Interference Management via User Clustering in Two-Stage Precoder Design

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***Abstract*—We consider a single cell downlink (DL) massive multiple-input multiple-output (MIMO) set-up with user clustering based on statistical information. The problem is to design a fully digital two-stage beamforming aiming to reduce the complexity involved in the conventional MIMO processing. The fully digital two-stage beamforming consists of a slow varying channel statistics based outer beamformer (OBF) and an inner beamformer (IBF) accounting for fast channel variations. Two different methods are presented to design the OBF matrix, so as to reduce the size of effective channel used for IBF design. A group specific two-stage optimization problem with weighted sum rate maximization (WSRM) objective is formulated to find the IBF for fixed OBF. We begin by proposing centralized IBF design were the optimization is carried out for all sub group jointly with user specific inter-group interference constraints. In order to further reduce the complexity, we also propose a group specific IBF design by fixing the inter group interference to a constant or by ignoring them from the problem altogether. In spite of incurring a small loss in performance, the computational complexity can be saved to a large extent with the group specific processing. Numerical experiments are used to demonstrate the performance of various proposed schemes by comparing the total sum rate of all users and the design complexity.**

I. Introduction

Massive multiple-input multiple-output (MIMO) is considered to be the future enabling technology for 5G cellular communication standards [1]–[3]. This system can support increased data rate, reliability, diversity due to increased degrees of freedom (DoF) and beamforming gain. However, the downside of massive MIMO is the increased computational complexity, since the conventional MIMO processing involves higher dimensional matrix operations to determine transmit beamformers based on user channel state information (CSI). Hence, complexity reduction in massive MIMO has gained lot of attention among researchers [4]–[6]. This is done in both hybrid (analog/digital) beamforming [5] and fully digital beamforming [6]. As we know, hybrid beamforming is a combination of analog outer beamformer (OBF) implemented with analog radio frequency (RF) front-end and a digital inner beamformer (IBF). The analog beamformer sets pre-beams to the spatially separated users and reduces the effective channel dimensions to reduce the complexity.

In recent years, the most noticed fully digital two stage beamforming is joint spatial division and multiplexing (JSDM) [6]. The main idea of JSDM lies in grouping users based on similar transmit correlation matrices to form OBF. In [6], author discusses various methods to form the OBF and analyze the performance using the techniques of deterministic equivalents for namely, joint group and per-group processing. In [7]–[9] JSDM was studied extensively for user grouping, whereas in [10] the OBF and the IBF were used to control the inter and intra-cell interference, respectively. In [11], two-stage precoding was explored for different heuristic OBF methods and the performance was evaluated as a function of statistical pre-beams.

In practice, the cellular users tend to be collocated geographically, leading to a user grouping that can be considered by the base station (BS) while designing the transmission strategy. Unlike in [11], we focus on group specific two-stage beamformer design. For this design, the OBF is based on long term channel statistics and these beams are used to effectively reduce the dimensions of the equivalent channels (product between the antenna specific channels and OBF). The main advantage of the statistics based OBF is that it varies over long time scales compared to the IBF that requires more frequent updates. This motivates us to consider different methods to form the outer beamformer by choosing a) Eigenvectors corresponding to the strongest eigenvalues of the (group specific) channel covariance matrices and b) discrete fourier transform (DFT) based orthogonal beams that aligns with the precise channel covariance matrix. The IBF in turn is applied for spatial multiplexing on the equivalent channel and helps to manage both intra- and inter-group interference (IGI) similarly to handling inter-cell interference in a multi-cell scenario [12]. Weighted sum rate maximization (WSRM) problem is formulated to optimize the IBF for a fixed OBF. We propose a centralized IBF design wherein the optimization is carried out for all sub-groups jointly. The inter-group interference is managed by introducing user specific IGI constraints To further reduce the complexity, we also considered a group specific IBF design by fixing the inter-group interference to a fixed predetermined value or by completely ignoring them from the IBF problem formulation.

