Performance Analysis Framework for NOMA Systems over Non-Identical Nakagami-m Fading Channels

Anderson Tregancini Jr*, Carlos H. M. Lima†, Edgar E. Benitez Olivo*, and Hirley Alves†
*São Paulo State University (UNESP), São João da Boa Vista, Brazil
†Centre for Wireless Communications (CWC), University of Oulu, Finland
{anderson.tregancini, edgar.olivo}@unesp.br, {carlos.lima, hirley.alves}@oulu.fi

***Abstract*—This paper proposes an analytical framework based on the H-function distribution to study the performance of non-orthogonal multiple access (NOMA)-based wireless networks. More specifically, herein we investigate the outage performance and the ergodic capacity of a three-node linear network topology where a source uses power-domain NOMA to communicate with two destinations over independent, but not necessarily identically distributed Nakagami-m fading channels. A Monte Carlo simulations campaign was carried out to support the proposed framework.**

***Index Terms*—ergodic capacity, H-function distribution, Nakagami-m, NOMA, outage probability**

I. Introduction

Different from previous iterations, the next generation of wireless communications will not be a straightforward evolution of current systems, instead it will disrupt longstanding design principles and revolutionize the way we live, interact and make business. Hitherto, legacy networks were designed as general purpose connecting platforms with very limited functionalities which were tailored to independent deployment scenarios and use cases. On the other hand, the new 5G infrastructure will consider specific requirements from the very beginning which is determinant to fully integrate digital technologies into the vertical markets and daily lives.

Actually, the next generation will combine physical and digital worlds and, by doing so, will realize the concept of a truly connected society [1]. To achieve this, four enabling technologies are converging, namely, 5G wireless systems, Artificial Intelligence (AI), data analysis (Big data), and Internet of Things (IoT). In fact, pundits advocate this convergence process will revolutionize both the communication and information technology ecosystems and drastically alter the main industry sectors.

The new 5G systems will become the catalyst of this technological revolution by making viable a new set of exclusive resources beyond nowadays high capacity services, namely, i) enhanced mobile broadband allowing continuous experience even at high mobility; ii) massive deployment of smart objects (internet of things); iii) critical mission services requiring very low latency and high reliability. Then, a transparent interaction between Information and Communications technology (ICT) and vertical industry sectors (with many new innovative use cases) will be enabled by 5G systems.

To meet such stringent requirements, several new technologies have emerged recently [2], [3]. In particular, Non-Orthogonal Multiple Access (NOMA) arises as a potential solution to significantly increase the network capacity in dense networks.

Different from Orthogonal Multiple Access (OMA) schemes, whereby each user is allocated in an individual orthogonal channel, NOMA enables multiple users to share the same radio resources by using code- or power-domain multiplexing [4]. Specifically, power-domain NOMA allocates different power levels to each users based on their channel conditions and jointly broadcast their information signals using a superposition coding. In fact, this strategy improves the network capacity at the expense of multiuser interference. Then, a Successive Interference Cancellation (SIC) algorithm can be applied at the receivers in order to cancel multiuser interference [4], [5].

Recently, innumerous NOMA-based works for 5G deployment scenarios have been proposed [4]–[6]. Motivated by these extensive research efforts, we propose an analytical framework based on the Fox-H function so as to derive closed-form expressions for the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) (*i.e.*, outage probability) of the Signal-to-Noise Ratio (SNR) distribution for the systems under investigation. In this contribution, the downlink of NOMA-based network is evaluated over various radio channel degradation effects and network configurations. We consider Nakagami-m fading since it allows for modeling a wide range of mobile multipath fading channels, including indoor and outdoor environments [7].

