The Effects of Firm Entry and Exit on Macroeconomic Fluctuations and Monetary Policy

Lauri Vilmi
University of Oulu

Working Paper No. 0904
September 2009
ISSN 1796-7961
The Effects of Firm Entry and Exit on Macroeconomic Fluctuations and Monetary Policy*

Abstract

This paper studies how endogenous firm entry affects the responses of the economy to the monetary policy, cost-push and demand shocks. Opposite to the earlier literature we assume also that firm default rate is not constant but subject to exogenous shocks. We show that increased bankruptcies have long-standing negative effect on output and positive effect on inflation. This raises the question how monetary policy should react to the variability in business defaults. Based on our simulations we propose two alternative monetary policy rules, which give lower variance than the traditional Taylor rule which reacts to inflation and consumption.

JEL Classification: E32, E52.

Keywords: Endogenous entry, monetary policy, bankruptcy shock

Lauri Vilmi

Department of Economics
University of Oulu
P.O.Box 4600
FIN-90014 University of Oulu
FINLAND

e-mail: lauri.vilmi@oulu.fi

* I thank Tor Jacobson, Seppo Orjasniemi, Mikko Puhakka, Virginia Queijo von Heideken and seminar participants at Oulu Macro Group and Sveriges Riksbank for useful comments. Especially I would like to thank Ulf Söderström for a helpful discussion. Financial support from Tauno Tönning foundation is gratefully acknowledged.
1. Introduction
Monopolistic competition is often assumed in dynamic stochastic general equilibrium (DSGE) models. Because firms have price setting power they earn positive profits even with nominal rigidities. Those rigidities cause time-varying mark-ups, but for shocks of reasonable size profits are positive. The profits should induce more firms to enter the market, but many DSGE models do not take into account this feature.

In the recent international trade literature the entry of new firms has played a central role since Melitz (2003). Bilbiie, Ghironi and Melitz (2007a) study endogenous entry in a business cycle model. They find that the value of firms has an important role in the dynamics of business cycles, because it affects saving decisions and the entry of firms. The number of firms responds sluggishly to the business cycle shocks and thus affects the length of the cycle.

Bilbiie, Ghironi and Melitz (2007b) have Taylor rule and price rigidity in the business cycle model with endogenous firm entry. There are two channels for monetary policy to affect the economy. The first channel is the traditional one where a decrease in interest rate has an expansionary effect, because it increases current consumption and labor supply. However, the return from savings responds to the changes in interest rate. There is a no-arbitrage condition between the real return on bonds and equity. Because of this fact the value of firms also changes as interest rate fluctuates. Thus monetary policy affects investments to the new firms. The latter channel is ignored in most of the monetary policy literature. This channel for the transmission of monetary policy through the extensive margin is studied also by Elkhoury and Mancini-Griffoli (2007).

Changes in the number of firms caused by monetary policy or separate exogenous shocks affect the efficiency of economy and inflation. Because final-goods are produced from individual varieties by a constant elasticity of substitution production technology, the number of varieties has a direct effect on the productivity of economy. Thus the number of firms directly affects aggregate consumption and inflation. We call the price index which is affected by the number of firms as a consumer price index. The prices of individual intermediate-goods varieties are called intermediate-goods price index and this index does not take into account the number of firms.

This paper studies the responses of an economy to the different shocks in a framework where the number of firms and firm entry are endogenous and monetary authority reacts according to a simple Taylor rule. Monetary authority might want to consider also the variations in the product variety
when making policy decisions, because the number of varieties has a direct effect on inflation. We study how monetary policy should take into account this issue. We assume that the economy faces three different shocks: cost-push, demand and bankruptcy shocks. Cost-push shock is a shock to the new Keynesian Phillips curve and affects the prices. Demand shocks are disturbances to the log-linear approximation of the Euler condition for bonds. These shocks are widely studied in new Keynesian frameworks, but as far as we know this is the first paper where the effects of these shocks are studied with endogenous firm entry. The average business default frequency varies a lot over time. Earlier models with endogenous entry e.g. Bilbiie et al. (2007b) and Elkhoury et al. (2007) assume constant business default rate. Contrary to them we assume that observed variability in the default rate can be caused by exogenous bankruptcy shocks. Thus, we study how increased business defaults affect the economy. Bankruptcy shocks can be for example shocks to the financial markets, which change the probability of firms to get external finance.

Instead of using the quadratic costs of adjusting prices as in Bilbiie, Ghironi and Melitz (2007b), we introduce the familiar staggered price setting due to Calvo (1983) to the DSGE model with endogenous firm entry. We find that when there is endogenous entry these two sources of price rigidities generate similar Phillips curves. These two curves are not however exactly the same as is the case for the models ignoring firm entry. Opposite to Bilbiie, Ghironi and Melitz (2007b) we assume that investments to the firms are made to the intermediate-goods firms, which produce and sell differentiated products to the perfectly competitive final-goods sector. More intermediate-goods varieties makes the production of final-goods more efficient as is assumed in a large body of new growth theory literature and especially in Ethier (1982).

We find that the firm entry has a strong effect on impulse responses of labor supply, profits, value of firms and wages. For these variables changes in responses differ in a considerable way from the responses of model ignoring firm entry. We also show that increased default rates have a long-standing depressing effect on economy and positive effect on inflation. Increased inflation and decreased economic activity raises the question how monetary policy should react to this kind of situation.

To evaluate different monetary policies under bankruptcy shocks, we simulate the model for different error processes and compare the variances of inflation and output different policy rules generate. We study the implications of three different policy rules: one which reacts to the consumer price index and another one which reacts to the prices of intermediate-goods. We also add
the number of firms to the Taylor rule and study how costly it is for the monetary authority to ignore the number of firms in their policy rule. This was one of the questions raised by Midrigan (2007) in his comment on Bilbiie et al.

We find out that the rule which reacts to consumer price index generates high variances. Much more stable and better outcome is achieved if monetary policy would react to the prices of intermediate-goods instead of consumer price index. Also policy rule which includes the number of firms generates better trade-off between two variances than rule which reacts only to consumer prices. If we assume that firm default rate is constant and there are no bankruptcy shocks in economy different policy rules give very similar results.

The paper is organised as follows. Section 2 presents the new Keynesian model with endogenous firm entry. Section 3 solves the model numerically. In this section we also study the impulse responses to the four different shocks: monetary policy, cost-push, demand and bankruptcy. In section 4 we study the effects of endogenous firm entry and exogenous bankruptcy shocks on optimal monetary policy. We evaluate different policies based on how well they stabilize consumption and consumer price inflation. Section 5 concludes.

2. Model

We construct a dynamic general equilibrium model, where firms producing intermediate-goods are monopolistically competitive. Intermediate-goods producers who plan to enter the market have to acquire finance for paying the entry cost. This finance is acquired from the stock market. In the stock market consumers can invest in the new entrants and buy and sell the ownerships of established firms. Firms cannot acquire finance from other sources except the stock market.

Entry costs cover development and setup costs for founding a new production line. The costs of start-up firms do not include only research and development costs but also costs due to government requirements and the regulation of entry. Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) study the costs from the regulation of entry. They find that these costs are extremely high in most countries.
2.1. Consumer

Consumers maximize the discounted value of expected infinite lifetime utility over consumption \( C_t \) and leisure \((1 - L_t^A)\). We assume a separable logarithmic utility function:

\[
U = \sum_{t=0}^{\infty} \beta^t (\log C_t + \eta \log(1 - L_t^A)),
\]

where \( \beta \) is the subjective discount factor and \( \eta > 0 \) is a preference parameter. Consumer receives wage income, \((1 + \tau^L)w_t\), for aggregate labour supply \( L_t^A \) and he can save either by buying bonds \( B_t \), stocks of old firms or investing in new firms. The main difference between new and old firms is that old firms are bought from stock market. Instead investment payments into the new firms cover the entry cost for firms. Establishing new firms use aggregate resources. The value of new firms is denoted by \( v_t^N \). The number of firms owned in period \( t \) is denoted by \( N_{D,t} \), \( N_{E,t} \) is invested amount into the new firms. We denote price of consumption by \( P_t \), value of old firms by \( v_t \) and dividends by \( d_t \).