II. System Model

We consider a downlink (DL) massive MIMO system as shown in Fig. 1 consisting of single BS equipped with N_T transmit antennas in a uniform linear array (ULA) pattern with $\frac{\lambda}{2}$ spacing between elements serving K single-antenna user terminal (UT). In this system, $N_T > K$, i.e, the users can be multiplexed in the spatial dimension. Even though the users are distributed uniformly around the BS, they tend to be collocated geographically, leading to a natural user clustering that can

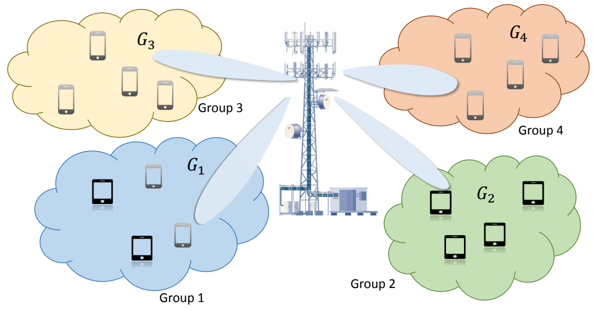

Fig. 1. System Model.

be considered by the BS while designing the transmission strategy. Thus, the users can be clustered into, say, G number of user clusters with $\mathcal{G} = \{1, 2, \ldots, G\}$ representing the set of user groups. Let $\mathcal{U}_g$ be the set of all users assigned to user group $g \in \mathcal{G}$ and $\mathcal{U} = \cup_{g\in\mathcal{G}} \mathcal{U}_g$ be the set of all users served by the BS. The channel seen between the BS and user $k \in \mathcal{U}$ is denoted by $\mathbf{h}_k \in \mathcal{C}^{N_T\times 1}$ while the beamforming vector for user k is given by $\mathbf{v}_k \in \mathbb{C}^{N_T\times 1}$. The transmitted data symbol for user k is denoted by x_k with $\mathbb{E}[|x_k|^2] \leq 1, \forall k \in \mathcal{U}$ and n_k corresponds to the additive white Gaussian noise with $n_k \sim \mathcal{CN}(0, N_0)$. Now, by using this setting, the signal y_k received by user $k \in \mathcal{U}_g$ is given by

$$y_k \;=\; \mathbf{h}_k^{\mathrm{H}}\mathbf{v}_k x_k + \sum_{i\in\mathcal{U}_g\setminus\{k\}} \mathbf{h}_k^{\mathrm{H}}\mathbf{v}_i x_i + \sum_{\substack{j\in\mathcal{U}_{\bar{g}},\\ \bar{g}\in\mathcal{G}\setminus\{g\}}} \mathbf{h}_k^{\mathrm{H}}\mathbf{v}_j x_j + \; n_k \tag{1}$$

where the first term in (1) is the desired signal while the second and third terms represent intra- and inter-group interference.

To design channel based on user location in the azimuthal direction, we model it using geometric ring model [13] as

$$\mathbf{h}_k = \frac{\beta_k}{\sqrt{L}} \sum_{l=1}^{L} e^{j\,\phi_{k,l}}\, \mathbf{a}(\theta_{k,l}) \tag{2}$$

where β_k represents the path loss between the BS and user k, L denotes the number of scatterers and $\phi_{k,l}$ corresponds to the random phase introduced by each scatterer l. The scatterers are assumed to be located uniformly around each user with certain angular spread, say, σ_k and the steering vector $\mathbf{a}(\theta_{k,l})$ corresponding to angle of departure (AoD) $\theta_{k,l} \in U(0, \sigma_{k,l})$ is given by $\mathbf{a}(\theta_{k,l}) = \left[1,\, e^{j\,\pi\cos(\theta_{k,l})}, \ldots, e^{j\,\pi\cos(\theta_{k,l})(N_T-1)}\right]^{\mathrm{T}}$.

