Pen-Shun Lu

DECODING AND LOSSY FORWARDING BASED MULTIPLE ACCESS RELAYING
Lu, Pen-Shun, Decoding and lossy forwarding based multiple access relaying.
University of Oulu Graduate School; University of Oulu, Faculty of Information Technology and Electrical Engineering, Department of Communications Engineering; Centre for Wireless Communications; Infotech Oulu; Japan Advanced Institute of Science and Technology, School of Information Science, Information Theory and Signal Processing Laboratory

Abstract
The goal of this thesis is to provide a unified concept of lossy-forwarding from the theoretical analysis to practical scheme design for the decode-and-forward-based multiple access relay channel (MARC) system. To improve the performance of MARC with the relay subject to resources or/and time constraints, the erroneous estimates output from simple detection schemes are used at the relay are forwarded and exploited. A correlation is then found between two sequences: one is the network-coded sequence sent from the relay, and the other is their corresponding exclusive-OR-ed information sequence. Several joint network-channel coding (JNCC) techniques are provided in which the correlation is utilized to update the log-likelihood ratio sequences during the iterative decoding process at the destination. As a result, the bit error rate (BER) and frame error rate (FER) are improved compared with those of MARC with select DF strategy (SDF-MARC). The MARC proposed above is referred to as erroneous estimates-exploiting MARC (e-MARC).

To investigate the achieved FER performance of the e-MARC system, the outage probability for e-MARC with two source nodes is theoretically derived. We re-formulate the e-MARC system and identify its admissible rate region according to the Slepian-Wolf theorem with a helper. Then, the outage probability is obtained by a set of integral over the rate region with respect to the probability density functions of all the links' instantaneous signal-to-noise power ratios. It is found through simulations that, as one of the source nodes is far away from both the relay and destination, e-MARC is superior to SDF-MARC in terms of outage performance. Furthermore, a joint adaptive network-channel coding (JANCC) technique is then proposed to support e-MARC with more source nodes. A vector is constructed at the destination in JANCC to identify the indices of the incorrectly decoded source node(s), and re-transmitted to the relay for requesting additional redundancy. The relay performs network-coding only over the estimates specified by the vector upon receiving the request. Numerical results show that JANCC-aided e-MARC is superior to e-MARC in terms of FER and goodput efficiency. In addition, compared iterative decoding is performed at relay with SDF-MARC, the use of differential detection with JANCC-aided e-MARC significantly reduces the computational complexity and latency with only a small loss in the FER.

Keywords: cooperative communication, decode-and-forward (DF), joint network-channel coding, multiple access relay channel (MARC), Slepian-Wolf theorem
Lu, Pen-Shun, Dekoodaukseen ja häviölliseen edelleenlähetykseen perustuvan monikäyttövälityksen


Asiasanat: monikäyttörelekanava, pura-ja-lähetä (DF), Slepian-Wolf -teoreema, yhdistetty verkko- ja kanavakoodaus, yhteistoiminnallinen viestintä
Dedicated to my parents
Preface

The dissertation is made based on a curriculum that is organized by the Collaborative Education Program of the Centre for Wireless Communications (CWC), University of Oulu, and Japan Advanced Institute Science and Technology (JAIST), Japan.

First of all, I wish to express my deepest gratitude my supervisor Professor Tad Matsumoto for providing me this opportunity of becoming a doctoral student. I deeply appreciate his supervision during my doctoral research career. His creative intuition and specific ideas constantly surprise me. Furthermore, his great enthusiasm for academic research always impresses me thoroughly. This thesis would not have been completed without his ideas, advice and criticism. I would also like to sincerely thank my other supervisor Professor Markku Juntti for the valuable advice on my research. I really appreciate all of the his precious time he spent on the paper work for me for credit and system transfer and dual degree issues. His tolerance and patience always encourages me to keep going ahead and to overcome any difficulties I encounter. In fact, words are unable to convey my sincere appreciation to Professor Juntti. My gratitude also goes to Professor Lajos Hanzo and Dr. Soon Xin Ng (Michael) for their academic advice during my master’s and doctorial studies.

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The stay in Finland and Japan gave me the opportunity to make very great friends. It seems that I have no secrets from Xiaobo. The Taiko-dinner-trip with Simon is still a vivid memory for me. Valtteri constantly friendly helps me experience Finnish culture. Many thanks to Xin for inviting me to join his organization. I really appreciate the help thoughtfully provided from Ken and Xiaojia in Japan and Finland. I sincerely wish Wu recover his health. Thanks to Aleksi Kolehmainen for his hospitality, and saving my life when I fainted. Also I would like to thank all my friends in Oulu and JAIST, especially Geza & Diane, Ming-Chung & Joyce & their family, and Janne Anttila & Yuki & their family. For sure I miss you, Kisho.

Finally, my warmest thanks belongs to my parents and brother for their endless love and encouragement.
## List of abbreviations

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ACC</td>
<td>accumulator</td>
</tr>
<tr>
<td>$ACC^{-1}$</td>
<td>decoder of ACC</td>
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<tr>
<td>ACK</td>
<td>acknowledgement</td>
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<td>ADD</td>
<td>addition</td>
</tr>
<tr>
<td>AF</td>
<td>amplify-and-forward</td>
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<td>ANCC</td>
<td>adaptive network coded cooperation</td>
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<td>ARQ</td>
<td>automatic repeat request</td>
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<td>AWGN</td>
<td>additive white Gaussian noise</td>
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<tr>
<td>BC</td>
<td>broadcast</td>
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<tr>
<td>BCJR</td>
<td>Bahl-Cocke-Jelinek-Raviv</td>
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<tr>
<td>BSC</td>
<td>binary symmetric channel</td>
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<tr>
<td>BER</td>
<td>bit error rate</td>
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<td>BPSK</td>
<td>binary phase-shift keying</td>
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<td>BP</td>
<td>belief propagation</td>
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<tr>
<td>CCC</td>
<td>constellation constraint capacity</td>
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<tr>
<td>CF</td>
<td>compress-and-forward</td>
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<tr>
<td>COMP</td>
<td>comparison</td>
</tr>
<tr>
<td>CRC</td>
<td>cyclic redundancy check</td>
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<td>CSIR</td>
<td>channel state information at the receiver</td>
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<tr>
<td>DD</td>
<td>differential detector</td>
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<tr>
<td>DF</td>
<td>decode-and-forward</td>
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<tr>
<td>DDEX</td>
<td>differential detector and extraction</td>
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<tr>
<td>DMT</td>
<td>diversity-multiplexing trade-off</td>
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<tr>
<td>DPSK</td>
<td>differential phase-shift keying</td>
</tr>
<tr>
<td>DTC</td>
<td>distributed turbo code</td>
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<tr>
<td>e-MARC</td>
<td>erroneous estimates-exploiting MARC</td>
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<tr>
<td>EM</td>
<td>electromagnetic</td>
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<tr>
<td>EXIT</td>
<td>extrinsic information transfer</td>
</tr>
<tr>
<td>FER</td>
<td>frame error rate</td>
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<tr>
<td>GANCC</td>
<td>generalized adaptive network coded cooperation</td>
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<td>GF</td>
<td>Galois field</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>JANCC</td>
<td>joint adaptive network-and-channel coding</td>
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<td>JNCC</td>
<td>joint network-and-channel coding</td>
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<tr>
<td>LAN</td>
<td>local area network</td>
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<tr>
<td>LDPC</td>
<td>low-density parity-check</td>
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<tr>
<td>LLR</td>
<td>log-likelihood ratio</td>
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<tr>
<td>LNC</td>
<td>linear network coding</td>
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<tr>
<td>LTE</td>
<td>long term evolution</td>
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<tr>
<td>LTE-A</td>
<td>long term evolution-advanced</td>
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<td>MA</td>
<td>medium access</td>
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<td>MAC</td>
<td>medium access control</td>
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<td>MARC</td>
<td>multiple-access relay channel</td>
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<tr>
<td>MAP</td>
<td>maximum a posteriori</td>
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<tr>
<td>MI</td>
<td>mutual information</td>
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<td>MIMO</td>
<td>multiple-input multiple-output</td>
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<td>MRC</td>
<td>multi-way relay channel</td>
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<tr>
<td>MUL</td>
<td>multiplication</td>
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<td>NC-MARC</td>
<td>network-coding-based MARC systems</td>
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<td>PCCC</td>
<td>parallel concatenated convolutional code</td>
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<tr>
<td>PLNC</td>
<td>physical-layer network coding</td>
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<tr>
<td>pdf</td>
<td>probability density function</td>
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<tr>
<td>QoS</td>
<td>quality of service</td>
</tr>
<tr>
<td>RD</td>
<td>relay-destination</td>
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<tr>
<td>RSC</td>
<td>recursive systematic convolutional</td>
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<tr>
<td>SCCC</td>
<td>serially concatenated convolutional code</td>
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<tr>
<td>$SCCC^{-1}$</td>
<td>iterative decoding of SCCC</td>
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<tr>
<td>SISO</td>
<td>single input single output</td>
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<tr>
<td>SD</td>
<td>source-destination</td>
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<tr>
<td>SDF</td>
<td>select decode-and-forward</td>
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<tr>
<td>SDF-MARC</td>
<td>MARC systems with SDF strategy</td>
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<tr>
<td>SNCC</td>
<td>separated network-and-channel coding</td>
</tr>
<tr>
<td>SNR</td>
<td>signal to noise ratio</td>
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<tr>
<td>SR</td>
<td>source-relay</td>
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<tr>
<td>TDMA</td>
<td>time division multiple access</td>
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<tr>
<td>TMRC</td>
<td>two-way relay channel</td>
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<tr>
<td>WLAN</td>
<td>wireless local area network</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>WSN</td>
<td>wireless sensor network</td>
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<tr>
<td>XOR</td>
<td>exclusive-OR</td>
</tr>
<tr>
<td>2D</td>
<td>two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>three-dimensional</td>
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<tr>
<td>3GPP</td>
<td>3rd generation partnership project</td>
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</table>
List of symbols

\( i \) Source node
\( R \) Relay
\( D \) Destination
\( \mathbf{u}_i \) information sequence of \( i \)
\( \mathbf{\hat{u}}_i \) estimates of \( \mathbf{u}_i \) received at \( R \)
\( \mathbf{\hat{u}}_i \) estimates of \( \mathbf{u}_i \) received at \( D \)
\( \mathbf{x}_i \) coded sequences of node \( i \)
\( \mathbf{y}_{iR} \) received signals at \( R \) sent from \( i \)
\( \mathbf{y}_{iD} \) received signals at \( D \) sent from \( i \)
\( \mathbf{y}_{RD} \) received signals at \( D \) sent from \( R \)
\( p_i \) error probability of \( iR \) link per transmission cycle
\( p_R \) error probability of \( RD \) link per transmission cycle
\( \hat{p}_i \) theoretical limit of \( p_i \)
\( \hat{p}_R \) theoretical limit of \( p_R \)
\( \tilde{p}_i \) non-zero value of \( \hat{p}_i \)
\( \tilde{p}_R \) non-zero value of \( \hat{p}_R \)
\( \gamma_{iR} \) instantaneous SNR of \( iR \) link
\( \gamma_{iD} \) instantaneous SNR of \( iD \) link
\( \gamma_{RD} \) instantaneous SNR of \( RD \) link
\( \gamma^*_{iR} \) \( p_i = 0 \) if \( \gamma_{iR} > \gamma^*_{iR} \)
\( \gamma^*_{iD} \) \( p_R = 0 \) if \( \gamma_{iD} > \gamma^*_{iD} \)
\( \Gamma_{iR} \) average SNR of \( iR \) link
\( \Gamma_{iD} \) average SNR of \( iD \) link
\( \Gamma_{RD} \) average SNR of \( RD \) link
\( \mathcal{E}_S \) signaling scheme at \( i \)
\( \mathcal{E}_R \) signaling scheme at \( R \)
\( \mathcal{D}_R \) receiver at \( R \)
\( \mathcal{D}_D \) receiver at \( D \)
\( \mathcal{E}_S^{-1} \) decoding of \( \mathcal{E}_S \) using the log-MAP algorithm
\( \mathcal{E}_R^{-1} \) decoding of \( \mathcal{E}_R \) using the log-MAP algorithm
\( \mathcal{R}_i \) source coding rate at \( i \)
source coding rate at $R$

spectrum efficiency of $E_S$

spectrum efficiency of $E_R$

entropy function

binary entropy function

mutual information between $w$ and $z$

mutual information between $w$ and $a priori$ LLR of $w$

mutual information between $w$ and $extrinsic$ LLR of $w$

distortion over $iR$ link transmission

outage probability

convolution, $\alpha(1-\beta) + \beta(1-\alpha)$

theoretical limit of $\delta$

$\delta$ with constant $p_i$

source correlation

network correlation; $p_{nc} = p_A \ast p_B$

three-fold integral over function $f$ with domain $V$

expectation

additional gain in dB

path loss in dB

logical ‘or’

logical ‘and’

the complement of event $\varepsilon$

LLR-updating function

exponential function

variance function

LLR values of sequence $u$

$a priori$ LLR values of sequence $u$

$extrinsic$ LLR values of sequence $u$

complex conjugation of a complex number $x$

taking real part of a complex number $x$

absolute value (magnitude) of a complex number $x$

interleaving by $\Pi_i$

de-interleaving from $\Pi_i$

interleaving by $\Pi_{i\alpha}$
\( \Pi_{ia}^{-1} [\cdot] \) de-interleaving from \( \Pi_{ia} \)

\( \Pi_{R} [\cdot] \) interleaving by \( \Pi_{R} \)

\( \Pi_{R}^{-1} [\cdot] \) de-interleaving from \( \Pi_{R} \)
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4.4 Conclusion

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Appendices
1 Introduction

Wireless communication systems and standards have evolved during the last decades. The main trigger for the evolution of the wireless communication systems and standards are demands for high data rate multimedia-based applications, high spectral efficiency, low power consumption and reliable services. The growth of mobile data traffic has been predicted to be more than 24-fold between 2010 and 2015, and more than 500-fold between 2010 and 2020 [1, 2]. On the other hand, many wireless mobile devices are by limited battery size, and thus the energy consumption is also critical for the design of wireless communication systems. However, achieving a high data rate usually requires high transmitting power levels; hence, there is a trade-off between communication performance and energy consumption.

Multiple-input multiple-output (MIMO) wireless technology [3, 4] is one solution for improving the data rate. However, several wireless mobile devices may not be able to deploy multiple antennas due to size, cost, or hardware limitation [5], and thus cooperative communication has emerged as a new means of emulating the strategies designed for multiple antenna systems. Wireless systems that use cooperative communication technique are also known as distributed or virtual MIMO systems [6].

1.1 Cooperative communication

Cooperative communication was initially introduced and studied by van der Meulen [7], and early formulations of general relaying problems appeared in the information theory community [8–10]. In conventional wireless transmission systems, a transmission link is built only by a source node and a destination. Hence, the successful probability of the transmission totally depends on the quality of the link. However, with using cooperative communication techniques, one or several additional nodes are introduced as relays to cooperate with the source node. Thus, multiple copies of an information sequence can be transmitted via independent fading channels, which significantly improves the probability that the information sequence is successfully received at the destination. In other words, spatial diversity gains can be exploited in cooperative communications.

In addition to the spatial diversity gains, a lot of design flexibility in the form of diversity-multiplexing trade-off (DMT), coverage extension, and multiple user quality of service (QoS) management are provided in wireless networks with cooperative com-
munications. Therefore, cooperative communication has emerged as a promising technique for improving the reliability and throughput of wireless multi-terminal networks and has attracted a lot of attention from the wireless communication research community recently. In fact, cooperative communication has been considered in several latest communication standards, for instance, in the Institute of Electrical and Electronics Engineers (IEEE) 802.16j WiMax standard [11, 12] and in the Long-Term Evolution-Advanced (LTE-A) of the Third Generation Partnership Project (3GPP) [13] multi-hop cellular networks [5, 14–17]. Nowadays cooperative communication techniques are playing an important part in modern communication systems, e.g., wireless mobile systems, device-to-device (D2D) communications, wireless sensor networks (WSNs) and ad-hoc networks.

![Basic models of cooperative communications. (a) single source relay channel, (b) two-way relay channel, (c) multiple access relay channel.](image)

The basic model of cooperative communications is the classic single source relay channel, shown in Fig. 1(a), where there are one source node, one relay and one destination. In the first time slot, the source node broadcasts its information sequence to the relay and destination. The relay processes the received signal vector sent via the source-relay (SR) link, and forwards the processed signal to the destination in the second time slot. Another two popular models of cooperative communications were extended based on the classic single source relay channel: two-way relay channel (TWRC) [18] and multiple access relay channel (MARC) [19].

The system model of a TWRC is shown in Fig. 1(b), where two source nodes communicate with each other through a relay, but there is no direct link between source nodes. In the first two time slots (the multiple access (MA) phase), each source node individually transmits its information sequence to the relay, and the relay can separately transmit the estimated sequence intended to the other source node in the last two time
slots, or performs network-coding on both estimates and broadcasts the network-coded sequence to the source nodes only in one time slot (the broadcast (BC) phase). The TWRC can be further extended to the multi-way relay channel where more than two source nodes communicate with each other through a relay. The MARC system model, on the other hand, is shown in Fig. 1(c), which consists of two source nodes, one relay and one common destination; the role of the relay is to assist the source nodes in improving the probability of successful transmission to the destination.

Multi-way relay channel systems can represent various practical communication scenarios. For example, in LTE networks, a set of mobile stations can form a group of users (i.e., source nodes) that multicast the information through the base station in a multi-way fashion. Another example can be found in satellite communication, where several ground stations can exchange information via the satellite which acts as a relay. In ad hoc networks, several users can contribute in building a distributed file sharing database via a central access point. In a WSN, sensor nodes can cooperatively pass their information to a fusion center. Similarly, the MARC system also can represent several scenarios and applications. For instance, in WSNs, the sensors are too weak to cooperate but they can send their information to the fusion center with the help of more powerful nodes. In data gathering networks, a larger cache is provided by the relay and thus more compressed data can be forwarded to the destination.

Designing energy-efficient functions for the relay for signal processing and relaying protocols is one of the important issues for cooperative communications, especially for cooperative communications in resource-constrained wireless networks. For example, it is difficult to replace or recharge the batteries for sensor nodes in WSN in some scenarios [20]. In addition, as the basic models exemplified above are expanded to form a larger network, multiple cooperative relays may be employed in cooperative communications. Hence, the choice of a relay node (i.e., relay selection [21–25]) plays a significant role in cooperative communications.

1.2 Relaying strategy

The design of relaying protocols and strategies has attracted considerable attention for almost the entire last decade. Various protocols and strategies have been developed for cooperative communications to increase throughput or reduce energy consumption. We briefly introduce the relaying protocols and strategies in this section.
1.2.1 Relaying protocols

Relaying protocols in cooperative communications are mainly involved with the scheduling of the transmission. Diversity protocols were introduced by Laneman et al. [5, 26], and classified into the following categories:

Static protocols

The transmission from the source node and the relay follows a fixed pattern, in which the source node transmits an information sequence in the first time slot and the relay repeats the obtained estimates in the second time slot. Static protocols are simple and easy for implementation. However, if the information sequence is correctly received at the destination in the first time slot, the transmission for the second time slot becomes redundant. Besides, if the information sequence is incorrectly received at the relay in the first time slot, the transmission of the second time slot is wasted. Hence, to improve the efficiency of static protocols, adaptive protocols, such as selection relaying and incremental relaying, were proposed [5].

