HYPERBOLIC SPATIAL TEMPORAL GRAPH CONVOLUTIONAL NETWORKS

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ABSTRACT

Spatial-temporal graph convolutional networks (ST-GCNs) have been successfully applied for dynamic graphs representation learning, such as modeling skeleton-based human actions. However, ST-GCNs embed these non-Euclidean graph structures into Euclidean space, which is not the natural space to represent such structures as embedding them in this space incurs a large distortion. In this work, we use hyperbolic non-Euclidean geometry and construct compact ST-GCNs in the hyperbolic space. It can be shown that hyperbolic ST-GCNs (HST-GCNs) outperform the corresponding Euclidean counterparts. Additionally, these compact hyperbolic models can be used to increase the performance of large complex Euclidean models. Moreover, we show that the same or even better performance of large Euclidean models can be achieved by fusing the scores of smaller Euclidean models and a compact hyperbolic model. This in turn leads to reducing the total number of model parameters and hence model size. To validate the performance of these hyperbolic networks, we conducted extensive experiments on NTU RGB+D, NTU RGB+D 120 and Kinetics-Skeleton datasets for human action recognition.

Index Terms— Hyperbolic geometry, dynamic graphs, graph convolutional networks, human action recognition

1. INTRODUCTION

Human action recognition is an active area of research with many applications such as surveillance and human-computer interaction. Human actions can be predicted from RGB videos, depth maps or skeleton data. In particular, skeleton-based human action recognition has increasingly gained attention due to its robustness against different changes like illumination or viewpoints [1]. Skeleton data provide abstract information about the locations of human joints.

Earlier deep learning skeleton-based human action recognition approaches considered the human joints as a set of independent features and ignored the relationships between joints. These features are then fed into Convolutional Neural Networks (CNNs) [2, 3] or Recurrent Neural Networks (RNNs) [4, 5] to predict the action class.

A human skeleton can be represented as a graph structure. Recently, GCNs were proposed to generalize the convolution operation to any structured graph data [6, 7]. GCNs were used to model the skeleton data in the spatial domain to build the ST-GCN [8]. Many subsequent works achieved state-of-the-art performance on human action recognition datasets using some variants based on ST-GCN to model human joints sequences [9, 10, 11, 12].

However, all these models embed the features into the Euclidean space which has been shown to incur a large distortion [13]. That is because the ball volume grows polynomially with respect to the radius $r$ in Euclidean space ($V = \frac{\pi^n}{n!} r^n$) for n-dimensional Euclidean space where $\Gamma(z)$ is the Euler gamma function) whereas it grows exponentially in hyperbolic space which leads to lower distortion. This can make the model compact as low-dimensional embeddings are needed without losing much information due to distortion. The hyperbolic space is ideal for embedding trees as the number of tree nodes is growing exponentially with respect to the tree depth. The Gromov or $\delta$-hyperbolicity ($\delta \geq 0$) [14] is used to measure the tree-likeness of data (trees have $\delta = 0$) and the smaller the value is, the more hyperbolic the data. Since the commonly used graph topologies for human skeleton are with $\delta$-hyperbolicity $= 0$, we were motivated to use the hyperbolic space as the embedding space for the human action features. Extensive experiments are performed on three large scale human action recognition datasets, namely, NTU RGB+D, NTU RGB+D 120 and Kinetics-Skeleton and the results demonstrate that using HST-GCNs has great advantages.

2. RELATED WORK

2.1. Skeleton-based action recognition

Human action recognition has been studied extensively and the first approaches used hand-crafted features for this task [15]. With the advances achieved in deep learning, the next approaches used CNNs or RNNs [4, 3, 2, 5, 16]. For example, the work in [3] uses a multi-scale CNN and fine-tune pre-trained CNNs, e.g., AlexNet, ResNet on human action datasets. The work in [4] divides the human skeleton into five parts which are fed into five subnets that can be fused to form a hierarchical RNN.

GCNs model the spatial relationships in graphs and generalize the convolution operation to any graph. ST-GCN [8] used GCNs to model the spatial relationship between joints. Adaptive GCN (AGCN) [9] used a learnable matrix to model the connections for a general graph for all sequences and also used an input-dependent matrix to learn the connections for each input sequence. In a subsequent work [10], the authors added attention modules in the spatial, temporal and channel dimensions to improve the performance. MSGD [11] designed a multi-scale sophisticated model which has parallel branches with multi-scale disentangled feature aggregation. The authors introduced across space-time connections to extract more relevant and enhanced features. There are many other works that built models based on GCNs [17, 18, 19, 20, 12]. For example, the work in [20] proposes shift-GCN which uses shift graph operations and point-wise convolutions. Dynamic GCNs were introduced in [18] to learn the skeleton topology automatically. However, all GCN-based previous works use the Euclidean space to embed the tree-like non-Euclidean human action recognition data. In this work, we exploit the hyperbolic geometry and use it to embed the features in the natural non-Euclidean hyperbolic space.

