

Sodankylä Geophysical Observatory  
Reports



UNIVERSITY of OULU  
OULUN YLIOPISTO

## **19th Inverse Days 2013**

**with a Special Emphasis on the Mathematics of Planet Earth**

Sodankylä 10th December 2013

Inari 11th-13th December 2013

## **Abstracts**

Edited by Lassi Roininen

Report No 61  
Sodankylä 2013



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**THEMATIC DAY ON MATHEMATICS WITH APPLICATIONS IN  
GEOSPACE AND ATMOSPHERIC RESEARCH**

Sodankylä Geophysical Observatory of University of Oulu in collaboration with Arctic Research Centre of Finnish Meteorological Institute are organising a one-day thematic seminar on mathematics with applications in geospace and atmospheric research. The day includes a number of invited talks on hot topics within the domain.

The event is part of the 100 year anniversary jubileum of Sodankylä Geophysical Observatory.

**TUESDAY 10 DECEMBER 2013**

<b>Tuesday</b>	<b>Polaria Lecture Hall</b>
09:00-10:00	<i>Coffee</i>
10:00-10:30	Johanna Tamminen
10:30-11:00	Antti Kero
11:00-11:30	Timo Lähivaara
11:30-12:00	Rigel Kivi
12:00-13:00	<i>Lunch</i>
13:00-13:30	Ozone sounding experiment by Finnish Meteorological Institute
13:30-14:00	Site visit: High-power EISCAT incoherent scatter radar receiver
14:00-14:30	Tuomo Kauranne
14:30-15:00	Heikki Haario
15:00-15:30	Elena Kozlovskaya
15:30-16:00	Mykhaylo Yudytskiy
16:00	<i>Coffee and transportation to Inari</i>

## INVERSE DAYS

Inverse days will be held in the Sámi Cultural centre Sajos situated on the banks of river Juutuanjoki in Inari. Sajos is cultural and administrative centre of Sámi people in Finland.

Sajos offers modern premises with latest technology. It offers for the Inverse Days the natural surroundings of Lake Inari with living culture of the Sámi people, Europe's only indigenous people.

Advised talk durations:

- Keynote talk: 45 minute talk + 5-10 mins discussion
- Contributed talk: 25 minute talk + 5 mins discussion

### WEDNESDAY 11 DECEMBER 2013

<b>Wednesday</b>	<b>Main Hall</b>	<b>Parliament Hall Solju</b>
11:00-12:45	<i>Lunch</i>	
12:45-13:00	Opening of the Inverse Days	
13:00-14:00	Erkki Somersalo	
14:00-14:30	Aku Seppänen	
14:30-15:00	Per Christian Hansen	
15:00-15:30	<i>Coffee</i>	
15:30-16:00	Matti Lassas	Maaria Rantala
16:00-16:30	Michel Cristofol	Petri Varvia
16:30-17:00	Keith Rogers	Matti Leinonen
17:00-18:00	Annual meeting of the Finnish Inverse Problems Society	
19:00-23:30	<i>Conference dinner in Hotel Inari</i>	



**THURSDAY 12 DECEMBER 2013**

<b>Thursday</b>	<b>Main Hall</b>	<b>Parliament Hall Solju</b>
09:00-10:00	Nuutti Hyvönen	
10:00-10:30	<i>Coffee</i>	
10:30-11:00	Daniela Calvetti	Matti Viikinkoski
11:00-11:30	Samuli Siltanen	Melessew Nigussie
11:30-12:00	Marcel Ullrich	Kristoffer Hoffmann
12:00-13:00	<i>Lunch</i>	
13:00-13:30	Hanne Kekkonen	Joonas Ilmavirta
13:30-14:00	Pia Heins	Tapio Helin
14:00-14:30	Joseph Volzer	Esa Vesalainen
14:30-15:00	<i>Coffee</i>	
15:00-15:30	Dong Liu	Alberto Sorrentino
15:30-16:00	Kimmo Karhunen	Sampsa Pursiainen
16:00-16:30	Tommi Brander	Harri Auvinen
16:30-17:00	Hanna Kiili	Sarah Hamilton
19:00-23:30	<i>Sauna in Hotel Korpikartano</i>	

**FRIDAY 13 DECEMBER 2013**

<b>Friday</b>	<b>Main Hall</b>	<b>Parliament Hall Solju</b>
09:00-10:00	Josef Durech	
10:00-10:30	<i>Coffee</i>	
10:30-11:00	Matthias Joachim Ehrhardt	Jussi Toivanen
11:00-11:30	Derek McKay-Bukowski	Meghdoot Mozumder
11:30-12:00	Ismael Bleyer	Mikhail Romanov
12:00-13:00	<i>Lunch</i>	

## PARTICIPANTS

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3. Ismail Bleyer, University of Helsinki, Finland
4. Eemeli Blåsten, University of Helsinki, Finland
5. Roberta Bosi, University of Helsinki, Finland
6. Tommi Brander, University of Jyväskylä, Finland
7. Daniela Calvetti, Case Western Reserve University, USA
8. Pedro Caro, University of Helsinki, Finland
9. Lucas Chesnel, Aalto University, Finland
10. Michel Cristofol, Aix-Marseille University, France
11. Yiqiu Dong, Technical University of Denmark
12. Josef Durech, Charles University in Prague, Czech Republic
13. Matthias Joachim Ehrhardt, University College London, United Kingdom
14. Georgios Fotopoulos, University of Oulu, Finland
15. Andoni Garcia, University of Jyväskylä, Finland
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19. Sarah Hamilton, University of Helsinki, Finland
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21. Markus Harju, University of Oulu, Finland
22. Andreas Hauptmann, University of Helsinki, Finland
23. Markus Hauru, University of Helsinki, Finland
24. Pia Heins, University of Münster, Germany
25. Tapio Helin, University of Helsinki, Finland

26. Kristoffer Hoffmann, Technical University of Denmark
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68. Zenith Purisha, University of Helsinki, Finland
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70. Lassi Päivärinta, University of Helsinki, Finland
71. Maaria Rantala, University of Helsinki, Finland
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99. Mykhaylo Yudytskiy, Johann Radon Institute for Computational and Applied Mathematics, Austria
100. Miren Zubeldia, University of Helsinki, Finland
101. Markku Åkerblom, Tampere University of Technology, Finland

## ABSTRACTS

# PARALLEL GLOTTAL INVERSE FILTERING (GIF) USING MCMC METHODS

HARRI AUVINEN, SAMULI SILTANEN

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TUOMO RAITIO, PAAVO ALKU

Aalto University, Finland

The physiological process of speech production can be divided into two stages: excitation and filtering. The excitation corresponds to the air flow that streams from the lungs and generates oscillations in the vocal folds. The second process, filtering, corresponds to the effects of the vocal tract, that is, the physiological filter that starts at the vocal folds and ends at lips and nostrils. The vocal tract is influenced, for example, by the positioning of the tongue and the movement of the lips. Glottal inverse filtering (GIF) is a technique for estimating the glottal volume velocity waveform (i.e. the glottal source) from a speech signal. Glottal inverse filtering involves first estimating the vocal tract filter, which is then used to cancel the effect of the vocal tract. The resulting signal is the time-domain waveform of the estimated glottal source. Glottal inverse filtering is an inverse problem that is difficult to solve accurately.