In order to establish a unified framework to capture the radio channel statistics [8], as well as assessing the performance of the proposed wireless communication schemes, we derive analytical expressions by means of the Fox H-function distribution, which is a generalized function given in terms of contour integrals involving products of gamma functions. Lately, this approach has attracted a lot of attention after the study carried out in [9] in which the error probability and capacity were analyzed by using the H-transform. Recently, the authors in [10] propose a novel H-function-based analytical framework to derive the End-to-End (E2E) SNR distribution for a cooperative dual-hop Amplify-and-Forward (AF) relaying scheme over Weibull-fading channels and authors in [11] present a novel framework for the sum of independent and non identically distributed H-function random variables. Besides, with the advances in the scientific standard softwares (*e.g*, open source SciPy [12]), the Fox H-function computing can be easily implemented. Hereinafter, H-function distribution is employed to derive a closed-form expression for the outage probability of our proposed downlink NOMA scenario.

The main contributions of this work are summarized as follows:

- An analytical framework based on the Fox H-function distribution for computing the SNR distribution for downlink NOMA scenarios is introduced.
- Exact expressions for the outage probability of each NOMA destination considering independent, but not necessarily identical Nakagami-m multipath fading channels are derived.
- Approximate expressions of the ergodic capacity of each NOMA destination are derived using the proposed framework, which are shown to be highly accurate over the entire range of SNR values.

The remainder of this paper is organized as follows. Our analytical framework based on the Fox H-function is discussed in Section II. Thereafter, the system and signal models are introduced in Section III. This framework is then used to carry out an outage analysis in Section IV, and an ergodic capacity analysis in Section V. Numerical results are illustrated in Section VI by assessing the performance of NOMA deployment scenarios. Finally, the conclusions of this work are drawn in Section VII.

II. Preliminaries and Analytical Framework

We begin this section by introducing the Fox H-function distribution, which is defined by means of a Mellin-Barnes integral [13], and then present some useful mathematical proprieties and identities which are used through this work.

A. The Fox H-function

The Fox H-function is based on a Mellin-Barnes type integral and can be defined in the following manner [14], [15]

$$H(x) = H_{p,q}^{m,n}\left[x \middle| \begin{matrix}(a_i, A_i)_{1\ldots p}\\(b_i, B_i)_{1\ldots q}\end{matrix}\right] = \frac{1}{2\pi j}\int_{\mathcal{L}} \frac{\prod_{i=1}^{m}\Gamma(b_i + B_i s)\prod_{i=1}^{n}\Gamma(1-a_i-A_i s)x^{-s}}{\prod_{i=n+1}^{p}\Gamma(a_i + A_i s)\prod_{i=m+1}^{q}\Gamma(1-b_i-B_i s)}ds \quad (1)$$

where $\Gamma(\cdot)$ is the gamma function; m,n,p,$q \in \mathbb{Z}^+$ with $0 \leq m \leq q$, $0 \leq n \leq p$; x,$\{a_i\}_{i=1}^{p}$,$\{b_i\}_{i=1}^{q} \in \mathbb{C}$, $\{A_i > 0\}_{i=1}^{p}$, $\{B_i > 0\}_{i=1}^{q}$; and $\mathcal{L}$ is the contour in the complex s-plane running from $\omega - j\infty$ to $\omega + j\infty$, such that all left half-plane poles of $\prod_{i=1}^{m}\Gamma(b_i + B_i s)$ are defined as

$$b_{il} = \frac{-(b_i + l)}{B_i} \quad (i = 1, \ldots, m;\ l = 0, 1, 2, \ldots), \quad (2)$$

lie to the left of $\mathcal{L}$, and all right half-plane poles of $\prod_{i=1}^{m}\Gamma(1-a_i-A_i s)$, defined by

$$a_{ir} = \frac{1-a_i+r}{A_i} \quad (i = 1, \ldots, n;\ r = 0, 1, 2, \ldots), \quad (3)$$

lie to the right of $\mathcal{L}$. Furthermore, if occurs an empty product in (1), it is taken to be one [14], [15] [1].

B. The Fox H-function distribution

The PDF of a random variable X following a Fox H-function distribution is given by

$$f_{eX}(x) = \frac{k}{e}H_{p,q}^{m,n}\left[\frac{c}{e}x \middle| \begin{matrix}(a_i, A_i)_{1\ldots p}\\(b_i, B_i)_{1\ldots q}\end{matrix}\right] \quad (4)$$

for $x > 0$, where k is a normalization constant so that $\int_0^\infty f_X(x)dx = 1$, $c > 0 \in \mathbb{C}$ and $e \neq 0 \in \mathbb{R}$ [16]–[18].

