Geotech Geol Eng (2021) 39:4427–4455
https://doi.org/10.1007/s10706-021-01772-5

ORIGINAL PAPER

Empirical Estimation of Uniaxial Compressive Strength of Rock: Database of Simple, Multiple, and Artificial Intelligence-Based Regressions

Adeyemi Emman Aladejare · Emmanuel Damola Alofe · Moshood Onifade · Abiodun Ismail Lawal · Toochukwu Malachi Ozoji · Zong-Xian Zhang

Received: 15 September 2020 / Accepted: 9 March 2021 / Published online: 1 April 2021

Abstract Empirical relationships for estimating Uniaxial Compressive Strength (UCS) of rock from other rock properties are numerous in literature. This is because the laboratory procedure for determination of UCS from compression tests is cumbersome, time consuming, and often considered expensive, especially for small to medium-sized mining engineering projects. However, these empirical models are scattered in literature, making it difficult to access a considerable number of them when there is need to select empirical model for estimation of UCS. This often leads to bias in estimated UCS data as there may be underestimation or overestimation of UCS, because of the site-specific nature of rock properties. Therefore, this study develops large database of empirical relationships between UCS and other rock properties that are reported in literatures. Statistical analysis was performed on the regression equations in the database developed. The typical ranges and mean of data used in developing the regressions, and the range and mean of their R^2 values were evaluated and summarised. Most of the regression equations were found to be developed from reasonable quantity of data with moderate to high R^2 values. The database can be easily assessed to select appropriate regression equation when there is need to estimate UCS for a specific site.

Keywords Regression analysis · Uniaxial compressive strength · Rock properties · Models · Database

A. E. Aladejare (✉) · T. M. Ozoji · Z.-X. Zhang
Oulu Mining School, University of Oulu, Oulu, Finland
e-mail: adeyemi.aladejare@oulu.fi

E. D. Alofe
Faculty of Earth Sciences, University of Uppsala, Uppsala, Sweden

M. Onifade
Department of Mining and Metallurgical Engineering, University of Namibia, Windhoek, Namibia

A. I. Lawal
Departent of Mining Engineering, Federal University of Technology, Akure, Nigeria

1 Introduction

The uniaxial compressive strength (UCS) is a mechanical property of intact rocks that is important in civil and mining engineering works (Aladejare 2020; Aladejare et al. 2020; Wang and Aladejare 2016a). Design and stability analysis of underground excavations and other geotechnical structures require the input of data like UCS on the geomechanical behaviour of rocks (Ulusay et al. 1994). Adebayo and Aladejare (2013) explained that UCS of rock has effect on excavation-loading operation of rock fragments. According to Hoek (1977), UCS is a required property when considering a variety of problems encountered during blasting, excavation, and support in engineering works. In addition, UCS is essential for classification of rock masses into different groups for engineering applications, and these classifications are used to determine their suitability for different construction purposes (Sachpazis 1990). For example, UCS is used as input in rock mass classification systems like rock mass rating (RMR) (Bieniawski 1974; Aladejare and Wang 2019a; Aladejare and Idris 2020) and rock mass index (RMi) (Palmstrøm 1996), and in predicting strength parameters of rock masses through Hoek–Brown failure criterion (Hoek et al. 2002). In the probabilistic characterization of Hoek–Brown *mi*, Aladejare and Wang (2019b) used UCS data in a Bayesian framework to simulate samples of Hoek–Brown m_i, which are useful for probability-based estimation of rock mass properties through the Hoek–Brown failure criterion. The UCS also serve as input data when using empirical equations to predict deformation modulus of rock masses (Aladejare and Wang 2019b) and characteristic impedance of rocks (Zhang et al. 2020). All these make UCS an important parameter to most rock and mining engineering designs and analyses. According to a survey reported by Bieniawski (1976), mining engineers request the UCS more often than any other rock material property. From surface to underground mine design and construction, UCS is a key parameter and it is required that UCS be known with certainty to a great extent for engineering analysis.

The guidelines and method for laboratory determination of UCS have been suggested by International Society of Rock Mechanics (ISRM) (Ulusay and Hudson 2007). However, the laboratory determination of UCS is expensive and time consuming. Therefore, for most mining projects, especially small to medium-sized projects, data of UCS are not often available (Aladejare 2016). For this reason, numerous regression equations have been developed in literature for estimation of UCS, when they cannot be directly obtained through laboratory testing (Sachpazis 1990; Gökçeoglu 1996; Chatterjee and Mukhopadhyay 2002; Yılmaz and Sendır 2002; Dincer et al. 2004, 2008; Gokceoglu and Zorlu 2004; Hudyma et al. 2004; Sabatakakis et al. 2008; Tiryaki 2008; Diamantis et al. 2009; Khandelwal and Singh 2009; Moradian and Behnia 2009; Yasar et al. 2010; Mishra and Basu 2012; Khandelwal 2013; Minaeian and Ahangari 2013; Mohamad et al. 2015; Kallu and Roghanchi 2015; Fereidooni 2016; Sharma et al. 2017; Heidari et al. 2018; Aliyu et al. 2019). Results of some physical and mechanical tests have been recommended for indirect estimation of UCS. Numerous studies of empirical equations developed for indirect estimation of UCS in the literature generally include those using physical properties such as Schmidt hardness number, shore hardness, density, water content, porosity, P-wave velocity, S-wave velocity, unit weight, Equotip hardness number (also referred to as Leeb hardness number) and slake durability index, and mechanical properties such as block punch index, Young's modulus, Brazilian tensile strength, and point load strength as inputs (Tugrul and Zarif 1999; Vasarhelyi 2005; Shalabi et al. 2007; Cobanoglu and Celik 2008; Török and Vasarhelyi 2010; Mishra and Basu 2013; Tandon and Gupta 2015; Mohamad et al. 2015; Najibi et al. 2015; Kahraman et al. 2016; Sharma et al. 2017; Uyanik et al. 2019). Simple and multiple regressions are available in literature for estimating UCS from these properties. In the recent past, artificial intelligence has been used to develop models for estimation of UCS, using techniques such as artificial neural network (ANN), support vector machine (SVM), Fuzzy inference system (FIS), genetic programming (GP) and hybrid ANN (Monjezi et al. 2012; Rezaei et al. 2014; Jalali et al. 2017; Aboutaleb et al. 2018; Armaghani et al. 2018; Mohamad et al. 2018; Ren et al. 2019).

With the numerous regression equations available in literature, there is a need to systematically select equations which suit specific sites. Wang and Aladejare (2015, 2016a) developed methods for selecting models and estimation of UCS. The Bayesian frameworks developed in studies such as Wang and Aladejare (2015, 2016a, b) need empirical equations as input. However, lack of accessibility to a great number of equations is a drawback. This is because when decision is to be made on the regression equation to be used for estimation of UCS, only equations that are readily assessed in literatures are considered. The regression equations developed are scattered in literatures, with no study yet that has systematically compiled them together for use during selection and estimation of UCS of rock. In order to solve this problem, this paper develops a database, which is a global compilation of empirical equations for estimating UCS from physical and mechanical properties of rocks. To provide a global compilation of different forms of regression equations, an extensive review of previous studies is performed to collect and compile information of different regression equations for estimation of the UCS of rock. This study is particularly beneficial for engineering projects when considering any analysis that involves the use of UCS as an input. This is because it serves as the equations bank from which different regression equations can be assessed for selection and their subsequent use for estimation of UCS.

2 Database Development and Description

A total of 163 research articles from internationally leading journals such as International Journal of Rock Mechanics and Mining Sciences, Rock Mechanics and Rock Engineering, Bulletin of Engineering Geology and the Environment, Journal of Rock Mechanics and Geotechnical Engineering, Engineering Geology, Neural Computing and Applications, Geotechnical and Geological Engineering, Applied Soft Computing, Environmental Earth Sciences, International Journal of Mining Science and Technology, Tunnelling and Underground Space Technology, Measurement, and Engineering with Computers were used to compile information of regression equations for estimating UCS, ranging from simple to multiple regression and artificial intelligence-based models. The regression equations that are documented in the database only includes those whose data were obtained according to testing procedure standards set by ISRM or American Society for Testing and Materials (ASTM). This ensures that all equations in the database were developed from test results involving consistent sample length to diameter ratio and testing conditions (Aladejare and Wang 2017). Note that only equations developed for rocks are considered in the database, soil and other weathered rocks which behave as soil are not considered in the database. Geo-materials whose equations are included in the database are generally referred to as rock samples in the original literatures. They generally include grade I–III weathered rocks (i.e., ranging from fresh rocks to slightly weathered rock and moderately weathered rocks. Grade IV or above weathered geo-material is generally referred to as soil (e.g. Ehlen 2002; Aladejare and Wang 2017) and are not considered in the database.

In the database, there are different types of regression equations ranging from simple to multiple regressions and artificial intelligence-based regressions such as ANN, SVM, FIS, GP, and hybrid ANNs. In addition, there are different modes of equations such as linear, power, exponential, logarithmic and polynomial functions in the database. The equations contained in the database include those developed for estimating UCS from rock properties such as Schmidt hardness number (N), shore hardness (SH), density (ρ), porosity (n), P-wave velocity (V_p), S-wave velocity (V_s), unit weight (γ), equotip hardness number (L_D), slake durability index (I_{d2}), block punch index (BPI), Young's modulus (E), Brazilian tensile strength (BTS) and point load strength ($Is_{(50)}$). Equations between UCS and other less frequently measured rock properties such as grain size (GS), shape factor (SF), quartz content (Qtz), particle diameter (D), single compressive strength index (SCSI) among others that are available in literature are also included in the database.

For each regression equation, number of data from which it was developed and the correlation coefficient (R^2) are documented. The mean (μ) of a group of data is calculated as:

$$\mu = \frac{1}{n_t}\sum_{i=1}^{n_t}\theta_i, \quad \text{for} \quad i = 1, 2, 3, \ldots n_t \tag{1}$$

where θ_i is a set of rock property data and n_t is the total number of rock data present in a group of data. The range and mean of number of data used in equation development and their R^2 for each regression equation are also included in the database.

3 Simple Regression

Simple regression is a statistical method for studying relationships between two continuous variables, in which one variable is regarded as the predictor or independent variable, and the other variable is regarded as the outcome or dependent variable (Freedman 2009). Assuming two groups of data (Y_i; X_{ai}); i = 1, …, n, where $X_{ai} = (X_{a1},\ldots X_{an})$ is a vector of independent variable and Y_i a real-valued dependent variable for the ith observation, a regression equation f is a model that makes a prediction $\dot{Y}$ of Y for a potentially new input vector X_a, written as:

$$\dot{Y} = f(X_a) \tag{2}$$

Simple regression for estimating UCS can take any form such as linear, logarithmic, exponential, power,

Table 1 Empirical equations for estimating UCS based on Equotip hardness number (L_s, or L_D)

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 8 \times 10^{-6} {L_s}^{2.5}$	33	0.77	Mixed	Japan and Indonesia	Aoki and Matsukura (2008) Based on part of dataset by Verwaal and Mulder (1993)
2	$UCS = 15.7 {L_D}^{2.42} \times 10^{-6}$	31	0.70	Mixed	Various countries	Corkum et al. (2018)
3	$UCS = 0.1 {L_D}^{3.18} \times 10^{-6}$	31	0.71	Sedimentary	Various countries	Corkum et al. (2018)
4	$UCS = 0.3 {L_D}^{2.98} \times 10^{-6}$	31	0.79	Metamorphic	Various countries	Corkum et al. (2018)
5	$UCS = 3 {L_D}^{2.64} \times 10^{-6}$	31	0.65	Igneous	Various countries	Corkum et al. (2018)
6	$UCS = 1.75 \times 10^{-9} {L_D}^{3.8}$	194	0.81	Mixed	Spain	Meulenkamp (1997)
7	$UCS = 4.906 \times 10^{-7} {L_D}^{2.974}$	28	NA	Mixed	Netherlands	Verwaal and Mulder (2000)
8	$UCS = 2.3007 e^{0.0057 L_D}$	62	0.82	Mixed	USA	Lee et al (2014) exclusive of shale rocks
9	$UCS = 2.1454 e^{0.0058 L_D}$	86	0.81	Mixed	USA	Lee et al (2014) inclusive of shale rocks
10	$UCS = 4.5847 L_D - 142.22$	18	0.82	Sedimentary	Turkey	Yilmaz (2013)

and polynomial forms (Diamantis et al. 2009; Yasar et al. 2010; Nefeslioghu 2013; Azimian et al. 2014; Kallu and Roghanchi 2015), and the difference in the models is the way that $f(X_a)$ in Eq. (2) is expressed for each regression equation. In this study, the simple regressions are grouped under two headings into those regressions derived from physical properties and those derived from mechanical properties as discussed in the following subsections.

3.1 Simple Relationship Between UCS and Physical Properties

Physical tests are generally easier and less expensive to perform, and for this reason many simple regressions are available in literature for estimating UCS from physical properties of rock (Cobanoglu and Celik 2008; Heidari et al. 2018; Aliyu et al. 2019; Aladejare 2020). Tables 1, 2, 3, 4, 5, 6, 7, 8 and 9 list regression equations for estimating UCS based on equotip number, Schmidt rebound number, shore hardness, density, porosity, P-wave velocity, S-wave velocity, unit weight, and slake durability index, respectively. For each regression equation listed in the tables, the number of data used to develop them, R^2 value and the rock type from which the equation was developed are presented. The tables show that regression equations using physical properties to estimate UCS for the three types of rock (i.e., igneous, sedimentary, and metamorphic rocks) are numerous and also for cases where different rock types are mixed together to develop regression equations. The different regression equations available in literature as can be observed from Tables 1, 2, 3, 4, 5, 6, 7, 8 and 9 indicate that not all equations will be suitable for specific site. Having database of regression equations will give mining engineers and other practitioners the opportunity to fairly assess all regression equations before deciding on the regression equations for a specific site. Recent studies in mining and geotechnical engineering have developed model selection approaches to select appropriate model from candidate models (e.g., Wang and Aladejare, 2015, 2016a). With many regression equations available in a paper, mining practitioners can subject many regression equations to assessment before deciding on the appropriate regression equation. Table 10 shows the information about the statistics of the regression equations in Tables 1, 2, 3, 4, 5, 6, 7, 8, and 9 that were used to develop the regression equations and the range and mean of their R^2 values. The mean of group data ranges from 24 to 210, while the lowest and highest R^2 values are 0.11 and 0.98, respectively. The quantity of data in a group and R^2 values shows that the equations collated in Tables 1, 2, 3, 4, 5, 6, 7, 8, and 9 may produce satisfactory estimation of UCS when they are used to estimate UCS for deposits of similar rock type.