In this work we introduce a new GIF method capable of estimating the glottal source from a speech pressure signal recorded outside the lips. For sustained non-nasalized vowels produced with normal phonation of low pitch, the existing GIF methods are, in general, capable of estimating the glottal source with tolerable accuracy. However, the performance of current GIF methods typically deteriorates in the analysis of high-pitch voices. This accuracy degradation is explained by the prominent harmonic structure in high-pitch voices which gives rise to biasing of the formant estimates. Biased formant estimates, in turn, result in insufficient cancellation of the true resonances of the vocal tract, which distorts the estimated voice source.

In our approach, advanced computational inversion methods are utilized in order to improve the accuracy of GIF. More specifically, we take advantage of Markov chain Monte Carlo (MCMC) methods in order to determine new vocal tract models for GIF. The GIF method is based on first finding an initial estimate of the vocal tract filter using an existing inverse filtering algorithm, iterative adaptive inverse filtering (IAIF), and then refining the vocal tract model parameters within the MCMC method in order to obtain a more accurate glottal flow estimate. The goodness of the results is evaluated by comparing the original speech signal to a synthetic one created by convolving the Rosenberg-Klatt (RK) glottal flow model based excitation with the estimated vocal tract filter. The GIF model parameters include a few

first poles of the vocal tract filter and the RK excitation parameter. Since it is not possible to observe the true glottal source non-invasively in the production of natural speech, it is difficult to validate the accuracy of any GIF method. Therefore, the GIF methods in this work are evaluated with vowels synthesized using physical modeling of the voice production mechanism. Experiments, demonstrated at the conference, show that the MCMC-based GIF method improves the accuracy of the results significantly at the higher fundamental frequency of the vowels. Furthermore we introduce a concept of parallel glottal inverse filtering using Techila high-performance computing (HPC) environment.



## A DETERMINISTIC APPROACH TO THE GLOTTAL INVERSE FILTERING PROBLEM

ISMAEL BLEYER

University of Helsinki, Finland

"Digital Speech Processing" refers to the study of a speech signal. Namely, these signals are processed in a digital representation, as for example, synthesis, analysis, enhancement, compression and recognition may refer to this process.

In this talk we are interested on solving the core problem known as "Glottal Inverse Problem". Commonly this problem can be modelled by convolving a pressure function (input signal) with an impulse response function (filter). Our approach is done in a deterministic setup and numerical realisation are presented for both simulated and real data.

## BOUNDARY DETERMINATION FOR P-LAPLACIAN

TOMMI BRANDER

University of Jyväskylä, Finland

We recover the gradient of a scalar conductivity defined on a smooth bounded open set in  $\mathbb{R}^d$  from the Dirichlet to Neumann map arising from the  $p$ -Laplace equation. For any boundary point we recover the gradient using Dirichlet data supported on an arbitrarily small neighbourhood of the boundary point. We use a Rellich-type identity in the proof. Our results are new when  $p \neq 2$ . In the  $p = 2$  case boundary determination plays a role in several methods for recovering the conductivity in the interior.

**BAYESIAN PRECONDITIONING FOR TRUNCATED KRYLOV  
SUBSPACE REGULARIZATION WITH AN APPLICATION TO  
MAGNETOENCEPHALOGRAPHY (MEG)**

DANIELA CALVETTI

Case Western Reserve University, USA

We consider the computational problem arising in magnetoencephalography (MEG), where the goal is to estimate the electric activity within the brain non-invasively from extra-cranial measurements of the magnetic field components. The problem is severely ill-posed due to the intrinsic non-uniqueness of the solution, and suffer further from the challenges of starting from a weak data signal, its high dimensionality and complexity of the noise, part of which is due to the brain itself. We propose a new algorithm that is based on truncated conjugate gradient algorithm for least squares (CGLS) with statistically inspired left and right preconditioners. We demonstrate that by carefully accounting for the spatiotemporal statistical structure of the brain noise, and by adopting a suitable prior within the Bayesian framework, we can design a robust and efficient method for the numerical solution of the MEG inverse problem which can improve the spatial and temporal resolution of events of short duration.

# INVERSE PROBLEM FOR AN ENERGY BALANCE MODEL WITH MEMORY (EBMMs)

MICHEL CRISTOFOL

Aix-Marseille University, France

We study parameter estimation for one-dimensional energy balance models with memory (EBMMs) given localized temperature measurements. These models introduced a delay mechanism in order to take into account the long response times of the ice sheets to temperature changes. Our results apply to a wide range of nonlinear, parabolic partial differential equations (PDEs) with integral memory terms. We show that a space-dependent parameter (here the albedo) can be determined uniquely everywhere in the PDE's domain of definition  $\mathcal{D}$ , using only temperature information in a small subdomain  $\mathcal{E} \subset \mathcal{D}$  (see [2]).

## References

- [1] Bhattacharya, K., Ghil, M. & Vulis, I. L. 1982 Internal variability of an energy-balance model with delayed albedo effects. *Journal of the Atmospheric Sciences*, **39**, 1747–1773.
- [2] L. Roques, Mickaël D. Chekroun, Michel Cristofol, Samuel Soubeyrand, Michael Ghil, Parameter estimation for energy balance models with memory, submitted to Philosophical Transactions of Royal Society A, 2013.

## REVEALING THE NATURE OF ASTEROIDS BY INVERSION OF REMOTE-SENSING DATA

JOSEF DURECH

Charles University in Prague, Czech Republic

In the last decade, the asteroid research has been significantly influenced by theoretical results obtained in the field of applied mathematics. The first step was so called "lightcurve inversion", which is an inversion method that reconstructs a shape of an asteroid from its disk-integrated photometry. With this method, hundreds of asteroid models have been derived. Its potential is much larger - it will enable us to reconstruct shapes and spin states of a significant fraction of the whole asteroid population ( 500,000 known asteroids so far). Together with photometry, other disk-resolved (low-resolution images, silhouettes from occultations,...) or disk-integrated data (thermal infrared flux, for example) can be analyzed and detailed models can be derived by so called "multi-modal data inversion". In this talk, I will (i) review the mathematical background of these methods, (ii) highlight the main achievements accomplished in the asteroid research with this approach, (iii) describe the current problems we working on, and (iv) mention the plans and prospects for the future.