There is time-to-build in firm entry since period \( t-1 \) entrants \( N_{E,t-1} \) start producing in period \( t \). Firms also face an exogenous probability \( \delta \) of exit. During time-to-build period new firms develop their products and set up organisation. We assume that part \( \delta^N \) of the new firms fail during this period and disappear before actual production has started. We assume that probability to fail during the development period is bigger than the exit rate of established firms, i.e. \( \delta^N > \delta \). It follows that the number of operating firms in period \( t \) is

\[
N_{D,t} = \left[(1 - \delta)N_{D,t-1} + (1 - \delta^N)N_{E,t-1}\right]e^{\varepsilon_t},
\]

where \( \varepsilon_t \sim N(0, \sigma^2_t) \) is a bankruptcy shock. Thus consumer’s budget constraint is

\[
P_tC_t = (1 + \tau^L)L_t^AW_t + (1 + \tau^L)B_{t-1} - B_t + (v_t + d_t)\left[(1 - \delta)N_{D,t-1} + (1 - \delta^N)N_{E,t-1}\right]e^{\varepsilon_t}
- v_t^N N_{E,t} - v_t N_{D,t} - T_t.
\]

Here \( \tau^L \) is the subsidy to labor supply whose role we discuss below and \( T_t \) is a lump-sum tax for financing the subsidy. Thus government budget constraint is \( \tau^L L_t^AW_t = T_t \). Because of monopolistic competition prices are higher than marginal costs. This skews demand towards leisure. The purpose of the subsidy for labor is to fix this inefficiency. Subsidy turns flexible price economy and steady-state efficient and allows us to make welfare comparisons around the steady-state.
Maximizing the objective function (1) with respect to $L_t^A, B_t, N_{D,t}$ and $N_{E,t}$, and subject to the budget constraint, yields the following first-order conditions

$$
(1 + r_t)w_t = \frac{\eta}{1 - L_t^A} \quad (3)
$$

$$
\frac{1}{P_tC_t} = \beta(1 + r_t) \quad \frac{1}{P_{t+1}C_{t+1}} \quad (4)
$$

$$
\frac{v_t^N}{P_tC_t} = \beta(1 - \delta^N)(v_{t+1} + d_{t+1}) \quad \frac{1}{P_{t+1}C_{t+1}} \quad (5)
$$

$$
\frac{v_t}{P_tC_t} = \beta(1 - \delta)(v_{t+1} + d_{t+1}) \quad \frac{1}{P_{t+1}C_{t+1}} \quad (6)
$$

Equation (3) is the standard intra-temporal first-order condition for labor supply. Equations (4), (5) and (6) are Euler equations for bonds, new firms and old firms. Expected returns for investments to all three investment possibilities must be the same. Thus we can write a no-arbitrage condition between investments to new and old firms as

$$
v_t = \frac{(1 - \delta)}{(1 - \delta^N)} v_t^N.
$$

Thus established firms are more expensive than new firms, because the probability of default is higher for new entrants. Similarly a no-arbitrage condition between returns to savings into the firms and the bonds must hold. This condition is the essential part of the channel for monetary policy. As monetary authority changes interest rate, it also affects the value of firms and changes investments to the firms. This channel is missing from standard DSGE models, which ignore endogenous entry.

Naturally, the transversality conditions $\lim_{S \to -\infty} \beta^S U(C_{t+S}) B_{t+S} = 0$ and $\lim_{S \to -\infty} \beta^S U(C_{t+S}) N_{D,t+S} = 0$ for bonds and firms must also be satisfied to ensure optimality.

Because our economy is in financial autarky, in the aggregate bonds are in zero net supply $B_t=0$ and the representative consumer owns all the firms $N_{D,t} = [(1 - \delta)N_{D,t-1} + (1 - \delta^N)N_{E,t-1}] e^{\delta t}$. From these it follows that aggregate budget constraint can be written as

$$
P_tC_t + v_t^N N_{E,t} = L_t^A w_t + d_t N_{D,t}. \quad (7)
$$

On the left-hand side are aggregate consumption and net investments. On the right hand-side there is aggregate output, which can be divided into the wage income and profits from the firms.
2.2. Final-goods producers

Final-goods firms produce consumption goods $Y^F_i$ in a perfectly competitive market. In equilibrium $Y^F_i = C_i$ must hold. Only inputs they use are the intermediate-goods from which they produce final-goods by using a constant elasticity of substitution production technology

$$Y^F_i = \left( \int Y_i(\psi) \frac{\theta}{\theta} \frac{\theta}{\theta} d\psi \right)^{\theta-1},$$

where $\psi$ is a differentiated variety and $Y_i(\psi)$ is the amount of individual intermediate-goods variety. From the chosen production technology it follows that more varieties there are more efficient is the final-goods production. The assumptions that intermediate inputs are differentiated and an increase in the number of intermediate-goods varieties make final-goods production more efficient were first introduced by Ethier (1982).

In final-goods sector there is perfect competition and profits are zero, thus

$$d^f_i = P_i C_i - \int_0^{N_{D,i}} P_i(\psi) Y_i(\psi) d\psi = 0.$$ 

Here $P_i(\psi)$ denotes the price of individual variety and $Y_i(\psi)$ denotes the amount of individual variety used in the production. Profit maximization gives the demand for an individual variety to be

$$Y_i(\psi) = \left( \frac{P_i(\psi)}{P_i} \right)^{-\theta} C_i.$$ (8)

This yields consumer price index

$$P_i = \left( \int_0^{N_{D,i}} P_i(\psi)^{-\theta} d\psi \right)^{1-\theta}. $$

Because the number of intermediate-goods varieties is $N_{D,i}$, we can define the average price of an individual intermediate-good to be

$$P^A_i(\psi) = P_i N_{D,i}^{\theta-1}. $$ (9)

2.3. Intermediate-goods firms

As Ghironi and Melitz (2005), we also assume that each firm produces an individual intermediate-goods variety. These firms are monopolistically competitive. The production function of individual variety is $Y_i(\psi) = A_i L_i(\psi)$, where $A_i$ is the aggregate productivity level and $L_i(\psi)$ is labor used in
the production of variety $\psi$. Thus it is obvious that marginal cost, $\rho_i$, with linear production function is $\frac{W_i}{A_t}$. Because we think of firm $i$ as a production line for an individual variety, we can denote $\psi=\mu_i$. The profits of a firm can be written as $d_i(i)=[P_i(i)-w_i/A_t]Y_i(i)$. All the profits are distributed to the shareholders as dividends. Thus firms cannot found a new subsidiary with its profits. Profit maximization gives the optimal flexible price to be a constant mark-up over the marginal costs $P_i(i)=\frac{\theta}{\theta-1} w_i/A_t$.

Firms are not allowed to reset their prices freely, but intermediate-goods producers set prices in staggered contracts with timing like that of Calvo (1983). As in standard Calvo-pricing model we assume that in every period intermediate-goods producers face probability $1-\omega$ to reset their price. Differently to the standard Calvo pricing model firms take now into account their probability to survive,$(1-\delta)$, in their discounting by valuing less future profits lower the probability to survive. Thus when resetting the prices at period $t$ firms maximize their discounted expected profits

$$E_t \sum_{j=0}^{\infty} \omega^j \beta^j (1-\delta)^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \left[ \frac{P_i(i)}{P_{t+j}} Y_{t+j}(i) - \rho_{t+j} Y_{t+j}(i) \right],$$

where $\sigma=1$ when the utility function is logarithmic. Using demand function (6), taking the first order condition with respect to $P_i(i)$ and denoting $P_i(i)=P_{it}^*$, gives after rearranging familiar Calvo-pricing result for optimal pricing:

$$\frac{P_{it}^*}{P_t} = \frac{\theta E_t \sum_{j=0}^{\infty} \omega^j \beta^j (1-\delta)^j C_{t+j}^{1-\sigma} \left( \frac{P_{t+j}}{P_t} \right)^{\theta} \rho_{t+j}}{E_t \sum_{j=0}^{\infty} \omega^j \beta^j (1-\delta)^j C_{t+j}^{1-\sigma} \left( \frac{P_{t+j}}{P_t} \right)^{\theta-1} \rho_{t+j}}. \quad (10)$$

Price index is $P_t=(\int(P_i(i))^{1-\theta} di)^{1/(1-\theta)}$. In period $t$ a fraction $\omega$ of firms cannot change their price and thus their price is the same as last period. Share $1-\omega$ of firms set a new optimal price $P_{it}^*$. For simplicity we assume that the prices of firms arriving to the market have the same distribution as the prices of the older firms. Thus we can write price index as

$$P_{it}^{1-\theta} = ((1-\omega)P_{it}^{1-\theta} N_{D,t} + \omega \frac{P_{t-1}^{1-\theta}}{N_{D,t-1}} N_{D,t}).$$
We denote \( Q_t = \frac{P_t^e}{P_t} \) and rearrange terms to get \( (1 - \omega)Q_t^{1-\theta} = \frac{1}{N_{D,t}} - \omega \left( \frac{P_{t-1}}{P_t} \right)^{1-\theta} \frac{1}{N_{D,t-1}} \).

