Analyses of Beamspace MIMO Channels at 142GHz

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Abstract—This paper presents the analyses of a single-user beamspace MIMO on measured indoor and outdoor channels at 142 GHz. The rank is evaluated under different antenna sizes, number of beams, and thresholds. We assume a total power constraint at the transmitter which results in a decrease in signal-to-noise ratio as the link distance increases. When using spatial multiplexing, the indoor and outdoor sites demonstrate an average capacity gain of 2x and 1.5x at link distances below 60 m. Also, the rank for our measured 142 GHz channels is comparable to that at 60 GHz channels but significantly lower than the rank at 5 GHz channels reported in the literature. We also found that at 142 GHz, the indoor and outdoor sites have median ranks of 3.0 and 1.7 for the small antenna case, and 4.9 and 2.4 for the large antenna case assuming a rank threshold of 20 dB. The indoor site has a rank higher by 1.8 than the outdoor site, regardless of antenna size. The rank decreases by only 20% and 15% for indoor and outdoor scenarios when beam density is halved, allowing a significant reduction in implementation complexity of the beamspace MIMO without remarkably reducing the rank.

Keywords—Beamspace, MIMO, propagation, spatial multiplexing, sub-terahertz.

I. INTRODUCTION

The use of sub-Terahertz (sub-THz) frequency band has been one of the research hotspots for future general cellular networks due to their capability of short-range high-data-rate communications [2]. It is commonly thought that sub-THz radio links require directive beams at both link ends using a phased antenna array or radio lenses. Directive beams, especially in non-line-of-sight (NLOS) links, are essential to compensate for the high propagation loss, i.e., diffraction and scattering. Furthermore, directive antennas at both link ends allow spatial multiplexing through LOS connection and reflections on flat, smooth surfaces when available. Seminal works [3], [4], [5] indicate that there is more than one independent beam and multiple spatial clusters leading to the possibility of spatial multiplexing in sub-THz channels. Capacity improvement of element-space sub-THz multiple-input multiple-output (MIMO) radio links was reported in [3] where the number of antenna elements in an array is identical to the number of transceiver chains. However, since the rank of sub-THz MIMO channels remains much less than the number of antenna elements, it would be more practical to consider a MIMO architecture with fewer fully equipped radio transceiver chains (digital signal processing paths, converters, mixers). One such architecture is the beamspace MIMO, which can have a much smaller number of transceiver chains than the number of antennas in an array, as illustrated in Fig. 1. The multiple narrow high-gain beams are produced through an analog phase shifting network [6], [7] and/or an antenna-array-fed lens [6], [8], [9].

We evaluated the beamspace MIMO link using double-directional channel data derived from measurements [1]. The single-directional multipath data is first obtained through measurement campaigns in four different environments. We then applied a measurement-based ray-launcher (MBRL) to estimate the double-directional multipath data. To the best of our knowledge, this is the first paper to investigate the benefits of spatial multiplexing of beamspace MIMO links using measurement-derived double-directional sub-THz channels. Furthermore, the effect of varying configurations, e.g., antenna array size, beam density, and rank threshold, on channel rank are evaluated. In this paper, we show that the channel rank at 142 GHz channels is almost similar to that at 60 GHz channels [10] but significantly lower than at 5 GHz channels [11], [12].

II. BEAMSPACE MIMO

A. System Model

A narrowband beamspace single-user MIMO system can be represented as,

$$y = \mathbf{H}x + \mathbf{n},$$  

(1)
where \( y \) is the received signal vector, \( \mathbf{H} \) is an \( N_R \times N_T \) beamspace channel matrix, \( x \) is the input signal vector, and \( n \) is a received complex Gaussian white noise vector with a power of \( N_0 \). \( N_R \) and \( N_T \) are the number of transmit and receive beams, respectively.

The double-directional propagation channel of a link is represented here as a set of discrete \( P \) multipath components (MPCs) as

\[
\mathcal{P} = \{ \alpha_p, \Omega_p, \hat{\theta}_p, \phi_p \}_{p=1}^P,
\]

where \( \alpha_p \) is the complex amplitude, \( \Omega_p = [\phi, \theta] \) is a vector of azimuth and polar angles on the Tx side, while \( \Omega_p = [\hat{\phi}, \hat{\theta}] \) is on the Rx side, and \( \tau_p \) is the propagation delay of the \( p \)th path. These multipath parameters are derived from single-directional channel measurements complemented by an MBRL, as discussed in Section III. The amplitude and angular information of each path, i.e., (2), are then used to calculate the entries of the \( k \)th row and \( l \)th column of the beamspace channel matrix of a link as

\[
\bar{h}_{k,l} = \sum_{p=1}^{N_p} \alpha_p \sqrt{A_l(\Omega_p) A_k(\Omega_p)} e^{j\xi_p},
\]