# MULTI-MODALITY IMAGE RECONSTRUCTION WITH PARALLEL LEVEL SETS

MATTHIAS JOACHIM EHRHARDT, KRIS THIELEMANS, SIMON R. ARRIDGE  
University College London, UK

**Introduction:** Recent advantages in technology have enabled us to combine imaging systems. Among others positron emission tomography (PET) and magnetic resonance imaging (MRI) scanners can be combined to simultaneously image function and anatomy of the human body [1]. As function follows anatomy the images to be reconstructed are expected to share a lot of information. By coupling these two inverse problems we aim to exploit the shared information in the data [2]. Similarly shared structures have been exploited in a joint framework in colour image processing [3,4] and geophysics [5].

**Reconstruction with Parallel Level Sets:** The joint reconstruction of two imaging modalities can be posed as a joint minimization problem

$$(u^\#, v^\#) \in \operatorname{argmin}_{(u,v)} \Phi(u, v)$$

with the objective function having the form

$$\Phi(u, v) = d_{1,f}(\mathcal{P}u) + d_{2,g}(\mathcal{B}v) + \alpha\Psi(u, v)$$

where the data fidelities depend on the noise models considered and  $\Psi(u, v)$  expresses our prior knowledge about the state of  $u$  and  $v$  [2]. One way of modelling the shared structure in the two modalities is to say that the spatial gradients in both images align or equivalently the images having parallel level sets [4] which can be achieved by setting the prior to be

$$\Psi(u, v) = \int \|\nabla u\| \|\nabla v\| - |\langle \nabla u, \nabla v \rangle|.$$

As  $\Psi$  vanishes when one of the images is flat this prior is not guaranteed to have a regularizing effect. This can be introduced for instance by combining joint structural information and local smoothness as

$$\Psi_\beta(u, v) = \frac{1}{\beta} \int \|\nabla u\|_\beta \|\nabla v\|_\beta - \|\langle \nabla u, \nabla v \rangle\|_{\beta^2}$$

where  $\|x\|_\beta = \sqrt{\|x\|^2 + \beta^2}$  [4].

**Results:** Here we restrict ourselves to estimate only one image given a second. We will apply this prior to reconstruct from highly undersampled MRI data [6] assuming our image is similar to another image either obtained from PET or from another MRI contrast [7]. It can be shown numerically that by exploiting this similarity far less data is needed. Even in the case of noise far less radial samplings are needed to reconstruct the Shepp Logan phantom than by exploiting sparsity in the gradients as in [8]. In addition the results indicate that the side information does not help only locally but globally to improve the image quality.

### References

- [1] D. W. Townsend, Multimodality Imaging of Structure and Function, *Physics in Medicine and Biology*, 53(4):R1–R39, 2008.
- [2] E. Haber and D. W. Oldenburg. Joint Inversion: A Structural Approach. *Inverse Problems*, 13:63–77, 1997.
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## METHODS FOR UNDERSTANDING UNCERTAINTIES OF WEATHER AND CLIMATE MODELS

HEIKKI HAARIO

Lappeenranta University of Technology, Finland

We start with the problem of determining uncertainties of the 'closure' parameters of climate or weather models. Such parameters are needed to characterize subgrid level physical phenomena, such as clouds or precipitation. The MCMC methods developed apply to any models characterized by heavy CPU demands. A special problem here is the chaoticity, how to formulate a likelihood for chaotic dynamics. We also discuss another option to tune weather model parameters, by monitoring operational ensemble prediction systems, with essentially no additional CPU demands.



# A DIRECT RECONSTRUCTION METHOD FOR 2D ANISOTROPIC ELECTRICAL IMPEDANCE TOMOGRAPHY

SARAH HAMILTON

University of Helsinki, Finland

A novel non-iterative and noise-robust reconstruction method is introduced for the planar anisotropic inverse conductivity problem. The method is based on bypassing the unstable steps required in earlier works. The resulting algorithm helps to explain the observed distortions that occur when isotropic D-bar algorithms are used on anisotropic conductivities. The first reconstructions from noisy simulated anisotropic EIT data using an anisotropic D-bar algorithm are presented. Key aspects of the approach involve D-bar methods and inverse scattering theory, complex geometrical optics solutions, and quasi-conformal mapping techniques.

## SEMI-CONVERGENCE PROPERTIES OF KACZMARZ'S METHOD

PER CHRISTIAN HANSEN

Technical University of Denmark

Kaczmarz's method – sometimes referred to as the Algebraic Reconstruction Technique (ART) – is an iterative method that is widely used in tomographic imaging, due to its favorable semi-convergence properties. Specifically, when applied to a problem with noisy data, during the early iterations it converges very fast toward a good approximation to the exact solution, and thus produces a regularized solution. While this property is generally accepted and utilized, there is surprisingly little theoretical justification for it. The purpose of this paper is to present a rigorous analysis of the semi-convergence of Kaczmarz's method as well as its projected counterpart (and their block versions). To do this we study how the data errors propagate into the iteration vectors and we derive bounds for this noise propagation, thus providing a solid justification of their use as regularization methods. Our bounds are compared with numerical results obtained from tomographic imaging.

## LOCALLY SPARSE RECONSTRUCTION USING $\ell^{1,\infty}$ -NORMS

PIA HEINS

University of Münster, Germany

Sparse reconstructions based on minimizing  $\ell^1$ -norms have gained huge attention in signal and image processing, inverse problems, and compressed sensing recently. However, the overall sparsity enforced by minimal  $\ell^1$ -norm is not the only kind of prior information available in practice. Strong recent direction of research are related to unknowns being matrices, with prior information being e.g. low rank incorporated via nuclear norm minimization or block sparsity (or collaborative sparsity) incorporated by minimization of  $\ell^{p,1}$ -norms with  $p \in (1, \infty)$ .

In this talk we consider another type of sparsity-functionals, namely  $\ell^{1,\infty}$ -norms. Our motivation is a *local sparsity* that frequently appears in inversion with some spatial dimensions and at least one additional dimension such as time or spectral information in imaging.

First we will motivate the use of the  $\ell^{1,\infty}$ -norm as regularization functional for dictionary based reconstruction of matrix completion problems. In order to obtain computational results we will propose a reformulation of the problem on which we will do some analysis. Finally some basic results will be presented using a double splitting.

An additional TV-regularization incorporates prior information in space and may even improve the results.

