

Performance Analysis for Multi-Source Multi-Relay Transmission over κ - μ Fading Channels

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Abstract—We derive the outage probability of a multi-source multi-relay transmission system, where all the links experience κ - μ fading variations. The source-relay links are assumed to be non-orthogonal multiple access channels (MACs). Two transmission schemes are considered for relay-destination transmission, i.e., non-orthogonal maximum ratio transmission (MRT) and orthogonal transmission with joint-decoding (JD) at the destination. The outage probability is formulated by taking into consideration the different decoding results at the relays. It is found that, with or without the impact of line-of-sight (LOS) component or the number of multipath clusters in channels, the outage performance of the system with JD is superior to that with MRT. Furthermore, we investigate the impact of the geometric gain based relay location and power allocation for relays on the performance.

I. INTRODUCTION

The performance degradation in wireless communications is usually caused by signal fading in transmission channels. The statistical properties of fading are well modelled by a number of distributions, e.g. Rayleigh, Rician, and Nakagami-m models. Recently, Yacoub *et al.* proposed two more generalized fading models, κ - μ and η - μ , which are more compliant with measurement data than the other models [1].

Cooperative communication is a scheme which could offer spatial diversity gains to mitigate the effects of fading in the propagation medium [2]. Various protocols have been proposed to achieve the benefits of cooperative communication, and the analysis for relaying schemes has been conducted over κ - μ and η - μ fading channels [3]–[5]. However, the extension to lossy decode-and-forward (DF) [6], also known as lossy-forwarding (LF), has not been studied yet.

The LF relaying scheme developed from the conventional DF relaying. Unlike the DF relaying, the LF relay always forwards the erroneously decoded information to the destination. In this sense, source coding with side information can be utilized to obtain the achievable rate region [7, Section 10.4]. It has been proved that LF significantly outperforms the conventional DF in terms of outage probability [8]. He *et al.* first introduced the LF technique to multi-source multi-relaying system with non-orthogonal multiple access channels (MACs) [9]. However, only Rayleigh fading with non line-of-sight (LOS) component is assumed for the channel model. Moreover, the geometric gain of the links and power allocation for the nodes are not considered, which are crucial in topology changing environment.

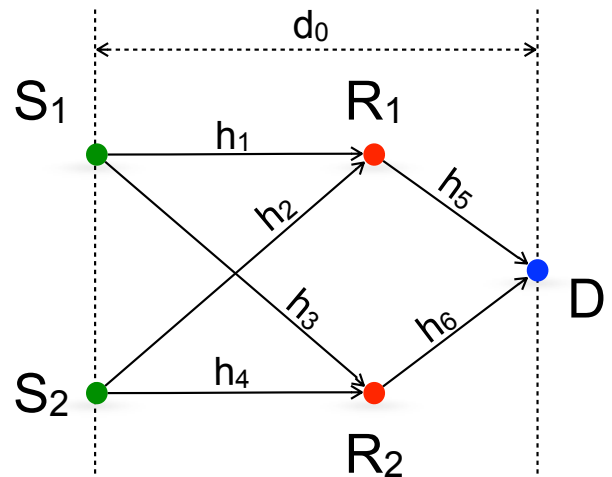


Fig. 1. Multi-source multi-relay transmission system.

In this paper, we consider a multi-source multi-relay single-destination (MSMR) transmission with all the links experiencing κ - μ fading, which better represents the small-scale variations of fading in LOS conditions. The multiple sources communicate with the relays over non-orthogonal MACs. For the relay-destination transmission, two transmission schemes are considered, i.e., non-orthogonal maximum ratio transmission (MRT) and orthogonal transmission with joint-decoding (JD). The outage probabilities of MSMR system with MRT (MSMR-MRT) and that with orthogonal transmission with JD (MSMR-JD) are derived, respectively. The optimal relay location and power allocation between relays for minimizing the outage probability are also investigated.

It is found that the larger the ratio of LOS component or the number of multipath clusters of the links, the lower outage probability can be achieved. The MSMR-JD outperforms MSMR-MRT in terms of outage performance. Even though the MSMR-JD requires two time slots for relay-destination transmission, it does not need to have the knowledge of channel state information (CSI) at the relays. Moreover, MSMR-JD also does not need to have a available backhaul network between the two relays to exchange the CSI.