where \( l = 1, 2, ..., N_T \) and \( k = 1, 2, ..., N_R \) are indices representing the Tx and Rx beam pointing angles, and \( A \) is a beam gain pattern whose magnitude for \( p \)th path depends on the pointing angle of its main lobe and path’s departure and arrival angles; \( \xi_p \) is a random phase of \( [0, 2\pi] \) assigned to each path to represent small-scale fading. Note that the path delay from (2) is not included in generating the channel matrix this study assumes a narrowband system. \( M_T \) and \( M_R \) baseband chains are available at Tx and Rx, respectively. The beam selector on each side chooses \( M_{T/R} \) beams with the highest powers, which are found by

\[
\tilde{P}_k = \sum_{l=1}^{N_T} |\bar{h}_{k,l}|^2, \quad \tilde{P}_l = \sum_{k=1}^{N_R} |\bar{h}_{k,l}|^2,
\]

then the resulting beamspace channel matrix \( \mathbf{H} \) with the selected beams is given by entries

\[
h_{m,n} = \{ \bar{h}_{k,l} | k \in K, l \in L \},
\]

where \( K \) and \( L \) are sets of rows and columns providing \( M_R \) and \( M_T \) strongest beams at the Rx and Tx.

### B. Beam Pattern

We consider here a uniform circular array (UCA) both in Tx and Rx. The antenna elements are Hertzian dipoles oriented on the \( z \)-axis and are arranged uniformly on the \( xy \)-plane with element spacing of \( \lambda/2 \), where \( \lambda \) is the signal wavelength. For a number of antenna elements \( Q \), the radius of UCA is \( r = \lambda/(4\sin(\pi/Q)) \). The number of discrete beam directions is set to

\[
N = \left\lfloor \frac{2\pi\beta}{\phi_{3dB}} \right\rfloor,
\]

where \( \lfloor \cdot \rfloor \) is the round operator, \( \beta \) is the beam density coefficient, and \( \phi_{3dB} \) is the azimuth half-power beamwidth (HPBW). The \( N \) beams are uniformly allocated around the horizontal plane with an azimuth step between adjacent beams given by

\[
\Delta\phi = 2\pi/N.
\]

Substituting (6) to (7) gives \( \Delta\phi \approx \phi_{3dB}/\beta \). When \( \beta = 1 \), the overall radiation pattern, i.e., combined radiation pattern of all the beams, has a gain flatness of about 3 dB. Meanwhile, \( \beta < 1 \) and \( \beta > 1 \) respectively mean sparser and denser beams. In terms of implementation complexity, higher \( \beta \) translates to a greater number of required discrete beams which is equivalent to a larger discrete Fourier transform matrix for the conventional phased array architecture [6], [7], or more antenna elements if an antenna-array-fed lens is used [6].

### C. Capacity and Rank

Singular value decomposition is applied to the beamspace channel matrix \( \mathbf{H} = \mathbf{U}\Sigma \mathbf{V}^H \), where \( \mathbf{U}^H \) and \( \mathbf{V} \) are the decoding and precoding matrices respectively and \( \{ \cdot \}^H \) is the Hermitian transpose operator. The capacity of a single-user beamspace MIMO system with spatial multiplexing can then be calculated as

\[
C = \sum_{i=1}^{K'} \log_2 \left( 1 + \frac{P_i}{N_0 s_i^2} \right),
\]

where \( K' \) is the rank of the beamspace channel matrix, \( P_i \) is the power allocated, and \( s_i \) is the singular value at \( i \)th substream. Note that \( s_i \) is a non-negative real number. Assuming that there is a perfect channel state information available at Tx, the capacity can be maximized through water-filling power allocation of the Tx power \( P_T \) as

\[
P_i = \max \left( \mu' - \frac{N_0}{s_i^2}, 0 \right),
\]

where \( \mu' \) is selected to satisfy the power constraint \( P_T = \sum_{i=1}^{K'} P_i \), and \( P_T \) is the total Tx power fed to the antenna elements.

For each link, \( L \) random channel realizations are generated where each realization has a unique set of random phases \( \xi_p \) applied to the channel coefficients as indicated in (3). The ergodic capacity \( C \) of each link is then calculated by taking the average of the instantaneous capacity \( C \) across \( L \) channel realizations. The resulting rank \( K \) of these \( L \) realizations of the complete beamspace channel matrix \( \mathbf{H} \) is calculated as

\[
K = \min \left( \text{rank} \left( \mathbf{R}_T \right), \text{rank} \left( \mathbf{R}_R \right) \right),
\]

where \( \mathbf{R}_T \) and \( \mathbf{R}_R \) are the Tx and Rx covariance matrices calculated as \( \mathbf{R}_T = \mathbb{E}[\mathbf{H}^H \mathbf{H}] \) and \( \mathbf{R}_R = \mathbb{E}[\mathbf{H} \mathbf{H}^H] \) where \( \mathbb{E}[\cdot] \) is the expectation operator. The rank is defined here as the number of singular values within a threshold from the strongest singular value.
III. DOUBLE-DIRECTIONAL MULTIPATH DATA