The representation in (4) offers several advantages for manipulating probability distributions, since it provides a common framework to characterize many continuous and non-negative random variables, such as Gamma, Exponential, Beta, Weibull, and Rayleigh distributions [17], [18], which are widely used to model and simulate wireless communication systems. Equally important, the Fox-H representation allows to develop the product, quotient, power, integral and (Laplace, Fourier and Mellin) transforms of random variables in a direct manner, as well as making it easier to derive probability distributions characteristics, such as the moments and CDF, which can also be written in terms of the Fox H-function distribution [17], [18].

Property 1: The rth moment about the origin of a real valued Random Variable (RV) X, given by $\mathrm{E}[X^r] = \int_0^\infty x^r f_X(x)dx$, can be expressed in terms of Fox H-function using the Mellin transform [14], [15] as

$$\mathrm{E}[X^r] = \frac{k}{c^{r+1}}\frac{\prod_{i=1}^{m}\Gamma(b_i+B_i+B_i r)\prod_{i=1}^{n}\Gamma(1-a_i-A_i-A_i r)}{\prod_{i=n+1}^{p}\Gamma(a_i+A_i+A_i r)\prod_{i=m+1}^{q}\Gamma(1-b_i-B_i-B_i r)}. \quad (5)$$

III. System Model

Consider the downlink of a NOMA-based network deployment scenario where a source S intends to communicate with two NOMA users $\{D_\ell\}_{\ell=1}^{2}$ [2]. The nodes S, D_1, and D_2 are single-antenna devices that operate with omni-directional radiation pattern. It is also assumed that all links are affected by additive white Gaussian Noise (AWGN) with mean power N_0, and occur over slow/non-selective fading channels, with

[1]For more details on the Fox H-function regarding additional properties and identities (with existence conditions), we refer the reader to [14], [15].

[2]It has been reported in the literature that NOMA with a large number of users multiplexed in the power domain may achieve higher spectrum efficiency [4]. However, it may not be feasible in practice since the processing complexity of SIC receivers grows non-linearly with the number of users, which is most evident when SIC error propagation is assumed, therefore two NOMA users is a practical assumption [19].

complex channel coefficients $\{h_\ell\}_{\ell=1}^2$ following a Nakagami-m distribution with channel mean power $\Omega_\ell = \mathrm{E}\left\{|h_\ell|^2\right\}$.

The source S broadcasts the composed information signal $v(t)$ to both D_1 and D_2, which is given by

$$v(t) = \sqrt{a_1 P} x_1(t) + \sqrt{a_2 P} x_2(t), \tag{6}$$

where a_1 and a_2 are the power allocation coefficients of the far user D_1 and near user D_2, such that $a_1 > a_2$ and $a_1 + a_2 = 1$, $x_1(t)$ and $x_2(t)$ are the unity-power information signals of the users D_1 and D_2, respectively (*i.e.* $\left\{\mathrm{E}\left[x_\ell(t)^2\right]\right\}_{\ell=1}^2 = 1$), P is the transmit power at S and t denotes time.

The received signal at the destination D_ℓ is given by

$$y_\ell(t) = h_\ell v(t) + n_\ell(t), \tag{7}$$

where $\{n_\ell(t)\}_{\ell=1}^2$, is the AWGN noise signal at the link between S and D_ℓ.

