An Iterative Approach for Inter-Group Interference Management in Two-Stage Precoder Design

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***Abstract*—We consider a single cell downlink (DL) massive multiple-input multiple-output (MIMO) set up with user clustering based on statistical information. The problem is to design a fully digital two stage beamforming consisting of slow varying channel statistics based outer beamformer (OBF) and an inner beamformer (IBF) accounting for fast channel variations aiming to reduce the complexity involved in the conventional MIMO processing. Two different methods are considered to design the OBF matrix, so as to reduce the size of the effective channel used for IBF design. A group specific two-stage optimization problem with weighted sum rate maximization (WSRM) objective is formulated to find the IBF for fixed OBF. We begin by proposing centralized IBF design were the optimization is carried out for all sub group jointly with user specific inter-group interference constraints. In order to further reduce the complexity, we propose an iterative solution for group-specific beamformer design via the Karush-Kuhn-Tucker (KKT) conditions for fixed inter group interference (IGI) values with per group transmit power constraint. A low complexity heuristic iterative method is also proposed for managing the inter-group interference. In spite of incurring a small loss in performance, the computational complexity can be saved to a large extent with the group specific processing. The sum rate behavior of various proposed schemes are illustrated using numerical simulations.**

I. Introduction

Massive multiple-input multiple-output (MIMO) is considered to be the future enabling technology for 5G cellular communication standards [1]–[3]. This system can support increased data rate, reliability, diversity due to increased degrees of freedom (DoF) and beamforming gain. However, it possess increased computational complexity while performing conventional MIMO processing involves higher dimensional matrix operations. Hence, complexity reduction has gained lot of attention among researchers [4]–[6]. This is done in both hybrid beamforming [5] and fully digital beamforming [6].

The most noticed fully digital two stage beamforming is joint spatial division and multiplexing (JSDM) [6]. The main idea of JSDM lies in grouping users based on similar transmit correlation matrices to design outer beamformer (OBF). In [6], the outer beamformers were designed by choosing discrete fourier transform (DFT) columns corresponding to the angular position of user groups, which provides almost the same sum rate as that of full channel state information (CSI) at the transmitter. In [7]–[9] JSDM was studied extensively for user grouping, whereas in [10] both the OBF and the inner beamformer (IBF) were used to control the inter and intra-cell interference, respectively. In [11], two-stage precoding was explored for various heuristic OBF vectors and the sum rate was analyzed as a function of number of statistical pre-beams.

In practice, the cellular users tend to be collocated geographically, leading to a user grouping that can be considered by the base station (BS) while designing the transmission strategy. To further reduce the complexity [11], we focus on group specific two-stage beamformer design. For this design, the OBF is based on long term channel statistics and these beams are used to effectively reduce the dimensions of equivalent channels (product between the antenna specific channels and OBF). The main advantage is that it varies over long time scales compared to the IBF that requires more frequent updates. The IBF in turn is applied for spatial multiplexing on the equivalent channel and helps to manage both intra- and inter group interference (IGI) similarly to handling inter-cell interference in a multi-cell scenario [12]. Weighted sum rate maximization (WSRM) problem is formulated to optimize the IBF for a fixed OBF.

We propose a group specific optimization based IBF design wherein the optimization is carried out for all sub-groups jointly. The inter-group interference is managed by introducing user specific IGI constraints. To further reduce the complexity, we also considered a group specific IBF design by fixing the IGI to a fixed predetermined value or by completely ignoring them from the IBF problem formulation. Furthermore, to come up with a reduced complexity iterative method amenable for practical implementation, we propose an iterative solution via the Karush-Kuhn-Tucker (KKT) expressions to obtain the IBF precoder for a fixed IGI value with per group transmit power constraint. The KKT expressions are obtained by associating the coupling variables across the respective groups. We also propose a heuristic iterative method by fixing the dual variable corresponding to the IGI terms to a constant. By doing so, the complexity is comparable to that of isolated beamformer design method wherein IGI value is ignored from the IBF formulation. All the proposed methods provides insight into the trade-off between dimensionality reduction and sum rate performance.

II. System Model

We consider a downlink (DL) massive MIMO system as shown in Fig. 1 consisting of single BS equipped with N_T transmit antennas in uniform linear array (ULA) pattern serving K single-antenna user terminal (UT). In this system, $N_T > K$, i.e, the users can be multiplexed in the spatial dimension. Even though the users are distributed uniformly

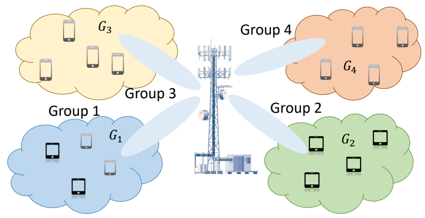

Fig. 1. System Model.