2.2. Hyperbolic neural networks

Using the Lorentz model of hyperbolic space gives enhanced embeddings specially for spaces with small dimensions [21]. HGCNs
were proposed by [13] and they were able to achieve better performance on node classification and link prediction tasks when compared to the Euclidean analogs. Concurrently to this work, [22] proposed the Hyperbolic Graph Neural Networks (HGNNs) which performed well on graph classification tasks. However, both methods only considered the spatial configurations between graph nodes on static graph tasks. In this work, we also consider the temporal domain for dynamic graphs. For dynamic graphs, it is also important to build compact models for computation efficiency. Another work which is most related to our work is Poincaré-GCN [23] which used Poincaré geometry. However, this work is not mathematically rigorous since they assumed that the input embedding features lie on the Poincaré model, which is not naturally satisfied. Besides, the learned model is much bigger than ours. The work in [24] presented an interesting survey on hyperbolic neural networks.

3. METHODS

3.1. Notations

A human skeleton can be represented by a graph \( G = \{V, \mathcal{E}\} \) where \( V = \{v_1, v_2, \ldots, v_n\} \) is the set of \( n \) human joints (graph nodes) in the human body and \( \mathcal{E} \) is the set of connections or bones (graph edges) between human joints. The edge set \( \mathcal{E} \) can be encoded in an adjacency matrix \( A \in \mathbb{R}^{n \times n} \) where \( A_{i,j} \in [0, 1] \) if there is a link between \( v_i \) and \( v_j \) otherwise, \( A_{i,j} = 0 \). Each node \( v_i \) has a feature vector \( x_i \in \mathbb{R}^d \) of dimension \( d \). Initially, the feature vector is the vector \( x \) at point \( t \) which is encoded as a function of its input feature vector \( x_i \). The set of feature vectors of all nodes at time step \( t \) where \( X_t \in \mathbb{R}^{n \times d} \). A human action can be observed over \( T \) frames.

3.2. Spatial Temporal Graph Convolutional Networks

For the spatial domain, the GCN update step can be formulated as:

\[
X_t^{out} = \sigma(A^{-1/2}(A + I)A^{-1/2}(X_t^inW^in + B^in))
\]  

where \( \sigma \) is an activation function, \( A^{ij} = 1 + \sum_j A^{ij} \) and \( A \) is a diagonal matrix. \( A \) is the identity matrix to identify identity features. \((A + I)\) then makes the output feature vector \( x_i^{out} \) for every node \( i \) as a function of its input feature vector \( x_i^{in} \) and the feature vector \( x_j^{in} \) for any node \( j \) in neighboring set for node \( i \) which is encoded in matrix \( A \). \( A^{-1/2}(A + I)A^{-1/2} \) is the normalized adjacency matrix to normalize the weights for the nodes in the neighboring set. \( W^in \) is the weight matrix corresponding to \( X_t^in \) and \( B^in \) is the bias translation matrix. To achieve better performance and to increase model capacity, a partitioning strategy can be applied and \( A + I \) can be decomposed into a number of matrices \( A_j \) such that \( A + I = \sum_j A_j \). For the temporal domain, a simple convolution can be applied to nodes in consecutive frames.

3.3. HST-GCNs

A hyperbolic space is a non-Euclidean space with a constant negative curvature. Many models were introduced to represent and model a hyperbolic space such as the Lorentz model, the Poincaré model and the Klein model. We use the Lorentz model (also called the hyperboloid model) as it is simple and numerically more stable[21].

Let \( \langle \cdot, \cdot \rangle_L : \mathbb{D}^{d+1} \times \mathbb{D}^{d+1} \rightarrow \mathbb{R} \) represents the Minkowski inner product where \( \langle x, y \rangle_L := \sum_{i=1}^{d} x_i y_i - x_0 y_0 \). Let \( \mathbb{H}^{d,K} \) be a \( d \) dimensional hyperboloid model with a constant negative curvature \( -1/K \) where \( K > 0 \). Then we have:

\[
\mathbb{H}^{d,K} := \{ x \in \mathbb{R}^{d+1} : \langle x, x \rangle_L = -K, x_0 > 0 \}
\]  