Now, the instantaneous received Signal-to-Interference-plus-Noise Ratio (SINR) at users D_1 and D_2 can be defined. The far user D_1 treats the information $x_2(t)$ as interference. Thus, the instantaneous SINR at D_1 is given by

$$\gamma_{D_1} = \frac{a_1 X_1}{a_2 X_1 + 1}, \tag{8}$$

where $X_1 = \gamma_0 |h_1|^2$ is the instantaneous received SNR of the link between S and D_1, and $\gamma_0 = P/N_0$ is the average transmit SNR. According to the NOMA scheme, the nearest user D_2 must first decode the user D_1 message, which is possible since $a_1 > a_2$; thus, similarly to (8), the instantaneous SINR at D_2 to detect $x_1(t)$ is given by

$$\gamma_{D_{12}} = \frac{a_1 X_2}{a_2 X_2 + 1}, \tag{9}$$

where $X_2 = \gamma_0 |h_2|^2$ is the instantaneous received SNR of the link between S and D_2. Thus, once D_1 message is decoded, D_2 is able to remove $x_1(t)$ and decode its own message, $x_2(t)$, using SIC. Therefore, the instantaneous received SINR at D_2 to detect its own message is given by

$$\gamma_{D_2} = a_2 X_2. \tag{10}$$

IV. Outage Probability Analysis and Framework Application

In this section, we first formulate the outage probability for the D_1 and D_2 users, and later on we apply them to our proposed framework. For the sake of simplicity, the SNR thresholds used to define the outage events of both D_1 and D_2, γ_{th_1} and γ_{th_2}, are assumed to be equal and given by γ_{th}.

A. Outage Probability at D_1

The event that defines the outage probability for the user D_1 can be expressed as

$$\begin{aligned}
\mathrm{OP}_1 &= \Pr\left(\gamma_{D_1} < \gamma_{\mathrm{th}}\right) \\
&= \Pr\left(X_1 < \frac{\gamma_{\mathrm{th}}}{a_1 - a_2\gamma_{\mathrm{th}}}\right) \\
&= F_{X_1}\left(\widetilde{\gamma}_{\mathrm{th}}\right),
\end{aligned} \tag{11}$$

where $\widetilde{\gamma}_{\mathrm{th}} = \gamma_{\mathrm{th}} / \left(a_1 - a_2\gamma_{\mathrm{th}}\right)$ and $F_{X_1}(\cdot)$ is the CDF of X_1.

B. Outage Probability at D_2

The user D_2 will be in outage when: (i) $\gamma_{D_{12}}$ is below the threshold γ_{th}, implying that SIC in D_2 fails; or (ii) even after successfully executing the SIC process, γ_{D_2} is below γ_{th}. Consequently, the event that defines the outage probability of the user D_2 can be given by

$$\begin{aligned}
\mathrm{OP}_2 &= \Pr\left(\gamma_{D_{12}} < \gamma_{\mathrm{th}}\right) + \Pr\left(\gamma_{D_{12}} > \gamma_{\mathrm{th}}, \gamma_{D_2} < \gamma_{\mathrm{th}}\right) \\
&\overset{(a)}{=} \Pr\left(X_2 < \widetilde{\gamma}_{\mathrm{th}}\right) + \Pr\left(X_2 < \breve{\gamma}_{\mathrm{th}}\right) \\
&\quad - \Pr\left(X_2 < \widetilde{\gamma}_{\mathrm{th}}, X_2 < \breve{\gamma}_{\mathrm{th}}\right) \\
&\overset{(b)}{=} \Pr\left(X_2 < \max\left(\widetilde{\gamma}_{\mathrm{th}}, \breve{\gamma}_{\mathrm{th}}\right)\right) \\
&= F_{X_2}\left(\theta\right),
\end{aligned} \tag{12}$$

where $\breve{\gamma}_{\mathrm{th}} = \gamma_{\mathrm{th}}/a_2$, $\theta = \max\left(\widetilde{\gamma}_{\mathrm{th}}, \breve{\gamma}_{\mathrm{th}}\right)$ and $F_{X_2}(\cdot)$ is the CDF of X_2. Furthermore, (a) is obtained by applying the identity $\Pr(A,B) = \Pr(A) - \Pr(A,\bar{B})$ [20] and (b) is obtained since the occurrence of an outage event with respect to the maximum between $\widetilde{\gamma}_{\mathrm{th}}$ and $\breve{\gamma}_{\mathrm{th}}$ implies the occurrence of the other event.

