Ultra-Reliable Cooperative Short-Packet Communications with Wireless Energy Transfer

Onel L. Alcaraz López, Evelio Martín García Fernández, Richard Demo Souza, and Hirley Alves

Abstract—We analyze a cooperative wireless communication system with finite block length and finite battery energy, under quasi-static Rayleigh fading. Source and relay nodes are powered by a wireless energy transfer (WET) process, while using the harvested energy to feed their circuits, send pilot signals to estimate channels at receivers, and for wireless information transmission (WIT). Other power consumption sources beyond data transmission power are considered. The error probability is investigated under perfect/imperfect channel state information (CSI), while reaching accurate closed-form approximations in ideal direct communication system setups. We consider ultra-reliable communication (URC) scenarios under discussion for the next fifth-generation (5G) of wireless systems. The numerical results show the existence of an optimum pilot transmit power for channel estimation, which increases with the harvested energy. We also show the importance of cooperation, even taking into account the multiplexing loss, in order to meet the error and latency constraints of the URC systems.

I. INTRODUCTION

A. Motivation

The 5G concept goes beyond broadband connectivity by bringing new kinds of services to life, enabling mission-critical control through ultra-reliable, low-latency links and connecting a massive number of smart devices, enabling the Internet of Things (IoT) [1]. It will not only interconnect people, but also interconnect and control a massive number of machines, objects, and devices, with applications having stringent requirements on latency and reliability. In fact, a novel operation mode under discussion for 5G is Ultra-Reliable Communication over a Short Term (URC-S) [2]–[4], which focuses on scenarios with stringent latency, e.g., $\leq 10\text{ms}$ and error probability, e.g., 10^{-3} , requirements [2].

Powering and keeping uninterrupted operation of such massive number of nodes is a major challenge [5], which could be addressed through the WET technique. WET constitutes an attractive solution because of the coverage advantages of radio-frequency (RF) signals, specially for IoT scenarios where replacing or recharging batteries require high cost and/or can be inconvenient or hazardous (e.g., in toxic environments), or highly undesirable (e.g., for sensors embedded in building structures or inside the human body) [6]–[8]. Also, RF signals can carry both energy and information, which enables energy

constrained nodes to harvest energy and receive information [9], [10], allowing to prolong their lifetime almost indefinitely.

The most important characteristics of WET systems are [11]: i) power consumption of the nodes on the order of μW ; ii) strict requirements on the reliability of the energy supply and of the data transfer; iii) information is conveyed in short packets due to intrinsically small data payloads, low-latency requirements, and/or lack of energy resources to support longer transmissions [12]. This agrees well with several URC-S scenarios with stringent latency requirements. Even though, most of the work in the field is made under the ideal assumption of communicating with large enough blocks in order to invoke Shannon theoretic arguments to address error performance, which contradicts the third characteristic of the WET systems. In fact, although performance metrics like Shannon capacity, and its extension to non-ergodic channels, have been proven useful to design current wireless systems since the delay constraints are typically above 10 ms [13], they are not necessarily appropriate in a short-packet scenario [14]. Also, for short-packet scenarios where the sizes of the preamble, the metadata and the data of the frame structure, are of the same order of magnitude, the Shannon capacity becomes an inaccurate metric for assessing the necessary blocklength to achieve a certain reliability. Instead, an essential quantity is the maximum coding rate for which [15] (and references therein) developed nonasymptotic bounds and approximations.

Our goal in this paper is to investigate wireless-powered communication with short packets, which is of great interest for URC-S use cases in 5G systems. We analyze a cooperative setup, where source and relay nodes are powered by a WET process from the destination, and practical issues such as the finite battery energy, other power consumption sources beyond data transmission, and imperfect CSI, are taken into account.

B. Related Work

The most common WET techniques are based on time-switching or power splitting [16]. Authors in [17] propose two WET protocols called Time Switching-based Relaying (TSR) and Power Splitting-based Relaying (PSR) to be implemented by an energy constrained relay node while assisting the communications of a single link. The implementation of the TSR protocol is further analyzed later in [18]. In all cases, there is not a direct link between source and destination, while only the relay is energy constrained and powered by the source. Alternatively, authors in [19] consider a system where energy-constrained sources have independent information to transmit to a common destination, which is responsible for

O. L. A. López and H. Alves are with Centre for Wireless Communications, University of Oulu, Finland. {onel.alcarazlopez, hirley.alves}@oulu.fi.

E. M. G. Fernández is with Federal University of Paraná (UFPR), Curitiba, Brazil. evelio@ufpr.br.

R. D. Souza is with Federal University of Santa Catarina (UFSC), Florianópolis, Brazil. richard.demo@ufsc.br.

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TABLE I: Summary of Main Symbols

Symbol	Definition	Symbol	Definition
S, R, D	Source, Relay and Destination nodes, respectively	$B_{\max}$	Battery capacity of S and R
v	WET blocklength	u	Pilot signal blocklength
n	Information blocklength in the DC scheme	n^*	Optimum information blocklength in the DC scheme
n_1, n_2	WIT blocklength in the 1st, 2nd phase with relaying	ν	Number of data and pilot symbols in the relaying scheme
k	Message length in bits	h_{ij}	Normalized channel in $i \rightarrow j$, $i, j \in \{S, R, D\}$
g_{ij}	Power gain coefficients in $i \rightarrow j$, $i, j \in \{S, R, D\}$	P_{csi}	Pilot signal transmit power for both S and R
T_c	Duration of each channel use	E_i	Energy harvested at $i \in \{S, R\}$
B_i	Battery charge in $i \in \{S, R\}$ before transmitting	η	Energy conversion efficiency
d_{ij}	Distance of the link $i \rightarrow j$, $i, j \in \{S, R, D\}$	α	Path loss exponent
κ_{ij}	Path loss factor in $i \rightarrow j$, $i, j \in \{S, R, D\}$	P_i	Transmit power of $i \in \{S, R, D\}$
λ_i	Threshold for the battery saturation in $i \in \{S, R\}$	P_c	Circuit and baseband processing power consumption
E_i^t	Available energy for transmission at $i \in \{S, R\}$	x_i	Signal transmitted by $i \in \{S, R\}$
ω_j	Gaussian noise vector at $j \in \{D, R\}$	σ_j^2	Variance of ω_j , $j \in \{D, R\}$
y_D	Received signal at D from S in the DC scheme	y_R	Received signal at R
y_{D1}, y_{D2}	Received signal at D from S and R with relaying	γ	Instantaneous SNR in $S \rightarrow D$ in the DC scheme
γ_{D1}, γ_{D2}	Instantaneous SNR in $S \rightarrow D$, $R \rightarrow D$ with relaying	γ_R	Instantaneous SNR in $S \rightarrow R$
δ, δ^*	Delay and minimum delay	β, β^*	Time sharing parameter and the optimum one
p_{out}	Outage probability	p_i	Energy insufficiency probability at $i \in \{S, R\}$
ϵ	Error probability	$C(\gamma)$	Shannon capacity for a SNR equal to γ
$V(\gamma)$	Shannon dispersion for a SNR equal to γ	$\hat{h}_{ij}, \bar{h}$	Estimate of h_{ij} and error in the estimation
x_i^p, y_j^p	Pilot symbols transmitted/received by i/j	γ_{imp}	Instantaneous SNR at the receiver with imperfect CSI
ϵ_0	Target error probability	δ_0	Maximum allowable delay
ξ	Error metric for the approximations in (16) and (17)	$i_M + 1$	Number of terms for summation in (16)

transferring energy wirelessly to the sources. The source nodes may cooperate with each other, under either decode-and-forward (DF) or network coding-based protocols, and even though the achievable diversity order is reduced due to wireless energy transfer process, it is very close to the one achieved for a network without energy constraints. Those results are extended in [20] by considering a more realistic consumption model where the circuitry power consumption is taken into account. In [21], both relay and source are powered by a WET process and the protocols: Harvest-then-Transmit (HTT) for a direct communication scenario, and Harvest-then-Cooperate (HTC) for relaying scenarios, are evaluated. However, all these works are under the ideal assumption of communicating with infinite blocklength.

Wireless-powered communication networks at finite blocklength regime have received attention in the scientific community recently. Authors of [22] attain tight approximations for the outage probability/throughput in an amplify-and-forward relaying scenario, while in [11] the authors implement retransmission protocols, in both energy and information transmission phases, to reduce the outage probability compared to open-loop communications. The impact of the number of channel uses for WET and for WIT on the performance of a system where a node charged by a power beacon attempts to communicate with a receiver, is investigated in [12]. Also, subblock energy-constrained codes are investigated in [23], while authors provide a sufficient condition on the subblock length to avoid energy outage at the receiver. In [24] we optimize a single-hop wireless system with WET in the downlink and WIT in the uplink, under quasi-static Nakagami-m fading in URC-S scenarios. The impact of a non-cooperative dual-hop setup is further evaluated in [25]. Finally and as an extension of [24], the error probability and energy consumption at finite block length and finite battery energy are characterized in [26] for scenarios with/without energy accumulation between transmission rounds with transmit power control.

The only source of energy consumption in all above works is the transmission power, which is a very impractical simplification for energy-limited systems. Also, all these works consider perfect CSI acquisition, which is a common situation in current scientific literature. However, the analysis under perfect CSI could be misleading for a wireless-powered communication network, due to its inherent energy constraints, and even more on systems with limited delay. Only few works have given a step forward on this direction in order to characterize different scenarios. Particularly interesting is the work in [27] where authors attain a closed-form approximation for the error probability at finite blocklength and imperfect CSI while showing the convenience of adapting the training sequence length and the transmission rate to reach higher reliability. However, new scenarios require deeper understanding, and battery capacity of devices and other energy consumption sources beyond transmission must be taken into account for a more realistic analysis.

C. Contributions

This paper aims at filling the above gaps in the state of the art. Next we list the main contributions of this work:

- We model an URC-S system under Rayleigh quasi-static fading at finite blocklength and finite battery constraint with other power consumption sources beyond data transmission power. We show that the infinite battery assumption is permissible for the scenarios under discussion since the energy harvested is very low; but considering other power consumption sources, e.g., circuit and baseband processing power, is crucial because they constitute a non-negligible cause of outage.
- We analyze the impact of imperfect CSI acquisition from pilot signals on the system performance. Numerical results show the existence of an optimum pilot transmit power for channel estimation, which increases with the harvested energy and decreases with the information

blocklength. We show that the energy devoted to the CSI acquisition, which depends on the power and utilized time, has to be taken into account due to the inherent energy and delay constraints of the discussed scenarios.

- For ideal direct communication, we reach accurate closed-form approximations for the error probability for finite and infinite battery devices.
- Also, cooperation appears as a viable solution to meet the reliability and delay constraints of URC-S scenarios, and it is advisable transmitting with shorter blocklengths during the broadcast phase.

The rest of this paper is organized as follows. Section II presents the system model. Sections III and IV discuss the delay and error probability metrics for scenarios with/without perfect CSI acquisition, respectively. Section V presents some numerical results, while Section VI concludes the paper.

Notation: Let $\mathbb{P}[A]$ denote the probability of event A , while $X \sim \text{Exp}(1)$ is a normalized exponentially distributed random variable with Probability Density Function (PDF) $f_X(x) = e^{-x}$ and Cumulative Distribution Function (CDF) $F_X(x) = 1 - e^{-x}$. Then, $Y \sim \mathcal{CN}(0,1)$ is a zero-mean circularly symmetric complex random variable with unit variance. Let $\mathbb{E}[\cdot]$ denote expectation and $|\cdot|$ is the absolute value operator. The Gaussian Q-function is denoted as $Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ [28, §8.250-4], $\text{Ei}(i,j) = \int_1^\infty \frac{e^{-jt}}{t^i} dt$ [28, §8.21] is the exponential integral and $K_t(\cdot)$ is the modified Bessel function of second kind and order t [28, §8.407]. Finally, $\min(x,y)$ represents the minimum value between x and y . The main symbols along the paper and their meaning are summarized in TABLE I.

