INVESTIGATING 10-13-YEAR-OLD FINNISH AND INDONESIAN STUDENTS' PROPORTIONAL REASONING IN A PAINT-BUCKET TASK

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In this report, the focus is on 10-13-year old Finnish and Indonesian students' proportional reasoning skills in a task, which requires an understanding of equality between two ratios. Ideally, every mathematic lesson, even at the primary school level, should develop students' skills in communicating their ideas by using 'mathematical language'. Explaining one's thinking is not easy: for example, 71% of Finnish students were not able to describe in the test item, how they ended up in their answer, whether it was correct or incorrect. On the other hand, 34% of Indonesian students left their explanations out. In addition to cognitive elements, social and cultural factors play an important role in student's performance and strategies.

Introduction to proportional reasoning study

My doctoral study will focus on 10-13-year-old students' proportional reasoning skills in Finland and Indonesia. The core question is to investigate if children from different cultural and educational backgrounds have differences in their strategic competence. Two mentioned countries have different performance profiles for example in the international PISA research, Indonesia traditionally being among the low, and Finland high performing countries.

In this presentation, I will describe some of the findings with one particular test item called paint-bucket task. 73 Finnish and 44 Indonesian fifth- and sixth-graders solved a problem, which required an understanding of equality between two ratios: "Orange paint is made by mixing four buckets of yellow paint and one bucket of red paint. To get exactly the same shade of orange, how many buckets of red would the painter need to mix to six buckets of yellow?". The correct answer was 1,5 buckets of red. The item was a part of a larger set of context-based, mathematical problem-solving tasks.

Proportional reasoning

Proportional reasoning skills and understanding of proportionality are considered to be cornerstones of cognitive development. Proportional reasoning abilities are a marker of a move towards more developed forms of reasoning, and the understanding develops slowly over the years. (Inhelder & Piaget, 1958; Lesh et al., 1988; Noelting, 1980a, 1980b). In developmental psychology, proportional reasoning has traditionally been seen as a global, general cognitive structure, but there are indications that the skills are more individually arranged (Lesh, Post & Behr, 1988). Children generate ideas and pre-proportional reasoning skills through everyday activities before they start formal education. Even small children understand that “daddy teddybear” needs bigger clothes than “baby teddybear”.

In textbooks and mathematics dictionary the word ‘proportion’ is defined as an equivalence of ratios or statement of equal ratios or fractions, written as follows:

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d\ .$$

‘Ratio’ is often used as an equivalent to proportional reasoning. (Carraher, 2001; Lobato et al., 2010.) Understanding proportionality requires reasoning with ratios. Even though the ratio is commonly described as a comparison of two quantities, it is important to pay attention to whether the comparison is multiplicative or additive (Lobato, Ellis & Charles, 2010, 21). Students often focus on one-dimensional comparison on “how much more” or “how much less”. Additive strategy is the most commonly reported erroneous strategy, whereas correct problem-solving processes include multiplicative ideas and relative thinking. To be able to operate with proportional reasoning problems, students should have an understanding of partitioning, unitising and changing quantities (Lamon, 1992; 1999; 2007, 636-637).

Reasoning skills develop slowly over the years

From 11-12 years onwards the child reaches the formal operational stage and is capable of thinking logically about abstract or hypothetical concepts without the aid of concrete representations (Inhelder & Piaget, 1958; Moseley, 2005). Ability to plan systematically and complete tasks requiring deductive reasoning are markers for reaching this stage. Children can consider possible outcomes and consequences of actions. Strategies develop from trial-and-error –attempts to planning and more organised approaches.

To be able to determine student’s skills and strategies, the performance should be observable. If students attend traditional answer-only tests, it can lead to overestimation of student’s ability. Sometimes correct answers can be generated by using other than proportional reasoning strategies (Tourniaire & Pulos, 1985, 183). Paper-and-pencil tasks are not always able to provide accurate information on how student ended up in the solution. It is easier to assess actual skills if students have a possibility to explain their thinking through the task.