Unlike the traditional MIMO transmission techniques, we adopt a two-stage precoder design consisting of OBF and IBF, which together characterize the total precoder matrix used to transmit respective data to all users in $\mathcal{U}$. Due to the clustering of users geographically, beamformers can be designed efficiently with significantly reduced complexity. Let $\mathbf{V} = \mathbf{BW}$ be the total precoding matrix with $\mathbf{V} \in \mathbb{C}^{N_T\times K}$, which is obtained by combining OBF matrix $\mathbf{B} \in \mathbb{C}^{N_T\times S}$ and IBF matrix $\mathbf{W} \in \mathbb{C}^{S\times K}$, where S represents the number of statistical beams used by the BS to serve user groups that are separated geographically in the azimuthal dimension. Let S_g be the number of statistical beams that are oriented towards each user group $g \in \mathcal{G}$, leading to $\sum_{g\in\mathcal{G}} S_g = S$ number of statistical beams in total. Similarly, let $\mathbf{B}_g$ contain all statistical beams corresponding to the users in group g, such that $\mathbf{B} = [\mathbf{B}_1, \ldots, \mathbf{B}_G]$. Hence, the signal-to-interference-plus-noise-ratio (SINR) for user $k \in \mathcal{U}_g$ is given by

$$\gamma_k = \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_{g_k}\mathbf{w}_k|^2}{\sum\limits_{i\in\mathcal{U}_g\setminus\{k\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_{g_k}\mathbf{w}_i|^2 + \sum\limits_{\substack{j\in\mathcal{U}_{\bar{g}},\\ \bar{g}\in\mathcal{G}\setminus\{g\}}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_{\bar{g}}\mathbf{w}_j|^2 + N_0}. \tag{3}$$

where index g_k indicates the user group of user k.

We consider the problem of weighted sum rate maximization (WSRM) objective for designing the transmit precoders, which is given by

$$R = \sum_{g\in\mathcal{G}} \sum_{k\in\mathcal{U}_g} \alpha_k \log_2(1+\gamma_k) \Rightarrow \sum_{k\in\mathcal{U}} \alpha_k \log_2(1+\gamma_k) \tag{4}$$

where $\alpha_k \geq 0$ is a user specific weight, which determines the scheduling priority.

III. Precoder Design

In this section we briefly discuss the design of both the OBF $\mathbf{B}$ and the IBF $\mathbf{W}$ with the objective of maximizing the total sum rate. In order to reduce the complexity involved in the design of transmit precoders, we fix the outer beamformers while optimizing for the inner precoders. We begin by describing the design of outer beamformers, which is then followed by the IBF design based on the WSRM objective .

A. Outer Precoder Design using Statistical Channel

Unlike the conventional MIMO, the two-stage beamformer design involves both the inner and the outer beamformers so as to reduce the computational complexity. The OBF plays a major role in determining the overall performance as it is common to all users in the group. Designing both outer and inner beamformers is a challenging task as they are inter-dependent. Thus, we adopt a sub-optimal strategy wherein the OBF is designed based on long-term channel statistics followed by the IBF design with fixed outer beamforming vectors. We present two well known heuristic methods to find outer beamformers, namely, Eigen and greedy DFT beams.

1) Eigen Beam Selection: The channel statistics of all users are assumed to remain relatively constant for a period of time. In such cases, the Eigenvectors of the channel covariance matrix can be used to form the outer precoding matrix via Eigen-value decomposition (EVD) [6], [11]. Let $\mathbf{H}_g = [\mathbf{h}_{\mathcal{U}_{g(1)}}, \ldots, \mathbf{h}_{\mathcal{U}_g(|\mathcal{U}_g|)}]$ be the stacked channel matrix of all users in group g and let $\mathbf{R}_g = \mathbb{E}\big[\mathbf{H}_g\mathbf{H}_g^{\mathrm{H}}\big]$ be the corresponding channel covariance matrix. Now, by decomposing $\mathbf{R}_g$ using EVD, we obtain $\mathbf{R}_g = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{U}_g^{\mathrm{H}}$, where the column vectors of $\mathbf{U}_g \in \mathbb{C}^{N_T\times S_g}$ correspond to the eigenvectors and the respective eigenvalues are stacked diagonally in $\mathbf{\Lambda}_g \in \mathbb{C}^{S_g\times S_g}$. Now, by choosing S_g columns of $\mathbf{U}_g$, which is denoted by $\mathbf{U}_g(S_g)$, corresponding to the S_g largest eigenvalues in $\mathrm{diag}(\mathbf{\Lambda}_g)$, we obtain the outer precoding matrix $\mathbf{B}_g = \mathbf{U}_g(S_g) \in \mathbb{C}^{N_T\times S_g}$ containing S_g predominant spatial signatures.

