UNIVERSITY of OULU

DEPARTMENT
OF ECONOMICS

Working Papers

Externalities and Multiple Equilibria in Cournot Competition

Seppo Orjasniemi
University of Oulu

Mikko Puhakka
University of Oulu

Lauri Vilmi
University of Oulu

and

Jennifer P. Wissink
Cornell University

Working Paper No. 0804
April 2008
ISSN 1796-7961

P.O.BOX 4600, FIN-90014 OULU FINLAND, TEL +358 8 553 2905, FAX +358 8 553 2906, E-MAIL firstname.lastname@oulu.fi

Department of Economics
University of Oulu
Working Paper No. 0804

Externalities and Multiple Equilibria in Cournot Competition*

Abstract

Industry-wide externalities have been recognized as important phenomena, and are topics for a considerable body of research. In much of this work emphasis is placed on the relationship between the externality and the existence and properties of multiple equilibria that, subsequently, arise. We introduce an externality into a firm's unit cost function and study the conditions for multiple equilibria. The existence of such externalities can create situations where strategic complementarities engender multiple equilibria and coordination failures. We provide an example for multiple equilibria. We also relate the existence of multiple equilibria to the magnitudes of price elasticity of demand and the elasticity of unit cost with respect to industry output. We also perform some welfare comparisons between different equilibria.

JEL Classification: L10, L13, L16

Keywords: externality, multiple equilibria, strategic complementarity

Seppo Orjasniemi
Department of Economics
University of Oulu
P.O.Box 4600
FIN-90014 University of Oulu
FINLAND

seppo.orjasniemi@oulu.fi

Mikko Puhakka
Department of Economics
University of Oulu
P.O.Box 4600
FIN-90014 University of Oulu
FINLAND

mikko.puhakka@oulu.fi

Lauri Vilmi
Department of Economics
University of Oulu
P.O.Box 4600
FIN-90014 University of Oulu
FINLAND

lauri.vilmi@oulu.fi

Jennifer P. Wissink
Department of Economics
Cornell University
468 Uris Hall, Ithaca, NY
14853-7601
USA

jpw6@cornell.edu

* We thank the audience of the FDPE IO seminar (Helsinki), and especially Otto Toivanen, for comments. We also appreciate the comments from Jinhwan Oh. Puhakka thanks the Yrjö Jahnsson and OKO Foundations, Orjasniemi and Vilmi the Tonning Foundation for financial support.

1. Introduction

Industry-wide externalities have been recognized as important phenomena and they have also been topics for a considerable body of theoretical research. In much of this work an emphasis is placed on examining the relationship between the externality and the existence and properties of multiple equilibria that, subsequently, arise. In addition to being theoretically important, the empirical relevance of such externalities has been recognized in a wide variety of applied research in several different fields of economics.[1]

In this paper we focus explicitly on the role of an externality in a static Cournot model.[2] We introduce an externality into a firm's unit cost function in, what is otherwise, a standard model of oligopoly. In particular, we assume that an individual firm's unit cost depends on the aggregate industry output. A reason for such an externality could be industry wide learning, where unit costs decrease as the industry output becomes larger. Stokey (1986) studied such a model but in a different context from ours. We submit, however, that such a cost externality might arise also for a variety of other reasons, though we do not model more precisely the emergence of the externality itself.

One might rightfully wonder if we treat two apparent externalities of our model in a non-symmetric way. Why do the firms neglect the cost externality, but see the effect of their own actions on the price? In the context of imperfect competition firms obviously understand the effects of their own actions on the price. But given e.g. the learning interpretation above and spillovers discussed below, it is not at all so straightforward that they fully understand quickly the effects of the industry-wide output on their own costs.

Cost externalities are often introduced when research and development issues are studied. When industry output increases, firms devote more resources to R&D projects. Successful innovation reduces the costs of the firm that initiated the project. The knowledge, however, spills over to other firms quite quickly thereby reducing their costs as well. Other sources of such external economies can be traced to the

---

[1] For example, Griliches (1992) surveys R&D spillovers; Caballero and Lyons (1990, 1992) analyze the effects of external economies at the industry level; Henderson (1997) examines externalities in industrial development; and evidence on industrial complementarities is considered by Cooper and Haltiwanger (1993). Brandt (2007) argues that spillovers have an effect on the observed productivity growth.