Setting and background of the study

The participants for the study came from different cultural backgrounds and educational systems. There were 73 students from a local school in Northern Finland, and another group of participants was from Indonesia: 25 of them studied in a local school in central Jakarta, and 19 were from an international school, located in Tangerang. The mean age of Finnish students was 11 years and eight months. Indonesian students were approximately the same age than their Finnish peers: mean age in the group was 11 years and four months.

Three participating schools were described as modern and advanced compared to the other schools in the same area. They had an active approach in curriculum-related issues and for example, the Indonesian local school was among the first ones to implement the New National Curriculum in 2013.

Individual paper-and-pencil tasks consisted of ten items. In this article, the focus is on the paint-mixture item, in which students had to operate with a missing value task and the practical context was familiar for children from different cultures. In paint-mixture tasks students needed to understand the equivalence of ratios and determine, if the mixture had a certain shade of colour or not.

9. Orange paint is made by mixing **four** buckets of yellow paint and **one** bucket of red paint.

To get exactly the same shade of orange**, how many** buckets of red would the painter need to mix to **six** buckets of yellow?

This is how I solve the problem:

My answer: The painter would need ________ buckets of red paint and six buckets of yellow paint.

Figure 1: Paint-bucket task

Neither Finnish nor International school students were introduced strategies, such as cross-multiplication algorithm, prior to the test. Indonesian 5th graders had already some understanding of the algorithm, but only one student was able to utilise it partially in this particular problem.

Investigating correct answers

To solve the task correctly, the student either needed to form a composed unit or operate with the given values by multiplicative comparison (see for example Lobato et al., 2010, 19-21). Multiplicative comparison required the understanding on how many times greater the amount of yellow paint was compared to the amount of red paint. Some of the students recognised that two buckets of yellow paint, as they stated, "needed" a half of a bucket of red paint. Because six buckets of yellow could be grouped into three groups of two buckets in each, they multiplied three times ½ buckets of red. None of the students seemed to be using a composed unit in this item, even though it is a useful strategy in many tasks, which require reasoning with equivalent ratios.

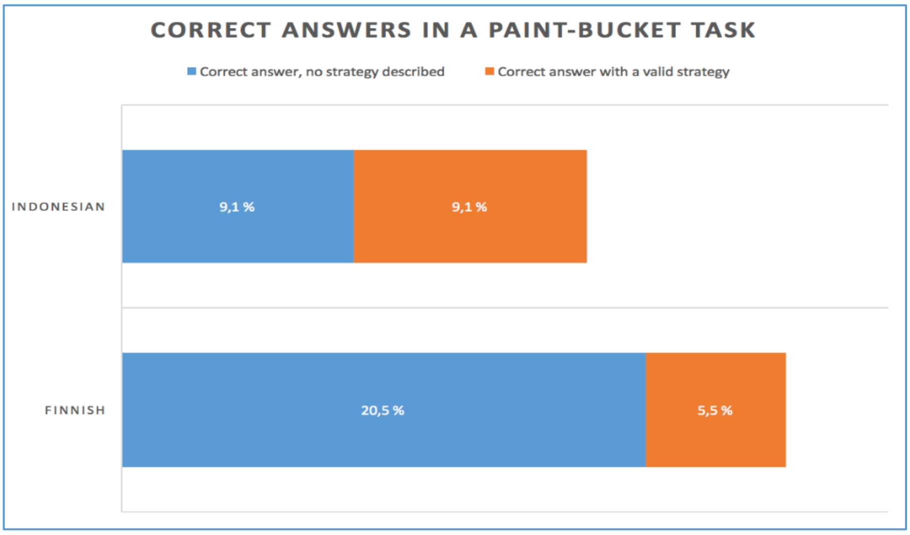


Figure 2. Correct answers in a paint-bucket task

26% (19/73 students) of Finnish students solved this particular problem correctly. Investigating their strategies was not straightforward: only four out of the whole group of 73 were able to describe and justify their thinking either by using words, numbers or drawings. Two of them relied on an intuitive strategy of solving the problem by drawing a picture. One formed a ratio, and one had an approach of multiplicative reasoning in the task. It seems that if Finnish students were not sure on how to express their thinking in 'mathematical language', they chose to leave the explanation completely out.

