements and phase shifts among them is included in $\exp(j2\pi\lambda_0^{-1}(\varphi_{n,m} \cdot \bar{r}_{R,u}))$ and $\exp(j2\pi\lambda_0^{-1}(\hat{\varphi}_{n,m} \cdot \bar{r}_{T,s}))$ in (46), where $\bar{r}_{R,u} = [x_u, y_u, z_u]^T$ is the location of the $u$th receive antenna element and AoA unit vector is

$$\varphi_{n,m} = [\cos \theta_{n,m} \cos \vartheta_{n,m}, \sin \theta_{n,m} \cos \vartheta_{n,m}, \sin \vartheta_{n,m}]^T, \quad (48)$$

where $\theta_{n,m}$ is the arrival azimuth angle and $\vartheta_{n,m}$ is the arrival elevation angle of subpath $n$, $m$. The phase offset of element $u$ can be calculated as

$$\Delta \psi_{u,n,m} = 2\pi\lambda_0^{-1} \varphi_{n,m} \cdot \bar{r}_{R,u}$$

$$= 2\pi\lambda_0^{-1}(x_u \cos \theta_{n,m} \cos \vartheta_{n,m}$$

$$+ y_u \sin \theta_{n,m} \cos \vartheta_{n,m} + z_u \sin \vartheta_{n,m}). \quad (49)$$

The transmit antenna array and AoD are similar. An illustration of the antenna coordination system and incident wave is shown in Fig. 29.

For a UPA with $N_R = N_{RxV}N_{RxH}$ elements, the channel vector is $h_{k,i}^{UPA} \in \mathbb{C}^{N_R \times 1}$, where $N_{RxV}$ and $N_{RxH}$ is the number of vertical and horizontal elements,
respectively. For a ULA with same amount of elements, the channel vector is $h_{k,i}^{\text{ULA}} \in \mathbb{C}^{N_{\text{RxH}} \times 1}$, where the output of the $u$th port is the sum of the vertical co-phased signals as

$$h_{k,i,u}^{\text{ULA}} = \sum_{j=(u-1)\times N_{\text{RxV}}+1}^{u\times N_{\text{RxV}}} h_{k,i,j}^{\text{UPA}}, \quad 1 \leq u \leq N_{\text{RxH}}. \quad (50)$$

With smaller size of beamforming vector, the noise power of ULA in [43], $\|w_{k,i}\|^2 \sigma^2$, is $N_{\text{RxV}}$ times smaller than UPA.
With the above expressions, a 16-elements spatial ULA antenna or UPA antenna can be easily modeled as in Fig. 30. It can be seen in the figure that the element geometries of these two are the same but the vertical column elements of ULA are co-phased. Thus, elements from the same column of ULA are not capable of forming beams vertically. In UPA, each antenna element output can be weighted by a complex weight, $w_u$, and it is capable of forming the beams in both domains. The drawback of the UPA antenna is that it needs $N_{RxV}$ times the amount of amplifiers than the ULA antenna, which causes $N_{RxV}$ times larger noise power, where $N_{RxV}$ is the number of vertical column elements. The antenna array patterns of 16-elements ULA and UPA are plotted in Fig. 31. Cable losses and combining losses in the circuit are omitted, which practically are larger in ULA than in UPA. Amplifiers in UPA can be placed right after the antenna elements and in ULA after the signal combiner, which makes the feeder cables longer in ULA. Thus, the ULA model could be used as an upper bound of the practical system.

Since the signal power of ULA can be at best the same as that of UPA after co-phased combining, for $N_{RxV} = 4$, the post processing SNR of ULA is at best 6dB larger than for UPA. For interference limited case, the spatial distributed interference plays a bigger role than noise. Because UPA has a better freedom in vertical domain to form the beams, the SINR would benefit from vertical
beamforming and interference avoidance. The UPA/ULA performance under noise and interference limited cases are evaluated in Section 5.3.

## 5.2 Uplink power control and BS cooperation

Multiple BS joint reception is interesting in 3D channel models. The UL power control, multi-BS cooperation and receive beamforming with per MS SINR can be formulated as a minimum transmit power problem \[15, 94\].