[2] Such a standard model is not used exclusively in textbooks. E.g. Genesove and Mullin (1998) use a static oligopoly model in an interesting recent empirical study on the sugar industry in the end of the 19th and the beginning of the 20th centuries.

effects of expansion and contraction of industry output on factor price levels or on technological returns. The externality can also be viewed as a reduced form of some other fundamental process associated with, for example, phenomena like learning-by-doing as emphasized above.

In an interesting empirical work Morrison Paul and Siegel (1999) find that externalities have an important role in explaining scale effects in U.S. manufacturing. And furthermore, and more importantly, they argue that external factors or agglomeration effects might have a significant effect on cost savings of firms.

Following terminology in Bulow, Geanakoplos and Klemperer (1985) and Cooper and John (1988), externalities can cause the firm's profit function to exhibit either positive or negative spillovers or either strategic complementarity or substitutability with respect to their rivals' output. To generate multiple equilibria in our model it is necessary for firms to regard each other's output levels as strategic complements. Strategic complementarity itself, however, will not guarantee multiple equilibria. The importance of strategic complementarity is not surprising. Cooper and John (see also Cooper, 1999) investigate the relationship between strategic interaction and multiple equilibria and point out that strategic complementarity is necessary for multiple equilibria and, therefore, critical to any discussion of coordination failures, i.e., situations where an inferior (in terms of welfare) equilibrium attains.[3] In a recent work Lagerlöf (2007) shows that uncertainty in the linear demand function (making expected demand essentially convex) can generate multiple equilibria in static Cournot competition.

Interestingly, in our example, we can relate the emergence of multiple equilibria to the magnitudes of two important elasticities. If the product of the absolute value of the price elasticity of demand and the elasticity of the unit cost function with respect to industry output exceeds unity, there can be multiple equilibria. We also submit that the emergence of multiple equilibria is interesting, since it can be interpreted as a coordination failure as emphasized e.g. by Cooper and John, and also because they can be interpreted as resulting from pessimism and optimism.

We proceed as follows. First we present the basics of our model. Then we study in detail a robust example of Cournot competition, where we can precisely relate the existence of multiple equilibria to strategic complementarity, and also in particular, to

[3] The issue of externalities and multiple equilibria have been investigated in many fields of economics, e.g. in macroeconomics as explained by Cooper (1999) and also in development economics; see e.g. Basu (1997, especially chapter 2).

the price elasticity of demand and the elasticity of the unit cost function with respect to industry output. We also do some welfare comparisons.

2. Externality in the Cournot Competition

We utilize an otherwise standard Cournot oligopoly model with identical firms except we assume that the typical firm's costs depend on the total quantity of output produced in the market as well as on the individual firm's own level of output. This introduces an externality into the firm's cost function and, thereby into its profit function. A typical firm's cost function is $c(Q)q_i$, where $c(Q)$ is the unit cost function, which depends on the level of industry output, $Q = \sum_{j=1}^{n} q_j$, and q_i is the output of the firm i. The unit cost function, $c(Q)$, is assumed to be continuously differentiable, strictly decreasing, strictly convex, and fulfilling $c(Q) \geq 1$ for all Q.[4] More industry output will decrease the marginal costs of each firm. We discussed a number of explanations for such an externality in the introduction, but, essentially, we interpret the inclusion of industry output in the individual firm's cost function as a reduced-form of some fundamental production or cost externality, which, in this analysis, we leave unspecified. And we also assume that the unit cost cannot fall below some number (here normalized to unity) for any level of production, which actually means that there are ultimately constant returns to scale in technology.

Firms produce a homogenous good. Often in the literature it is assumed that the inverse market demand function, $P(Q)$, is twice continuously differentiable and has the following properties:

(Pl) $P'(Q) < 0$ and finite for all $Q \geq 0$

and

(P2) $P'(Q) + P''(Q)Q \leq 0$.

The first property states that the demand curve satisfies the "law of demand" and has bounded prices while the second is a sufficient condition ensuring that each firm's marginal revenue declines with output. Actually, if the marginal costs are constant, condition (P2) guarantees the uniqueness of equilibrium.[5] We also assume

---

[4] These assumptions mean that there is strategic complementarity between the firm and its competitors. We elaborate this issue more precisely below. Our formulation of the cost function such that the externality enters via a unit cost is not crucial for our results. It is the strategic complementarity, which is essential.