around the BS, they tend to be collocated geographically, leading to a natural user clustering that can be considered by the BS while designing the transmission strategy. Thus, the users can be clustered into, say, G number of user clusters with $\mathcal{G} = \{1, 2, \ldots, G\}$ representing the set of user groups. Let $\mathcal{U}_g$ be the set of all users assigned to user group $g \in \mathcal{G}$ and $\mathcal{U} = \cup_{g\in\mathcal{G}} \mathcal{U}_g$ be the set of all users served by the BS. The channel seen between the BS and user $k \in \mathcal{U}$ is denoted by $\mathbf{h}_k \in \mathcal{C}^{N_T\times 1}$. To design channel based on user location in the azimuthal direction, we model it using geometric ring model [13] as

$$\mathbf{h}_k = \frac{\mu_k}{\sqrt{L}} \sum_{l=1}^{L} e^{j\,\phi_{k,l}}\, \mathbf{a}(\theta_{k,l}) \tag{1}$$

where μ_k represents the path loss between the BS and user k, L denotes the number of scatterers and $\phi_{k,l}$ corresponds to the random phase introduced by each scatterer l. The scatterers are assumed to be located uniformly around each user with certain angular spread, say, σ_k and the steering vector $\mathbf{a}(\theta_{k,l})$ corresponding to angle of departure (AoD) $\theta_{k,l} \in U(0, \sigma_{k,l})$ is given by $\mathbf{a}(\theta_{k,l}) = \left[1,\, e^{j\,\pi\cos(\theta_{k,l})}, \ldots, e^{j\,\pi\cos(\theta_{k,l})(N_T-1)}\right]^{\mathrm{T}}$.

Unlike the traditional MIMO transmission techniques, we adopt a two-stage precoder design consisting of OBF and IBF, which together characterize the total precoder matrix used to transmit respective data to all users in $\mathcal{U}$. Moreover, due to the clustering of users geographically, beamformers can be designed efficiently with significantly reduced complexity.

Let S_g be the number of statistical beams that are oriented towards each user group $g \in \mathcal{G}$, leading to $\sum_{g\in\mathcal{G}} S_g = S \le N_T$ number of statistical beams in total. Similarly, let $\mathbf{B}_g \in \mathbb{C}^{N_T\times S_g}$ contain all statistical beams corresponding to the users in group g, such that $\mathbf{B} = [\mathbf{B}_1, \ldots, \mathbf{B}_G]$ and $\mathbf{w}_k \in \mathbb{C}^{S_g\times 1}$ corresponds to the IBF to the user $k \in \mathcal{U}_g$. The transmitted data symbol for user k is denoted by x_k with $\mathbb{E}[|x_k|^2] \le 1, \forall k \in \mathcal{U}$ and n_k corresponds to the additive white Gaussian noise with $n_k \sim \mathcal{CN}(0, N_0)$. Thus, signal y_k received by user $k \in \mathcal{U}_g$ is

$$y_k = \mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k x_k + \sum_{i\in\mathcal{U}_g\setminus\{k\}} \mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_i x_i + \sum_{\substack{j\in\mathcal{U}_{\bar g},\\ \bar g\in\mathcal{G}\setminus\{g\}}} \mathbf{h}_k^{\mathrm{H}}\mathbf{B}_{\bar g}\mathbf{w}_j x_j + n_k \tag{2}$$

where the first term in (2) is the desired signal while the second and third terms represent intra- and inter-group interference.

Hence, the signal-to-interference-plus-noise-ratio (SINR) for user $k \in \mathcal{U}_g$ is given by

$$\gamma_k = \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k|^2}{\sum\limits_{i\in\mathcal{U}_g\setminus\{k\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_i|^2 + \sum\limits_{j\in\mathcal{U}_{\bar g}, \bar g\in\mathcal{G}\setminus\{g\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_{\bar g}\mathbf{w}_j|^2 + N_0} \tag{3}$$

where index g indicates the user group of user k.

We consider the problem of weighted sum rate maximization (WSRM) objective for designing the transmit precoders, which is given by

$$R = \sum_{g\in\mathcal{G}} \sum_{k\in\mathcal{U}_g} \alpha_k \log_2(1+\gamma_k) \Rightarrow \sum_{k\in\mathcal{U}} \alpha_k \log_2(1+\gamma_k) \tag{4}$$

where $\alpha_k \ge 0$ is a user specific weight, which determines the scheduling priority.

III. Outer Precoder Design using Statistical Channel

Unlike the conventional MIMO, the two-stage beamformer design involves both the inner and the outer beamformers so as to reduce the computational complexity. The OBF plays a major role in determining the overall performance as it is common to all the users in the group. Designing both outer and inner beamformers is a challenging task as they are inter-dependent. Thus, we adopt a sub-optimal strategy wherein the OBF is designed based on long-term channel statistics followed by the IBF design with fixed outer beamforming vectors. We present two well known heuristic methods to find outer beamformers, namely, Eigen and greedy DFT beams. We assume that the users are grouped based on their channel statistics, hence we design group specific OBF matrix.