# INVERSE PROBLEM FOR THE WAVE EQUATION WITH A RANDOM SOURCE

TAPIO HELIN

University of Helsinki, Finland

We consider a smooth Riemannian metric tensor  $g$  on  $\mathbb{R}^n$  and study the stochastic wave equation for the Laplace-Beltrami operator  $\partial_t^2 u - \Delta_g u = F$ . Here,  $F = F(t, x, \omega)$  is a random source that has white noise distribution supported on the boundary of some smooth compact domain  $M \subset \mathbb{R}^n$ . We study the following formally posed inverse problem with only one measurement. Suppose that  $g$  is known only outside of a compact subset of  $M^{int}$  and that a solution  $u(t, x, \omega_0)$  is produced by a single realization of the source  $F(t, x, \omega_0)$ . We ask what information regarding  $g$  can be recovered by measuring  $u(t, x, \omega_0)$  on  $\mathbb{R}_+ \times \partial M$ ? We prove that such measurement together with the realization of the source determine the scattering relation of the Riemannian manifold  $(M, g)$  with probability one.

# THE STABILITY PROPERTIES OF A CLASS OF LINEARISED HYBRID INVERSE PROBLEMS

KRISTOFFER HOFFMANN

Technical University of Denmark

Hybrid inverse problems are mathematical descriptions of novel imaging methods that make use of coupled physical phenomena to provide images with high contrast and high resolution. A certain class of hybrid imaging methods related to electrical impedance tomography can be expressed mathematically as a system of generalised Laplace equations augmented with an additional data functional which depends on the magnitude of the spatially varying electrical field. The inverse problem is to reconstruct the scalar electrical conductivity using this functional and the chosen boundary potentials. A way to approach such a non-linear inverse problem is to consider the corresponding linearisation. It turns out that ellipticity and the associated stability properties of the linearised problem are highly dependent on the chosen boundary potentials.

I will give an overview of the theory related to the stability properties of the linearised inverse problem and present the situations in which the problem is elliptic and, if not, how the loss of ellipticity will manifest itself as propagating singularities. The results are verified by a numerical implementation.

**SIMULTANEOUS RECONSTRUCTION OF BODY SHAPE AND  
CONDUCTIVITY DISTRIBUTION IN ELECTRICAL IMPEDANCE  
TOMOGRAPHY**

NUUTTI HYVÖNEN

Aalto University, Finland

The simultaneous reconstruction of the exterior shape and the internal conductivity of an examined body by electrical impedance tomography is considered. The reconstruction method is built in the framework of the complete electrode model and it is based on the Frechet derivative of the corresponding electrode current-to-voltage map with respect to the body shape. The reconstruction problem is cast into the Bayesian framework and maximum a posteriori estimates for the conductivity and the boundary geometry are computed. The feasibility of the approach is evaluated by experimental data. The results suggest that the proposed algorithm may have potential for handling an unknown measurement geometry in practical electrical impedance tomography.

## A GEOMETRIC APPROACH TO THE BROKEN RAY TRANSFORM

JOONAS ILMAVIRTA

University of Jyväskylä, Finland

Consider a Euclidean domain or a Riemannian manifold with boundary, whose boundary is split in two parts. We ask: is a function on the domain uniquely determined by its integrals over all lines (geodesics) which have endpoints in part one and reflect on part two? In other words, is the so defined broken ray transform injective?

The broken ray transform is the usual X-ray transform with reflections on a part of the boundary, and the reflections are what makes analysis difficult. We present two geometrical methods to reduce injectivity of the broken ray transform to injectivity of the X-ray transform on another domain. Our results imply new results for Calderón's problem with partial data.

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# **ELECTRICAL CAPACITANCE TOMOGRAPHY MONITORING OF NONSTATIONARY GAS-SOLID FLOW IN A HORIZONTAL DUCT**

KIMMO KARHUNEN

University of Eastern Finland

In this work, we consider a novel method for monitoring of nonstationary gas-solid flow in a horizontal duct using electrical capacitance tomography (ECT). The method is based on state estimation technique in which the time varying nature of the target can be taken into account. The information about the gas-solid interactions is incorporated into the state estimation through process evolution model that is constructed on the basis of two-phase flow model. The numerical simulations indicate that solid volume fraction and mass flow rate can be reconstructed quite accurately from the ECT measurements using the proposed method.



**CAPTURING CARBON EMISSIONS OF TROPICAL FORESTS:  
FACING SPACE ODDITY AND RUNNING FROM TIGERS IN THE  
JUNGLE?**

VIRPI JUNTILA AND TUOMO KAURANNE

Lappeenranta University of Technology, Finland

Tropical forests are a major carbon sink on Planet Earth, but their destruction is also a major source of carbon emissions to the atmosphere. Quantifying their net effect is not only an important research goal, but also a key element in the global pursuit of mitigating climate change. This mission has been codified in the United Nations REDD+ program, short for Reducing Emissions from Deforestation and forest Degradation.

Measuring the carbon content and carbon change in tropical forests must be accomplished with high spatial resolution and without significant statistical bias. Moreover, the areas to be measured are huge, often millions of hectares. This calls for a combination of many data sources: ground measurements, airborne laser scanning, and space borne imaging. Target areas are challenging in every way: by access, by international and national politics and by security concerns.

The LUT team has worked closely together with Arbonaut Ltd., a spin-off company in Joensuu, to complete carbon measurement missions on six continents. The mathematical keys to the success of such missions are two algorithmic methods developed at LUT: LAMP, or Lidar-Assisted Multi-source Program for forest inventory, and Bayesian truncated Singular Value Decomposition, or Bayesian tSVD. Both combine sound statistical estimation with regularization of a challenging data assimilation problem. In the talk, we shall discuss both algorithms and illustrate their application in Finland, USA, Ghana and Nepal.

# RESOLVING THE WHITE NOISE PARADOX IN THE REGULARISATION OF INVERSE PROBLEMS

HANNE KEKKONEN

University of Helsinki, Finland

Let us consider an indirect noisy measurement  $M$  of a physical quantity  $U$

$$M = AU + \mathcal{E}\delta, \quad \delta > 0 \tag{1}$$

where the realisation  $u$  of  $U$  is a function on a domain of  $\mathbb{R}^n$  and  $\mathcal{E}$  is normalised Gaussian white noise. The inverse problem is to find  $U$  if we are given a realisation  $m$  of the measurement  $M$ .

Tikhonov regularisation with chosen penalty function gives us an estimate

$$\mathbf{u} = \arg \min_{u \in H^1} \left\{ \frac{1}{2\sigma^2} \|Au - m\|_{L^2}^2 + \frac{\alpha}{2} \|(I - \Delta)^{1/2}u\|_{L^2}^2 \right\}. \tag{2}$$

Above we use  $L^2$  norm even though the realisations of  $\mathcal{E}$  are in  $L^2$  only with probability zero. On the other hand realisations of white noise are in  $H^s$  with probability one when  $s < -n/2$ . That is why we will show in this talk what happens when we use Tikhonov regularisation for the noise that is a realisation of the white noise having realisations in Sobolev space with negative smoothness index. We will also consider the question in which space does the estimate convergence to a correct solution when the noise variance goes to zero and what is the speed of the convergence.