II. SYSTEM MODEL

We consider a dual-hop cooperative network where a DF relay node, R , is available for assisting the transmissions from the source, S , to the destination, D . All the nodes are single antenna, half-duplex devices, where D is assumed to be externally powered and acts as an interrogator, e.g., an access point or a base station, requesting information from S , which along with R , may be seen as sensor nodes with very limited energy supply and finite battery of capacity $B_{\max}$. The communication scheme is illustrated in Fig. 1. First, D powers S and R during v channel uses through a WET process. Right after, S broadcasts a known pilot signal of u_1 channel uses before sending its data during a WIT phase over the next n_1 channel uses. The pilot is used by the receivers, D and R , to acquire CSI. Then, R tries to decode the information from S and if succeeds, transmits a pilot signal of u_2 channel uses to D for CSI acquisition and then forwards information received during the last WIT phase over the next n_2 channel uses. Finally, D combines the received information from S and R to decode the message. At each WIT phase, k information bits constitute the transmitted message. We assume $u = u_1 = u_2$, and notice that an equivalent direct communication (DC) scheme can be easily obtained from Fig. 1 without the assistance of R , with only one pilot signal transmission (u channel uses) and one WIT (n channel uses) phase.

Fig. 1: Relaying communication scheme: D powers S and R using v channel uses, then S broadcasts the pilots in u_1 channel uses and the information into n_1 channel uses, while R repeats the processes using, respectively, u_2 and n_2 channel uses.

We assume Rayleigh quasi-static fading channels, where the fading process is constant over a transmission round, which spans over all the phases of the communication scheme shown in Fig. 1 ($v + u_1 + u_2 + n_1 + n_2$ channel uses), and it is independent and identically distributed from round to round. The normalized channel gains from node i to node j , where $i, j \in \{S, D, R\}$ and $i \neq j$, are denoted by h_{ij} , while $g_{ij} = h_{ij}^2 \sim \text{Exp}(1)$ is the power gain coefficient. Similarly, distance between nodes is denoted by d_{ij} . In addition, no energy accumulation is allowed between consecutive transmission rounds, thus all the harvested energy at S and R is used for powering their circuits and for pilot and data transmission. The pilot signal transmit power is assumed fixed and equal to P_{csi} for both S and R . Also, the duration of each channel use is denoted by T_c .

III. SYSTEM PERFORMANCE WITH PERFECT CSI

First, as a benchmark, we assume that regardless the resources used for CSI acquisition, u and P_{csi} , receivers get a perfect channel estimation (pCSI). We note that scenarios with pCSI are quite frequent in the literature, e.g., [11], [12], [17], [18], [21]–[25]. Moreover, we start analyzing the outage probability for the DC scheme and then we extend the results for the cooperative scheme.

A. DC scheme

The energy harvested at S in the WET phase, which lasts for vT_c , is [21]

$$E_S = \frac{\eta P_D g_{DS}}{\kappa_{DS}} v T_c, \quad (1)$$

and the charging state of the battery before transmitting is

$$B_S = \min(E_S, B_{\max}) = \begin{cases} \frac{v T_c \eta P_D g_{DS}}{\kappa_{DS}}, & \text{if } g_{DS} < \lambda_S \\ B_{\max}, & \text{if } g_{DS} \geq \lambda_S \end{cases} \\ = \frac{v T_c \eta P_D \min(g_{DS}, \lambda_S)}{\kappa_{DS}}, \quad (2)$$

since no energy is accumulated between transmission rounds and because of the finite battery at S , it is not allowed to store more energy than the allowable limit $B_{\max}$. P_D is the transmit power of D when wirelessly powering S (and R), $0 < \eta < 1$ is the energy conversion efficiency and $\kappa_{ij} = \kappa d_{ij}^\alpha$ accounts for the path loss in the $i \rightarrow j$ link, where α is the path loss exponent, and κ represents a combination of other losses as due to carrier frequency and the antenna gains [29]. Also, λ_S is the $D \rightarrow S$ channel power gain threshold for the saturation of the battery in S , which is given by

$$\lambda_S = \frac{B_{\max} \kappa_{DS}}{v T_c \eta P_D}. \quad (3)$$

In addition, we assume that P_D is sufficiently large such that the energy harvested from noise is negligible. Notice that (2) is a valid expression if and only if all the energy harvested by S at each round is completely used during the pilot signal transmission and WIT phases of the same round. We acknowledge that energy accumulation between transmission rounds with adequate power allocation strategies could improve the system performance, however that scenario is out of the scope of this work.

Right after the WET phase, S transmits a pilot signal of u channel uses and power P_{csi} to be used by D for CSI acquisition. Also, we here take into account the circuit and baseband processing power consumption, namely P_c , which is assumed to be constant without loss of generality. When the harvested energy is insufficient for pilot transmission, WIT and other consumption sources, there is an outage and transmission fails, otherwise all the remaining energy is used for those processes. Notice that an outage event happens when the transmitter has not the sufficient power to feed its circuits and send the estimating pilot or when there is data transmission but the receiver is unable to successfully decode the received message. Thus, the available energy for transmission at S and its transmit power are stated, respectively, as

$$E_S^t = B_S - P_{\text{csi}} u T_c - P_c (u + n) T_c, \quad (4)$$

$$P_S = \frac{E_S^t (a)}{n T_c} = \frac{v \eta P_D \min(g_{DS}, \lambda_S)}{n \kappa_{DS}} - \frac{u}{n} (P_{\text{csi}} + P_c) - P_c, \quad (5)$$

where (a) in (5) comes from substituting (2) into (4). In addition, the received signal at D is

$$y_D = \sqrt{\frac{P_S g_{SD}}{\kappa_{SD}}} x_S + \omega_D, \quad (6)$$

where x_S is the codebook transmitted by S , which is assumed Gaussian with zero-mean and unit-variance, $\mathbb{E}[|x_S|^2] = 1$. Notice that there is no guarantee that Gaussian codebooks will provide the best reliability performance at finite blocklength, and here they are merely used to gain in mathematical tractability (authors in [15] derive accurate performance approximations). ω_D is the Gaussian noise vector at D with variance σ_D^2 . Thus, by using (6) and (5), we can write the instantaneous SNR at D as

$$\begin{aligned} \gamma &= \frac{v \eta P_D g_{SD} \min(g_{DS}, \lambda_S)}{n \kappa_{DS}^2 \sigma_D^2} - \frac{g_{SD}}{\kappa_{DS} \sigma_D^2} \left[\frac{u}{n} (P_{\text{csi}} + P_c) + P_c \right] \\ &= \phi g_{SD} \min(g_{DS}, \lambda_S) - \varphi g_{SD}, \end{aligned} \quad (7)$$

where

$$\phi = \frac{v \eta P_D}{n \kappa_{DS}^2 \sigma_D^2}, \quad (8)$$

$$\varphi = \frac{1}{\kappa_{DS} \sigma_D^2} \left[\frac{u}{n} (P_{\text{csi}} + P_c) + P_c \right]. \quad (9)$$

Let δ be the delay in delivering a message of k bits, while δ^* is the minimum delay that satisfies a given reliability constraint. Moreover, β is the time sharing parameter representing the fraction of δ devoted to WET only. Therefore,

$$\delta = v + u + n, \quad (10)$$

$$\beta = v / \delta. \quad (11)$$

Notice that δ is measured in channel uses, while δT_c would be the delay in seconds. Finally, we define the optimum WIT blocklength, in the sense of minimizing δ^* , as n^* . Both δ^* and n^* are numerically investigated in Section V.

On the other hand, the reliability analysis comes from evaluating the outage probability. An outage event may be due to a low harvested energy, precluding the whole WIT process, or if after the WIT process there is a decoding error at D . Hence, the outage probability is given as follows

$$p_{\text{out}} = p_S + \mathbb{E}[\epsilon(\gamma, k, n)], \quad (12)$$

where p_S is the probability that the harvested energy at S is insufficient simultaneously for channel estimation and for satisfying other consumption requirements at S , and it is obtained through

$$\begin{aligned} p_S &= \mathbb{P}[B_S < P_{\text{csi}} u T_c + P_c (u + n) T_c] \\ &= \mathbb{P}\left[\frac{v T_c \eta P_D \min(g_{DS}, \lambda_S)}{\kappa_{DS}} < P_{\text{csi}} u T_c + P_c (u + n) T_c\right] \\ &= \mathbb{P}\left[\min(g_{DS}, \lambda_S) < \frac{\kappa_{DS}}{v \eta P_D} (P_{\text{csi}} u + P_c (u + n))\right] \\ &\stackrel{(a)}{=} \mathbb{P}\left[\min(g_{DS}, \lambda_S) < \frac{\varphi}{\phi}\right] = 1 - \mathbb{P}\left[g_{DS} \geq \frac{\varphi}{\phi}\right] \mathbb{P}\left[\lambda_S \geq \frac{\varphi}{\phi}\right] \\ &\stackrel{(b)}{=} 1 - \mathbb{P}\left[g_{DS} \geq \frac{\varphi}{\phi}\right] = \mathbb{P}\left[g_{DS} < \frac{\varphi}{\phi}\right] \\ &\stackrel{(c)}{=} 1 - \exp\left(-\frac{\varphi}{\phi}\right). \end{aligned} \quad (13)$$

Notice that (a) comes from using the definitions of ϕ and φ given in (8) and (9), respectively, while (b) comes from the fact that λ_S is not a random variable and it should be greater than $\frac{\kappa_{DS}}{v \eta P_D} (P_{\text{csi}} u + P_c (u + n))$ for every practical system since otherwise the system is in outage all the time. Equality in (c) is obtained by using the CDF expression of g_{DS} . Also, $\epsilon(\gamma, k, n)$ is the error probability when transmitting a message of k information bits over n channel uses and being received with SNR equal to γ at the destination, thus $\mathbb{E}[\epsilon(\gamma, k, n)]$ is the average probability. Both terms are accurately approximated

by (14) [15], and (15) for quasi-static fading channel [30], when $n \geq 100$,¹ as shown next

$$\epsilon(\gamma, k, n) \approx Q\left(\frac{C(\gamma) - r}{\sqrt{V(\gamma)/n}}\right), \quad (14)$$

$$\begin{aligned} \mathbb{E}[\epsilon(\gamma, k, n)] &\approx \mathbb{E}\left[Q\left(\frac{C(\gamma) - r}{\sqrt{V(\gamma)/n}}\right)\right] \\ &\approx \int_{\gamma} Q\left(\frac{C(\gamma) - r}{\sqrt{V(\gamma)/n}}\right) f_{\gamma}(\gamma) d\gamma, \end{aligned} \quad (15)$$

where $r = k/n$ is the source fixed transmission rate, $C(\gamma) = \log_2(1 + \gamma)$ is the Shannon capacity, $V(\gamma) = \left(1 - \frac{1}{(1+\gamma)^2}\right) (\log_2 e)^2$ is the channel dispersion, which measures the stochastic variability of the channel relative to a deterministic channel with the same capacity [15].

Lemma 1. *The average error probability is given in (16) and (17) for finite and infinite battery, respectively.*

$$\begin{aligned} \mathbb{E}_{\text{fin}}[\epsilon(\gamma, k, n)] &\approx \int_0^{\lambda_S} \int_{\frac{\phi}{\phi}}^{\lambda_S} q_1(g_{DS}, g_{SD}, n) e^{-g_{DS}} dg_{DS} dg_{SD} + \\ &+ \int_0^{\lambda_S} \int_{\lambda_S}^{\infty} q_1(\lambda_S, g_{SD}, n) e^{-g_{DS}} dg_{DS} dg_{SD}, \end{aligned} \quad (16)$$

$$\mathbb{E}_{\text{inf}}[\epsilon(\gamma, k, n)] \approx \int_0^{\lambda_S} \int_{\frac{\phi}{\phi}}^{\infty} q_1(g_{DS}, g_{SD}, n) e^{-g_{DS}} dg_{DS} dg_{SD}, \quad (17)$$

where

$$q_1(x_1, x_2, n) = Q\left(\frac{C((\phi x_1 - \phi)x_2) - k/n}{\sqrt{V((\phi x_1 - \phi)x_2)/n}}\right) e^{-x_2}. \quad (18)$$

Proof. Substituting (7) into (15) and using the PDF expression for g_{SD} and g_{DS} , we attain (16) and (17). Notice that for the infinite battery case, (7) becomes $\gamma = \phi g_{SD} g_{DS} - \phi g_{SD}$. $\square$

We consider as ideal system, one where all the energy harvested by S is used only during the WIT phase ($P_{\text{csi}} = P_c = 0 \rightarrow \phi = 0$) while, even so, D has perfect knowledge of the channel.