Adaptive protocols

To avoid the waste of the transmission in the second time slot, in selection relaying, the relay remains silent if the information sequence is incorrectly received at the relay in the first time slot. Instead, the source node re-transmits a copy of the original information sequence directly to the destination in the second phase [5]. On the other hand, in incremental relaying, the destination is assumed to be able to give feedback to the source and the relay nodes after each transmission. With this assumption, the transmission in the second time slot is not necessarily required if the transmission from the source to the destination was successful in the first time slot. Thus, incremental relaying has the best performance among the proposed protocols in terms of spectral efficiency [5].

1.2.2 Forwarding behavior

According to forwarding behaviors, relay strategies can be mainly classified into following categories:
Amplify and Forward

In the amplify-and-forward (AF) strategy, the received signal vector sent via $SR$ link is simply amplified by a relay, and forwarded to the destination [27]. No decoding and re-encoding is performed on the received signal vector at the relay. In the AF strategy, in addition to amplification, several practical issues such as sampling, and retransmitting analog values have to be taken into account. Furthermore, any noise is also amplified with the received signal vector and retransmitted, which degrades the performance of the system.

Decode and Forward

In the decode-and-forward (DF) strategy, the received signal vector sent via $SR$ link is decoded at the relay [27]. After that, error detection, such as a cyclic redundancy check (CRC) [28], is used to check whether the estimated sequence contains error(s) or not. The correctly decoded estimates are re-encoded at the relay and forwarded to the destination, while the estimates decoded in error are discarded by the relay to avoid error propagation [29, 30]. In the scenario where multiple source nodes are served by a common relay, the DF strategy may be modified to the select DF (SDF) strategy [29, 31–34], in which a header has to be added to identify the correctly decoded source node(s).

Compress and Forward

In the compress-and-forward (CF) strategy, the received signal vector sent via $SR$ link is quantized and compressed at a relay before being forwarded to the destination [35]. The CF strategy is sometimes referred to as estimate-and-forward [36]. In fact, the received signals at the relay and the destination are correlated due to the nature of broadcasting. Hence, the correlation is utilized by the relay to compress the received signal vector, and the compressed version of the received signals is forwarded to the destination. Compressing the received signal vector at the relay may be computationally expensive and thus adaptive protocols are suitable for the CF strategy.
1.3 Network coding

In a multicast network, an information sequence is transmitted from a source node to the destination via several intermediate nodes. To improve the throughput efficiency and multicast capacity of a network, Ahlswede et al. introduced a network coding technique by which the intermediate nodes in networks can encode the received information sequences sent from its neighboring nodes [37]. Assuming an error-free point-to-point network, Ahlswede et al. [37] proved that a source node is able to multicast $k$ information sequences to several destinations if the min-cut between the source node and each destination has the capacity of $k$ per unit time. [38, 39] have demonstrated that the bandwidth efficiency for a wireless mesh is significantly improved with network coding. In 2003, Li et al. explicitly constructed a linear network code (LNC) [40] and demonstrated that the min-cut capacity for the multicast problem can always be achieved. The network coding for LNC is assumed to be operated at a higher layer protocol. In 2006, physical-layer network coding (PLNC) was proposed where network coding is operated at the physical layer by superimposing electromagnetic (EM) waves which carry the information sequences transmitted from source nodes. The concepts of LNC and PLNC are briefly described in the following sections.

1.3.1 Linear network coding

In LNC, each intermediate node in the network randomly generates its encoding vector [41], and uses the encoding vector to encode the previously received information sequences in a linear combination. Then, the encoded sequence with the corresponding encoding vector is forwarded to the next routing node. To be able to recover the original information sequences, as many encoded sequences as possible should be received at the destination, and the encoding vectors associated with the received sequences have to be linearly independent. Thus, by solving a system of linear equations with Gaussian elimination, the original information sequences can be obtained at the destination. A detailed discussion with respect to LNC or random network coding are available in [42, 43].

It should be noticed that the exclusive-OR (XOR) network-coding is a special case of LNC [40]. The coded sequences that are transmitted in the network are elements in the Galois field (GF) $\mathbb{F}_q$ with $q = 2$ (i.e., GF(2)), and bit-wise XOR in GF(2) is used as an operation.
1.3.2 Physical-layer network coding

Unlike LNC operating at a higher layer, PLNC directly superimposes the EM waves which carry the information sequences in the physical layer. For example, assuming source nodes $A$ and $B$ attempt to exchange their complex information symbols $u_A = a_A + jb_A$ and $u_B = a_B + jb_B$ with each other via a relay $R$ in a TWRC system. In the first time slot, nodes $A$ and $B$ simultaneously broadcast their symbols modulated on the same radio frequency $\omega$ to the relay $R$. The combined bandpass signal $y_R$ received at $R$ during one symbol period $t$ is

\[
y_R(t) = \Re \left\{ u_A e^{j\omega t} + u_B e^{j\omega t} \right\} \\
= \Re \left\{ (a_A + jb_A)e^{j\omega t} + (a_B + jb_B)e^{j\omega t} \right\} \\
= (a_A + a_B) \cos(\omega t) - (b_A + b_B) \sin(\omega t)
\]

(1)

where $\Re \{ \cdot \}$ denotes a function that takes the real part of its argument. Hence, the baseband in-phase and quadrature components of $y_R$ are $y_R^I = a_A + a_B$ and $y_R^Q = b_A + b_B$, respectively. Suppose $u_A$ and $u_B$ are QPSK symbols, the arithmetic sum in $y_R^I$ and $y_R^Q$ is equivalent to the bit-wise XOR operation; in other words, the network coding is performed by nature. Hence, as several source nodes transmit simultaneously, the transmission efficiency can be significantly improved using PLNC. Several critical issues for PLNC, such as synchronization and channel estimation, are studied in [44–46].

1.3.3 Network and channel coding

We have briefly reviewed network coding for multi-casting in error-free networks in the previous subsection. Nevertheless, error-free networks may be less practical in real scenarios. Several reasons such as link outage, collision or buffer overflow may cause errors to occur in the links of the network. Hence, channel coding is in general combined with network coding to eliminate errors occurring in the links of the network. Existing research on unifying channel and network coding can be roughly classified into the following two categories.
Separated network-and-channel coding (SNCC)

Network coding and channel coding are separately designed and the redundant information is not jointly exploited. More specifically, received sequences are firstly channel decoded in the physical layer to obtain the estimates of information sequences. After that, the estimates are directly passed to the network layer to be network-coded. The output from the network coding operation is directly delivered back to the physical layer for channel encoding again. Thus, in the design of SNCC, the overall problems are separated into two independent problems and thus researchers can focus on one of these two independent problems. For example, Larsson et al. [47] introduced an automatic repeat request (ARQ) scheme for multiple unicast flows in a multi-user system. Berger et al. theoretically analyzed the optimization problem in joint erasure-correction and error-correction coding schemes [48].

Joint network-and-channel coding (JNCC)

Network coding and channel coding are merged in such a way that the network coding is able to contribute to error protection. Instead of guaranteeing the error-free transmission for each point-to-point link, the aim is to guarantee error-free decoding at the destination. Hence, in the design of JNCC, the network-coded sequence is designed to carry the additional redundancy to help decode the received sequences at the destination. Many studies jointly design network-channel codes to exploit the redundancy in both channel and network codes. For example, Bao et al. proposed adaptive network coded cooperation (ANCC) [49] and generalized adaptive network coded cooperation (GANCC) [50] where channel coding and network coding are unified in a general framework. Hausl et al. introduced iterative network and channel decoding on a Tanner graph [51], and also proposed joint network-channel coding [52, 53] for both MARC and TWRC based on distributed turbo code (DTC) [54, 55]. Besides, Duyck et al. proposed a structured full-diversity joint network-channel code in [56] and applied it in large networks [57]. Zhang et al. uses low-density parity-check (LDPC) codes [58] and network coding to approach the capacity of TWRC [59].
1.4 Motivation

Cooperative communications have attracted a lot of attention from the wireless communication research community, since spatial diversity gain is provided. In cooperative communications, one or several nodes may act as relays to help other nodes forward their data to a common destination. Typically, the relay performs the decoding of powerful codes, such as turbo codes [60] and LDPC codes to ensure that the received estimates are correctly decoded with a high probability.

However, in practice, the source nodes are not geographically always close enough to the relay in cooperative communications, and the average received signal-to-noise power ratio (SNR) is determined by the pathloss. Furthermore, the signal/signals received by the nodes may suffer from deep fade, depending on their locations. Errors occurring in the $SR$ links may well be eliminated by using powerful codes in the signaling scheme. Nevertheless, in many scenarios, the use of powerful codes per link does not always provide a reasonable solution, especially for scenarios where the relay may operate with resources or/and time constraints.

For example, the sensor nodes in WSN are typically with a finite energy and data buffer (memory) [20], and replacing or recharging the batteries is difficult in some scenarios. Additionally, exhaustive energy and memory are required to perform complicated decoding algorithms, such as the maximum a posteriori (MAP) [61] or belief propagation (BP) [62] algorithms. Also, several real-time systems operate with tight time constraints, which may prevented powerful codes from being used in real-time systems due to their large latency resulting from the iterative decoding process.

In above exemplifying situations, the relay may be only allowed to use simple detection schemes. However, when using low computational complexity detection, there is a high probability of receiving erroneous estimates at the relay, and forwarding these erroneous estimates will result in error propagation in the decoding process at the destination. As mentioned in [29, 30], performance and diversity gain are dramatically degraded due to the error propagation. Therefore, utilizing the forwarded erroneous estimates and mitigating error propagation caused by the estimates has been gaining more and more attention recently.

DF (or SDF) is one the most simple and popular relaying strategies used in cooperative communications to avoid the error propagation, and many excellent DF-based JNCC techniques for TWRC/MARC systems have been developed [52, 63–69] and [70–73], respectively. Most DF strategies used at the relay discard estimated sequence(s)
containing errors, and thus, either perfectly decoding or CRC is assumed in [52, 63–67]. However, CRC-based selection relaying/combining are not bandwidth efficient and would incur decoding delays, even though they are effective in controlling error propagation. Furthermore, much power is wasted on decoding unreliable received signals especially when the quality of SR links is low.

On the other hand, erroneous estimates still contain a lot of useful information, which is helpful in reconstructing the transmitted signals from the source nodes at the destination. Hence, several emerging design schemes have been proposed for preserving the information of the erroneous estimates at the relay, and forwarding the estimates in an analog form or their quantized versions. For example, in [30, 74–78], the relay forwards the log likelihood ratio (LLR) values of the network-coded sequence to the destination in case of erroneous SR links. The soft-DF technique was introduced in [79–81] where the relay performs a soft decoding of the received signal and re-encodes it softly. However, relaying the signals in an analog form requires more bandwidth for the representation of the soft bits, and thus applying the above techniques at the relay may not be suitable for the resource-constrained wireless networks. Motivated by this, a simple DF-based MARC system with the concept of lossy-forwarding is proposed and studied in this thesis.

1.5 Objectives and outline for the thesis

The target of this thesis is to provide a unified concept of lossy-forwarding from the theoretical analysis to practical scheme design for the DF-based MARC system where the relay is assumed to be subject to resources or/and time constraints. Due to the fact that source-channel separation theorem does not hold in general for sending correlated sources over multiuser networks [82], to facilitate the theoretical analysis, time-division multiple-access (TDMA) is used for the transmission of each node and thus the MARC system is orthogonal (i.e., the orthogonal MARC system). The time allocation parameters are assumed to be fixed and the relay and every source node have their own transmission interval. The channels of the MARC system are assumed to suffer from additive white Gaussian noise (AWGN) and block Rayleigh fading, respectively. The relay is assumed to operate in a half-duplex mode [83]. Channel state information at the receiver (CSIR) is assumed to be available at the destination, but not necessarily required at the relay if non-coherent differential detection is performed on the received signal vector sent via the SR link.
In Chapter 2 [84, 85], a simple DF-based lossy-forwarding MARC system, referred to as an erroneous estimates-exploiting MARC (e-MARC) system, is proposed. The aim of the e-MARC system is to exploit the erroneous estimates resulting from a low computational complexity detection scheme applied at the relay. Both estimates, regardless of whether or not they are correctly received at the relay, are always XOR-coded and forwarded to the destination. Due to performing the XOR coding at the relay, a correlation is found between the two sequences: one is the XOR-coded sequence sent from the relay, and the other is their corresponding XOR-ed information sequence. The correlation is referred to as network correlation in this thesis. Then, several JNCC and corresponding decoding schemes are developed for the practical realization of the e-MARC system, in which the knowledge of the network correlation is exploited at the destination by using a log-likelihood ratio (LLR)-updating function [86] in the iterative JNCC decoding process. Furthermore, extrinsic information transfer (EXIT) analysis [87, 88] is used to verify the simulated bit error rate (BER) results of the JNCC scheme used in the e-MARC system. It is found that, even though a very simple detection scheme is used at the relay, the BER and frame error rate (FER) performances of the e-MARC system are superior to those of MARC systems with the SDF strategy (SDF-MARC) [33].

Chapter 3 [89] theoretically derives the outage probability for the e-MARC system with two source nodes proposed in Chapter 2. The theoretical analyses are based on the techniques presented in [90, 91]. We re-formulate the e-MARC system according to the Slepian-Wolf theorem [92] for correlated source coding with a helper, and analyze the e-MARC system’s admissible rate region according to the re-formulation. After that, we derive the outage probability of the e-MARC system where all five links in the system (two $SD$ links, two $SR$ links and one relay-destination ($RD$) link) suffer from statistically independent block Rayleigh fading.

The probability distribution of the $SR$ links’ error probabilities in practice depends on the signaling scheme applied at the source and relay nodes. Nevertheless, to be able to make comparisons of the outage performances with other relaying strategies used in the MARC system, it is necessary for the theoretical outage probability of the e-MARC to be independent of any signaling scheme. Therefore, in this chapter, we also derive the theoretical limit of the $SR$ links’ error probabilities by using rate distortion and inverse entropy functions [93]. Following this, it is shown that the outage probability of the e-MARC system, independent of signaling schemes, can be theoretically derived by

\[^1\) Any MARC system with the SDF strategy is viewed as the SDF-MARC system in this thesis.
a fivefold-integral over the admissible rate region with respect to the probability density functions (pdfs) of the five links’ instantaneous SNRs.

The process for deriving the probability is then applied to the following two special cases: 1) Two SR links are assumed to be binary symmetric channels (BSCs), and 2) The e-MARC system with a practical signaling scheme [85] presented in Chapter 2. Then the fivefold-integral needed to obtain the outage probability can be reduced to simpler expressions, corresponding to their error probabilities of SR links. Numerical results show that theoretical outage probabilities of the e-MARC applying differential detection at the relay are roughly 3.5 and 4.5 dB away from the probabilities of the e-MARC system independent of signaling schemes in the Symmetric and Asymmetric scenarios, respectively. This performance loss is because we aim at reducing the computational complexity. Finally, we investigate the impact of the correlation between the two source nodes on the outage probability of the e-MARC system. It is found that for the case two source nodes are highly correlated, the outage performance can still be improved as long as one of the SD links is reliable. However, to improve the outage performance by exploiting the network correlation, the RD link and at least one of the SD links needs to be reliable.

Chapter 4 [94] proposes a novel joint adaptive network-channel coding (JANCC) technique to aid the e-MARC system with more than two source nodes. As the number of the source nodes in the orthogonal MARC system is increased, unnecessary erroneous estimates may be included in the XOR coding process at the relay, which leads to low network correlation and degrades the decoding performance of the JNCC schemes for e-MARC. Hence, to efficiently exploit the network correlation, in the proposed JANCC technique, the destination constructs a vector identifying the indices of the incorrectly decoded source nodes, and sends it to the relay to request a re-transmission. Upon receiving the request, the relay performs network-coding only over the stored estimates specified by the identifier vector, rather than over all estimates as the e-MARC. In addition, an algorithm is proposed to estimate the knowledge of the network correlation during the iterative decoding process of JANCC, by which the destination is able to exploit the knowledge of the network correlation without the aid of a higher layer protocol. Compared to the e-MARC in terms of FER and goodput efficiency, it has been observed that the performance of a JANCC-aided e-MARC is improved with three source nodes. In addition, compared with the SDF-MARC system where the iterative decoding is performed at the relay, the use of differential detection with a JANCC-aided e-MARC significantly reduces the computational complexity and latency with only a small loss.
in the FER performance.

Chapter 5 concludes the thesis and summarizes the main results. The open issues and suggestions for future research are presented.

1.6 Author’s contribution

This thesis is based on two journal papers [89, 94], and two published conference papers [84, 85]. The first journal paper [94] has already been published and the second one [89] is under revision. The author has had the main responsibility for performing the analysis, programming the simulation, generating the numerical results, and writing all the papers [84, 85, 89, 94]. Other authors provided ideas, help, comments and criticism during the writing process.

In summary, the main contributions of the thesis are summarized in the following,

– The e-MARC system is proposed for low computational complexity detection schemes applied at the relay. In the proposed system, the received erroneous estimates, instead of being discarded by the relay with the SDF strategy, are forwarded to the destination to help the recovery of the information sequences sent via the SR links. As a result, by using very simple detection at the relay, it is found that roughly $0.2 - 0.4$ dB and $0.7 - 2.2$ dB gain can be obtained from the forwarded erroneous estimates received at relay in AWGN and fading scenarios, respectively.

– The theoretical outage probability of the e-MARC system, independent of signaling schemes, is derived to investigate the achieved FER performance of the e-MARC system. It is found through simulations that lossy-forwarding improves the outage performance in the case that the sources are far away from both the relay and the destination.

– The JANCC and its decoding techniques are proposed to improve the e-MARC system, especially when the number of the source node increases. Compared with the SDF-MARC system where fully-iterative decoding is performed at the relay, the utilization of differential detection with JANCC-aided e-MARC at least reduces the computational complexity to $1/200$, which leads to meaningful power savings with only a $0.5 - 1.5$ dB loss in the FER performance.
2 Erroneous estimates-exploiting MARC

This chapter concentrates on the performances of e-MARC in AWGN and fading channels when very simple detection schemes are applied at the relay to obtain estimates sent from source nodes. The chapter is organized as follows. The assumptions and the proposed e-MARC system model are described in Section 2.1. The exploitation of erroneous estimates received at the relay in the proposed e-MARC system is introduced in Section 2.2. In addition, for the practical realization of the e-MARC system, a JNCC framework and its corresponding decoding scheme are developed in Section 2.2 to support the relay using very simple detection. Section 2.3 provides an EXIT analysis for the convergence property evaluation of the decoder of the JNCC in terms of the extrinsic information exchange. Section 2.4 shows the BER performance of the e-MARC system with JNCC schemes based on the EXIT analysis; furthermore, we compare the BER and FER of the proposed e-MARC system with those of the SDF-MARC system. Finally, Section 2.5 concludes the chapter.