2) Greedy Beam Selection: As the number of users in the system increases, the probability of finding a user in the azimuthal direction follows the uniform distribution, i.e., $\theta_k \in [-\pi, \pi]$. Thus, in the limiting case, the column vectors of $\mathbf{U} \in \mathbb{C}^{N_T \times N_T}$ corresponding to the channel covariance $\mathbf{R}, \forall k \in \mathcal{U}$ can be approximated to the columns of DFT matrix $\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_S] \in \mathbb{C}^{N_T \times S}$ with $\mathbf{D}\mathbf{D}^{\mathrm{H}} = \mathbf{I}_{N_T}$, where the $k\mathrm{th}$ column vector of $\mathbf{D}$ is given by $\mathbf{d}_k = \frac{1}{\sqrt{N_T}} \left[\mathbf{1}, e^{j2\pi k/N_T}, \ldots, e^{j2\pi k(N_T-1)/N_T}\right]^{\mathrm{H}}$.

The OBF matrix based on DFT columns aids in multiplexing data into multiple high directional (high gain) beams [6]. Thus, the problem reduces to finding a subset of column vectors from the unitary DFT matrix. To do so, we select S_g DFT column vectors that maximizes the following metric for each group g by initializing $\mathcal{D} = \{1, \ldots, N_T\}$ and $\mathcal{B}_g = \emptyset$ as

$$\begin{aligned} k &= \underset{i}{\operatorname{argmax}} \, (\mathbf{d}_i^{\mathrm{H}} \, \mathbf{R}_g \, \mathbf{d}_i), \, \forall i \in \mathcal{D} \\ \mathcal{B}_g &= \mathcal{B}_g \cup \{k\}, \quad \mathcal{D} = \mathcal{D} \backslash \mathcal{B}_g. \end{aligned} \tag{5}$$

Upon finding subset $\mathcal{B}_g$, the group specific OBF is given by $\mathbf{B}_g = [\mathbf{d}_{\mathcal{B}(1)}, \ldots, \mathbf{d}_{\mathcal{B}(|\mathcal{B}|)}]$. Thus, the resulting OBF matrix $\mathbf{B}_g$ consisting of orthogonal DFT beams include strongest signal paths of each group.

B. Group-Specific Inner Beamformer Design

Unlike the conventional MIMO beamforming wherein the precoders are designed by considering only the instantaneous user channels, the inner beamformers in the two-stage precoding considers both the user's channel and the group specific outer beamformers that alters the effective channel seen by the BS. Instead of serving all users in a single group possibly with reduced dimensions $S \leq N_{\mathrm{T}}$ as in [11], herein, a group specific beamformer design is proposed so as to further reduce the computational complexity (i.e., IBF size S_g) involved in the beamformer design. The main objective of inner beamformers is to maximize the received signal power at the intended user terminal in a given group while minimizing the interference caused to the other terminals in the same group and the ones in other groups. In the following, we first introduce a centralized design where the inter-group interference is handled via IGI constraints/variables. Then, a group specific IBF design with fixed (or ignored) IGI values is presented.