Table 2 Empirical equations for estimating UCS based on Schmidt Hammer rebound number

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 6.59N - 212.63$	150	0.65	Sedimentary	Turkey	Cobanoglu and Celik (2008)
2	$UCS = 2N$	30	0.72	Sedimentary	USA	Singh et al. (1983)
3	UCS = 0.4 N-3.6	20	0.94	Sedimentary	USA	Shorey et al. (1984)
4	$UCS = 0.994N - 0.383$	10	0.7	Sedimentary	USA	Haramy and DeMarco (1985)
5	$UCS = 0.88N - 12.11$	13	0.87	Sedimentary	India	Ghose and Chakraborti (1986)
6	$UCS = 4.85N - 76.18$	NA	0.77	Sedimentary	USA	O'Rourke (1989)
7	$UCS = 2.98e^{(0.06N)}$	NA	0.95	Metamorphic	NA	Xu et al. (1990)
8	$UCS = 1.31N - 2.52$	30	0.55	Igneous	Greece	Aggistalis et al. (1996)
9	$UCS = 0.0001N^{3.2658}$	NA	0.84	Sedimentary	Japan	Gökçeoglu (1996)
10	$UCS = \exp(0.818 + 0.059N)$	20	0.98	Sedimentary	Turkey	Yilmaz and Sendir (2002)
11	$UCS = 2.75N - 36.83$	24	0.95	Igneous	Turkey	Dincer et al. (2004)
12	$UCS = 104.3\ln(N) - 308.6$	24	0.96	Igneous	Turkey	Dincer et al. (2004)
13	$UCS = 13.02e^{0.0414N}$	24	0.96	Igneous	Turkey	Dincer et al. (2004)
14	$UCS = 0.267N - 2.210$	19	0.64	Sedimentary	Turkey	Dincer et al. (2008)
15	$UCS = 7.044\ln N - 17.96$	19	0.60	Sedimentary	Turkey	Dincer et al. (2008)
16	$UCS = 4.6 \times 10^{-2}N^{1.406}$	19	0.66	Sedimentary	Turkey	Dincer et al. (2008)
17	$UCS = 1.143e^{0.051N}$	19	0.65	Sedimentary	Turkey	Dincer et al. (2008)
18	$UCS = 1246N - 34890$	257	0.88	Mixed	USA	Deere and Miller (1966) (UCS in psi)
19	$UCS = 4.29N - 67.52$	29	0.96	Sedimentary	Greece and England	Sachpazis (1990)
20	$UCS = 4.5 \times 10^{-4}N^{2.46}$	10	0.93	Mixed	Turkey	Kahraman (1996)
21	$UCS = 8.36N - 416$	19	0.87	Igneous	Turkey	Tugrul and Zariff (1999)
22	$UCS = 6.97e^{0.014N}$	48	0.78	Mixed	Turkey	Kahraman (2001)
23	$UCS = 4 \times 10^{-6}N^{4.2917}$	9	0.89	Mixed	Turkey	Yasar and Erdogan (2004a, b)
24	$UCS = 1.4459e^{0.0706N}$	40	0.92	Igneous	Hong Kong	Aydin and Basu (2005)[a]
25	$UCS = 3.20N - 46.59$	58	0.76	Sedimentary	USA	Shalabi et al. (2007)
26	$UCS = 0.0028N^{2.584}$	9	0.92	Mixed	Turkey	Yagiz (2009)
27	$UCS = 2.262N - 29.38$	21	0.91	Metamorphic	India	Tandon and Gupta (2015)
28	$UCS = 2.729N - 41.78$	9	0.96	Igneous (Granitoid)	India	Tandon and Gupta (2015)
29	$UCS = 2.547N - 33.08$	12	0.71	Igneous (Gneiss)	India	Tandon and Gupta (2015)
30	$UCS = 2.722N - 30.19$	12	0.93	Metamorphic	India	Tandon and Gupta (2015)
31	$UCS = 1.233N - 2.846$	6	0.89	Sedimentary	India	Tandon and Gupta (2015)
32	$UCS = 1.910N - 10.30$	60	0.75	Mixed	India	Tandon and Gupta (2015)
33	$UCS = 0.994N - 0.383$	10	0.70	Mixed	USA	Haramy and DeMarco (1985)
34	$\ln UCS = 1.8X10^{-2}(N \times \rho_d) + 2.9$	14	0.98	Sedimentary	USA	Cargill and Shakoor (1990)[c]
35	$UCS = \exp(1.332 + 0.053N)$	99	0.94	Sedimentary	Spain	Morales et al. (2004)
36	$UCS = 3.1e^{0.09N}$	75	0.79	Sedimentary	Greece	Sabatakakis et al. (2008)
37	$UCS = 3.201N - 46.59$	58	0.76	Sedimentary	USA	Shalabi et al. (2007)
38	$UCS = 3.6468N - 98.777$	1700	0.81	Metamorphic	Turkey	Yavuz et al. (2005)
39	$UCS = \exp(0.818 + 0.059N)$	20	0.98	Metamorphic	Turkey	Yilmaz and Sendir (2002)
40	$UCS = 5.3466N - 99.878$	53	0.76	Sedimentary	Iran	Heidari et al (2017)

Table 2 continued

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
41	$UCS = 0.25N^{1.77}$	200	0.88	Igneous	USA	Kallu and Roghanchi (2015)
42	$\text{In}UCS = 0.792 + 0.067N \pm 0.231$	7	0.96	Mixed	Israel and USA	Katz et al. (2000)
43	$UCS = 0.0137N^{2.2721}$	19	0.94	Mixed	Turkey	Kılıç and Teymen (2008)
44	$UCS = 0.64N + 37.5$	3	0.96	Metamorphic	India	Gupta (2009)
45	$UCS = \exp(-4.04 + 2.28.\ln N)$	95	0.97	Sedimentary	Various countries	Bruno et al. (2012)
46	$UCS = 4.24e^{0.059N}$	11	0.81	Mixed	Turkey	Fener et al. (2005)
47	$UCS = 0.02N^{2.28}$	8	0.92	Metamorphic	Iran	Fereidooni (2016)
48	$UCS = 0.0465N^2 - 0.1756N + 27.682$	41	0.86	Metamorphic	Iran	Torabi et al. (2010)
49	$UCS = 1.15N - 15$	7	0.91	Igneous	India	Gupta (2009)
50	$UCS = 0.9165e^{0.0669N}$	40	0.94	Igneous	Hong Kong	Aydin and Basu (2005)[b]
51	$\ln UCS = 4.3X10^{-2}(N \times \rho_d) + 1.2$	14	0.93	Sedimentary	USA	Cargill and Shakoor (1990)[c]

psi pounds per square inch

[a]L-type Schmidt Hammer

[b]N-type Schmidt Hammer

[c]Dry density is in Mg/m^3

Table 3 Empirical equations for estimating UCS based on Shore hardness

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 0.397SH + 0.332$	19	0.71	Sedimentary	Turkey	Dincer et al. (2008)
2	$UCS = 4.830\ln(SH) - 6.546$	19	0.67	Sedimentary	Turkey	Dincer et al. (2008)
3	$UCS = 0.461SH^{0.957}$	19	0.72	Sedimentary	Turkey	Dincer et al. (2008)
4	$UCS = 1.918e^{0.074SH}$	19	0.68	Sedimentary	Turkey	Dincer et al. (2008)
5	$UCS = 514SH - 6213$	275	0.90	Mixed	USA	Deere and Miller (1966)
6	$UCS = 3.54(SH - 12)$	NA	0.57	–	USA	Atkinson (1993)
7	$UCS = 0.895SH + 41.977$	30	0.57	Sedimentary	USA	Koncagul and Santi (1999)
8	$UCS = 1 \times 10^{-8}(SH)^{5.555}$	9	0.91	Mixed	Turkey	Yasar and Erdogan (2004a, b)
9	$UCS = 3.326SH - 79.76$	8	0.80	Sedimentary (high density dolomite)	USA	Shalabi et al. (2007)
10	$UCS = 1.581SH - 62.2$	9	0.85	Sedimentary (Shale rock)	USA	Shalabi et al. (2007)
11	$UCS = 14.868e^{0.042SH}$	1700	0.84	Metamorphic	Turkey	Yavuz et al. (2005)

3.2 Simple Relationship Between UCS and Mechanical Properties

Mechanical tests are generally more difficult and expensive to perform than physical tests, because most mechanical tests require rigorous sample preparation. Despite the difficulties in performing the mechanical tests, some of these tests are easier to be performed than UCS. For this reason, researchers have developed simple regressions for estimating UCS from mechanical properties of rock (Kahraman and Gunaydin 2009; Mishra and Basu 2012; Moradian and Behnia 2009; Fereidooni 2016). Tables 11, 12, 13 and 14 list regression equations for estimating UCS based on

Table 4 Empirical equations for estimating UCS based on Density

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 55.57\rho - 100.75$	22	0.89	Sedimentary	India (Krishna-Godavari basin)	Chatterjee and Mukhopadhyay (2002)
2	$UCS = 37.47\rho - 63.11$	22	0.98	Sedimentary	India (Cauvery basin)	Chatterjee and Mukhopadhyay (2002)
3	$UCS = 178.33 \times \rho - 384.65$	44	0.11	Mixed	England and Turkey	Tiryaki (2008)
4	$UCS = (28812.5\rho - 52.586) \times 0.0069$	257	0.90	Mixed	USA	Deere and Miller (1966)
5	$UCS = 10^{-5}\rho^{16.7}$	12	0.97	Igneous (basalts)	Turkey	Tugrul and Gurpinar (1997)
6	$UCS = 139.34\rho - 272.25$	94	0.87	Sedimentary	India	Sharma et al. (2017)
7	$UCS = -47454.4 + 35905.6\rho - 671.68\rho^2$	7	0.90	Sedimentary	UK, France and Denmark	Aliyu et al. (2019)

Table 5 Empirical equations for estimating UCS based on Porosity

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 34.44e^{-0.044n}$	22	0.83	Sedimentary	India (Krishna–Godavari basin)	Chatterjee and Mukhopadhyay (2002)
2	$UCS = 64.23e^{-0.085n}$	22	0.92	Sedimentary	India (Cauvery basin)	Chatterjee and Mukhopadhyay (2002)
3	$UCS = -33.13\ln(n) + 64.6$	32	0.82	Metamorphic	Greece	Diamantis et al. (2009)
4	$UCS = 97.77\exp^{-0.40n}$	32	0.76	Metamorphic	Greece	Diamantis et al. (2009)
5	$UCS = -21.58n + 91.87$	32	0.80	Metamorphic	Greece	Diamantis et al. (2009)
6	$UCS = -49.36\ln(n) + 189.35$	8	0.62	Igneous tuff	USA	Hudyma et al. (2004)
7	$UCS = 78.22n + 201$	19	0.81	Igneous	Turkey	Tugrul and Zarif (1999)
8	$UCS = 274 - 8.51n$	20	0.98	Igneous	Saudi Arabia	Al-Harthi et al. (1999) $n < 20\%$
9	$UCS = 104 - 1.01n$	33	0.96	Igneous	Saudi Arabia	Al-Harthi et al. (1999) $n > 20\%$
10	$UCS = -0.439n + 16.717$	19	0.78	Sedimentary	Turkey	Dincer et al. (2008)
11	$UCS = -10.960\ln(n) - 40.826$	19	0.80	Sedimentary	Turkey	Dincer et al. (2008)
12	$UCS = 3439.38n^{-2.02}$	19	0.76	Sedimentary	Turkey	Dincer et al. (2008)
13	$UCS = 42.111e^{-0.083n}$	19	0.77	Sedimentary	Turkey	Dincer et al. (2008)
14	$UCS = 149.33n^{-0.53}$	8	0.89	Metamorphic	Iran	Fereidooni (2016)
15	$UCS = -2.270n^2 + 33.88n + 16.30$	11	0.96	Sedimentary (sandstone)	Australia	Yasar et al. (2010)
16	$UCS = -2.135n^2 + 28.74n + 18.82$	11	0.90	Sedimentary (siltstone)	Australia	Yasar et al. (2010)
17	$UCS = -0.663n^2 + 9.648n + 21.01$	11	0.92	Sedimentary (mudstone)	Australia	Yasar et al. (2010)
18	$UCS = 123\exp^{-0.12n}$	95	0.63	Sedimentary	Greece	Sabatakakis et al. (2008)
19	$UCS = 16.55n + 183$	19	0.83	Igneous	Turkey	Tugrul and Zarif (1999)

Table 6 Empirical equations for estimating UCS based on P-wave Velocity

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 56.71V_p - 192.93$	150	0.67	Sedimentary	Turkey	Cobanoglu and Celik (2008)
2	$UCS = 6 \times 10^{-3}V_p - 0.556$	19	0.91	Sedimentary	Turkey	Dincer et al. (2008)
3	$UCS = 5.136\ln(V_p) - 28.337$	19	0.91	Sedimentary	Turkey	Dincer et al. (2008)
4	$UCS = 9 \times 10^{-3}{V_p}^{0.963}$	19	0.89	Sedimentary	Turkey	Dincer et al. (2008)
5	$UCS = 2.054e^{0.001V_p}$	19	0.82	Sedimentary	Turkey	Dincer et al. (2008)
6	$UCS = 0.0642V_p - 117.99$	49	0.90	Mixed	India	Sharma and Singh (2008)
7	$UCS = 9.95{V_p}^{1.21}$	27	0.83	Mixed	Turkey	Kahraman (2001)
8	$UCS = 165.05e^{[-4.452/V_p]}$	64	0.70	Sedimentary	Iran	Moradian and Behnia (2009)
9	$UCS = 0.033V_p - 34.83$	13	0.87	Mixed	India	Khandelwal (2013)
10	$UCS = 133.3V_p - 227.19$	12	0.96	Mixed	India	Khandelwal and Singh (2009)
11	$UCS = 0.005V_p$	140	0.94	Sedimentary	Iran	Minaeian and Ahangari (2013)
12	$UCS = 110V_p - 515.56$	32	0.81	Metamorphic	Greece	Diamantis et al. (2009)
13	$UCS = 0.78e^{[0.88V_p]}$	171	0.53	Igneous	United Kingdom	Entwisle et al. (2005)
14	$UCS = 0.032V_p - 44.227$	40	0.83	Mixed	Malaysia	Mohamad et al. (2015)
15	$UCS = 64.2V_p - 117.99$	49	0.90	Mixed	India	Sharma and Singh (2008)
16	$UCS = 35.54V_p - 55$	19	0.80	Igneous	Turkey	Tugrul and Zariff (1999)
17	$UCS = 31.5V_p - 63.7$	9	0.80	Mixed	Turkey	Yasar and Erdogan (2004a, b)
18	$UCS = 0.14V_p - 899.23$	32	0.90	Metamorphic	Greece	Diamantis et al. (2011)
19	$UCS = 0.0675V_p - 245.13$	20	0.92	Igneous	Turkey	Kurtulus et al. (2012) (across foliation)
20	$UCS = 0.0188V_p - 71.04$	20	0.83	Igneous	Turkey	Kurtulus et al. (2012) (along foliation)
21	$UCS = 0.005V_p$	140	0.94	Sedimentary	Iran	Minaeian and Ahangari (2013)
22	$UCS = 6.6{V_p}^{1.6}$	46	0.92	Sedimentary	Turkey	Uyanik et al. (2019)
23	$UCS = 0.11V_p - 515.56$	32	0.81	Metamorphic	Greece	Diamantis et al. (2009)
24	$UCS = 2.6 \times 10^{-3}exp^{0.0019V}$	32	0.80	Metamorphic	Greece	Diamantis et al. (2009)
25	$UCS = 570.94\ln(V_p) - 4840.1$	32	0.79	Metamorphic	Greece	Diamantis et al. (2009)
26	$UCS = 0.457983e^{1.504268(V_p)}$	66	0.82	Sedimentary	Turkey	Nefeslioglu (2013)
27	$UCS = 2.258013(V_p) + 0.060749$	66	0.92	Sedimentary[a]	Turkey	Nefeslioglu (2013)
28	$UCS = 0.499138e^{1.575579(V_p)}$	66	0.91	Sedimentary[b]	Turkey	Nefeslioglu (2013)
29	$UCS = 3.313262(V_p) - 0.814776$	66	0.92	Sedimentary[c]	Turkey	Nefeslioglu (2013)
30	$UCS = 1.779459(V_p)^{1.409563}$	66	0.96	Sedimentary[d]	Turkey	Nefeslioglu (2013)
31	$UCS = 1.902589(V_p)^{1.031474}$	66	0.87	Sedimentary[e]	Turkey	Nefeslioglu (2013)
32	$UCS = 4.751294(V_p) - 2.354974$	66	0.92	Sedimentary[f]	Turkey	Nefeslioglu (2013)
33	$UCS = 4.585574(V_p) - 2.230556$	66	0.87	Sedimentary[g]	Turkey	Nefeslioglu (2013)
34	$UCS = 1.642474(V_p)^{1.277730}$	66	0.87	Sedimentary[h]	Turkey	Nefeslioglu (2013)
35	$UCS = 35.54V_p - 55$	19	0.64	Igneous	Turkey	Tugrul and Zarif (1999)
36	$UCS = 0.026V_p - 20.207$	40	0.91	Sedimentary	Iran	Azimian et al. (2014)
37	$UCS = 0.0375V_p - 50.969$	53	0.67	Sedimentary	Iran	Heidari et al. (2018)
38	$UCS = 22.032{V_p}^{1.247}$	9	0.72	Igneous	Portugal	Sousa et al. (2005)
39	$UCS = 0.039V_p - 50.01$	94	0.93	Mixed	India	Sarkar et al. (2012)