2.1 System model

Fig. 2 illustrates a basic model of the orthogonal e-MARC system assumed in this thesis, where there are two source nodes $A$ and $B$, one common relay $R$, and one common destination $D$. The $K$-bit length independent identically distributed (i.i.d.) binary information sequences generated from nodes $A$ and $B$ are denoted as $u_A = \{u_A(k)\}_{k=1}^K$ and $u_B = \{u_B(k)\}_{k=1}^K$, respectively. The signaling scheme used at the source node is denoted as $E_S(\cdot)$, which consists of a serial concatenation of encoding and modulation. There are three time slots in one transmission cycle. In the first two

![Fig 2. Orthogonal e-MARC system model, where there are three time slots in a transmission cycle, [89] (© 2015 IEEE).](image)

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time slots, nodes $A$ and $B$ respectively broadcast their $M$-bit length symbol sequences $x_A = E_S(u_A) = \{x_A(m)\}_{m=1}^{M}$ and $x_B = E_S(u_B) = \{x_B(m)\}_{m=1}^{M}$ to the relay $R$ and destination $D$, and their corresponding received signals obtained at $R$ and $D$ are respectively written as

$$y_{iR} = h_{iR} \cdot x_i + n_{iR}, \quad i \in \{A, B\}$$

where $h_{iR}$ and $h_{iD}$ indicate the channel coefficients of $iR$ and $iD$ links with the source node $i$, respectively. $n_{iR}$ and $n_{iD}$ indicate the vectors of independent zero-mean complex AWGN of the $iR$ and $iD$ links, respectively, with variance $\sigma_{iR}^2 = \sigma_{iD}^2 = \sigma^2$ per dimension. The $iR$ and $iD$ links are also referred to as the intra and direct links, respectively, in this thesis.

The receiver applied at the relay $R$ is denoted as $D_R(\cdot)$, composed of demodulation and signal detection/decoding, which corresponds to the inverse structure of $E_S(\cdot)$. The estimates $\hat{u}_i = D_R(y_{iR})$ of $u_i$ obtained at $R$ may contain errors due to the variation of the $iR$ link. The error probability of the $iR$ link is represented by

$$p_i = B(u_i, \hat{u}_i) = \frac{\sum_{k=1}^{K} |u_i(k) - \hat{u}_i(k)|}{K}, \quad i \in \{A, B\}.$$  

(3)

If the estimates $\hat{u}_i$ is found to contain errors, it will be discarded at the relay in the SDF-MARC system. However, in the e-MARC system, the estimates $\hat{u}_A$ and $\hat{u}_B$, are always joint network-channel coded at the relay $R$ regardless of whether they are correct or not, as

$$x_R = E_R(u_R) = E_R(\hat{u}_A \oplus \hat{u}_B) = \{x_R(m)\}_{m=1}^{M},$$

(4)

where the notation $\oplus$ denotes bit-wise XOR operation and $E_R(\cdot)$ represents the signaling scheme applied at $R$, including channel encoding and modulation. The destination $D$ obtains the signal vector $y_{RD}$ of $x_R$ sent via the $RD$ link as

$$y_{RD} = h_{RD} \cdot x_R + n_{RD},$$

(5)

where $h_{RD}$ and $n_{RD}$ indicate the channel coefficient and AWGN vector of the $RD$ link with variance $\sigma_{RD}^2 = \sigma^2$, respectively. The estimated sequence of $u_R$ obtained at the
destination is denoted as \( \hat{u}_R \), and the error rate of the \( RD \) link is \( p_R = \mathcal{B}(u_R, \hat{u}_R) \).

Finally, to obtain the estimated sequences \( \hat{u}_A \) and \( \hat{u}_B \) of the information sequences \( u_A \) and \( u_B \), respectively, at the destination, decoding of JNCC is performed on the received signal vectors \( y_{AD} \) and \( y_{BD} \) with the help of the signal vector \( y_{RD} \).

If all the links are assumed to suffer from block Rayleigh fading, \( h_{iR} \), \( h_{iD} \) and \( h_{RD} \) are assumed to be constant over one symbol sequence but vary independently transmission-by-transmission and link-by-link. Without loss of generality, we assume that \( E[|h_{iR}|^2] = E[|h_{iD}|^2] = E[|h_{RD}|^2] = 1 \). The error probabilities \( p_i \) and \( p_R \), thus, vary in each transmission cycle. The instantaneous SNRs \( \gamma_{iR}, \gamma_{iD} \) and \( \gamma_{RD} \) of the links are then given by

\[
\gamma_{iR} = |h_{iR}|^2 \cdot \Gamma_{iR} \\
\gamma_{iD} = |h_{iD}|^2 \cdot \Gamma_{iD} \\
\gamma_{RD} = |h_{RD}|^2 \cdot \Gamma_{RD}
\]

where \( \Gamma_{iR}, \Gamma_{iD} \) and \( \Gamma_{RD} \) represent the average SNRs of the intra, direct and \( RD \) links, respectively.

2.2 AWGN MARC

To better understand the fundamental idea of the e-MARC system, all the links in e-MARC are assumed to be AWGN (i.e., \( h_{iR} = h_{iD} = h_{RD} = 1 \)) in this section. With this assumption, the average value of the probability \( p_i \) in (3) is equivalent to a bit-flipping model [95, 96]. In addition, the average value of \( p_i \) depends on the signaling and detection scheme applied at the source node and relay, respectively. For the purpose of reducing the computational complexity in the encoding process, the source node in this thesis employs a serially concatenated convolutional code (SCCC), composed of a rate-1/2 recursive systematic convolutional (RSC) code and a rate-1, memory-1
accumulator (ACC) [97]. The generator polynomials of the RSC and ACC encoder are \((1, 5/7)_8\) and \((2/3)_8\), respectively, where the notation \((\cdot)_8\) represents the argument is an octal number.

As shown in Fig. 3, the interleaved version of the information sequence \(u_i\) is first encoded by the RSC code to produce the coded sequence \(c_i\), and the interleaved version of the sequence \(c_i\) is further encoded by the ACC and modulated using binary phase-shift keying (BPSK) to produce the symbol sequence \(x_i\). The use of the ACC aims to ensure that the EXIT convergence tunnel is open until a point very close to the \((1.0, 1.0)\) mutual information (MI) point [98]. The notations \(\Pi_i[\cdot]\) and \(\Pi_{ia}[\cdot]\) shown in Fig. 3 denote interleaving by \(\Pi_i\) and by \(\Pi_{ia}\), respectively.

According to the encoder structure applied at the source node, several detection strategies are able to be used at the relay:

- IR: the relay obtains the estimated sequence \(\hat{u}_i\) by performing iterative decoding between the decoders of the ACC and RSC code with the log-MAP algorithm on vector \(y_{iR}\), as shown in Fig. 4(a). Here the notations \(\Pi_i^{-1}[\cdot]\) and \(\Pi_{ia}^{-1}[\cdot]\) denote de-interleaving by \(\Pi_i\) and by \(\Pi_{ia}\), and 10 iterations are set for the decoding.

- DACC: to eliminated heavy computational complexity caused by iterative decoding, the relay only performs the decoding of the ACC using the log-MAP algorithm, and extracts the systematic part output from the ACC decoder to obtain \(\hat{u}_i\) as the decoding of the ACC is completed. The detection scheme of DACC is depicted in Fig. 4(b).

- DDEX: the relay simply extracts the systematic part output from the differential detector (DD), as shown in Fig. 4(c). Hence, the computational complexity of detection is further reduced.

Note that the bits in sequence \(u_i\) may be correlated caused by the shift registers of the signaling chain. To eliminate the correlation, the information sequence \(u_i\) is interleaved by \(\Pi_i\) before the encoding process at the source node, and the estimated sequence \(\hat{u}_i\) is the de-interleaved output after detection, by which the relationship between \(u_i\) and \(\hat{u}_i\) is guaranteed to be equivalent to an equivalent bit-flipping model.

The error probability \(p_i\) of the three detection strategies are demonstrated in Fig. 5, where 100 information sequences are used in the simulation, and the length of the information sequence is 10000 bits. It can be observed in Fig. 5 that, even in the relatively low SNR regime, the IR strategy can recover the sequence \(u_i\) at the relay with a high
probability, this is because powerful decoding is performed at the relay. On the contrary, the DACC and DDEX strategies obtain the estimated sequence $\tilde{u}_i$ containing errors with a high probability although heavy computational complexity is eliminated. Hence, in the following subsection, a technique is proposed to exploit the erroneous estimates.

### 2.2.1 Network correlation

As mentioned above, the average value of the $SR$ link’s error probability $p_i$ is equivalent to the bit-flipping model. Then, since bitwise XOR coding is always performed at the relay in the proposed e-MARC system, we found a new bit-flipping probability $p_{nc}$ between the two sequences: one is the forwarded XOR-coded sequence generated by the relay by performing bitwise XOR coding on $\tilde{u}_A$ and $\tilde{u}_B$, and the other is their corresponding XOR-ed information sequences $u \oplus (i.e., u_{\oplus} = u_A \oplus u_B)$. Hence, by utilizing $p_A$ and $p_B$, the probability $p_{nc}$ between sequences $u_{\oplus}$ and $u_R$ can be calculated as
Average error prob. vs. Average SNR of AR link, $\Gamma_{AR}$ [dB]

IR (10 iterations)
DACC
DDEX
Limit for code rate = 1/2

Fig 5. Average value of probabilities $p_{nc}$ of detection strategies listed in Fig. 4. [84] (c) 2011 IEEE.

We refer the probability $p_{nc}$ as a network correlation in this thesis. Note that in this chapter, the values of $p_A$ and $p_B$ are assumed to be known to the relay, and the knowledge of $p_{nc}$ is available at the destination with the aid of higher layer protocol setting. In fact, $p_{nc}$ can be directly estimated at the destination during the iterative JNCC decoding process, which will be introduced in Chapter 4.

To exploit the erroneous estimates $\bar{u}_i$, in the following subsection, a JNCC framework and its corresponding decoding scheme are developed to support the relay using very simple detection, such as DACC and DDEX, in which the knowledge of the network correlation $p_{nc}$ is utilized at the destination to help recover the information sequences sent via $iD$ links.
2.2.2 e-MARC scheme

Fig 6. JNCC framework in proposed e-MARC scheme.

Fig 7. Decoder of JNCC applied at destination in proposed e-MARC scheme, [84] (© 2011 IEEE).

Fig. 6 shows the JNCC framework developed for the e-MARC system, where the relay first performs bit-wise XOR coding on both the estimates $\hat{u}_A$ and $\hat{u}_B$, and the interleaved version of $u_R$ is encoded by a RSC code, and modulated using BPSK. Here $\Pi_R[\cdot]$ denotes interleaving by $\Pi_R$.

The corresponding decoder of the JNCC coding scheme is shown in Fig. 7. To exploit the erroneous estimated sequence $\hat{u}$, obtained at the relay, the LLR-updating function $f_e(\cdot)$ [86] is used in the JNCC decoding scheme, where the knowledge of
network correlation \(p_{\text{nec}}\) is exploited by the function \(f_c(\cdot)\) to avoid the error propagation if the XOR-coded sequence \(u_R\) is erroneous. The function \(f_c(\cdot)\) is defined as follows:

\[
\mathcal{L}' = f_c(\mathcal{L}, p_{\text{nec}}) = \ln \left(\frac{1 - p_{\text{nec}}}{1 - p_{\text{nec}}} + p_{\text{nec}} \cdot e^{\mathcal{L}}\right),
\]

(8)

where \(\mathcal{L}\) represents the input LLR sequence to be updated by the bit-wise function \(f_c(\cdot)\), and \(\mathcal{L}'\) is the updated LLR sequence output from \(f_c(\cdot)\).

The received signal vectors \(y_{iD}\) sent from the two source nodes are respectively demodulated bit-wisely at \(D\) to obtain the corresponding soft channel values, as

\[
L(y_{iD}|x_i) = 2 \cdot \Re \left(\frac{y_{iD} h_{iD}^*}{\sigma^2}\right), \quad i \in \{A, B\},
\]

(9)

where the notations \(L(\cdot)\) and \(\Re(\cdot)\) denote the LLR\(^2\) and a function that takes the real part of its argument, respectively, and \(*\) indicates complex conjugation.\(^3\)

The computation from the received vector \(y_{RD}\) to \(L(y_{RD}|x_R)\) also applies (9).

As shown in Fig. 7, given the soft channel values, the soft-in-soft-out (SISO) SCCC and RSC decoders compute the extrinsic LLR values \(L_e(u_A)\) and \(L_e(u_R)\), respectively, using the log-MAP algorithm. Then, since XOR coding is performed at the relay, the extrinsic LLR sequence \(L_e(u_\oplus)\) is obtained by performing the bit-wisely “box-plus” operation [99] on \(L_e(u_A)\) and \(L_e(u_B)\), as

\[
L_e(u_\oplus) = L_e(u_A) \oplus L_e(u_B)
= \ln \frac{\exp(L_e(u_A)) + \exp(L_e(u_B))}{1 + \exp(L_e(u_A) + L_e(u_B))},
\]

(10)

Since the correlation between bits \(u_\oplus(k)\) and \(u_R(k)\) is \(p_{\text{nec}}\), the a priori LLR values for \(u_R\) can be obtained by using \(f_c(\cdot)\) to bit-wisely modify \(L_e(u_\oplus)\), as

\[
L_a(u_R) = f_c(\Pi_R [L_e(u_\oplus)] , p_{\text{nec}})
\]

(11)

Similarly, with using the bit-wise “box-plus” operation, \(L_e(u_A)\) and \(L_e(u_B)\) can be

\[L(x) = \ln \frac{\Pr(x=1)}{\Pr(x=0)}, \text{ where } \Pr(\cdot) \text{ denotes the probability of its argument.}\]

\[\text{Please refer Appendix 1 for the derivation of (9).}\]
extracted from $L_c(u_R)$ computed by the RSC decoder, and be modified by function $f_c(\cdot)$ to become a priori LLR values for $u_A$ and $u_B$, as

$$L_a(u_A) = \Pi_R^{-1}[f_c(L_c(u_R), p_{nc}) \oplus L_c(u_B)]$$

(12)

and

$$L_a(u_B) = \Pi_R^{-1}[f_c(L_c(u_R), p_{nc}) \oplus L_c(u_A)],$$

(13)

where $\Pi_R^{-1}[\cdot]$ indicates de-interleaving from $\Pi_R$. For the sake of simplicity, the combination of the JNCC and its decoding scheme developed for the e-MARC system is referred to as an e-MARC scheme. It should be emphasized that the proposed e-MARC scheme is not a unique practical realization for the e-MARC system. It is possible to exploit the network correlation with more sophisticated coding and modulation schemes.

### 2.2.3 SDF-MARC scheme

Fig 8. Decoder of SDF-MARC scheme as only one correct estimated sequence is forwarded. Here $u_B = u_B$ and $u_A \neq u_A$ for example.

For the purpose of fair comparison, the JNCC and its decoding schemes, respectively shown in Fig. 6 and Fig. 7, are also used for the SDF-MARC system with simple modifications. Unlike e-MARC, the relay discards the erroneous received estimates in the SDF-MARC system. Therefore, in the case that both estimates are correctly de-
coded, the encoding and decoding processes of JNCC is same as that of the proposed e-MARC scheme. As one of the estimates is decoded in error, instead of performing JNCC encoding, the relay simply encodes the interleaved version of the correct estimated sequences using RSC code, and the JNCC decoder shown in Fig. 7 is reduced to a parallel concatenated convolutional code (PCCC) decoder, as shown in Fig. 8. The relay remains silent as both estimated sequences are incorrect. For consistency, the modified JNCC and its decoding scheme described above is referred to as a SDF-MARC scheme.

2.3 EXIT analysis

![Fig 9. Detailed flows of LLR values for 3D EXIT analysis, [84] (© 2011 IEEE).]

In this section, we analyze the JNCC decoder of the proposed e-MARC scheme illustrated in Fig. 7 by utilizing a three-dimensional (3D) EXIT chart for tracking the convergence property of the information estimates while the two SCCC decoders simultaneously exchange the message in the from of LLR values with the RSC decoder. The
analysis focuses on the both SCCC decoders since our aim is to perfectly retrieve the information estimates $\hat{u}_A$ and $\hat{u}_B$. In Fig. 9, we temporarily remove one of the SCCC decoders from the JNCC decoder to compute the mutual information of the extrinsic bit probabilities $I_e(u_B)$; since the JNCC decoder is symmetric in structure, the computation for $I_e(u_A)$ is same as for $I_e(u_B)$. As shown in Fig. 9, the EXIT function $T_u^P(\cdot)$ composed of the two RSC decoders can be characterized as follows

$$I_e(u_B) = T_u^P(I_a(c_B), I_e(u_A)|p_{nc}, \Gamma_{RD})$$

in which

$$I_e(u_B) = I(u_B; L_e(u_B))$$

$$I_a(c_B) = I(c_B; L_a(c_B))$$

$$I_e(u_A) = I(u_A; L_e(u_A))$$

As shown in (14), the function $T_u^P(\cdot)$ has two input parameters $I_a(c_B)$ and $I_e(u_A)$, and thus we use a 3D EXIT chart to draw the output mutual information $I_e(u_B)$ given the SNR of the $RD$ link and the value of $p_{nc}$.

### 2.3.1 Consistency condition

As mentioned in [87], the Gaussian distributed LLR sequence with symmetric and consistency conditions are fundamental requirements for a tight EXIT chat analysis. In general, the LLR values output from the decoder using Bahl-Cocke-Jelinek-Raviv (BCJR) decoding algorithm are Gaussian distributed. However, the LLR values updated by the function $f_c(\cdot)$ are not; moreover, the consistency condition does not hold for the distribution of the updated LLR values. Hence, we numerically measure the distribution of $L_e(u_B)$ and $L_e(c_B)$ output from the function $T_u^P(\cdot)$ to verify the robustness of the function $f_c(\cdot)$ to EXIT analysis.

As shown in Fig. 9, the two inputs $L_a(c_B)$ and $L_e(u_A)$ come from the ACC and RSC decoders, respectively, and thus, it is reasonable to artificially generate the values of $L_a(c_B)$ and $L_e(u_A)$ with Gaussian distribution.
u_B = 1 and u_B = −1 (or, the distribution of L_e(c_B) given c_B = 1 and c_B = −1) are measured after completing 40 iterations for the information exchange between the two RSC decoders of T_u^P(·). Figs. 10(a)-10(d) show the conditional distributions of L_e(u_B) and L_e(c_B) with different input parameters. It can be observed that the conditional distribution of L_e(u_B) output from the T_u^P(·) is approximated to Gaussian; furthermore, for the conditional distribution of L_e(u_B), the variance is nearly two times that of the mean. This states that the consistency condition is still maintained even though the function f_c(·) is embedded in the function T_u^P(·).

2.3.2 Impact of pnc

Figs. 11(a) and 11(b) show the EXIT planes of T_u^P(·) with small and large pnc values, respectively, where \( \Gamma_{RD} = 0 \) dB. It can be seen from Fig. 11(a) that when pnc is small
(i.e., $u_R$ and $u_A$ are highly correlated), the extrinsic information provided from the other SCCC decoder, $I_e(u_A)$, has a significantly effect on $T_P^R(\cdot)$; however, when $p_{nc}$ is large, as indicated by Fig. 11(b), that $I_e(u_B)$ is nearly a constant value over the entire range of $I_e(u_A)$. Therefore, when $p_{nc}$ is small, we have to consider the impact of the other SCCC decoder when performing an EXIT analysis for the JNCC decoder; on the contrary, when $p_{nc}$ is large, the 3D EXIT analysis for the JNCC decoder can be simplified to a two-dimensional (2D) EXIT analysis.