1) Centralized Formulation: In order to design beamformers for each group with reduced dimensions, the interference term in the denominator of the SINR constraint in (3) can be rewritten for each user $k \in \mathcal{U}_g$ as

$$\textstyle\sum_{i \in \mathcal{U}_g \backslash \{k\}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_i|^2 + \sum_{m \in \mathcal{G} \backslash \{g\}} \zeta_{m,k} + N_0 \tag{6a}$$
$$\textstyle\sum_{j \in \mathcal{U}_{\bar{g}}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{\bar{g}} \mathbf{w}_j|^2 \; \leq \; \zeta_{\bar{g},k}, \; \forall \bar{g} \in \mathcal{G} \backslash \{g\} \tag{6b}$$

where $\zeta_{\bar{g},k}$ limits the interference caused by the neighboring group $\bar{g} \in \mathcal{G} \backslash \{g\}$ to user $k \in \mathcal{U}_g$. By introducing new variable $\zeta_{\bar{g},k}$, the inner beamformer can be designed for each group independently by exchanging only the group specific interference $\zeta_{\bar{g},k}$ threshold across the groups. Thus, the problem of inner beamformer design is given by

$$\begin{aligned} \underset{\gamma_k, b_k, \mathbf{w}_k, \boldsymbol{\zeta}_g}{\text{maximize}} \quad & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \alpha_k \log(1 + \gamma_k) \\ \text{subject to} \quad & \frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{g_k} \mathbf{w}_k|^2}{b_k} \geq \gamma_k, \; \forall k \in \mathcal{U} && \text{(7a)} \\ & \sum_{i \in \mathcal{U}_{g_k} \backslash \{k\}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{g_k} \mathbf{w}_i|^2 + \sum_{\bar{g} \in \mathcal{G} \backslash \{g_k\}} \zeta_{\bar{g},k} + N_0 \leq b_k && \text{(7b)} \\ & \sum_{k \in \mathcal{U}_{g_k}} |\mathbf{h}_i^{\mathrm{H}} \mathbf{B}_{g_k} \mathbf{w}_k|^2 \leq \zeta_{g_k,i}, \; \forall i \in \mathcal{U} \backslash \mathcal{U}_{g_k} && \text{(7c)} \\ & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \|\mathbf{B}_g \mathbf{w}_k\|^2 \leq P_{\mathrm{tot}} && \text{(7d)} \end{aligned}$$

where $\zeta_{\bar{g},k}, \forall \bar{g} \in \mathcal{G} \backslash \{g_k\}$ are the inter-group interference terms, which couples the IBF design problem.

In spite of relaxing the SINR expression in (3) using (7a) and (7b), (7) is still nonconvex due to the *quadratic-over-linear* constraint (7a) [14]. Thus, to solve problem (7) efficiently, we resort to the successive convex approximation (SCA) technique wherein the nonconvex constraint is replaced by a sequence of approximate convex subsets, which is then solved iteratively until convergence [15]. We note that the LHS of (7) is convex, therefore we resort to the first order Taylor approximation of *quadratic-over-linear* function around some operating point, say, $\{\mathbf{w}_k^{(i)}, b_k^{(i)}\}$, is given by

$$\frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{g_k} \mathbf{w}_k|^2}{b_k} \geq \mathcal{F}_k^{(i)}(\mathbf{w}_k, b_k; \mathbf{w}_k^{(i)}, b_k^{(i)}) \tag{8}$$

where the first order approximation $\mathcal{F}_k^{(i)}(\mathbf{w}_k, b_k; \mathbf{w}_k^{(i)}, b_k^{(i)})$ is an under-estimator for the LHS term in (8) is given by [12]

$$\begin{aligned} \mathcal{F}_k^{(i)}(\mathbf{w}_k, b_k; \mathbf{w}_k^{(i)}, b_k^{(i)}) &\triangleq 2 \, \frac{\mathbf{w}_k^{(i)\mathrm{H}} \mathbf{B}_{g_k}^{\mathrm{H}} \mathbf{h}_k \mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{g_k}}{b_k^{(i)}} \left(\mathbf{w}_k - \mathbf{w}_k^{(i)}\right) \\ &+ \quad \frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{g_k} \mathbf{w}_k^{(i)}|^2}{b_k^{(i)}} \left(1 - \frac{b_k - b_k^{(i)}}{b_k^{(i)}}\right). \end{aligned} \tag{9}$$