Log-linearizing this equation yields
\[
\bar{Q}_t = \frac{\omega}{1 - \omega}(\hat{P}_t - \hat{P}_{t-1}) - \frac{1}{(1 - \omega)(1 - \theta)} \frac{1}{N_D} \hat{N}_{D,t} + \frac{\omega}{(1 - \omega)(1 - \theta)} \frac{1}{N_D} \hat{N}_{D,t-1},
\]
where a hat over variables denotes percentage deviation from the steady-state.

By log-linearizing equation (10) we get
\[
\bar{Q}_t = (1 - \omega)\beta(1 - \delta)\hat{P}_t + \omega \beta(1 - \delta)(\bar{Q}E_t\hat{q}_{t+1} + \pi_t\hat{\pi}_{t+1}).
\]

Here \( \pi_t \) denotes inflation. Plugging (11) to (12) gives the form of a new Keynesian Phillips curve with firm entry:
\[
\hat{\pi}_t = \frac{(1 - \omega)(1 - \omega)\beta(1 - \delta)}{\omega} \hat{P}_t + \beta(1 - \delta)E_t\hat{\pi}_{t+1} - \frac{\beta(1 - \delta)}{(1 - \theta)N_D} E_t\hat{N}_{D,t+1}
\]
\[
+ \frac{1 + \omega^2 \beta(1 - \delta)}{\omega(1 - \theta)N_D} \hat{N}_{D,t} - \frac{1}{(1 - \theta)N_D} \hat{N}_{D,t-1}.
\]

From equation (13) we see that the main differences between standard new Keynesian and our Phillips curve are that equation (13) takes into account exogenous probability of exit and the effect of changes in the number of firms on the inflation. Interestingly we see that when the probability of exit, \( \delta \), increases, price stickiness decreases with given \( \omega \). In the extreme case where firms produce only in one period (i.e. \( \delta = 1 \)), expected future inflation does not affect price-setting at all. Similar result is in view in Bilbiie et al. (2007b) for Phillips curve derived from the quadratic cost of adjusting prices.

As in Bilbiie et al. (2007b) there is also predetermined state variable, the number of firms, in the Phillips curve. The number of firms exists there because consumer price index is affected not only by individual prices but also by the number of different intermediate-goods varieties. This existence generates endogenous persistence to the price inflation, because even the numbers of firms in period \( t-1 \), which is determined at time \( t-2 \), affects inflation.

Bilbiie et al. (2007b) use the quadratic cost of adjusting prices to get following Phillips curve:
\[ \hat{\xi}_t = \frac{\theta - 1}{\kappa} \hat{\omega}_t + \beta(1 - \delta)E_i \hat{\xi}_{t+1} - \frac{1}{\kappa} \hat{N}_{D,t} - \frac{1}{\theta - 1} \left[ \hat{N}_{D,t} - \hat{N}_{D,t-1} - \beta(1 - \delta)(E_i \hat{N}_{D,t+1} - \hat{N}_{D,t}) \right], \]

where \( \kappa \) is adjustment cost parameter. It is generally known that without firm entry and exit it is possible to set \( \kappa \) such that these two price rigidity models lead to the same Phillips curve. When comparing Philips curve from Bilbiie et al. to the equation (13), we see that even if these two Philips curve are similar, they are not exactly same. If we set

\[ \kappa = \frac{(\theta - 1)\omega}{(1 - \omega)(1 - \omega \beta(1 - \delta))}, \]

the only difference between two Phillips curves is that equation (13) includes steady-state value for the number of firms, \( \bar{N}_D \).

Because all the firms are similar in their technology and production function is homogenous of degree 1 we can write

\[ Y_t = \frac{1}{2} [Y_t(i)di = A_t [L_t(i)di = A_tL_t]. \quad (14) \]

Using this fact average profits can be written as

\[ d_t^A = \left[ p_t^A(i) - w_i / A_t \right] Y_t / N_{Dt}. \quad (15) \]

### 2.4. Entry of firms

To enter the market firms must hire labor to produce requirements for entry \( f_E \), which is measured in units of effective labor. Thus the cost of entry for a firm is \( w_i f_E / A_t \). Entry cost covers the development and setup costs for founding the firm. Because of free entry firms will enter the market until entry costs are equal to the market value of a new firm

\[ N_t^N = w_i f_E / A_t. \quad (16) \]

Total labor supply \( L_t^A \) is divided into the two sectors: production sector and entry sector

\[ L_t^A = L_t + \frac{f_E N_{E,t}}{A_t}, \quad (17) \]

where \( L_t \) is the share of labor working in the production and \( f_E N_{E,t} / A_t \) is the share of labor working in the entry sector. Labor force can move from one sector to another sector without cost. This implies that wages must be the same in both sectors.
2.5. Summary of the model

The equilibrium equations constitute a system of twelve equations in twelve endogenous variables. Of these endogenous variables the number of firms and the number of new firms are state variables. The model equations are agent’s first order conditions (3)–(6), resource constraints (7), (14) and (17) and equation for the accumulation of firm stock (2). In addition we have free entry condition (16), equations for profits (15) and consumer price index (9).

The model is closed by adding a monetary policy rule for nominal interest rate setting. Monetary policy is assumed to react to the fluctuations in inflation and aggregate final-goods production, \( Y_t^F \), according to a Taylor rule with interest rate smoothing

\[
\hat{r}_t = (1 - g_R) \left[ g_\pi \hat{\pi}_t + g_{\pi F} \hat{Y}_t^F \right] + g_{\hat{r}F} \hat{r}_{t-1} + \epsilon_{Rt},
\]

where \( \epsilon_{Rt} = \rho_{R} \epsilon_{Rt-1} + e_{Rt}, \quad e_{Rt} \sim N(0, \sigma_{\epsilon_R}^2) \) is a monetary policy shock. Monetary policy shock captures the non-systematic component of monetary policy. Naturally aggregate final-goods production equals to consumption, i.e. \( \hat{Y}_t^F = \hat{C}_t \).

From these twelve conditions we can solve the steady state by setting \( \bar{P} = 1 \) and \( \bar{A} = 1 \), where bar over variable means steady state value. In appendix 1 we describe how to solve the steady-state number of firms. We can write the model in real terms and rewrite equilibrium conditions as described in appendix 2.

The closest paper to our study is Bilbiie et al. (2007b). In our model firms set prices according to Calvo pricing. We assume also that there is a stochastic element affecting the number of firms. This contingency can suddenly cause firms to go to bankruptcy. These shocks can occur e.g. from the changes in the circumstances in financial markets and prevent firms getting external finance. This kind of variability in the number of firms has not been studied before in the business cycle or new Keynesian environments. We study also cost-push and demand shocks, which are standard shocks in new Keynesian models but as far as we know are not studied before in the framework with endogenous firm entry.

2.6. Benchmark model

To make model more understandable we compare the impulse responses of the model with the entry and exogenous exit to the responses of the benchmark model which does not have these
characteristics. In this benchmark model firm exit rate $\delta = 0$ and the number of firms is fixed to the steady-state level, but parameters are exactly the same than in our main model.

Benchmark model is very near to the basic new Keynesian model. There are traditional Phillips curve with future inflation expectations and marginal cost. Benchmark model differs from traditional model in the sense that the number of firms is different from one and agents can buy the stocks of old firms from the stock market. The value of old firms is determined by Euler equation

$$\frac{v_t}{P_tC_t} = \beta(v_{t+1} + d_{t+1})$$.