18 % (8/44) of Indonesian students got the correct answer of 1,5 buckets, and three of them were able to calculate the ratio and use that information in their problem-solving process. One student had an interesting way to solve the problem. First, she doubled both amounts, getting eight buckets of yellow and two buckets of red. The task stated that for the new paint there were six buckets of yellow, and she reasoned that the amount of red has to be "halfway between one and two". None of the students from Indonesian local school left the explanation part out, whether the answer was correct or incorrect. Some of the international school students, on the other hand, left the explanation out or stated that they "don't know".

	Indonesian students	Finnish students	All
Correct answer, but no strategy described	4 / 44 9,1 %	15 / 73 20,5 %	19 / 117 16,2 %
Correct answer with a valid strategy	4 / 44 9,1 %	4 / 73 5,5 %	8 / 117 6,8 %
Correct answers altogether	8 / 44 18,2 %	19 / 73 26,0 %	27 / 117 23,1 %

Table 1. Frequencies of correct answers

Erroneous strategies and tendency to additive reasoning

Additive reasoning is the most commonly reported strategy in tasks requiring proportional reasoning. Almost 30% of Indonesian students applied this erroneous approach, focusing on "how much less" or "how much more" the quantities were when compared with each other. The percentage of Finnish was 18%.

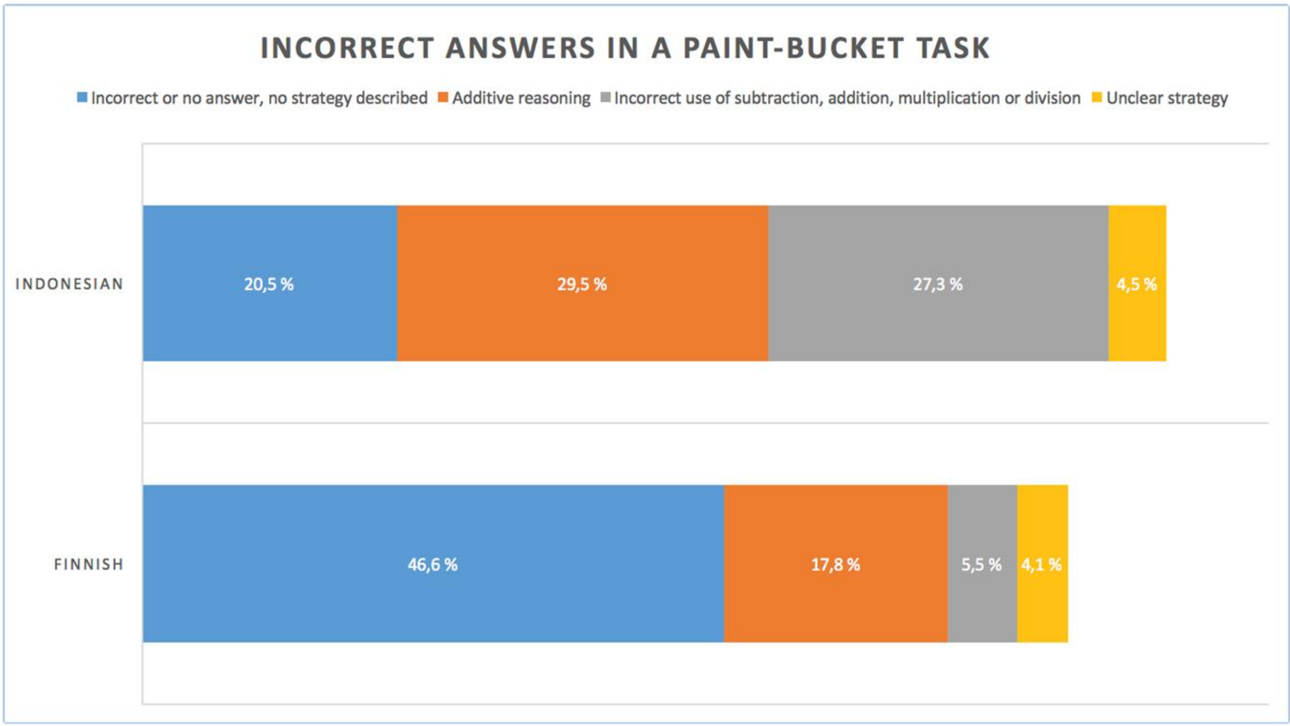


Figure 3. Incorrect answers and strategies used in the task

	Indonesian students	Finnish students	All
Incorrect answer, no strategy described or no answer	9 / 44 20,5 %	34 / 73 46,6 %	43 / 117 36,8 %
Additive reasoning	13 / 44 29,5 %	13 / 73 17,8 %	26 / 117 22,2 %
Incorrect use of subtraction, addition, multiplication or division	12 / 44 27,3 %	4 / 73 5,5 %	16 / 117 13,7 %