[5] See e.g. Shapiro (1989) p. 335-336.

that $P(0) > c(0)$, which apparently holds for the constant elasticity inverse demand function, $P(Q) = AQ^{-1/\varepsilon}$, since $\lim_{Q\to 0} P(Q) = \infty$ Note interestingly that (P1) and (P2) do not hold for the above inverse demand function, which we use below in our example, since it is a strictly convex function.

We now examine the model with the cost externality present assuming its effect is not directly recognized by the individual firm, i.e. firms maximize their profits without taking into account the effect of industry output on costs. Firm i's problem is as follows:

(PF) $\max_{q_i} \pi_i = P(Q)q_i - c(Q)q_i$.

The first-order condition for this problem is

(1) $$\left.\frac{\partial \pi_i}{\partial q_i}\right|_{c'(Q)=0} = P(Q) + P'(Q)q_i - c(Q) = 0 .$$

The second-order condition for a maximum is that the marginal revenue curve must be downward-sloping, i.e. $2P'(Q) + P''(Q)q_i < 0$. We consider only symmetric equilibria, i.e. $q_i = q$ for all $i = 1,...,n$, and drop the firm subscript whenever convenient. Focusing exclusively on symmetric equilibria is natural because we assume that firms are identical. Define the function $F(q)$ as follows

(2) $$F(q) \equiv P(nq) + P'(nq)q - c(nq) .$$

Cournot equilibria are all q^C such that $F(q^C) = 0$. Below when studying in detail our example, we verify that the equilibrium of that model without externality is unique. Next we present a necessary condition for multiple Cournot equilibria in the model with the externality present.

Proposition 1. If P2 holds, it is necessary to have $c'(Q)$ negative for there to be multiple Cournot equilibria. Furthermore, if the demand function has constant elasticity, the same conclusion holds.

Proof: Consider equation (2). The limit of $F(q)$ as q approaches infinity is negative, because, as q gets large, $P(nq)$ approaches zero and all the remaining terms are negative. Differentiating equation (2) we obtain $F'(q) = (n+1)P'(nq) + P''(nq)nq - nC'(nq)$. $F'(q)$ is negative for every q, if P2 holds and $c'(Q)$ is positive. For the constant elasticity demand we can express $F'(q)$ as $F'(q) = (1/\varepsilon)A(nq)^{-\frac{1}{\varepsilon}-1}\left[-n + (1/\varepsilon)\right] - nc'(nq)$. If $c'(Q)$ is positive, $F'(q)$ is negative, since $n\varepsilon$ is the price elasticity faced by an individual firm, and thus it must be greater than unity. To generate multiple equilibria, $F'(q)$ must be positive for some values of q. Thus, it is necessary that $c'(nq)$ is negative for at least some range of output. Q.E.D.

This proposition states that for there to be any chance of multiple equilibria, the individual firms' marginal costs must decrease with total industry output. This implies that some type of strategic complementarity must exist across firms' outputs. This is an example-specific analogue to what is proven in Cooper and John (1988). Bulow, Geanakoplos and Klemperer (1985) as well as Cooper and John define strategic complementarity to be a situation in which the marginal payoff with respect to a firm's action is enhanced by increases in its rival's action. Consider what happens to marginal profit, $\partial\pi_i / \partial q_i$, in our model, when a competitor's output changes. In our notation, strategic complementarity implies that $\partial^2\pi_i / \partial q_i \partial q_j$ is positive. By straightforward differentiation of the firm's profits, and now taking the cost externality into account, we obtain the following

(3) $$\frac{\partial^2\pi_i}{\partial q_i \partial q_j} = P'(Q) + P''(Q)q_i - c'(Q) - c''(Q)q_i .$$

Strategic complementarity would then mean that $-c'(Q) - c''(Q) > -P'(Q) - P''(Q)q_i$. Since the unit cost function is strictly convex, this certainly is possible only when $c'(Q)$ is negative. The fact that $\partial^2\pi_i / \partial q_i \partial q_j$ is positive, however, does not guarantee that $F'(q)$ is positive. So in fact, Proposition 1 requires, in some sense, a "strong" strategic complementarity for multiple equilibria. It should be noted that this proposition tells us nothing about whether or not equilibrium exists; that particular question is the one we do not address here except in our example below.