II. SYSTEM MODEL

In this paper, we consider specifically two sources (S_1 and S_2) communicate with one common destination (D) with the help of two relays (R_1 and R_2) as illustrated in Fig. 1. The relays are shared to both of the sources and no direct link exists between the sources and the destination. All nodes are equipped with a single antenna and operate in a half-duplex mode. The transmission is divided into two hops. In the first hop, S_1 and S_2 broadcast binary data to R_1 and R_2 simultaneously with MAC assumption. In the second hop, the relays decode the received information of both sources and forward the recovered data to D. We consider two transmission schemes for the second hop (relay-destination): 1) Non-orthogonal maximum ratio transmission (MRT) scheme; 2) Orthogonal transmission with JD at D. Orthogonal transmission needs two time slots to send data from R_1 and R_2 , respectively. However, it does not request CSI at the relays compared to MRT.

With the index 1...6 representing the corresponding links shown in Fig. 1, the signal received at R_i ($i = 1, 2$) can be written as

$$y_{R_1} = \sqrt{G_1}h_1s_1 + \sqrt{G_2}h_2s_2 + n_{R_1}, \quad (1)$$

$$y_{R_2} = \sqrt{G_3}h_3s_1 + \sqrt{G_4}h_4s_2 + n_{R_2}, \quad (2)$$

respectively, where G_i is the gain related to transmit distance of each link, h_i is the complex channel gain, and n_{R_i} is the additive white Gaussian noise (AWGN) with zero mean and equal variance of $N_0/2$ per dimension. The modulated symbols of S_1 and S_2 are denoted by s_1 and s_2 , respectively.

With the non-orthogonal transmission in the relay-destination link, the signal received at D over one time slot can be written as

$$y_D = \sqrt{G_5}h_5s_{R_1} + \sqrt{G_6}h_6s_{R_2} + n_D, \quad (3)$$

where s_{R_1} and s_{R_2} denote the modulated symbols of R_1 and R_2 , respectively.

With the orthogonal transmission in the relay-destination link, the signal received at D during two time slots can be written as

$$y_{D_1} = \sqrt{G_5}h_5s_{R_1} + n_{D_1}, \quad (4)$$

$$y_{D_2} = \sqrt{G_6}h_6s_{R_2} + n_{D_2}, \quad (5)$$

respectively, where n_{D_1} and n_{D_2} denote the AWGN noise for two time slots with equal distribution.

All the links are assumed to experience block κ - μ fading. The probability density function (*pdf*) with instantaneous signal-to-noise ratio (SNR) of link i , γ_i , is given by

$$f_{\kappa\mu}(\gamma_i) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} \cdot \gamma_i^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)\Gamma_i^{\frac{\mu+1}{2}}} \cdot \exp\left(-\frac{\mu(1+\kappa)\gamma_i}{\Gamma_i}\right) \cdot I_{\mu-1} \left[2\mu \cdot \sqrt{\frac{\kappa(1+\kappa)\mu}{\Gamma_i}} \right], \quad (i = 1 \dots 6), \quad (6)$$

where Γ_i represents the average SNR of the link i . The parameter κ is related to the ratio of the total power of the dominant components to the total power of the scattered waves. The parameter μ represents the number of multipath clusters,

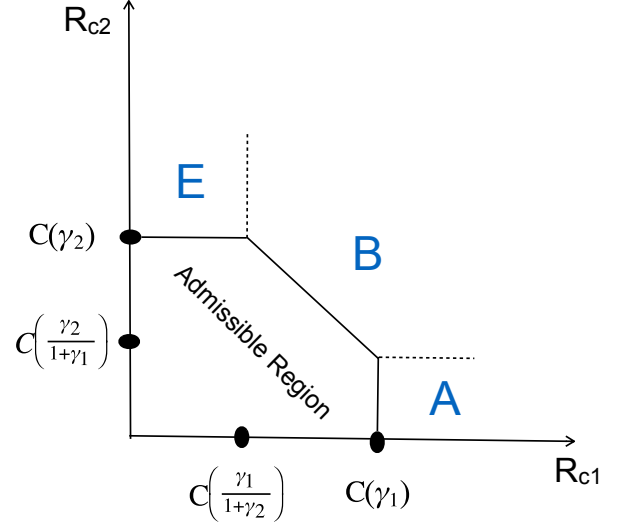


Fig. 2. MAC rate region for source-relay transmission. $C(\gamma) = \log_2(1+\gamma)$.