A. Eigen Beam Selection

The channel statistics of all users are assumed to remain relatively constant for a period of time. In such cases, the Eigenvectors of the channel covariance matrix can be used to form the outer precoding matrix via Eigenvalue decomposition (EVD) [6], [11]. Let $\mathbf{H}_g = [\mathbf{h}_{\mathcal{U}_{g(1)}}, \ldots, \mathbf{h}_{\mathcal{U}_g(|\mathcal{U}_g|)}]$ be the stacked channel matrix of all users in group g and let $\mathbf{R}_g = \mathbb{E}\left[\mathbf{H}_g\mathbf{H}_g^{\mathrm{H}}\right]$ be the corresponding channel covariance matrix averaged over significant channel realizations / coherence times. Now, by decomposing $\mathbf{R}_g$ using EVD, we obtain $\mathbf{R}_g = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{U}_g^{\mathrm{H}}$, where the column vectors of $\mathbf{U}_g \in \mathbb{C}^{N_T\times S_g}$ correspond to the Eigenvectors and the respective Eigenvalues are stacked diagonally in $\mathbf{\Lambda}_g \in \mathbb{C}^{S_g\times S_g}$. Now, by choosing S_g columns of $\mathbf{U}_g$, which is denoted by $\mathbf{U}_g(S_g)$, corresponding to the S_g largest Eigenvalues in diag$(\mathbf{\Lambda}_g)$, we obtain the outer precoding matrix $\mathbf{B}_g = \mathbf{U}_g(S_g) \in \mathbb{C}^{N_T\times S_g}$ containing S_g predominant spatial signatures.

B. Greedy Beam Selection

As the number of users in the system increases, the probability of finding a user in the azimuthal direction follows the uniform distribution, i.e., $\theta_k \in [-\pi, \pi]$. Thus, in the limiting case, the column vectors of $\mathbf{U} \in \mathbb{C}^{N_T\times N_T}$ corresponding to the channel covariance $\mathbf{R}, \forall k \in \mathcal{U}$ can be approximated to the columns of DFT matrix $\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_S] \in \mathbb{C}^{N_T\times S}$ with $\mathbf{D}\mathbf{D}^{\mathrm{H}} = \mathbf{I}_{N_T}$, where the kth column vector of $\mathbf{D}$ is given by $\mathbf{d}_k = \frac{1}{\sqrt{N_T}}\left[\mathbf{1}, e^{j2\pi k/N_T}, \ldots, e^{j2\pi k(N_T-1)/N_T}\right]^{\mathrm{H}}$. The OBF matrix based on DFT columns aids in multiplexing data into multiple high directional (high gain) beams [6]. Thus, the problem reduces to finding a subset of column vectors from the unitary DFT matrix. To do so, we select S_g DFT column vectors that maximizes the following metric for each group g by initializing $\mathcal{D} = \{1, 2, \ldots, N_T\}$ and $\mathcal{B}_g = \emptyset$ as

$$\begin{aligned} k &= \underset{i}{\text{argmax}}\ (\mathbf{d}_i^{\mathrm{H}} \mathbf{R}_g \mathbf{d}_i),\ \forall i \in \mathcal{D} \\ \mathcal{B}_g &= \mathcal{B}_g \cup \{k\}, \quad \mathcal{D} = \mathcal{D} \backslash \mathcal{B}_g. \end{aligned} \tag{5}$$

Upon finding subset $\mathcal{B}_g$, the group specific OBF is given by $\mathbf{B}_g = [\mathbf{d}_{\mathcal{B}(1)}, \ldots, \mathbf{d}_{\mathcal{B}(|\mathcal{B}|)}]$. Thus, the resulting OBF matrix $\mathbf{B}_g$ consisting of orthogonal DFT beams include strongest effective number of group specific beams S_g signal paths of each group.

IV. Group-Specific Inner Beamformer Design

Unlike the two-stage beamforming with single user group as in [11], we consider a group specific OBF matrices to reduce the computational complexity further by restricting the effective number of group specific beams S_g. The main objective of inner beamformers is to maximize the received signal power at the intended user terminal in a given group while minimizing the interference caused to the other terminals in the same group and the ones in other groups. Having, a finite number of transmit antennas N_T causes significant leakage to adjacent groups due to the side lobes from the statistical beams. Therefore, to handle this inter-group interference and to improve the system sum rate, we first introduce a group interference optimization design where the inter-group interference is handled via IGI constraints/variables. An iterative solution via KKT expressions is also presented for the centralized design, which is then followed by an iterative low complexity heuristic solution.