C. Framework Application in NOMA Scenario

As aforesaid, the radio channel coefficients follow a Nakagami-m distribution, such that the PDF of the instantaneous received SNR follows a gamma distribution given by

$$f_X(x) = \frac{m_s^{m_s} x^{m_s - 1}}{\left(\gamma_0\Omega\right)_s^m \Gamma(m_s)} e^{\left(-\frac{m_s x}{\gamma_0\Omega}\right)}, \qquad \text{for } x > 0, \tag{13}$$

where $m_s > 1/2$ is the fading severity parameter. It is worth mentioning that this formulation can model Rayleigh ($m_s = 1$), Hoyt ($m_s < 1$) and Rice ($m_s > 1$) fading.

As a special case of the H-function distribution, the instantaneous SNR given in (13) can be rewritten by means of the H-function representation in (4) as follows

$$f_X(x) = \frac{m_s}{\gamma_0\Omega\Gamma(m_s)} \mathsf{H}_{0,1}^{1,0}\left[\frac{m_s}{\gamma_0\Omega}x \middle| \begin{matrix} - \\ (m_s - 1, 1) \end{matrix}\right], \tag{14}$$

Next, the CDF corresponding to (14), can be obtained in H-function representation by applying [17, Theorem 4.5], thus

$$F_X(x) = 1 - \frac{1}{\Gamma(m_s)} \mathsf{H}_{1,2}^{2,0}\left[\frac{m_s}{\gamma_0\Omega}x \middle| \begin{matrix} (1,1) \\ (0,1), (m_s, 1) \end{matrix}\right]. \tag{15}$$

Finally, the outage probability is derived using the H-function representation for both the farthest and nearest users by mapping (11) and (12) into (15), respectively.

V. Ergodic Capacity Analysis and Framework Application

In this section, we first formulate the normalized ergodic capacity for the D_1 and D_2 users, and then we apply them to our proposed framework. The ergodic capacity at $\{D_\ell\}_{\ell=1}^2$ user can be given by

$$\bar{C}_\ell = \mathrm{E}\left[\log_2\left(1 + \gamma_{D_\ell}\right)\right]. \tag{16}$$

A. Framework Application to the Ergodic Capacity at D1

For the far user D_1, using Jensen's inequality [7, Eq. 4.5], the ergodic capacity can be approximated as

$$\bar{C}_1 \approx \log_2 \left(1 + \mathrm{E}\left[\frac{a_1}{a_2} \left(1 + \frac{1}{a_2 X_1} \right)^{-1} \right] \right). \tag{17}$$

To express (17) in terms of the H function, we make use of the mathematical properties presented in Section II in order to compute the reciprocal of the sum of a random variable and a constant. To this end, we first use [16, Theorem 4.2] in order to determine the PDF of $Y = (a_2 X_1)^{-1}$ as

$$f_Y(y) = \frac{a_2 \gamma_0 \Omega}{m_s \Gamma(m_s)} \mathsf{H}_{1,0}^{0,1} \left[\frac{a_2 \gamma_0 \Omega}{m_s} y \middle| \begin{matrix} (-m_s, 1) \\ - \end{matrix} \right]. \tag{18}$$

Afterwards, we use the moment-match approach [18] to derive the PDF of the sum $Z = Y + 1$. Thus, using [10, Proposition 1] and the binomial expansion, we compute the rth (non-central) moment of Z as

$$\mu_{Z_r} = \mathrm{E}\left[(Y+1)^r \right] = \sum_{i=0}^{r} \binom{r}{i} \mathrm{E}\left[Y^i \right]. \tag{19}$$

Then, by using Property 1, we can build a nonlinear system of $2(p+q)+2$ equations so as to solve and estimate the parameters of the H-function distribution of Z, denoted by a_z, A_z, k_z, and c_z. Since Y is characterized by the parameters $[m, n, p, q] = (0, 1, 0, 1)$, Z also fit to these parameters.