and $I_{\mu-1}$ is the $(\mu-1)$ th order modified Bessel function of the first kind. All the links are assumed mutually independent and identically distributed. The cumulative distribution function (*cdf*) of κ - μ distribution is obtained as,

$$F_{\kappa\mu}(\gamma_i) = 1 - Q_{\mu} \left(\sqrt{2\kappa\mu}, \frac{\sqrt{2\kappa(1+\kappa)\gamma_i}}{\Gamma_i} \right), \quad (7)$$

where $Q_{\mu}(\cdot, \cdot)$ is generalized Marcum Q-function.

It is assumed that the relays can move simultaneously between the sources and the destination horizontally as shown in Fig. 1. We set the horizontal distance between the sources and the destination to d_0 . With the gain of the link with length d_0 being normalized to one, the geometric gains of the link i with length d_i can be defined as [10]

$$G_i = \left(\frac{d_0}{d_i} \right)^{\alpha}, \quad (8)$$

where α is the path loss factor.

III. OUTAGE PROBABILITY DERIVATION

In this section, we provide the definition and derivation of the outage probability for the MSMR system. The outage probabilities for S_1 and S_2 are the same due to the symmetry of the system topology. Therefore, we only focus on the derivation of the outage probability for S_1 . The derivation for the outage probability of S_2 is straightforward.

Since the transmission includes two hops, the overall outage is calculated based on the law of Bayes' rule as,

$$P_{\text{out}} = \Pr(\text{outage}|\text{Case I})\Pr(\text{Case I}) + \Pr(\text{outage}|\text{Case II})\Pr(\text{Case II}) + \Pr(\text{outage}|\text{Case III})\Pr(\text{Case III}), \quad (9)$$

where Case I, II, and III indicate that: In Case I, the information of S_1 cannot be decoded error-free at both of the relays; In Case II, information of S_1 can be recovered at both of the relays without error; In Case III, only one of the relays can decode the information of S_1 with an arbitrary low error probability.

A. Outage Probability Calculation for Case I

For the sources-relay transmission hop, the MAC rate region for (S_1-R_1, S_2-R_1) links is shown in Fig. 2. If the rate pair R_{c1}, R_{c2}^1 falls into the inadmissible area A or B shown in Fig. 2, error-free transmission for S_1 and S_2 can not be guaranteed. Therefore,

$$\Pr(\text{Case I}) = (P_A + P_B) * (P'_A + P'_B), \quad (10)$$

where P_A and P_B denote the probability that (R_{c1}, R_{c2}) falls into the inadmissible areas A and B, respectively, whereas P'_A and P'_B denote the probability that rate pair for S_1-R_2 and S_2-R_2 links falls into the inadmissible areas in their corresponding MAC rate region, respectively.

With the Gaussian codebook being assumed, P_A and P_B can be expressed as

$$P_A = \Pr[R_{c1} > \log(1 + \gamma_1), R_{c2} \leq \log(1 + \frac{\gamma_2}{1 + \gamma_1})], \quad (11)$$

$$P_B = \Pr[R_{c1} > \log(1 + \frac{\gamma_1}{1 + \gamma_2}), R_{c2} > \log(1 + \frac{\gamma_2}{1 + \gamma_1}), \\ R_{c1} + R_{c2} > \log(1 + \gamma_1 + \gamma_2)]. \quad (12)$$