3. Simulation

To be able to solve the model we have to log-linearize it. Log-linearized equations, which are used in the numerical solution, are listed in appendix 3. We solve the model by the solution method described in Söderlind (1999). This method requires that we have to divide variables to the predetermined variables and forward-looking variables. In this model predetermined variables, denoted by $z_{1,t}$, are $e_t, e_R, e_P, e_{C,i}, \hat{r}_{i-1}, \hat{N}_{E,i-1}, \hat{v}_{i-1}, \hat{v}_{i-1}, \hat{d}_{i-1}, \hat{L}_{i-1}, L^A_{i-1}, \hat{w}_{i-1}, \hat{N}_{D,i}$ and forward-looking variables, denoted by $z_{2,t}$ are $\hat{r}_t, \hat{C}_i, \hat{\epsilon}_t$. The number of predetermined variables is $n_1$ and the number of nonpredetermined variables is $n_2$. Now we can write our system as

$$\begin{bmatrix} z_{1,t+1} \\ E_t z_{2,t+1} \end{bmatrix} = G_{n \times n} \times z_{t} + H_{n \times 1} u_t + err_{t+1},$$

where $n = n_1 + n_2$. Here $u_t$ is a policy instrument $\hat{r}_t$. The model can be solved by using the method of generalized Schur decomposition (see also Klein 1997) as described by Söderlind (1999). Note that all the variables except inflation and interest rate are expressed in the real term.

3.1. Parameterization

The period in our model is a quarter as in most of business cycle and monetary policy literature. We set discount factor $\beta$ to be 0.99, which is a standard choice for quarterly business cycle models. We follow Bilbiie, Ghironi and Melitz (2007b) in setting $\delta = 0.025$ and $\theta = 3.8$, which means that the steady-state mark-up is about 1.36. The chosen value for $\delta$ matches the level of 10 percent annual job destruction observed in U.S.. Sometimes as in Galí (2002) steady-state mark-up is set lower than here. However, it is good to notice that in our model firms must also pay entry costs and thus
firms set prices at average cost (See discussion in Bilbiie et al. 2007b, 24). For these reasons and making comparison to Bilbiie et al. easier we choose $\theta$ to be 3.8.

During the first period new firms do not produce anything but they set up production line and develop product and market it. We assume that there is big probability to fail doing these operations and new firms can tumble into situation where their production line is inefficient or their product is not able to compete with other products. In this kind of situations firms must exit the market before they have produced anything. We suppose that this default rate is twice as high as default rate for old firms and thus we set $\delta^N = 0.05$. The chosen value for $\delta^N$ does not affect noticeable impulse responses. Entry cost $f_E$ is set to 1 and $\eta = 1.5$, which implies the steady-state labor supply to be 0.4554. We assume $\omega = \frac{2}{3}$, which implies an average price duration of three quarters. Clarida, Galí and Gertler (2000) estimate Taylor rule with interest rate smoothing. Their estimations for parameter $g_R$ vary between 0.63 and 0.88, for parameter $g_C$ between 0.27 and 1.5 and for $g_\pi$ between 0.68 and 2.15. Based on these results we set $g_R = 0.8$, $g_{yF} = 0.8$ and $g_\pi = 1.5$.

3.2. Monetary policy shock

In this section we study the effects of expansionary monetary policy on macroeconomic variables. Also Bilbiie et al. (2007b), Elkhoury et al. (2007) and Lewis (2006) study the same shock in the framework with firm entry. We consider this shock, because it is informative to see how policy changes affect economy, and also because we want to study how introduction of firm entry changes responses to monetary policy shock compared to the model without entry. Expansionary monetary policy is modelled as a drop in interest rate, which is caused by a negative one percent monetary policy shock, $\epsilon_{R,t}$, to the Taylor rule. We plot the impulse responses for a nonpersistent (i.e. $\rho_R = 0$) shock. Thus all the persistence into the interest rate comes from the interest rate smoothing in the policy rule. Figure 1 shows the impulse responses of variables to the 1 percentage negative shock to the interest rate with parameters given above for the model with firm entry and the benchmark model. All the variables except inflation and interest rate in the following figures are expressed in real terms.
Impulse responses to a monetary policy shock are pretty similar to the impulse responses obtained by Bilbiie et al. (2007b). The magnitude of responses is the only main difference between the two models. The shock lowers interest rate in the period 1. As interest rate decreases, today’s consumption rises, which in turn reduces labor supply, because demand for leisure has increased. The entire drop in the labor occurs because of decreased investments to new firms. Decrease in interest rate requires the expected return on equity to fall because of no-arbitrage condition between investment in bonds and equity. This causes today’s price of equity to increase. Note that the price of equity is directly connected with the wage level by free entry condition and wages react procyclically to the expansionary policy shock. Increased cost of firm entry dampens firm entry. The number of firms stays long under its steady-state value. This causes long-term inflation, because the number of intermediate-goods varieties affects directly the effectiveness of final-goods production and consumer prices as discussed earlier. Profits decrease initially because real wages have risen. Note that firms are more precious than earlier even if future profits are lower because monetary policy changes stochastic discount factor via Euler equations.

One of the most important things here is that in addition to increasing consumption expansionary monetary policy affects asset prices. In the case of price rigidity and entry costs measured in the
terms of labor drop in interest rate has a positive effect on the real value of firms (i.e. asset prices). Increase in the value of firms in turn depresses investments in the new firms. Thus expansionary monetary policy decreases the number of operating firms. Similar phenomenon is present also in Bilbiie et al. (2007b). The drop in the number of varieties reduces the efficiency of final-goods production because of a constant elasticity of substitution production technology.

Comparing our model with the benchmark model, we see that including firm entry into the model changes impulses responses for most of the macroeconomic variables in a considerable way. For the benchmark model only the variables in the two lowest rows in figure 1 exist. When comparing the responses of these variables, we can see that one big difference between models is that in the benchmark model labor supply jumps up and stays higher for a long time. This occurs because in this case there is no drop in labor demand caused by decrease in investments to the new firms. Higher wages in benchmark model cause bigger drop in profits. Now equity prices are not anymore connected to the wage level but monetary policy still affects them via no-arbitrage condition. In the benchmark model future profits decrease more and the value of firms drops instead of decrease in investments. Note that inflation reacts initially more in the benchmark model, but later inflation is higher in the model with firm entry because the number of firms stays below the steady-state.

3.3. Cost-push shock
Next we consider the effects of a sudden increase in the prices of intermediate-goods, which occurs as one percent nonpersistent increase in $p_t$. This shock is a disturbance to the new Keynesian Phillips curve. Cost-push shock is widely used in the new Keynesian literature and is called an inflation shock, a cost shock or a price shock (see Walsh 2003, 517). However, according to our knowledge this type of shock has not been studied before in the environment, where firm entry is endogenous. Impulse responses to the cost-push shock occurring in period 1 are plotted in figure 2.
Figure 2. Impulse responses to the cost-push shock. Solid lines represent the responses in the model with firm entry and dashed lines the responses in the benchmark model.

A temporary rise in today’s prices accelerates inflation initially. Because today’s prices are high compared to tomorrow’s prices agents reduce today’s consumption and save via founding new firms. Next period the number of firms increases and inflation becomes negative. This causes the gradual recover of consumption. Depressed consumption boosts labor supply. Because labor demand in production has decreased, firms can lower real wages. Initially inflation is high and consumption decreases only little. Thus monetary authority raises interest rate. Due to persistence in Taylor rule, monetary policy stays restrictive in the following periods, even if consumption is under its steady-state value and there is deflation.

To summarize, a price increase depresses consumption as in the DSGE-models without firm entry. In addition to this risen prices decrease real wages and lower the real costs of entry. Thus positive price shocks induce firm entry. This increases labor demand in the entry sector and causes higher wages in the model with firm entry than in the benchmark model. Thus supply is remarkable lower in the benchmark model. Firm’s real marginal costs are lower in the benchmark model and due to this profits are higher. The most significant difference between two models is in the reaction of value of firms. This difference occurs even if stochastic discount factor behaves in a similar way. In
the model taking into account firm entry the value of firms first decreases and then rises gradually back. This occurs because value of equity is directly linked to the real wages. This does not happen in the benchmark model, where value of firms initially jumps up as there are big future profits. As profits decrease in the future, the value of firms decreases in later periods and generates loss to the owners via decrease in the value.