To obtain the PDF of γ_{D_1} in terms of the H-function distribution, we compute the PDF of $W = (a_1/a_2)\ Z^{-1}$ from (18) as follows

$$f_W(w) = \frac{a_2 k_z}{a_1 {c_z}^2} \mathsf{H}_{0,1}^{1,0} \left[\frac{a_2 w}{a_1 c_z} \middle| \begin{matrix} - \\ (1 - a_z - 2A_z, A_z) \end{matrix} \right]. \tag{20}$$

Finally, we apply Property 1 to (20) to obtain $\mathrm{E}[W]$. Then, the ergodic capacity for the user D_1 can be computed as

$$\bar{C}_1 \approx \log_2 \left(\frac{a_1 k_z}{a_2} \Gamma(1 - a_z) + 1 \right). \tag{21}$$

B. Framework Application to the Ergodic Capacity for D2

The ergodic capacity for D_2 can be approximated by

$$\bar{C}_2 \approx \log_2 \left(\mathrm{E}\left[a_2 X_2 + 1 \right] \right). \tag{22}$$

To determine (22), we compute the moment-match approach to find the distribution of $Z_2 = a_2 X_2 + 1$. Then, similarly to the process used in Section V-A, we first use (19) to calculate the moments of $a_2 X_2$; then, we use Property 1 to construct the nonlinear equations system in order to estimate the new parameters of Z_2, defined by $b_{z_2}, B_{z_2}, k_{z_2}$, and c_{z_2}. Finally, using Property 1 in Z_2 and substituting in (14), we obtain

$$\bar{C}_2 \approx \log_2 \left(\frac{k_{z_2}}{c_{z_2}^2} (b_z + 2B_z) \right) \tag{23}$$

$$= \log_2 \left(\frac{a_2 \Omega_2 \gamma_0}{m_{s_2} \Gamma(m_{s_2})} \Gamma(m_{s_2} + 1) + 1 \right). \tag{24}$$

The attained expressions in (21) and (24) (for $m_{s_2} >> 1$) prove to be very tight to the corresponding exact ergodic capacities, as shall be seen in Section VI.

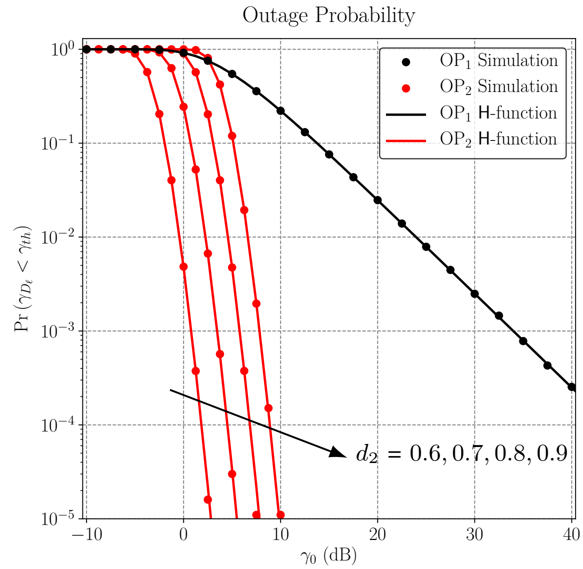

Fig. 1. Outage probability of the users D_1 and D_2 versus transmitted SNR for different values of normalized distances d_2 = (0.6, 0.7, 0.8, 0.9) and d_1 = 1.0, power allocation factors (a_1, a_2)=(0.7; 0.3), fading severity parameters m_{s1} = 1 and m_{s2} = 13.313 (Rician parameter $K \approx$ 14 dB).