Table 6 continued

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
40	$UCS = 12.743{V_p}^{1.194}$	72	0.76	Sedimentary	Several countries	Altindag (2012)
41	$UCS = 0.026V_p - 20.47$	40	0.91	Sedimentary	Iran	Abdolazim and Rassoul (2015)
42	$\ln UCS = 3.94 InV_p - 28.12$	10	0.92	Igneous	USA	Kallu and Roghanchi (2015)
43	$UCS = 165.05\exp(-4.452/V_p)$	64	0.70	Mixed	Iran	Moradian and Behnia (2009)
44	$UCS = 0.0389V_p - 50.009$	94	0.93	Sedimentary	India	Sharma et al. (2017)
45	$UCS = 0.91V_p - 4500.6$	7	0.87	Sedimentary	UK, France and Denmark	Aliyu et al. (2019)

[a–h]Genetic rock type codes representing varying spectral absorptions using reflectance spectroscopy

Table 7 Empirical equations for estimating UCS based on S-wave Velocity

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 16{V_s}^{1.6}$	46	0.82	Sedimentary	Turkey	Uyanik et al. (2019)
2	$UCS = 0.14V_s - 336.05$	32	0.80	Metamorphic	Greece	Diamantis et al. (2009)
3	$UCS = 0.057e^{0.0025V_s}$	32	0.79	Metamorphic	Greece	Diamantis et al. (2009)
4	$UCS = 391.38\ln(V_s) - 3043.2$	32	0.79	Metamorphic	Greece	Diamantis et al. (2009)

Table 8 Empirical equations for estimating UCS based on unit weight/dry unit weight

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 56.71\gamma + 16.471$	19	0.79	Sedimentary	Turkey	Dincer et al. (2008)
2	$UCS = 21.035\ln\gamma - 56.81$	19	0.76	Sedimentary	Turkey	Dincer et al. (2008)
3	$UCS = 2.60 \times 10^{-5}\gamma^{4.108}$	19	0.80	Sedimentary	Turkey	Dincer et al. (2008)
4	$UCS = 0.0737e^{0.217\gamma}$	19	0.81	Sedimentary	Turkey	Dincer et al. (2008)
5	$UCS = 0.0574e^{2.9168\gamma_d}$	154	0.74	Sedimentary	Turkey	Cobanoglu and Celik (2012)
6	$UCS = 0.0063e^{3.813\gamma}$	40	0.90	Sedimentary	Hungary	Török and Vasarhelyi (2010)
7	$UCS = 0.4182\gamma^{6.037}$	15	0.97	Sedimentary	France	Moh'd (2009)
8	$UCS = 42.63\gamma_d - 1057.8$	32	0.80	Metamorphic	Greece	Diamantis et al. (2009)
9	$UCS = 2 \times 10^{-7}\exp^{0.75\gamma_d}$	32	0.79	Metamorphic	Greece	Diamantis et al. (2009)
10	$UCS = 1115.6\ln(\gamma_d) - 3558.2$	32	0.79	Metamorphic	Greece	Diamantis et al. (2009)
11	$UCS = 461\gamma - 52586$	257	0.60	Mixed	USA	Deere and Miller (1966) (UCS in psi, γ in pcf)
12	$UCS = 7.3\gamma - 110.32$	43	0.62	Sedimentary	USA	Shalabi et al. (2007)
13	$UCS = 60.75\gamma - 1430$	19	0.81	Igneous	Turkey	Tugrul and Zarif (1999)
14	$UCS = 57.72\gamma_d - 1347$	19	0.82	Igneous	Turkey	Tugrul and Zarif (1999)

Dry density indicated by subscript d

psi pounds per square inch

pcf pounds per cubic feet

Table 9 Empirical equations for estimating UCS based on slake durability index

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 0.211 I_{d2} - 13.815$	19	0.47	Sedimentary	Turkey	Dincer et al. (2008)
2	$UCS = 16.636 \ln(I_{d2}) - 69.552$	19	0.43	Sedimentary	Turkey	Dincer et al. (2008)
3	$UCS = 4.9 \times 10^{-7} {I_{d2}}^{3.578}$	19	0.55	Sedimentary	Turkey	Dincer et al. (2008)
4	$UCS = 0.084 e^{0.45 I_{d2}}$	19	0.58	Sedimentary	Turkey	Dincer et al. (2008)
5	$UCS = 0.6581 I_{d2} + 9.081$	30	0.63	Sedimentary	USA	Koncagul and Santi (1999)
6	$UCS = 29.631 I_{d4} - 2858$	10	0.94	Sedimentary	Turkey	Yagiz (2011)
7	$UCS = 0.047 e^{0.065 I_{d4}}$	31	0.92	Igneous	Turkey	Kahraman et al (2016)
8	$UCS = 0.453 I_{d4} - 26.22$	31	0.89	Igneous[a]	Turkey	Kahraman et al (2016)
9	$UCS = 7.751 I_{d4} - 711.4$	31	0.93	Igneous[b]	Turkey	Kahraman et al (2016)
10	$UCS = 26.21 I_{d2} - 2476.20$	94	0.86	Sedimentary	India	Sharma et al. (2017)

Subscript figure denote the number of cycles for the SDI

[a,b]Represent Igneous with UCS below and above 20 MPa respectively

Table 10 Statistics of regression equations for estimating UCS from Tables 1, 2, 3, 4, 5, 6, 7, 8 and 9

S/N	Input property for regression equations	No of data		Range of R^2
		Range	Mean	
1	Equotip number	18–194	55	0.65–0.82
2	Schmidt hardness number	3–1700	73	0.55–0.98
3	Shore hardness	9–1700	210	0.57–0.91
4	Density	7–257	65	0.11–0.98
5	Porosity	8–95	24	0.62–0.98
6	P-wave velocity	7–171	51	0.53–0.96
7	S-wave velocity	32–46	36	0.79–0.82
8	Unit weight	15–257	51	0.64–0.97
9	Slake durability index	10–94	30	0.43–0.93

block punch index, Young's modulus, tensile strength and point load strength, respectively. The tables show that regression equations using mechanical properties for estimating UCS for the three types of rock are also numerous like those using physical properties. Table 15 shows the information about the statistics of the regression equations in Tables 11, 12, 13 and 14, which includes the range and mean of group of data that were used to develop the regression equations and the range and mean of their R^2 values. The mean of group data ranges from 46 to 150, while the lowest and highest R^2 values are 0.33 and 0.99, respectively. The R^2 value for regression equations using physical properties are higher than those using mechanical properties. This may indicate that the regression equations using physical properties produce low errors when they are used to estimate UCS.

4 Multiple Regression

Multiple regression is an extension of simple regression. It is used to predict the value of a variable based on the value of two or more other variables. The concept of multiple regression reflects the likelihood that a variable may have relationship with more than one variable. In such case, all the independent variables can be systematically combined to estimate a dependent variable (Aiken et al. 1991). Assuming groups of data (Y_i; $X_{ai} \ldots X_{zi}$); where $X_{ai} \ldots X_{zi}$ are vector of independent variables from $X_a \ldots X_z$, $i = 1, \ldots, n$

Table 11 Empirical equations for estimating UCS based on block punch index

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 8.9217BPI - 1.2334$	53	0.77	Sedimentary	Iran	Heidari et al. (2018)
2	$UCS = 23.49BPI^{0.68}$	55	0.82	Igneous	USA	Kallu and Roghanchi (2015)
3	$UCS = 6.1BPI - 3.3$	1150	0.86	Mixed	Netherlands	Van der Schrier (1988)
4	$UCS = 5.5BPI$	23	0.94	Mixed	Turkey	Ulusay and Gokceoglu (1997)
5	$UCS = 5.25BPI$	127	0.95	Mixed	Turkey	Gokceoglu and Aksoy (2000)
6	$UCS = 5.1BPI$	41	0.90	Mixed	Turkey	Sulukcu and Ulusay (2001)
7	$UCS = 2.72BPI + 13.7$	82	0.71	Sedimentary (Greywacke)	Turkey	Gokceoglu and Zorlu (2004)
8	$UCS = 4.93BPI$	60	0.93	Mixed	India	Mishra and Basu (2012)
9	$UCS = 4.02BPI + 36.16$	20	0.89	Igneous	India	Mishra and Basu (2012)
10	$UCS = 1.35BPI + 10.89$	20	0.85	Metamorphic	India	Mishra and Basu (2012)
11	$UCS = 4.99BPI + 10.69$	20	0.87	Sedimentary	India	Mishra and Basu (2012)

Table 12 Empirical equations for estimating UCS based on Young's modulus

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 4.31 \times \left(\frac{E}{10}\right)^{1.705}$	152	0.33	Mixed	Canada	King (1983)[a]
2	$UCS = \frac{122.11 \times E}{39.37 + E}$	64	0.59	Sedimentary	Iran	Moradian and Behnia (2009)
3	$UCS = 12.8 \times \left(\frac{E}{10}\right)^{1.32}$	45	0.88	Sedimentary	Iran	Najibi et al. (2015)[b]
4	$UCS = 0.0084E$	10	0.66	Sedimentary (sandstone)	Pakistan	Malik and Rashid (1997)
5	$UCS = 0.0073E$	10	0.43	Sedimentary (siltstone)	Pakistan	Malik and Rashid (1997)
6	$UCS = 0.0072E$	10	0.35	Sedimentary (claystone)	Pakistan	Malik and Rashid (1997)
7	$UCS = 0.0033E - 2886$	28	0.83	Mixed	USA	Deere and Miller (1966)

[a,b]E is dynamic elastic modulus

representing the number of data for each independent variable, and Y_i a real-valued dependent variable for the ith observation, a regression equation f is a model that makes a prediction $\dot{Y}$ of Y for a potentially new input vectors $X_a \ldots X_z$, written as:

$$\dot{Y} = f(X_a \ldots X_z) \tag{3}$$

Like simple regression, multiple regression for estimating UCS can take any form such as linear, logarithmic, exponential, power, and polynomial forms (Majdi and Rezaei 2013; Cheshomi et al. 2015; Ng et al. 2015; Madhubabu et al. 2016; Armaghani et al. 2018), and the difference in the models will reflect how $f(X_a \ldots X_g)$ $f(X_a \ldots X_z)$ in Eq. (3) is expressed for each regression equation.

Table 16 lists multiple regression equations for estimating UCS based on different properties of rock, including physical and mechanical properties. The number of data per group for the equations ranges between 5 and 600 with a mean of 78 data per group. The R^2 values for the equations range from 0.53 to 0.99. The table shows that multiple regressions for estimating UCS for the three types of rock are numerous, and for cases where different rock types are mixed to develop multiple regressions.

5 Artificial Intelligence

Artificial intelligence refers to the simulation of human intelligence in machines that are programmed

Table 13 Empirical equations for estimating UCS based on Brazilian tensile strength

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 10.33TS^{0.89}$	22	0.94	Sedimentary	India	Chatterjee and Mukhopadhyay (2002)
2	$UCS = 6.89TS + 5.39$	22	0.93	Sedimentary	India	Chatterjee and Mukhopadhyay (2002)
3	$UCS = 10.61BTS$	46	0.54	Mixed	Turkey	Kahraman et al. (2012)
4	$UCS = 7.86BTS - 447.63$	37	0.92	Sedimentary	USA	Farah (2011)
5	$UCS = 6.8TS + 13.5$	82	0.65	Sedimentary (Greywacke)	Turkey	Gokceoglu and Zorlu (2004)
6	$UCS = 12.308TS^{1.0725}$	143	0.90	Mixed	Several countries	Altindag and Guney (2010)
7	$UCS = 9.25TS^{0.947}$	20	0.90	Sedimentary	Malaysia	Nazir et al. (2013)
8	$UCS = 15.361TS - 10.303$	40	0.82	Mixed	Malaysia	Mohamad et al. (2015)
9	$UCS = 12.195BTS$	406	NA	Sedimentary	Nigeria	Clifford (1991)
10	$UCS = 7.53BTS$	60	0.45	Sedimentary	Pakistan	Tahir et al. (2011)
11	$UCs = 6.75BTS^{1.08}$	22	0.80	Igneous	USA	Kallu and Roghanchi (2015)
12	$UCS = 10.03BTS + 55.19$	8	0.92	Metamorphic	Iran	Fereidooni (2016)
13	$UCS = 10.4TS + 18.2$	7	0.63	Sedimentary	UK, France and Denmark	Aliyu et al. (2019)
14	$UCS = 12.4TS - 9$	10	0.76	Sedimentary	USA	Gunsallus and Kulhawy (1984)

TS tensile strength, *BTS* Brazilian tensile strength

to think like humans and mimic their actions (Carbonell 2003; Cawsey and Aylett 2009; Lawal and Kwon 2020). This reasoning capability is valuable in applications where repetition, complexity, or tediousness makes human-like intervention impractical. Dershowitz and Einstein (1984) explained that artificial intelligence is applicable in rock mechanics, even where some complex decision-making is required. There are many approaches of artificial intelligence that are being used in rock mechanics and mining engineering, such as ANN, SVM, FIS, GP, and hybrid artificial intelligence such as Genetic Algorithm Artificial Neural Network (GA-ANN), Particle Swarm Optimisation Artificial Neural Network (PSO-ANN), Particle Swarm Optimisation Artificial Neural Network (PSO-ANN), Imperialist Competitive Algorithm Artificial Neural Network (ICA-ANN) and Adaptive neuro-fuzzy inference system (ANFIS) (Majdi and Beiki 2010; Manouchehrian et al. 2012; Beiki et al. 2013; Rezaei et al. 2014; Mohamad et al. 2015; Sharma et al. 2017; Aboutaleb et al. 2018; Lawal and Kwon 2020).