3.4. Demand shock
Next we consider the effects of a temporary one percent positive demand shock. Demand shock means that suddenly agents want to consume more today than Euler equation for bonds would imply. This disturbance represents variation in spending that is not associated with changes in the real interest rate. This type of shock might be for example a disturbance to the willingness to hold bonds. Earlier Amato and Laubach (2004) have used this kind of shock in their study about the effects of habit formation on optimal monetary policy. As far as we know the effects of demand shock on firm entry have not been studied earlier. The impulse responses to the demand shock are plotted in figure 3.

![Figure 3. Impulse responses to the demand shock.](image)

---

**Figure 3. Impulse responses to the demand shock.** Solid lines represent the responses in the model with firm entry and dashed lines the responses in the benchmark model.
Shock in period 1 causes an immediate increase in demand. This jump in consumption is financed by increased labor supply and decrease in the investments to the new firms. In period 2 disturbance does not affect anymore and because there are too few firms, investments into the new firms increase. This causes next period positive investments and negative consumption. Decrease in the number of firms forces consumption to drop even more. Because of persistence in monetary policy, interest rate reaction stays positive also next period. In the long run we can see that demand shock causes the number of firms to stay under the steady-state value for a long time. Because consumption and inflation rise initially, interest rate also increases and stays above steady-state value due to persistence in monetary policy.

Comparison of the model with firm entry and benchmark model shows that biggest differences in the impulses responses are again in the responses of labor supply, profits, value of firms and wages. In the benchmark model labor is not used for the establishing new firms. Thus labor supply jumps first up and then in the second period down as consumption changes. These changes in the labor demand and supply cause stronger movement in the wages and profits. Stronger movements in profits cause just opposite responses in the value of firms to the response when firm entry is taken into account.

3.5. Bankruptcy shock

Earlier new Keynesian literature on the entry of firms and monetary policy has assumed that firm default or exit rate is constant over time and business cycle. Contrary to this we assume that firm default rate faces exogenous disturbances, which we call bankruptcy shocks. Bankruptcy shock means that in period one, before any decisions or production are made one percent more of current firms exit the market. This occurs e.g. for firms notice that their production is not anymore efficient or they cannot acquire finance for their operations. However, we do not try to model the exact reason of increased default, but study what implications risen default has on economy.

The implications of these shocks are rarely studied in the new Keynesian macroeconomic models. Indeed they deserve a more accurate explanation. The average business default frequency varies a lot over time (see e.g. Jacobson, Kindell, Lindé and Roszbach (2008)). Contrary to this finding models with firm entry assume that default rate is constant over time. We presume that the variability of default occurs because of exogenous bankruptcy shock. Jacobson et al. find that macroeconomic factors strongly affect business defaults. In our model there is no such endogenous exit channel which could explain the phenomenon studied by Jacobson et al. Opposite to their view
we study which are the effects of increased defaults on macroeconomic variables. In figure 4 are plotted impulse responses for one percent nonpersistent bankruptcy shock emerging in period 1.

Figure 4. Impulse responses to the bankruptcy shock.

Bankruptcy shock causes immediate one percent drop in the number of firms producing in period 1. According to the constant elasticity of substitution production technology, decrease in the number of intermediate-goods varieties affects directly effectiveness to produce final-goods and thus increase consumer price inflation. As a result monetary authority reacting according to Taylor rule raises interest rate. To compensate for decreased efficiency in the production labor demand for production increases, even if final-goods production has not changed a lot. This causes wage level to go up, which in turn has negative effect on firm profits and positive effect on the value of firms. Responses in profits and equity value make investments to new firms less profitable. Thus initial consumption increases and investment to new firms drop. This slows down recovery from the bankruptcy shock and makes recovery from the shock very slow.

Due to interest rate persistence in the monetary policy rule interest rate peaks just after 4 periods. Note that monetary policy becomes restrictive even though there is a negative shock to the
economy. This stems from the effect which the number of varieties has on inflation. This restrictive reaction gives a reason to study whether policy rule should somehow take into account the variability in the number of firms.

4. Optimal monetary policy
In chapter 3 we found that monetary policy reacts restrictively to the negative exit shock, which we called bankruptcy shock. This gives a reason to think that better and perhaps more realistic monetary policy rule would somehow take into account the role which the number of firms and intermediate-goods varieties have in the economy. In this chapter we study such policy rules. We compare the outcomes of different rules and evaluate which rule would generate best outcome. First we compare traditional rule considered in the earlier chapters and the Taylor rule which reacts to the price inflation of individual intermediate-goods. Later we call this kind of inflation intermediate-goods inflation. Next we study which are optimal Taylor rule parameters. Then we study what would be consequences for the variances of output and inflation, if central bank added the number of firms into its policy rule or followed intermediate-goods price index instead of consumer price index.

4.1. To which price index to react?
In this section we examine whether monetary authority should react to the consumer price inflation or intermediate-goods inflation, when following Taylor rule. Now monetary authority has two possible Taylor rules to follow:

\[
\hat{r}_t = (1 - g_R) \left[ g_\pi \hat{\pi}_t + g_{y,F} \hat{Y}_t^F \right] + g_R \hat{r}_{t-1} + \varepsilon_{R,t} \quad (R1)
\]

or

\[
\hat{r}_t = (1 - g_R) \left[ \hat{\pi}_t + g_\pi \hat{\pi}_t + \frac{1}{\theta-1} (\hat{N}_{Dt} - \hat{N}_{D,t-1}) \right] + g_{y,F} \hat{Y}_t^F \left[ + g_R \hat{r}_{t-1} + \varepsilon_{R,t} \right] . \quad (R2)
\]

Former rule reacts to CPI-inflation and latter rule considers intermediate-goods inflation. We evaluate the supremacy of rule by comparing the variances of final-goods production (or consumption) and consumer price inflation. The variances of these variables are commonly concerned as welfare measures in literature on optimal monetary policy. Also often (e.g. Taylor (1979)) used quadratic loss function for the monetary authority is based on comparison of these variances. Thus we report in addition to the variances the values of the quadratic loss function

\[
LOSS = \sum_{i=0}^{\infty} \beta^i \left( \sigma_{\pi,i+1}^2 + \lambda \sigma_{C,i+1}^2 \right)
\]
to make evaluation clearer. Here $\sigma^2_{\pi,t}$ and $\sigma^2_{C,t}$ denote the variances of inflation and consumption and $\lambda$ is the weight for consumption. In simulations we assume that monetary authority places more weight on inflation stability and set $\lambda = 0.5$. The best interest rate rule is concerned to be the one which minimizes the loss function. Rotemberg and Woodford (2001) derive quadratic loss function as a second-order Taylor approximation to a household’s welfare. Unfortunately, their method cannot be applied to the model with firm entry.

Next we assume different values for the variances of shocks $\sigma^2_{\varepsilon,\pi}$, $\sigma^2_{\varepsilon,R}$, $\sigma^2_{\varepsilon,C}$, $\sigma^2_{\varepsilon,P}$ and evaluate different policy rules by comparing the unconditional variances of consumption and inflation they give in the model with firm entry. Parameter values are the same as those described earlier. The variances of error terms and variables and the values of loss function are in table 1. All the error terms are independent and identically-distributed random variables, whose distribution follow $N(0, \sigma^2)$.

<table>
<thead>
<tr>
<th>$\sigma^2_{\varepsilon}$</th>
<th>$\sigma^2_{\varepsilon,R}$</th>
<th>$\sigma^2_{\varepsilon,C}$</th>
<th>$\sigma^2_{\varepsilon,P}$</th>
<th>$\sigma^2_{\pi,\text{int}}$</th>
<th>$\sigma^2_{C,\text{int}}$</th>
<th>LOSS$_\pi$</th>
<th>LOSS$_{\pi,\text{int}}$</th>
<th>% Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.072</td>
<td>0.071</td>
<td>0.081</td>
<td>11.018</td>
<td>11.045</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.086</td>
<td>0.083</td>
<td>0.087</td>
<td>12.271</td>
<td>12.165</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.206</td>
<td>0.186</td>
<td>0.138</td>
<td>23.552</td>
<td>22.250</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0.9</td>
<td>0.643</td>
<td>0.635</td>
<td>0.344</td>
<td>79.527</td>
<td>79.572</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>0.1</td>
<td>0.9</td>
<td>0.656</td>
<td>0.647</td>
<td>0.350</td>
<td>80.781</td>
<td>80.692</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.1</td>
<td>0.9</td>
<td>0.776</td>
<td>0.749</td>
<td>0.401</td>
<td>92.062</td>
<td>90.777</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td>0.1</td>
<td>0.080</td>
<td>0.079</td>
<td>0.468</td>
<td>30.651</td>
<td>30.874</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>0.9</td>
<td>0.1</td>
<td>0.093</td>
<td>0.090</td>
<td>0.473</td>
<td>31.904</td>
<td>31.994</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.9</td>
<td>0.1</td>
<td>0.213</td>
<td>0.193</td>
<td>0.525</td>
<td>43.185</td>
<td>42.079</td>
</tr>
</tbody>
</table>

Table 1. Variances and value of loss functions between two policy rules.