A. Group Interference Optimization

In order to design beamformers for each group with reduced dimensions, the interference term in the denominator of the SINR constraint in (3) is expressed for each user $k \in \mathcal{U}_g$ as

$$\textstyle\sum_{i \in \mathcal{U}_g \backslash \{k\}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_i|^2 + \sum_{m \in \mathcal{G} \backslash \{g\}} \zeta_{m,k} + N_0 \le \beta_k \tag{6a}$$

$$\textstyle\sum_{j \in \mathcal{U}_{\bar{g}}} |\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_{\bar{g}} \mathbf{w}_j|^2 \ \le\ \zeta_{\bar{g},k},\ \forall \bar{g} \in \mathcal{G} \backslash \{g\} \tag{6b}$$

where $\zeta_{\bar{g},k}$ limits the interference caused by the neighboring group $\bar{g} \in \mathcal{G} \backslash \{g\}$ to user $k \in \mathcal{U}_g$. By introducing new variable $\zeta_{\bar{g},k}$, the inner beamformer can be designed for each group independently by coordinating only the group specific interference $\zeta_{\bar{g},k}$ threshold across the groups. Thus, the problem of inner beamformer design is given by

$$\begin{aligned} \underset{\gamma_k, \beta_k, \mathbf{w}_k, \boldsymbol{\zeta}_g}{\text{maximize}} \quad & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \alpha_k \log(1 + \gamma_k) \\ \text{subject to} \quad & \frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_k|^2}{\beta_k} \ge \gamma_k,\ \forall k \in \mathcal{U} && \text{(7a)} \\ & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \|\mathbf{B}_g \mathbf{w}_k\|^2 \le P_{\text{tot}} && \text{(7b)} \\ & \text{(6a), and (6b)} && \text{(7c)} \end{aligned}$$

where $\zeta_{\bar{g},k}, \forall \bar{g} \in \mathcal{G} \backslash \{g\}$ are the inter-group interference terms, which couples the IBF design problem.

In spite of relaxing the SINR expression in (3) using (7a) and (6), (7) is still nonconvex due to the *quadratic-over-linear* constraint (7a) [14]. Thus, to solve problem (7) efficiently, we resort to the successive convex approximation (SCA) technique wherein the nonconvex constraint is replaced by a sequence of approximate convex subsets, which is then solved iteratively until convergence [15]. We note that the LHS of (7) is convex, therefore we resort to the first order Taylor approximation of *quadratic-over-linear* function around some operating point, say, $\{\mathbf{w}_k^{(i)}, \beta_k^{(i)}\}$, is given by

$$\begin{aligned} \mathcal{F}_k(\mathbf{w}_k, \beta_k; \mathbf{w}_k^{(i)}, \beta_k^{(i)}) \triangleq\ & 2\, \frac{\mathbf{w}_k^{(i)\mathrm{H}} \mathbf{B}_g^{\mathrm{H}} \mathbf{h}_k \mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g}{\beta_k^{(i)}} \big(\mathbf{w}_k - \mathbf{w}_k^{(i)}\big) \\ & + \frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{B}_g \mathbf{w}_k^{(i)}|^2}{\beta_k^{(i)}} \Big(1 - \frac{\beta_k - \beta_k^{(i)}}{\beta_k^{(i)}}\Big). \end{aligned} \tag{8}$$

where the first order approximation $\mathcal{F}_k(\mathbf{w}_k, \beta_k; \mathbf{w}_k^{(i)}, \beta_k^{(i)})$ is an under-estimator for the LHS term in (8) is given by [12].

Now, by using the above approximation in (8), an approximate convex reformulation of (7) is given by

$$\begin{aligned} \underset{\gamma_k, \beta_k, \mathbf{w}_k, \boldsymbol{\zeta}_g}{\text{maximize}} \quad & \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{U}_g} \alpha_k \log(1 + \gamma_k) \\ \text{subject to} \quad & \mathcal{F}_k(\mathbf{w}_k, \beta_k; \mathbf{w}_k^{(i)}, \beta_k^{(i)}) \ge \gamma_k && \text{(9a)} \\ & \text{(6a), (6b), and (7b).} && \text{(9b)} \end{aligned}$$

The resulting problem (9) is solved iteratively until convergence by updating the operating point with the solution obtained from the previous iteration. Thus, upon convergence, the resulting inner beamforming vectors $\mathbf{w}_k \in \mathbb{C}^{S_g \times 1}, \forall k \in \mathcal{U}_g$ determine the linear combination of OBF column vectors of $\mathbf{B}_g$ that maximizes the overall sum rate of all users.

Unlike the approach presented in (9), the beamformers can also be designed independently by either fixing the inter-group interference to a fixed value or by ignoring them from the formulation. By doing so, the complexity involved in the design of inner beamformers reduces significantly as the number of optimization variables is limited.

Upon replacing $\mathbf{B}_g$ by $\mathbf{B} = [\mathbf{B}_{\mathcal{G}(1)}, \ldots, \mathbf{B}_{\mathcal{G}(|\mathcal{G}|)}]$ in (9), we obtain two-stage beamformer design without user grouping, i.e., $\mathbf{w}_k \in \mathbb{C}^{S \times 1}$, leading to a fully connected design (FC). This is equivalent to [11] when $G = 1$ is used in (9). By doing so, the inner beamformer finds a linear combination of all the available outer beamforming vectors, i.e., S spatial beams to serve any user in the system.