This is joint work with Matti Lassas and Samuli Siltanen (University of Helsinki).

# IONOSPHERIC ELECTRON DENSITY PROFILE ESTIMATION IN SPECTRAL RIOMETRY WITH MODEL REDUCTION AND MCMC

ANTTI KERO

Sodankylä Geophysical Observatory, Finland

ANTTI SOLONEN, JUHA VIERINEN

Massachusetts Institute of Technology, USA

In the so-called spectral riometry, cosmic radio noise absorption is measured simultaneously at multiple radio wave frequencies, instead of a single band used in the traditional riometry. The main advantage of this approach is a possibility to invert the electron density height profile of the D-region ionosphere based on the frequency dependence of the absorption. However, this inversion turns out to be both nonlinear and highly ill-posed, hence needing some strong prior information on the unknown.

We study what kind of information the riometer measurements have about the electron density profile. We apply a smoothing prior and use MCMC to sample from the posterior distribution. To make MCMC sampling feasible, we reduce the dimension of the inference problem by concentrating the sampling in the directions of the leading eigenvectors of the prior covariance matrix. Simulations reveal that the electron density profiles can, indeed, be determined with some reasonable accuracy in the altitudes of relatively strong absorption.

## TM-WAVES GUIDED BY A NONLINEAR FILM

HANNA KIILI, VALERY SEROV

University of Oulu, Finland

The propagation of TM-waves in a planar waveguide with a nonlinear film surrounded by semi-infinite linear media is studied. The film is assumed to be nonmagnetic, anisotropic and it is defined by complex permittivity with a Lipschitz continuous nonlinearity. In the film the Maxwell's equations are reduced to a system of two Volterra integral equations. By Banach fixed point theorem it is shown that the unique solution exists as a uniform limit of iterations. The dispersion relation is presented and some numerical examples are given.

## OZONESONDE MEASUREMENTS OVER SODANKYLÄ AND MARAMBIO

RIGEL KIVI

Finnish Meteorological Institute

RICARDO SÁNCHEZ

Servicio Meteorológico Nacional, Argentina

Ozonesondes provide vertical profiles of ozone in troposphere and stratosphere. Long term changes in ozone are important for the earth UV shield and for the climate. A decrease in tropospheric ozone leads to warming in the troposphere and a decrease in stratospheric ozone would cause cooling in the stratosphere. In Sodankylä the first ozonesonde measurements were performed in late 1980s and since 1989 the Finnish Meteorological Institute (FMI) has made ozonesonde measurements on regular basis. The frequency of ozone soundings over Sodankylä has typically been once per week, while during measurement campaigns soundings have been performed in some cases on daily basis and with even less time difference to observe short-term variability. In spring 2011 we observed significant ozone depletion in the Arctic stratospheric vortex. This is in contrast to the Antarctic vortex, where stratospheric ozone hole has been observed regularly. A similar ozonesonde measurement program has been run at Marambio, Antarctic Peninsula. The measurements over Marambio are a result of long term cooperation between the FMI and the Servicio Meteorológico Nacional, Argentina. Here we present measurements from Sodankylä and Marambio to report extent of observed ozone depletion and long term changes at both sites.

## SEISMIC STUDIES AT THE SGO: FROM SEISMIC TOMOGRAPHY TO SEISMIC INTERFEROMETRY

ELENA KOZLOVSKAYA

Sodankylä Geophysical Observatory, Finland

Seismological studies at the University of Oulu and SGO have been carried out since 1965. The Observatory has its own pool of portable seismic instruments. With these instruments, the seismic group of the Observatory participated in numerous international passive seismic array experiments aiming at studying deep structure of the lithosphere and lithosphere-asthenosphere boundary. Studying the lithosphere-asthenosphere boundary is one of the fundamental tasks in solid Earth geosciences. The knowledge about the deep structure of the lithosphere is also important for better understanding of processes of mineral deposits formation and for diamond exploration. In passive seismic array experiments large 2D arrays of seismic stations are deployed over the target area and are recording continuous seismic data for several years. One of the main methods used for interpretation of passive seismic arrays data is seismic tomography, in which travel times of seismic waves from remote earthquakes are inverted in order to estimate 3D distribution of seismic velocities in the lithosphere. Seismic travel time tomography was used in SVEKALAPKO project (1996-1997) in order to obtain the lithosphere structure beneath southern and central Finland. It was also applied to the data of PASSEQ 2007-2008 experiment in central Europe and to the data of the POLENET/LAPNET experiment in northern Fennoscandia during the International Polar Year 2007-2009. In the last experiment a novel method of seismic data interpretation, namely, a passive seismic interferometry, was used to obtain a 3D structure of the crust beneath the POLENET/LAPNET array down to a depth of 20 km. The method is based on inversion of empirical Green functions estimated by cross-correlating ambient seismic noise recorded by different stations of the array. The Laboratory of Applied Seismology of the SGO is now developing passive seismic interferometry methods with application to mineral exploration tasks.

## INVERSE PROBLEMS OF NON-LINEAR WAVE EQUATIONS

MATTI LASSAS

University of Helsinki, Finland

We consider inverse problem for a non-linear wave equation with a time-dependent metric tensor on manifolds. In addition, we study the question, do the observation of the solutions of coupled Einstein equations and matter field equations in an open subset  $U$  of the space-time  $M$  corresponding to sources supported in  $U$  determine the properties of the metric in a larger domain  $W \subset M$  containing  $U$ .

To study these problems we define the concept of light observation sets and show that these sets determine the conformal class of the metric.

The results have been done in collaboration with Yaroslav Kurylev and Gunther Uhlmann.

# STOCHASTIC GALERKIN FINITE ELEMENT METHOD FOR ELECTRICAL IMPEDANCE TOMOGRAPHY

MATTI LEINONEN

Aalto University, Finland

The task in electrical impedance tomography is to determine the internal conductivity distribution of a physical body from boundary value measurements of current and voltage. The most accurate model for practical impedance tomography is the complete electrode model. In this work, we have developed a two-dimensional stochastic Galerkin hp-FEM solver for the complete electrode model forward problem with the conductivity and the contact resistances treated as a random field and random variables, respectively. Moreover, the reconstruction task of impedance tomography is recast as a Bayesian inference problem for which our sGFEM solver is applied.