VI. Numerical Results

In this section, we assess the performance of the system under study in terms of the outage probability and the ergodic capacity using the Fox-H-based analytical framework developed in previous sections. To corroborate our analytical results, an extensive simulation campaign using the Monte Carlo approach (10^6 snapshots) were carried out. We suppose a linear topology, with d_ℓ being the normalized distance of the link between S and D_ℓ. Furthermore, the average channel gain of all links are considered proportional to the path-loss, in other words, $\Omega_\ell = d_\ell^{-\alpha}$, where α is the path-loss exponent. Fig. 1 compares the outage probability for NOMA users D_1 and D_2 as a function of increasing transmit SNR for different values of normalized distances between S and D_2, namely, d_2 = (0.6, 0.7, 0.8, 0.9). In this case, we consider a hypothetical scenario where the near user D_2 undergoes, on average, a better channel condition than the far user D_1 (due to normalized distances values $d_1 > d_2$), as well as being subject to a strong Line-of-Sight (LoS) component in comparison to D_1 (*i.e.*, $m_{s2} > m_{s1}$). Accordingly, we consider power allocation factors of (a_1, a_2)=(0.7, 0.3), normalized distance d_1 = 1.0, fading severity parameters (m_{s1}, m_{s2}) = (1.0, 13.313), path-loss exponent α = 4 and outage threshold γ_{th} = 0 dB. From Fig. 1, we can observe that, for the highlighted scenario, even with such a large disparity between the power allocated to users D_1 and D_2, the nearest user D_2 is still able to show outage performance significantly higher than far user D_1. This condition draws us attention to the fact that taking advantage of the Channel State Information (CSI) experimented by the users, we can exploit NOMA to substantially increase the network capacity while handling

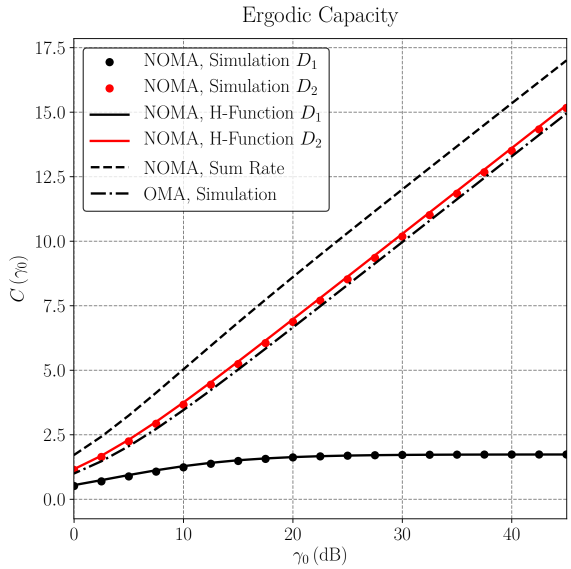

Fig. 2. Ergodic Capacity of the NOMA-based users D_1 and D_2 versus transmitted SNR with normalized distances d_1 = 1.0 and d_2 = 0.7, power allocation factors (a_1, a_2)=(0.7; 0.3), fading severity parameters m_{s_1} = 1 and m_{s_2} = 13.313 (Rician parameter $K \approx 14$ dB). The sum rate for users D_1 and D_2, as well as the ergodic capacity for a OMA-based system with $d = m_s = 1$ are also shown.

multiuser interference.

Fig. 2 shows a comparison of the ergodic capacity for NOMA users D_1 and D_2 versus the transmit SNR, considering the same parameters as in Fig. 1, but setting $d_2 = 0.7$. Moreover, for comparison, the sum rate of the proposed system and the ergodic capacity for a single OMA user with $d = d_1$ and $m_s = m_{s_1}$ are also presented. We can observe from Fig. 2 that the ergodic capacity for D_2 is comparable to that obtained for the OMA user, while the user D_1 performance is saturated due to the interference from D_2. However, the sum rate of our system outperforms that of the OMA-based one in all the SNR range, demonstrating the benefits of NOMA in the highlighted scenario. Note that the curves obtained from our H-Function-based analytical framework match the Monte Carlo simulation results, thus validating our analysis.