### 2.4 Numerical results

<table>
<thead>
<tr>
<th>Table 1. Parameters for BER simulations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Length of information sequence ($K$)</td>
</tr>
<tr>
<td>Generator polynomial of RSC code</td>
</tr>
<tr>
<td>Generator polynomial of ACC</td>
</tr>
<tr>
<td>Spectrum efficiency of $\mathcal{E}_R$ ($K/M$)</td>
</tr>
<tr>
<td>Spectrum efficiency of $\mathcal{E}_R$ ($K/M$)</td>
</tr>
<tr>
<td>Signaling scheme $\mathcal{E}_R$</td>
</tr>
<tr>
<td>Receiver $\mathcal{D}_R$</td>
</tr>
<tr>
<td>Global iteration (GI)</td>
</tr>
<tr>
<td>Local iteration (LI)</td>
</tr>
<tr>
<td>Loops between GI and LI</td>
</tr>
<tr>
<td>Decoding algorithm</td>
</tr>
<tr>
<td>Interleavers</td>
</tr>
</tbody>
</table>

In this section, we apply the analytical method presented in Sec. 2.3 to identify the
SNR values required for the $AD$ and $BD$ links to be able to retrieve the information estimates $\hat{u}_A$ and $\hat{u}_B$ successfully given $\Gamma_{iR}$ and $\Gamma_{RD}$. All the detection strategies numerated in Section 2.2 use the same iterative decoder illustrated in Fig. 7, since the same structure in signaling scheme is applied at the source node. The procedure of the decoding process is described as follows

- (Step1) $L_e(u_j)$ for $j \in \{A, B, R\}$ is computed from $L(y_{jD} | x_j)$. No iterative decoding is performed between the ACC and RSC decoders for the computation of $L_e(u_A)$ and $L_e(u_B)$.

- (Step2) The two SCCC decoders simultaneously exchange the message in the from of LLR values with the RSC decoder. The behavior is denoted as a global iteration (GI). The number of global iterations is set to 40.

- (Step3) The RSC decoder of the two SCCC obtains the apriori $L_a(u_i)$ for $i \in \{A, B\}$ after completing Step2. Then, iterative decoding is performed between the ACC and RSC decoders, which is referred to as a local iteration (LI). The number of local iterations is set to 6. Go back to Step2 when Step3 is finished, and the number of loops between Step2 and Step3 is set to 3.

The BER here is defined as follows: total number of errors $\hat{u}_A$ and $\hat{u}_B$ contain, divided by the total number of transmission bits. The main parameters are listed in Table 1.

### 2.4.1 AWGN links

With the assumption of relatively high SNR intra links, $\Gamma_{AR} = \Gamma_{BR} = 2$ dB and $\Gamma_{RD} = 0$ dB were assumed. The 3D EXIT chart analysis for the JNCC decoder with the DACC strategy is shown in Fig. 12, where the 3D EXIT planes of the function $T^P_u(\cdot)$ and ACC decoder are demonstrated. Note that the same structure of the SCCC is used for the decoding of the signal vectors sent from $A$ and $B$, and thereby, we use the same EXIT analysis to trace the decoding trajectories of the both information estimates $\hat{u}_A$, $\hat{u}_B$, and placed them together in Fig. 12.

Given $\Gamma_{AR}$, $\Gamma_{BR}$ and $\Gamma_{RD}$, we simultaneously change the values of $\Gamma_{AD}$ and $\Gamma_{BD}$ and draw the corresponding decoding trajectories. It can be observed in Fig. 12 that as $\Gamma_{AD}$ and $\Gamma_{BD}$ achieve -2.3 dB, the convergence tunnel, seen in the 2D EXIT chart
when $I_e(u_A) = 0$, is closed, but the 3D tunnel ($0 \leq I_e(u_A) \leq 1$) is slightly open; moreover, the trajectories for the $\hat{u}_A$ and $\hat{u}_B$ match the EXIT planes and both reach the $(1, 1, 1)$ mutual information point. Therefore, the turbo cliff of the DACC strategy with the proposed e-MARC scheme is expected to occur when $\Gamma_{AD}$ and $\Gamma_{BD}$ achieve around -2.3 dB.

Fig. 13 shows the BER performances of all the strategies listed in Fig. 4 with the proposed e-MARC scheme. The turbo cliff of the DACC strategy happens when $\Gamma_{AD}$ and $\Gamma_{BD}$ are around at -2.3 dB. The BER curve of the IR strategy is provided as a limit for those of the DACC and DDEX strategies, since both the $SR$ links with using the IR strategy are nearly error-free as $\Gamma_{AR} = \Gamma_{BR} = 2$ dB. As shown in Fig. 13, there is a 0.55-dB gap between the IR and DACC/DDEX strategies; however, it should be noticed that even when $\Gamma_{AR} = \Gamma_{BR} = 2$ dB, as shown in Fig. 5, the estimates $\hat{u}_i$ received at the relay still contain errors with a very high probability with using the DACC and DDEX strategies; furthermore, to achieve nearly error-free $SR$ links as $\Gamma_{AR} = \Gamma_{BR} = 2$ dB, iterative decoding requiring heavy computational complexity has to be performed at the relay. Such a high complexity requirement can be eliminated with the DACC/DDEX detection strategy, since only ACC decoding/differential detection has to be performed at the relay, for which computational complexity is very low. In summary, compared to a high complexity decoding strategy at the relay, the DACC/DDEX strategies combined with the proposed e-MARC scheme significantly reduces the relay complexity, which
Average BER

<table>
<thead>
<tr>
<th>Scheme</th>
<th>DACC</th>
<th>DDEX</th>
<th>IR</th>
<th>Without relay (or SDF-MARC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{iR}$</td>
<td>2 dB</td>
<td>0.8 dB</td>
<td>$-0.5$ dB</td>
<td>0.55 dB</td>
</tr>
</tbody>
</table>

Fig 13. BER performances of all strategies with e-MARC and SDF-MARC schemes. All links in MARC are AWGN. $\Gamma_{RD}$ is fixed to 0 dB. [84] (© 2011 IEEE).

is in exchange for a 0.55 dB loss in the BER performance from the case of nearly error-free $SR$ links.

With the assumption of relatively low SNR intra links, $\Gamma_{AR} = \Gamma_{BR} = -0.5$ dB and $\Gamma_{RD} = 0$ dB. In this case the quality of the intra links is worse (i.e., $p_i \approx 0.1564, 0.2015$ for DACC and DDEX, respectively); therefore, the performance of the two strategies with the proposed e-MARC scheme are, as shown in Fig. 13, worse than the case of relatively high SNR intra links. Moreover, it can be observed in Fig. 13 that, compared with no cooperation, forwarding erroneous estimates achieves only a slightly better performance, but requires higher power consumption. This implies a trade-off between the performance and energy efficiency when the e-MARC scheme is used in the poor quality of both intra links.

### 2.4.2 $SR$ links with constant error probability

In this subsection, comparisons of the BER performances are made between the e-MARC and SDF-MARC schemes when the $SR$ links are modeled by BSCs while the

---

4 The code used at the sources is shown in Fig. 3.

5 The detection strategy needs not to be taken into account since the $SR$ links are modeled by BSCs.
Fig 14. BER performances of e-MARC and SDF-MARC, where SR links are modeled by BSCs while SD and RD links are AWGN.

Fig 15. EXIT analysis of LLR-updating function $f_c(\cdot)$.
other links are assumed to be AWGN. It is found in Fig. 14 that e-MARC is superior to SDF-MARC except for the case that \((p_A, p_B) = (0, 0)\). This is because with applying the SDF relay strategy, the recovery of the information sequences totally depends on the error correction capability of the schemes applied in the direct link (i.e, SCCC), and according to [100], the minimum SNR for the SCCC, composed of a \((1, 5/7)_{8}\) RSC code and a \((2/3)_{8}\) ACC, to achieve convergence after performing 40 iterations is -1.8 dB.

Nevertheless, the BER performance is significantly improved with the proposed e-MARC scheme due to the exploitation of the erroneous estimates \(\hat{u}_A\). Furthermore, it can be observed in Fig. 14 that the fewer number of errors \(\hat{u}_A\) and \(\hat{u}_B\) contain, the better BER performance can be achieved. This is because the large value of \(p_{nc}\) degrades the output amount of mutual information from the LLR-updating function \(f_c(\cdot)\), as shown in Fig. 15.

### 2.4.3 Fading channels

**Table 2. Settings of Symmetric and Asymmetric scenarios, [89] (© 2015 IEEE).**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\Gamma_{AD})</th>
<th>(\Gamma_{BD})</th>
<th>(\Gamma_{AR})</th>
<th>(\Gamma_{BR})</th>
<th>(\Gamma_{RD})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>(X)</td>
<td>(X)</td>
<td>(X + \Delta)</td>
<td>(X + \Delta)</td>
<td>(X + \Delta)</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>(X)</td>
<td>(X - L)</td>
<td>(X + \Delta)</td>
<td>(X + \Delta - L)</td>
<td>(X + \Delta)</td>
</tr>
</tbody>
</table>

**Table 3. Parameters for BER/FER simulations in fading channels, [89] (© 2015 IEEE).**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of information sequence (K)</td>
<td>2048 bits</td>
</tr>
<tr>
<td>Generator polynomial of RSC code</td>
<td>((1, 5/7)_{8})</td>
</tr>
<tr>
<td>Generator polynomial of ACC</td>
<td>((2/3)_{8})</td>
</tr>
<tr>
<td>Spectrum efficiency of (E_S (K/M))</td>
<td>(1/2)</td>
</tr>
<tr>
<td>Spectrum efficiency of (E_R (K/M))</td>
<td>(1/2)</td>
</tr>
<tr>
<td>Signaling scheme (E_R)</td>
<td>SCCC</td>
</tr>
<tr>
<td>Receiver (D_R)</td>
<td>DDEX</td>
</tr>
<tr>
<td>Receiver (D_D)</td>
<td>[85, Fig. 4]</td>
</tr>
<tr>
<td>Global iteration (GI)</td>
<td>1 iteration</td>
</tr>
<tr>
<td>Local iteration (LI)</td>
<td>10 iterations</td>
</tr>
<tr>
<td>Loops between GI and LI</td>
<td>10</td>
</tr>
<tr>
<td>Decoding algorithm</td>
<td>log-MAP</td>
</tr>
<tr>
<td>Interleavers</td>
<td>Random</td>
</tr>
</tbody>
</table>
The SR links are assumed to be AWGN and BSCs in subsections 2.4.1 and 2.4.2, respectively. To be more practical for the real scenarios, in this subsection, all the links in MARC are assumed to suffer from block Rayleigh fading. We compare the e-MARC and SDF-MARC schemes in terms of the FER and BER performances, where DDEX is used as an example. The aim of utilizing the low-complexity and non-coherent DDEX is to reduce the power consumption and large latency caused by complicated decoding algorithms. The settings of the both scenarios are summarized in Table 2 for testing, where $X$ represents the average SNR of the AD link in dB; $\Delta$ and $L$ denoting additional gain and loss due to the shorter and longer distance, respectively, and are set to 3 dB and 10 dB in this subsection. The main parameters are listed in Table 3.

Results of the FER performances in the Symmetric and Asymmetric scenarios are demonstrated in Fig. 16 for comparison. Here FER is defined as follows: the number of the transmission cycles where either one or both of the information sequences sent from the nodes $A$ and $B$ cannot successfully recovered at $D$ even with the help of $R$, divided by the total number of transmission cycles.

It can be observed in Fig. 16 that in the Symmetric and Asymmetric scenarios, using DDEX, e-MARC respectively obtains 0.7 dB and 2.2 dB gain from SDF-MARC.
This indicates that even though the estimates $\hat{u}_i$ contain errors with a high probability due to the use of DDEX, the erroneous estimates received at the relay, instead of being discarded, can still be effectively exploited in the proposed e-MARC system for reconstructing the information sequences $u_i$, sent via the $iD$ link, at the destination.

The theoretical outage probabilities of SDF-MARC in both scenarios are also included in Fig. 16 as the limits, and obtained using the results of [33, eqs. (4)-(6)] with a modification of [101, eq. (12)]. Here the outage probabilities are assumed to use Gaussian codebook with an infinite sequence length. It can be observed from Fig. 16 that the FER performances of the proposed e-MARC with DDEX are roughly 6 dB and 5 dB away from the outage of SDF-MARC in the Symmetric and Asymmetric scenarios, respectively. The loss is due to several reasons, such as the very short sequence length used in the simulation, and very simple DDEX; furthermore, optimal code design is not taken into account here.

The BER performances of DDEX with e-MARC and with SDF-MARC schemes are also demonstrated in Fig. 17. Compared to the FER performance shown in Fig. 16, the e-MARC is more advantageous in the BER performance, and provides roughly 2.3 dB and 3.8 dB gain over SDF-MARC at $\text{BER} = 10^{-3}$ in the Symmetric and Asymmetric
scenarios, respectively. This is because the lower amount of mutual information, caused by the larger value of \( p_{nc} \), is still helpful in correcting several error bits, even though it is not enough to correct all the error bits.

2.5 Conclusions

The e-MARC system has been proposed to improve the BER/FER performances by exploiting the erroneous information estimates received at the relay, especially for the case where low-computational complexity detection strategies are applied at the relay due to several constraints, such as the large latency induced by the iterative decoding process and/or the battery and memory resources of the relay may be limited to perform complicated decoding algorithms. The key factors of the e-MARC system are: I) the relay always performs network-coding on both estimates and forwards the coded sequence, and II) the knowledge of the network correlation \( p_{nc} \) is utilized by using the LLR-updating function in the JNCC iterative decoding process at the destination. According to the simulated results, the BER and FER performances are apparently improved in the proposed e-MARC system. A significant improvement has been observed for the cases: 1) where two \( SR \) links are assumed to be BSCs with constant small crossover probabilities while other links are assumed to be AWGN, and 2) where one source node is far away from both the relay and destination when all links suffer from block Rayleigh fading.

The knowledge of the network correlation \( p_{nc} \) is assumed to be known in the destination, which may be less practical in the real scenario. In addition, the limit of the proposed e-MARC system has not been explored. We will demonstrate these in the later chapters of this thesis.
3 Outage probabilities of erroneous estimates-exploiting MARC

In this chapter, we derive the outage probability of the proposed e-MARC system where all five links in the system (two SD links, two SR links and one RD link) suffer from statistically independent block Rayleigh fading. A theoretical limit of the SD link’s error probability is first found by utilizing the rate-distortion and inverse capacity functions. After that, we identify the e-MARC system’s admissible rate region according to the Slepian-Wolf theorem [92] for correlated source coding with a helper. Then, the outage probability of the proposed e-MARC system, independent of signaling schemes, can be theoretically derived by a fivefold-integral over the admissible rate region with respect to the pdfs of the five links’ instantaneous SNRs.

This chapter is organized as follows: The outage probability for the proposed e-MARC system is theoretically derived in Section 3.1. Numerical results of the outage probabilities for the universal case and two special cases are presented in Section 3.2. Furthermore, the results of simulations are presented to evaluate the performance of the DDEX with the proposed e-MARC scheme introduced in Chapter 2 in terms of FER in Section 3.2. Also, the theoretical outage probabilities of network-coding-based MARC systems (NC-MARC) [29] and that of SDF-MARC are included in Section 3.2 for comparison. The impact of the source correlation and network correlation on the outage probability is discussed in Section 3.3. Finally, Section 3.4 concludes the chapter.

3.1 Derivation for outage probability of e-MARC

For the e-MARC system depicted in Fig. 2, the two source nodes aim to be losslessly recovered given the side information provided by the relay. Furthermore, the information sequences $u_A$, $u_B$ and relay sequence $u_R$ are correlated. Therefore, the e-MARC system model can be viewed as correlated source coding with a helper [82, Section 10.4], where the relay helps reduce the source coding rate. In the following, we establish the admissible rate region for the e-MARC system, and define the outage event according to the region.
Fig 18. Admissible rate region of rate pair \((R_A, R_B)\) given \(R_R \geq I(u_R; \hat{u}_R)\) for lossless compression, where \(\delta = H_b(p_A \ast p_B \ast p_R)\) [89] (© 2015 IEEE).

3.1.1 Outage event for each transmission cycle

The proposed e-MARC system model shown in Fig. 2 can be viewed as a system having two source nodes \(A\) and \(B\), and one helper \(R\). Letting \(R_A\) and \(R_B\) be the source coding rate of \(A\) and \(B\), respectively. To investigate the admissible rate region for the two independent source nodes with one helper, we thus invoke the theorem [82, Theorem 10.4.], according to which the information sequences \(u_A\) and \(u_B\) can be successfully recovered at the destination if

\[
\begin{align*}
R_A &\geq H(u_A|u_B, \hat{u}_R), \\
R_B &\geq H(u_B|u_A, \hat{u}_R), \\
R_A + R_B &\geq H(u_A, u_B|\hat{u}_R), \\
R_R &\geq I(u_R; \hat{u}_R),
\end{align*}
\]

(18)

where \(R_R\) represents the source coding rate at \(R\), and the coded side information provided by the relay \(R\) (helper) helps reduce the rate \(R_A\) and \(R_B\). Then, by using the
chain rule, (18) can be re-formulated (see Appendix 2) as follows

\[ R_A \geq \delta, \]  
\[ R_B \geq \delta, \]  
\[ R_A + R_B \geq \delta + 1, \]  
\[ R_R \geq 1 - H_b(p_R). \]

where

\[ \delta = H_b(p_A^* p_B^* p_R). \]  

The operation \( \alpha * \beta \) is defined as \( \alpha(1 - \beta) + \beta(1 - \alpha) \), and \( H_b(\cdot) \) denotes the binary entropy function [102]. To simplify the derivation, we assume that the inequality (19d) is satisfied. With a given \( R_R \), the admissible rate region of \((R_A, R_B)\) for each transmission cycle is obtained, as shown in Fig. 18.

For a transmission cycle, the outage event is defined as: one or both of the information sequences \( u_A \) and \( u_B \) cannot be successfully recovered at the destination. Therefore, given a value of \( R_R \), the outage event happens when the pair \((R_A, R_B)\) falls outside the admissible rate region. As shown in Fig. 18, the entire admissible rate region can be divided into two parts \( T_1 \) and \( T_2 \). \( \epsilon_1 \) and \( \epsilon_2 \) denote the events that the rate pair \((R_A, R_B)\) falls into \( T_1 \) and \( T_2 \), respectively, as

\[ \epsilon_1 = \{(R_A, R_B) \in T_1\} = \{\delta \leq R_A \leq 1\} \land \{R_A + R_B \geq \delta + 1\} \]
\[ \epsilon_2 = \{(R_A, R_B) \in T_2\} = \{R_A \geq 1\} \land \{R_B \geq \delta\}. \]  

where the symbols ‘\( \lor \)’ and ‘\( \land \)’ denote as the logical ‘or’ and ‘and’ operators, respectively. Therefore, with the assumption that (19d) is satisfied, the outage event of the e-MARC system for each transmission cycle is obtained, as

\[ \text{OUT} = \{\epsilon_1 \lor \epsilon_2\}. \]  

where the \( \{\epsilon_1 \lor \epsilon_2\} \) denotes the complement of event \( \{\epsilon_1 \lor \epsilon_2\} \).

According to Shannon’s source-channel separation theorem, the relationship be-
between the instantaneous SNR $\gamma_{iD}$ and its corresponding source coding rate $R_i$ is given by [96]

$$R_i = \frac{C(\gamma_{iD})}{R_{ci}} = f_i(\gamma_{iD}), \quad i \in \{A, B\}$$

(23)

where it is assumed that a capacity-achieving channel code is used in the $iD$ link (see Appendix 3). Here $R_{ci}$ represents the spectrum efficiency of the signaling scheme $E_S(\cdot)$, including the coding rate and bit-per-symbol modulation, and $C(\alpha) = \log_2(1 + \alpha)$. It should be emphasized that the optimality of source-channel separation holds for Rayleigh fading MARC systems where all the links are orthogonal [103, 104].