Not possible to define the strategy	2 / 44 4,5 %	3 / 73 4,1 %	5/ 117 4,3 %
Incorrect answers altogether	36/ 44 81,8 %	54 / 73 74,0 %	90 / 117 76, 9 %

Table 2. Frequencies of incorrect answers and strategies used in the task

If the student focused on one-dimensional additive reasoning, one of the very common misunderstandings was that for the larger amount of paint, whatever the amount was, the painter always needed two buckets of red.

In the following example, the student explained her solution

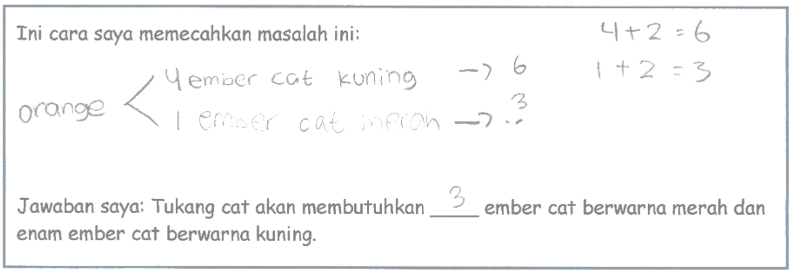
4 buckets of yellow → 6 *1 bucket of red → 3* and with adding up *4+2=6* *1+2=3* ended up in answer, *3 buckets of red.*	 Ini cara saya memecahkan masalah ini: 4+2=6 1+2=3 orange < 4 ember cat kuning → 6 1 ember cat merah → 3 Jawaban saya: Tukang cat akan membutuhkan 3 ember cat berwarna merah dan enam ember cat berwarna kuning.

Figure 4. Additive reasoning in a paint-bucket task (Indonesian student)

More sophisticated version of additive reasoning was "building up". The student had an intuitive understanding of ratio and numbers changing together but did not understand, that the situation involved a relative change.

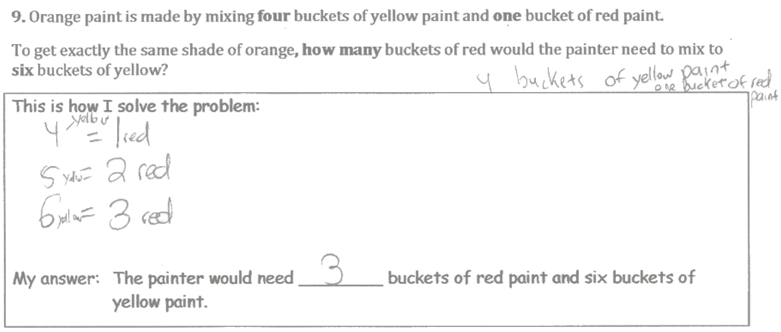
4 buckets of yellow → 1 red *5 yellow → 2 red* *6 yellow → 3 red.*	 9. Orange paint is made by mixing **four** buckets of yellow paint and **one** bucket of red paint. To get exactly the same shade of orange, **how many** buckets of red would the painter need to mix to **six** buckets of yellow? 4 buckets of yellow paint one bucket of red paint This is how I solve the problem: 4 yellow = 1 red 5 ylw = 2 red 6 ylw = 3 red My answer: The painter would need 3 buckets of red paint and six buckets of yellow paint.