3. Multiple Equilibria: An Example

We now illustrate our results with an example. We assume that the market demand curve has a constant price elasticity, i.e. the inverse demand is $P(Q) = AQ^{-1/\varepsilon}$. $A > 0$, and ε is the price elasticity of demand. The unit cost function of each firm is assumed to depend on the industry's aggregate output in the following fashion: $c(Q) = B(Q+1)^{-\eta}$. $B > 0$, and $-\eta$ is roughly the elasticity of firm's marginal cost with respect to industry output. We assume, however, that the unit cost can never fall below unity. This means that $c(Q) = 1$, whenever $Q > \overline{Q}$. $\overline{Q}$ is defined from $1 = B(\overline{Q}+1)^{-\eta}$, i.e. $\overline{Q} = B^{1/\eta} - 1$.[6]

We write the profits of a representative firm as

(4) $$\pi_i = P(Q)q_i - c(Q)q_i \;\Rightarrow\; \pi_i = AQ^{-1/\varepsilon}q_i - B(Q+1)^{-\eta}q_i .$$

The firm does not take into account the external effects in the unit cost function. Thus the first-order condition for maximum is

(5) $$\frac{\partial \pi_i}{\partial q_i} = AQ^{-\frac{1}{\varepsilon}} - \frac{1}{\varepsilon}AQ^{-\frac{1}{\varepsilon}-1}q_i - B(Q+1)^{-\eta} = 0 ,$$

which says that output should be chosen in such a fashion that marginal revenue equals marginal cost recognized by the firm. The second partial derivative of the profit function is

(6) $$\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2}{\varepsilon}AQ^{-\frac{1}{\varepsilon}-1} + \frac{1}{\varepsilon}(\frac{1}{\varepsilon}+1)AQ^{-\frac{1}{\varepsilon}-2}q_i .$$

Rearranging we can express (6) as

(7) $$\frac{\partial^2 \pi_i}{\partial q_i^2} = \left(-\frac{2}{\varepsilon} + \frac{q_i}{Q}\frac{1}{\varepsilon^2} + \frac{q_i}{Q}\frac{1}{\varepsilon}\right)AQ^{-\frac{1}{\varepsilon}-1} .$$

This is negative at least for any $\varepsilon > 1$. If we evaluate the sign of (7) in a symmetric equilibrium, where $nq = Q$, the second-order condition holds if $n\varepsilon > 1$, which holds in standard Cournot models, since $n\varepsilon$ is the elasticity faced by a particular firm.

Before proceeding to multiple equilibria, we check whether the equilibrium without externality is unique. We denote the constant marginal cost by $\overline{c}$. A firm's first-order condition will be

(8) $$AQ^{-\frac{1}{\varepsilon}} - \frac{1}{\varepsilon}AQ^{-\frac{1}{\varepsilon}-1}q_i = \overline{c} .$$

Expressing this in a symmetric equilibrium we obtain

(9) $$\left(1 - \frac{1}{n\varepsilon}\right)AQ^{-\frac{1}{\varepsilon}-1} = \overline{c} .$$

---

[6] The unit cost function we use is exactly the same as that used by Stokey (1986). The elasticity is exactly $-\eta Q/(Q+1)$, which is close to $-\eta$ for large Q.

If $n\varepsilon > 1$, the left-hand-side of (9) is positive, and a decreasing function of the per firm output, and furthermore the limit of the left-hand side is zero when industry output approaches infinity. Thus, the equilibrium is unique.