Now, by using the above approximation in (9), an approximate convex reformulation of (7) is given by

$$\begin{aligned} \underset{\gamma_k, b_k, \mathbf{w}_k, \boldsymbol{\zeta}_g}{\text{maximize}} \quad & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \alpha_k \log(1 + \gamma_k) \\ \text{subject to} \quad & \mathcal{F}_k^{(i)}(\mathbf{w}_k, b_k; \mathbf{w}_k^{(i)}, b_k^{(i)}) \geq \gamma_k && \text{(10a)} \\ & \text{(7b) – (7d).} && \text{(10b)} \end{aligned}$$

The resulting problem (10) is solved iteratively until convergence by updating the operating point with the solution obtained from the previous iteration. Thus, upon convergence, the resulting inner beamforming vectors $\mathbf{w}_k \in \mathbb{C}^{S_g \times 1}, \forall k \in \mathcal{U}_g$ determine the linear combination of OBF column vectors of $\mathbf{B}_g$ that maximizes the overall sum rate of all users.

2) Group Specific Beamformer Design: Unlike the centralized approach presented in Section III-B1, the beamformers are designed independently by either fixing the inter-group interference to a fixed value or by ignoring them from the formulation. By doing so, the complexity involved in the design of inner beamformers reduces significantly as the number of optimization variables is limited. Thus, the group specific beamformer design for group $g \in \mathcal{G}$ is given by

$$\begin{aligned}
&\underset{\gamma_k, b_k, \mathbf{w}_k}{\text{maximize}} \quad \sum_{k \in \mathcal{U}_g} \alpha_k \log(1+\gamma_k) \\
&\text{subject to} \quad \sum_{i \in \mathcal{U}_g \setminus \{k\}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_i|^2 + \sum_{\bar{g} \in \mathcal{G} \setminus \{g\}} \bar{\zeta}_{\bar{g},k} + N_0 \leq b_k \qquad \text{(11a)} \\
&\qquad \sum_{k \in \mathcal{U}_g} |\mathbf{h}_i^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_k|^2 \leq \bar{\zeta}_{g,i}, \ \forall i \in \mathcal{U} \setminus \mathcal{U}_g \qquad \text{(11b)} \\
&\qquad \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \|\mathbf{B}_g \mathbf{w}_k\|^2 \leq \frac{P_{\mathrm{tot}}}{G}, \text{ and (9)} \qquad \text{(11c)}
\end{aligned}$$

where $\bar{\zeta}_{g,i}$ is the constant interference value that is fixed before solving the optimization problem. Setting $\bar{\zeta}_{g,i} = 0, \forall i \in \mathcal{U} \setminus \{\mathcal{U}_g\}$ yields group zero-forcing solution. The proposed group specific beamformer design with fixed inter-group interference limit reduces the problem complexity significantly compared to that of the centralized design. However, due to the fixed interference threshold, the performance will be inferior compared to the centralized approach. A choice of $\bar{\zeta}_{g,i}$ can be obtained, e.g., from the statistics of inter-group interference.

Finally, by replacing $\mathbf{B}_g$ by $\mathbf{B} = [\mathbf{B}_{\mathcal{G}(1)}, \ldots, \mathbf{B}_{\mathcal{G}(|\mathcal{G}|)}]$ in (10) or (11b), we obtain two-stage beamformer design without user grouping, i.e., $\mathbf{w}_k \in \mathbb{C}^{S \times 1}$, leading to a fully connected design (FC). This is equivalent to [11] when $G = 1$ is used in (10). By doing so, the inner beamformer finds a linear combination of all the available outer beamforming vectors, i.e., S spatial beams to serve any user in the system.

IV. Complexity Analysis

To compare the complexity of various designs, we first identify the number of optimization variables present in each of the problems. Since, the solutions are obtained iteratively by solving a convex sub-problem in each iteration, the complexity is proportional to the number of SCA iterations required times sub-problem complexity. We ignore the complexity of OBF design as it can be computed off-line. The number of variables used in the centralized design (Section III-B1) is $S_g \times K \times 2 + G \times (K - |\mathcal{U}_g|) + 2 \times K$, where the first term corresponds to complex $\mathbf{w}_k$ size, second term denotes the number of $\zeta_{g,i}$ and the last term includes both b_k and γ_k. On the contrary, number of variables present in the group specific beamformer design counts up to $S_g \times K \times 2 + 2 \times K$ variables only, since $\bar{\zeta}_{g,i}$ is a fixed constant.