Given the receiver beamformers \(w_{k,i}\) and the cooperation BS set \(\pi_k\), the effective SINR of MS \(k\) can be written as

\[
\gamma_{k,\pi_k} = \frac{\left| \sum_{i \in \pi_k} w_{k,i}^H h_{k,i} \right|^2 p_k}{\sum_{k' \neq k, i \in \pi_k} \left| \sum_{i \in \pi_k} w_{k',i}^H h_{k',i} \right|^2 p_{k'} + \sum_{i \in \pi_k} \|w_{k,i}\|^2 \sigma^2}, \tag{51}
\]

where \(|\cdot|\) denotes the absolute value and \(\|\cdot\|\) the standard Euclidean vector norm. The power control, BS cooperation reception and beamforming problem can be reformulated as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} p_k \\
\text{subject to} & \quad \log (1 + \gamma_{k,\pi_k, w_k}) \geq \Gamma_k, \forall k \tag{52}
\end{align*}
\]

where \(\Gamma_k\) is the minimum rate requirement of MS \(k\) and \(P_k\) is the maximum transmit power of MS \(k\). \(P_k\) is the maximum transmit power of MS \(k\) and the optimal beamforming vector is given by

\[
\{w_{k,1}, w_{k,2}, \cdots, w_{k,|\pi_k|}\} = \arg \max_{w_{k,i}} \frac{\left| \sum_{i \in \pi_k} \tilde{w}_{k,i}^H h_{k,i} \right|^2 \tilde{p}_k}{\sum_{k' \neq k, i \in \pi_k} \left| \sum_{i \in \pi_k} \tilde{w}_{k',i}^H h_{k',i} \right|^2 \tilde{p}_{k'} + \sum_{i \in \pi_k} \|\tilde{w}_{k,i}\|^2 \sigma^2}. \tag{53}
\]

Although the problem (53) is a non-convex problem in general, the minimum total transmit power solution, the optimal receiving BSs set and the optimal beamforming vector can be found after iterative search, as long as the power vectors of the MSs are feasible. A more detailed description of the algorithm can be found in [76].
5.3 Simulation results

Consider a cellular system containing 19 BSs with 3 sectors per BS, i.e., 57 sectors in total. As shown in Fig. 9, the central 57 sectors are the original sectors while the outer sectors are the copies of the central sectors. The blue circles indicate MSs and the black bars indicate the antenna board directions of selected sectors. Multiple BSs can jointly processing the received signals from one MS. The units of the x and y axes are meters. The edge effect is eliminated by wrapping around the network [89]. Other simulation parameters can be found in Table 2.

The system of UPA/ULA with multiple BS joint reception and beamforming are to be simulated. The performance will be evaluated in terms of average transmit power, and the number of feasible links that can be supported by the network. The performance of the $4 \times 4$ ULA and UPA are studied in two scenarios, i.e., noise limited scenario and interference limited scenario.

In Fig. 32 a noise limited case is studied, where 30 MSs are uniformly distributed in 57 sectors. Single BS, two and three BSs joint reception are simulated. It is clear that despite of how many BSs are involved in the reception, ULA always performs better than UPA. It is not surprising, since UPA has 6dB larger noise power than ULA. With a larger beamforming gain, UPA is only 1.8dB worse than ULA in the single BS reception case.

The performances of an interference limited case are plotted in Fig. 33, where 160 MSs are generated. As can be seen from the figure, no matter how many BS are receiving, UPA performs better than ULA. When single BS is receiving, the beamforming gain of UPA is larger compared with ULA. As much as 5 dB gain can be get from UPA when CDF of transmit power is 0.5, bearing in mind that UPA has 6 dB larger noise power at the combiner than ULA. If more BSs are involved in the reception, the vertical beamforming gain is smaller, because two separated cooperating ULA antennas can also form the beam in vertical domain. The more BSs are involved in the cooperation, the better/narrower vertical beams can be shaped. When three BSs are used for joint reception, UPA is slightly better than ULA.

The performance of noise limited case with SINR of 8dB is plotted in Fig. 34. As the SINR constraint gets higher, the performance gap between ULA and UPA reduces. In an interference limited scenario, 160 MSs with SINR constraint
of 8 dB per MS is plotted in Fig. 35. The single BS and two BSs, joint reception curves are not plotted, because with a high SINR constraint and small size of cooperation BS cluster, the feasibility of the link can not be guaranteed, i.e., the maximum transmit power constraint is exceeded. Compared with the 0 dB constraint in Fig. 33, the UPA beamforming gain of three BSs is huge.