The problem in (9) can also be solved in a distributed manner among the groups via primal / dual decomposition or alternating directions method of multipliers (ADMM) technique as outlined in [12], [16], [17]. However, the number of iterations and the overhead involved in the signaling exchange among user groups limits the practical viability of those algorithms. In [18], unlike the centralized method the beamformers were designed independently by fixing the IGI term $\zeta_{\bar{g},k}$ to a fixed value or by ignoring them from the formulation. However, the performance for a fixed IGI constraint in [18] was observed to be fairly close to the group optimization problem. Hence, instead of [18], we consider an iterative solution via the system of KKT expressions for the fixed IGI value and propose to find the corresponding dual variable iteratively.

B. Iterative Solution

A practical iterative solution for (9) can be obtained by solving the system of KKT conditions [14]. To do so, we express (9) along with the dual variables as follows

$$\underset{\gamma_k,\beta_k,\mathbf{w}_k}{\text{maximize}} \quad \sum_{g\in\mathcal{G}}\sum_{k\in\mathcal{U}_g} \alpha_k \log(1+\gamma_k)$$

subject to

$$a_k: \quad \mathcal{F}_k(\mathbf{w}_k,\beta_k;\mathbf{w}_k^{(i)},\beta_k^{(i)}) \geq \gamma_k, \forall k \in \mathcal{U}_g \tag{10a}$$

$$b_k: \quad \sum_{i\in\mathcal{U}_g\backslash\{k\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_i|^2 + \sum_{\bar{g}\in\mathcal{G}\backslash\{g\}} \zeta_{\bar{g},k} + N_0 \leq \beta_k, \forall k \in \mathcal{U}_g \tag{10b}$$

$$c_{g,i}: \quad \sum_{k\in\mathcal{U}_g} |\mathbf{h}_i^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k|^2 \leq \zeta_{g,i}, \ \forall i \in \mathcal{U}\backslash\mathcal{U}_g \tag{10c}$$

$$d_g: \quad \sum_{g\in\mathcal{G}}\sum_{k\in\mathcal{U}_g} \|\mathbf{B}_g\mathbf{w}_k\|^2 \leq \frac{P_{\mathrm{tot}}}{G}, \forall g \in \mathcal{G} \tag{10d}$$

where $a_k, b_k, c_{g,i}$ and d_g are dual variables corresponding to constraints (10a), (10b), (10c) and (10d). The dual variable d_g in (10) is associated with the total power constraint of each group, whereas the dual variables a_k, b_k are associated with each user in the system. The dual variable $c_{g,i}$ belongs to the constraint (10c) ensuring inter-group interference within a fixed $\zeta_{g,i}$ value. Thus the Lagrangian is given by

$$\underset{a_k,b_k,c_{g,i},d_g}{\text{maximize}} \ \underset{\gamma_k,\beta_k,\mathbf{w}_k}{\text{minimize}} \quad L(\gamma_k,\beta_k,\mathbf{w}_k,a_k,b_k,c_{g,i},d_g) \tag{11}$$

where the objective is reversed in the Lagrangian expression due to the negative operator before the actual sum rate objective in (11). The solution is obtained by differentiating (11) with respect to each of the associated optimization variable

$$\nabla_{\gamma_k}: \frac{-1}{1+\gamma_k} + a_k = 0 \tag{12a}$$

$$\nabla_{\beta_k}: \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k^{(i)}|^2}{\beta_k^{(i)2}} a_k - b_k = 0 \tag{12b}$$

$$\nabla_{\mathbf{w}_k}: \mathbf{w}_k^{\mathrm{H}}\Big\{ \sum_{i\in\mathcal{U}_g\backslash k} b_i \mathbf{B}_g^{\mathrm{H}}\mathbf{h}_i\mathbf{h}_i^{\mathrm{H}}\mathbf{B}_g + \sum_{j\in\mathcal{U}_{\bar{g}}} c_{g,j}\mathbf{B}_g^{\mathrm{H}}\mathbf{h}_j\mathbf{h}_j^{\mathrm{H}}\mathbf{B}_g + d_g\mathbf{B}_g^{\mathrm{H}}\mathbf{B}_g\Big\} = \frac{a_k\mathbf{w}_k^{(i)\mathrm{H}}\mathbf{B}_g^{\mathrm{H}}\mathbf{h}_k\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g}{\beta_k^{(i)}} \tag{12c}$$

In addition to (12) and primal and dual feasibility constraints, KKT conditions also include the complementary slackness conditions as

$$a_k\Big\{\gamma_k - \mathcal{F}_k(\mathbf{w}_k,\beta_k;\mathbf{w}_k^{(i)},\beta_k^{(i)})\Big\} = 0 \tag{13a}$$

$$b_k\left\{\sum_{i\in\mathcal{U}_g\backslash\{k\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_i|^2 + \sum_{\bar{g}\in\mathcal{G}\backslash\{g\}} \bar{\zeta}_{\bar{g},k} + N_0 - \beta_k\right\} = 0 \tag{13b}$$

$$c_{g,i}\Big(\sum_{k\in\mathcal{U}_g} |\mathbf{h}_i^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k|^2 - \bar{\zeta}_{g,i}\Big) = 0 \tag{13c}$$

$$d_g\left(\sum_{g\in\mathcal{G}}\sum_{k\in\mathcal{U}_g} \|\mathbf{B}_g\mathbf{w}_k\|^2 - \frac{P_{\mathrm{tot}}}{G}\right) = 0. \tag{13d}$$

By fixing $a_k > 0$ and $b_k > 0$, we obtain a tractable solution for γ_k and β_k, respectively, by using the complementary slackness conditions in (13). However, to meet IGI constraints, dual variable $c_{g,i}$ must be found using an iterative sub-gradient update, which is similar to guaranteeing the inter-cell interference constraints in [12], [16], [19].