With the assumption that all links suffer from statistically independent block κ - μ fading, P_A and P_B can be calculated by integral with respect to the *pdf* of the instantaneous SNRs of corresponding links, as,

$$P_A = \int_{\gamma_1=0}^{2^{R_{c1}}-1} f_{\kappa\mu}(\gamma_1) d\gamma_1 \int_{\gamma_2=(2^{R_{c2}}-1)(1+\gamma_1)}^{\infty} f_{\kappa\mu}(\gamma_2) d\gamma_2 \\ = \int_{\gamma_1=0}^{2^{R_{c1}}-1} f_{\kappa\mu}(\gamma_1) \cdot (1 - F_{\kappa\mu} [(2^{R_{c2}} - 1)(1 + \gamma_1)]) d\gamma_1 \quad (13)$$

and

$$P_B = \int_{\gamma_1=0}^{2^{R_{c1}}-1} f_{\kappa\mu}(\gamma_1) d\gamma_1 \int_{\gamma_2=0}^{(2^{R_{c2}}-1)(1+\gamma_1)} f_{\kappa\mu}(\gamma_2) d\gamma_2 \\ + \int_{\gamma_1=2^{R_{c1}}-1}^{2^{R_{c2}}(2^{R_{c1}}-1)} f_{\kappa\mu}(\gamma_1) d\gamma_1 \int_{\gamma_2=\frac{\gamma_1}{2^{R_{c1}}-1}-1}^{(2^{R_{c1}+R_{c2}}-1)-\gamma_1} f_{\kappa\mu}(\gamma_2) d\gamma_2 \\ = \int_{\gamma_1=0}^{2^{R_{c1}}-1} f_{\kappa\mu}(\gamma_1) \cdot F_{\kappa\mu} [(2^{R_{c2}} - 1)(1 + \gamma_1)] \\ + \int_{\gamma_1=2^{R_{c1}}-1}^{2^{R_{c2}}(2^{R_{c1}}-1)} f_{\kappa\mu}(\gamma_1) \left\{ F_{\kappa\mu} \left(\frac{\gamma_1}{2^{R_{c1}}-1} - 1 \right) \right. \\ \left. - F_{\kappa\mu} \left(2^{(R_{c1}+R_{c2})} - 1 - \gamma_1 \right) \right\} d\gamma_1. \quad (14)$$

Similarly, P'_A and P'_B can be calculated following the same technique based on their corresponding MAC rate region.

Since in Case I, both of the relays cannot decode S_1 's information without error, the outage calculation problem at D falls into the chief executive officer (CEO) problem for the relay-destination hop. For simplicity, we assume

$$\Pr(\text{outage}|\text{Case I}) = 1. \quad (15)$$

¹ R_{ci} denotes the spectrum efficiency of link i including the channel coding rate and the modulation order.

B. Outage Probability Calculation for Case II

For Case II where the information of both sources are fully recovered at R_1 and R_2 , it is easy to have

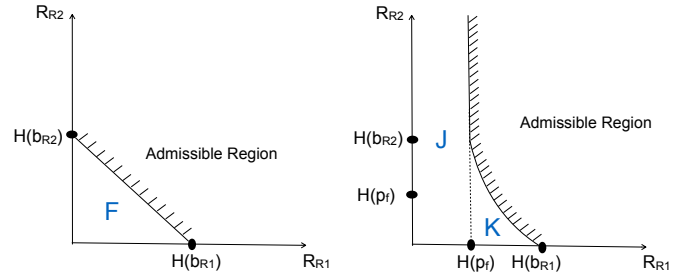
$$\Pr(\text{Case II}) = (1 - P_A - P_B) * (1 - P'_A - P'_B). \quad (16)$$

We consider two transmit schemes for the second hop (relay-destination) in Case II, i.e, non-orthogonal MRT and orthogonal transmission with JD.

1) *MRT scheme*: With MRT, the information sequences are forwarded by two relays simultaneously to D. The received signal at D could refer to (3). The outage probability is given by

$$\Pr(\text{outage}|\text{Case II}) = \Pr[R_c > \log(1 + \gamma_5 + \gamma_6)] \\ = \int_{\gamma_5=0}^{2^{R_{c5}}-1} f_{\kappa\mu}(\gamma_5) d\gamma_5 \int_{\gamma_6=0}^{(2^{R_{c6}}-1)-\gamma_5} f_{\kappa\mu}(\gamma_6) d\gamma_6 \\ = \int_{\gamma_5=0}^{2^{R_{c5}}-1} f_{\kappa\mu}(\gamma_5) F_{\kappa\mu} (2^{R_{c6}} - 1 - \gamma_5) d\gamma_5. \quad (17)$$

2) *JD scheme*: D performs joint decoding to retrieve the original information of sources. The successfully recovered information sequences at the relays, denoted by b_{R_1} and b_{R_2} , are interleaved before sending to D. The admissible rate region of R_1 -D and R_2 -D links are shown in Fig. 3(a).