In table 1 $\sigma^2_{\pi}$ is the variance of inflation when central banks follows rule (R1) and $\sigma^2_{\pi,\text{int}}$ is the variance when policy reacts according to rule (R2). Similar notation stands for the variance of consumption $\sigma^2_{C}$. Similarly LOSS$_{\pi}$ is the value of loss function if central bank follows rule (R1) and LOSS$_{\pi,\text{int}}$ is the value of loss function from another rule. The last column shows percentage gain from following the rule (R1).

From table 1 we see that the differences in variances between two rules are not very large. However there are two clear findings. The variance of inflation is always lower when monetary policy reacts to the rule (R2) and on the other hand the variance of consumption stays lower with the rule (R1).
Thus central bank who is more concerned about inflation than aggregate production should react to intermediate-goods inflation. This result stems from the different reactions to the demand shock.

More interestingly we see from table 1 that existence and size of bankruptcy shocks as a cause for uncertainty has effect on the selecting the rule. For the chosen loss function we see that bigger role bankruptcy shocks have in the economy, more favourable trade-off between the variances of inflation and consumption the central bank following rule (R2) faces compared to the following rule (R1). Actually this result is independent from the chosen value of $\lambda$. The explanation for the result is following. When variability in bankruptcy cost is high CPI-inflation jumps up with negative shock. This negative shock has a negative effect on consumption as seen earlier. Then increase in interest rate because of increased inflation decreases consumption further and causes a welfare loss. If central bank followed intermediate price index, central bank would not react to increased inflation caused by the bankruptcies and consumption would not drop further.

In table 2 we repeat simulations for more persistent shocks. We assume that cost-push shocks and demand shocks follow the processes $\varepsilon_{D,t} = \rho_D^* \varepsilon_{D,t-1} + \varepsilon_{D,t}$ and $\varepsilon_{P,t} = \rho_P^* \varepsilon_{P,t-1} + \varepsilon_{P,t}$. We use the values $\rho_P = \rho_D = 0.8$ for parameters. The results in table 2 show that persistence in the shocks does not change the main implications found earlier.

<table>
<thead>
<tr>
<th>$\sigma_e^2$</th>
<th>$\sigma_R^2$</th>
<th>$\sigma_C^2$</th>
<th>$\sigma_P^2$</th>
<th>$\sigma_{\pi,G}^2$</th>
<th>$\sigma_{\pi,G}^2$</th>
<th>$\sigma_{C,G}^2$</th>
<th>$\sigma_{C,G}^2$</th>
<th>LOSS$_\pi$</th>
<th>LOSS$_{\pi,G}$</th>
<th>% Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.01</td>
<td>0.1</td>
<td>1.832</td>
<td>1.718</td>
<td>1.458</td>
<td>1.582</td>
<td>240.490</td>
<td>235.915</td>
<td>-1.939</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.1</td>
<td>1.845</td>
<td>1.729</td>
<td>1.464</td>
<td>1.587</td>
<td>241.744</td>
<td>237.035</td>
<td>-1.986</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>0.1</td>
<td>1.965</td>
<td>1.832</td>
<td>1.515</td>
<td>1.639</td>
<td>253.025</td>
<td>247.120</td>
<td>-2.390</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.1</td>
<td>0.09</td>
<td>16.005</td>
<td>14.959</td>
<td>10.352</td>
<td>11.310</td>
<td>1983.069</td>
<td>1932.193</td>
<td>-2.633</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.09</td>
<td>16.018</td>
<td>14.970</td>
<td>10.357</td>
<td>11.316</td>
<td>1984.323</td>
<td>1933.314</td>
<td>-2.638</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.09</td>
<td>2.313</td>
<td>2.220</td>
<td>4.232</td>
<td>4.505</td>
<td>421.835</td>
<td>426.956</td>
<td>1.200</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.9</td>
<td>0.1</td>
<td>2.326</td>
<td>2.231</td>
<td>4.237</td>
<td>4.511</td>
<td>423.088</td>
<td>428.077</td>
<td>1.165</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
<td>2.446</td>
<td>2.334</td>
<td>4.288</td>
<td>4.563</td>
<td>434.369</td>
<td>438.161</td>
<td>0.865</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Variances and value of loss functions between two policy rules with persistent shocks.

4.2. The effects of bankruptcy shocks on optimal policy rule

In this section we compare optimal monetary policy to the three different optimal policy rules: traditional rule (R1), one reacting to intermediate prices (R2) and one which reacts directly to the number of firms. As in the earlier chapter we evaluate different policies by comparing the variances of consumption and inflation and the value of loss function. Rule is chosen to be optimal in the
sense that it minimizes central bank’s loss function \( \text{LOSS} = \sum_{i=0}^{\infty} \beta^i \left( \sigma_{\pi,i}^2 + \lambda \sigma_{C,i}^2 \right) \). The optimal monetary policy is referred to be the commitment solution. Commitment case means that central bank is able to precommit to a path for current and future inflation and output gap (see e.g. Walsh 2003, 524). It commits to a path which minimizes the loss function subject to the new Keynesian Phillips curve and other equations in the model which are defined in section 2.5.

In table 3 we present the values of loss functions for optimal (R1)-rule and optimal policy when using parameters \( \rho_p = \rho_D = 0, \rho_R = 0, \sigma_{\pi}^2 = 0.1, \sigma_{\pi,R}^2 = 0, \sigma_{\pi,C}^2 = 0.1 \) and \( \sigma_{\pi}^2 = 0.1 \). In table 4 we report same values, when there are no bankruptcy shocks, i.e. \( \sigma_{\pi}^2 = 0.0 \). Results in table 4 can be thought as the results from basic Bilbiie et al (2007b) model, where economy’s exit rate is constant. In tables we report in addition to the rules where parameters are chosen freely, the optimal rules in the case where policy rule does not include smoothing (this type of rule is used e.g. by Taylor (1999)) and where smoothing parameter gets the value often used in literature \( g_R = 0.8 \). Rule coefficients with an asterisk (*) are imposed values. Also \( g_\pi \) is restricted to be bigger than 1 thus that Taylor principle holds and persistence parameter is restricted to be \( 0 \leq g_R < 1 \).

<table>
<thead>
<tr>
<th></th>
<th>( g_\pi )</th>
<th>( g_{Y,F} )</th>
<th>( g_R )</th>
<th>( \sigma_{\pi}^2 )</th>
<th>( \sigma_{C}^2 )</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor</td>
<td>505.500</td>
<td>242.724</td>
<td>0.999</td>
<td>0.077</td>
<td>0.096</td>
<td>10.390</td>
</tr>
<tr>
<td></td>
<td>7.118</td>
<td>4.829</td>
<td>0</td>
<td>0.108</td>
<td>0.168</td>
<td>17.726</td>
</tr>
<tr>
<td></td>
<td>4.720</td>
<td>1.970</td>
<td>0.8</td>
<td>0.084</td>
<td>0.172</td>
<td>15.943</td>
</tr>
</tbody>
</table>

Table 3. Optimal policy rule (R1) with bankruptcy shocks.

From table 3 we see that the variances of inflation and consumption are much higher from following traditional Taylor rule than from optimal policy. Also the value of loss function is almost 50 percent higher, which is very high and much higher than noticed in the traditional new Keynesian models. Optimal persistence in Taylor rule is extremely large. Also Amato and Laubach (2003) find high persistence for optimal policy rule. Anyway if we restrict persistence to be empirically relevant but still high 0.8 we see that the variances of inflation and consumption do not change much from the unrestricted rule. Thus it seems that it is optimal to include interest rate smoothing to the Taylor rule.
To study which effects the bankruptcy shock has on the high variances of inflation and consumption and the value of loss function noticed in table 3 we present results without bankruptcy shocks in table 4. From table 4 we observe that the value of loss function in optimal policy rule is much nearer to the optimal policy case than in the case where bankruptcy shocks exist. Percentage loss from the policy rule is now only about 25 percent, variance of inflation is only a little bit higher and variance of consumption is even lower than in optimal policy case. Thus most of the high variances in Taylor rule (R1) are caused by bankruptcy shocks. Again it is optimal to have high persistence in optimal rule but variances are not much higher when we restrict interest rate smoothing parameter.