C. Heuristic Solution

Instead of iteratively finding the exact value for dual variable $c_{g,i}$ that ensures the equality of IGI constraints, fixed $c_{g,i}$ values are used to heuristically control the amount of interference caused from one group to another. By doing so, the beamformer expression $\mathbf{w}_k$ contains both inter- and intra-group interference channel components weighted by their respective dual variables. By fixing $c_{g,i}$, the optimal interference level cannot be guaranteed, leading to a slightly reduced sum rate performance. However, the complexity is greatly reduced as the sub-gradient search to meet the IGI constraints is avoided. Therefore, by solving the KKT expressions using (12), (13), and by setting $c_{g,i}$ to a fixed value, we obtain the system of update equations to design transmit precoders with fixed operating point

$$a_k^{(i)} = \frac{\alpha_k}{1+\gamma_k^{(i-1)}}, \qquad b_k^{(i)} = \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k^{(i-1)}|^2}{(\beta_k^{(i-1)})^2} a_k^{(i)} \tag{14a}$$

$$\mathbf{w}_k^{(i)\mathrm{H}} = \frac{a_k^{(i)}\mathbf{w}_k^{(i-1)\mathrm{H}}\mathbf{B}_g^{\mathrm{H}}\mathbf{h}_k\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g}{\beta_k^{(i-1)}}\left\{\sum_{i\in\mathcal{U}_g\backslash k} b_i^{(i)}\mathbf{B}_g^{\mathrm{H}}\mathbf{h}_i\mathbf{h}_i^{\mathrm{H}}\mathbf{B}_g + \sum_{j\in\mathcal{U}_{\bar{g}}} c_{g,j}^{(i)}\mathbf{B}_g^{\mathrm{H}}\mathbf{h}_j\mathbf{h}_j^{\mathrm{H}}\mathbf{B}_g + d_g\mathbf{B}_g^{\mathrm{H}}\mathbf{B}_g\right\}^{-1} \tag{14b}$$

$$\zeta_{g,i}^{(i)} = \sum_{k\in\mathcal{U}_g} |\mathbf{h}_i^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k^{(i)}|^2 \tag{14c}$$

$$\beta_k^{(i)} = \sum_{i\in\mathcal{U}_g\backslash\{k\}} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_i^{(i)}|^2 + \sum_{\bar{g}\in\mathcal{G}\backslash\{g\}} \zeta_{\bar{g},k}^{(i)} + N_0 \tag{14d}$$

$$\gamma_k^{(i)} = \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}_g\mathbf{w}_k^{(i)}|^2}{\beta_k^{(i)}}. \tag{14e}$$

Since the dual variable $a_k^{(i)}$ depends on $\gamma_k^{(i-1)}$, the initial operating point $\gamma_k^{(i-1)}$ is fixed by using some fixed feasible transmit precoder $\mathbf{w}_k^{(i-1)}$. It follows from the fact that $\gamma_k^{(i-1)}$ and $\beta_k^{(i-1)}$ can be obtained for a fixed $\mathbf{w}_k^{(i-1)}$. Upon fixing the $\mathbf{w}_k^{(i-1)}$, $\gamma_k^{(i-1)}$ and $\beta_k^{(i-1)}$ rest of the variables are updated as outlined in (14). The dual variable d_g is found by bisection search for each group, such that the total power constraint $\frac{P_{\mathrm{tot}}}{G}$ is satisfied by the transmit precoder $\mathbf{w}_k$.

We note that (14b) is the only constraint that involves the dual variable $c_{g,i}$ corresponding to inter-group interference constraints. The transmit precoder $\mathbf{w}_k^{(i)}$ can be evaluated using (14b), since the dual variable $c_{g,i}$ is fixed to a constant value. Before proceeding with $\beta_k^{(i)}$, we evaluate $\zeta_{g,i}^{(i)}$ for each group g using (14c) and exchange the real valued scalar IGI values between groups to provide better rate approximation for the iterative process. Upon, exchanging the $\zeta_{g,i}^{(i)}$ between the groups, we find $\beta_k^{(i)}$ using (14d). Finally, solving $\gamma_k^{(i)}$ using

(8) may not provide a suitable approximation due to fixed $c_{g,i}$, hence, we use the actual SINR expression in (14e) to find $\gamma_k^{(i)}$.

This procedure is performed until convergence [1] in the same sequence as outlined in (14). The algorithm for this procedure is provided in Algorithm 1, and is repeated until the sum rate between two successive iterations is below 10^{-3}.