Theorem 1. *The outage probability given in (12) can be approximated as in (20) and (21) (on the top of the next page) for an ideal system as described in Section III-A, where $\theta = 2^{k/n} - 1$, $\psi = \sqrt{\frac{n}{2\pi}}(2^{2k/n} - 1)^{-\frac{1}{2}}$, $\varrho = \theta - \frac{1}{\psi}\sqrt{\frac{\pi}{2}}$, $\vartheta = \theta + \frac{1}{\psi}\sqrt{\frac{\pi}{2}}$, for finite and infinite battery devices, respectively. i_M is a parameter to limit the number of terms in the summation when evaluating (20).*

Proof. See Appendix A.

¹See [15, Figs. 12 and 13] and related analyses for more insight on the accuracy of (14). Authors show that the approximate achievable rate matches almost perfectly its true value for $n \geq 100$. Many other works, such as [11]–[13], [22], [24]–[27], [31]–[33], use this accurate approximation and/or (15) to gain in tractability.

Fig. 2: $\xi(\%)$ as a function of i_M with $v = 1000$ channel uses.

Corollary 1. (21) reduces to [24, Eq.(10)] for Rayleigh fading.

To measure the accuracy of (20) and (21), we evaluate the following error metric

$$\xi = \frac{|\mathbb{E}[\epsilon_{\text{ex}}] - \mathbb{E}[\epsilon_{\text{ap}}]|}{\mathbb{E}[\epsilon_{\text{ex}}]}, \quad (19)$$

where $\mathbb{E}[\epsilon_{\text{ex}}]$ is the exact error probability according to (16) and (17), for finite and infinite battery, respectively, while $\mathbb{E}[\epsilon_{\text{ap}}]$ is the approximated value according to (20) and (21). Notice that i_M is a parameter that must be carefully selected when evaluating the finite battery approximation given in (20) because it establishes the quantity of elements taken into account for the summation. As $i_M \rightarrow \infty$, expression (20) becomes more accurate but harder to evaluate and computationally heavier. Besides, a relatively small value conduces to an inaccurate approximation.

Fig. 2 shows numerically the impact of i_M on ξ , when $v = 1000$ and $n \in \{500, 1000\}$ channel uses, and $B_{\text{max}} \in \{10^{-8}, 10^{-7}, \infty\}$ J. The remaining system parameters were the ones chosen in Section V. The larger the battery capacity, the larger the required i_M for a good approximation using (16). Note that for all cases $i_M \geq 10$ provides a good accuracy with $\xi < 1\%$. The accuracy of (16) and (17) was also measured for many other different setups, and in all the cases we reached similar results, e.g. $\xi < 3.5\%$ for any (n, v) pair with $n, v \in [100 \ 5000]$ and $k > 64$ bits. The accuracy is good because we used the linearization (first order approximation) of (14) to attain (20) and (21), which is symmetrical with respect to $\gamma = 2^r - 1$, and lies beneath and above of (14) for $\gamma < 2^r - 1$ and $\gamma > 2^r - 1$, respectively. Thus, when integrating over all the channel realizations, the error tends to vanish.

B. Relaying scheme

Cooperative technique has rekindled enormous interests from the wireless communication community over the past decade. As shown in [31], the attained spatial diversity can improve communication reliability, thus it can also reduce the system delay for a target error constraint. For an energy-constrained URC system, cooperation seems advisable to reduce the instantaneous consumption power of devices while meeting the reliability/delay constraints. Consequently, herein

$$\mathbb{E}_{\text{fin}}[\epsilon(\gamma, k, n)] \approx 1 - \left(\frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\theta - \varrho - \lambda_s \phi) \right) e^{-\lambda_s - \frac{\varrho}{\lambda_s \phi}} - \left(\frac{1}{2} + \frac{\psi}{\sqrt{2\pi}}(\theta - \vartheta - \lambda_s \phi) \right) e^{-\lambda_s - \frac{\vartheta}{\lambda_s \phi}} + \sum_{i=0}^{i_M} \frac{(-1)^i \lambda_s^{i+1}}{i!} \cdot \left[\frac{\psi \lambda_s \phi}{\sqrt{2\pi}} \left(\text{Ei}(i+3, \frac{\vartheta}{\lambda_s \phi}) - \text{Ei}(i+3, \frac{\varrho}{\lambda_s \phi}) \right) - \left(\frac{1}{2} - \frac{\psi(\theta - \varrho)}{\sqrt{2\pi}} \right) \text{Ei}(i+2, \frac{\varrho}{\lambda_s \phi}) - \left(\frac{1}{2} + \frac{\psi(\theta - \vartheta)}{\sqrt{2\pi}} \right) \text{Ei}(i+2, \frac{\vartheta}{\lambda_s \phi}) \right], \quad (20)$$

$$\mathbb{E}_{\text{inf}}[\epsilon(\gamma, k, n)] \approx 1 - \sqrt{\frac{\varrho}{\phi}} \left[1 + \left(\frac{2\psi}{\sqrt{2\pi}} \right) (\varrho + \phi - \theta) \right] K_1 \left(2\sqrt{\frac{\varrho}{\phi}} \right) - \frac{2\psi\varrho}{\sqrt{2\pi}} K_0 \left(2\sqrt{\frac{\varrho}{\phi}} \right) + \sqrt{\frac{\vartheta}{\phi}} \left[1 - \left(\frac{2\psi}{\sqrt{2\pi}} \right) (\vartheta + \phi - \theta) \right] K_1 \left(2\sqrt{\frac{\vartheta}{\phi}} \right) + \frac{2\psi\vartheta}{\sqrt{2\pi}} K_0 \left(2\sqrt{\frac{\vartheta}{\phi}} \right). \quad (21)$$

we consider the presence of node R , which is willing to assist the $S \rightarrow D$ communication all the time, while operating under the DF protocol. Under the finite blocklength regime, decoding errors may occur. We assume that R reliably detects the errors, and consequently it does not forward the message to D when an error occurs, as in [32]. In that condition, an outage event is declared for the $S \rightarrow R$ link, and R is inactive during the time reserved for its transmission, e.g., the last $u + n_2$ channel uses according to Fig. 1. The energy that could be saved in those cases is out of the scope of our analysis². This behavior reduces the system complexity compared to other works, e.g. [18] for infinite blocklength and preset power relay. Nodes S and R are assumed to have the same characteristics, e.g., battery capacity, pilot power, power consumption profile.

Similar than in (1) and (2), the energy harvested at R and the battery charge before transmitting are, respectively,

$$E_R = \frac{\eta P_D g_{DR}}{\kappa_{DR}} v T_c, \quad (22)$$

$$B_R = \min(E_R, B_{\max}) = \begin{cases} \frac{v T_c \eta P_D g_{DR}}{\kappa_{DR}}, & \text{if } g_{DR} < \lambda_R \\ B_{\max}, & \text{if } g_{DR} \geq \lambda_R \end{cases} = \frac{v T_c \eta P_D \min(g_{DR}, \lambda_R)}{\kappa_{DR}}, \quad (23)$$

where λ_R is the $D \rightarrow R$ channel power gain threshold for R battery saturation given by

$$\lambda_R = \frac{B_{\max} \kappa_{DR}}{v T_c \eta P_D}. \quad (24)$$

The expressions for the energy harvested at S , the charge of its battery, the available energy for transmission and its transmit power are the same as in the analysis of the DC scheme, respectively (1), (2), (4) and (5), but with $n = n_1$. Now, for R , they are given by

$$E_R^t = B_R - P_{\text{csi}} u T_c - P_c \nu T_c, \quad (25)$$

$$P_R = \frac{E_R^t}{n_2 T_c} = \frac{v \eta P_D \min(g_{DR}, \lambda_R)}{n_2 \kappa_{DR}} - \frac{1}{n_2} (u P_{\text{csi}} + \nu P_c), \quad (26)$$

where $\nu = 2u + n_1 + n_2$ (note that R consumes circuit power for receiving the pilots and the data from S , besides the power

consumed in the transmission of pilots and data to D). During the first WIT phase, S broadcasts its data to D and R . The expression of the signal received at D in this phase is $y_{D1} = y_D$, equal to (6) but with $n = n_1$, and the signal received at R is

$$y_R = \sqrt{\frac{P_S g_{SR}}{\kappa_{SR}}} x_S + \omega_R, \quad (27)$$

where P_S is given in (5) and ω_R is the Gaussian noise vector at R with variance σ_R^2 . When R successfully decodes the message during the first WIT phase, it re-encodes it in n_2 channel uses, and after the pilot signal transmission, it transmits the message to D during the second WIT phase. The received signal at D at this phase is thus given by

$$y_{D2} = \sqrt{\frac{P_R g_{RD}}{\kappa_{RD}}} x_R + \omega_D, \quad (28)$$

where P_R is given in (26) and x_R is the zero-mean, unit-variance Gaussian codebook transmitted by R . R uses the same codebook, which is defined a priori, and when $n_1 = n_2 \rightarrow x_R = x_S$.

Let γ_{D1} and γ_R be the instantaneous SNRs at D and R , respectively, for the signal transmitted by S during the first WIT phase. Also, let γ_{D2} be the instantaneous SNR at D for the signal transmitted by R during the second WIT phase, as long as R achieved a successful decoding of the message transmitted by S . Expression (7) is still useful for γ_{D1} but with $n = n_1$ ($\phi_{D1} = \phi|_{n=n_1}$ and $\varphi_{D1} = \varphi|_{n=n_1}$ according to (8) and (9), respectively) since now the transmission time depends on n_1 . Using (27) and (28) we attain the expressions for γ_R and γ_{D2} , which are

$$\gamma_R = \frac{\eta v P_D g_{SR} \min(g_{DS}, \lambda_S)}{n_1 \kappa_{DS} \kappa_{SR} \sigma_R^2} - \frac{g_{SR}}{\kappa_{SR} \sigma_R^2} \left(\frac{u}{n_1} (P_{\text{csi}} + P_c) + P_c \right) = \phi_R g_{SR} \min(g_{DS}, \lambda_S) - \varphi_R g_{SR}, \quad (29)$$

$$\gamma_{D2} = \frac{v \eta P_D g_{RD} \min(g_{DR}, \lambda_R)}{n_2 \kappa_{DR}^2 \sigma_D^2} - \frac{g_{RD}}{\kappa_{RD} n_2 \sigma_D^2} (u P_{\text{csi}} + \nu P_c) = \phi_{D2} g_{RD} \min(g_{DR}, \lambda_R) - \varphi_{D2} g_{RD}, \quad (30)$$

²Notice that in URC-S scenarios the probability of such events should be kept low, specially if relaying is crucial in achieving the ultra-reliability, and the impact of that remaining energy on the system performance can be negligible, as shown in [25] for a system without a direct link.

where

$$\phi_R = \frac{\eta v P_D}{n_1 \kappa_{DS} \kappa_{SR}^\alpha \sigma_R^2}, \quad (31)$$

$$\phi_{D2} = \frac{\eta v P_D}{n_2 \kappa_{DR}^2 \sigma_D^2}, \quad (32)$$

$$\varphi_R = \frac{1}{\kappa_{SR} \sigma_R^2} \left(\frac{u}{n_1} (P_{\text{csi}} + P_c) + P_c \right), \quad (33)$$

$$\varphi_{D2} = \frac{1}{\kappa_{RD} n_2 \sigma_D^2} (u P_{\text{csi}} + \nu P_c). \quad (34)$$