Note that \( x_0 > 0 \) to indicate the upper half of the hyperboloid manifold. Let \( T_e \mathbb{H}^{d,K} \) be the Euclidean tangent space centered at point \( x \in \mathbb{H}^{d,K} \). Then we have:

\[
T_e \mathbb{H}^{d,K} := \{ v \in \mathbb{R}^{d+1} : \langle v, x \rangle_L = 0 \}
\]  

To map a point \( y \in \mathbb{H}^{d,K} \) to the tangent space \( T_y \mathbb{H}^{d,K} \) centered at point \( x \in \mathbb{H}^{d,K} \) such that \( x \neq y \), the logarithmic map can be used which is defined as:

\[
\log^K_y(x) = d^K_y(x, y) \frac{y + \sqrt{1/K} \langle x, y \rangle_L x}{\|y + \sqrt{1/K} \langle x, y \rangle_L x\|_L}
\]  

where \( \|x\|_L = \sqrt{(x, x)_L} \) is the norm of \( x \). \( d^K_y(x, y) \) is the Minkowski distance between two points \( x \) and \( y \) in \( \mathbb{H}^{d,K} \) and is given by:

\[
d^K_y(x, y) = \sqrt{K} \arccosh((-\langle x, y \rangle_L)/K)
\]  

To map a point \( v \in T_y \mathbb{H}^{d,K} \) to the hyperboloid manifold such that \( v \neq 0 \), we use the exponential map defined as:

\[
\exp^K_y(v) = \cosh\left(\frac{\|v\|_L}{\sqrt{K}}\right)x + \sqrt{K} \sinh\left(\frac{\|v\|_L}{\sqrt{K}}\right) \frac{v}{\|v\|_L}
\]  

The logarithmic and exponential maps represent a bijection between the tangent space at a point and the hyperboloid. Figure 1 illustrates the mapping between the hyperbolic space \( \mathbb{H}^{d,K} \) and the tangent space at the origin \( o \) which is \( T_o \mathbb{H}^{d,K} \) for \( d = 2 \).

Parallel transport is used to perform translation in the hyperbolic space. \( P_{x \rightarrow y}(\cdot) \) maps a point \( u \in T_y \mathbb{H}^{d,K} \) to a point \( u^\prime \in T_y \mathbb{H}^{d,K} \) for \( x, y \in \mathbb{H}^{d,K} \) and \( x \neq y \). The parallel transport of a point \( u \in T_y \mathbb{H}^{d,K} \) to the tangent space \( T_o \mathbb{H}^{d,K} \) is:

\[
P_{x \rightarrow y}(u) = u - \frac{\langle \log^K_y(x), u \rangle_L}{d^K_y(x, y)^2} (\log^K_y(x) + \log^K_y(x))
\]  

The hyperbolic bias addition can then be defined as:

\[x^K \odot b := \exp^K_{oH}(P_{o \rightarrow H}(b))
\]  

where \( o \) is the origin and the bias \( b \) is a learnable Euclidean vector defined at the tangent space of the origin.
For the hyperbolic space, Eq. 1 can be rewritten as:

$$X_{t}^{out,H} = \exp_{o}^{K} \left( \sigma \left( \log_{o}^{K} \left( \sum_{j} \exp_{o}^{K} \left( A_{j}^{\Lambda_{j}^{-1/2}} X_{in}^{\Lambda_{j}^{-1/2}} \right) \right) \right) \right)$$

(9)

where the input and output features are in the hyperbolic space (denoted by $\mathbb{H}$), $\sum_{j}^{\oplus_{K_{j}}}$ is the summation in the hyperboloid model (Möbius addition of points on the hyperboloid) over the partitioning sets where $j \in \{1,2,\ldots,J\}$. We call this module the Hyperbolic Spatial GCN (HS-GCN). Note that for the 2D or 3D input skeleton joints positions or any initial input feature $X_{in}^{l}$, we have $(0, X_{in}^{l}) \in \mathbb{T}_{d=2,3}^{\mathbb{H}}$. As from Eq. 3, we get $\langle (0, X_{in}^{l}), o \rangle \epsilon = 0$. Using the exponential map (Eq. 6 at the origin $o$), we can obtain the initial hyperbolic input feature $X_{in}^{l}$. For $j = 1$, $\log_{o}^{K_{j}}(X_{in}^{l})$ maps $X_{in}^{l}$ to the tangent space of the manifold used in the previous layer which can be the initial input feature vector.