5.1 Artificial Neural Network

ANN is an approach of artificial intelligence, introduced by McCulloch and Pitts (1943). ANN is trained using a set of real inputs and their corresponding outputs. A neural network must be trained so that a known set of inputs produces the desired outputs. Once the network is trained with enough sample dataset, for a new input of relatively similar patterns, predictions can be made based on previous learning. Many researchers have used ANN to predict UCS from other rock properties (Yagiz et al. 2012; Jalali et al. 2017; Aboutaleb et al. 2018; Ren et al. 2019). Table 17 presents some ANN-based models for prediction of UCS for different types of rock and when different types of rock are mixed. It can be deduced that ANN provide promising performances with most R^2 values in the database greater than 0.86. However, the performance of an ANN model depends on many factors including the number of dataset and the training algorithm of ANN used.

Table 14 Empirical equations for estimating UCS based on Point load strength

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
1	$UCS = 8.66Is_{(50)} + 10.85$	75	0.76	Sedimentary[a]	Turkey	Cobanoglu and Celik (2008)
2	$UCS = 7.18Is_{(50)} + 27.78$	15	0.80	Sedimentary[b]	Turkey	Cobanoglu and Celik (2008)
3	$UCS = 11.78Is_{(50)} - 9.17$	15	0.91	Sedimentary[c]	Turkey	Cobanoglu and Celik (2008)
4	$UCS = 10.73Is_{(50)} - 5.50$	15	0.88	Sedimentary[d]	Turkey	Cobanoglu and Celik (2008)
5	$UCS = 8.87Is_{(50)} + 4.11$	15	0.86	Sedimentary[e]	Turkey	Cobanoglu and Celik (2008)
6	$UCS = 8.25Is_{(50)} + 14.02$	15	0.67	Sedimentary[f]	Turkey	Cobanoglu and Celik (2008)
7	$UCS = 5.0961Is_{(50)} - 0.533$	19	0.83	Sedimentary	Turkey	Dincer et al. (2008)
8	$UCS = 6.088\ln\left(Is_{(50)}\right) + 4.833$	19	0.81	Sedimentary	Turkey	Dincer et al. (2008)
9	$UCS = 4.413Is_{(50)}^{1.162}$	19	0.82	Sedimentary	Turkey	Dincer et al. (2008)
10	$UCS = 1.662e^{0.932Is_{(50)}}$	19	0.77	Sedimentary	Turkey	Dincer et al. (2008)
11	$UCS = 8.41I_{s(50)} + 9.51$	27	0.85	Mixed	Turkey	Kahraman (2001)
12	$UCS = 15.31I_{s(50)}$	23	0.83	Mixed	Turkey	Sulukcu and Ulusay (2001)
13	$UCS = 7.3I_{s(50)}^{1.71}$	188	0.82	Sedimentary	Greece	Tsiambaos and Sabatakakis (2004)
14	$UCS = 10.22I_{s(50)} + 24.31$	23	0.75	Mixed[g]	Turkey	Kahraman et al. (2005)
15	$UCS = 24.83I_{s(50)} - 39.64$	15	0.72	Mixed[h]	Turkey	Kahraman et al. (2005)
16	$UCS = 18I_{s(50)}$	40	0.97	Igneous	Hong Kong	Basu and Aydin (2006)
17	$UCS = 13.4I_{s(50)}$	39	0.89	Mixed	Indonesia	Agustawijaya (2007)
18	$UCS = 12.4I_{s(50)} - 9.0859$	39	0.81	Sedimentary (gypsum)	Turkey	Yilmaz and Yuksek (2008)
19	$UCS = 19.79I_{s(50)}$	32	0.74	Metamorphic	Greece	Diamantis et al. (2009)
20	$UCS = 14.63I_{s(50)}$	60	0.88	Mixed	India	Mishra and Basu (2012)
21	$UCS = 16.4I_{s(50)}$	329	0.92	Mixed	Japan	Kohno and Maeda (2012)
22	$UCS = 12.291I_{s(50)} + 5.892$	40	0.96	Mixed	Malaysia	Mohamad et al. (2015)
23	$UCS = 20.7I_{s(50)} + 4.299$	22	0.92	Mixed	USA	Deere and Miller (1966)
24	$UCS = 12.5I_{s(50)}$	21	0.73	Igneous	Hong Kong	Chau and Wong (1996)
25	$UCS = 22.792I_{s(50)} + 13.295$	35	0.88	Sedimentary (hard rocks)	Pakistan	Akram and Bakar (2007)
26	$UCS = 11.076I_{s(50)}$	16	0.89	Sedimentary (soft rocks)	Pakistan	Akram and Bakar (2007)
27	$UCS = 10.92I_{s(50)} + 24.24$	52	0.56	Mixed	Turkey	Kahraman and Gunaydin (2009)
28	$UCS = 22.8I_{s(50)}$	7	0.99	Metamorphic (quartzite)	India	Singh et al. (2012)
29	$UCS = 15.8I_{s(50)}$	19	0.91	Metamorphic (khondalite)	India	Singh et al. (2012)
30	$UCS = 22.2I_{s(50)}$	6	0.78	Metamorphic (quartzite[l])	India	Singh et al. (2012)
31	$UCS = 21.9I_{s(50)}$	10	0.89	Sedimentary (sandstone)	India	Singh et al. (2012)
32	$UCS = 16.1I_{s(50)}$	7	0.71	Sedimentary (rock salts)	India	Singh et al. (2012)
33	$UCS = 14.4I_{s(50)}$	6	0.82	Sedimentary (shale)	India	Singh et al. (2012)

Table 14 continued

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
34	$UCS = 23.3I_{s(50)}$	21	0.97	Igneous (gabbro)	India	Singh et al. (2012)
35	$UCS = 23.5I_{s(50)}$	7	0.98	Metamorphic (amphibolite)	India	Singh et al. (2012)
36	$UCS = 21I_{s(50)}$	6	0.96	Metamorphic (epidiorite)	India	Singh et al. (2012)
37	$UCS = 22.3I_{s(50)}$	8	0.68	Sedimentary (limestone)	India	Singh et al. (2012)
38	$UCS = 22.7I_{s(50)}$	9	0.82	Sedimentary (dolomite)	India	Singh et al. (2012)
39	$UCS = 10.99I_{s(50)} + 7.042$	15	0.92	Sedimentary[m]	Iran	Heidari et al. (2012)
40	$UCS = 11.96I_{s(50)} + 10.94$	15	0.94	Sedimentary[n]	Iran	Heidari et al. (2012)
41	$UCS = 13.29I_{s(50)} + 5.251$	15	0.90	Sedimentary[o]	Iran	Heidari et al. (2012)
42	$UCS = 4.792I_{s(50)} + 44.37$	21	0.75	Metamorphic	India	Tandon and Gupta (2015)
43	$UCS = 5.602I_{s(50)} + 4.380$	9	0.96	Igneous (granitoid)	India	Tandon and Gupta (2015)
44	$UCS = 3.103I_{s(50)} + 17.95$	12	0.40	Igneous (gneiss)	India	Tandon and Gupta (2015)
45	$UCS = 2.479I_{s(50)} + 24.68$	12	0.37	Metamorphic	India	Tandon and Gupta (2015)
46	$UCS = 10.53I_{s(50)} + 7.615$	6	0.91	Sedimentary	India	Tandon and Gupta (2015)
47	$UCS = 3.125I_{s(50)} + 40.08$	60	0.41	Mixed	India	Tandon and Gupta (2015)
48	$UCS = 24I_s$	390	NA	Mixed[p]	Several countries	Bieniawski (1975)
49	$UCS = 21I_s$	240	NA	Mixed[q]	Several countries	Bieniawski (1975)
50	$UCS = 18I_s$	255	NA	Mixed[r]	Several countries	Bieniawski (1975)
51	$UCS = 17.81I_{s(50)}^{1.06}$	32	0.82	Metamorphic	Greece	Diamantis et al. (2009)
52	$UCS = 16.45\exp^{0.39I_s}$	32	0.80	Metamorphic	Greece	Diamantis et al. (2009)
53	$UCS = 21.54I_{s(50)} - 6.02$	32	0.74	Metamorphic	Greece	Diamantis et al. (2009)
54	$UCS = 7.62I_{s(50)}^{1.74}$	240	0.81	Sedimentary (general)	Greece	Sabatakakis et al. (2008)
55	$UCS = 25.3I_{s(50)}$	240	0.71	Sedimentary	Greece	Sabatakakis et al. (2008)
56	$UCS = 13I_{s(50)}$	240	0.49	Sedimentary[i]	Greece	Sabatakakis et al. (2008)
57	$UCS = 24I_{s(50)}$	240	0.36	Sedimentary[j]	Greece	Sabatakakis et al. (2008)
58	$UCS = 28I_{s(50)}$	240	0.53	Sedimentary[k]	Greece	Sabatakakis et al. (2008)
59	$UCS = 15.25I_{s(50)}$	19	0.98	Igneous	Turkey	Tugrul and Zarif (1999)
60	$UCS = 56.939\ln\left(I_{s(50)}\right) - 1.6551$	40	0.93	Sedimentary	Iran	Azimian et al. (2014)
61	$UCS = 43.8981I_{s(50)} - 57.134$	53	0.76	Sedimentary	Iran	Heidari et al. (2018)
62	$UCS = 90.14I_{s(50)}^{0.92}$	143	0.91	Igneous	USA	Kallu and Roghanchi (2015)
63	$UCS = 16.5I_{s(50)} + 51$	10	0.69	Sedimentary	USA	Gunsallus and Kulhawy (1984)
64	$UCS = 16I_{s(50)}$	11	NA	Igneous	India	Ghosh and Srivastava (1991)
65	$UCS = 23I_{s(50)}$	30	NA	Sedimentary	USA	Smith (1997)
66	$UCS = 9.08I_{s(50)} + 39.32$	11	0.85	Mixed	Turkey	Fener et al. (2005)
67	$UCS = 8.2I_{s(50)} + 36.43$	17	0.68	Igneous	Turkey	Kahraman and Gunaydin (2009)
68	$UCS = 18.45I_{s(50)} - 13.63$	16	0.77	Metamorphic	Turkey	Kahraman and Gunaydin (2009)

Table 14 continued

S/N	Relationship	No of data	R^2	Rock types	Country of origin	References
69	$UCS = 29.77I_{s(50)} - 51.49$	19	0.78	Sedimentary	Turkey	Kahraman and Gunaydin (2009)
70	$UCS = 11.103I_{s(50)} + 37.659$	34	0.86	Metamorphic	India	Basu and Kamran (2010)
71	$UCS = 17.6I_{s(50)} + 13.5$	7	0.88	Sedimentary	UK, France and Denmark	Aliyu et al. (2019)
72	$UCS = 24I_{s(50)}$	15	0.88	Igneous	United Kingdom	Broch and Franklin (1972)[y]
73	$UCS = 9.459I_{s(50)}$	419	0.68	Mixed	United Arab Emirates	Salah et al. (2014)
74	$UCS = 18.71I_{s(50)}$	35	0.60	Mixed	Several countries	Thuro et al. (2001)
75	$UCS = 2.59I_{s(50)} + 0.21$ [t]	22	0.65	Sedimentary	UAE	Elhakim (2015)
76	$UCS = 2.86I_{s(50)}$ [u]	22	0.64	Sedimentary	UAE	Elhakim (2015)
77	$UCS = 24.36I_{s(50)} - 2.14$	8	0.99	Metamorphic	Iran	Fereidooni (2016)
78	$UCS = 5.575I_{s(50)} + 21.92$	15	0.93	Sedimentary[v]	Iran	Heidari et al. (2012)
79	$UCS = 7.557I_{s(50)} + 23.68$	15	0.94	Sedimentary[w]	Iran	Heidari et al. (2012)
80	$UCS = 3.495I_{s(50)} + 24.84$	15	0.89	Sedimentary[x]	Iran	Heidari et al. (2012)
81	$UCS = 23I_{s(54)} + 13$	14	0.94	Sedimentary (Limestone)	USA	Cargill and Shakoor (1990)

[a]Combination of 54, 48, 42, 30 and 21 mm core diameter sizes

[b]54 mm core diameter size only

[c]48 mm core diameter size only

[d]42 mm core diameter size only

[e]30 mm core diameter size only

[f]21 mm core diameter size only

[g]Rocks with porosity $> 1\%$

[h]Rocks with porosity $< 1\%$

[i]Rocks with Is $<$ 2 MPa

[j]Rocks with Is = 2–5 MPa

[k]Rocks with Is $>$ 5 MPa

[l]Quartzite sample is finer-grained with higher porosity when compared to the other quartzite sample considered in that study

[m]Point load determined axially for saturated state

[n]Point load determined diametrically for saturated state

[o]Point load determined using irregular samples for saturated state

[p]Core diameter size is 54 mm (NX)

[q]Core diameter size is 42 mm (BX)

[r]Core diameter size is 21.5 mm (EX)

[s]Rocks with $I_{s(50)} > 3.5$ MPa

[t]Linear correlation with non-zero intercept

[u]Linear correlation with zero intercept

[v]Point load determined axially for air-dried state

[w]Point load determined diametrically for air-dried state

[x]Point load determined using irregular samples for air-dried state

[y]UCS and $I_{s(50)}$ in MN/m^2

Table 15 Statistics of regression equations for estimating UCS from Tables 11, 12, 13, and 14

S/N	Input property for regression equations	No of data		Range of R^2
		Range	Mean	
1	Block punch index	23–1150	150	0.71–0.95
2	Young's modulus	10–152	46	0.33–0.88
3	Brazilian tensile strength	7–406	66	0.45–0.94
4	Point load strength	6–419	58	0.36–0.99

5.2 Support Vector Machine

SVM models are supervised learning models with associated learning algorithms that analyse data used for classification and regression analysis (Aboutaleb et al. 2018). It is an approach of artificial intelligence that enables non-linear mapping of an *n*-dimensional input space into a higher-dimensional feature space where, for example, a linear classifier can be used. The method can train non-linear models based on the structural risk minimization principle that seeks to minimize an upper bound of the generalization error rather than minimize the empirical error as implemented in other neural networks (Khandelwal et al. 2010). The approach has been used in rock mechanics to estimate UCS (Ceryan 2014; Ren et al. 2019). Table 18 lists some SVM-based estimation of UCS from other rock properties. The analysis of the R^2 of the studies compiled show that SVM models have R^2 value ranging from 0.60 to 0.99.