The delay for the cooperative scheme is

$$\delta = v + \nu = v + 2u + n_1 + n_2, \quad (35)$$

while the time sharing parameter still obeys (11), but using (35) for the delay. An outage event for the cooperative scheme can be due to: i) a low harvested energy at S , precluding the whole transmission process; ii) communication error in the $S \rightarrow D$ link at the same time that R is unable to perform a transmission due to its low harvested energy or when it fails in decoding the information from S ; iii) both S and R were capable of completing their transmissions (R succeeded in decoding the message from S) but after combining both signals at D there is still a decoding error. Thus, the outage probability can be mathematically written as³

$$p_{\text{out}} = p_S + p_R \mathbb{E}[\epsilon(\gamma_{D1}, k, n_1)] + \mathbb{E}[(1 - \epsilon(\gamma_R, k, n_1)) \epsilon_{\text{comb}}] + (1 - p_R) \mathbb{E}[\epsilon(\gamma_R, k, n_1) \epsilon(\gamma_{D1}, k, n_1)], \quad (36)$$

where p_S can be calculated as in (13) but with $n = n_1$, and p_R is the probability that the harvested energy at R is insufficient simultaneously for channel estimation and for satisfying other consumption requirements at R , and can be easily obtained similarly to p_S as

$$\begin{aligned} p_R &= \mathbb{P}[B_R < P_{\text{csi}} u T_c + P_c \nu T_c] \\ &= 1 - \exp\left(-\frac{\kappa_{DR}}{v \eta P_D} (P_{\text{csi}} u + P_c \nu)\right) \\ &= 1 - \exp\left(-\frac{\varphi_{D2}}{\phi_{D2}}\right). \end{aligned} \quad (37)$$

$\mathbb{E}[\epsilon(\gamma_{D1}, k, n_1)]$ can be obtained through (16) (or (17) for the infinite battery case) with $\gamma = \gamma_{D1}$, $n = n_1$. Also, $\epsilon_{\text{comb}} \in \{\epsilon_{\text{sc}}, \epsilon_{\text{mrc}}\}$ is the error probability when D tries to decode the information from the combination of signals received from both WIT phases and depends on the used combination technique: Selection Combining (SC) or Maximal Ratio Combining (MRC). Notice that

$$\begin{aligned} \mathbb{E}[\epsilon(\gamma_R, k, n_1) \epsilon(\gamma_{D1}, k, n_1)] &\neq \mathbb{E}[\epsilon(\gamma_R, k, n_1)] \mathbb{E}[\epsilon(\gamma_{D1}, k, n_1)], \\ \mathbb{E}[(1 - \epsilon(\gamma_R, k, n_1)) \epsilon_{\text{comb}}] &\neq \mathbb{E}[(1 - \epsilon(\gamma_R, k, n_1))] \mathbb{E}[\epsilon_{\text{comb}}], \end{aligned}$$

since γ_R , γ_{D1} and γ_{comb} are correlated through the variable g_{DS} . In the first case, the expectation can be evaluated through

³Note that in (36) and (12) when a complementary event, e.g. $1 - p_S$ or $1 - p_R$, does not appear explicitly in the equation is because its effect is implicit when evaluating the communication errors by properly setting the integration limits.

(38) and (39) for finite and infinite battery, respectively, as shown next

$$\begin{aligned} &\mathbb{E}_{\text{fin}}[\epsilon(\gamma_R, k, n_1) \epsilon(\gamma_{D1}, k, n_1)] \\ &\approx \int_0^\infty \int_0^\infty \int_{\frac{\varphi}{\phi}}^{\lambda_S} q_1(g_{DS}, g_{SD}, n_1) q_2(g_{DS}, g_{SR}) e^{-g_{DS}} dg_{DS} dg_{SD} dg_{SR} + \\ &+ \int_0^\infty \int_0^\infty \int_{\lambda_S}^\infty q_1(\lambda_S, g_{SD}, n_1) q_2(\lambda_S, g_{SR}) e^{-g_{DS}} dg_{DS} dg_{SD} dg_{SR}, \quad (38) \\ &\mathbb{E}_{\text{inf}}[\epsilon(\gamma_R, k, n_1) \epsilon(\gamma_{D1}, k, n_1)] \\ &\approx \int_0^\infty \int_0^\infty \int_{\frac{\varphi}{\phi}}^\infty q_1(g_{DS}, g_{SD}, n_1) q_2(g_{DS}, g_{SR}) e^{-g_{DS}} dg_{DS} dg_{SD} dg_{SR}, \quad (39) \end{aligned}$$

where

$$q_2(x_1, x_2) = Q\left(\frac{C((\phi_R x_1 - \varphi_R) x_2) - k/n_1}{\sqrt{V((\phi_R x_1 - \varphi_R) x_2)/n_1}}\right) e^{-x_2}. \quad (40)$$

The classical SC technique establishes that the signal with the highest SNR is selected since its error probability is the lowest [29]. Nonetheless, when the blocklength is small and different for each link, $n_1 \neq n_2$, this assumption does not hold always as is induced from (14), which motivates us to state the outage probability of the SC combining scheme as

$$\epsilon_{\text{sc}} = \min(\epsilon(\gamma_{D1}, k, n_1), \epsilon(\gamma_{D2}, k, n_2)), \quad (41)$$

where $\epsilon(\gamma_{D1}, k, n_1)$ and $\epsilon(\gamma_{D2}, k, n_2)$ can be obtained through (14) for each channel realization. Thus, D has to select the signal to be decoded based on

$$\frac{C(\gamma_{D1}) - k/n_1}{\sqrt{V(\gamma_{D1})/n_1}} \stackrel{y_{D1}}{\geq} \frac{C(\gamma_{D2}) - k/n_2}{\sqrt{V(\gamma_{D2})/n_2}}. \quad (42)$$

Notice that this is the optimum strategy when D selects only one signal; although it can be difficult to implement. In addition, it seems intractable to find a closed-form expression for (41), that is why in Section V we estimate it numerically.

On the other hand, the MRC technique reaches better results than SC, although with higher hardware complexity [29], and limited to the case where $n_1 = n_2$. The error probability for that case is given by

$$\epsilon_{\text{mrc}} = \epsilon(\gamma_{D1} + \gamma_{D2}, k, n_1), \quad (43)$$

and the last term in (36) for finite and infinite battery is given in (44) and (45) (on the top of the next page), respectively, where $q_3(x_1, x_2, x_3, x_4, x_5)$ is defined in (46).

C. Impact of the Combining Technique

As we mentioned above, the MRC technique reaches better results than SC for a conventional network without WET [29]. Notice that without WET and assuming that nodes S and R transmit the information with fixed power, the links

$$\begin{aligned} \mathbb{E}_{\text{fin}} \left[(1 - \epsilon(\gamma_R, k, n_1)) \epsilon_{\text{MRC}} \right] &\approx \int_0^\infty \int_0^\infty \int_0^\infty \int_{\frac{\varphi_{D1}}{\phi_{D1}}}^{\lambda_S} \int_{\frac{\varphi_{D2}}{\phi_{D2}}}^{\lambda_R} q_3(g_{DR}, g_{DS}, g_{RD}, g_{SD}, g_{SR}) e^{-g_{DS} - g_{DR}} dg_{DR} dg_{DS} dg_{RD} dg_{SD} dg_{SR} + \\ &+ \int_0^\infty \int_0^\infty \int_0^\infty \int_{\lambda_S}^\infty \int_{\lambda_R}^\infty q_3(\lambda_R, \lambda_S, g_{RD}, g_{SD}, g_{SR}) e^{-g_{DS} - g_{DR}} dg_{DR} dg_{DS} dg_{RD} dg_{SD} dg_{SR} \end{aligned} \quad (44)$$

$$\mathbb{E}_{\text{inf}} \left[(1 - \epsilon(\gamma_R, k, n_1)) \epsilon_{\text{MRC}} \right] \approx \int_0^\infty \int_0^\infty \int_0^\infty \int_{\frac{\varphi_{D1}}{\phi_{D1}}}^\infty \int_{\frac{\varphi_{D2}}{\phi_{D2}}}^\infty q_3(g_{DR}, g_{DS}, g_{RD}, g_{SD}, g_{SR}) e^{-g_{DS} - g_{DR}} dg_{DR} dg_{DS} dg_{RD} dg_{SD} dg_{SR} \quad (45)$$

$$q_3(x_1, x_2, x_3, x_4, x_5) = \left(1 - Q \left(\frac{C(x_2(\phi_R x_5 - \varphi_R)) - k/n_1}{\sqrt{V(x_2(\phi_R x_5 - \varphi_R))/n_1}} \right) \right) Q \left(\frac{C(x_4(\phi_{D1} x_2 - \varphi_{D1}) + x_3(\phi_{D2} x_1 - \varphi_{D2})) - k/n_1}{\sqrt{V(x_4(\phi_{D1} x_2 - \varphi_{D1}) + x_3(\phi_{D2} x_1 - \varphi_{D2}))/n_1}} \right) e^{-x_3 - x_4 - x_5} \quad (46)$$

Fig. 3: SC vs MRC for and ideal system with equal mean SNR ($\bar{\gamma}$) in all links and equal blocklengths in the broadcast and cooperation phases, for the case of without WET and with WET. For scenarios with WET, $\bar{\gamma} = \phi = \phi_{D1} = \phi_{D2} = \phi_R$.

$S \rightarrow D$ and $S \rightarrow R$ are completely independent. However, for the model with WET analyzed in this work, those links are correlated through channel coefficient g_{DS} and therefore, the diversity cannot be fully achieved. Based on that, it is not surprising that the impact of the combining scheme used at D is much lower for the case with WET than for a conventional network as shown in Fig. 3. As shown in the figure, there is not a significant performance difference between SC and MRC, being practically indistinguishable for $p_{\text{out}} < 10^{-2}$, which is our study case since we are analyzing URC-S scenarios. Also, notice that the performance gap between DC and the cooperative scheme is much smaller for the case with WET. Nonetheless, using cooperation remains advantageous and further discussion takes place in Section V. Additionally, Fig. 3 also corroborates the accuracy claimed when giving (21).

IV. SYSTEM PERFORMANCE WITH IMPERFECT CSI

Herein we consider a more realistic case where the channel estimate $\hat{h}_{ij}$ made by node $j \in \{R, D\}$ of the true channel

coefficient h_{ij} is imperfect (iCSI), i.e., $h_{ij} \neq \hat{h}_{ij}$. The pilot signal received by j due to the transmission of i is

$$y_j^p = \sqrt{\frac{P_{\text{csi}}}{\kappa_{ij}}} h_{ij} x_i^p + \omega_j, \quad (47)$$

where $i \in \{S, R\}$ with $i \neq j$, and x_i^p are the u pilot symbols transmitted by i with $\mathbb{E}[|x_i^p|^2] = 1$. Supposing minimum mean square error (MMSE) estimation at the receivers, the relation between the true channel coefficients, the estimates, and the estimation error can be modeled as [34]

$$h_{ij} = \hat{h}_{ij} + \tilde{h}_{ij}, \quad (48)$$

where $h_{ij} \sim \mathcal{CN}(0, 1)$,

$$\hat{h}_{ij} \sim \mathcal{CN}\left(0, \frac{P_{\text{csi}} u \kappa_{ij}^{-1}}{P_{\text{csi}} u \kappa_{ij}^{-1} + \sigma_j^2}\right), \quad (49)$$

and $\tilde{h}_{ij}$, the error in the channel estimation, is

$$\tilde{h}_{ij} \sim \mathcal{CN}\left(0, \frac{\sigma_j^2}{P_{\text{csi}} u \kappa_{ij}^{-1} + \sigma_j^2}\right). \quad (50)$$