High variances in the rule (R1) and bankruptcy shock case occur due to restrictive monetary policy response to the increased business defaults discussed in chapter 3.5. Following policy rule which reacts to the intermediate-goods price index instead of consumer price index might solve this puzzle. In tables 5 and 6 we repeat results in tables 3 and 4 with the policy rule

\[ \hat{r}_t = (1 - g_R) \left[ g_\pi \left( \hat{\kappa}_t + \frac{1}{\theta - 1} (\hat{N}_{D,t} - \hat{N}_{D,t-1}) \right) + g_{\gamma F} \hat{\gamma}_t \right] + g_R \hat{r}_{t-1} + \varepsilon_{R,t}. \]

where parameters \( g_R, g_{\gamma F} \) and \( g_\pi \) get optimal value in the sense that it minimizes loss function.

<table>
<thead>
<tr>
<th>( g_\pi )</th>
<th>( g_{\gamma F} )</th>
<th>( g_R )</th>
<th>( \sigma^2_\pi )</th>
<th>( \sigma^2_C )</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>0.077</td>
<td>0.096</td>
<td>10.390</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor</td>
<td>2086.263</td>
<td>3680.691</td>
<td>0.970</td>
<td>0.072</td>
<td>0.111</td>
</tr>
<tr>
<td>7.235</td>
<td>4.972</td>
<td>0*</td>
<td>0.103</td>
<td>0.167</td>
<td>17.297</td>
</tr>
<tr>
<td>7.955</td>
<td>5.151</td>
<td>0.8*</td>
<td>0.080</td>
<td>0.159</td>
<td>14.948</td>
</tr>
</tbody>
</table>

**Table 5. Optimal policy rule (R2) with bankruptcy shocks.**

<table>
<thead>
<tr>
<th>( g_\pi )</th>
<th>( g_{\gamma F} )</th>
<th>( g_R )</th>
<th>( \sigma^2_\pi )</th>
<th>( \sigma^2_C )</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>0.073</td>
<td>0.020</td>
<td>7.243</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor</td>
<td>647.160</td>
<td>1608.499</td>
<td>0.990</td>
<td>0.086</td>
<td>0.013</td>
</tr>
<tr>
<td>39.793</td>
<td>101.127</td>
<td>0*</td>
<td>0.086</td>
<td>0.013</td>
<td>9.156</td>
</tr>
<tr>
<td>15653.249</td>
<td>50350.026</td>
<td>0.8*</td>
<td>0.089</td>
<td>0.009</td>
<td>9.179</td>
</tr>
</tbody>
</table>

**Table 6. Optimal policy rule (R2) without bankruptcy shocks.**
By comparing tables 3 and 5 we see that the variances of both inflation and consumption are lower when monetary authority follows rule (R2). Naturally, loss function with any reasonable $\lambda$ gets also lower values in table 5. The percentage loss from following optimal policy rule is now even under 20 percent as it was with rule (R1) almost 50 percent. Thus we see much better trade-off between variances from the rule (R2), when bankruptcy shocks exist. The results without bankruptcy shock in table 6 do not differ remarkable from results with rule (R1). We can say that the policy rule reacting to the intermediate prices gives much more stable inflation and output than the policy rule reacting to the consumer prices, when bankruptcy shocks exist.

Second possibility for monetary policy to take into account variability in business defaults is to react directly to the number of firms. The number of firms is also an observable variable. Thus it would be possible for central bank to react to the number of firms in addition to inflation and output. In tables 7 and 8 are reported simulations equivalent to the earlier tables for the policy rule

$$\hat{r}_t = (1-g_R)(g_\pi \hat{\pi}_t + g_{F} \hat{Y}_t + g_N \hat{N}_D) + g_R \hat{r}_{t-1}. \tag{R3}$$

<table>
<thead>
<tr>
<th>$g_\pi$</th>
<th>$g_{F}$</th>
<th>$g_R$</th>
<th>$g_N$</th>
<th>$\sigma^2_\pi$</th>
<th>$\sigma^2_C$</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>15.191</td>
<td>35.794</td>
<td>0.724</td>
<td>-7.661</td>
<td>0.090</td>
<td>0.093</td>
</tr>
<tr>
<td>Taylor</td>
<td>7.895</td>
<td>20.254</td>
<td>0*</td>
<td>-4.359</td>
<td>0.091</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>25.507</td>
<td>61.253</td>
<td>0.8*</td>
<td>-13.099</td>
<td>0.090</td>
<td>0.092</td>
</tr>
</tbody>
</table>

Table 7. Optimal monetary policy (R3) with bankruptcy shock.

<table>
<thead>
<tr>
<th>$g_\pi$</th>
<th>$g_{F}$</th>
<th>$g_R$</th>
<th>$g_N$</th>
<th>$\sigma^2_\pi$</th>
<th>$\sigma^2_C$</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>29.167</td>
<td>68.394</td>
<td>0.817</td>
<td>2.938</td>
<td>0.086</td>
<td>0.014</td>
</tr>
<tr>
<td>Taylor</td>
<td>26.375</td>
<td>67.817</td>
<td>0*</td>
<td>2.568</td>
<td>0.087</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>31.618</td>
<td>75.155</td>
<td>0.8*</td>
<td>3.198</td>
<td>0.086</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Table 8. Optimal monetary policy (R3) without bankruptcy shock.

From table 7 we see that policy rule (R3) reacting directly to the number of firms leads higher variability in inflation but lower variability in consumption than the policy rule in table 3, which ignores the number of firms. When $\lambda = 0.5$ loss function gets smaller values than with the policy rule (R1) but a little bit higher values than with the policy rule (R2). According to the results in table 8 variances do not differ remarkable from rules (R1) or (R2) reported in tables 4 and 6, if bankruptcy shocks do not exist.
Based on results reported in tables 3–8 we can conclude that variances of inflation and consumption significantly increase if monetary authority ignores the number of firms in its policy response and reacts to the consumer price inflation. This result occurs only if there are bankruptcy shocks in the economy.

5. Conclusions
This paper studies how responses to the macroeconomic shocks change when firm entry is endogenous. We also examine how monetary policy should take into account the variability in default rates. Our framework is based on Bilbiie et al. (2007b), where firm entry is endogenous and occurs as a result of agent’s investment decision. Thus the numbers of firms and intermediate-goods varieties respond to the shocks. This has a direct effect on the productivity of perfectly competitive final-goods sector and consumer prices. When monetary policy reacts to inflation, its policy affects directly the entry via changing asset prices. This occurs because bonds and equity are substitute instruments for savings. This channel is not present in traditional monetary policy literature, which ignores endogenous firm entry.

In the first part of paper we study how impulse responses to the monetary policy, cost-push and demand shocks change when firm entry is introduced to the model. Cost-push and demand shocks are widely studied in the new Keynesian literature, but they are not studied before in the framework where firm entry is endogenous. We find that the inclusion of firm entry in the model affects most strongly impulse responses of labor supply, profits, value of firms and wages. For these variables changes in responses differ in a remarkable way from the responses of model ignoring firm entry.

Differently to the earlier literature we assume that firm default rate is not constant over time. This is achieved by assuming exogenous bankruptcy shocks, which can occur e.g. from the shocks to financial markets. These shocks change the amount of defaulting firms. We show that increased default rates have a long-run depressing effect on economy and positive effect on inflation. Increased inflation and decreased economic activity raises the question how monetary policy should react under this kind of circumstances.

To evaluate different monetary policies under bankruptcy shocks, we simulate the model for different error processes and compare the variances of inflation and output different policy rules generate. We find out that the rule, which reacts to consumer price index, generates high variances.
Much more stable and better outcome would be achieved if monetary policy reacted to the prices of intermediate-goods instead of consumer price index. Also policy rule which includes the number of firms generates better trade-off between two variances than the rule which reacts only to consumer prices. If we assume that firm default rate is constant and there are no bankruptcy shocks in economy different policy rules give very similar results.