(a) Rate region for R_1 and R_2 in Case II for JD scheme. (b) Rate region for R_1 and R_2 in Case III.

Fig. 3. Rate region for R_1 and R_2 in relay-destination transmission. $H(\cdot)$ denotes the binary entropy function.

The outage happens when the information coding rate pair of R_1 and R_2 (R_{R1}, R_{R2}) falls into the inadmissible area F shown in Fig. 3(a), with probability P_F . Therefore, we have

$$\Pr(\text{outage}|\text{Case II}) = P_F = \int_{\gamma_5=0}^{2^{R_{c5}}-1} f_{\kappa\mu}(\gamma_5) \\ \cdot (1 - F_{\kappa\mu}(\gamma_3 = 1)) \cdot F_{\kappa\mu} \left(2^{\left(1 - \frac{\log(1+\gamma_5)}{R_{c5}}\right) R_{c6}} - 1 \right) d\gamma_5. \quad (18)$$

C. Outage Probability Calculation for Case III

In Case III, only R_1 or R_2 can losslessly decode the information sent from S_1 . Therefore, the possibility of Case III is given as

$$\Pr(\text{Case III}) = (1 - P_A) * P_B + (1 - P'_A) * P'_B. \quad (20)$$

Even if decoding error is detected at one relay, the two information sequences received at both relays are correlated

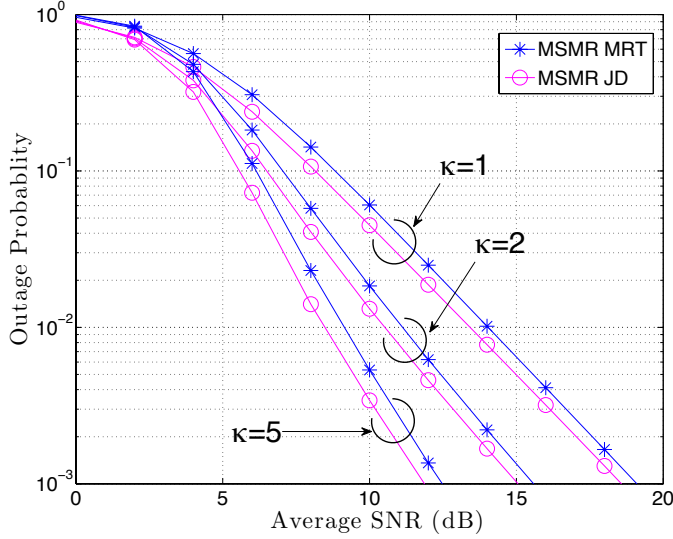


Fig. 4. Outage probabilities of MSMR with MRT, and MSMR with JD; $\mu = 1$.

since they are from the same source. Therefore, the problem falls into the category of source coding with a helper. The outage probability is defined as the probability that (R_{R_1}, R_{R_2}) falls into the inadmissible areas J and K in Fig. 3(b), denoted as P_J and P_K , respectively. p_f represents the bit flipping probability between the information sequences obtained after decoding at R_1 and R_2 .

We assume the recovered information of S_1 at R_2 is erroneous and the message are fully recovered at R_1 . According to Shannon's lossless source channel separation theorems, the relationship between γ_5 and R_{R_1} is established as

$$\gamma_5 \geq (2^{R_{R_1} R_{c5}} - 1). \quad (21)$$

With the Hamming distortion measure for binary sources, the relationship between p_f and γ_3 can be established as

$$p_f = H^{-1}(1 - \log_2(1 + \gamma_3)), \quad (22)$$

according to lossy source channel separation theorems. Let $H^{-1}(\cdot)$ denote the inverse function of the binary entropy.