Using (48), (49) and (50), the signal received at j from i during a WIT phase is

$$\begin{aligned} y_j &= \sqrt{\frac{P_i}{\kappa_{ij}}} h_{ij} x_i + \omega_j = \sqrt{\frac{P_i}{\kappa_{ij}}} \hat{h}_{ij} x_i + \sqrt{\frac{P_i}{\kappa_{ij}}} \tilde{h}_{ij} x_i + \omega_j \\ &= \sqrt{\frac{P_i}{\kappa_{ij}}} \hat{h}_{ij} x_i + w_{\text{eq}}, \end{aligned} \quad (51)$$

where w_{eq} is the effective noise due to both the estimation error and AWGN. That noise is neither Gaussian nor independent on the data signal in general. However, it is been shown in [35] that treating it as a circularly symmetric zero mean complex Gaussian process with variance $\sigma_{\text{eq}}^2 = P_i \kappa_{ij}^{-1} \sigma_{\hat{h}_{ij}}^2 + \sigma_j^2$ provides the worst case scenario for the channel capacity (more precisely, for the mutual information

between a Gaussian channel input and the channel output)⁴. This implies that the equivalent SNR under such assumption is a lower bound of the actual instantaneous SNR, thus (and based on (14) and (15)), this assumption also provides the worst case scenario for the system performance in terms of error probability. Based on (51), the equivalent instantaneous SNR at j in the iCSI case is thus given by

$$\gamma^{\text{imp}} = \frac{P_i |h_{ij}|^2 \kappa_{ij}^{-1}}{\sigma_{\text{eq}}^2} = \frac{P_i \hat{g}_{ij} \kappa_{ij}^{-1}}{P_i \kappa_{ij}^{-1} \sigma_{h_{ij}}^2 + \sigma_j^2}. \quad (52)$$

A. DC scheme

For the DC scheme, substituting (5), and $\sigma_{h_{SD}}^2 = \sigma_D^2 / (P_{\text{csi}} u \kappa_{SD}^{-1} + \sigma_D^2)$ from (50), into (52), we attain the following equivalent instantaneous SNR at D

$$\begin{aligned} \gamma^{\text{imp}} &= \frac{P_S g_{SD}}{\kappa_{SD} \sigma_D^2} \left(\frac{P_{\text{csi}} u}{P_{\text{csi}} u + P_S + \kappa_{SD} \sigma_D^2} \right) \\ &= (\phi \min(g_{DS}, \lambda_S) - \varphi) \cdot \\ &\quad \cdot g_{SD} \frac{P_{\text{csi}} u}{P_{\text{csi}} u + \kappa_{SD} \sigma_D^2 (\phi \min(g_{DS}, \lambda_S) - \varphi + 1)}. \end{aligned} \quad (53)$$

Thus, the effect of imperfect channel estimation can be seen as a decrease in the instantaneous SNR seeing at D . Thus, to find the information error probability it is just necessary to evaluate (16) and (17) for finite and infinite battery, respectively, but with $\gamma = \gamma^{\text{imp}}$.

Substituting (8) and (9) into (53) and calculating its second derivative yields (54) at the top of the next page, where subscripts of σ_D^2 , λ_S are omitted to shorten the notation, and $\kappa_{\cdot} = \kappa_{DS} = \kappa_{SD}$ for convenience. We know that $B_S > P_c(u + n)T_c$ is the condition required for the CSI estimation phase to take place; otherwise there is an outage due to the insufficiency energy for transmission, which is counted in the term p_S of (12). Based on (2), that equality is equivalent to $\min(g_{DS}, \lambda) \frac{\eta P_D v}{\kappa_{\cdot}} > P_c(n + u)$, thus the fraction in (54) is positive and $\frac{\partial^2 \gamma^{\text{imp}}}{\partial P_{\text{csi}}^2} < 0$. Therefore γ^{imp} is concave on P_{csi} , and since $\epsilon(\gamma^{\text{imp}}, k, n)$ is decreasing on γ^{imp} (the greater the SNR, the smaller the chance of error), there is only one value of P_{csi} that minimizes the error probability by maximizing the SNR for given channel realizations g_{DS} and g_{SD} . However, when averaging over all channel realizations it becomes intractable to prove mathematically the existence of a unique optimum pilot transmit power, let alone find it, fundamentally because $\epsilon(\gamma^{\text{imp}}, k, n)$ is not convex on γ^{imp} . Thus, we resort to numerical evaluation. We found that $p_{\text{out}}(P_{\text{csi}})$ has the shape of an inverted bell, in part because for small P_{csi} , the mean SNR is low and $\mathbb{E}[\epsilon(\gamma^{\text{imp}}, k, n)] \rightarrow 1$; while for large P_{csi} , $p_S \rightarrow 1$. In both cases $p_{\text{out}} \rightarrow 1$ (see (12)), with equality from a point onwards and from a point backwards. Based on the inverted bell shape, the minimum is unique, and it is found numerically in Subsection V-B2, while analyzing its dependence on some system parameters.

B. Relaying scheme

The equivalent instantaneous SNRs at D and R during the first and second WIT phases with iCSI are

$$\gamma_{D1}^{\text{imp}} = (\phi_{D1} \min(g_{DS}, \lambda_S) - \varphi_{D1}) g_{SD} \cdot \frac{P_{\text{csi}} u}{P_{\text{csi}} u + \kappa_{SD} \sigma_D^2 (\phi_{D1} \min(g_{DS}, \lambda_S) - \varphi_{D1} + 1)}, \quad (55)$$

$$\gamma_R^{\text{imp}} = (\phi_R \min(g_{DS}, \lambda_S) - \varphi_R) g_{SR} \cdot \frac{P_{\text{csi}} u}{P_{\text{csi}} u + \kappa_{SR} \sigma_R^2 (\phi_R \min(g_{DS}, \lambda_S) - \varphi_R + 1)}, \quad (56)$$

$$\gamma_{D2}^{\text{imp}} = (\phi_{D2} \min(g_{DR}, \lambda_R) - \varphi_{D2}) g_{RD} \cdot \frac{P_{\text{csi}} u}{P_{\text{csi}} u + \kappa_{RD} \sigma_D^2 (\phi_{D2} \min(g_{DR}, \lambda_R) - \varphi_{D2} + 1)}, \quad (57)$$

where γ_{D1}^{imp} is obtained by using (53) with $n = n_1$, γ_R^{imp} comes from (52) with $i = S$, $j = R$ and using (5), and γ_{D2}^{imp} comes from (52) with $i = R$, $j = D$ and using (26). Now, the information error probability for each link can be easily calculated as in the pCSI case, but with $\gamma_{D1} = \gamma_{D1}^{\text{imp}}$, $\gamma_R = \gamma_R^{\text{imp}}$ and $\gamma_{D2} = \gamma_{D2}^{\text{imp}}$.

V. NUMERICAL RESULTS

In this section, we present numerical results to investigate the performance of the system at finite blocklength with pCSI and iCSI at the receivers. Let $T_c = 2 \mu\text{s}$, thus $\sigma_D^2 = -110 \text{ dBm}$ is a valid assumption if a bandwidth around 1 MHz is assumed. We consider scenarios with stringent error probability and delay requirements, which are expected to be typical of URCS services in future 5G systems. Therefore, being ε_0 the target error probability and δ_0 the maximum allowable delay, $p_{\text{out}} \leq \varepsilon_0$ and $\delta \leq \delta_0$ must be satisfied. Let $\delta_0 = 8 \text{ ms} \rightarrow 4000$ channel uses and $\varepsilon_0 \in \{10^{-3}, 10^{-4}\}$.

Results are obtained by setting $\alpha = 2$, $d_{SR} = d_{SD} = d_{DR} = d$ and $\kappa = 10^3$, what is equivalent to 30 dB average signal power attenuation at a reference distance of 1 meter. Sensor nodes, S and R , are ultra-low consumption devices⁵ with $P_c = -30 \text{ dBm}$, and, following the state-of-the-art in circuit design, we consider $\eta = 0.5$ [38]. Moreover, $P_D = 50 \text{ dBm}$ and, unless stated otherwise, $k = 256$ bits. The utilized values of the system parameters are summarized in Table II.

TABLE II: System parameters

Parameter	Value	Parameter	Value
T_c	$2 \mu\text{s}$	κ	10^3
σ_D^2	-110 dBm	P_c	-30 dBm
δ_0	8 ms	η	0.5
ε_0	$\{10^{-3}, 10^{-4}\}$	P_D	50 dBm
α	2	k	256
d	$\{10, 40\} \text{ m}$	u	1

⁴This result has been widely used, e.g., [34], [36].

⁵Notice that sensors in that order of consumption in active mode already exist. Some examples can be found in [37].

$$\frac{\partial^2 \gamma^{\text{imp}}}{\partial P_{\text{csi}}^2} = -2g_{SD} n u^2 \frac{(\kappa \sigma^2 - P_c(n+u) + \min(g_{DS}, \lambda) \frac{\eta P_D^v}{\kappa})(\kappa \sigma^2 n - P_c(n+u) + \min(g_{DS}, \lambda) \frac{\eta P_D^v}{\kappa})}{\sigma^2 (\kappa n \sigma^2 - P_c(n+u) + \min(g_{DS}, \lambda) \frac{\eta P_D^v}{\kappa}) + (n-1)u P_{\text{csi}}}^3 \quad (54)$$

A. Ideal System

Herein we analyze the performance of an ideal system, where all the harvested energy is used for information transmission and the receiver has perfect knowledge of the channel while $P_{\text{csi}} = P_c = 0$ and $p_S = p_R = 0$. Notice that the outage probability equals the error probability for such systems. The performance under such assumptions, which are the most common in the literature, offers an upper-bound for the performance of practical systems. Numerical results are obtained with $d = 40\text{m}$.

Fig. 4a shows the minimum delay, $\delta^* = \min_{p_{\text{out}} \leq \varepsilon_0} \delta$, required to deliver messages of $k = 256$ bits while meeting the reliability constraints given by ε_0 . DC, and cooperative schemes with MRC ($n_1 = n_2 = n/2$) and SC ($n_1 = n_2 = n/2$, $n_1 = 0.2n$ and $n_2 = 0.8n$, $n_1 = 0.8n$ and $n_2 = 0.2n$), are compared. Here we assume infinite batteries while some analyzes assuming finite batteries are discussed later. Notice that the DC scheme is unable to reach a reliability around 99.99% ($\varepsilon_0 = 10^{-4}$) for a maximum allowable delay of 4000 channel uses. However, cooperation through R solves that problem reaching the desirable results. This is because the attained spatial diversity can improve either communication reliability, or can reduce the system delay for a target outage constraint as is the case here. Among the cooperative schemes, the SC setup with $n_1 < n_2$ presents the worst performance since the chances of communication errors increase in both $S \rightarrow D$ and $S \rightarrow R$ links simultaneously. While the opposite occurs for $n_1 > n_2$, although slightly. We can see that the optimum value for n is small, $n^* \leq 600$ channel uses, for the cooperative scenarios. On the other hand, the performance in terms of the time sharing parameter, β , is shown in Fig. 4b. When n increases, the required value for v tends to decrease, and therefore β decreases too. As the reliability requirements are more stringent, the WET phase must be larger, which is even more accentuated for the DC scheme. Notice that we use the same line to denote the MRC and SC techniques with $n_1 = n_2 = n/2$ since their performance is almost identical, which is expected from the comments made in Section III-C.

The minimum reachable outage probability, $p_{\text{out}}^* = \min_{\delta} p_{\text{out}}$, as a function of the system delay is shown in Fig. 5a. According to the figure, it is possible to reach a reliability of 99.995% ($\varepsilon_0 = 5 \times 10^{-5}$) with a delay of 8ms (4000 channel uses) when R cooperates with S . Without its assistance, the required delay would be higher than 10ms (5000 channel uses) and it is not shown in the figure. As shown in Fig. 5b, while reducing the message length, the minimum delay, required for a given reliability, decreases. Thus, it is possible to achieve a reliability around 99.99% without the relay assistance as long as the messages have no more than $k = 80$ bits. For a given reliability and based on (14), when k increases, n should also increase to avoid increasing the rate r , and consequently γ and

Fig. 4: (a) δ^* , (b) β^* , as a function of n with $B_{\text{max}} = \infty$.

$C(\gamma)$ tend to increase. In addition, increasing v decelerates the capacity reduction and n is not required to be so high, thus there is a trade-off between increasing n and v . Whatever the case, increasing k for a given ε_0 renders an unavoidable increase in the total number of required channel uses, hence larger delay. On the other hand, Figs. 4 and 5 corroborate the accuracy of expression (20), claimed when discussing Fig. 2.