The feasibility performance of the two antenna arrays and BS cooperations are plotted in Fig. 36 and Fig. 37. In the interference limited scenario, for single BS reception, the supported number of MSs nearly doubles. For the interference limited case, UPA can support more MSs than ULA with the same amount of receiving BSs. With single BS reception, UPA supports about three times the amount of MSs as ULA at feasibility of 90%.

Fig. 38 examines the performance of UPA and ULA in different elevation angular spread (EAS) cases with single BS reception. The small, medium and large EAS corresponds to the rural, urban macro and suburban scenarios, respectively. As it is shown, UPA always performs better than ULA. In the small EAS case, UPA has the largest gain. If EAS becomes larger, there are more diversity gains coming from elevation domain and beamforming gains becomes less.
5.4 Summary and discussions

A 3D multicell channel model and spatial antenna array models including ULA and UPA were presented. The UL system level simulation were carried out with multiple BS joint reception, power control and beamforming. In a noise limited scenario, ULA was observed to outperform the UPA, because the latter has more amplifiers than the former. This fact increases noise power. With the help of vertical coherent combining in UPA, the loss due to noise is partly compensated. In an interference limited scenario, UPA is better than ULA due to the large vertical beamforming gain and elevation domain interference avoidance. However, the performance gap between UPA and ULA reduces as the number of cooperating BSs increases. The performance of UPA and ULA are also compared in different scenarios with different elevation angular spread, from where a conclusion is that UPA performs significantly better than ULA in rural scenarios.
Fig 34. Comparison of the transmit power of ULA and UPA with the SINR constraint of 8dB in a noise limited scenario (30 MSs in 57 sectors) ([80], published by permission of IEEE).

Fig 35. Comparison of the transmit power of ULA and UPA with the SINR constraint of 8dB in an interference limited scenario (160 MSs in 57 sectors) ([80], published by permission of IEEE).
Fig 36. Probability of feasible connections of ULA vs. UPA with the SINR constraint of 0 dB ([80], published by permission of IEEE).

Fig 37. Probability of feasible connections of ULA vs. UPA with the SINR constraint of 8 dB ([80], published by permission of IEEE).
Fig 38. Transmit power of ULA vs. UPA with the SINR constraint of 0 dB with different elevation angular spread in an interference limited scenario (160 MSs in 57 sectors) ([80], published by permission of IEEE).
6 Conclusions and future work

The resource allocation strategies for the 3D cellular uplink were studied in this thesis. In particular, the focus was on the sum power minimization with per MS rate constraint in the coordinated multicell multicarrier systems. Attention was also on the development of 3D channel and antenna array models and application of the proposed power control algorithms in such environment. Each chapter focused on a different but related problem. The starting point was the simple scenario of MIMO multicarrier MAC with nonlinear receivers. Next, a more complicated but still optimality-achievable single-carrier coordinated multicell scenario was considered. Finally, the NP-hard nonconvex $P_{\text{min}}$ problem in the coordinated multicell multicarrier scenario was considered.

Chapter 2 studied the $P_{\text{min}}$ problem in MIMO MAC with the SIC receiver. Two existing methods were reviewed and modified to fit the multicarrier scenario. The proposed geometry aided algorithms transform the $P_{\text{min}}$ problem into a series of convex WSRmax problems. The convexity of the capacity bound is utilized to avoid full search on the rate bound. In the original GA algorithm, once the optimal user weight is found to be inside or outside the constructed feasible/infeasible region, the user weight search with current power constraint is stopped. The power is bisected and then user weight search starts from equal weights again. In the IGA algorithm, once the optimal user weight is confirmed to be located in the feasible/infeasible region, the weight search will not go back to the equal weight point. Instead, it starts from the nearby region where the search stopped at the last iteration. Thus, the searching effort on the optimal user weights is dramatically reduced. The IGA algorithm is shown to converge faster than the two existing methods.

The $P_{\text{min}}$ problem in the single-carrier coordinated multicell MIMO uplink was studied in Chapter 3. Transmit power, beamforming and BSC were jointly optimized. The global optimal solution can be found by the proposed iterative method. With the flexibility of the cluster size, the tradeoff between the computational complexity and performance can be made. The algorithm was also applied to the scenario of fixed cluster intra-cell/inter-cell CoMP. The
simulation results showed that the proposed algorithm has large performance gain than the intra-cell/inter-cell CoMP.