Then, the outage probability in Case III can be written as

$$\begin{aligned} \Pr(\text{outage}|\text{Case III}) &= P_J + P_K = \int_{\gamma_3=0}^{2^{R_{c3}}-1} f_{\kappa\mu}(\gamma_3) \\ &\cdot F_{\kappa\mu} \left(2^{\left(1 - \frac{\log(1+\gamma_3)}{R_{c5}}\right) R_{c5}} - 1 \right) d\gamma_3 \\ &+ \int_{\gamma_3=0}^1 \int_{\gamma_5=2^{\lceil 1-H_2^{-1}(1-\log(1+\gamma_3)) \rceil}-1}^1 f_{\kappa\mu}(\gamma_3) f_{\kappa\mu}(\gamma_5) \\ &\cdot F_{\kappa\mu} \left(2^{\xi(\gamma_3, \gamma_5)} - 1 \right) d\gamma_3 d\gamma_5, \end{aligned} \quad (23)$$

where $\xi(\gamma_3, \gamma_5) = H \left\{ H^{-1} \left[1 - \frac{C(\gamma_3)}{R_{c3}} \right] * H^{-1} \left[1 - \frac{C(\gamma_5)}{R_{c5}} \right] \right\}$.

IV. NUMERICAL RESULTS

The outage performance of the MSMR system is illustrated in this section. The path loss factor $\alpha = 2$ is used [10]. R_{c_i} is

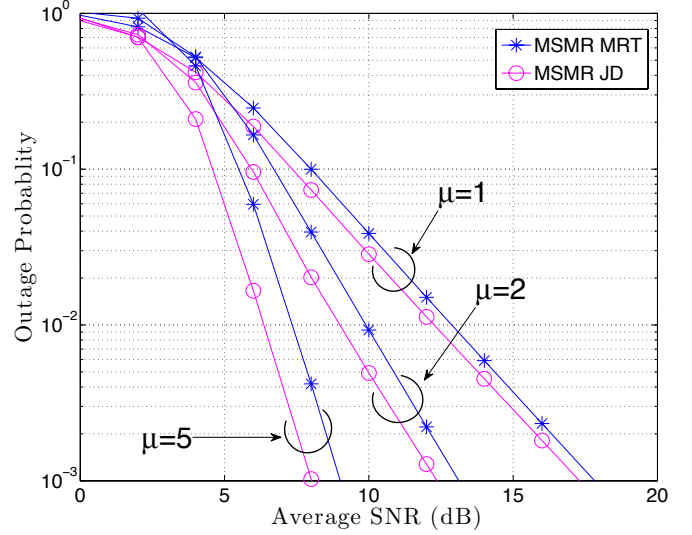


Fig. 5. Outage probabilities of MSMR with MRT, and MSMR with JD; $\kappa = 1$.

set to be one. We assume that S_1 and S_2 as well as R_1 and R_2 are symmetrical along the horizontal line through D. The distance between S_1 and S_2 as well as that between R_1 and R_2 are equal to $d_0/2$. Hence, the distance of each link can be easily calculated by the Pythagorean theorem.

Fig. 4 shows the theoretical outage probabilities of the MSMR system versus average SNR with different κ values. The outage probabilities are calculated with MRT and JD schemes in the relay-destination hop respectively. It can be found that the outage probability curves exhibit the tendency that the larger the κ values, the smaller the outage probability. With $\mu = 1$, the κ - μ fading becomes Rician fading, where κ equals to Rician factor K [11, Chapter 19]. Therefore, as the channel variation becomes milder (larger K value), lower outage probability can be achieved. Interestingly, the outage probability with JD scheme is superior compared to that with MRT scheme.

Fig. 5 also compares the outage performance of the MSMR system with JD and MRT schemes. The value of parameter μ changes with fixed κ . It can be observed from Fig. 5 that, when $\mu = 1$ second order diversity can be achieved for both the schemes in terms of the outage probabilities. This result is consistent with the observation in [9]. This is because κ - μ distribution becomes Nakagami- m distribution with shape factor $m = \mu$ when $\kappa = 0$. Moreover, Nakagami- m fading reduces to Rayleigh fading when m (μ) = 1. The outage curves can achieve sharper decay than that with 2nd order diversity, when the m (μ) value increases. However, the outage performance of MSMR with JD always outperforms that with MRT, with 0.5-1.0 dB gain.