B. Non-Ideal channel estimation

Herein, we analyze a more realistic case, where part of the harvested energy is required by the devices (namely S and R) to feed their circuits and to transmit the pilot signals in order to acquire CSI at the receivers. The imperfection of the acquired CSI is also taken into account.

1) *Outages due to insufficient energy harvested:* Fig. 6 shows the probability that the harvested energy at S is insufficient simultaneously for channel estimation and for satisfying other consumption requirements at S . Notice that the curves appear as functions of n , where $n = n_1$ for the cooperative case, and $v \in \{1000, 3000\}$ channel uses. In Fig. 6a, different power levels for channel estimation were tested, $P_{\text{csi}} = \{0, -20\}$ dBm, with $u = 1$ channel use. Thus, $P_{\text{csi}} T_c$ is equivalent to the energy used for CSI estimation.

(a)

(b)

Fig. 5: (a) p_{out}^* , (b) δ^* , as a function of δ , k , with $B_{\text{max}} = \infty$.

Notice that when n increases, p_S also increases, which is expected from (13), since the sensor remains active longer and its circuits require more energy. Moreover, a larger value of v leads to more energy being harvested, decreasing p_S . Also, we can notice that it is impossible to reach a high reliability when $d = 40\text{m}$ and consequently from now on we use $d = 10\text{m}$. The performance increases by decreasing P_{csi} , specially for a small n . Although it is not shown in the figure, the performance improves much slower by decreasing P_{csi} below -20dBm . Thus, it seems convenient to always use an energy no greater than $uT_c P_{\text{csi}}|_{u=1, P_{\text{csi}}=-20\text{dBm}}$, although we have to keep in mind that when P_{csi} decreases, the communication errors increase due to a greater imperfection on the channel estimates, which is analyzed later. On the other hand, Fig. 6b shows p_S when $P_{\text{csi}} = -20\text{dBm}$ and $u = \{1, 10, 30, 50\}$ channel uses. When u increases, the required energy for CSI estimate increases thus, the probability that the harvested energy is insufficient for attending those requirements also increases. This dependency starts decreasing when n increases since the influence of u on the delay and the active sensor phase reduces. We argue that, from the standpoint of p_S , the more suitable strategy is to set $u = 1$ channel use and properly choose P_{csi} in order to minimize the impact on P_c , which is confirmed when analyzing the communication error expressions.

2) *On the imperfect estimates and finite battery:* Fig. 7 shows the DC system performance in terms of outage probability as a function of n , and considering $u = 1$, $v = 2000$

(a)

(b)

Fig. 6: p_S as a function of n with $B_{\text{max}} = \infty$. (a) $d \in \{10, 40\}\text{m}$, $P_{\text{csi}} \in \{0, -20\}\text{dBm}$ and $u = 1$ channel use, (b) $d = 10\text{m}$ and $P_{\text{csi}} = -20\text{dBm}$.

channel uses. In Fig. 7a, we set $B_{\text{max}} = \infty$ and evaluate the performance for $P_{\text{csi}} \in \{-30, -20, -10\}\text{dBm}$ when the estimates are considered perfect and imperfect. The performance gap when assuming pCSI and iCSI is very perceptible for low pilot power, which means that the used energy is still insufficient and the number of pilot symbols and/or its transmit power must be increased. Notice that a proper energy value is around $-10\text{dBm} \cdot T_c$, which is larger than the one we had recommended in Section V-B1 since now the channel estimation affects the error probability when there is communication, and a relatively larger value is required. Consequently, the results shown in Fig. 7b are with $P_{\text{csi}} = -10\text{dBm}$, but now considering the finite battery case. Notice that, when $B_{\text{max}} = 10^{-7}\text{J}$ the system performance approximates the case when $B_{\text{max}} = \infty$ (blue and red curves are almost coincident). The general performance gets worse when $B_{\text{max}} = 10^{-8}\text{J}$, although practical batteries have storage capacities much greater than the values under consideration here, which means that models assuming infinite battery are adequate in this context.

Fig. 8a shows the optimum power for pilot transmission, $P_{\text{csi}}^* = \arg \min_{P_{\text{csi}}} p_{\text{out}}$, when $u = 1$ channel use, and Fig. 8b shows the outage probability reached for those values. Interestingly, P_{csi}^* depends on the pair (n, v) . Configurations with a large v also require a large P_{csi} , and an even larger pilot transmission power is needed when n is small. On the

Fig. 7: p_{out} as a function of n with $u = 1$ and $v = 2000$ channel uses. (a) $P_{\text{csi}} \in \{-30, -20, -10\} \text{ dBm}$ and $B_{\text{max}} = \infty$, (b) $P_{\text{csi}} = -10 \text{ dBm}$ and $B_{\text{max}} \in \{10^{-8}, 10^{-7}, \infty\} \text{ J}$.

other hand, for large n a small P_{csi} is adequate, which is evinced when v is small. As shown in Section V-A, the optimal performance occurs for relatively small and large n and v , respectively, and from now on we use -15 dBm as the pilot transmit power, which seems appropriate according to Fig. 8. Once again, a small performance gap can be noticed when comparing configurations with finite and infinite battery, reinforcing the convenience of the analysis with infinite battery assumption for these scenarios.

3) *On the performance of the cooperative scheme:* In this subsection we analyze scenarios where devices are assumed to be equipped with infinite battery. Fig. 9a shows the minimum delay required to deliver the messages while reaching the reliability constraints. Notice that a reliability around 99.99% ($\varepsilon_0 = 10^{-4}$), is impossible to be reached without the relay assistance within the allowable delay (10ms). Among the different combination configurations at D , the SC scheme⁶ with $n_1 < n_2$ performs better for a given n . Obviously, the smaller n_1 is, the longer is the time that S remains inactive, reducing the energy consumption of its circuits. Differently from the ideal case discussed in Section V-A, $n^* < 200$ channel uses and it is not shown in the figure. SC and MRC with $n_1 = n_2$ perform similarly, while SC with $n_1 > n_2$ forces a higher

⁶Notice that $n \geq 500$ channel uses since $n_1 = 0.2 \cdot 500 = 100$ channel uses, which is the lower bound to an accurate approximation when using (14) as demonstrated in [39, eq.(59)] for quasi-static fading channels and sufficiently large values of $n \geq 100$.

Fig. 8: (a) P_{csi}^* and (b) p_{out} , as functions of n and v with $u = 1$ channel use.

energy consumption of the S circuits, remaining less energy resources for pilot and information transmission, although its performance still overcomes the DC case. Increasing P_c , the performance gap among the SC schemes with $n_1 > n_2$, $n_1 = n_2$ and $n_1 < n_2$ also increases, since the impact of n would be greater. As shown in Fig. 9b, the larger n is, the smaller is the required fraction of time for WET, although a slight decrease is noticeable⁷ due to the non-trivial value of P_c . In order to achieve a high reliability, the larger energy expenditure is required, thus β gets closer to unity. Finally, the performance gap among the cooperative schemes with different blocklength in the broadcast and cooperation phase becomes more relevant when reliability constraints are more stringent.

The minimum reachable outage probability is shown in Fig. 10a, while the relative frequency of the outages due to the energy insufficiency at S as a function of the message length is shown in Fig. 10b. We set $n = 500$ channel uses. According to Fig. 10a, a delay of 10ms (5000 channel uses) when using the DC or cooperative schemes with $n_1 \geq n_2$, is insufficient to reach a reliability around 99.99%. For those schemes, the required delay would have to be much higher to fulfill such requirement, specially for the DC case. With SC and $n_1 = 100$, $n_2 = 400$ channel uses, that requirement can be attended with a delay around 6.4ms (~ 3200 channel uses). The convenience of choosing $n_1 < n_2$ was already discussed above. On the other hand, a main conclusion coming from

⁷Notice that in the ideal system scenario discussed in Section V-A, that decrease is much pronounced.

(a)

(b)

Fig. 9: (a) δ^* , (b) β^* , as a function of n with $B_{\max} = \infty$.

Fig. 10b is that increasing k would not significantly degrade the performance for the cooperative schemes since the main cause of outage is due to energy insufficiency, meanwhile for the DC scheme, increasing k up to 300 bits could decrease the performance since the communication errors become almost 18% of the outage events.

VI. CONCLUSION

We evaluated a cooperative wireless-powered communication network at finite blocklength regime and with limited battery capacity. We modeled a realistic system where other power consumption sources beyond data transmission, such as circuit and baseband processing and pilot transmission, are taken into account along with imperfections on channel estimates. We also characterized the error probability for direct communication and cooperative schemes, while attaining closed-form expressions in ideal direct communication systems.

Our results demonstrate that there is an optimum pilot transmit power for channel estimation, which depends on system parameters such as n , v , P_D and k . We show that, whenever more energy is harvested, the optimum pilot energy must be greater, and even more for small n . An important remark is that the energy devoted to the CSI acquisition, which depends on the power and utilized time, has to be taken into account due to the inherent energy and delay constraints of the discussed scenarios. Also, while infinite battery assumption is shown to be permissible for the scenarios under discussion since the

(a)

(b)

Fig. 10: (a) Minimum reachable outage probability as a function of the delay, (b) Relative frequency of outages due to energy insufficiency at S as a function of the message length, with $P_{\text{csi}} = -15\text{dBm}$, $B_{\max} = \infty$, $u = 1$ and $n = 500$ channel uses.

energy harvested is very low; considering other power consumption sources beyond data transmission is crucial because they constitute a non-negligible cause of outage. Moreover, we show that cooperation is important in order to reach URC-S requirements, while also illustrating the convenience of using a small blocklength during the first WIT phase, since S stays active for less time and thus decreases its power consumption. Finally, a greater flexibility regarding the latency target allows for reaching stringent reliability constraints and vice-versa.

As a future work we intend to analyze multi-relay scenarios for short message communications, while proposing practical schemes. It could be also interesting to incorporate other strategies to these scenarios such as power allocation and/or HARQ.

APPENDIX A PROOF OF THEOREM 1

Let $x = g_{DS}$ and $y = g_{SD}$ for shortening notation. Since the system is considered ideal, we have $p_s = 0 \rightarrow p_{\text{out}} = \mathbb{E}[\epsilon(\gamma, k, n)]$, and $\gamma = \phi y \min(x, \lambda_s)$. We resort to the first order approximation of $Q(f(\gamma))$, $f(\gamma) = \frac{C(\gamma)-r}{\sqrt{V(\gamma)/n}}$, given by [11], [33]

$$Q(f(\gamma)) \approx \Omega(\gamma) = \begin{cases} 1, & \gamma \leq \varrho \\ \frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\gamma - \theta), & \varrho < \gamma < \vartheta \\ 0, & \gamma \geq \vartheta \end{cases}, \quad (58)$$

where $\theta = 2^{k/n} - 1$, $\psi = \sqrt{\frac{n}{2\pi}}(2^{2k/n} - 1)^{-\frac{1}{2}}$, $\varrho = \theta - \frac{1}{\psi}\sqrt{\frac{\pi}{2}}$ and $\vartheta = \theta + \frac{1}{\psi}\sqrt{\frac{\pi}{2}}$.