Chapter 4 considered multicell multicarrier multiantenna systems. Because the problem is nonconvex, bit loading algorithms and BSW algorithms were proposed. At each iteration of the bit loading algorithm, the best link candidate that causes the minimum sum power increment with fixed rate increment in the system is calculated. The problem at each iteration is then decoupled into a single carrier fixed rate target subproblems that can be optimally solved by the algorithms in Chapter 3. To reduce the high computational complexity in the link search process, the gradient based method was proposed. Instead of calculated exact power increment on all possible links, the sum rate-power gradient is calculated. The greedy nature of the two bit loading algorithms makes the final rate allocation sub-optimal. To find better rate allocations, the BSW that starts from full rate allocation was proposed. The BSW adjusts bits between link candidates such that the sum power is reduced with per user rate unchanged. To get more insights into how far the proposed algorithm is from the nonlinear capacity achieving receiver, the performance of the proposed algorithms were compared to those of capacity achieving algorithms from Chapter 2 and the equal rate allocation over subcarriers in a simplified single-cell scenario. The numerical results showed that the BSW with equal rate allocation achieves fast convergence speed and good sum power performance.

Chapter 5 studied the 3D channel and antenna array model. Two types of antenna arrays were studied, including UPA and ULA. UPA has degree of freedom in the elevation domain but also suffers more noise caused by a larger number of amplifiers. The simulation showed that in the interference limited scenario, the benefits of vertical DoF in UPA compensates the problems caused by noise. The performance was also compared in difference scenarios, including different elevation angular spread and noise limited scenarios. The conclusion is that UPA performs significantly better than the ULA in rural scenarios.

The resource allocation in multicarrier multicell multiantenna systems is a complicated problem. How to find the optimal solution of the SPmin or WSRmax is not trivial. Obtaining a computationally efficient algorithm that provides close performance to the optimal is still an open problem. The resource allocation considered in this thesis assumes centralized processing. Theoretically, it is expected to have better performance than the single BS processing. However,
the practical constraints, e.g., backhaul capacity and signalling delay, must be taken into account. The requirement of minimum amount of signalling overhead between coordinated BSs and maximized system performance has always been an interesting issue.

In the large cell scenario, the centralized processing is rather difficult to realize due to the large signalling traffic and long delay. The distributed resource allocation is a way to solve these problems. Either the amount of exchanged neighboring cells information is kept minimum, or the intercell interference is treated as noise. Obviously, more precise the information of the interference is, the better the unitization of the information is and the better performance it can achieve. The tradeoff between the amount of signalling data and system performance needs more studies.

With the extra dimension in the elevation domain, the multiple BS antennas are able to perform the elevation beamforming. In the latest 3GPP LTE standard development, the ‘3D MIMO’ is a new research item [120]. The 3D network and antenna simulator developed in this thesis provide a suitable base for further research in these areas. In the coordinated cellular system, how to design antennas with proper element spacing and layout is interesting. Because the antenna design depends on the cell layout, user distribution, elevation/horizontal domain angular distribution, antenna element beam pattern, resource allocation strategy, etc., the closed form expression of the antenna spacing and layout is difficult to derive. Thus, the antenna design that takes these factors into account is a future direction. By the time of this thesis, the channel modeling in vertical domain is only limited in certain scenarios. The measurement and parameters extraction for more scenarios are needed to be done. The system performance in such scenarios is thus needed to be evaluated.
References


120. (2013) 3GPP TR 37.840 V12.0.0, study of radio frequency (RF) and electromagnetic compatibility (EMC) requirements for active antenna array system (AAS) base station (Release 12). Technical report, 3GPP.
Appendix 1 Modified ellipsoid method

In order to guarantee the convexity of (5), it is required that $\mu \succeq 0$ at each iteration. Thus, a slight modification is made to the original ellipsoid method and a factor $F$ is introduced to scale the step size such that $\mu \succeq 0$. The ellipsoid method is also modified from the symmetric rate case. A rate target ratio $\left[\Gamma \right]_{K}^{-1}$ is inserted to enable the searching towards any desired direction. The convergence of the ellipsoid method is proved in [96] and $g = R - \Gamma$ is shown to be the gradient of weighted sum rate as a function of $\mu$ [9].