Since the function $f_i(\cdot)$ is one-to-one mapping, the events $\varepsilon_1$ and $\varepsilon_2$ can be, respectively, expressed as

$$\varepsilon_1 = \{f_A^{-1}(\delta) \leq \gamma_{AD} \leq f_A^{-1}(1)\} \land \{f_B^{-1}(\omega) \leq \gamma_{BD}\}$$

$$\varepsilon_2 = \{f_A^{-1}(1) \leq \gamma_{AD}\} \land \{f_B^{-1}(\delta) \leq \gamma_{BD}\},$$

(24)

where

$$\omega = \delta + 1 - f_A(\gamma_{AD}).$$

(25)

### 3.1.2 Theoretical limits of $p_A$ and $p_B$

The boundaries of the events $\varepsilon_1$ and $\varepsilon_2$ in (24) are involved with the $iR$ link error probability $p_i$, and the value of the $p_i$ is a function of $\gamma_{it}$. However, the relationship between $p_i$ and $\gamma_{it}$ depends on the signaling schemes employed in the $iR$ link, as shown in Fig. 5. Furthermore, it is quite common that $p_i$ cannot be explicitly expressed as a function of $\gamma_{it}$, if specific channel coding or modulation schemes are used. Therefore, instead, we aim to derive a theoretical limit for the value of $p_i$, given a $\gamma_{it}$ value in the following.

According to Shannon’s lossy source-channel separation theorem [105], the infor-
received SNR of at R, $\gamma_i R$ [dB]
Average error prob.
Scheme IR (10 iterations)
Scheme DACC
Scheme DDEX
Limit of err prob. (Gaussian capacity)
Limit of err prob. (CC capacity)

Fig 19. Theoretical limit $\hat{p}_i$ of $p_i$ for given $\gamma_{at}$. CC is the abbreviation of constellation constraint.

A message sequence $u_i$ can be transmitted over the $iR$ link with a distortion level $D_i$ if

$$R_i(D_i)R_{ci} \leq C(\gamma_{at}), \quad i \in \{A, B\}. \quad (26)$$

With the Hamming distortion measure, the distortion $D_i$ is equivalent to the error probability $p_i$, and thus according to [102], the rate-distortion function $R_i(D_i)$ is represented as

$$R_i(D_i) = 1 - H_b(p_i). \quad (27)$$

By assuming that a capacity-achieving channel code is applied at the $iR$ link, the equality in (26) holds, yielding the theoretical limit $\hat{p}_i$ of $p_i$ for any given $\gamma_{at}$, as

$$\hat{p}_i(\gamma_{at}) = \begin{cases} H_b^{-1}(1 - f_i(\gamma_{at})) = \hat{p}_i(\gamma_{at}), & 0 \leq \gamma_{at} < \gamma_i^* \\ 0, & \gamma_{at} \geq \gamma_i^* \end{cases} \quad (28)$$

where $H_b^{-1}(\cdot)$ denotes the inverse function of $H_b(\cdot)$, and $\gamma_i^*$ is a threshold value of the
instantaneous SNR $\gamma_{\text{rd}}$ such that $f_i(\gamma^*) = 1$. Fig. 19 shows the result of the theoretical limit $\hat{p}_i$ of $p_i$ derived by (28).

### 3.1.3 Theoretical limit of $p_R$

In this subsection, the assumption that the inequality (19d) holds in (24) is eliminated, such that the variation of the rate $R_R$ can be taken into account when deriving the outage probability in the next subsection. In the same way as deriving (23), the relationship between the instantaneous SNR $\gamma_{\text{rd}}$ and rate $R_R$ can be expressed, as

$$R_R = \frac{C(\gamma_{\text{rd}})}{R_{cR}} = f_R(\gamma_{\text{rd}}) \geq 1 - H_b(p_R) \quad (29)$$

where $R_{cR}$ represents the spectrum efficiency of the signaling scheme $E_R(\cdot)$. Considering the constraint imposed on $R_R$ (i.e., $R_R \geq 1 - H_b(p_R)$), it is found from (29) that, although the variation range of the rate $R_R$ is $[0, \infty)$, $p_R$ is reduced to zero when the value of $\gamma_{\text{rd}}$ is larger than $f_R^{-1}(1)$. Therefore, by (29), the theoretical limit $\hat{p}_R$ of $p_R$ for any given instantaneous SNR value $\gamma_{\text{rd}}$, is

$$\hat{p}_R(\gamma_{\text{rd}}) = \begin{cases} H_b^{-1}(1 - f_R(\gamma_{\text{rd}})) = \hat{p}_R(\gamma_{\text{rd}}), & 0 \leq \gamma_{\text{rd}} < \gamma^*_R \\ 0, & \gamma_{\text{rd}} \geq \gamma^*_R, \end{cases} \quad (30)$$

where $\gamma^*_R = f_R^{-1}(1)$.

By replacing $p_i$ and $p_R$ with $\hat{p}_i(\gamma_{\text{rd}})$ and $\hat{p}_R(\gamma_{\text{rd}})$ in (24), respectively, the boundaries of events $\varepsilon_1$ and $\varepsilon_2$ in (24) are converted to the domains of instantaneous SNR, as

$$\varepsilon_1 = \{f_A^{-1}(\delta) \leq \gamma_{\text{ad}} \leq f_A^{-1}(1)\} \land \{f_B^{-1}(\hat{\omega}) \leq \gamma_{\text{bd}}\}$$

$$\varepsilon_2 = \{f_A^{-1}(1) \leq \gamma_{\text{ad}}\} \land \{f_B^{-1}(\delta) \leq \gamma_{\text{bd}}\}, \quad (31)$$

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where
\[
\hat{\delta} = H_b(\hat{p}_A(\gamma_{AR}) \ast \hat{p}_B(\gamma_{BR}) \ast \hat{p}_R(\gamma_{RD}))
\]
\[
\hat{\omega} = \hat{\delta} + 1 - f_A(\gamma_{AD}).
\]

(32)

3.1.4 Outage probability of e-MARC

Recall that in this chapter all the links are assumed to be statistically independent and their corresponding instantaneous SNRs are Rayleigh distributed, transmission-by-transmission and link-by-link. Hence, the probabilities of \( \varepsilon_1 \) and \( \varepsilon_2 \) are calculated, as

\[
\Pr(\varepsilon_1) = \Pr \left( \{ f_A^{-1}(\gamma_{AD}) \leq \gamma_{AD} \leq f_A^{-1}(1) \} \land \{ f_B^{-1}(\hat{\omega}) \leq \gamma_{BD} \} \right)
\]

\[
= \int \int \int \int_{V} \int_{f_A^{-1}(\delta)}^{f_A^{-1}(1)} \int_{f_B^{-1}(\omega)}^{\infty} \frac{1}{\Gamma_{AD}} \frac{1}{\Gamma_{BD}} \exp \left( -\frac{f_B^{-1}(\omega)}{\Gamma_{BD}} - \frac{\gamma_{AD}}{\Gamma_{AD}} \right) d\gamma_{AD} d\gamma_{BD} d\gamma_{AR} d\gamma_{BR} d\gamma_{RD}
\]

\[
= g_1 \cdot p(\gamma_{AR}, \gamma_{BR}, \gamma_{RD})
\]

(33)

\[
p(\gamma_q) = \frac{1}{\Gamma_q} \exp \left( -\frac{\gamma_q}{\Gamma_q} \right), \quad q \in \{ AR, BR, AD, BD, RD \}
\]
and

\[
\Pr(\varepsilon_2) = \Pr \left( \{ f_A^{-1}(1) \leq \gamma_{AD} \} \land \{ f_B^{-1}(\hat{\delta}) \leq \gamma_{BD} \} \right)
\]

\[
= \iiint_V p(\gamma_{AD}) d\gamma_{AD} \int_{f_B^{-1}(\hat{\delta})}^{\infty} p(\gamma_{RD}) d\gamma_{RD} p(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}) d\gamma_{AR} d\gamma_{BR} d\gamma_{RD}
\]

\[
= \iiint_V \exp \left( -\frac{f_A^{-1}(1)}{\Gamma_{AD}} - \frac{f_B^{-1}(\hat{\delta})}{\Gamma_{BD}} \right) p(\gamma_{AR}) p(\gamma_{BR}) p(\gamma_{RD}) d\gamma_{AR} d\gamma_{BR} d\gamma_{RD}
\]

\[
= \mathcal{I}[g_2; V],
\]

where the domain of the threefold integral is

\[
V = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}) : \gamma_{AR} \in \mathbb{R}^+, \gamma_{BR} \in \mathbb{R}^+, \gamma_{RD} \in \mathbb{R}^+\},
\]

\[
\mathbb{R}^+ = [0, \infty).
\]

Finally, the outage probability of the e-MARC system can be obtained, as

\[
P_{\text{out}} = 1 - (\Pr(\varepsilon_1) + \Pr(\varepsilon_2)).
\]

It may be difficult to calculate the integrals shown in (33) and (34) in closed form. Hence, the results of (33) and (34) in this thesis are numerically obtained by using functions provided in [106]. Moreover, (33) and (34) can be respectively divided into eight sub-integrals according to different domains, which makes the numerical calculation of (33) and (34) tractable. The divisions of (33) and (34) are listed in Appendix 4.
3.2 Numerical results

Fig. 20. Outage probabilities of e-MARC that are independent of signaling schemes, where $\Gamma_{AD} = \Gamma_{BD}$ and $\Gamma_{RD} = \Gamma_{AD} + 3$ dB is assumed, and $R_{ci} = R_{cR} = 1/2$, [89] (© 2015 IEEE).

3.2.1 Outage probability of e-MARC

Fig. 20 shows the numerical results of the outage probability for the e-MARC system that are independent of signaling schemes, obtained by calculating from (36) with (33) and (34) shown in Section 3.1, where $\Gamma_{AD} = \Gamma_{BD}$ and $\Gamma_{RD} = \Gamma_{AD} + 3$ dB is assumed. The values of $R_{ci}$ and $R_{cR}$ are set to 1/2. The curve of the probability with perfect (i.e., error-free) intra links is obtained from [91]. It is found that as the quality of the intra links degrades, the gap increases between the probabilities with perfect and imperfect intra links.

The curves of the outage probabilities with constellation constraint capacity (CCC) are also provided in Fig. 20, where the capacity function $C(\alpha) = \log_2(1+\alpha)$ is replaced with $C(\alpha) = 2J(\sqrt{4\alpha})$. Here the $J$ function can be closely approximated by
\[ J(\alpha) \approx (1 - 2^{-H_1 \alpha^2 H_2})^{H_3}, \]  

(37)

where \( H_1 = 0.3073 \), \( H_2 = 0.8935 \) and \( H_3 = 1.1064 \) [88].

As shown in Fig. 19, the theoretical limit of \( p_i \) obtained by using CCC function is slightly different with that obtained by using Gaussian capacity function. Furthermore, as the average SNRs of the \( iR \) links increase, the probability that the instantaneous SNR is less than the threshold \( \gamma_{i}^* \) is significantly reduced. Hence, it can be observed in Fig. 20 that there is only a slight difference in the low SNR regime between the curves with Gaussian capacity and with CCC, while in the high SNR regime the curves overlap.

### 3.2.2 Outage performance comparisons

![Outage probability graph](image)

Fig 21. Outage probabilities of e-MARC in comparisons with those with NC-MARC [29], and with SDF-MARC [33]. \( R_{ci} = R_{cR} = 1/2 \) [89] (© 2015 IEEE).

To evaluate the impact of the utilization of the erroneous estimates received at the re-
Table 4. Relaying behavior.

<table>
<thead>
<tr>
<th>System</th>
<th>Estimates obtained at $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{u}_A = u_A$</td>
</tr>
<tr>
<td>e-MARC</td>
<td>$\hat{u}_A \oplus \hat{u}_B$</td>
</tr>
<tr>
<td>NC-MARC (static mode) [29]</td>
<td>$\hat{u}_A \oplus \hat{u}_B$</td>
</tr>
<tr>
<td>NC-MARC (adaptive mode) [29]</td>
<td>$\hat{u}_A \oplus \hat{u}_B$</td>
</tr>
<tr>
<td>SDF-MARC [33]</td>
<td>$\hat{u}_A \oplus \hat{u}_B$</td>
</tr>
</tbody>
</table>

lay, we also compare the outage probability of the e-MARC system with the theoretical outage probabilities with NC-MARC [29] and SDF-MARC [33]. Results of the outage probability analysis in the Symmetric and Asymmetric scenarios are demonstrated in Fig. 21 for comparison. The scenarios settings exemplifying the symmetric and asymmetric cases are summarized in Table 2, where $X$ represents the average SNR of the $AD$ link in dB; $\Delta$ and $L$ denoting additional gain and loss due to the shorter and longer distance, respectively, and are set to 3 dB and 10 dB in this subsection.

NC-MARC is a special case of the SDF-MARC system where a maximal ratio combining (MRC) technique is used at the destination. Furthermore, according to [29], the NC-MARC system is able to operate in static and adaptive modes. In the static mode, the relay forwards the sequence only when both the estimates sent from the two sources are decoded correctly at the relay. On the other hand, in the adaptive mode, except for the case that the estimates sent from the two sources are decoded correctly, the relay also forwards the correct estimate if only one source node is correctly decoded at the relay. The relaying behaviors of e-MARC, SDF-MARC and NC-MARC are summarized in Table 4.

The theoretical outage probabilities of NC-MARC, analyzed in a SNCC framework, were included in Fig. 21. The outage probabilities\(^8\) in static and adaptive modes are obtained by [29, eqs. (4)-(5)] and [29, eqs. (4),(8)-(9),(11)], respectively. It is found that the e-MARC system outperforms NC-MARC in terms of outage performance.

In principle, as stated in [33], a header (i.e., flag) is used at the relay in SDF-MARC to identify the correctly decoded source nodes, and furthermore, the header has to be protected with a very powerful error correction code. On the contrary, the relay in e-MARC always performs network-coding and does not add a header to the forwarded XOR-ed sequence. Therefore, it is not reasonable to make comparisons between SDF-MARC and e-MARC, because SDF-MARC and e-MARC belong to different categories of MARC (i.e., SDF-MARC is “address-based”, while e-MARC is

\(^8\)The probabilities are computed at a system level [29].
To the best of our knowledge, there is no explicit mathematical expression to calculate the theoretical outage probability of SDF-MARC. Nevertheless, it is still meaningful to include SDF-MARC performance curves as a reference. The performance results with SDF-MARC, shown in Fig. 21, are all based on the Monte-Carlo method according to [33, eqs. (4)-(6)] with a modification of [101, eq. (12)], while all the curves for e-MARC are the theoretical results.

Deriving an explicit mathematical expression of the outage probability with SDF-MARC is an open future problem and out of the scope of this thesis; hence, we emphasize again that the conclusions for the superiority/inferiority of e-MARC related to SDF-MARC in the Asymmetric/Symmetric scenarios are based on simulations. Rigorous mathematical analysis for the results shown in Fig. 21 are left as future study.

### 3.2.3 Special cases

#### Static intra links

In the case that intra links are static (not suffering from fading), $AR$ and $BR$ links can be modeled as BSCs with constant crossover probabilities $p_A$ and $p_B$, respectively. This scenario may be exemplified by sensor networks where only the destination moves. In that case, averaging the $AR$ and $BR$ intra link variations over their pdfs can be eliminated from (33) and (34). Accordingly, the derivations of $Pr(\varepsilon_1)$ and $Pr(\varepsilon_2)$ can be reduced to

$$Pr(\varepsilon_1) = \frac{1}{\Gamma_{AD}} \int_0^\infty \int_0^\infty \frac{f_A^{-1}(1)}{f_A^{-1}(\delta)} \exp \left(-\frac{f_B^{-1}(\tilde{\omega})}{\Gamma_{BD}} - \frac{\gamma_{AD}}{\Gamma_{AD}}\right) d\gamma_{AD} p(\gamma_{RD}) d\gamma_{RD}$$

$$Pr(\varepsilon_2) = \int_0^\infty \exp \left(-\frac{f_A^{-1}(1)}{\Gamma_{AD}} - \frac{f_B^{-1}(\delta)}{\Gamma_{BD}}\right) p(\gamma_{RD}) d\gamma_{RD}, \quad (38)$$

---

9Text-based explanation for the superiority/inferiority of e-MARC related to SDF-MARC in the Asymmetric/Symmetric scenarios is provided in Appendix 5
The results of the theoretical outage probabilities calculated in this special case are shown in Fig. 22. It can be clearly observed in Fig. 22 that the outage probability achieves second-order diversity when \( p_A = p_B = 0 \), and matches the probability curve with perfect intra links in Fig. 20. When both intra links are imperfect but with relatively small crossover probabilities (e.g. when \( p_A = p_B = 0.01 \)), it is found that the decay of the outage probability follows the second order diversity in the low \( \Gamma_{ID} \) regime. However, the diversity order asymptotically converges to the first order as \( \Gamma_{ID} \) increases. It should be noticed that for the case that \( p_A \neq 0 \) and \( p_B \neq 0 \), the relay remains silent with SDF-MARC or NC-MARC; however, with e-MARC, the erroneous estimates forwarded from the relay still contribute to reducing the outage probability.

In the case that one of the intra links is perfect but the other one is imperfect, the helper’s contribution, provided by the relay, depends on the quality of the imperfect

\[
\tilde{\omega} = \tilde{\delta} + 1 - f_A(\gamma_{AD}).
\]
link. This is because in (19a)-(19d) provided in Section 3.1, the value of \( p_A \cdot p_B \) is a dominating factor that determines the size of the admissible rate region of \((R_A, R_B)\). Hence, \( p_A \) dominates the value of \( p_A \cdot p_B \) if \( p_B = 0 \), and vice versa.

**DDEX strategy with e-MARC**

The derivation for the outage probability of the e-MARC system shown in Section 3.1 can further be used to evaluate the efficiency of the DDEX strategy with the proposed e-MARC scheme [85] where the erroneous estimates are also utilized. As mentioned in Chapter 2, with using DDEX, the information sequences \( u_i \) for \( i \in \{A, B\} \) are first encoded using RSC codes, and then broadcasted from the source nodes using binary differential phase-shift keying (DPSK) modulation.\(^{10}\) To achieve low computational complexity and a small degree of latency, instead of performing fully-iterative decoding between the decoders of the ACC and RSC code, the relay \( R \) obtains the estimates \( \hat{u}_i \) of \( u_i \) by employing the DDEX detection strategy, where the relay simply extracts the systematic part output from the differential detector. The estimates \( \hat{u}_i \) for \( i \in \{A, B\} \) are detected in error with a high probability because no error correction is performed at \( R \), in exchange for saving battery and memory resources due to low computational complexity. The obtained estimates are XOR-coded at \( R \) and forwarded to the destination \( D \) for improving the FER performance.

As shown in [85], DPSK modulation and differential detection are used at the source node and relay, respectively, and hence according to [107], the error probability of the extracted estimates of the information sequence is expressed as

\[
p_i(\gamma_m) = \frac{1}{2} \exp(-\gamma_m), \quad i \in \{A, B\}.
\]

The theoretical outage probability (i.e., the theoretical achievable FER performance) for the DDEX with the proposed e-MARC scheme is obtained by replacing (28) with (40) in (36). Results of outage probabilities in the Symmetric and Asymmetric scenarios are demonstrated in Fig. 23 for comparison. The settings of both scenarios are same as those in Section 3.2.2.