## REGION OF INTEREST IMAGING IN ELECTRICAL IMPEDANCE TOMOGRAPHY

DONG LIU, AKU SEPPÄNEN, VILLE KOLEHMAINEN

University of Eastern Finland

SAMULI SILTANEN

University of Helsinki, Finland

ANNE-MARIA LAUKKANEN

University of Tampere, Finland

In this talk, we present a novel region of interest (ROI)-based method that reconstructs the changes in a target conductivity from electrical impedance tomography (EIT) measurements. Our method reconstructs the *absolute* images of the target before and after the change simultaneously. The key feature of the ROI-based method is that the conductivity after the change is expressed as a linear combination of the initial state and the change of the conductivity. This ROI-based method has the advantage of allowing the independent modeling of the spatial properties of the background conductivity and the change of the conductivity - by separate prior models. The proposed approach is evaluated with simulated studies, and also with experimental data from laboratory setting. The performance is examined and compared with several conventional absolute and difference imaging approaches. The results show that the accuracy of the EIT reconstructions can be improved significantly by the new approach - especially if the change of the conductivity is known to occur in a relatively small subdomain.

## ESTIMATION OF AQUIFER DIMENSIONS FROM SEISMIC SIGNALS

TIMO LÄHIVAARA

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NICHOLAS F. DUDLEY WARD

Otago Computational Modelling Group Ltd, New Zealand

TOMI HUTTUNEN

University of Eastern Finland, Kuava Ltd., Finland

JARI P. KAPIO

University of Auckland, New Zealand

This study focuses on developing computational tools to estimate aquifer dimensions from seismic measurements. The poroelastic signature from an aquifer is simulated and methods to use this signature to estimate the level of the water table and aquifer thickness are investigated. Here, the spectral-element method (SEM) is used for solving the forward model that characterizes propagation of seismic waves. The SEM combines the accuracy of the global pseudospectral method with the flexibility of the classical finite element method. The inverse problem is solved in the Bayesian framework, which makes efficient use of a priori information related to modeling and measurement uncertainties of the problem.

## USING KAIRA AS AN INTERFEROMETRIC IMAGING RIOMETER

DEREK MCKAY-BUKOWSKI

Sodankylä Geophysical Observatory, Finland

In 2011-2012, Sodankylä Geophysical Observatory undertook the construction of a new phased-array radio system for HF- and VHF-radio observations. Known as the "Kilpisjärvi Atmospheric Imaging Receiver Array" (KAIRA), this facility became operational in 2012 and has been operating continuously since then.

One of KAIRA's main experiments is the measurement of the relative radio opacity of the ionosphere, a technique known as riometry. It can accomplish directional riometry by phasing the individual antennas of KAIRA together with different time delays to form "beams" on the sky – sensitivity to a radio waves from a particular pointing direction. With multiple beams, a crude pixellation of the sky's radio opacity can be achieved. This is known as "multibeam imaging riometry".

However, KAIRA also has a cross-correlator that can be used to measure visibilities between the individual antennas of the array. Combined with the all-sky field-of-view of the constituent elements we can, for the first time, undertake full "interferometric imaging riometry".

This presentation examines the inverse methods required for this technique and demonstrates some of the first results.

**COMPENSATION OF MEASUREMENT UNCERTAINTIES IN  
ILL-POSED PROBLEMS USING THE APPROXIMATION ERROR  
APPROACH - APPLICATION TO DIFFUSE OPTICAL  
TOMOGRAPHY**

MEGHDOOT MOZUMDER

University of Eastern Finland

Diffuse optical tomography (DOT) is a technique in which 3D images of the optical properties of a target object are formed from light transport measurements using modeling and image reconstruction techniques. In a DOT setup, the surface of the body under investigation is illuminated with a focused laser beam and the transmitted light is measured at various locations on the body's surface. From the data obtained at the boundary of the body, the optical parameters, typically scattering and absorption coefficients are reconstructed. Given the ill-posed nature of the image reconstruction problem, measurement and model uncertainties can cause large artifacts in the reconstructed images. Typically these uncertainties result from uncertainty of the source and detector position on the object boundary and other system uncertainties known as coupling constants that include the source strength, coupling losses of fibers and detector efficiency or gain. Recently a theory known as the approximation error theory in the Bayesian inversion context has been developed to handle model uncertainties such as these. In this framework the key idea is to represent the model uncertainties as additive noise processes and marginalize approximately over the noise processes. In the talk we will demonstrate that it is possible to refrain from model uncertainties due to source and detector coupling and position in DOT imaging using this approach. We will describe the modeling and implementation of this method and also the tolerance of this method against poorly specified priors.

# THREE-DIMENSIONAL NON-TOMOGRAPHIC IMAGING OF THE EAST-AFRICAN EQUATORIAL IONOSPHERE USING GPS TEC

MELESSEW NIGUSSIE, BAYLIE DAMTIE

Bahir Dar University, Ethiopia

ENDAWOKE YIZENGAW

Boston College, USA

SANDRO M. RADICELLA

International Centre for Theoretical Physics, Italy

Different ionospheric tomography techniques, which include complicated algorithm, have been applied to reconstruct time dependent three-dimensional ionospheric electron density for different regions from a chain of GPS receiver measurements. However, the East-African ionosphere, which is very dynamic and complex medium, is the least imaged ionosphere due to lack of enough GPS receivers. In this study we have demonstrated new non-tomographic technique, which require GPS receiver data collected at an epoch, to reconstruct time dependent three dimensional electron density map for East-African region. The slant total electron content (sTEC) estimated from fifteen GPS receivers corresponding to the link between each receiver and visible GPS satellites has been used to adapted NeQuick 2 ionospheric model. The adaptation has been carried out by computing the effective ionization level, Az (the driver of NeQuick 2) which provides the minimum absolute difference between experimental and NeQuick 2 modeled sTEC corresponding to each GPS signal ray- path. Then after, first Az values are represented by deterministic and stochastic models, which can be obtained by applying the Generalized Least Square (GLS) and Kriging estimation techniques, respectively. The instantaneous Az map of the East- African region has been obtained by adding the outputs of these models (Universal Kriging) at any grid point continuously epoch by epoch. The Az map has been used to drive NeQuick 2 model to reproduce instantaneous time dependent three dimensional electron density maps for East-African region. We have shown that the reconstructed electron density maps are nicely mimic the small scale horizontal gradient of experimental TEC at the ionospheric pierce points, which shows the reconstruction methods are well representing the spatiotemporal variability of the equatorial ionosphere. In addition, we will show different case studies for different days and demonstrate the Universal Kriging interpolation technique together with sTEC ingestion into NeQuick 2 are the best way to image the characteristics of the ionosphere of the regions that are devoid of ground based instrument, like East-Africa region.