In the future the welfare effects of different policy rules should be studied with more accurate welfare criteria. It would also be interesting to study how the functioning of banking sector and especially shocks emerging there would affect firm entry. Studying impulse responses to the macroeconomic shocks with the model including endogenous firm exit would perhaps give new insights to the interaction between macroeconomic circumstances and the number of firms.

References


Appendix 1. Solving the steady-state number of firms

Assume \( P = 1 \) and \( A = 1 \). We have budget constraint \( \bar{C} = \bar{L}^D \bar{w} + \bar{d}N_D - \bar{v}^N \bar{N}_E \). Use equation \((1 + \tau^L) \bar{L}^D \bar{w} = (1 + \tau^L) \bar{w} - \eta \bar{C} \) and we have \( \bar{C} = \bar{w} - \frac{\eta \bar{C}}{(1 + \tau^L)} + \bar{d}N_D - \bar{v}^N \bar{N}_E \). In the steady-state profits are \( \frac{1}{\theta} \frac{\bar{C}}{\bar{N}_D} \). Thus plug \( \bar{C} = \theta \bar{d}N_D \) into this equation and we get \( \theta \bar{d}N_D = \bar{w} - \eta \theta \bar{d}N_D \frac{(1 + \tau^L)}{(1 + \tau^L)} + \bar{d}N_D - \bar{v}^N \bar{N}_E \).

Using \( \bar{N}_E = \frac{\delta}{1 - \delta^N} \bar{N}_D \) and \( \bar{v}^N = \bar{w} \bar{f}_E \) we get \( \theta \bar{d}N_D = \bar{w} - \eta \theta \bar{d}N_D \frac{(1 + \tau^L)}{(1 + \tau^L)} + \bar{d}N_D - \frac{\delta}{1 - \delta^N} \bar{w} \bar{f}_E \bar{N}_D \). We have \( \bar{d} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{w} \) and \( \bar{v} = \frac{1 - \delta}{1 - \delta^N} \bar{w} \bar{f}_E \) and plugging these to the equation above gives

\[
\theta(1 + \frac{\eta}{(1 + \tau^L)}) \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \frac{1 - \delta}{1 - \delta^N} \bar{w} \bar{f}_E \bar{N}_D = \bar{w} + \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \frac{1 - \delta}{1 - \delta^N} \bar{w} \bar{f}_E \bar{N}_D - \frac{\delta}{1 - \delta^N} \bar{w} \bar{f}_E \bar{N}_D.
\]

Dividing by \( \bar{w} \) and rearranging terms give \(-\theta(1 + \frac{\eta}{(1 + \tau^L)}) (1 - \beta(1 - \delta)) \bar{f}_E \bar{N}_D + (1 - \beta) \bar{f}_E \bar{N}_D + \beta(1 - \delta^N) = 0 \), where \( \bar{N}_D \) can be solved as the function of parameters.
Appendix 2. Model equations in real terms

Euler for bonds
\[
\frac{1}{P_t C_t} = \beta (1 + r_t) \frac{P_{t+1} C_{t+1}}{P_t C_t}
\]  
(A1)

Euler for old stocks
\[
\frac{v_t}{C_t} = \beta (1 - \delta) \frac{v_{t+1}}{C_{t+1}}
\]  
(A2)

No arbitrage
\[
v_t = \frac{(1 - \delta^N)}{(1 - \delta)} v_N^t
\]  
(A3)

Future income from firm
\[x_t = d_t + v_t
\]  
(A4)

Labour supply
\[
\frac{(1 + \tau^L)w_t}{C_t} = \frac{\eta}{1 - L_t^A}
\]  
(A5)

Number of firms
\[N_{D,t} = (1 - \delta) N_{D,t-1} + (1 - \delta^N) N_{E,t-1} \]  
(A6)

Aggregate constraint
\[C_t = L_t^A w_t + \pi_t N_{D,t} - v_t^N N_{E,t} \]  
(A7)

Labor constraint
\[L_t^A = L_t + \frac{f_E}{A_t} N_{E,t} \]  
(A8)

Free entry
\[v_t^N = \frac{w_t f_{E,t}}{A_t}
\]  
(A9)

Profits
\[d_t^A = \left[ P_t^A (i) - \frac{w_t}{A_t} \right] \frac{A_t L_t}{N_{D,t}}
\]  
(A10)

Real price
\[P_t(i) = N_{D,t}^{\delta - 1}
\]  
(A11)

In these equations we have used no-arbitrage condition (A3) instead of Euler equation for new firms. In equation (A2) we have used auxiliary variable \(x_t\) to denote income from the ownership of firm. Otherwise equations (A1)–(A11) are equations described in chapter 2.5 written in real terms.
Appendix 3. Log-linearized model

\[ \epsilon_{t+1} = \epsilon_t \]

\[ \epsilon_{Rt+1} = \epsilon_{Rt} \]

\[ \epsilon_{Pt+1} = \epsilon_{Pt} \]

\[ \epsilon_{Ct+1} = \epsilon_{Ct} \]

\[ \hat{p}_t = \hat{p}_t \]

\[ \hat{V}^N N^N_E \hat{N}_{E,t} = L^{A} \hat{w}_t \hat{w}_t + L^{A} \hat{L}_t + \hat{N}_D \hat{N}_P \hat{P}_t + \hat{N}_D \hat{N}_P \hat{P}_{\text{Prod}} - C \hat{C}_t - \hat{V}^N \hat{N}_E \hat{V}^N_t \]

\[ \hat{\psi}_t = \hat{w}_t \]

\[ \hat{\psi}_t = \hat{\psi}_t \]

\[ \ddot{d}_t = (\ddot{d} + \ddot{v}) \hat{\psi}_t - \ddot{w}_t \]

\[ \ddot{d}_t = \ddot{d}_t + \frac{\hat{w}_L}{\hat{N}_D} \hat{w}_t + (\ddot{d} - \frac{1}{\theta - 1} LN_\theta^D ) \hat{P}_{\text{Prod}} \]

\[ \hat{L}^A \hat{L}_t = \hat{L}_t + \hat{f}_E \hat{A}_E \hat{A}_{E,t} \]

\[ \hat{w}_t = \frac{\hat{L}^A}{1 - \hat{L}^A} \hat{L}_t + \hat{C}_t \]

\[ \hat{N}_{\text{Prod}}^t = \hat{N}_{D,t} + \epsilon_t \]

\[ \hat{N}_{D,t+1} = (1 - \delta) \hat{N}_{D,t} + \delta \hat{N}_{E,t} \]

\[ \hat{x}_i = \frac{(1 - \omega)}{\omega} \hat{w}_t \beta(1 - \delta) \hat{E}_t \hat{x}_{t+1} - \frac{\beta(1 - \delta)}{(1 - \theta) \hat{N}_D} \hat{E}_t \hat{N}_{D,t+1} \]

\[ + \frac{1 + \omega^2 \beta(1 - \delta)}{\omega(1 - \theta) \hat{N}_D} \hat{N}_{\text{Prod}}^t - \frac{1}{(1 - \theta) \hat{N}_D} \hat{N}_{\text{Prod}}^t + \epsilon_{P,t} \]

\[ \hat{E}_t \hat{C}_{t+1} = \hat{C}_t + \hat{C}_t - \hat{E}_t \pi_{t+1} - \epsilon_{D,t} \]

\[ \hat{E}_t \hat{X}_{t+1} = \hat{V}_t + \hat{E}_t \hat{C}_{t+1} - \hat{C}_t \]

\[ \hat{p}_t = (1 - \hat{g}_R)^t \left[ g_{\pi} \hat{w}_t + g_{vF} \hat{V}_t \right] + \hat{g}_R \hat{p}_{t-1} + \epsilon_{R,t} \]

where \( \epsilon_{P,t} = \rho_P \epsilon_{P,t-1} + \epsilon_P \) \( \epsilon_{P,t} \sim N(0, \sigma_{\epsilon_P}^2) \) and \( \epsilon_{C,t} = \rho_C \epsilon_{C,t-1} + \epsilon_C \) \( \epsilon_{C,t} \sim N(0, \sigma_{\epsilon_C}^2) \). Here \( \epsilon_{C,t} \) and \( \epsilon_{P,t} \) are cost-push and demand shock.