Algorithm 1: Heuristic Method

Input: $\alpha_k, \mathbf{h}_k, \forall k \in \mathcal{U}$.
Output: $\mathbf{w}_k, \forall k \in \mathcal{U}$
Initialize: $i = 1$, $\gamma_k^{(0)}$, and $\beta_k^{(0)}$ using feasible IBF $\mathbf{w}_k^{(0)}$ for fixed OBF $\mathbf{B}_g$, $\forall g \in \mathcal{G}$. Fix the inter-group dual variable $c_{g,i}$ to some constant value.
repeat
  Solve for $a_k^{(i)}$ with (14a) using fixed $\gamma_k^{(i-1)}$
  Solve for $b_k^{(i)}$ using $a_k^{(i)}, \mathbf{w}_k^{(i-1)}$ and $\beta_k^{(i-1)}$
  Upon finding $\mathbf{w}_k^{(i)}$ using (14b) via bisection search over d_g, update $\bar{\zeta}_{g,i}$, $\forall i \in \mathcal{U}_{\bar{g}}$, $\forall g \in \mathcal{G}$ using (14c)
  Once $\bar{\zeta}_{g,i}$ is obtained, the inter-group interference values are exchanged between groups in order to provide better rate approximation
  Upon exchanging the $\bar{\zeta}_{g,i}$ value, the total interference $\beta_k^{(i)}$ and SINR $\gamma_k^{(i)}$ are found by solving (14d) and (14e), respectively.
until perform (14) until convergence

V. Complexity Analysis

In this section, we compare the computational complexity of different IBF design methods. Since, the solutions are obtained iteratively by solving a convex sub-problem in each iteration, the complexity is proportional to the number of iterations required. We ignore the complexity of the OBF design as it can be computed off-line. However, we observe that as the number of groups increases, then the number of OBF dimensions per group S_g is reduced similarly.

It is worth noting that the fully connected (FC) design can also be solved using KKT expressions without fixed IGI constraints, which involves inversion of matrix sized $S \leq N_T$. It requires roughly $\mathcal{O}(S^3 \times K \times \Delta)$ complexity per iteration with Δ being the number of bisection search required to assign transmit powers to satisfy power constraint. The complexity involved in estimating other variables are noticeably less. On the contrary, group specific heuristic design with fixed dual variables $c_{g,i}$, the complexity per group and per iteration is dominated by the beamformer expression in (14b), which is in the order of $\mathcal{O}(S_g^3 \times K \times \Delta)$. As an example, when $S_g = 16, (G = 4)$ and $S = 64, (G = 1)$ for per group and FC design, respectively, the total computational complexity of per group scheme is $4 \times \frac{16^3}{64^3} = 4^{-3} = 4 \times 2\%$ fraction of FC design without sacrificing much on the performance.

VI. Numerical Results

Similar to [11], we consider a single cell DL massive MIMO BS equipped with $N_T = 64$ ULA antenna elements

[1]Formal convergence proof is omitted, since it is shown in [12], [17].

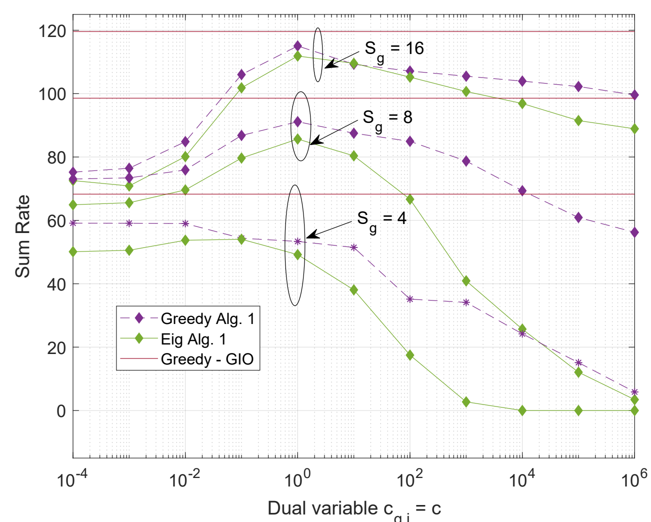

Fig. 2. Sum rate vs. Dual variable $c_{g,i}$ for $S_g = 4, 8, 16$.

serving a total of $K = 16$ single-antenna users. The users are naturally partitioned into 4 segments each with $45°$, and with 4 users randomly placed in each segment. The angular spread for each user is considered to be $15°$ degree with 20 independent paths per user. The groups are distributed uniformly within $[\frac{-\pi}{2}, \frac{\pi}{2}]$. The user specific weights in the WSRM objective are fixed to $\alpha_k = 1 \forall k \in \mathcal{U}$. The plots are obtained by varying the number of statistical beams employed at the BS. If $G > 1$, then the number of statistical beams are divided equally among the user groups, i.e., $S_g = \frac{S}{G}$ while using (9). The results are averaged over 200 channel realizations and the noise variance is assumed to be unity. The transmit power is set to $P_{\text{tot}} = 20\text{dB}$ with respect to noise variance.