## SINGLE VS. MULTIPLE TRANSPONDERS FOR RADIO TOMOGRAPHY OF ASTEROIDS

SAMPSA PURSIAINEN

Aalto University, Finland

The purpose of this study was to advance numerical methods for radio tomography in which asteroid's internal electric permittivity distribution is to be recovered from radio frequency data gathered by an orbiter. The tomography approach examined was closely related to that of the CONSERT experiment aiming at recovery of a comet nucleus structure as a part of the ROSETTA mission. The focus was on signal generation via multiple sources (transponders) providing one potential, or even essential, scenario to be implemented in a challenging in situ measurement environment and within tight payload limits. The permittivity was reconstructed with a combination of the iterative alternating sequential (IAS) inverse algorithm and finite-difference time-domain (FDTD) forward simulation. Single and multiple source scenarios were compared in two-dimensional localization of permittivity anomalies.

## MODELLING AND ANALYSING ORIENTED FIBROUS STRUCTURES

MAARIA RANTALA, MATTI LASSAS, JOUNI SAMPO,  
JOUNI TAKALO, JUSSI TIMONEN, SAMULI SILTANEN

University of Helsinki, Finland

A mathematical model for fibrous structures using a direction dependent scaling law is presented. The orientation of fibrous nets (e.g. paper) is analysed with a method based on the curvelet transform. The curvelet-based orientation analysis has been tested successfully on real data from paper samples: the major directions of fibre orientation can apparently be recovered. Similar results are achieved in tests on data simulated by the new model, allowing a comparison with ground truth.

## ROUGH POTENTIAL RECOVERY IN THE PLANE

KEITH ROGERS

Instituto de Ciencias Matemáticas, Spain

We will consider the inverse scattering problem at a fixed energy in the plane. That is to recover a potential from the scattered plane waves with a fixed frequency, incoming from all directions and then measured in all directions. Following the pioneering work of Bukhgeim, we will provide explicit formulae with which one can recover compactly supported potentials with half a derivative in  $L^2$  and show that below this level of regularity the recovery process can fail. This is joint work with Kari Astala and Daniel Faraco.



# SIMULTANEOUS RECONSTRUCTION AND SEGMENTATION WITH PROBABILISTIC HIDDEN MARKOV MODEL REGULARIZATION

MIKHAIL ROMANOV

Technical University of Denmark

We formulate a general bayesian framework for the simultaneous reconstruction and segmentation to enhance the robustness of the reconstruction and segmentation for the noise and for the lack of data and to enhance the material separation on the tomogram. Chosen the optimization strategy, we do experimental validation of our technique by comparison with a Filtered Back Projection (FBP) and a Total Variation (TV) methods for a simple problem instance with a grey-scale segmentation.

**ELECTRICAL IMPEDANCE TOMOGRAPHY -BASED SENSING  
SKIN FOR DAMAGE DETECTION IN CONCRETE**

AKU SEPPÄNEN

University of Eastern Finland

MILAD HALLAJI, MOHAMMAD POUR-GHAZ

North Carolina State University, USA

We have developed a low cost sensing skin for damage detection in concrete. The sensing skin consists of a thin layer of electrically conductive copper paint that is applied to the surface of the concrete structure. Cracking of the concrete substrate results in the rupture of the sensing skin, decreasing the electrical conductivity of the sensing skin locally. Electrical Impedance Tomography (EIT) is used for detecting the local change in the conductivity of the sensing skin. In this talk, we show experimental results of painted sensing skins applied on the surfaces of polymeric substrates and a concrete beam. We also discuss modeling, the EIT inverse problem and computational aspects related to this application.

## THE NOVIKOV-VESELOV EQUATION AND SOLITONS

SAMULI SILTANEN

University of Helsinki, Finland

The Novikov-Veselov equation is a  $(2+1)$ -dimensional generalization of the celebrated Korteweg-de Vries equation describing nonlinear water waves and optical communication. Recent progress in the study of the medical imaging technique called Electrical Impedance Tomography provides a surprising connection between conductivity-type Schrödinger potentials and solitary wave solutions of the Novikov-Veselov equation.

## SEQUENTIAL MONTE CARLO AND PARTICLE METHODS IN INVERSE PROBLEMS

ERKKI SOMERSALO

Case Western Reserve University, USA

In sequential Monte Carlo methods, the posterior distribution of an unknown of interest is explored in a sequential manner, by updating the Monte Carlo sample as new data arrive. In a similar fashion, particle filtering encompasses different sampling techniques to track the time course of a probability density that evolves in time based on partial observations of it. Methods that combine particle filters and sequential Monte Carlo have been developed for some time, mostly in connection with estimating unknown parameters in stochastic differential equations. In his talk, some of the ideas will be reviewed and new ideas suitable for treating non-stochastic severely stiff and large systems of differential equations are discussed.

## REMOTE SENSING OF ATMOSPHERIC COMPOSITION, HIGHLIGHTS AND CHALLENGES

JOHANNA TAMMINEN

Finnish Meteorological Institute, Finland

Satellites play an important role for air quality, climate and ozone research, monitoring and forecasting. The Dutch-Finnish Ozone Monitoring Instrument on-board NASA's EOS-Aura satellite was launched in 2004 and is still working. The European Space Agency's Envisat satellite with three instruments dedicated to atmospheric measurements made observations for ten years in 2002 - 2012. In this presentation we show highlights of recent remote sensing observations of atmospheric composition, including the Arctic ozone hole in 2011, volcanic eruptions in Iceland in 2011 and monitoring ship emissions in Baltic sea.

The interpretation of the satellite measurements requires mathematical and statistical methods as well as detailed understanding of the physics related to the measurements like radiative transfer modeling. The need and methods for improved retrieval methods, uncertainty quantification, model selection and model discrepancy of satellite measurements is discussed.

### 3D THERMAL TOMOGRAPHY WITH EXPERIMENTAL MEASUREMENT DATA

JUSSI M. TOIVANEN, JANNE M. J. HUTTUNEN, T. SAVOLAINEN

University of Eastern Finland

TANJA TARVAINEN

University of Eastern Finland and University College London, UK

HELICIO ORLANDE

Federal Univesity of Rio de Janeiro, Brazil

JARI P. KAIPIO

University of Eastern Finland and University of Auckland, New Zealand

In thermal tomography, a target is non-destructively imaged using surface measurements of heat transfer. In the measurement setup, the target is sequentially heated at different surface heating locations and the induced temperature evolutions are measured at multiple surface measurement locations. Based on these measurements, the thermal properties of the target are estimated as spatially distributed parameters.