In the Symmetric scenario, as shown in Fig. 23, the outage probability of the DDEX e-MARC scheme is roughly 3.5 dB away from the outage probability of e-MARC that is independent of the signaling scheme. The gap is due to the relatively high \( p_i \) value \(^{10}\)The ACC followed by coherent PSK is equivalent to DPSK.
with the DDEX strategy. In the Asymmetric scenario, the gap is further increased by 1 dB, resulting in a 4.5-dB gap from the outage probability of e-MARC. However, a large number of addition, multiplication and comparison operations can be eliminated on obtaining the estimates $\tilde{u}_i$, according to [108, Section 4.1].

The FER performance results of the DDEX e-MARC scheme obtained through simulations are also included in Fig. 23. Here FER is defined as follows: the number of the transmission cycles where either one or both of the information sequences sent from the nodes A and B cannot successfully recovered at D even with the help of R, divided by the total number of transmission cycles. The parameters used in the simulations are summarized in Tables 2 and 3. It is found from Fig. 23 that both in the Symmetric and Asymmetric scenarios, there is a roughly 1.8-dB loss with the practical FER performance from the corresponding theoretical outage probabilities. The loss is because the theoretical outage probabilities are derived assuming the use of capacity-achieving codes in the iD and RD links.
3.3 Impact of correlation

In some scenarios, for example in WSNs, an object could be observed by several source nodes, which causes the correlation among the observations of these nodes. Hence, in this subsection, we further extended our analysis to the e-MARC system where the correlation between two source nodes is taken into account. The correlation of source nodes is modeled by random bit-flipping with the flipping probability \( p_s = B(u_A, u_B) \).

The admissible rate region for the e-MARC system with the correlation \( p_s \) between source nodes is analyzed in (67) in Appendix 2. To investigate the impact of the source correlation \( p_s \) and the network correlation \( p_{nc} = p_A * p_B = B(u_R, u_{\oplus}) \), three extreme scenarios: \((\Gamma_{AD}, \Gamma_{BD}, p_R) = (30 \text{ dB}, -30 \text{ dB}, 0), (30 \text{ dB}, -30 \text{ dB}, 0.5) \) and \((-30 \text{ dB}, -30 \text{ dB}, 0.5) \) are investigated, respectively. The values of \( R_{ci} \) and \( R_{eR} \) are set to \( 1/2 \).

3.3.1 Source correlation \( p_s \)

We first focus on the impact of source correlation \( p_s \) on the e-MARC outage probability. For the scenarios \((\Gamma_{AD}, \Gamma_{BD}, p_R) = (30 \text{ dB}, -30 \text{ dB}, 0) \) and \((30 \text{ dB}, -30 \text{ dB}, 0.5) \), the soft channel values of the received signal vector \( y_{AD} \) are reliable while the values of \( y_{BD} \) are unreliable at the destination \( D \). Even in this case, the information sequence \( u_B \) can still be recovered with a high probability if two source nodes are highly correlated. This is because when \( u_A \) and \( u_B \) are highly correlated, reliable a priori information for the recovery of \( u_B \) can be directly acquired from extrinsic information related to \( u_A \) during the decoding process. Hence, it can be concluded that, with or without the relay, when at least one of the direct links is reliable and the two source nodes are highly correlated, a very low probability can be achieved. This situation is demonstrated in Figs. 24(a) and 24(b), where the probability is shown as a function of \( p_s \) and \( p_{nc} \).

3.3.2 Network correlation \( p_{nc} \)

Next, let us focus on the impact of the network correlation \( p_{nc} \). Assuming \( u_A \) and \( u_B \) are uncorrelated, but \( u_R \) and \( u_{\oplus} \) are fully correlated (i.e., \( u_R = u_{\oplus} \) and \( p_{nc} = 0 \)). For the scenario \((\Gamma_{AD}, \Gamma_{BD}, p_R) = (30 \text{ dB}, -30 \text{ dB}, 0) \), the soft channel values of \( y_{AD} \) and \( y_{RD} \) at the destination are reliable while the values of \( y_{BD} \) are unreliable. In this scenario, \( u_A \) and \( u_R \) can be recovered with a high probability after the decoding of \( y_{AD} \) and \( y_{RD} \), respectively. \( u_B \) can also be recovered with the aid of reliable a priori
Fig 24. Impacts of source correlation $p_s$ and network correlation $p_{nc}$ on outage probability of e-MARC. Here $R_{ci} = R_{cR} = 1/2$, [89] (© 2015 IEEE).
information using boxplus operation \[99\] on both extrinsic information corresponded to \(u_A\) and \(u_B\). However, for the scenario \((\Gamma_{AD}, \Gamma_{BD}, p_R) = (-30 \text{ dB}, -30 \text{ dB}, 0)\) with the same assumption, only the soft channel values of \(y_{RD}\) are reliable, and thus the recoveries of \(u_A\) and \(u_B\) require apriori information output from the boxplus operation. Nevertheless, the extrinsic information related to \(u_A\) and \(u_B\) is unreliable with high probabilities, and hence, the output from the boxplus operation is still unreliable for the decoding of \(y_{iD}\) for \(i \in \{A, B\}\). Therefore, as shown in Fig. 24(c), the value of the probability cannot be reduced to a very small value, although both intra links and the \(RD\) link are perfect (i.e., \(p_A = p_B = p_R = 0\)).

### 3.4 Conclusion

We have derived the outage probability for the e-MARC system. The theoretical outage probabilities of NC-MARC [29] and SDF-MARC [33] were also included for comparison in the Symmetric/Asymmetric scenarios listed in Table 2. It has been observed through simulations that the outage probability of NC-MARC is inferior to that of e-MARC. Moreover, it has been found through simulations that in the Asymmetric scenario where one of the source nodes is far away from both the relay and the destination, e-MARC performs better than SDF-MARC. However, in the Symmetric scenario, e-MARC is inferior to SDF-MARC.

We have further applied the derivation process of the outage probabilities to two special cases: 1) static intra links and 2) the DDEX with the proposed e-MARC scheme. According to the numerical results shown in case 1), we verified that erroneous estimates received at the relay are useful for the recovery of the information sequences at the destination. To evaluate the DDEX with the proposed e-MARC scheme, comparisons are provided between the e-MARC outage probability and the outage probability of the DDEX with e-MARC in case 2). It was found that a 3.5 to 4.5-dB loss occurs in exchange for a significant reduction in computational complexity. Finally, we recognized that for the case where two source nodes are highly correlated, the source correlation can be exploited to improve the e-MARC outage performance as long as one of the direct links is reliable. However, the \(RD\) link and at least one of the direct links have to be reliable for the exploitation of the network correlation \(p_{nc}\).
4 Joint adaptive network-channel coding

This chapter aims to further improve our previously proposed e-MARC scheme in Chapter 2. The system model described in Chapter 2 is first generalized with an arbitrary number of source nodes. We then propose a novel joint adaptive network-channel coding (JANCC) technique to avoid unnecessary erroneous estimates being network-coded at the relay. In the proposed JANCC technique, after completing the decoding of all the information sequences transmitted from the source nodes, the destination constructs a vector identifying the indices of the incorrectly decoded source nodes. The identifier vector is then sent to the relay via a feedback channel to request a re-transmission. Upon receiving the request, the relay performs network-coding by taking the XOR-operation only over its received estimates of the information sequences specified by the identifier vector. The decoding of JANCC is performed at the destination, where the knowledge of network correlation $p_{nc}$ is directly estimated by using the proposed algorithm and effectively utilized in the decoding process.

This chapter is organized as follows. Section 4.1 introduces the system model. A detailed description of the proposed JANCC and its corresponding decoding techniques are presented in Section 4.2. Section 4.3 presents simulation results to demonstrate the performance of JANCC-aided e-MARC, in terms of average FER and the computational complexity. The average goodput evaluation results are also provided in Section 4.3. Finally, Section 4.4 concludes this chapter.

4.1 System model

Fig. 25 shows a block diagram of the generalized e-MARC scheme assumed in this chapter, where there are $N$ source nodes, one common relay node $R$, and one common destination node $D$. Each node is equipped with a single antenna, and $R$ is assumed to be connected to $D$ via a half-duplex link. As shown in Fig. 25, each source node $S_i$, $i = 1, 2, ..., N$ generates its binary and CRC encoded information sequence $u_i = \{u_i(k)\}_{k=1}^{K}$. All the information sequences are assumed to be statistically independent.

Each sequence $u_i$ is interleaved and encoded by an SCCC where the SCCC is composed of a rate-$K/M$ RSC encoder $C_S$ and rate-1 ACC. The corresponding coded bit sequence $x_i = \{x_i(m)\}_{m=1}^{M}$ is modulated by BPSK and broadcasted to $R$ and $D$ at the each source node’s dedicated independent time slot.
Fig 25. Orthogonal Multiple Access Relay Channel model with $N$ source nodes combined with ARQ, [94] (© 2014 IEEE).

Without loss of generality, all transmitted signals have unit power. All the links are assumed to suffer from block Rayleigh fading, and thus the signal vectors received at $R$ and $D$ are

$$
y_{iR} = h_{iR} \cdot x_i + n_{iR}$$
$$y_{iD} = h_{iD} \cdot x_i + n_{iD}
$$

(41)

with $i = 1, 2, ..., N$, where $h_{iR}$ and $h_{iD}$ indicate the channel coefficients of $SR$ and $SD$ links with the source node $S_i$. $n_{iR}$ and $n_{iD}$ indicate the vectors of the independent zero-mean complex AWGN at $R$ and $D$ with the source node $S_i$, respectively, with variance $\sigma^2_{iR} = \sigma^2_{iD} = \sigma^2$ per dimension. Block Rayleigh fading is assumed with which $h_{iR}$ and $h_{iD}$, and assumed to be constant over one coded sequence, but to vary independently transmission-by-transmission and link-by-link.

Unlike the previously proposed e-MARC scheme, the re-transmission technique is utilized in the new system model shown in Fig. 25. Hence, the forwarding behavior of the relay $R$ depends on the decoding status of the destination $D$. If the destination $D$ identifies that all the users’ information sequences are successfully decoded by the CRC error detection, acknowledgement (ACK) is broadcasted back to the source nodes and relay, and the source nodes broadcast their following information sequences in their dedicated time slots. Otherwise $D$ sends to the relay $R$ a redundancy re-transmission request via the feedback channel,\footnote{The feedback channel is assumed to be error-free. This assumption is reasonable because the re-} and once receiving the re-transmission request, the
relay \( R \) transmits BPSK-modulated symbol sequence \( x_R = \{ x_R(m) \}_{m=1}^{M} \) to the destination \( D \) via \( RD \) link, as

\[
y_{RD} = h_{RD} \cdot x_R + n_{RD}
\]  

(42)

where \( h_{RD} \) and \( n_{RD} \) indicate the channel coefficient and AWGN vector of the \( RD \) link with variance \( \sigma_{RD}^2 = \sigma^2 \), respectively. In this chapter, it is assumed that \( E[|h_{iR}|^2] = E[|h_{iD}|^2] = E[|h_{RD}|^2] = 1 \), with \( i = 1, 2, \ldots, N \). The instantaneous SNR \( \gamma_{iR} \), \( \gamma_{iD} \) and \( \gamma_{RD} \) of the links are then given by

\[
\gamma_{iR} = |h_{iR}|^2 \cdot \Gamma_{iR}
\]

\[
\gamma_{iD} = |h_{iD}|^2 \cdot \Gamma_{iD}
\]

\[
\gamma_{RD} = |h_{RD}|^2 \cdot \Gamma_{RD}
\]  

(43)

where \( \Gamma_{iR} \) and \( \Gamma_{iD} \) represent the average SNRs of the \( SR \) and \( SD \) links connected to the source node \( S_i \), respectively. \( \Gamma_{RD} \) is the average SNR of the \( RD \) link.

Instead of performing the iterative decoding of the SCCC, the relay \( R \) utilizes very simple DDEX strategy to detect the received signal vector \( y_{iR} \) transmitted from the source node \( S_i \). The aim of utilizing DDEX has been addressed.

With the process for estimating the information sequences at the relay \( R \), it is highly probable that the sequences \( \tilde{u}_i \) obtained at \( R \) contain errors by using DDEX detection strategy because the decoding of \( C_S \) is not performed. The average FER performance of the \( SR \) link with using DDEX and with performing the iterative decoding of the SCCC were evaluated by [109], respectively. It can be observed in Fig. 26 that when \( R \) performs the iterative decoding of the SCCC, the FER performance of the \( SR \) link is very close to the outage probability, however, it is about 12 dB away from the outage when using the DDEX strategy although the decoding complexity of the SCCC is eliminated.

### 4.2 Proposed techniques

The drawback of the DDEX detection strategy is that the estimates of the information sequences \( \tilde{u}_i \) obtained at \( R \) contain errors with a high probability. Therefore, a JANCC transmission request can be represented by \( N \) bits, as detailed in Section 4.2, which is protected by using a powerful channel coding.
and its corresponding decoding techniques are proposed here, where the erroneous estimates obtained at $R$ after the DDEX strategy are further efficiently exploited to help recover the information sequences decoded in error at $D$.

### 4.2.1 JANCC

The received signal vector $y_{iD} = \{y_{iD}(m)\}_{m=1}^{M}$ of the $i$th slot at $D$ is demodulated to the corresponding soft channel values $L(y_{iD}|x_i) = \{L(y_{iD}(m)|x_i(m))\}_{m=1}^{M}$ by (9). The values $L(y_{iD}|x_i)$ for the $i$th slot, are iteratively decoded by the decoder of SCCC with the log-MAP [108] algorithm at the destination $D$. The estimated sequence $\hat{u}_i$ of $u_i$ is obtained by making a binary hard decision on the a posteriori LLR $\{L(u_i(k)|y_{iD})\}_{k=1}^{K}$ output from the decoder of $C_S$, when no relevant increase in LLR is obtained by the iterations.

The sequences $\hat{u}_i$ are CRC decoded for error detection after completing the iterative decoding of the SCCC, and the decoding results for each user are stored in an $N$-bit
identifier vector $F = \{f(i)\}_{i=1}^{N}$, where

$$f(i) = \begin{cases} 0, & \hat{u}_i = u_i \\ 1, & \hat{u}_i \neq u_i \end{cases}$$

and a set $\mathcal{F}$ is defined as

$$\mathcal{F} = \{i : f(i) = 1\}. \quad (45)$$

If $\mathcal{F} \neq \emptyset$, the destination $D$ sends to the relay $R$ a redundancy-retransmission request with the vector $F$ via the feedback channel and attempts to recover the information sequences decoded in error with additional redundancy forwarded from $R$.

As shown in Fig. 25, upon receiving the re-transmission request, the relay $R$ performs network-coding by taking a sequence-by-sequence XOR-operation, notated as $\oplus$, only over the received estimates of the information sequences specified by $F$, not over all the estimates as e-MARC, although the specified estimates may contain errors, as

$$u_R = \bigoplus_{i_j \in \mathcal{F}} \tilde{u}_{i_j}. \quad (46)$$

The relay node $R$ then re-encodes the interleaved version of the network-coded bit sequence $u_R = \{u_R(k)\}_{k=1}^{K}$ by using a rate-$K/M$ RSC encoder $C_R$, and finally, the corresponding symbol sequence $x_R = \{x_R(m)\}_{m=1}^{M}$ is forwarded to $D$ using BPSK modulation as additional redundancy.

### 4.2.2 Decoding scheme and algorithm of JANCC

Fig. 27 shows a factor graph representation of the proposed JANCC decoding scheme, where $C_S^{-1}$ and $C_R^{-1}$, respectively, indicate the RSC decoders corresponding to $C_S$ and $C_R$ used in the source nodes and relay $R$. The notations $L_a$ and $L_e$ shown in Fig. 27 denote a priori and extrinsic LLR of their argument variables, respectively. Once $D$ receives additional redundancy forwarded from $R$, the decoding of JANCC is initiated to recover the information sequences decoded in error at $D$ via the $SD$ links.

As shown in Fig. 27, the LLR-updating function $f_c(\cdot)$ is utilized in the proposed JANCC decoding scheme with the knowledge of the network correlation $p_{nc}$, where


Fig 27. Proposed JANCC decoding scheme represented by a factor graph, where $ACC^{-1}$ and $SCCC^{-1}$ denote the ACC decoder and the decoder of the SCCC applied by $S_i$, respectively, [94] (© 2014 IEEE).

$p_{nc}$ is defined as

$$p_{nc} = \frac{\sum_{k=1}^{K} |u_R(k) - u_{\oplus}(k)|}{K}$$  \hspace{1cm} (47)

where the sequence $u_{\oplus}$ is generated by the source nodes specified by $F$ as

$$u_{\oplus} = \bigoplus_{j \in F} u_j.$$  \hspace{1cm} (48)

Knowledge of correlation $p_{nc}$ is exploited during the JANCC decoding process in the LLR-updating function $f_c(\cdot)$ to avoid error propagation if one/some of the sequences $\tilde{u}_j$ obtained at $R$ is/are erroneous. It should be emphasized that correlation $p_{nc}$ can be directly estimated at $D$ without having to involve any higher layer protocols, as [96, Eq. (9)]
\[ \hat{p}_{nc} = G(L_e(\Pi_R[u_R]), L_e(\Pi_R[u_R]), \mathcal{G}) \]
\[ = G(L_e(u_R), L_e(u_R), \mathcal{G}) \]
\[ = \frac{1}{|\mathcal{G}|} \sum_{g \in \mathcal{G}} \Pr(u_R(g) = 1) \Pr(u_R(g) = 0) \Pr(u_R(g) = 1) \]
\[ = \frac{1}{|\mathcal{G}|} \sum_{g \in \mathcal{G}} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} + \frac{1}{(1 + e^{L_e(u_R(g))})} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \]
\[ = \frac{1}{|\mathcal{G}|} \sum_{g \in \mathcal{G}} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \]
\[ = \frac{1}{|\mathcal{G}|} \sum_{g \in \mathcal{G}} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \frac{e^{L_e(u_R(g))}}{(1 + e^{L_e(u_R(g))})} \] (49)

where \( \hat{p}_{nc} \) is the estimated value of the probability \( p_{nc} \), and the set \( \mathcal{G} \) is defined as
\[ \mathcal{G} = \{ g : |L_e(u_R(g))| \geq T \} \] (50)

with \( T \) representing a threshold value. However, since the \( SD \) links suffer from block fading, we modified [96, eq. (9)]. Algorithm 1 summarizes the JANCC decoding process.

The out-most “for-loop” in Algorithm 1 is mainly to exchange the extrinsic information between the decoders \( C_R^{-1} \) and \( SCCC_j^{-1} \) specified by the identifier \( \mathcal{F} \). To achieve a precise estimation of \( p_{nc} \), we refer to [96] to introduce a threshold value \( T \) to eliminate some unreliable positions of the sequence \( L_e(u_R) \). However, unlike the assumption of the AWGN channels in [96], all the links here are assumed to suffer from block fading. Therefore, in the second out-most “for-loop”, the value of \( T \) decreases (i.e., \( T - \epsilon \), where \( \epsilon = 0.05 \) in this chapter) during every iterative operation of the loop until a sufficiently large number of \( L_e(u_R(g)) \) in the sequence \( L_e(u_R) \) is guaranteed (three-quarters of the entire sequence-length at least in this chapter). The sequence \( L_e(u_R) \) will not be punctured if the value of \( T \) has already decreased at most \( n \) times (\( n = 10 \) in this chapter).