Fig. 6 shows the impact of the power allocation between the two relays on outage performance. The outage probability is given as a function of the allocated power ratio for R_1 under the assumption that the total transmit power for the two relays is fixed. With the noise variance being normalized to the unity, the average SNRs are equivalent to their corresponding transmit power, allocated to R_1 and R_2 , respectively. It can be

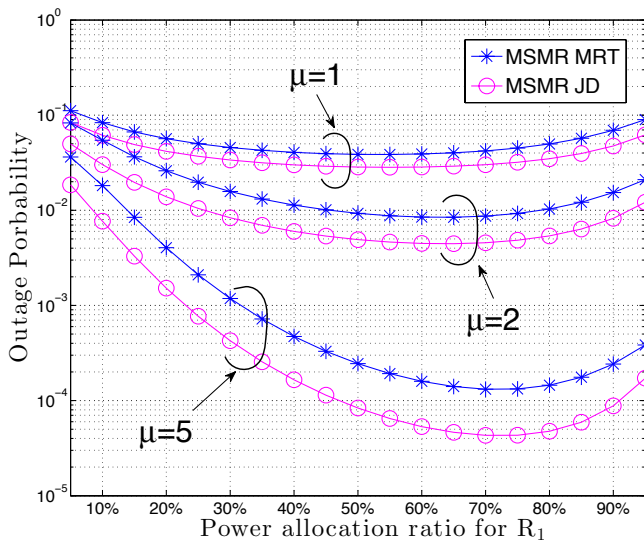


Fig. 6. Outage probability of MSMR system with different power allocation between relays (equal power allocation for two sources).

observed from Fig. 6 that, when $\mu = 1$ the outage probability curves are symmetric to the equal power allocation point. Namely, the lowest outage probability can be achieved with transmit power being equally allocated to the two relays. As the μ value increases, lower outage probability can be achieved. However, for harvesting better outage performance, the power ratio for R_1 which can recover the information without error, has to be increased with either JD or MRT scheme in the relay-destination transmission.

Fig. 7 plots the theoretical outage probabilities versus the position of the relays. The horizontal axis is the normalized distance from sources to relays. Increasing the value of horizontal axis means the relays is closer to the destination. It can be found that if the relays is close to the sources, the outage probability of MSMR with JD can be significantly reduced compared to that of MSMR with MRT. While if relays are near to the destination, the outage performance with the both schemes are almost same. This indicates that JD scheme has bigger advantage when the source-relay distance is smaller.

V. CONCLUSION

We have derived the exact outage probability expressions for a MSMR cooperative system over κ - μ fading channels. The overall outage probability has been derived according to the decoding results at the relay nodes. Both non-orthogonal MRT and orthogonal transmission with JD are considered for the relay-destination transmission. Diversity order for the different fading parameters has been investigated. The theoretical analysis has demonstrated the improved performance of MSMR with JD scheme as compared to the that with MRT scheme. Optimal power allocation for the relay nodes has been studied. We observe that the performance improvement is more obvious by increasing the power ratio of the relay which can decode the information error-freely. Additionally, the impact of the location of the relays to the outage performance has been investigated. The outage performance gap between MSMR-MRT and MSMR-JD is relatively large when the relays are close to the source nodes.

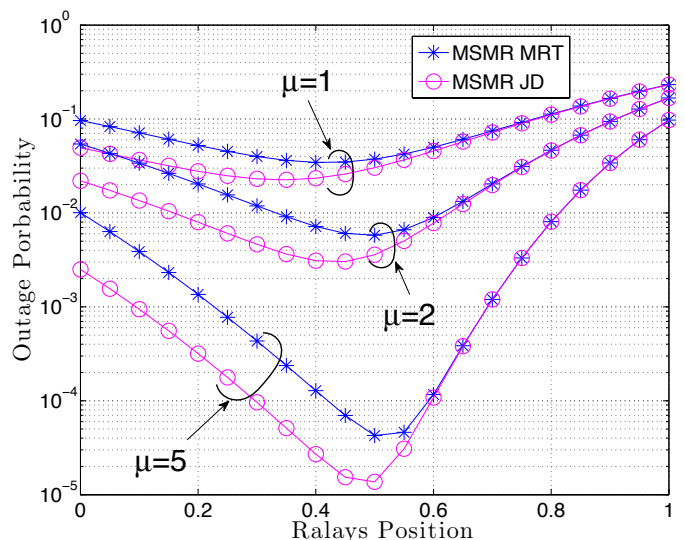


Fig. 7. Outage probability of MSMR system with different relay locations.

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