VII. Conclusions

In this paper, an analytical framework to compute the performance of a downlink NOMA system was introduced so as to evaluate the performance of modern wireless communication systems. We introduced an approach based on the Fox H-function distribution and highlighted its flexibility to assess several evaluation scenarios ranging from indoor to outdoor deployments. To do so, we derived closed-form expressions for the outage probability and the ergodic capacity under various radio channel characteristics. Then, we employ our framework to investigate the performance of a NOMA system in scenarios where the radio channel undergo independent, but not necessarily identical Nakagami-m fading. It is worth noting that the H-function representation simplify the analysis and allows for robust-closed-form solutions for many emergent analytical problems using NOMA.

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References

[1] 5GPP, "White paper: The 5G infrastructure public private partnership: the next generation of communication networks and services," [Online]. Avaliable: https://5g-ppp.eu/wp-content/uploads/2015/02/5G-Vision-Brochure-v1.pdf.

[2] M. Agiwal, A. Roy, and N. Saxena, "Next generation 5G wireless networks: A comprehensive survey," *IEEE Comm. Surveys Tuts.*, vol. 18, no. 3, pp. 1617–1655, thirdquarter 2016.

[3] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 74–80, Feb. 2014.

[4] S. M. R. Islam, N. Avazov, O. A. Dobre, and K. Kwak, "Power-domain non-orthogonal multiple access NOMA in 5G systems: Potentials and challenges," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 721–742, Secondquarter 2017.

[5] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, "A survey of non-orthogonal multiple access for 5G," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 2294–2323, thirdquarter 2018.

[6] A. Kiani and N. Ansari, "Edge computing aware NOMA for 5G networks," *IEEE Internet Things J.*, vol. 5, no. 2, pp. 1299–1306, April 2018.

[7] A. Goldsmith, *Wireless Communications*. Cambridge Univ. Press, 2005.

[8] K. Yao, M. K. Simon, and E. Bigileri, "Unified theory on wireless communication fading statistics based on SIRP," in *IEEE 5th Workshop Signal Process. Adv. Wireless Commun., 2004.*

[9] Y. Jeong, H. Shin, and M. Z. Win, "H-transforms for wireless communication," *IEEE Trans. Inf. Theory*, vol. 61, no. 7, pp. 3773–3809, 2015.

[10] C. H. de Lima, H. Alves, and P. H. Nardelli, "Fox H-function: A study case on variate modeling of dual-hop relay over weibull fading channels," in *Proc. WCNC*. IEEE, 2018, pp. 1–5.

[11] Y. Abo Rahama, M. H. Ismail, and M. S. Hassan, "On the sum of independent fox's H-function variates with applications," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 6752–6760, Aug 2018.

[12] E. Jones, T. Oliphant, P. Peterson *et al.*, "SciPy: Open source scientific tools for Python," 2001–, [Online; accessed July 3, 2019]. Avaliable: http://www.scipy.org/.

[13] C. Fox, "The G and H functions as symmetrical fourier kernels," *Trans. American Math. Soc.*, vol. 98, no. 3, pp. 395–429, 1961.

[14] A. A. Kilbas, H*-transforms: Theory and Applications*. CRC Press, 2004.

[15] A. M. Mathai, R. K. Saxena, and H. J. Haubold, *The* H*-function:Theory and Applications*. Springer Science & Business Media, 2009.

[16] B. D. Carter and M. D. Springer, "The distribution of products, quotients and powers of independent H-function variates," *SIAM J. Appl. Mathematics*, vol. 33, no. 4, pp. 542–558, 1977.

[17] I. D. Cook Jr, "The H-function and probability density functions of certain algebraic combinations of independent random variables with H-function probability distribution," Ph.D. dissertation, University of Texas, Oct. 1981.

[18] C. D. Bodenschatz, "Finding an H-function distribution for the sum of independent H-function variates," Ph.D. dissertation, University of Texas, May 1992.

[19] O. L. A. López, H. Alves, P. H. J. Nardelli, and M. Latva-aho, "Aggregation and resource scheduling in machine-type communication networks: A stochastic geometry approach," *IEEE Trans. Wireless Commun.*, vol. 17, no. 7, pp. 4750–4765, July 2018.

[20] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*. Tata McGraw-Hill Edu., 2002.