Now we study the symmetric equilibria with an externality, and thus assume $q_i = q$ for all i. This also means that $Q = nq$. Rewriting and rearranging we can express (5) as

$$(10) \qquad LHS(q) \equiv A^{\frac{1}{\eta}}(1-\frac{1}{\varepsilon n})^{\frac{1}{\eta}}(1+nq) = B^{\frac{1}{\eta}}(nq)^{\frac{1}{\varepsilon\eta}} \equiv RHS(q).$$

Cournot equilibrium is a level of per firm output, q^C, such that $LHS(q^C) = RHS(q^C)$. The left-hand side is a linear function, and increasing, because $n\varepsilon > 1$. $LHS(q)$ is not strictly increasing for every level of output. This is because unit cost cannot fall below unity. Thus for all $q \geq \overline{Q}/n$, the value of the function stays constant. The right-hand side function can be strictly convex, linear or strictly concave, if the product of elasticities is less than unity, unity or more than unity. We draw both functions in Figure 1, when $RHS(q)$ is strictly concave. It is apparent from the Figure that only in the case, where the right-hand side function is strictly concave there can be multiple equilibria. If the price elasticity of demand (ε) and the unit cost elasticity with respect to industry output (η) fulfill the inequality $\eta\varepsilon > 1$, there can be multiple equilibria. A high price elasticity requires a relatively low cost elasticity and vice versa. We denote the three equilibria in the Figure by q_i ($i = 1,2,3$).

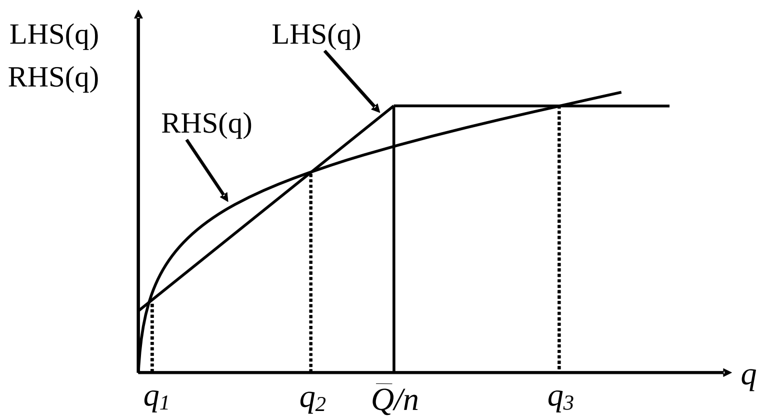

Figure 1. Multiple equilibria.

We next present a numerical example with three equilibria. We use the following parameter values: $A = 2$, $\varepsilon = 4$, $B = 3$, $n = 2$ and $\eta = 0.5$.[7] With these values marginal cost is unity when total industry production is 8 or above. Now there will be three equilibria in Cournot competition. For the sake of comparison we also compute the price-taking equilibria, where firms set their output such that price equals marginal cost. The three symmetric equilibria (also denoted by E1, E2 and E3) are: $q_1 = .077$, $q_2 = 3.241$ and $q_3 = 4.689$. We describe the reaction functions in Figure 2. Interestingly equilibrium E3 is the one, which would be obtained in a standard model without externality, and with the unit cost equaling unity. In E3 the reaction curves are downward sloping, which would be expected in standard model. We can also interpret E3 as an optimistic equilibrium or equilibrium prevailing in a mature industry. There the learning effect has been diminished, or has even completely disappeared. In fact, Stokey (1986, especially p. 82) points out that the dependence of a firm's unit cost on the cumulative industry output captures perhaps something about the early development of infant industries.

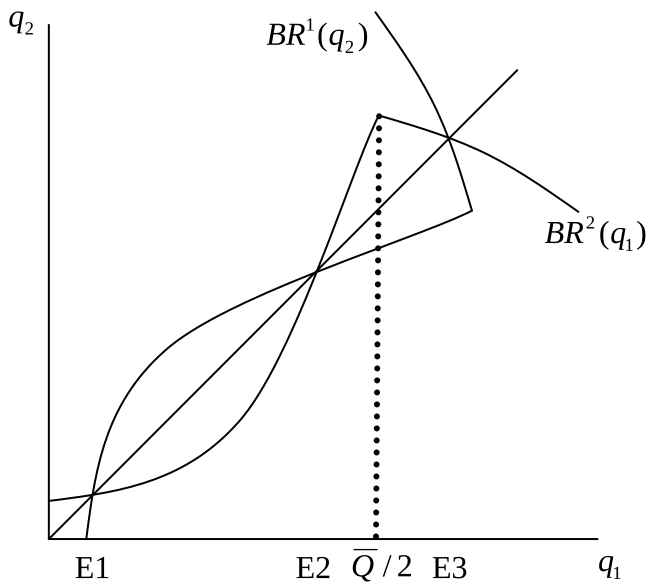

Figure 2. Reaction functions.