To enlarge the receptive field and obtain features from farther away joints, we use a disentangling approach [11] to get features from up to $M$-hop neighbors which effectively enlarge the receptive field of nodes. The $m$-th adjacency matrix $A_{m}$ is then given by:

$$[A_{m}]_{i,j} = \begin{cases} 1 & \text{if } d(v_{i}, v_{j}) = m, \\ 0 & \text{otherwise} \end{cases}$$

(10)

where $d(v_{i}, v_{j})$ is the shortest distance between node $v_{i}$ and node $v_{j}$.

To obtain features from up to $M$-hop neighbors, we use $A_{m}$ for $m = 0,\ldots,M$. Using the spatial configuration partitioning strategy, we get $2M+1$ subsets for $M$-hop neighbors. These adjacency matrices can be used in the HS-GCN module (Eq. 9) to enlarge the receptive field and to obtain enhanced features.

For the temporal domain, the hyperbolic features are mapped to the tangent space ($\exp_{o}^{K} \left( X_{t}^{out,H} \right)$) where the convolution across time domain is performed then the resulting features are mapped back to the hyperboloid using $\exp_{o}^{K_{j}}(\ldots)$ to generate the output for the next layer. This module is the Hyperbolic Temporal Convolutional Network (HTCN). Figure 2 shows the full model architecture.

### 4. EXPERIMENTS

Here, we provide extensive experiments on three skeleton datasets.

#### 4.1. Datasets

NTU RGB+D 60 is a large scale dataset for 3D human activity analysis which has 60 human action classes. The authors recommend two benchmarks for this dataset: (1) Cross-Subject (X-Sub) and (2) Cross-View (X-View). NTU RGB+D 120 extends NTU RGB+D 60 to have 120 action classes and he authors recommend replacing the Cross-View setting with a Cross-Setup (X-Set) setting. Kinetics-Skeleton is a large-scale human action dataset that has 400 classes. The top-1 and top-5 accuracy on the testing set are reported.

#### 4.2. Ablation study

##### 4.2.1. Number of neighbors for feature aggregation

Table 1 shows the accuracy and number of parameters for a 2-layer HST-GCN to determine $M$. Using a 4-hop neighbors for disentangled feature aggregation gives the best performance. More information can be captured from further joints away which is particularly important for shallow models.

#### 4.2.2. Network configuration on different datasets

Table 2 shows the performance of HST-GCNs using different configurations. For the NTU RGB+D 120 and the Kinetics-Skeleton 400 datasets, we conducted similar experiments and found that the best performance can be obtained using the configuration (C,L) = (40,2).

#### 4.3. Performance of hyperbolic vs Euclidean models

We use the NTU RGB+D 60 Cross-Subject benchmark in this experiment. Table 2 shows this comparison for different network configurations. HST-GCNs clearly outperform the corresponding Euclidean ST-GCNs (EST-GCNs) by about 5% for most of the network configurations specially the low-dimensional ones. This shows that enhanced discriminative features can be obtained and embedded in the hyperbolic manifold which increases model performance.

Table 3 shows the performance of HST-GCNs on the NTU datasets and Kinetics dataset. These are light-weight compact models when compared to large existing Euclidean models. For example, the number of parameters in HST-GCN is only about 0.5% and 0.6%
<table>
<thead>
<tr>
<th>Model</th>
<th>Dataset</th>
<th>NTU RGB+D 60</th>
<th>NTU RGB+D 120</th>
<th>Kinetics-Skeleton 400</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X-Sub</td>
<td>X-View</td>
<td>X-Sub</td>
</tr>
<tr>
<td>EST-GCN</td>
<td>Acc(%)</td>
<td>70.9</td>
<td>80.2</td>
<td>63.2</td>
</tr>
<tr>
<td>HST-GCN</td>
<td>Acc(%)</td>
<td>76.1</td>
<td>81.1</td>
<td>67.8</td>
</tr>
<tr>
<td></td>
<td>Params(M)</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3. HST-GCNs and the Euclidean counterparts (EST-GCNs) performance on different datasets.