The number of iterations required by interior point method to find an ϵ optimal solution is proportional to the number of constraints, which is same for both the proposed schemes, is given by $\mathcal{O}((2 \times K + G \times (K - |\mathcal{U}_g|))^{0.5} \log(\epsilon^{-1})$. Now, the worst case complexity involved in solving the convex subproblem in each step using interior point method is given by $\mathcal{O}((S_g \times K \times 2 + G \times (K - |\mathcal{U}_g|) + 2 \times K)^3)$ for the centralized design and $\mathcal{O}((S_g \times K \times 2 + 2 \times K)^3)$ for the group-specific method [16], [17]. It is worth noting that as the number of group increases, complexity of centralized design

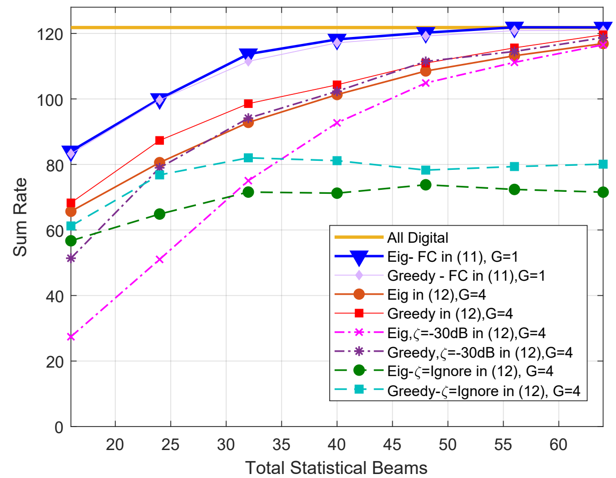

Fig. 2. Sum rate vs. total statistical beams with $P_{\mathrm{tot}} = 20$dB.

scales-up to the power of three. In the case of two-stage beamformer design with fully connected system, i.e., by using outer beamformer as B, the worst case complexity is given by $\mathcal{O}((S \times K \times 2 + G \times (K - |\mathcal{U}_g|) + 2 \times K)^3)$, which is noticeably greater than that of the centralized design (10). Finally, the arithmetic complexity of conventional beamformer method comes out to be $\mathcal{O}((N_T \times K \times 2 + 2 \times K)^3)$, which scales up with the number of transmit antennas.

V. Numerical Results

Similar to [11], we consider a single cell DL massive MIMO BS equipped with $N_T = 64$ ULA antenna elements serving a total of $K = 16$ single-antenna users. The users are naturally partitioned into 4 non-overlapping segments each with $45°$ degree, and with 4 users randomly placed in each segment. The angular spread for each user is considered to be $15°$ degree with 20 independent paths per user. The groups are distributed uniformly within $[\frac{-\pi}{2}, \frac{\pi}{2}]$. The user specific weights in the WSRM objective are fixed to $\alpha_k = 1 \, \forall k \in \mathcal{U}$. The plots are obtained by varying the number of statistical beams employed at the BS. If $G > 1$, then the number of statistical beams are divided equally among the user groups, i.e., $S_g = \frac{S}{G}$ while using (10) and (11). The results are averaged over 200 channel realizations and the noise variance N_0 is assumed to be unity. Both transmit power P_{tot} and the interference constraint $\bar{\zeta}_{g,i}$ are represented in dB with respect to noise variance.

Before proceeding further, we define the legends used in Figs. 2 and 3. The OBF is defined explicitly followed by the type of IBF design. The fully connected (FC) system is obtained via solving (10) by setting $\mathbf{B}_g = \mathbf{B}, \forall g \in \mathcal{G}$, thus all the outer beams are utilized by the IBF while serving a user in any group. We refer $\zeta = -30$dB when IGI term $\bar{\zeta}_{g,i} = -30$dB in (11). Finally, by ignoring (11b) together setting $\bar{\zeta}_{g,i} = 0$, we obtain the isolated design, referred as ζ-ignore in figures.