5.3 Fuzzy Inference System

A fuzzy inference system (FIS) is a system that uses fuzzy set theory to map inputs to outputs (Gokceoglu and Zorlu 2004). Fuzzy logic accomplishes machine intelligence by providing a mean for representing and reasoning about human knowledge that is imprecise by nature (Gupta and Kulkami 2013). Fuzzy inference is a method that interprets the values in the input vector and based on some sets of rules, assigns values to the output vector. In fuzzy logic, the truth of any statement becomes a matter of a degree. FIS has been used in rock mechanics to estimate rock properties. Specifically, the technique has been to estimate UCS from other rock properties (Grima and Babuška 1999; Gokceoglu and Zorlu 2004; Karakus and Tutmez 2006; Heidari et al. 2018). Table 19 lists some FIS-based estimation of UCS of different rock types from other rock properties. The analysis of the R^2 of the studies compiled show that FIS models have R^2 values ranging from 0.64 to 0.98.

5.4 Genetic Programming

Genetic programming (GP) is a technique of evolving programs, starting from a population of usually random programs, fit for a task by applying operations analogous to natural genetic processes to the population of programs. It is a technique for the automatic generation of computer programs by means of natural selection (Beiki et al. 2013). The GP process starts by creating a large initial population of programs that are random combinations of elements from the problem-specific function sets and terminal sets. Improvements are made possible by stochastic variation of programs and selection according to pre-specified criteria for judging the quality of a solution (Brameier and Banzhaf 2001). GP has been used in rock mechanics for estimating UCS from other properties (Canakci et al. 2009; Armaghani et al. 2018). Table 20 lists some studies where GP has been used to estimate UCS from other rock properties. The statistics of the R^2 values of the models generated for the studies listed in the table shows a range of 0.63–0.97.

5.5 Hybrid Artificial Neural Network

ANN has several disadvantages such as long training time, unwanted convergence to local instead of global optimal solution, and large number of parameters (Liou et al. 2009). To overcome these drawbacks, there have been attempts to remedy some of these disadvantages by combining ANN with another algorithm that can take care of a specific problem. Hybrid forms of ANN such as ANFIS, PSO-ANN, ICA-ANN,

Table 16 Multiple regression equations for estimating UCS from other rock properties

S/N	Relationship	No of data	Input parameters	R^2	Rock types	Country of origin	References
1	$UCS = -6.319 + 4.27\rho + 4.418V_p + 0.427\gamma$	19	Density ρ, P-wave velocity V_p, unit weight γ	0.90	Sedimentary	Turkey	Dincer et al. (2008)
2	$UCS = 142.47 \times e^{-9.561/\rho.V_p}$	64	Density ρ, P-wave velocity V_p	0.56	Sedimentary	Iran	Moradian and Behnia (2009)
3	$UCS = -7.708 + 92.722v + 0.866E_d$	482	Poisson ratio (v), dynamic Young's modulus (E_d)	0.897	Sedimentary	Iran	Aboutaleb et al. (2018)
4	$UCS = 0.079e^{-0.039n}L_s^{1.1}$	9	Porosity n, equotip hardness number L_s	0.88	Mixed	Japan and Indonesia	Aoki and Matsukura (2008)
5	$UCS = 69.505\rho_{dry} + 0.025V_p - 0.479Qtz - 1.439Plg - 158.796$	45	Dry density (ρ_{dry}), P-wave velocity V_p, quartz content (Qtz), plagioclase content (Plg)	0.55	Igneous	Malaysia	Armaghani et al. (2015)
6	$UCS = 6.24Is_{(50)} + 25.8V_p - 90.3$	150	Point load strength $Is_{(50)}$, P-wave velocity	0.85	Sedimentary	Turkey	Cobanoglu and Celik (2008)
7	$UCS = 4.14Is_{(50)} + 29.8V_p + 0.54(N) - 116$	150	Point load strength $Is_{(50)}$, P-wave velocity V_p, Schmidt hardness rebound (N)	0.99	Sedimentary	Turkey	Cobanoglu and Celik (2008)
8	$UCS = 6.9 \times 10^{[0.0087\gamma N + 0.16]}$	28	Schmidt hardness rebound (N), unit weight γ	0.94	Mixed	USA	Deere and Miller (1966)
9	$UCS = 6.9 \times 10^{[1.348\log(\gamma N) - 1.325]}$	25	Unit weight γ, Schmidt hardness number	0.80	Mixed	USA	Aufmuth (1973)
10	$UCS = 12.74e^{(0.185\gamma N)}$	20	Unit weight γ, Schmidt hardness Rebound (N)	NA	Mixed	USA	Berverly et al. (1979)
11	$UCS = 0.447e^{[0.045(N+3.5)+\gamma]}$	5	Unit weight γ, Schmidt hardness rebound (N)		Sedimentary	USA	Kidybinski (1980)
12	$UCS = 4.5 \times 10^{-4}(N\gamma)^{2.46}$	10	Unit weight γ, Schmidt hardness rebound (N)	0.93	Sedimentary	Turkey	Kahraman (1996)
13	$UCS = -6.319 + 4.418 \times 10^{-3}V_p + 0.427\gamma$	19	P-wave velocity V_p, unit weight γ	0.95	Sedimentary	Turkey	Dinçer et al. (2008)
14	$UCS = 3V_p^4 V_s^{-2.85}$	46	P-wave V_p and S-wave V_s velocities	NA	Sedimentary	Turkey	Uyanik et al. (2019)
15	$UCS = 52.214 - 527.77GS + 80.86SF + 0.526Qtz^2$	30	Grain size (GS), shape factor (SF), quartz content (Qtz)	0.84	Metamorphic	China	Ali et al. (2014)

Table 16 continued

S/N	Relationship	No of data	Input parameters	R^2	Rock types	Country of origin	References
16	$UCS = (-25.8\ln(D) + 153.5)\ln(SCSI) - (83.51\ln(D) + 310.2)$	600	Particle diameter (D), single compressive strength index ($SCSI$)	0.91	Sedimentary	Iran	Cheshomi and Sheshde (2013)
17	$UCS = (-0.3D + 1.92)SCSI + (1.24D + 6.72)$	300	Particle diameter (D), single compressive strength index (SCSI)	0.96	Sedimentary	Iran	Cheshomi et al. (2015)
18	$UCS = 0.121SCSI - 7.462D + 63.98$	10	Single compressive strength index, particle diameter	0.66	Sedimentary	Iran	Ashtari et al. (2019) (D = 3-10 mm)
19	$UCS = 10.61I_{s(50)} + 6.8710^{-2}V_p - 339.48$	32	Point load strength, P-wave velocity	0.88	Metamorphic	Greece	Diamantis et al. (2009)
20	$UCS = 10.51I_{s(50)} + 27.45\gamma_d - 675.50$	32	Point load strength, dry unit weight	0.86	Metamorphic	Greece	Diamantis et al. (2009)
21	$UCS = 12.15I_{s(50)} - 1.78\beta + 169.72$	32	Point load strength, degree of serpentinization (β)	0.83	Metamorphic	Greece	Diamantis et al. (2009)
22	$UCS = 8.0710^{-2}V_p - 0.92\beta - 295.49$	32	P-wave velocity, degree of serpentinization	0.82	Metamorphic	Greece	Diamantis et al. (2009)
23	$UCS = 6.5710^{-2}V_p + 17.50\gamma_d - 739.38$	32	P-wave velocity, dry unit weight	0.81	Metamorphic	Greece	Diamantis et al. (2009)
24	$UCS = 36.31\gamma_d - 0.59\beta - 821.85$	32	Dry unit weight, degree of serpentinization	0.80	Metamorphic	Greece	Diamantis et al. (2009)
25	$UCS = 5.01I_{s(50)} + 5.52e^{0.0004V_p} - 3.53$	85	Point load strength, P-wave velocity	0.83	Igneous (Grade 3 weathering)	Macau	Ng et al. (2015)
26	$UCS = \exp(-0.08008h + 0.01630e - 0.28813d + 4.12057$	65	Nail penetration depth (h) (mm), NailGun energy (e) (J), nail diameter (d) (mm)	0.95	Mixed	Turkey	Selcuk and Kayabali (2015)
27	$UCS = 0.88 \times \rho^{2.24} \times SH^{0.22} \times CI^{0.89}$	44	Density, shore hardness (SH), cone indenter (CI)	0.55	Mixed	England and Turkey	Tiryaki (2008)
28	$\ln UCS = 4.3 \times 10^{-2}(N\gamma_d) + 1.2$	7	Schmidt hardness number, Dry density	0.93	Sedimentary (sandstones)	USA	Cargill and Shakoor (1990)
29	$\ln UCS = 1.8 \times 10^{-2}(N\gamma_d) + 2.9$	7	Schmidt hardness number, Dry density	0.98	Sedimentary (carbonates)	USA	Cargill and Shakoor (1990)
30	$UCS = 0.476PD - 0.017CC - 0.049Q + 0.065$	138	Packing density (PD), concavo-convex (CC), quartz content (Q)	0.53	Sedimentary (sandstones)	Turkey	Zorlu et al. (2008)

Table 16 continued

S/N	Relationship	No of data	Input parameters	R^2	Rock types	Country of origin	References
31	$UCS = 13.244I_{s(50)} + 0.13V_p - 16.987$	40	Point load index, P-wave velocity	0.94	Sedimentary (marlstone)	Iran	Azimian et al. (2014)
32	$UCS = 1.277N + 2.186BPI + 16.41I_{s(50)} + 0.011V_p - 82.436$	108	Schmidt hardness number, Block punch index, point load strength, P-wave velocity	0.91	Sedimentary	Iran	Heidari et al. (2018)
33	$UCS = 47.11I_{s(50)} + 0.006i + 1.59JO$	5	Point load strength, asperity angle (i), joint orientation (JO)	0.68	Mixed	India	Kabilan et al. (2017) JO = 0°
34	$UCS = 13.371I_{s(50)} + 0.005i + 0.62JO$	5	Point load strength, asperity angle, joint orientation	0.90	Mixed	India	Kabilan et al. (2017) JO > 0°
35	$UCS = -595.303 - 442.363V_p + 45.338V_p^2 - 6.1n + 0.52n^2 + 28.314I_{s(50)} - 4.061I_{s(50)}^2 + 115.822N - 2.007N^2$	30	P-wave velocity, porosity, point load strength, Schmidt hardness number	0.64	Sedimentary	Iran	Dehghan et al. (2010)
36	$UCS = 38 - 352.26n - 5.3Cfc + 10.67Cf + 93.15M$	44	Void Percent, ferroan calcitic cement (Cfc), ferruginous cement (Cf) mica percentage (M)	0.57	Sedimentary	Nepal	Manouchehrian et al. (2012)
37	$UCS = 0.035V_p + 3.158I_{d2} - 0.954\rho - 342.729$	94	P-wave velocity, slake durability index (2nd cycle), density	0.94	Sedimentary (coal)	India	Sharma et al. (2017)
38	$UCS = -727 + 0.0427UPV + 19.3WA + 33DD + 95SD + 86BD$	52	Ultrasound pulse velocity (UPV), water absorption (WA), dry density (DD), saturated density (SD), bulk density (BD)	0.64	Igneous	Turkey	Canakci et al. (2009)
39	$UCS = -229 + 3.74N + 76.2\rho - 3.24n$	93	Schmidt hardness number, density, porosity	0.90	Mixed	Iran	Majdi and Rezaei (2013)
40	$UCS = 1.277N + 2.86BPI + 16.41I_{s(50)} + 0.011V_p - 82.436$	53	Schmidt hardness number, Block punch index, point load strength, P-wave velocity	0.91	Sedimentary	Iran	Jalali et al (2017)
41	$UCS = -11.813 - 2.572n + 23.665I_{s(50)} + 41.654v + 12.197\rho - 0.001V_p$	163	Porosity, point load strength, Poisson's ratio, density and P-wave velocity	0.91	Sedimentary	India	Madhubabu et al (2016)
42	$UCS = 34.186DD + 0.838I_{d2} + 2.308BTS - 109.184$	47	Dry Density, Slake durability Index, Brazilian Tensile Strength	0.93	Sedimentary	Malaysia	Armaghani et al. (2018)

Table 17 ANN-based models for prediction of UCS from other rock properties

S/N	Output	Input	No of data	R^2	Rock types	Country of origin	References
1	UCS	Dynamic poisson ratio	425	0.56	Sedimentary	Iran	Aboutaleb et al. (2018)
2	UCS	Dynamic poisson ratio	425	0.58	Sedimentary	Iran	Aboutaleb et al. (2018)
3	UCS	Dynamic poisson ratio, Young's modulus	425	0.90	Sedimentary	Iran	Aboutaleb et al. (2018)
4	UCS	Dynamic poisson ratio, Young's modulus	425	0.92	Sedimentary	Iran	Aboutaleb et al. (2018)
5	UCS	Equotip number, porosity, density, grain size	33	0.97	Mixed	Spain	Meulenkamp and Grima (1999)
6	UCS	Petrography study values (mineral composition, grain size, aspect ratio, form factor, area weighting and orientation of foliation planes of weakness	112	NA	Metamorphic	India	Singh et al. (2001)
7	UCS	Quartz content, packing density, concavo convex	138	0.87	Sedimentary	Turkey	Zorlu et al. (2008)
8	UCS	P-wave velocity, point load strength, Schmidt hardness number, porosity	30	0.86	Sedimentary	Iran	Dehghan et al. (2010)
9	UCS	Porosity, bulk density, water saturation	5000	0.98	Mixed	Iran	Rabbani et al. (2012)
10	UCS	Porosity, slake durability index, P-wave velocity in solid part of the sample, effective porosity, petrography study values	55	0.88	Sedimentary	Turkey	Ceryan et al. (2012)
11	UCS	Dry density, P-wave velocity, quartz content, plagioclase content	45	0.99	Igneous	Malaysia	Armaghani et al. (2015)
12	UCS	Density, shore hardness, cone indenter hardness		0.40	Mixed	England and Turkey	Tiryaki (2008)
13	UCS	Effective porosity, slake durability index, point load strength	39	0.93	Sedimentary	Turkey	Yilmaz and Yuksek (2008)
14	UCS	Origin of rocks, two/four-cycle slake durability index and clay content	56	0.98	Sedimentary	Turkey	Cevik et al. (2011)
15	UCS	Unit weight, shore hardness, porosity, P-wave velocity, slake durability index	54	0.50	Sedimentary	Turkey	Yagiz et al. (2012)
16	UCS	Porosity, density, P-wave Velocity, Poisson ratio, point load strength	NA	0.97	-	India	Madhubabu et al. (2016)
17	UCS	Grain size, shape factor, quartz content	30	0.95	Metamorphic	China	Ali et al. (2014)
18	UCS	Cone indenter, density and Shore hardness	44	0.63	Mixed	England & Turkey	Tiryaki (2008)
19	UCS	Packing density, concavo-convex, quartz content	138	0.82	Sedimentary (sandstones)	Turkey	Zorlu et al. (2008)
20	UCS	Amplitude attenuation coefficient, high and low frequency ratio	1614	0.99	Mixed	China	Ren et al. (2019)
21	UCS	P-wave velocity, porosity, point load strength	30	0.93	Sedimentary	Iran	Dehghan et al. (2010)
22	UCS	Void percent, ferroan calcitic cement, ferruginous cement, mica percentage	44	0.77	Sedimentary	Nepal	Manouchehrian et al. (2012)