<table>
<thead>
<tr>
<th>Model</th>
<th>CL</th>
<th>Params(M)</th>
<th>NTU RGB+D 60</th>
<th>NTU RGB+D 120</th>
<th>Kinetics-Skeleton 400</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X-Sub</td>
<td>X-View</td>
<td>X-Sub</td>
<td>X-View</td>
</tr>
<tr>
<td>AAGCN</td>
<td></td>
<td>32.6</td>
<td>0.26</td>
<td>81.8/83.7</td>
<td>90.8/91.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.6</td>
<td>0.56</td>
<td>83.7/85.8</td>
<td>91.1/91.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>32.9</td>
<td>0.98</td>
<td>84.7/85.8</td>
<td>91.6/92.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.9</td>
<td>2.15</td>
<td>85.8/86.8</td>
<td>92.5/93.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>64.9</td>
<td>3.87</td>
<td>86.6/87.7</td>
<td>94.8/94.8</td>
</tr>
<tr>
<td>MS-G3D</td>
<td></td>
<td>56.3</td>
<td>1.36</td>
<td>88.8/88.9</td>
<td>94.5/94.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>72.3</td>
<td>1.98</td>
<td>89.2/89.5</td>
<td>94.8/95.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>96.3</td>
<td>3.20</td>
<td>89.3/89.4</td>
<td>95.0/95.0</td>
</tr>
<tr>
<td>Shift-GCN</td>
<td></td>
<td>64.9</td>
<td>0.69</td>
<td>87.8/88.2</td>
<td>95.1/95.1</td>
</tr>
</tbody>
</table>

Table 4. Boosting the performance of existing methods using HST-GCNs. The numbers separated by / represent the Euclidean model accuracy and the boosted accuracy using HST-GCN model, respectively.

4.4. Boosting the performance of existing methods

We show that HST-GCNs can be used to boost the performance of other methods. In addition, we show that by using smaller versions of these Euclidean models combined with HST-GCNs is comparable to or outperforms the larger versions of these models. Table 4 shows the performance of different sizes of AAGCN models [10] and MS-G3D models [11] on different datasets. The table also shows the boosted performance of these models and shift-GCN model [20] using the corresponding HST-GCN model from Table 3. For each method, the last row is the original model introduced by the authors. For the AAGCN model, a comparable or better performance with 45% parameters reduction. For the MS-G3D model, we achieved comparable or better performance with 40% parameters reduction. For the NTU RGB+D 120 cross-set benchmark, a better performance was achieved with 60% parameters reduction. Similarly, HST-GCNs can be used to boost the performance of any other models.

4.5. Comparison with SOTA

Table 5 shows the comparison between different methods on the NTU RGB+D 60 dataset. Our method achieved comparable or better performance using smaller or comparable number of parameters.

<table>
<thead>
<tr>
<th>Method</th>
<th>X-Sub</th>
<th>X-View</th>
<th>Params(M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST-GCN [8]</td>
<td>81.5</td>
<td>88.3</td>
<td>3.10</td>
</tr>
<tr>
<td>SR-TSL [16]</td>
<td>84.8</td>
<td>92.4</td>
<td>19.07</td>
</tr>
<tr>
<td>RAGCN [25]</td>
<td>85.9</td>
<td>93.5</td>
<td>6.21</td>
</tr>
<tr>
<td>AAGCN [10]</td>
<td>86.6</td>
<td>94.8</td>
<td>3.87</td>
</tr>
<tr>
<td>AS-GCN [26]</td>
<td>86.8</td>
<td>94.2</td>
<td>9.50</td>
</tr>
<tr>
<td>NAS-GCN [17]</td>
<td>87.4</td>
<td>94.6</td>
<td>6.57</td>
</tr>
<tr>
<td>Poincaré-GCN [23]</td>
<td>87.8</td>
<td>95.0</td>
<td>2.62</td>
</tr>
<tr>
<td>Shift-GCN [20]</td>
<td>87.8</td>
<td>95.1</td>
<td>0.69</td>
</tr>
<tr>
<td>DC-GCN+ADG [19]</td>
<td>88.2</td>
<td>95.2</td>
<td>1.24</td>
</tr>
<tr>
<td>Ours (small)</td>
<td>88.2</td>
<td>95.1</td>
<td>0.71</td>
</tr>
<tr>
<td>AGC-LSTM [27]</td>
<td>89.2</td>
<td>95.0</td>
<td>22.89</td>
</tr>
<tr>
<td>PL-GCN [28]</td>
<td>89.2</td>
<td>95.0</td>
<td>20.70</td>
</tr>
<tr>
<td>MS-G3D [11]</td>
<td>89.3</td>
<td>95.0</td>
<td>3.20</td>
</tr>
<tr>
<td>Ours</td>
<td>89.5</td>
<td>95.0</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Table 5. Comparison between different methods on the NTU RGB+D 60 dataset.

5. CONCLUSION

In this work, we showed that using the hyperbolic space to embed human action features is more superior than using the Euclidean space as in classical ST-GCNs. At the same time, HST-GCNs can be used with existing methods to build compact models to achieve comparable performance. We believe that this work has a great potential and hope it motivates researchers to take advantage of the hyperbolic embedding space in different research fields.

6. ACKNOWLEDGEMENTS

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7. REFERENCES