When the second out-most “for-loop” is completed, the estimation of correlation \( p_{nc} \) is performed using (49) where the extrinsic information produced by the decoder \( C_R^{-1} \) is utilized. The intermediate a priori LLR sequence \( L_a(u_R) \) is then produced using the LLR-updating function \( f_c(\cdot) \). In the following “for-loop”, the a priori information to be provided to each specified decoder \( SCCC_j^{-1} \) is calculated by performing a boxplus operation with the updated intermediate a priori information, and the iter-
Algorithm 1: Decoding process of JANCC

Pre-defined: iterations \( I \), interval \( \epsilon \), ratio \( \rho \), threshold \( T^{(0)} \), times \( n \)

Input: \( L_c(u_j), \forall j \in F \)

Initialization: \( \hat{p}_{ncc} = 0.25 \)

for 1 to \( I \) do

\[ L_c(u) = \sum_{j \in F} L_c(u_j); \]

for \( l = 1 \) to \( n \) do

Find \( \forall q \) such that \( |L_c(u \oplus (q))| > T^{(l-1)} \);

Store \( \forall q \) to set \( Q \);

if \( |Q| \geq \rho \cdot \#L_c(u) \) then

- Exit for;

end

\[ T^{(l)} = T^{(l-1)} - \epsilon; \]

if \( T^{(l)} \leq 0 \) then

- Store all the indices of \( L_c(u) \) to set \( Q \);  
- Exit for;

end

end

\[ L_a(\Pi R[u]) = f_c(L_c(\Pi R[u]), \hat{p}_{ncc}); \]

Calculate \( L_c(\Pi R[u]) \) by \( C^{-1}_R \);

\[ \hat{p}_{ncc} = G(L_c(\Pi R[u]), L_c(\Pi R[u]), Q); \]

\[ L_a(u) = \Pi R^{-1}[f_c(L_c(\Pi R[u]), \hat{p}_{ncc})]; \]

for all \( j \in F \) do

- Calculate \( L_a(u_j) \) by boxplus operation;

- Feed \( L_a(u_j) \) to \( SCCC_j^{-1} \);

- Calculate \( L_c(u_j) \) by \( SCCC_j^{-1} \) with \( I \) iterations;

end

end

Calculate \( \bar{u}_j, \forall j \in F \);
ative decoding process is performed in each specified $SCCC^{-1}_j$ utilizing the updated 
$a priori$ information.
4.3 Numerical results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of information sequence $K$</td>
<td>1024 bits</td>
</tr>
<tr>
<td>$C_S$</td>
<td>$(1, 5/7)_8$</td>
</tr>
<tr>
<td>$C_R$</td>
<td>$(1, 5/7)_8$</td>
</tr>
<tr>
<td>Code rate ($K/M$)</td>
<td>1/2</td>
</tr>
<tr>
<td>Decoding algorithm</td>
<td>log-MAP</td>
</tr>
<tr>
<td>Interval $\epsilon$</td>
<td>0.05</td>
</tr>
<tr>
<td>Ratio $\rho$</td>
<td>3/4</td>
</tr>
<tr>
<td>Threshold $T^{(0)}$</td>
<td>3</td>
</tr>
<tr>
<td>Global iteration (GI) $I$</td>
<td>1</td>
</tr>
<tr>
<td>Local iteration (LI)</td>
<td>10</td>
</tr>
<tr>
<td>Loops between GI and LI</td>
<td>10</td>
</tr>
<tr>
<td>Interleavers</td>
<td>Random</td>
</tr>
<tr>
<td>$\Gamma_{RD}$</td>
<td>20 dB</td>
</tr>
</tbody>
</table>

Here, we provide the results of the simulations conducted to evaluate the JANCC-aided e-MARC scheme. The number of the re-transmission is a parameter and is set at 1 in the simulations, and the other parameters assumed in the simulations are summarized in TABLE 5. The text-based procedure of the decoding process can be referred in Section 2.4.

We use the e-MARC scheme with the DDEX strategy as an example. Furthermore, two other alternative orthogonal MARC systems are also provided in the simulations for comparison. One is the SDF-MARC scheme with the IR strategy, and the other is the SDF-MARC scheme with DDEX. The outage probability of the SDF-MARC system is numerically calculated using Monte Carlo methods, where we generated a sufficient number of random fading coefficients ($h_{iR}$, $h_{iD}$ and $h_{RD}$) and measured the probability that the conditions provided in [33, Eqs. (4)-(6)] are not satisfied.

The structure for the three MARC systems are assumed to be symmetric in geometric gain where all the $SR$ links have the same average SNR ($\Gamma_{iR} = \text{constant } \forall i$) and all the $SD$ links as well ($\Gamma_{iR} = \text{constant } \forall i$). Finally, we define FER as the number of the information sequences that cannot be correctly decoded at $D$, even with the help of $R$, divided by the total number of the sequences transmitted by all the source nodes.

Fig. 28 demonstrates the FER performance with three source nodes, where $\Gamma_{iR} =$
Fig 28. FER performance when $\Gamma_{iR} = \Gamma_{iD} + 0.5 \text{ dB}$, $\Gamma_{RD} = 20 \text{ dB}$, and the number of source nodes $N = 3$, [94] (© 2014 IEEE).

Fig 29. Estimation of $p_{nc}$ using Algorithm 1 and using [96, eq. (9)] with fixed threshold $T$ values, respectively.
First, it can be observed that, with the DDEX strategy, the e-MARC scheme is worse than the SDF-MARC scheme as the number of the source nodes is increased. This is because XOR coding is always performed at the relay in e-MARC, by which the XOR-coded sequence may be corrupted by unnecessary erroneous estimates and thus the performance is degraded.

Next, we investigate the effect of the detection strategy applied at $R$ with the SDF-MARC scheme. For the DDEX with the SDF-MARC scheme, as can be observed in Fig. 28, there is roughly a 3-dB loss in average $\Gamma_{ID}$ at $\text{FER} = 10^{-3}$ compared with the IR with the SDF-MARC scheme. The loss is due to the fact that DDEX only extracts the estimates of the information sequences output from the different detector. However, with the aid of the JANCC technique, the loss can be reduced to only 1.5 dB by efficiently exploiting the erroneous estimates of the transmitted source information received at $R$.

Furthermore, it can also be observed in Fig. 28 that by employing the proposed JANCC decoding algorithm (see Algorithm 1), the FER performance of JANCC with the estimated $\hat{p}_{nc}$ is very close to the FER curve with correlation $p_{nc}$ being known at $D$. Fig. 29 shows the results of the FER performance estimated by Algorithm 1 and by [96, eq. (9)] with the fixed values of the threshold $T$, respectively, while Fig. 30
evaluates the initial $\hat{p}_{nc}$ settings in Algorithm 1. It can be observed from Fig. 29 that neither a large or small fixed value of $T$ can accurately estimate the correlation $p_{nc}$, while from Fig. 30 similar results are obtained for other values of $\hat{p}_{nc}$ initially set in Algorithm 1. In fact, these observations, together with the application of low-complexity DDEX at $R$, makes the DDEX with the JANCC-aided e-MARC scheme practical in exchange for a 1.5-dB loss in average $\Gamma_{iD}$, and a significant reduction in power consumption can be achieved due to the low computational complexity required.
4.3.1 Impact of SR link quality

To consider the impact of the quality of the SR links on the FER performances of the three MARC systems, another scenario was tested, where $\Gamma_{IR} = \Gamma_{iD} + 6.5$ dB. It can be clearly observed in Fig. 31 that the gap in FER between the DDEX and IR strategies with SDF-MARC is reduced, because the better quality of the SR links achieves the higher accuracy of the sequences $\tilde{u}_i$ obtained at $R$, even by using the DDEX strategy. With the aid of JANCC, the DDEX with the e-MARC scheme can achieve 1-dB gain over the DDEX with the SDF-MARC scheme at FER = $10^{-3}$, and only 0.5 dB away from the IR with SDF-MARC scheme. However, the DDEX with the e-MARC scheme is roughly 3.5 dB away from the outage probability of SDF-MARC [33] even with the aid of JANCC. This observation indicatively means that the optimal code design for the SR links is left as a future study, once the quality of the SR links is good.
4.3.2 Computational complexity evaluation

We also make a comparison between the two systems in terms of computational complexity: one is the IR with the SDF-MARC scheme, and the other is the DDEX with a JANCC-aided e-MARC scheme. The main difference between the two schemes is due mainly to the detection strategy applied at $R$, so we calculate the number of addition (ADD), multiplication (MUL) and comparison (COMP) operations needed to obtain one estimated information sequence $\hat{u}_i$, where all the operations are on real numbers.

It is assumed $\hat{u}_i$ is obtained by performing the IR strategy on the received vector $y_{iR}$, where the log-MAP algorithm is used on the inner and outer codes with memory lengths $m_1$ and $m_2$, respectively. The number of operations needed per bit in the log-MAP decoding algorithm is provided in [108, Section 4.1]. Since the lengths of sequences decoded by the inner and outer decoders are $M$ and $K$ bits, respectively, the required number of operations per sequence of each constituent code is approximately $M$ and $K$ times higher, respectively.

However, with using the DDEX strategy, only one complex multiplication per bit, equivalent to two ADDs and four MULs, is required by the differential detection. Thus, $2M$ ADDs and $4M$ MULs in total are required in the detection of $M$-bit $y_{iR}$. The sequence $\hat{u}_i$ is simply extracted from the output of the differential detector, and thus, no operations are required.

The total computational costs for performing the IR and DDEX strategies are summarized in Table 6. With the parameters $m_1 = 1$, $m_2 = 2$, $K/M = 1/2$ and $I = 10$ iterations set in this chapter, the complexity of the IR strategy is at least 200 times higher than DDEX, which leads to significant power and memory saving for computation at $R$.

<table>
<thead>
<tr>
<th>Operation</th>
<th>IR</th>
<th>DDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMP</td>
<td>$(5 \cdot 2^{m_1} - 2) \cdot M + (5 \cdot 2^{m_2} - 2) \cdot K \cdot I$</td>
<td>0</td>
</tr>
<tr>
<td>ADD</td>
<td>$(15 \cdot 2^{m_1} + 9) \cdot M + (15 \cdot 2^{m_2} + 11) \cdot K \cdot I$</td>
<td>$2 \cdot M$</td>
</tr>
<tr>
<td>MUL</td>
<td>$(8 \cdot M + 8 \cdot K) \cdot I$</td>
<td>$4 \cdot M$</td>
</tr>
<tr>
<td>Look-up table</td>
<td>$(5 \cdot 2^{m_1} - 2) \cdot M + (5 \cdot 2^{m_2} - 2) \cdot K \cdot I$</td>
<td>0</td>
</tr>
</tbody>
</table>
### 4.3.3 Average goodput

To identify the probability for recovering the information sequence correctly at $D$ with the SDF-MARC scheme and with the JANCC-aided e-MARC scheme, we also evaluate the average goodput of the three MARC systems used in the simulations. Tables 7 and 8 summarize all the combinations for the cases that require a re-transmission request for the SDF-MARC and JANCC-aided e-MARC schemes, respectively. The achievable goodput is defined as the number of correctly decoded information sequences at $D$ totalling over all the source nodes, and the average goodput is calculated by averaging the achievable goodput under all the combinations according to the corresponding occurrence probabilities of those cases.


<table>
<thead>
<tr>
<th>$D$ sends request</th>
<th>X</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ forwarding</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Correctly decoded msgs at $D$</td>
<td>$N$</td>
<td>$N - 1$</td>
<td>$\cdots$</td>
<td>$1$</td>
<td>$0$</td>
<td>$N$</td>
<td>$N - 1$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>Achievable goodput</td>
<td>$1$</td>
<td>$\frac{N - 1}{N}$</td>
<td>$\frac{1}{N}$</td>
<td>$0$</td>
<td>$\frac{N}{N + 1}$</td>
<td>$\frac{N - 1}{N + 1}$</td>
<td>$\cdots$</td>
<td>$\frac{1}{N + 1}$</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>$D$ sends request</th>
<th>X</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
<th>✓</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ forwarding</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Correctly decoded msgs at $D$</td>
<td>$N$</td>
<td>$N$</td>
<td>$N - 1$</td>
<td>$\cdots$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>Achievable goodput</td>
<td>$1$</td>
<td>$\frac{N}{N + 1}$</td>
<td>$\frac{N - 1}{N + 1}$</td>
<td>$\cdots$</td>
<td>$\frac{1}{N + 1}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
Fig. 32. Average goodput when $\Gamma_{iR} = \Gamma_{iD} + 0.5$ dB, $\Gamma_{RD} = 20$ dB, and the number of source nodes $N = 3$, [94] (© 2014 IEEE).

Fig. 32 shows the average goodput efficiencies of all the systems for $\Gamma_{iR} = \Gamma_{iD} + 0.5$ dB. The average goodput of the JANCC-aided e-MARC scheme is even lower than the point-to-point transmission in the low SD SNR regime, which indicates even with the aid of the JANCC technique, the e-MARC system for the recovery of the information sequence at $D$ is limited when the $SR$ link quality is poor. The probability of $\tilde{u}_j$, $\forall j \in F$ after the detection of DDEX containing a large amount of errors is relatively high when the quality of the $SR$ links is poor, and thus, the correlation between $u_{R}$ and $u_{R}$ degrades if any one of the sequences $\tilde{u}_j$ contains a large amount of errors, which decreases the correction capability of JANCC decoding.
As the quality of the $SR$ links improves, as shown in Fig. 32 and Fig. 33, the performance of the JANCC-aided e-MARC scheme is apparently improved and outperforms the performance of scheme SDF-MARC scheme in terms of the average goodput efficiency as DDEX is applied. Furthermore, the average goodput of the DDEX with the JANCC-aided e-MARC scheme asymptotically approaches to that of the IR with the SDF-MARC scheme, as shown in Fig. 33.

4.4 Conclusion

This chapter has investigated the developed energy-efficient orthogonal MARC system. A low-complexity and non-coherent detection DDEX strategy was applied at the relay $R$ to reduce the energy consumption. The proposed JANCC and its corresponding decoding techniques were employed to exploit the erroneous estimates output from DDEX. A vector is constructed in the JANCC technique to identify the indices of the source nodes whose information sequences are decoded in error at the destination $D$.

The relay $R$ performs network coding only on the estimates specified by the vector upon receiving the re-transmission request, which aims to avoid unnecessary erroneous
estimates being network coded. The proposed JANCC decoding algorithm estimated and exploited the correlation $p_{nc}$ to update the extrinsic LLR sequences during the iterative decoding process. In the case of the system setup assumed in this chapter, computational complexity of the DDEX is at least 200 times lower than the iterative decoding of the SCCC performed at $R$, in exchange for only a $0.5 - 1.5$ dB loss in the FER performance. However, a significant reduction in power consumption can be achieved due to low computational complexity required and the elimination of channel estimation. Furthermore, large latency incurred by the iterative decoding performed at the relay can be eliminated. Hence, the DDEX with the JANCC-aided e-MARC scheme is suitable for the relaying systems where the availability of the resources is limited, or to be used in real-time systems.
5 Conclusions and future studies

A thorough analysis of the DF-based MARC system based on the idea of lossy-forwarding has been provided in this thesis, where we aim to improve the performance of the MARC system with a relay subject to resources or/and time constraints by efficiently exploiting the erroneous estimates obtained at the relay.

A simple DF-based lossy-forwarding MARC system model, named e-MARC, has been proposed in Chapter 2 with its corresponding JNCC scheme. Network correlation in the e-MARC scheme has been utilized to exploit the forwarded erroneous estimates sent from the relay at the destination. Numerical results have shown that the erroneous estimates were helpful for the reconstruction of the information sequences sent via SD links, especially when the estimates contain only a few errors. As a result, by using very simple detection at the relay, it has been found that the proposed e-MARC system obtained roughly $0.2 - 0.4$ dB and $0.7 - 2.2$ dB gain from the SDF-MARC in AWGN and fading scenarios, respectively.

Chapter 3 theoretically derived the outage probability for the e-MARC system with two source nodes, and the derived outage probability was independent of signaling schemes. Comparisons with NC-MARC and SDF-MARC were also made. It was found through simulations that when one of the source nodes is far away from both the relay and the destination, the e-MARC system was superior to SDF-MARC in terms of outage performance. The impact of source correlation and network correlation on the outage probability of e-MARC was investigated. It was observed that for the case two source nodes are highly correlated, the outage performance can still be improved by exploiting the source correlation as long as one of the SD links is reliable. However, to improve the outage performance by exploiting the network correlation, the RD link and at least one of the SD links have to be reliable.

Chapter 4 proposed a JANCC technique to aid the e-MARC system with more source nodes. With using an identifier vector re-transmitted from the destination to the relay via a feedback channel, instead of all the estimates, only the specified estimates were network-coded and forwarded to the destination. This avoided the network correlation being corrupted by unspecified erroneous estimates in the XOR-coding process, and thus the network correlation can be exploited more effectively at the destination. Computer simulations demonstrated that JANCC-aided e-MARC is su-
perior to e-MARC in terms of FER and goodput efficiency. In the case of the system setup assumed in this thesis, the computational complexity of the DDEX strategy is at least 200 times lower than the iterative decoding of the SCCC (i.e., the IR strategy) performed at $R$, in exchange for only a $0.5 - 1.5$ dB loss in the FER performance. However, a significant reduction in power consumption can be achieved due to low computational complexity required and the elimination of channel estimation. Furthermore, large latency incurred by the iterative decoding process can be eliminated. Hence, the DDEX with JANCC-aided e-MARC scheme is quite useful for cooperative communications in resource-constrained wireless networks.

Several interesting topics based on this thesis can be further explored. We list several possible topics in the following as future studies:

- More sophisticated coding and modulation schemes could possibly be used for the practical realization of the e-MARC system. For example, Denoise-and-Forward strategy [110] and Joint-over-Antenna [111] detection could be applied at the relay and destination, respectively, for multi-access transmission with high order modulation.

- A derivation of the theoretical outage probability of SDF-MARC.

- The theoretical outage probability of the JANCC-aided e-MARC system could also possibly be investigated, where the re-transmission is assumed to suffer from shadowing. Shadowing variation could be modeled by a time-correlated log-Normal random process.

- The average goodput of the JANCC-aided e-MARC system is inefficient when the $SR$ link quality is poor, but it is highly possible that this could be improved by a well designed protocol for the coordination between $R$ and $D$. For example, before the re-transmission, the error probabilities of the $SR$ links, $\mathcal{P} = \{p_i\}_{i=1}^N$, could be estimated at $R$ and forwarded to $D$. The set $\mathcal{F}$ could be updated by removing the indices of the sources whose estimates may contain a large number of errors at $R$.

- The average length of the identifier vector can be reduced if the vector could be designed in the form of variable length. The utilization of the identifier vector with a variable length could be more meaningful for a large number of sources contained in
the network, such as the MARC scenario considered in ANCC or GANCC [49, 50].