Table 17 continued

S/N	Output	Input	No of data	R^2	Rock types	Country of origin	References
23	UCS	P-wave velocity, porosity, density	133	0.96	Sedimentary (Sandstones)	UAE	Jahanbakhshi et al. (2011)
24	UCS	P-wave velocity, density, porosity	105	0.95	Sedimentary	Iran	Torabi-Kaveh et al. (2015)
25	UCS	P-wave velocity, point load strength, Schmidt hardness number, porosity	30	0.86	–	NA	Garret (1994)
26	UCS	Density, P-wave velocity, point load strength, Schmidt hardness number	66	0.71	Mixed	Malaysia	Momeni et al. (2015)
27	UCS	P-wave velocity, slake durability index, density	70	0.95	Sedimentary (Coal)	India	Sharma et al. (2017)
28	UCS	Ultrasound pulse velocity, water absorption, dry density, saturated density, bulk density	52	0.98	Igneous	Turkey	Canakci et al. (2009)
29	UCS	Schmidt hardness number, density, porosity	93	0.97	Mixed	Iran	Majdi and Rezaei (2013)
30	UCS	Schmidt hardness number, Block punch index, Point load strength, P-wave velocity	106	0.96	Sedimentary	Iran	Jalali et al (2017)
31	UCS	Dry density, moisture content, P-wave velocity, point load strength, slake durability index	228	0.94	Sedimentary	Malaysia	Mohamad et al. (2018)

Table 18 Support vector machine

S/N	Output layer	Input layer	No of data	R^2	Rock types	Country of origin	References
1	UCS	Poisson ratio, Young's modulus	397	0.60	Sedimentary	Iran	Aboutaleb et al. (2018)
2	UCS	Young's modulus	397	0.85	Sedimentary	Iran	Aboutaleb et al. (2018)
3	UCS	Poisson ratio, Young's modulus	397	0.92	Sedimentary	Iran	Aboutaleb et al. (2018)
4	UCS	Porosity, durability index	47	0.77	Mixed	Turkey	Ceryan (2014)
5	UCS	Amplitude attenuation coefficient, high and low frequency ratio	1614	0.99	Mixed	NA	Ren et al. (2019)

and GA-ANN have been used to predict UCS of rocks by many studies (Monjezi et al. 2012; Armaghani et al. 2016; Jalali et al. 2017; Mohamad et al. 2018). Table 21 lists some studies where Hybrid forms of ANN have been used to estimate UCS from other rock properties. The statistics of the R^2 values of the models generated for the studies range from 0.60 to 0.99. Compared to other forms of artificial intelligence approaches, the hybrid ANNs produced higher R^2 values, indicating that they have more prediction capability compared to those forms of artificial intelligence that are not hybrid.

6 Summary and Conclusions

This study made a compilation of empirical relations for estimating UCS from other rock properties reported in the literature for the three types of rock and for cases where different rock types are mixed.

Table 19 Fuzzy inference system

S/N	Output layer	Input layer	No of data	R^2	Rock types	Country of origin	References
1	UCS	Schmidt hardness number, density, porosity	93	0.95	Sedimentary	Iran	Rezaei et al. (2014)
2	UCS	Petrographic composition	102	0.92	Igneous	Turkey	Gokceoglu (2002)
3	UCS	P-wave velocity, block punch index, point load strength, tensile strength	82	0.67	Sedimentary	Turkey	Gokceoglu and Zorlu (2004)
4	UCS	Petrographic composition	NA	0.64	Igneous	Turkey	Sonmez et al. (2004)
5	UCS	Point load strength, shore hardness, P-wave velocity	NA	0.97	Mixed		Karakus and Tutmez (2006)
6	UCS	Clay content, slake durability index	68	0.88	Sedimentary	Turkey	Gokceoglu et al. (2009)
7	UCS	Block punch index, point load strength, shore hardness, P-wave velocity	60	0.98	Mixed	India	Mishra and Basu (2012)
8	UCS	Grain size, shape factor, quartz content	30	0.91	Metamorphic	China	Ali et al. (2014)
9	UCS	Block point index, Schmidt hardness number, point load strength, P-wave velocity	288	0.91	Sedimentary	Iran	Heidari et al. (2018)
10	UCS	Density, Equotip value, porosity	226	NA	Mixed		Grima and Babuška (1999)
11	UCS	Schmidt hardness number, Block punch index, Point load strength, P-wave velocity	106	0.91	Sedimentary	Iran	Jalali et al (2017)

Table 20 Genetic programming

S/N	Output layer	Input layer	No of data	R^2	Rock types	Country of origin	References
1	UCS	Density, porosity, P-wave velocity	72	0.83	Sedimentary	Iran	Beiki et al. (2013)
2	UCS	P-wave velocity, water absorption, density	106	0.86	Sedimentary	Turkey	Baykasoğlu et al. (2008)
3	UCS	Quartz content, density, porosity, shore hardness, cone indenter hardness	44	0.63	Sedimentary	Nepal	Manouchehrian et al. (2013)
4	UCS	Dry density, slake durability index, Brazilian tensile strength	47	0.97	Sedimentary	Malaysia	Armaghani et al. (2018)
5	UCS	Ultrasound pulse velocity, water absorption, dry density, saturated density, bulk density	52	0.88	Igneous	Turkey	Canakci et al. (2009)
6	UCS	Origin of rocks, two-cycle slake durability index and clay content	56	0.96	Sedimentary	Turkey	Cevik et al. (2011)
7	UCS	Origin of rocks, four-cycle slake durability index and clay content	56	0.97	Sedimentary	Turkey	Cevik et al. (2011)
8	UCS	Bottom ash dosage, dry unit weight, relative compaction, brittleness index, energy absorption capacity	70	0.85	Sedimentary	Turkey	Güllü (2014)

Table 21 Hybrid based ANN for prediction of UCS from other rock properties

S/N	Model type	Output layer	Input layer	No of data	R^2	Rock types	Country of origin	References
1	Genetic Algorithm Artificial Neural Network (GA-ANN)	UCS	Density, rock quality designation (RQD), porosity, number of joints per meter, geological strength index,	120	NA	Sedimentary	Iran	Majdi and Beiki (2010)
2		UCS	Porosity, density, Shore hardness	93	0.96	Mixed	Iran	Monjezi et al. (2012)
3	Particle Swarm Optimisation Artificial Neural Network (PSO-ANN)	UCS	Density, P-wave velocity, point load strength, Schmidt hardness number	66	0.97	Mixed	Malaysia	Momeni et al. (2015)
4		UCS	Point load strength, Brazilian tensile strength, bulk density, P-wave velocity	40	0.97	Mixed	Malaysia	Mohamad et al. (2015)
5		UCS	Dry density, moisture content, P-wave velocity, point load strength, slake durability index	228	0.96	Sedimentary	Malaysia	Mohamad et al. (2018)
6		UCS	Dry density, moisture content, P-wave velocity, point load strength, slake durability index	228	0.92	Sedimentary	Malaysia	Mohamad et al. (2018)
7	Imperialist Competitive Algorithm Artificial Neural Network (ICA-ANN)	UCS	Shore hardness, point load strength, P-wave velocity	124	0.94	Mixed	Malyasia	Armaghani et al. (2016)
8		UCS	Porosity, shore hardness, P-wave velocity, point load strength	124	0.92	Mixed	Malyasia	Armaghani et al. (2016)
9	Adaptive neuro-fuzzy inference system (ANFIS)	UCS	P-wave velocity, point load strength, Schmidt hardness number, water content	121	0.94	Sedimentary	Turkey	Yilmaz and Yuksek (2008)
10		UCS	Brazilian tensile strength, P-wave velocity	75	0.60	Igneous	Turkey	Yesiloglu-Gultekin et al. (2013)
11		UCS	Petrographic composition	75	0.83	Igneous	Turkey	Yesiloglu-Gultekin et al. (2013)
12		UCS	Dry density, P-wave velocity, quartz content, plagioclase content	45	0.99	Igneous	Malaysia	Armaghani et al. (2015)
13		UCS	P-wave velocity, slake durability index, density	70	0.98	Mixed	India	Sharma et al. (2017)
14		UCS	Schmidt hardness number, block punch index, point load strength, P-wave velocity	106	0.99	Sedimentary (coal)	Iran	Jalali et al (2017)

Based on the database developed, typical ranges and mean of data used in developing the regressions, and the range and mean of the R^2 values of regressions for estimating UCS from other rock properties were evaluated and summarised. The empirical relationships considered in this study include simple regressions, multiple regressions, and artificial intelligence-based relations for estimating UCS using approaches such as ANN, SVM, FIS, GP, and hybrid ANN like ANFIS, PSO-ANN, ICA-ANN, and GA-ANN.

The database of regression equations between UCS and other rock properties provides a systematic and logical assemblage of empirical relations that can be used in mining engineering practice. The relationships between UCS and other rock properties can be assessed to decide on the regression equation to be used for estimation of UCS at a specific site for a rock type. This will eliminate the problem of overestimation or underestimation of rock properties often encountered when regression equations are used to estimate the UCS. In addition, the database will serve as a useful companion to rock characterization approaches developed for mining and geotechnical application, especially when there is need to perform model selection and when quantifying the variability of UCS at a project site. The database will be particularly beneficial at small to medium-sized project sites, where rock properties data are often too sparse and there is need to estimate UCS of rock for mine planning and design purposes. A future study can investigate the possibility of developing an approach to rank the reliability of the regression equations in the database when they are used for estimation of UCS.

Funding Open access funding provided by University of Oulu including Oulu University Hospital.

Declarations

Conflict of interest The authors declare that there are no known conflicts of interest.

References

Abdolazim A, Rassoul A (2015) Empirical correlation of physical and mechanical properties of marly rocks with P wave velocity. Arab J Geosci 8:2069–2079

Aboutaleb S, Behnia M, Bagherpour R, Bluekian B (2018) Using non-destructive tests for estimating uniaxial compressive strength and static Young's modulus of carbonate rocks via some modeling techniques. Bull Eng Geol Environ 77(4):1717–1728

Adebayo B, Aladejare AE (2013) Effect of rock properties on excavation-loading operation in selected quarries. Adv Mater Res 824:86–90

Aggistalis G, Alivizatos A, Stamoulis D, Stournaras G (1996) Correlating uniaxial compressive strength with Schmidt hammer rebound number, point load index, Young's modulus, and mineralogy of gabbros and basalts (Northern Greece). Bull Eng Geol 54:3–11

Agustawijaya DS (2007) The uniaxial compressive strength of soft rock. Civ Eng Dimens 9(1):9–14

Aiken LS, West SG, Reno RR (1991) Multiple regression: testing and interpreting interactions. Sage, Thousand Oaks

Akram M, Bakar MZA (2007) Correlation between uniaxial compressive strength and point load index for salt-range rocks. Pak J Eng Appl Sci 1:1–8

Aladejare AE (2016) Development of Bayesian probabilistic approaches for rock property characterization. Doctoral dissertation, City University of Hong Kong, Hong Kong

Aladejare AE (2020) Evaluation of empirical estimation of uniaxial compressive strength of rock using measurements from index and physical tests. J Rock Mech Geotech Eng. https://doi.org/10.1016/j.jrmge.2019.08.001

Aladejare AE, Idris MA (2020) Performance analysis of empirical models for predicting rock mass deformation modulus using regression and Bayesian methods. J Rock Mech Geotech Eng 12(6):1263–1271

Aladejare AE, Wang Y (2017) Evaluation of rock property variability. Georisk Assess Manag Risk Eng Syst Geohazards 11(1):22–41

Aladejare AE, Wang Y (2019a) Estimation of rock mass deformation modulus using indirect information from multiple sources. Tunn Undergr Space Technol 85:76–83

Aladejare AE, Wang Y (2019b) Probabilistic characterization of Hoek–Brown constant mi of rock using Hoek's guideline chart, regression model and uniaxial compression test. Geotech Geol Eng 37(6):5045–5060

Aladejare AE, Akeju VO, Wang Y (2020) Probabilistic characterisation of uniaxial compressive strength of rock using test results from multiple types of punch tests. Georisk Assess Manag Risk Eng Syst Geohazards 15:1–2

Al-Harthi AA, Al-Amri RM, Shehata WM (1999) The porosity and engineering properties of vesicular basalt in Saudi Arabia. Eng Geol 54(3–4):313–320

Ali E, Guang W, Ibrahim A (2014) Empirical relations between compressive strength and microfabric properties of amphibolites using multivariate regression, fuzzy inference and neural networks: a comparative study. Eng Geol 183:230–240