We present numerical comparisons in Table 1. Note that the second Cournot equilibrium dominates the respective price-taking equilibrium by having higher total surplus. The conventional relationships hold at the other equilibria. As we increase the

7 Stokey uses the values 1.5 for the price elasticity and .32 for the cost elasticity.

number of competitors in the market, it can be shown that the equilibrium values of Cournot competition and price-taking competition converge closer to each other.

Cournot competition				**price-taking competition**			
	E1	E2	E3		E1	E2	E3
total production	0.154	6.482	9.379	total production	0.372	2.691	16.000
price	3.191	1.253	1.143	price	2.562	1.562	1.000
consumer surplus	0.164	2.708	3.573	consumer surplus	0.317	1.401	5.333
total profit	0.062	1.016	1.340	total profit	0.000	0.000	0.000
net-social-surplus	0.226	3.724	4.913	net-social-surplus	0.317	1.401	5.333

Table 1. Comparison of equilibria and welfare.

4. Conclusion

In this paper we investigate equilibria in a rather standard model with quantity-setting oligopolies and a cost externality. The existence of such externalities can create situations in which strategic complementarities give rise to multiple equilibria and coordination failures. It is also of interest that we can relate the emergence of multiple equilibria to the magnitudes of two important elasticities in our example. If the product of the price elasticity of demand and the elasticity of the unit cost function with respect to industry output exceeds unity, there can be multiple equilibria. Our model also points out that the standard Cournot model captures perhaps some features of more mature industries, where the external effect (due e.g. to learning) has disappeared.

References:

Basu, K., 1997, *Analytical Development Economics*. MIT Press. Cambridge, MA.

Brandt, N., 2007, Mark-ups, economies of scale and the role of knowledge spillovers in OECD industries. *European Economic Review* 51, 1708-1732.

Bulow, J., J. Geanakoplos and P. Klemperer, 1985, Multimarket oligopoly: strategic substitutes and complements, *Journal of Political Economy* 93, 488-511.

Caballero, R.J. and R.K. Lyons, 1990, Internal versus external economies in European industry, *European Economic Review* 34, 805-826.

Caballero, R.J. and R.K. Lyons, 1992, External effects in US procyclical productivity, *Journal of Monetary Economics* 29, 209-225.

Cooper, R. and J. Haltiwanger, 1993, Autos and the National Industrial Recovery Act: Evidence on Industry Complementarities. *Quarterly Journal of Economics* 108, 1043-1072.

Cooper, R. and A. John, 1988, Coordinating coordination failures in Keynesian models, *Quarterly Journal of Economics* 103, 441-463.

Cooper, R.W., 1999, *Coordination Games. Complementarities and Macroeconomics*. Cambridge University Press. New York, N.Y.

Genesove, D. and W.P. Mullin, 1998, Testing static oligopoly models: conduct and cost in the sugar industry, 1890-1914, *Rand Journal of Economics* 29, 355-377.

Griliches, Z., 1992, The search for R&D spillovers, *Scandinavian Journal of Economics* 94 (Supplement 1992), 29-47.

Henderson, V., 1997, Externalities and Industrial Development, *Journal of Urban Economics* 42, 449-470.

Lagerlöf, J., N., M., 2007, Insisting on a non-negative price: Oligopoly, uncertainty, welfare, and multiple equilibria, *International Journal of Industrial Organization* 25, 861-875.

Morrison Paul C., J. and D. S. Siegel, 1999, Scale Economies and Industry Agglomeration Externalities: A Dynamic Cost Function Approach, *American Economic Review* 89, 272-290.

Shapiro, C., 1989, Theories of oligopoly behavior, in: R. Schmalensee and R. Willig, eds., *Handbook of Industrial Organization*, Vol. 1 (North-Holland, Amsterdam), 329-415.

Stokey, N., 1986, The dynamics of industry-wide learning, in: W. Heller, R. Starr, and D. Starrett, eds., *Equilibrium analysis: Essays in honor of Kenneth J. Arrow*, Vol. 2, (Cambridge University Press, Cambridge), 81-104.