A. Derivation of (20)

The expression in (58) can be reformulated using $\gamma = \phi y \min(x, \lambda_s)$ as follows

$$\Omega(\phi y \min(x, \lambda_s)) = \begin{cases} 1, & y \leq \frac{\varrho}{x\phi}, x < \lambda_s \\ 1, & y \leq \frac{\varrho}{\lambda_s\phi}, x \geq \lambda_s \\ \frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\phi xy - \theta), & \frac{\varrho}{x\phi} < y < \frac{\vartheta}{x\phi}, x < \lambda_s \\ \frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\phi \lambda_s y - \theta), & \frac{\varrho}{\lambda_s\phi} < y < \frac{\vartheta}{\lambda_s\phi}, x \geq \lambda_s \\ 0, & \text{otherwise} \end{cases}. \quad (59)$$

Now, by substituting (59) into (20) since $Q(f(\phi y \min(x, \lambda_s))) \approx \Omega(\phi y \min(x, \lambda_s))$, we get

$$\begin{aligned} p_{\text{out}} &\approx \int_0^{\lambda_s} \int_0^{\frac{\varrho}{x\phi}} e^{-x-y} dy dx + \int_{\lambda_s}^{\infty} \int_0^{\frac{\varrho}{\lambda_s\phi}} e^{-x-y} dy dx + \\ &+ \int_0^{\lambda_s} \int_{\frac{\varrho}{x\phi}}^{\frac{\vartheta}{x\phi}} \left(\frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\phi xy - \theta) \right) e^{-x-y} dy dx + \\ &+ \int_{\lambda_s}^{\infty} \int_{\frac{\varrho}{\lambda_s\phi}}^{\frac{\vartheta}{\lambda_s\phi}} \left(\frac{1}{2} - \frac{\psi}{\sqrt{2\pi}}(\phi \lambda_s y - \theta) \right) e^{-y-x} dy dx \\ &\approx I_1 + I_2 + I_3 + I_4, \end{aligned} \quad (60)$$

where I_j is the j -th adding integral with $j = 1, 2, 3, 4$, and each of one has to be solved to find a closed-form expression. To be able to do so we first attain the following results

$$\begin{aligned} \int_0^c e^{-x-\frac{d}{x}} dx &\stackrel{(a)}{=} \int_0^c \sum_{i=0}^{\infty} \frac{(-1)^i x^i}{i!} e^{-\frac{d}{x}} dx = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} \int_0^c x^i e^{-\frac{d}{x}} dx \\ &\stackrel{(b)}{=} \sum_{i=0}^{\infty} \frac{(-1)^i x^{i+1} \text{Ei}(i+2, \frac{d}{x})}{i!} \Big|_0^c \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i c^{i+1} \text{Ei}(i+2, \frac{d}{c})}{i!} - \lim_{x \rightarrow 0} \sum_{i=0}^{\infty} \frac{(-1)^i x^{i+1} \text{Ei}(i+2, \frac{d}{x})}{i!} \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i c^{i+1} \text{Ei}(i+2, \frac{d}{c})}{i!}, \end{aligned} \quad (61)$$

$$\begin{aligned} \int_0^c x e^{-x-\frac{d}{x}} dx &\stackrel{(a)}{=} \int_0^c \sum_{i=0}^{\infty} \frac{(-1)^i x^{i+1}}{i!} e^{-\frac{d}{x}} dx = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} \int_0^c x^{i+1} e^{-\frac{d}{x}} dx \\ &\stackrel{(b)}{=} \sum_{i=0}^{\infty} \frac{(-1)^i x^{i+2} \text{Ei}(i+3, \frac{d}{x})}{i!} \Big|_0^c \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i c^{i+2} \text{Ei}(i+3, \frac{d}{c})}{i!} - \lim_{x \rightarrow 0} \sum_{i=0}^{\infty} \frac{(-1)^i x^{i+2} \text{Ei}(i+3, \frac{d}{x})}{i!} \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i c^{i+2} \text{Ei}(i+3, \frac{d}{c})}{i!}, \end{aligned} \quad (62)$$

where (a) comes from the Taylor series expansion of e^{-x} and (b) comes from the definition of the exponential integral [28]. Now we proceed solving each I_j integral as shown in (63), (64), (65), (66). The last two are on the top of the next page and the final expressions for I_1 and I_2 are obtained from using (61) and (62) as

$$\begin{aligned} I_1 &= \int_0^{\lambda_s} \int_0^{\frac{\varrho}{x\phi}} e^{-x-y} dy dx = \int_0^{\lambda_s} (e^{-x} - e^{-x-\frac{\varrho}{x\phi}}) dx = 1 - e^{-\lambda_s} - \int_0^{\lambda_s} e^{-x-\frac{\varrho}{x\phi}} dx \\ &= 1 - e^{-\lambda_s} - \sum_{i=0}^{\infty} \frac{(-1)^i \lambda_s^{i+1} \text{Ei}(i+2, \frac{\varrho}{\lambda_s\phi})}{i!}, \end{aligned} \quad (63)$$

$$\begin{aligned} I_2 &= \int_{\lambda_s}^{\infty} \int_0^{\frac{\varrho}{\lambda_s\phi}} e^{-x-y} dy dx = \int_{\lambda_s}^{\infty} (1 - e^{-\frac{\varrho}{\lambda_s\phi}}) e^{-x} dx \\ &= (1 - e^{-\frac{\varrho}{\lambda_s\phi}}) e^{-\lambda_s} = e^{-\lambda_s} - e^{-\lambda_s - \frac{\varrho}{\lambda_s\phi}}. \end{aligned} \quad (64)$$

Notice that the representations in series in (63) and (65) are convergent since all the results come from the Taylor series expansion of $e^{-f(x)}$. Now, substituting (63), (64), (65), (66) into (60), and by selecting a finite number of terms in the summation, $i_M + 1$, we attain the approximation in (20). $\square$

B. Derivation of (21)

Let $z = x \cdot y$, which according to [24, Eq. 5] with $m = 1$, has PDF given by

$$f_Z(z) = 2 K_0(2\sqrt{z}), \quad z > 0. \quad (67)$$

Now, working on (12), which is equal to (21) since $p_s = \varphi = 0$ from the ideal system assumption, we get

$$\begin{aligned} p_{\text{out}} &\approx \int_0^{\infty} Q\left(\frac{C(\phi z) - k/n}{V(\phi z)/n}\right) f_Z(z) dz \\ &= 2 \int_0^{\infty} Q\left(\frac{C(\phi z) - k/n}{V(\phi z)/n}\right) K_0(2\sqrt{z}) dz. \end{aligned} \quad (68)$$

Using $Q(f(\phi z)) \approx \Omega(\phi z)$ given in (58), (68) approximates to

$$\begin{aligned} p_{\text{out}} &\approx 2 \int_0^{\infty} K_0(2\sqrt{z}) \Omega(\phi z) dz \approx 2 \int_0^{\varrho/\phi} K_0(2\sqrt{z}) dz \\ &\approx 2 I_5(z) \Big|_{z=0}^{\frac{\varrho}{\phi}} + \left(1 + \frac{2\psi\theta}{\sqrt{2\pi}}\right) I_5(z) \Big|_{z=\frac{\varrho}{\phi}}^{\frac{\vartheta}{\phi}} - \frac{2\psi\phi}{\sqrt{2\pi}} I_6(z) \Big|_{z=\frac{\varrho}{\phi}}^{\frac{\vartheta}{\phi}}, \end{aligned} \quad (69)$$

where $I_5(z) = \int K_0(2\sqrt{z}) dz$ and $I_6(z) = \int z K_0(2\sqrt{z}) dz$ are solved next

$$\begin{aligned} I_5(z) &= \int K_0(2\sqrt{z}) dz \stackrel{(a)}{=} \frac{1}{2} \int q K_0(q) dq \stackrel{(b)}{=} -\frac{1}{2} q K_1(q) + c \\ &\stackrel{(c)}{=} -\sqrt{z} K_1(2\sqrt{z}) + c, \end{aligned} \quad (70)$$

$$\begin{aligned}
I_3 &= \int_0^{\lambda_s \frac{\vartheta}{x\phi}} \int_{\frac{\varrho}{x\phi}}^{\frac{\vartheta}{x\phi}} \left(\frac{1}{2} - \frac{\psi}{\sqrt{2\pi}} (\phi xy - \theta) \right) e^{-x-y} dy dx = \left(\frac{1}{2} + \frac{\psi\theta}{\sqrt{2\pi}} \right) \int_0^{\lambda_s \frac{\vartheta}{x\phi}} \int_{\frac{\varrho}{x\phi}}^{\frac{\vartheta}{x\phi}} e^{-x-y} dy dx - \frac{\psi\phi}{\sqrt{2\pi}} \int_0^{\lambda_s \frac{\vartheta}{x\phi}} \int_{\frac{\varrho}{x\phi}}^{\frac{\vartheta}{x\phi}} xy e^{-x-y} dy dx \\
&= \left(\frac{1}{2} + \frac{\psi\theta}{\sqrt{2\pi}} \right) \int_0^{\lambda_s} (e^{-x-\frac{\varrho}{\phi x}} - e^{-x-\frac{\vartheta}{\phi x}}) dx - \frac{\psi\phi}{\sqrt{2\pi}} \int_0^{\lambda_s} x e^{-x-\frac{\varrho}{\phi x}} dx - \frac{\psi\varrho}{\sqrt{2\pi}} \int_0^{\lambda_s} e^{-x-\frac{\varrho}{\phi x}} dx + \frac{\psi\phi}{\sqrt{2\pi}} \int_0^{\lambda_s} x e^{-x-\frac{\vartheta}{\phi x}} dx + \frac{\psi\vartheta}{\sqrt{2\pi}} \int_0^{\lambda_s} e^{-x-\frac{\vartheta}{\phi x}} dx \\
&= \sum_{i=0}^{\infty} \frac{(-1)^i \lambda_s^{i+1}}{i!} \left[\left(\frac{1}{2} + \frac{\psi(\theta-\varrho)}{\sqrt{2\pi}} \right) \text{Ei}\left(i+2, \frac{\varrho}{\lambda_s \phi}\right) - \left(\frac{1}{2} + \frac{\psi(\theta-\vartheta)}{\sqrt{2\pi}} \right) \text{Ei}\left(i+2, \frac{\vartheta}{\lambda_s \phi}\right) + \frac{\psi\phi\lambda_s}{\sqrt{2\pi}} \left(\text{Ei}\left(i+3, \frac{\vartheta}{\lambda_s \phi}\right) - \text{Ei}\left(i+3, \frac{\varrho}{\lambda_s \phi}\right) \right) \right] \quad (65)
\end{aligned}$$

$$\begin{aligned}
I_4 &= \int_{\lambda_s \frac{\varrho}{\phi}}^{\infty} \int_{\lambda_s \frac{\varrho}{\phi}}^{\lambda_s \frac{\vartheta}{\phi}} \left(\frac{1}{2} - \frac{\psi}{\sqrt{2\pi}} (\phi \lambda_s y - \theta) \right) e^{-x-y} dy dx = \left(\frac{1}{2} + \frac{\psi\theta}{\sqrt{2\pi}} \right) \int_{\lambda_s \frac{\varrho}{\phi}}^{\infty} \int_{\lambda_s \frac{\varrho}{\phi}}^{\lambda_s \frac{\vartheta}{\phi}} e^{-x-y} dy dx - \frac{\psi\phi\lambda_s}{\sqrt{2\pi}} \int_{\lambda_s \frac{\varrho}{\phi}}^{\infty} \int_{\lambda_s \frac{\varrho}{\phi}}^{\lambda_s \frac{\vartheta}{\phi}} y e^{-x-y} dy dx \\
&= \left(\frac{1}{2} + \frac{\psi\theta}{\sqrt{2\pi}} \right) (e^{-\lambda_s \frac{\varrho}{\phi}} - e^{-\lambda_s \frac{\vartheta}{\phi}}) e^{-\lambda_s} + \frac{\psi\phi\lambda_s}{\sqrt{2\pi}} \left(\left(\frac{\vartheta}{\lambda_s \phi} + 1 \right) e^{-\lambda_s \frac{\vartheta}{\phi}} - \left(\frac{\varrho}{\lambda_s \phi} + 1 \right) e^{-\lambda_s \frac{\varrho}{\phi}} \right) e^{-\lambda_s} \\
&= \left(\frac{1}{2} + \frac{\psi}{\sqrt{2\pi}} (\theta - \varrho - \lambda_s \phi) \right) e^{-\lambda_s - \lambda_s \frac{\varrho}{\phi}} - \left(\frac{1}{2} + \frac{\psi}{\sqrt{2\pi}} (\theta - \vartheta - \lambda_s \phi) \right) e^{-\lambda_s - \lambda_s \frac{\vartheta}{\phi}} \quad (66)
\end{aligned}$$

$$\begin{aligned}
I_6(z) &= \int z K_0(2\sqrt{z}) dz \stackrel{(d)}{=} \int \frac{q^3}{8} K_0(q) dq \stackrel{(e)}{=} \int \left(\frac{q^3}{8} K_2(q) - \frac{q^2}{4} K_1(q) \right) dq \\
&\stackrel{(f)}{=} -\frac{q^3}{8} K_3(q) + \frac{1}{4} q^2 K_2(q) + \varsigma \\
&\stackrel{(g)}{=} -\frac{q^3}{8} \left(\frac{8}{q^2} K_1(q) + \frac{4}{q} K_0(q) + K_1(q) \right) + \frac{q^2}{4} \left(\frac{2}{q} K_1(q) + K_0(q) \right) + \varsigma \\
&\stackrel{(h)}{=} -\frac{q}{2} \left(\frac{q^2}{4} + 1 \right) K_1(q) - \frac{q^2}{4} K_0(q) + \varsigma \\
&\stackrel{(i)}{=} \sqrt{z}(z+1) K_1(2\sqrt{z}) - z K_0(2\sqrt{z}) + \varsigma, \quad (71)
\end{aligned}$$

where (a) comes from the transformation $q = 2\sqrt{z}$, (b) is based on $\int q^{t+1} K_t(q) dq = -q^{t+1} K_{t+1}(q)$ [40], in (c) and (d) we reapply the same property as in (a), (e) is reached by using $K_{t+1}(q) - K_{t-1}(q) = \frac{2t}{q} K_t(q)$ [40], in (f) and (g) we reapply the same properties as in (b) and (e) respectively, obtaining (h) is straightforward through algebraic transformations, (i) comes from using property in (a) again, and ς is a constant.