Aliyu MM, Shang J, Murphy W, Lawrence JA, Collier R, Kong F, Zhao Z (2019) Assessing the uniaxial compressive

strength of extremely hard cryptocrystalline flint. Int J Rock Mech Min Sci 113:310–321
Altindag R (2012) Correlation between P-wave velocity and some mechanical properties for sedimentary rocks. J South Afr Inst Min Metall 112:229–237
Altindag R, Guney A (2010) Predicting the relationship between brittleness and mechanical properties (UCS, TS and SH) of rocks. Sci Res Essays 5(16):2107–2118
Aoki H, Matsukara Y (2008) Estimating the unconfined compressive strength of intact rocks from Equotip hardness. Bull Eng Geol Environ 67(1):23–29
Armaghani DJ, Mohamad ET, Momeni E, Narayanasamy MS (2015) An adaptive neuro-fuzzy inference system for predicting unconfined compressive strength and Young's modulus: a study on Main Range granite. Bull Eng Geol Environ 74(4):1301–1319
Armaghani DJ, Mohamad ET, Hajihassani M, Yagiz S, Motaghedi H (2016) Application of several non-linear prediction tools for estimating uniaxial compressive strength of granitic rocks and comparison of their performances. Eng Comput 32(2):189–206
Armaghani DJ, Safari V, Fahimifar A, Monjezi M, Mohammadi MA (2018) Uniaxial compressive strength prediction through a new technique based on gene expression programming. Neural Comput Appl 30(11):3523–3352
Ashtari M, Mousavi SE, Cheshomi A, Khamechian M (2019) Evaluation of the single compressive strength test in estimating uniaxial compressive and Brazilian tensile strengths and elastic modulus of marlstone. Eng Geol 248:256–266
Atkinson RH (1993) Hardness tests for rock characterization. In: JA Hudson (ed) Rock Testing and Site characterisation. Compressive rock engineering, vol 3, pp 105–117
Aufmuth RE (1973) A systematic determination of engineering criteria for rocks. Bull Assoc Eng Geol 11:235–245
Aydin A, Basu A (2005) The Schmidt hammer in rock material characterization. Eng Geol 81(1):1–14
Azimian A, Ajalloeian R, Fatehi L (2014) An Empirical correlation of uniaxial compressive strength with P-wave velocity and point load strength index on Marly rocks using statistical method. J Geotech Geol Eng 32:205–214
Basu A, Aydin A (2006) Predicting uniaxial compressive strength by point load test: significance of cone penetration. Rock Mech Rock Eng 39(5):483–490
Basu A, Kamran M (2010) Point load test on schistose rocks and its applicability in predicting uniaxial compressive strength. Int J Rock Mech Min Sci 5(47):823–828
Baykasoğlu A, Güllü H, Çanakçı H, Özbakır L (2008) Prediction of compressive and tensile strength of limestone via genetic programming. Expert Syst Appl 35(1–2):111–123
Beiki M, Majdi A, Givshad AD (2013) Application of genetic programming to predict the uniaxial compressive strength and elastic modulus of carbonate rocks. Int J Rock Mech Min Sci 63:159–169
Beverly BE, Schoenwolf DA, Brierly GS (1979) Correlations of rock index values with engineering properties and the classification of intact rock
Bieniawski ZT (1974) Estimating the strength of rock materials. J S Afr Inst Min Metall 74(8):312–320
Bieniawski ZT (1975) The point-load test in geotechnical practice. Eng Geol 9(1):1–11
Bieniawski ZT (1976) Exploration for rock engineering. In: Proceedings of the symposium on exploration for rock engineering, Johannesburg, Balkema
Brameier M, Banzhaf W (2001) Evolving teams of predictors with linear genetic programming. Genet Program Evolvable Mach 2(4):381–407
Broch E, Franklin JA (1972) The point-load strength test. Int J Rock Mech Min Sci Geomech Abstr 9(6):669–676
Bruno G, Vessia G, Bobbo L (2012) Statistical method for assessing the uniaxial compressive strength of carbonate rock by Schmidt hammer tests performed on core samples. Rock Mech Rock Eng 46(1):199–206
Canakci H, Baykasoglu A, Güllü H (2009) Prediction of compressive and tensile strength of Gaziantep basalts via neural networks and gene expression programming. Neural Comput Appl 18:1031–1041
Carbonell J (2003) Artificial intelligence 15-381 today: introduction to AI and search methods. www.cs.cmu.edu/~15381/Lectures/intro-search.ppt
Cargill JS, Shakoor A (1990) Evaluation of empirical methods for measuring the uniaxial compressive strength of rock. Int J Rock Mech Min Sci Geomech Abstr 27(6):495–503
Cawsey A, Aylett R (2009) Artificial intelligence introduction. www.macs.hw.ac.uk/~alison/ai3/ppt/l1.ppt
Ceryan N (2014) Application of support vector machines and relevance vector machines in predicting uniaxial compressive strength of volcanic rocks. J Afr Earth Sci 100:634–644
Ceryan N, Okkan U, Kesimal A (2012) Prediction of unconfined compressive strength of carbonate rocks using artificial neural networks. Environ Earth Sci 68(3):807–819
Cevik A, Akcapınar-Sezer E, Cabalar AF, Gokceoglu C (2011) Modeling of the uniaxial compressive strength of some clay-bearing rocks using neural network. Appl Soft Comput 11:2587–2594
Chatterjee R, Mukhopadhyay M (2002) Petrophysical and geomechanical properties of rocks from the oilfields of the Krishna-Godavari and Cauvery Basins, India. Bull Eng Geol Environ 61(2):169–178
Chau KT, Wong RHC (1996) Uniaxial compressive strength and point load strength. Int J Rock Mech Min Sci Geomech Abstr 33(2):183–188
Cheshomi A, Sheshde E (2013) Determination of uniaxial compressive strength of microcrystalline limestone using single particles load test. J Pet Sci Eng 111:121–126
Cheshomi A, Mousavi E, Ahmadi-Sheshde E (2015) Evaluation of single particle loading test to estimate the uniaxial compressive strength of sandstone. J Pet Sci Eng 135:421–428
Clifford ST (1991) An evaluation of the engineering properties of some Nigerian limestones as construction materials for highway pavements. Eng Geol 31(3–4):315–326
Cobanoglu I, Celik S (2008) Estimation of uniaxial compressive strength from point load strength, Schmidt hardness and P-wave velocity. Bull Eng Geol Environ 67:491–498
Corkum AG, Asiri Y, El Naggar H, Kinakin D (2018) The Leeb hardness test for rock: an updated methodology and UCS correlation. Rock Mech Rock Eng 51:665–675
Deere DU, Miller RP (1966) Engineering classification and index properties for intact rock. Technical Report AFNL-

TR-65-116. Albuquerque, USA: Air Force Weapon Laboratory

Dehghan S, Sattari GH, Chelgani SC, Aliabadi MA (2010) Prediction of uniaxial compressive strength and modulus of elasticity for Travertine samples using regression and artificial neural networks. Min Sci Technol (China) 20(1):41–46

Dershowitz WS, Einstein HH (1984) Application of artificial intelligence to problems of rock mechanics. In: 25th US symposium on rock mechanics

Diamantis K, Gartzos E, Migiros G (2009) Study on uniaxial compressive strength, point load strength index, dynamic and physical properties of serpentinites from Central Greece: test results and empirical relations. Eng Geol 108(3–4):199–207

Diamantis K, Bellas S, Migiros G, Gartzos E (2011) Correlating wave velocities with physical, mechanical properties and petrographic characteristics of peridotites from the Central Greece. Geotech Geol Eng 29(6):1049–1062

Dincer I, Acar A, Cobanoglu I, Uras Y (2004) Correlation between Schmidt hardness, uniaxial compressive strength and Young's modulus for andesites, basalts and tuffs. Bull Eng Geol Environ 63:141–148

Dincer I, Acar A, Ural S (2008) Estimation of strength and deformation properties of Quaternary caliche deposits. Bull Eng Geol Environ 63:141–148

Ehlen J (2002) Some effects of weathering on joints in granitic rocks. CATENA 49(1–2):91–109

Elhakim AF (2015) The use of point load trst for Dubai weak calcerous sandstones. J Rock Mech Geotech Eng 7(4):452–457

Entwisle DC, Hobbs RN, Jones LD, Gunn D, Raines MG (2005) The relationship between effective porosity, uniaxial compressive strength and sonic velocity of intact Borrowdale volcanic group core samples from Sella field. Geotech Geol Eng 23:793–809

Farah R (2011) Correlations between index properties and unconfined compressive strength of weathered ocala limestone. UNF Theses and Dissertations, Florida, USA

Fener M, Kahraman S, Bilgil A, Gunaydin O (2005) A comparative evaluation of indirect methods to estimate the compressive strength of rocks. Rock Mech Rock Eng 38(4):329–343

Fereidooni D (2016) Determination of the geotechnical characteristics of hornfelsic rocks with a particular emphasison the correlation between physical and mechanical properties. Rock Mech Rock Eng 49:2595–2608

Freedman DA (2009) Statistical models: theory and practice. Cambridge University Press, Cambridge

Garret JH (1994) Where and why artificial neural networks are applicable in civil engineering. J Comput Civ Eng ASCE 8(2):129–130

Ghose AK, Chakraborti S (1986) Empirical strength indices of Indian coals proceedings of 27th US symposium on rock mechanics Balkema. Rotterdam

Ghosh DK, Srivastava M (1991) Point-load strength: an index for classification of rock material. Bull Int Assoc Eng Geol 44(1):27–33

Gökçeoglu C (1996) Schmidt sertlik cekici kullamlarak tahmin edilen tek eksenli basinc dayanini verilerinin güvenirligi üzerine bir degerlendirme. Jeol Müh 48:78–81 ((**in Turkish**))

Gokceoglu C (2002) A fuzzy triangular chart to predict the uniaxial compressive strength of the Ankara agglomerates from their petrographic composition. Eng Geol 66(1–2):39–51

Gokceoglu C, Aksoy H (2000) New approaches to the characterization of clay-bearing, densely jointed and weak rock masses. Eng Geol 58(1):1–23

Gokceoglu C, Zorlu K (2004) A fuzzy model to predict the uniaxial compressive strength and the modulus of elasticity of a problematic rock. Eng Appl Artif Intell 17(1):61–72

Gokceoglu C, Sonmez H, Zorlu K (2009) Estimating the uniaxial compressive strength of some clay-bearing rocks selected from turkey by nonlinear multivariable regression and rule-based fuzzy models. Expert Syst 26(2):176–190

Grima MA, Babuška R (1999) Fuzzy model for the prediction of unconfined compressive strength of rock samples. Int J Rock Mech Min Sci 36(3):339–349

Güllü H (2014) Function finding via genetic expression programming for strength and elastic properties of clay treated with bottom ash. Eng Appl Artif Intell 35:143–157

Gunsallus KL, Kulhawy FH (1984) A comparative evaluation of rock strength measures. Int J Rock Mech Min Sci Geomech Abstr 21(5):233–248

Gupta V (2009) Non-destructive testing of some Higher Himalayan rocks in the Satluj Valley. Bull Eng Geol Environ 68(3):409–416

Gupta P, Kulkarni N (2013) An introduction of soft computing approach over hard computing. Int J Latest Trends Eng Technol 3(1):254–258

Haramy KY, DeMarco MJ (1985) Use of the Schmidt hammer for rock and coal testing. In: Proceedings of the 26th US symposium on rock mechanics (USRMS). American Rock Mechanics Association (ARMA)

Heidari M, Khanlari G, Torabi-Kaveh M, Kargarian S (2012) Predicting the uniaxial compressive and tensile strengths of gypsum rock by point load testing. Rock Mech Rock Eng 45(2):265–273

Heidari M, Mohseni H, Jalali SH (2017) Prediction of uniaxial compressive strength of some sedimentary rocks by fuzzy and regression models. Geotech Geol Eng 36:401–412

Hoek E (1977) Rock mechanics laboratory testing in the context of a consulting engineering organization. Int J Rock Mech Min Sci Geomech Abstr 14(2):93–101

Hoek E, Carranza-Torres C, Corkum B (2002) Hoek–Brown failure criterion-2002 edition. Proc NARMS-Tac 1(1):267–273

Hudyma N, Avar BB, Karakouzian M (2004) Compressive strength and failure modes of lithophysae-rich Topopah Spring Tuff specimens and analog models containing cavities. Eng Geol 73:179–190

Jahanbakhshi R, Keshavarzi R, Azinfar MJ (2011) Intelligent prediction of uniaxial compressive strength for sandstone. In: Proceedings of the 45th US rock mechanics/geomechanics symposium. ARMA

Jalali SH, Heidari M, Mohseni H (2017) Comparison of models for estimating uniaxial compressive strength of some sedimentary rocks from Qom Formation. Environ Earth Sci 76:753

Kabilan N, Muttharam M, Elamathi V (2017) Prediction of unconfined compressive strength for jointed rocks using point load index based on joint asperity angle. Geotech Geol Eng 35(6):2625–2636

Kahraman S (1996) Basınc direnci tahmininde Schmidt venokta yuk indeksi kullanmanın guvenirligi. In: Korkmazve S, Akcay M (eds) KTÜ Jeoloji Muhendisligi Bolumu 30. Yıl Sempozyumu BildirilerKitabı, Trabzon, pp 362–369 ((**in Turkish**))

Kahraman S (2001) Evaluation of simple methods for assessing the uniaxial compressive strength of rock. Int J Rock Mech Min Sci 38(7):981–994

Kahraman S, Gunaydin O (2009) The effect of rock classes on the relation between uniaxial compressive strength and point load index. Bull Eng Geol Environ 68(3):345–353

Kahraman S, Gunaydin O, Fener M (2005) The effect of porosity on the relation between uniaxial compressive strength and point load index. Int J Rock Mech Min Sci 4(42):584–589

Kahraman S, Fener M, Kozman E (2012) Predicting the compressive and tensile strength of rocks from indentation hardness index. J S Afr Inst Min Metall 112(5):331–339

Kahraman S, Fener M, Gunaydin O (2016) Estimating the uniaxial compressive strength of pyroclastic rocks from the slake durability index. Bull Eng Geol Environ 76(3):1107–1115

Kallu R, Roghanchi P (2015) Correlations between direct and indirect strength test methods. Int J Min Sci Technol 25(3):355–360

Karakus M, Tutmez B (2006) Fuzzy and multiple regression modelling for evaluation of intact rock strength based on point load, Schmidt hammer and sonic velocity. Rock Mech Rock Eng 39(1):45–57

Katz O, Reches Z, Roegiers JC (2000) Evaluation of mechanical rock properties using a Schmidt Hammer. Int J Rock Mech Min Sci 37(4):723–728

Khandelwal M (2013) Correlating P-wave velocity with the physico-mechanical properties of different rocks. Pure Appl Geophys 170:507–514

Khandelwal M, Singh TN (2009) Correlating static properties of coal measures rocks with P-wave velocity. Int J Coal Geol 79(1–2):55–60

Khandelwal M, Kankar PK, Harsha SP (2010) Evaluation and prediction of blast induced ground vibration using support vector machine. Min Sci Technol (China) 20(1):64–70

Kidybinski A (1980) Bursting liability indices of coal. Int J Rock Mech Min Sci Geomech Abstr 17:161–167

Kılıç A, Teymen A (2008) Determination of mechanical properties of rocks using simple methods. Bull Eng Geol Environ 67(2):237

King MS (1983) Static and dynamic elastic properties of rocks from the Canadian Shield. Int J Rock Mech Min Sci 20:237–241

Kohno M, Maeda H (2012) Relationship between point load strength index and uniaxial compressive strength of hydrothermally altered soft rocks. Int J Rock Mech Min Sci 50:147–157

Koncagul EC, Santi PM (1999) Predicting the unconfined compressive strength of the Breathitt shale using slake durability, Shore hardness and rock structural properties. Int J Rock Mech Min Sci 36(2):139–153

Kurtulus C, Bozkurt A, Endes H (2012) Physical and mechanical properties of serpentinized ultrabasic rocks in NW Turkey. Pure Appl Geophys. https://doi.org/10.1007/s00024-011-0394-z