Fig. 2 demonstrates the sum rate performance of all the proposed schemes with $P_{\mathrm{tot}} = 20$dB. In the FC designs, $G = 1$ whereas for all group specific techniques, number of groups $G = 4$, however, $K = 16$ is the same in both schemes. The total sum rate achieved by the FC Eigen beam and greedy

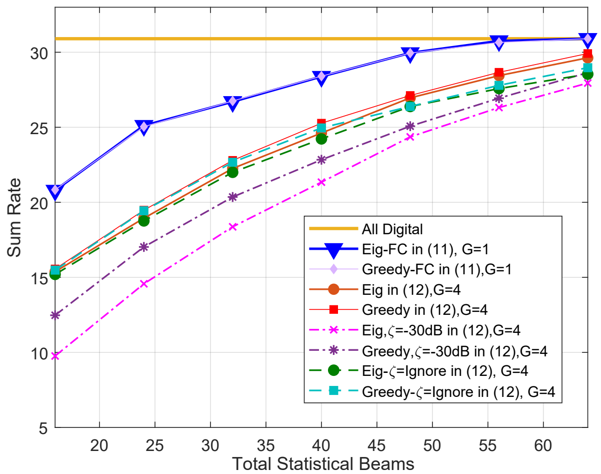

Fig. 3. Sum rate vs. total statistical beams with $P_{\text{tot}} = 0\text{dB}$.

designs is superior when comparing with the proposed group specific techniques, since the inner beamformer combines all the available beams linearly to serve any user in the system without any beam restriction. However, at higher number of beams, $S_g = 16$ per group for group specific beamformers and $S = 64$ for FC we observe almost similar performance with a small gap while the computational complexity is $(\frac{1}{(4)^3})$ of FC design. It is also worth noting that when the IGI value is $\zeta = -30dB$ then the performance is almost similar to that of (10). Finally, by ignoring the IGI term (11b) from the optimization problem, the achievable sum rate is noticeably inferior. This is due to the fact that the OBF may not be perfectly orthogonal, hence, the leakage to neighboring groups users that are located at the boundaries suffer from severe IGI as it is left uncompensated while designing the inner beamformers.

Unlike Fig. 2, the low SINR scenario is studied in Fig. 3 wherein P_{tot} is set to 0dB with respect to noise variance. In such a low power regime, only a subset of users are served, and therefore the inter-group interference has minimal impact on the total sum rate achieved by the system. This behavior can be seen in Fig. 3, where all the group specific schemes performs fairly similarly and the difference from the single group FC design is less while comparing with the difference seen in Fig. 2, i.e., in high SINR regime. It is also worth noticing that greedy maximization performs better compared to Eigen selection in the group specific design. It is due to the fact that Eigen selection may have much higher side-lobes than greedy maximization. This is not a problem for FC but harms group specific design. However, still FC design attains better sum rate due to the fact that it has more degrees of freedom to choose the best set of users from $K = 16$, wherein in the group specific scheme best set of users are chosen from each group.

VI. Conclusion

In this paper, we proposed a fully digital two-stage beamforming for a single cell downlink (DL) massive multiple-input multiple-output (MIMO) system with user grouping based on geographical location. This beamformer design consists of outer beamformer (OBF) and inner beamformer (IBF). We also proposed two different approaches to form the OBF matrix namely, Eigen selection and greedy energy maximization so as to reduce the effective channel dimensions for the IBF design. Upon fixing the OBF, The inner beamformers were designed by considering group specific interference constraint using weighted sum rate maximization (WSRM) objective. In order to further reduce the complexity, a group specific beamformer design was formulated by fixing the inter-group interference terms to a fixed predetermined value or by ignoring them from the formulation. Even though the sum rate performance of the proposed group specific designs were shown to have a small loss in performance when compared to the fully connected system, the computational complexity can be saved to a large extent with the group specific processing.

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