Substituting (70) and (71) into (69) yields

$$\begin{aligned}
p_{\text{out}} &\approx -2\sqrt{z} K_1(2\sqrt{z}) \Big|_{\frac{\varrho}{\phi}}^{\frac{\vartheta}{\phi}} - \left(1 + \frac{2\psi\theta}{\sqrt{2\pi}} \right) \sqrt{z} K_1(2\sqrt{z}) \Big|_{\frac{\varrho}{\phi}}^{\frac{\vartheta}{\phi}} \\
&\quad + \frac{2\psi\phi}{\sqrt{2\pi}} \left[\sqrt{z}(z+1) K_1(2\sqrt{z}) + z K_0(2\sqrt{z}) \right] \Big|_{\frac{\varrho}{\phi}}^{\frac{\vartheta}{\phi}} \\
&\approx 1 - \sqrt{\frac{\varrho}{\phi}} \left[1 + \left(\frac{2\psi}{\sqrt{2\pi}} \right) (\varrho + \phi - \theta) \right] K_1\left(2\sqrt{\frac{\varrho}{\phi}} \right) + \\
&\quad - \frac{2\psi\varrho}{\sqrt{2\pi}} K_0\left(2\sqrt{\frac{\varrho}{\phi}} \right) - \sqrt{\frac{\vartheta}{\phi}} \left[1 - \left(\frac{2\psi}{\sqrt{2\pi}} \right) (\vartheta + \phi - \theta) \right] \cdot \\
&\quad \cdot K_1\left(2\sqrt{\frac{\vartheta}{\phi}} \right) + \frac{2\psi\vartheta}{\sqrt{2\pi}} K_0\left(2\sqrt{\frac{\vartheta}{\phi}} \right), \quad (72)
\end{aligned}$$

where the last equality comes from $2 \lim_{z \rightarrow 0} \sqrt{z} K_1(2\sqrt{z}) = 2\frac{1}{2} = 1$ (easily checked by doing the series expansion at $z = 0$). $\square$

REFERENCES

- [1] K. Campbell, J. Diffley, B. Flanagan, B. Morelli, B. O'Neil, and F. Sideco, "The 5G economy : How 5G technology will contribute to the global economy," Tech. Rep. January, 2017.
- [2] P. Popovski, "Ultra-reliable communication in 5G wireless systems," in *2014 Int. Conf. on Ubiquitous Connectivity (5GU)*. IEEE, Nov 2014, pp. 146–151.
- [3] N. A. Johansson, Y. P. E. Wang, E. Eriksson, and M. Hessler, "Radio access for ultra-reliable and low-latency 5G communications," in *IEEE Int. Conf. on Commun. Workshop (ICCW)*, June 2015, pp. 1184–1189.
- [4] O. N. C. Yilmaz, Y. P. E. Wang, N. A. Johansson, N. Brahmi, S. A. Ashraf, and J. Sachs, "Analysis of ultra-reliable and low-latency 5G communication for a factory automation use case," in *IEEE Int. Conf. on Commun. Workshop (ICCW)*, June 2015, pp. 1190–1195.
- [5] A. Zanella, N. Bui, A. Castellani, L. Vangelista, and M. Zorzi, "Internet of things for smart cities," *IEEE IoT J.*, vol. 1, no. 1, pp. 22–32, 2014.
- [6] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 5, pp. 1989–2001, May 2013.
- [7] E. G. Kilinc, C. Baj-Rossi, S. Ghoreishizadeh, S. Riario, F. Stradolini, C. Boero, G. D. Micheli, F. Maloberti, S. Carrara, and C. Dehollain, "A system for wireless power transfer and data communication of long-term bio-monitoring," *IEEE Sensors Journal*, vol. 15, no. 11, pp. 6559–6569, Nov 2015.
- [8] W. Ejaz, M. Naeem, M. Basharat, A. Anpalagan, and S. Kandeepan, "Efficient wireless power transfer in software-defined wireless sensor networks," *IEEE Sensors Journal*, vol. 16, no. 20, pp. 7409–7420, Oct 2016.
- [9] L. R. Varshney, "Transporting information and energy simultaneously," in *IEEE Int. Symp. on Inf. Theory*, July 2008, pp. 1612–1616.
- [10] P. Grover and A. Sahai, "Shannon meets Tesla: Wireless information and power transfer," in *IEEE Int. Symp. on Inf. Theory (ISIT)*, June 2010, pp. 2363–2367.
- [11] B. Makki, T. Svensson, and M. Zorzi, "Wireless energy and information transmission using feedback: Infinite and finite block-length analysis," *IEEE Trans. Commun.*, vol. 64, no. 12, pp. 5304–5318, Dec 2016.
- [12] T. A. Khan, R. W. Heath Jr, and P. Popovski, "On wirelessly powered communications with short packets," in *IEEE Globecom 2016 - Workshops*, Washington DC, USA, 2016.
- [13] R. Devassy, G. Durisi, P. Popovski, and E. G. Ström, "Finite-blocklength analysis of the ARQ-protocol throughput over the Gaussian collision channel," in *Int. Symp. on Commun., Contr. and Sig. Process. (ISCCSP)*. IEEE, May 2014, pp. 173–177.
- [14] G. Durisi, T. Koch, and P. Popovski, "Toward massive, ultrareliable, and low-latency wireless communication with short packets," *Proceedings of the IEEE*, vol. 104, no. 9, pp. 1711–1726, Sept 2016.

- [15] Y. Polyanskiy, H. V. Poor, and S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Trans. on Inf. Theory*, vol. 56, no. 5, pp. 2307–2359, 2010.
- [16] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: Architecture design and rate-energy tradeoff," *IEEE Trans. on Commun.*, vol. 61, no. 11, pp. 4754–4767, 2013.
- [17] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 7, pp. 3622–3636, 2013.
- [18] —, "Wireless-powered relays in cooperative communications: Time-switching relaying protocols and throughput analysis," *IEEE Trans. on Commun.*, vol. 63, no. 5, pp. 1607–1622, May 2015.
- [19] G. L. Moritz, J. L. Rebelatto, R. D. Souza, B. F. Uchôa-Filho, and Y. Li, "Time-switching uplink network-coded cooperative communication with downlink energy transfer," *IEEE Trans. on Sig. Process.*, vol. 62, no. 19, pp. 5009–5019, Oct 2014.
- [20] —, "On the performance of network-coded cooperative communications with wireless energy transfer under a realistic power consumption model," in *2014 IFIP Wireless Days (WD)*, Nov 2014, pp. 1–6.
- [21] H. Chen, Y. Li, J. L. Rebelatto, B. F. Uchôa-Filho, and B. Vucetic, "Harvest-then-cooperate: Wireless-powered cooperative communications," *IEEE Trans. on Sig. Process.*, vol. 63, no. 7, pp. 1700–1711, April 2015.
- [22] M. Haghifam, B. Makki, M. Nasiri-Kenari, T. Svensson, and M. Zorzi, "Wireless-powered relaying with finite block-length codes," *arXiv preprint arXiv:1611.05995*, 2016.
- [23] A. Tandon, M. Motani, and L. R. Varshney, "Subblock energy-constrained codes for simultaneous energy and information transfer," in *IEEE Int. Symp. on Inf. Theory (ISIT)*, July 2016, pp. 1969–1973.
- [24] O. L. A. López, H. Alves, R. D. Souza, and E. M. G. Fernández, "Ultra-reliable short-packet communications with wireless energy transfer," *IEEE Sig. Process. Lett.*, vol. 24, no. 4, pp. 387–391, April 2017.
- [25] O. L. A. López, R. D. Souza, H. Alves, and E. M. G. Fernández, "Ultra reliable short message relaying with wireless power transfer," in *IEEE Int. Conf. on Communications (ICC)*, May 2017, pp. 1–6.
- [26] O. L. A. López, E. M. G. Fernández, H. Alves, and R. D. Souza, "Wireless Powered Communications with Finite Battery and Finite Blocklength," *ArXiv e-prints*, May 2017.
- [27] S. Schiessl, H. Al-Zubaidy, M. Skoglund, and J. Gross, "Analysis of wireless communications with finite blocklength and imperfect channel knowledge," *arXiv preprint arXiv:1608.08445*, 2016.
- [28] A. Jeffrey and D. Zwillinger, *Table of integrals, series, and products*. Academic Press, 2007.
- [29] A. Goldsmith, *Wireless communications*. Cambridge Uni. Press, 2005.
- [30] W. Yang, G. Durisi, T. Koch, and Y. Polyanskiy, "Quasi-static multiple-antenna fading channels at finite blocklength," *IEEE Trans. on Inf. Theory*, vol. 60, no. 7, pp. 4232–4265, 2014.
- [31] P. Nouri, H. Alves, R. D. Souza, and M. Latva-aho, "Ultra-reliable short message cooperative relaying protocols under Nakagami-m fading," in *Int. Symp. on Wireless Commun. Systems (ISWCS)*, Aug 2017.
- [32] Y. Hu, J. Gross, and A. Schmeink, "On the capacity of relaying with finite blocklength," *IEEE Trans. on Veh. Tech.*, vol. 65, no. 3, pp. 1790–1794, 2016.
- [33] B. Makki, T. Svensson, and M. Zorzi, "Finite block-length analysis of the incremental redundancy HARQ," *IEEE Wireless Commun. Lett.*, vol. 3, no. 5, pp. 529–532, Oct 2014.
- [34] M. C. Gursoy, "On the capacity and energy efficiency of training-based transmissions over fading channels," *IEEE Trans. on Inf. Theory*, vol. 55, no. 10, pp. 4543–4567, Oct 2009.
- [35] M. Medard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel," *IEEE Trans. on Inf. Theory*, vol. 46, no. 3, pp. 933–946, May 2000.
- [36] S. J. Haghighi, D. J. Dechene, A. Shami, S. Primak, and X. Wang, "On energy efficiency of pilot assisted modulation schemes," in *IEEE Sarnoff Symp.*, April 2010, pp. 1–5.
- [37] E. Hestnes, "Low power inertial measurement unit for internet of things applications," Master's thesis, NTNU, 2016.
- [38] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless networks with RF energy harvesting: A contemporary survey," *IEEE Commun. Surveys and Tut.*, vol. 17, no. 2, pp. 757–789, 2015.
- [39] J. Yang, "Achievable rate for energy harvesting channel with finite blocklength," in *IEEE Int. Symp. on Inf. Theory*. IEEE, June 2014, pp. 811–815.
- [40] M. Kreh, "Bessel functions," *Lecture Notes, Penn State-Göttingen Summer School on Number Theory*, vol. 82, 2012.