Algorithm 11 Modified ellipsoid method

Input: $R, \Gamma, K, E, \mu$

1: $g \leftarrow \left[ R \right]_{K}^{-1} - \left[ \frac{\Gamma}{\sqrt{\mu}} \right]_{K}^{-1} R_{WSR}$
2: $x \leftarrow \left[ \mu \right]^{-1}_{K} - E \frac{g}{K \sqrt{g^T E g}}$
3: if $x \succeq 0$ then
4: $[\mu]^{-1}_{K} \leftarrow x$
5: else
6: $F \leftarrow 1$
7: while $\exists x_k < 0, \forall k$ do
8: $F \leftarrow F + 1$
9: $[\mu]^{-1}_{K} \leftarrow [\mu]^{-1}_{K} - E \frac{g}{F K \sqrt{g^T E g}}$
10: end while
11: $\mu_{K} = \max \{1 - \sum_{k=1}^{K} \mu_{k}, 0\}$
12: $E \leftarrow \frac{(K-1)^2}{(K-1)^2 - 1} \left( E - \frac{2}{K} Eg^T E \right)$

Output: $\mu, E$
Appendix 2 Proof of Lemma 1

Proof 1) Positivity: The term $\sum_{k'=1, k' \neq k}^{K} | \sum_{i \in \pi_k} w_{k,i}^H h_{k',i} |^2 p_{k'} \geq 0$ and $\| w_{k,i} \|^2 \sigma_i^2 > 0$ due to the fact that the noise power and link gain cannot be zero. Thus, $m_k(p)$ is always positive. 2) Monotonicity: Given two power vectors $p$ and $p'$ with $p \succeq p'$, it is evident that for any $\pi_k$ and $w_k$,

$$I_k(p, \pi_k, w_k) \geq I_k(p', \pi_k, w_k).$$  

(54)

Since the term $| \sum_{i \in \pi_k} w_{k,i}^H h_{k,i} |^2$ is the same, we have

$$m_k(p) = \min_{\pi_k, w_k} I_k(p, \pi_k, w_k) \geq \min_{\pi_k, w_k} I_k(p', \pi_k, w_k) \geq m_k(p').$$  

(55)

The monotonicity holds.

3) Scalability: for any $\alpha > 1$, we have

$$\alpha m_k(p)$$

$$= \alpha \min_{\pi_k, w_k} \frac{I_k(p, \pi_k, w_k)}{\sum_{i \in \pi_k} | w_{k,i}^H h_{k,i} |^2}$$

$$\geq \min_{\pi_k, w_k} \frac{\alpha I_k(p, \pi_k, w_k)}{\sum_{i \in \pi_k} | w_{k,i}^H h_{k,i} |^2}$$

$$= \alpha \sum_{k' \neq k} \left| \sum_{i \in \pi_k} w_{k,i}^H h_{k',i} \right|^2 p_{k'} + \alpha \sum_{i \in \pi_k} \| w_{k,i} \|^2 \sigma_i^2$$

(56)

where the inequality comes from the fact that the term $| \sum_{i \in \pi_k} w_{k,i}^H h_{k,i} |^2$ is positive and $\alpha > 1$. 

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As the proof holds for all $k$, the proof completes.
Appendix 3 Proof of Theorem 1

Proof From the minimum SINR constraint of (17), after a minor modification, we have

\[
\begin{bmatrix}
p_1^* \\
p_2^* \\
\vdots \\
p_K^*
\end{bmatrix} \geq \begin{bmatrix} m_1(p^*) \\
m_2(p^*) \\
\vdots \\
m_K(p^*)\end{bmatrix} = m(p^*). \tag{57}
\]

Given problem (17) is feasible and \( p^* \) is the minimum power vector, without loss of generality, suppose the equality does not hold for MS 1, for example, we can always construct another feasible power vector

\[
\begin{bmatrix}
p_1^{**} \\
p_2^{**} \\
\vdots \\
p_K^{**}
\end{bmatrix} = \begin{bmatrix} m_1(p^*) \\
m_2(p^*) \\
\vdots \\
m_K(p^*)\end{bmatrix}. \tag{58}
\]