Before proceeding further, we define the legends used in figures. The OBF is defined explicitly followed by the type of IBF design. The FC system is obtained via solving (9) by setting $\mathbf{B}_g = \mathbf{B}, \forall g \in \mathcal{G}$, thus all the outer beams are utilized by the IBF for serving a user in any group. The group interference optimization (GIO) figures are obtained by solving (9) with optimal IGI, thus only the group specific outer beamformers are utilized by the IBF while serving group specific users. By setting $\zeta_{g,i} = 0$ and ignoring the constraint in (6b), we obtain the isolated design as in [6], referred as ζ = ignore in figures. Finally, by solving the set of KKT expressions in Algorithm 1 with a fixed dual variable $c_{g,i} = c$, $\forall g \in \mathcal{G}, i \in \mathcal{U} \backslash \mathcal{U}_g$, we obtain the heuristic solution, referred to as Alg. 1 in figures.

Fig. 2 illustrates the sum rate performance for the two different group specific OBF methods with $G = 4$ as a function of dual variables $c_{g,i} = c, \forall g, i$. This dual variable corresponds to the IGI constraint in (10c). It is worth noticing that the best sum rate performance is obtained when $c_{g,i} = 1$ considering both OBF scenarios. However, for lower number of statistical beams, say $S_g = 4$, the sum rate falls down to zero when $c_{g,i} \geq 10$, this is due to the fact that there is not enough DoF to null IGI in each group. Therefore, we fix the dual variable with the best value $c_{g,i} = 1$ to solve the KKT expressions as in Algorithm 1. Furthermore, we can observe that the heuristic

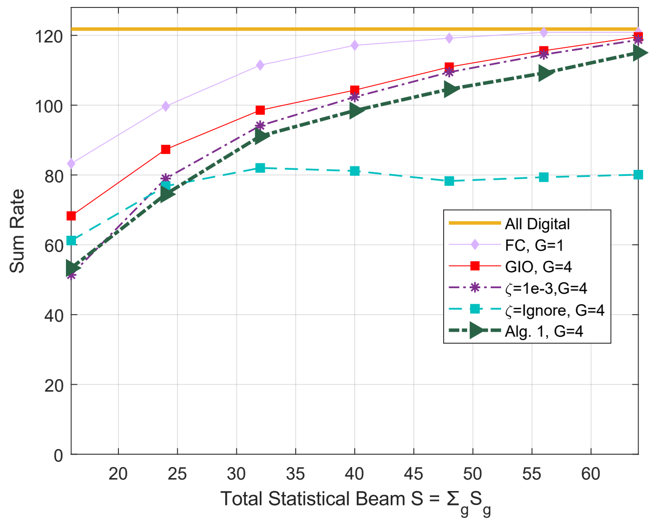

Fig. 3. Sum rate vs. total statistical beams for greedy OBF design.

Algorithm 1 provides almost the same sum rate as compared to the optimized IGI levels when proper dual variables $c_{g,i}$ are used. Since greedy maximization achieves better sum rate performance over Eigen selection, we ignore Eigen selection based OBF plots in Fig. 3 for clarity.

Fig. 3 demonstrates the sum rate performance of all the proposed schemes with $P_{\text{tot}} = 20\text{dB}$. The FC design is identical to the method proposed in [11], which is nothing but (9) with $G = 1$. On the contrary, the number of user group is fixed as $G = 4$ in all group specific techniques. It is also worth noting that setting $\zeta = 10^{-3}$ design performs almost similar to that of (9). However, upon, ignoring the IGI term altogether from the optimization problem, the achievable sum rate is noticeably inferior. This is due to the fact that the OBF beams have considerable sidelobes, hence, the leakage to neighboring groups users that are located at the boundaries suffer from severe IGI as it is left uncompensated while designing the inner beamformers. Finally, by fixing the dual variable $c_{g,i} = 1$ from Fig. 2, to solve the KKT expressions, we observe that the performance is very close to the methods with optimized IGI levels. Moreover, the complexity is comparable to that of isolated design, since, there is no need to update the dual variable corresponding to the IGI constraint.

VII. Conclusion

In this paper, we proposed a fully digital two-stage beamforming for a single cell downlink massive multiple-input multiple-output system with user grouping based on geographical location. This beamformer design consists of OBF and IBF. We also considered two different approaches to form the OBF matrix namely, Eigen selection and greedy energy maximization so as to reduce the effective channel dimensions. Upon fixing the outer beamformer, the inner beamformers were designed by considering group specific interference constraint using weighted sum rate maximization objective. In order to further reduce the complexity, a computationally efficient iterative solution was proposed via solving the Karush-Kuhn-Tucker optimality conditions. A heuristic iterative approach was also presented by fixing the dual variable corresponding to the inter-group interference constraint to a constant value. It is shown that the computational complexity can be reduced to a large extent as compared to the single group beamforming while preserving most of the sum rate performance with a minor loss.

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