Figure 5. Additive reasoning in paint-bucket task (International school student)

The less sophisticated problem-solution strategies included implementing simple subtraction, addition, multiplication or division algorithms, but ignoring the relative nature of this task. Among Indonesian students, this strategy was almost as prominent as additive reasoning (27,3%), but only few (5,5%) of the Finnish students used these erroneous approaches.

Majority of students struggled in expressing their thinking process and found it difficult to explain, how they ended up in their solutions. 71% of the Finnish students did not describe their problem-solving path, whether the answer was correct or incorrect. There was a contrast with local Indonesian students: 34% of Indonesian participants left the explanation out or it was not possible to determine which strategy they used. None of the students from a local school left the task completely blank or stated that they "didn't know" or are "just guessing", which some of the international school students and several Finnish students did.

Self-efficacy and problem-solving

Traditional paper-and-pencil tests are not always able to give insight on how students think. Explaining their problem-solving processes seemed to be difficult especially for

Finnish students, which should be, according to PISA-studies, the high-performers compared to Indonesian ones. Finnish students rather gave the correct answer and left the explanation out completely, if they were not sure on how to formulate it in a correct way. Instead of trying to explain by using numbers or drawings, in one example the student gave the right answer and just stated that the paint was made "in the same way than the painter made the first one". Having a closer look on students' beliefs on self-efficacy might provide valuable insight on why Finnish students struggled in describing their thinking.

Self-efficacy is linked to individual's understanding of problem-solving capabilities or success in mathematical tasks (Bandura, 1997). It would be interesting to know if Finnish students lack skills to formulate their thinking in mathematical form or was the issue rather culture-related? In Asia, it might not be culturally acceptable to admit that one is not knowing, how to solve the problem. In this study, the Asian students very rarely left the explanation out. For example, one of the students described his way of ending up into correct answer of 1 ½ the following way:

$$4 + 2 = 6,\ 2 = ½ + 4 = 1 ½ .$$

Discussion

The concept, context and strategies, which students have in their personal mathematics toolbox affect on their performance in solving proportional reasoning problems. Indonesian students were, in general, weaker in solving this particular test item, but they usually tried, even if they struggled in expressing their thoughts. They also seemed to be confident to present unfinished and incomplete steps from their problem-solving processes. That provides a good opportunity for the teacher to assess possible misunderstandings. In an open and accepting learning environment mistakes are a great source for investigating the nature of mathematics.

It would be important to encourage students to become confident with their ideas and describe their thinking in multiple ways. Open-ended problems and challenges with planted errors provide interesting sources to practise communicating about mathematical problems. Proportional reasoning is a context which provides numerous possibilities to develop mathematical problem-solving skills by utilising meaningful tasks from everyday mathematics.

Even though formal mathematical thinking skills are achieved in a mathematics classroom, it is important to remember, that it always overlaps with the culture of "common sense", that students possess (Prediger, 2001, 167). Mathematical strategies and concepts should be applicable to everyday thinking (Prediger, 2001, 168) and on the other hand, the formal mathematics teaching should employ problems arising from a daily life. Ideally, every mathematics lesson should offer students opportunities to develop their understanding by discovering different options to solve problems, which are meaningful from students' point of view. By listening on how students describe their thinking through the tasks, the teacher has a better chance to guide the learning processes.

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