It is evident that \( p_1^{**} = m_1(p^*) < p_1^* \) and \( \sum_{k=1}^K p_k^{**} \) is smaller than \( \sum_{k=1}^K p_k^* \), which conflicts with the assumption that \( p^* \) is the optimum. Therefore, the equality must hold, i.e., \( p^* \) is a fixed point of \( m(p) \). Next, following the approach in [16], we assume that there are two different fixed points \( p^{*1} \) and \( p^{*2} \), which are also positive. Without loss of generality, we also assume that \( p^{*1} \) has at least one element larger than \( p^{*2} \). Thus, we must be able to find an index

\[
l = \arg \max_k \frac{p_{k}^{*1}}{p_k^{*2}} \tag{59}
\]

and a scalar \( \alpha = \frac{p_{l}^{*1}}{p_l^{*2}} > 1 \). Then, we can construct a new vector \( \alpha p^{*2} \), where \( \alpha p_{k}^{*2} = p_{l}^{*1} \) and \( \alpha p_{k}^{*2} \geq p_k^{*2} \) for \( k \neq l \). However, for \( p_{l}^{*1} \), by the scalability and monotonicity properties, we also have

\[
p_{l}^{*1} = m_l(p^{*1}) < m_l(\alpha p^{*2}) < \alpha m_l(p^{*2}) = \alpha p_{l}^{*2}. \tag{60}
\]

The last equality comes from the fact that \( p_{l}^{*2} \) is also a fixed point. As this contradicts the fact that \( p_{l}^{*1} = \alpha p_{l}^{*2} \), the fixed point must be unique. The proof is completed.
Appendix 4 Proof of Theorem 2

Proof First, the SINR constraint determines that $\mathbf{p}_p^{[0]} = \mathbf{p}^\prime \succeq m(\mathbf{p}^\prime) = \mathbf{p}_p^{[1]}$ as $\mathbf{p}^\prime$ is a feasible solution. By the monotonicity property, we have $\mathbf{p}_p^{[0]} \succeq m(\mathbf{p}_p^{[0]}) = \mathbf{p}_p^{[1]} \succeq m(\mathbf{p}_p^{[1]}) = \mathbf{p}_p^{[2]} \succeq \cdots \succeq m(\mathbf{p}_p^{[t-1]}) = \mathbf{p}_p^{[t]}$. Obviously, $\mathbf{p}_p^{[t]}$ is monotonically decreasing with $t$. By the positivity property, the power vector is lower bounded by zero. Thus, it must converge to a fixed point as the index $t$ increases. Based on the fact that the fixed point is unique, the proof is completed.
Appendix 5 Proof of Theorem 3

Proof First, $\mathbf{0} \prec m(\mathbf{0})$ must hold because of the positivity property, where $\prec$ denotes the element-wise smaller. Then using the monotonicity property, we have $\mathbf{0} = \mathbf{p}_0^{[0]} \lneq m(\mathbf{p}_0^{[0]}) = \mathbf{p}_0^{[1]} \lneq m(\mathbf{p}_0^{[1]}) = \mathbf{p}_0^{[2]} \lneq \cdots \lneq m(\mathbf{p}_0^{[t-1]}) = \mathbf{p}_0^{[t]}$. Obviously, the value of the produced vector sequence is increasing. Assuming that the problem is feasible and $\mathbf{p}^*$ is the minimum power vector, we can easily obtain a sequence with Algorithm 5 with $\mathbf{p}' = \mathbf{p}^*$, i.e. $\mathbf{p}' = \mathbf{p}_p^{[0]} = m(\mathbf{p}') = \mathbf{p}_p^{[1]} = m(\mathbf{p}') = \mathbf{p}_p^{[2]} = \cdots = m(\mathbf{p}') = \mathbf{p}_p^{[t]}$. The equality is from the fact that $\mathbf{p}^*$ is the fixed point. By using the monotonicity property, we have $\mathbf{p}_p^{[t]} \succ \mathbf{p}_0^{[t]}$. It completes the proof.
Appendix 6 Proof of Theorem 4

Proof First, for any $p'$, we can always find a scalar $\alpha > 1$ satisfying $p' \preceq \alpha p^*$. Using the scalability property, we know that $\alpha p^*$ must be a feasible solution as well. Since $0 \preceq p' \preceq \alpha p^*$, we have $p_0^{[1]} \preceq p_0^{[1]} \preceq p_{op}^{[1]}$, $p_0^{[2]} \preceq p_0^{[2]} \preceq p_{op}^{[2]}$ and so on, based on the monotonicity property. According to Theorem 2 and 3, both $p_0^{[1]}$ and $p_{op}^{[2]}$ converge to $p^*$ as the index $t$ is increased, the middle part must converge to $p^*$ as well. The proof completes.
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