Lawal AI, Kwon S (2020) Application of artificial intelligence to rock mechanics: an overview. J Rock Mech Geotech Eng. https://doi.org/10.1016/j.jrmge.2020.05.010

Lee JS, Smallwood L, Morgan E (2014) New application of rebound hardness numbers to generate logging of unconfined compressive strength in laminated shale formations. In: 48th US rock mechanics/geomechanics symposium, vol 2, pp 972–978

Liou SW, Wang CM, Huang YF (2009) Integrative discovery of multifaceted sequence patterns by frame-relayed search and hybrid PSO-ANN. J UCS 15(4):742–764

Madhubabu N, Singh PK, Kainthola A, Mahanta B, Tripathy A, Singh TN (2016) Prediction of compressive strength and elastic modulus of carbonate rocks. Measurement 88:202–213

Majdi A, Beiki M (2010) Evolving neural network using a genetic algorithm for predicting the deformation modulus of rock masses. Int J Rock Mech Min Sci 47(2):246–253

Majdi A, Rezaei M (2013) Prediction of unconfined compressive strength of rock surrounding a roadway using artificial neural network. Neural Comput Appl 23:381–389

Malik MH, Rashid S (1997) Correlation of some engineering geological properties of the Murree formation at lower Topa (Murree district), Pakistan. Geol Bull Univ Peshawar 30:69–81

Manouchehrian A, Sharifzadeh M, Moghadam RH (2012) Application of artificial neural networks and multivariate statistics to estimate UCS using textural characteristics. Int J Min Sci Technol 22(2):229–236

McCulloch WS, Pitts W (1943) A logical calculus of the ideas immanent in nervous activity. Bull Math Biophys 5(4):115–133

Meulenkamp F, Grima MA (1999) Application of neural networks for the prediction of the unconfined compressive strength (UCS) from Equotip hardness. Int J Rock Mech Min Sci 36(1):29–39

Minaeian B, Ahangari K (2013) Estimation of uniaxial compressive strength based on P-wave and Schmidt hammer rebound using statistical method. Arab J Geosci 6:1925–1931

Mishra DA, Basu A (2012) Use of the block punch test to predict the compressive and tensile strengths of rocks. Int J Rock Mech Min Sci 51:119–127

Mishra DA, Basu A (2013) Estimation of uniaxial compressive strength of rock materials by index tests using regression analysis and fuzzy inference system. Eng Geol 160:54–68

Mohamad ET, Armaghani DJ, Momeni E, Abad SVANK (2015) Prediction of the unconfined compressive strength of soft rocks: a PSO-based ANN approach. Bull Eng Geol Environ 74(3):745–757

Mohamad ET, Armaghani DJ, Momeni E, Yazdavar AH, Ebrahimi M (2018) Rock strength estimation: a PSO-based BP approach. Neural Comput Appl 30(5):1635–1646

Mohd BK (2009) Compressive strength of vuggy oolitic limestones as a function of their porosity and sound propagation. Jordan J Earth Environ Sci 2(1):18–25

Momeni E, Nazir R, Armaghani DJ, Mohamad ET (2015) Prediction of unconfined compressive strength of rocks: a review paper. Jurnal Teknologi 77(11):43–50

Monjezi M, Khoshalan HA, Razifard M (2012) A neuro-genetic network for predicting uniaxial compressive strength of rocks. Geotech Geol Eng 30(4):1053–1062

Moradian ZA, Behnia M (2009) Predicting the uniaxial compressive strength and static Young's modulus of intact sedimentary rocks using the ultrasonic test. Int J Geomech 9:1–14

Morales T, Uribe-Etxebarria G, Uriarte JA, Valderrama IF (2004) Geomechanical characterisation of rock masses in Alpine regions: the Basque Arc (Basque-Cantabrian basin, Northern Spain). Eng Geol 71:343–362

Najibi AR, Ghafoori M, Lashkaripour GR, Asef MR (2015) Empricla relations between strength static and dynamic elastic properties of Asmari and Sarvak limestones, two main oil reservoirs in Iran. J Pet Sci Eng 126:78–82

Nazir R, Momeni E, Jahed Armaghani D (2013) Correlation between unconfined compressive strength and indirect tensile strength of limestone rock samples. Electr J Geotech Eng 18:1737–1746

Nefeslioglu HA (2013) Evaluation of geo-mechanical properties of very weak and weak rock materials by using non-destructive techniques: ultrasonic pulse velocity measurements and reflectance spectroscopy. Eng Geol 160:8–20

Ng IT, Yuen KV, Lau C (2015) Predictive model for uniaxial compressive strength for Grade III granitic rocks from Macao. Eng Geol 199:28–37

O'Rourke JE (1989) Rock index properties for geoengineering in underground development. Min Eng 41(2):106–109

Palmstrøm A (1996) Characterizing rock masses by the RMi for use in practical rock engineering: part 1: the development of the Rock Mass index (RMi). Tunn Undergr Space Technol 11(2):175–188

Rabbani E, Sharif F, Salooki MK, Moradzadeh A (2012) Application of neural network technique for prediction of uniaxial compressive strength using reservoir formation properties. Int J Rock Mech Min Sci 56:100–111

Ren Q, Wang G, Li M, Han S (2019) Prediction of rock compressive strength using machine learning algorithms based on spectrum analysis of geological hammer. Geotech Geol Eng 37:475–489

Rezaei M, Majdi A, Monjezi M (2014) An intelligent approach to predict unconfined compressive strength of rock surrounding access tunnels in longwall coal mining. Neural Comput Appl 24(1):233–241

Sabatakakis N, Koukis G, Tsiambaos G, Papanakli S (2008) Index properties and strength variation controlled by microstructure for sedimentary rocks. Eng Geol 97:80–90

Sachpazis CI (1990) Correlating Schmidt hardness with compressive strength and Young's modulus of carbonate rocks. Bull Int Assoc Eng Geol 42(1):75–83

Salah H, Omar M, Shanableh, (2014) Estimating unconfined compressive strength of sedimentary rocks in United Arab Emirates from Point Load Index. J Appl Math Phys 2:296–303

Sarkar K, Vishal V, Singh TN (2012) An empirical correlation of index geomechanical parameters with the compressional wave velocity. Geotech Geol Eng 30(2):469–479

Selcuk L, Kayabali K (2015) Evaluation of the unconfined strength of rocks using nail guns. Eng Geol 195:164–171

Shalabi FI, Cording EJ, Al-Hattamleh OH (2007) Estimation of rock engineering properties using hardness tests. Eng Geol 90:138–147

Sharma PK, Singh TN (2008) A correlation between P-wave velocity, impact strength index, slake durability index and uniaxial compressive strength. Bull Eng Geol Environ 67:17–22

Sharma LK, Vishal V, Singh TN (2017) Developing novel models using neural networks and fuzzy systems for the prediction of strength of rocks from key geomechanical properties. Measurement 102:158–169

Shorey PR, Barat D, Das MN, Mukherjee KP, Singh B (1984) Schmidt hammer rebound data for estimation of large scale insitu coal strength (Tech Note). Int J Rock Mech Min Sci Geomech Abstr 21:39–42

Singh RN, Hassani FP, Elkington PAS (1983) The application of strength and deformation index testing to the stability assessment of coal measures excavations. In: Proceeding of 24th US symposium on rock mechanics. Texas A and M University AEG, Balkema, Rotterdam, pp 599–609

Singh VK, Singh D, Singh TN (2001) Prediction of strength properties of some schistose rocks from petrographic properties using artificial neural networks. Int J Rock Mech Min Sci 38(2):269–284

Singh TN, Kainthola A, Venkatesh A (2012) Correlation between point load index and uniaxial compressive strength for different rock types. Rock Mech Rock Eng 45(2):259–264

Smith HJ (1997) The point load test for weak rock in dredging applications. Int J Rock Mech Min Sci 34(3–4):295.e1-295.e13

Sonmez H, Tuncay E, Gokceoglu C (2004) Models to predict the uniaxial compressive strength and the modulus of elasticity for Ankara agglomerate. Int J Rock Mech Min Sci 41(5):717–729

Sousa LMO, Del Rio LMS, Calleja L, de Argandona VGR, Rey AR (2005) Influence of microfractures and porosity on the physico-mechanical properties and weathering of ornamental granites. Eng Geol 77:153–168

Sulukcu S, Ulusay R (2001) Evaluation of the block punch index test with particular reference to the size effect, failure mechanism and its effectiveness in predicting rock strength. Int J Rock Mech Min Sci 38(8):1091–1111

Tahir M, Mohammad N, Din F (2011) Strength parameters and their inter-relationship for limestone of Cherat and Kohat areas of Khyber Pakhtunkhwa. J Himal Earth Sci 44(2):45–51

Tandon RS, Gupta V (2015) Estimation of strength characteristics of different Himalayan rocks from Schmidt hammer rebound, point load index and compressional wave velocity. Bull Eng Geol Environ 74:521–533

Thuro K, Plinninger RJ, Zäh S (2001) Scale effects in rock strength properties. Part 1: unconfined compressive test and Brazilian test. In: Särkkä P, Eloranta P (eds) Rock mechanics—a challenge for society. Proceedings of ISRM regional symposium on Eurock 2001, Espoo, Finland. Swets & Zeitlinger, Taylor and Francis Group, London

Tiryaki B (2008) Predicting intact rock strength for mechanical excavation using multivariate statistics, artificial neural networks, and regression trees. Eng Geol 99:51–60

Torabi SR, Ataei M, Javanshir M (2010) Application of Schmidt rebound number for estimating rock strength under specific geological conditions. J Min Environ 1(2):1–8

Torabi-Kaveh M, Naseri F, Saneie S, Sarshari B (2015) Application of artificial neural networks and multivariate statistics to predict UCS and E using physical properties of Asmari limestones. Arab J Geosci 8(5):2889–2897

Török A, Vasarhelyi B (2010) The influence of fabric and water content on selected rock and mechanical parameters of travertine, examples from Hungary. Eng Geol 115:237–245

Tsiambaos G, Sabatakakis N (2004) Considerations on strength of intact sedimentary rocks. Eng Geol 72(3–4):261–273

Tugrul A, Gurpinar O (1997) A proposed weathering classification for basalts and their engineering properties (Turkey). Bull Eng Geol Environ 55:139–149

Tugrul A, Zarif IH (1999) Correlation of mineralogical and textural characteristics with engineering properties of selected granitic rocks from Turkey. Eng Geol 51(4):303–317

Ulusay R, Gokceoglu C (1997) The modified block punch index test. Can Geotech J 34(6):991–1001

Ulusay R, Hudson JA (2007) The complete ISRM suggested methods for rock characterization, testing and monitoring. ISRM Turkish National Group, Ankara

Ulusay R, Türeli K, Ider MH (1994) Prediction of engineering properties of a selected litharenite sandstone from its petrographic characteristics using correlation and multivariate statistical techniques. Eng Geol 38(1–2):135–157

Uyanik O, Sabbag N, Uyanik NA, Oncu Z (2019) Prediction of mechanical and physical properties of some sedimentary rocks from ultrasonic velocities. Bull Eng Geol Environ 78(8):6003–6016

Van der Schrier JS (1988) The block point index test. Bull Int Assoc Eng Geol 38:121–126

Vasarhelyi B (2005) Statistical analysis of the influence of water content on the strength of the Miocene limestone. Rock Mech Rock Eng 38:69–76

Verwaal W, Mulder A (1993) Estimating rock strength with the Equotip hardness tester: technical note. Int J Rock Mech Min Sci Geomech 30:659–662

Verwaal W, Mulder A (2000) Rock and aggregate test procedures. Rock Aggreg Lab Man 13:14

Wang Y, Aladejare AE (2015) Selection of site-specific regression model for characterization of uniaxial compressive strength of rock. Int J Rock Mech Min Sci 75:73–81

Wang Y, Aladejare AE (2016a) Bayesian characterization of correlation between uniaxial compressive strength and Young's modulus of rock. Int J Rock Mech Min Sci 85:10–19

Wang Y, Aladejare AE (2016b) Evaluating variability and uncertainty of geological strength index at a specific site. Rock Mech Rock Eng 49(9):3559–3573

Xu S, Grasso P, Mahtab A (1990) Use of Schmidt hammer for estimating mechanical properties of weak rock. In: Proceeding of 6th international IAEG Congress, Balkema, Rotterdam, pp 511–519

Yagiz S (2009) Predicting uniaxial compressive strength, modulus of elasticity and index properties of rocks using the Schmidt hammer. Bull Eng Geol Environ 68(1):55–63

Yagiz S (2011) P-wave velocity test for assessment of geotechnical properties of some rock materials. Bull Mater Sci 34(4):947–953

Yagiz S, Sezer EA, Gokceoglu C (2012) Artificial neural networks and nonlinear regression techniques to assess the influence of slake durability cycles on the prediction of uniaxial compressive strength and modulus of elasticity for carbonate rocks. Int J Numer Anal Methods Geomech 36(14):1636–1650

Yasar E, Erdogan Y (2004a) Estimation of rock physio-mechanical properties using hardness methods. Eng Geol 71:281–288

Yasar E, Erdogan Y (2004b) Correlating sound velocity with the density, compressive strength and Young's modulus of carbonate rocks. Int J Rock Mech Min Sci 5:871–875

Yasar E, Ranjith PG, Perera MA (2010) Physico-mechanical behaviour of southeastern Melbourne sedimentary rocks. Int J Rock Mech Min Sci 47(3):481–487

Yavuz AB, Turk N, Koca MY (2005) Geological parameters affecting the marble production in quarries along the southern flank of the Menderes Massif, Turkey. Eng Geol 80:214–241

Yesiloglu-Gultekin N, Gokceoglu C, Sezer EA (2013) Prediction of uniaxial compressive strength of granitic rocks by various nonlinear tools and comparison of their performances. Int J Rock Mech Min Sci 62:113–122

Yilmaz N (2013) The influence of testing procedures on uniaxial compressive strength prediction of carbonate rocks from Equotip hardness tester (EHT) and proposal of a new testing methodology: hybrid dynamic hardness (HDH). Rock Mech Rock Eng 46(1):95–106

Yılmaz I, Sendır H (2002) Correlation of Schmidt hardness with unconfined compressive strength and Young's modulus in gypsum from Sivas (Turkey). Eng Geol 66(3–4):211–219

Yılmaz I, Yuksek AG (2008) An example of artificial neural network (ANN) application for indirect estimation of rock parameters. Rock Mech Rock Eng 41(5):781–795

Zhang ZX, Hou DF, Aladejare AE (2020) Empirical equations between characteristic impedance and mechanical properties of rocks. J Rock Mech Geotech Eng 12(5):975–983

Zorlu K, Gokceoglu C, Ocakoglu F, Nefeslioglu HA, Acikalin S (2008) Prediction of uniaxial compressive strength of sandstones using petrography-based models. Eng Geol 96(3–4):141–158