– The (network) correlation could be more efficiently utilized during the iterative decoding of LDPC with BP algorithm, since the process of updating the LLR sequence is inherent in the message passing process [112, 113].
18. Vaze R & Heath R (2011) On the capacity and diversity-multiplexing tradeoff of the two-


52. Hausl C & Dupraz P (2006) Joint network-channel coding for the multiple-access relay
Appendix 1 Derivation of (9)

Let us consider a very simple model of a block fading channel

\[ y = hx + n \]  

(51)

where the BPSK symbol \( x \in \{+1, -1\} \), complex channel coefficient \( h = h_R + jh_i \), and noise \( n = n_R + jn_i \) is a complex AWGN with variance \( \sigma^2 \) per dimension (i.e., \( \text{var}(n_R) = \text{var}(n_i) = \sigma^2 \)). Then, the destination performs coherent detection on the received signal \( y \) obtain the signal \( z \), as

\[
z = \Re \left\{ \frac{h^* y}{|h|^2} \right\} = x + \Re \left\{ \frac{h^* n}{|h|^2} \right\}
\]  

(52)

The variance of the signal after coherent detection becomes

\[
\text{var}(z) = \text{var} \left\{ \Re \left\{ \frac{h^* n}{|h|^2} \right\} \right\} \\
= \text{var} \left( \Re \left( \frac{(h_R - jh_i)(n_R + jn_i)}{|h|^2} \right) \right) \\
= \text{var} \left( \frac{h_R n_R + h_i n_i}{|h|^2} \right) \\
= \text{var} \left( \frac{h_R n_R}{|h|^2} \right) + \text{var} \left( \frac{h_i n_i}{|h|^2} \right) \\
= \frac{h_R^2}{(h_R^2 + h_i^2)^2} \text{var}(n_R) + \frac{h_i^2}{(h_R^2 + h_i^2)^2} \text{var}(n_i) \\
= \frac{h_R^2}{(h_R^2 + h_i^2)^2} \text{var}(n_R) + \frac{h_i^2}{(h_R^2 + h_i^2)^2} \text{var}(n_i) \\
= \frac{1}{|h|^2} \sigma^2 \\
= \sigma_z^2
\]  

(53)

After that, we convert the detected signal \( z \sim \mathcal{N}(0, \sigma_z^2) \) to its corresponding soft
channel values $L(z|x)$, as

$$L(z|x) = \ln \frac{\Pr(z|x = +1)}{\Pr(z|x = -1)}$$

$$= \ln \frac{1}{\sqrt{2\pi}z} \cdot \exp \left(\frac{-(z-1)^2}{2\sigma_z^2}\right)$$

$$= \ln \frac{1}{\sqrt{2\pi}z} \cdot \exp \left(\frac{-(z+1)^2}{2\sigma_z^2}\right)$$

$$= \frac{-(z - 1)^2 + (z + 1)^2}{2\sigma_z^2}$$

$$= \frac{4z}{2\sigma_z^2}$$

$$= \frac{4z}{2\frac{1}{|h|^2}\sigma^2}$$

$$= \frac{2\Re\{h^*y\}}{\sigma^2}$$

(54)
Appendix 2 Derivation of (19a)-(19d)

We start the derivation of the results of (19a)-(19d) with the general equivalent system model of e-MARC shown in Fig. 34, where source nodes A and B are assumed to be correlated. Variables $e_A$, $e_B$, $e_R$ and $e_s$ are i.i.d. binary random variables with $Pr(e_A = 1) = p_A$, $Pr(e_B = 1) = p_B$, $Pr(e_R = 1) = p_R$ and $Pr(e_s = 1) = p_s$, respectively. According to the definition, the mutual information $I(u_R; \hat{u}_R)$ is

\[ I(u_R; \hat{u}_R) = H(\hat{u}_R) - H(\hat{u}_R|u_R). \]

(55)

Then, given the model shown in Fig. 34, $I(u_R; \hat{u}_R)$ can be further expressed as follows

\[
I(u_R; \hat{u}_R) = H(u_A \oplus e_A \oplus u_B \oplus e_B \oplus e_R) - H_b(p_R)
= H(u_A \oplus e_A \oplus u_B \oplus e_B \oplus e_R) - H_b(p_R)
= H(e_A \oplus e_B \oplus e_R \oplus e_s) - H_b(p_R)
= H_b(p_A \ast p_B \ast p_R \ast p_s) - H_b(p_R),
\]

(56)

where the operation $\alpha \ast \beta$ is defined as $\alpha(1 - \beta) + \beta(1 - \alpha)$, and $H_b(\cdot)$ denotes the binary entropy function \[102\]. For the sake of notation simplicity, we defined $\mu$ as $\mu = H(\hat{u}_R) = H_b(p_A \ast p_B \ast p_R \ast p_s)$. Then, by using the chain rule, the joint entropy

![Fig 34. Generalized equivalent model of e-MARC, where source correlation is taken into account, [89] (© 2015 IEEE).](image-url)
$H(u_A, u_B, u_R, \hat{u}_R)$ can be represented as

$$H(u_A, u_B, u_R, \hat{u}_R) = H(u_A, u_B) + H(u_R|u_A, u_B) + H(\hat{u}_R|u_A, u_B, u_R) \quad (57a)$$
$$= H(\hat{u}_R) + H(u_A, u_B|\hat{u}_R) + H(u_R|u_A, u_B, \hat{u}_R). \quad (57b)$$

Hence, the conditional entropy $H(u_A, u_B|\hat{u}_R)$ is expressed with the combination of (57a) and (57b), as

$$H(u_A, u_B|\hat{u}_R) = H(u_A, u_B) + H(\hat{u}_R|u_A, u_B) - H(\hat{u}_R) - H(u_R|u_A, u_B, \hat{u}_R). \quad (58)$$

Also, the conditional mutual information $I(u_R; \hat{u}_R|u_A, u_B)$ can be expressed by the chain rule, as

$$I(u_R; \hat{u}_R|u_A, u_B) = H(u_R|u_A, u_B) - H(u_R|u_A, u_B, \hat{u}_R)$$
$$= H(\hat{u}_R|u_A, u_B) - H(\hat{u}_R|u_A, u_B, u_R). \quad (59)$$

By substituting $H(u_R|u_A, u_B)$ of (59) into (58), $H(u_A, u_B|\hat{u}_R)$ can be rewritten as

$$H(u_A, u_B|\hat{u}_R) = H(u_A, u_B) + H(\hat{u}_R|u_A, u_B) - H(\hat{u}_R) \quad (60)$$

where the conditional entropy $H(\hat{u}_R|u_A, u_B)$ is calculated as

$$H(\hat{u}_R|u_A, u_B) = H(u_A \oplus e_A \oplus u_R \oplus e_B \oplus e_R|u_A, u_B)$$
$$= H(e_A \oplus e_B \oplus e_R). \quad (61)$$

Therefore, $H(u_A, u_B|\hat{u}_R)$ shown in the third inequality of (18) can be obtained by substituting (61) into (60), as
\[
H(u_A, u_B | \hat{u}_R) = H(u_A, u_B) + H(e_A \oplus e_B \oplus e_R) - H(\hat{u}_R) \\
= 1 + H_b(p_s) + H_b(p_A * p_B * p_R) - \mu. \tag{62}
\]

Again, let \( \delta_s = H_b(p_s) + H_b(p_A * p_B * p_R) - \mu \) for simplicity. Since the conditional entropy \( H(u_A, u_B | \hat{u}_R) \) in (62) can also alternatively be expressed by the chain rule, as

\[
H(u_A, u_B | \hat{u}_R) = H(u_B | u_A, \hat{u}_R) + H(u_A | \hat{u}_R) \tag{63a}
\]
\[
= H(u_A | u_B, \hat{u}_R) + H(u_B | \hat{u}_R). \tag{63b}
\]

Since the sequence \( \hat{u}_R \) is composed of \( u_A \), the conditional entropy \( H(u_A | \hat{u}_R) \) in (63a) can be calculated as

\[
H(u_A | \hat{u}_R) = H(u_A | e_A \oplus u_B \oplus e_B \oplus e_R) \\
= H(e_A \oplus u_A \oplus e_s \oplus e_B \oplus e_R) \\
= 1. \tag{64}
\]

Therefore, we can obtain \( H(u_B | u_A, \hat{u}_R) \), shown in the second inequality of (18), by substituting (62) and (64) into (63a), as

\[
H(u_B | u_A, \hat{u}_R) = H(u_B, u_A | \hat{u}_R) - H(u_A | \hat{u}_R) \\
= \frac{H(u_A, u_B | \hat{u}_R) - H(u_A | \hat{u}_R)}{1 + \delta_s} \\
= \frac{1 + H_b(p_A * p_B * p_R) - \mu}{1 + \delta_s}. \tag{65}
\]

The conditional entropy \( H(u_A | u_B, \hat{u}_R) \) shown in the first inequality of (18) is also calculated in the same way, and the result is

\[
H(u_A | u_B, \hat{u}_R) = \delta_s. \tag{66}
\]

A set of inequalities is obtained by combining (56), (62), (65) and (66), as
\[ R_A \geq \delta_s, \]
\[ R_B \geq \delta_s, \]
\[ R_A + R_B \geq 1 + \delta_s, \]
\[ R_R \geq \mu - H_b(p_R). \]  

(67)

For the case that the two source nodes are uncorrelated (i.e., \( p_s = 0.5 \)), \( \mu = 1 \) and \( \delta = H_b(p_A \ast p_B \ast p_R) \), which leads (67) into (19a)-(19d).
Appendix 3 Explanation for (23) and (29)

The signaling chains $\mathcal{E}_S$ and $\mathcal{E}_R$ shown in Fig. 35 can be viewed as joint source-channel encoders, where $\mathbf{u}_i$ and $\mathbf{u}_R$ are the $K$-bit length i.i.d. binary information sequences generated from the source node $i$ for $i \in \{A, B\}$ and relay $R$, respectively. Here $\mathbf{x}_i = \mathcal{E}_S(\mathbf{u}_i) = \{x_i(m)\}_{m=1}^M$ and $\mathbf{x}_R = \mathcal{E}_R(\mathbf{u}_R) = \{x_R(m)\}_{m=1}^M$ are $M$-length symbol sequences. For the sake of simplicity, modulation is not shown in Fig. 35. The transmissions from $i$ and from $R$ to the destination $D$ are orthogonal since time-sharing is assumed in this thesis. According to [82, Section 14.1], the source-channel separation holds for the orthogonal multiple access channel. Thus, we apply the separation theorem to find the sufficient and necessary conditions for reliable transmissions for $\mathbf{u}_i$. By the separation theorem, the encoding processes of $\mathcal{E}_S$ and $\mathcal{E}_R$ are respectively treated as a combination of source coding and channel coding, as shown in Fig. 35. Here $\mathbf{b}_i$ and $\mathbf{b}_R$ are $L$-bit length i.i.d. sequences after the source coding of the encoders $\mathcal{E}_S$ and $\mathcal{E}_R$, respectively. Then, we define the source coding rates $R_i$ and $R_R$, channel coding rates.

![Fig 35. Source-channel separation is used to analyze the conditions for the recovery of $\mathbf{u}_i$.](image)
$\mathcal{R}_{ch_1}$ and $\mathcal{R}_{ch_R}$, and end-to-end rates $\mathcal{R}_{ci}$ of $\mathcal{E}_S$ and $\mathcal{R}_{cR}$ of $\mathcal{E}_R$ as follows

\begin{align*}
\mathcal{R}_i &= \mathcal{R}_R = L/K, \\
\mathcal{R}_{ch_1} &= \mathcal{R}_{ch_R} = L/M, \\
\mathcal{R}_{ci} &= \mathcal{R}_{cR} = K/M,
\end{align*}

(68)

To make sure the sequence $u_i$ can be recovered at the destination, the source coding rates $\mathcal{R}_i$ and $\mathcal{R}_R$ have to satisfy the inequalities shown in (19). Moreover, according to the channel coding theorem [102, Section 8.7], the channel coding rates $\mathcal{R}_{ch_1}$ and $\mathcal{R}_{ch_R}$ must satisfy the following inequality

\begin{align*}
\mathcal{R}_{ch_1} &= \mathcal{R}_i \mathcal{R}_{ci} \leq C(\gamma_{iD}); \\
\mathcal{R}_{ch_R} &= \mathcal{R}_R \mathcal{R}_{cR} \leq C(\gamma_{RD}),
\end{align*}

(69)

and equalities in (69) hold by assuming a capacity-achieving code is used in the $iD$ and $RD$ links.
Appendix 4 Divisions of integrals (33) and (34)

As shown in (28) and (30), the values of $\hat{p}_i(\gamma_{AR})$ and $\hat{p}_R(\gamma_{RD})$ become zero as the SNR $\gamma_{AR}$ and $\gamma_{RD}$ exceed the threshold value, and thus, the domain of the integral shown in (35) can be divided into eight intervals according to the threshold value $\gamma^*_A$, $\gamma^*_B$ and $\gamma^*_R$, as

$$\Pr(\varepsilon_1) = \mathcal{J}[g_1; V] = \sum_{j=1}^{8} \mathcal{J}[g_1; V_j]$$

$$\Pr(\varepsilon_2) = \mathcal{J}[g_2; V] = \sum_{j=1}^{8} \mathcal{J}[g_2; V_j],$$  \hspace{1cm} (70)

where

$$V_1 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^+, \gamma_{BR} \in B^+, \gamma_{RD} \in C^+\}$$

$$V_2 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^+, \gamma_{BR} \in B^-, \gamma_{RD} \in C^+\}$$

$$V_3 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^-, \gamma_{BR} \in B^+, \gamma_{RD} \in C^+\}$$

$$V_4 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^-, \gamma_{BR} \in B^-, \gamma_{RD} \in C^+\}$$

$$V_5 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^+, \gamma_{BR} \in B^+, \gamma_{RD} \in C^-\}$$

$$V_6 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^+, \gamma_{BR} \in B^-, \gamma_{RD} \in C^-\}$$

$$V_7 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^-, \gamma_{BR} \in B^+, \gamma_{RD} \in C^-\}$$

$$V_8 = \{(\gamma_{AR}, \gamma_{BR}, \gamma_{RD}): \gamma_{AR} \in A^-, \gamma_{BR} \in B^-, \gamma_{RD} \in C^-\}. \hspace{1cm} (71)$$

Here, the sets $A^-, A^+, B^-, B^+, C^-$ and $C^+$ are defined as

$$A^- = [0, \gamma^-_*), \hspace{1cm} A^+ = [\gamma^+_*, \infty)$$

$$B^- = [0, \gamma^-_*), \hspace{1cm} B^+ = [\gamma^+_*, \infty)$$

$$C^- = [0, \gamma^-_*), \hspace{1cm} C^+ = [\gamma^+_*, \infty). \hspace{1cm} (72)$$

The integrals of (33) and (34) with respect to each interval listed in (71) are expressed in (73) and (74), respectively. The arguments $\hat{\delta}_j$ and $\hat{\omega}_j$ corresponding to each interval in (73) and (74) are summarized in Table 9.
| \( \hat{\delta}_1 \) | \( H_b(0 \ast 0 \ast 0) \) |
| \( \hat{\delta}_2 \) | \( H_b(0 \ast \hat{p}_B(\gamma_{BR}) \ast 0) \) |
| \( \hat{\delta}_3 \) | \( H_b(\hat{p}_A(\gamma_{AR}) \ast 0 \ast 0) \) |
| \( \hat{\delta}_4 \) | \( H_b(\hat{p}_A(\gamma_{AR}) \ast \hat{p}_B(\gamma_{BR}) \ast 0) \) |
| \( \hat{\delta}_5 \) | \( H_b(0 \ast \hat{p}_R(\gamma_{RD})) \) |
| \( \hat{\delta}_6 \) | \( H_b(0 \ast \hat{p}_R(\gamma_{RD}) \ast \hat{p}_R(\gamma_{RD})) \) |
| \( \hat{\omega}_1 \) | \( \hat{\omega}_1 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_2 \) | \( \hat{\omega}_2 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_3 \) | \( \hat{\omega}_3 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_4 \) | \( \hat{\omega}_4 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_5 \) | \( \hat{\omega}_5 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_6 \) | \( \hat{\omega}_6 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_7 \) | \( \hat{\omega}_7 + 1 - f_A(\gamma_{AD}) \) |
| \( \hat{\omega}_8 \) | \( \hat{\omega}_8 + 1 - f_A(\gamma_{AD}) \) |
\[
\gamma_{[1;V_1]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{RD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_1}(d_{1}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{RD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[
\gamma_{[1;V_2]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{BD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_2}(d_{2}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{BD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[
\gamma_{[1;V_3]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{BD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_3}(d_{3}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{BD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[
\gamma_{[1;V_4]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{BD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_4}(d_{4}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{BD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[
\gamma_{[1;V_5]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{BD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_5}(d_{5}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{BD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[
\gamma_{[1;V_6]} = \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{B}}{\gamma_{BD}} - \frac{\gamma_{A}}{\gamma_{AR}} - \frac{\gamma_{B}}{\gamma_{BR}} \right) \int_0^{\gamma_{-1}(1)} f_{F_A}^{a_6}(d_{6}) \exp \left( -\frac{f_{-1}(2)}{\gamma_{BD}} - \frac{\gamma_{AD}}{\gamma_{AD}} \right) d\gamma_{AD}
\]

\[117\]
Appendix 5 Superiority/inferiority of e-MARC related to SDF-MARC in Asymmetric/Symmetric scenarios

As mentioned in Section 3.2.2, the SDF-MARC and e-MARC systems belong to different categories of MARC (SDF-MARC is “address-based”, while e-MARC is “non-address-based”). Here we try to provide more details to better understand how the e-MARC and SDF-MARC systems behave in asymmetric and symmetric topologies, let us consider Case A and Case B shown in the Fig. 36 in the following.

In Case A shown in Fig. 36, the information estimates sent via the \( BR \) and \( BD \) links both contain errors with high probability. The erroneous \( \tilde{u}_B \), instead of being discarded by the SDF-MARC system, can still be utilized in the e-MARC system at the destination, which contributes to the improvement of the outage performance. Hence, the proposed e-MARC system is superior to the SDF-MARC system in Case A. However, we cannot exactly evaluate how much better the proposed e-MARC system is than the SDF-MARC system, because as mentioned above, there is no explicit mathematical expressions to calculate the theoretical outage probability of the SDF-MARC system.

On the contrary, in Case B shown in Fig. 36, the information estimates sent via the \( AR \) and \( BD \) links both contain errors with high probability. Correctly decoded \( \tilde{u}_B \) is guaranteed in the SDF-MARC system to help recover the information sequence sent via the \( BD \) link. However, network coding is always performed in the proposed e-MARC system, which results in the correctly decoded \( \tilde{u}_B \) being corrupted by the erroneous

---

**Fig 36.** \( u_R = \tilde{u}_A \oplus \tilde{u}_B \) in e-MARC for both case; while \( u_R = \tilde{u}_A \) and \( u_R = \tilde{u}_B \) in SDF-MARC for Case A and Case B, respectively.
Node B is far away to both relay and destination and
Same distance for Node A and Node B to both the relay and destination

Fig 37. Outage probabilities of e-MARC and SDF-MARC are shown as functions of loss $L$, where $\Gamma_{AD}$ and $\Delta$ are fixed to 10 dB and 3 dB, respectively.

$\tilde{u}_A$ in the network-coding process, and the erroneous XOR-ed sequence degrades the outage performance. Therefore, the proposed e-MARC system is inferior to the SDF-MARC system in Case B. Nevertheless, we cannot exactly evaluate how worse the proposed e-MARC is than SDF-MARC.

For the Asymmetric scenario where the loss $L = 10$ dB, Case A is most likely to occur since node B is far away from both the relay and destination, and hence it is reasonable for the proposed e-MARC system to be advantageous, shown in Fig. 37. As the node $B$ gradually moves close to both the relay and destination (i.e. $L$ from 10 dB to 0 dB), the probability that Case A occurs decreases while the probability that Case B occurs increases. According to simulation results shown in Fig. 37, SDF-MARC outperforms e-MARC as the value of $L$ is less than 4 dB.

The explanation mentioned above for the superiority/inferiority of e-MARC related to SDF-MARC in the Asymmetric/Symmetric scenarios is only text-based, not based on mathematical analyses. In addition, deriving explicit mathematical expression of the outage probability with SDF-MARC is out of the scope of this thesis. Therefore, we do not include the explanation in the main body of the thesis.
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