Thermal tomographic imaging of thermally insulated targets was demonstrated with simulations in [1] where the thermal conductivity and heat capacity of an insulated 2D target were estimated as spatially distributed parameters using the Bayesian inversion framework [2,3]. The computational methods were extended in [4] to suit a more practical measurement setup where the target does not need to be thermally insulated. Simulations were used to show the feasibility of this approach where the surface heat transfer coefficient of the target is estimated as an additional spatially distributed parameter.

In this work, the computational methods of [4] are extended to suit estimation of thermal conductivity, heat capacity and surface heat transfer coefficient based on laboratory measurements. A prototype measurement device has been built and experimental measurement data is used to test the feasibility of thermal tomography.

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# RESOLUTION GUARANTEES IN EIT FOR REALISTIC SETTINGS INCLUDING RANDOM AND SYSTEMATIC ERRORS

MARCEL ULLRICH

University of Stuttgart, Germany

We discuss how to guarantee correct reconstruction features for inclusion detection in EIT for realistic electrode models. In a realistic setting only a finite number of electrodes, a finite measurement accuracy and the setting parameters (e.g. background conductivity as well as the contact conductivities at the electrodes) are given approximately. For this purpose we discuss a monotonicity relation for the complete electrode model (CEM).

Till recently, monotonicity based methods have been known to be able to reconstruct an upper or lower bound of inclusions. Current results show that (for continuous boundary data and piecewise analytic conductivities) monotonicity methods are in fact capable to reconstruct the exact outer shape.

In this talk it will be shown how an analogue of this result can be transferred to the CEM to prove the possibility of rigorous resolution guarantees. In an outlook we also want to sketch how monotonicity arguments could be extended to additionally handle shape errors of the geometry as well as imprecisely known electrode positions.

# A BAYESIAN APPROACH TO RETRIEVAL OF CANOPY LAI FROM HYPERSPECTRAL REMOTE SENSING DATA

PETRI VARVIA AND AKU SEPPÄNEN

University of Eastern Finland

MIINA RAUTIAINEN

University of Helsinki, Finland

Canopy leaf area index (LAI) is an important biophysical variable that is used as an input parameter in various climate and vegetation models. LAI is defined as half of the total leaf/needle area of a forest canopy per ground area. In this work we use Bayesian inversion and a physically based forest reflectance model to estimate canopy LAI from hyperspectral satellite imaging data. Forest reflectance models contain several other unknown variables in addition to LAI, and the uncertainty in these parameters is a substantial error source. The Bayesian approach is taken, because it allows taking into account the uncertainties in the model parameters. A Markov chain Monte Carlo (MCMC) method is used to compute posterior marginal densities and expected values for the model variables.

The feasibility of the proposed approach is tested with EO-1 Hyperion data captured during 2010 in Hyytiälä. The data consist of both homogenous and mixed boreal coniferous and deciduous forests imaged at three different occasions during the growing season. The estimates are compared to field-measured LAI values. Credible intervals for the estimates are calculated to quantify the precision of the estimates. The computed 90% credible intervals contain the field-measured LAI in nearly all cases.



## RELICH TYPE THEOREMS FOR UNBOUNDED DOMAINS

ESA VESALAINEN

University of Helsinki, Finland

Let  $u \in L^2_{\text{loc}}(\mathbb{R}^n)$  solve the equation  $(-\Delta - \lambda)u = f$  in  $\mathbb{R}^n$ , where  $\lambda \in \mathbb{R}_+$  and  $f \in L^2(\mathbb{R}^n)$  is compactly supported. The classical Rellich uniqueness theorem from scattering theory says that if

$$\frac{1}{R} \int_{B(0,R)} |u|^2 \longrightarrow 0, \quad \text{as } R \longrightarrow \infty,$$

then  $u$  is also compactly supported. We shall present generalizations of this result to the situation where  $f$  is no longer compactly supported.

# A FOURIER TRANSFORM METHOD FOR 3D SHAPE INVERSION FROM PIXEL-BASED IMAGES

MATTI VIKINKOSKI

Tampere University of Technology, Finland

Disk resolved astronomical observations are usually represented as pixel images. The finite size of pixels cannot be ignored, making the 3D shape reconstruction by gradient-based optimization methods problematic as the forward problem requires data binning which destroys gradient information. We demonstrate an alternative method in which the data and the model are compared on the Fourier plane, allowing the calculation of partial derivatives with respect to the shape parameters. Additionally, we present examples of asteroid shape reconstruction from range-doppler, adaptive optics and thermal infrared interferometry images.

# AN INVARIANT EMBEDDING APPROACH TO DOMAIN DECOMPOSITION

JOSEPH VOLZER

Case Western Reserve University, USA

We consider the problem of numerically solving the wave scattering problem in two dimensions, when the scatterer consists of a sound-soft compact scatterer surrounded by a compactly supported scattering medium. The scattering problem in the exterior domain is solved using boundary integral equations, while the solution near the scatterer is best treated by finite element methods. It is well known that these solutions can be glued together using non-reflecting boundary conditions, a common choice being the Dirichlet-to-Neumann (Steklov-Poincaré) map. If the support of the scattering medium is large, the interior problem may require a large mesh and become computationally intense. We consider an alternative method based on the idea of invariant imbedding: first numerically solve for the DtN map on a boundary of a small domain merely enclosing the sound-soft scatterer, and then radially propagate the map out of the support of the scattering medium.

# A FINITE ELEMENT-WAVELET HYBRID ALGORITHM FOR ATMOSPHERIC TOMOGRAPHY

MYKHAYLO YUDYTSKIY

Johann Radon Institute for Computational and Applied Mathematics, Austria

This is a joint work with Tapio Helin and Ronny Ramlau.

Reconstruction of the refractive index fluctuations in the atmosphere, or atmospheric tomography, is an underlying problem of many next generation adaptive optics (AO) systems in ground-based telescope imaging. The mathematical formulation of the problem resembles limited angle tomography. The availability of statistical information on the measurements and the turbulence of the atmosphere allows a natural formulation of the problem in the Bayesian framework. The dimension of the problem for the extremely large telescopes, such as the European Extremely Large Telescope (E-ELT), combined with the short allotted time for the reconstruction suggests the use of iterative schemes as an alternative to the currently employed direct methods.

Recently, a novel algorithm based on the wavelet representation of the turbulence has been introduced by the authors to solve the atmospheric tomography using the conjugate gradient iteration. In this talk we introduce a computationally efficient variation of the method, the finite element-wavelet hybrid algorithm (FEWHA), which utilizes techniques such as the dual domain discretization strategy, a scale-dependent preconditioner and a multi-scale method to obtain a method that is globally  $\mathcal{O}(n)$ , parallelizable and compact with respect to memory.

In this talk we give a short introduction to the topic, discuss the theoretical results, and present numerical examples in the context of a simulation for the E-ELT.



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