Measurements of radio channels were conducted in an entrance hall and three outdoor environments such as suburban, residential, and city center. The number of LOS and NLOS links and link distance ranges are summarized in Table I. A channel sounder based on a Vector Network Analyzer integrated with frequency converters is employed. It measures from 140 GHz to 144 GHz resulting in a delay resolution of 0.25 ns. The Tx and Rx are equipped with an omni-directional and rotator-mounted directional horn antenna with 10° azimuth HPBW, respectively. The azimuth on the Rx side is scanned with a 5° step while keeping the antenna’s main beam fixed in the horizontal plane. The channel sounder and measurements are further described in [13], [14]. The single-directional power-angular delay profile (PADP) which contains the received signal intensity as a function of the propagation delay and the horn’s broadside direction is obtained from these measurements. To determine the gain contributed by the wave propagation alone, the Tx and Rx antenna gains and the shape of radiation patterns were de-embedded from the PADP, leading to the discrete multipath parameters [15], [16].

The resulting single-directional multipath data is then mapped to the environment’s detailed geometry, in the form of a point cloud, to estimate the path trajectory using an MBRL [14]. This ray-based simulation method launches rays from the receiver in the directions where the measured multipath components are observed, and the measured propagation delay defines their length. The trajectory of each launched ray that ends up close to the transmitter is considered the estimated trajectory of the corresponding measured multipath. The double-directional path data as in (2) and their interaction point(s) on the physical environment are then derived from this estimated trajectory. Up to fourth-order reflections are allowed using the MBRL.

About 10% of the multipaths were not successfully mapped onto the geometry due to the limitations of the tool and the input data. The angles of these unmapped paths are determined by randomly drawing values from the best-fitting distribution observed in the mapped paths. The Laplace distribution is a good fit for the angular information we obtained regardless of the environment and link type. The Laplace probability density function is described by the mean of the distribution which is equal to the direction of Rx seen from Tx, and by the scale parameter \( b \) which varies with the environments and link types as listed in Table II. A random azimuth angle-of-departure is then generated by

\[ \phi_T = \Phi + \phi_{TR}, \]

where \( \Phi \) is a random value with Laplacian distribution, i.e. \( \Phi \sim \mathcal{L}_d(0, b) \), and \( \phi_{TR} \) is the direction of Rx seen from Tx.

IV. RESULTS AND DISCUSSION

A center frequency of 142 GHz, transmit power of 20 dBm, and 100 channel realizations are assumed in the simulations. The noise power per beam is calculated as \( N_0 = kTB \), where \( k \) is the Boltzmann constant, \( T \) is the ambient temperature, and \( B \) is the system bandwidth. Assuming a temperature and bandwidth of 290 K and 1 GHz, the noise power is \(-84 \text{ dBm}\). The UCA configurations \( C_1, C_2, C_3, \) and \( C_4 \) considered in the simulations are summarized in Table III. Each configuration has a different antenna size, resulting in varying HPBW and gain. We assume a downlink transmission scenario, where the base station serves as the Tx and the user equipment acts as the Rx. The Tx is set to have a larger antenna size than the Rx. In the following simulations, the configurations \( C_2 \) and \( C_4 \) for the Tx and Rx, a beam density of 1, and a rank threshold of 20 dB are assumed unless otherwise stated. The \( C_2-C_3 \) and \( C_4-C_2 \) configuration pairs are considered as the small and large antenna cases. The synthesized beam patterns with the mentioned UCA setup in Section II-B have a sidelobe level of 8 dB lower than the main lobe regardless of the antenna size, and no sidelobe suppression is applied. The rank presented in the following results corresponds to the rank \( K \) as defined in (10). In addition, the indoor site refers to the entrance hall environment, and the outdoor site refers to the combined results of those mentioned in Section III.

A. Capacity and Rank

The ergodic capacity using single-stream (\( M = 1 \)) and multi-stream (\( M = 8 \)) transmissions, and rank for both sites are shown in Fig. 2. The single-stream transmission is equivalent to beamforming where its capacity is calculated as \( C_s = \log_2 \left( 1 + \frac{\mathbb{E}[h]^2}{N_0} \right) \), where \( h \) is the highest channel coefficient obtained using (5). Meanwhile, multi-stream corresponds to spatial multiplexing where the capacity is calculated using (8). Fig. 2 shows that there is a significant capacity gain for both sites, especially for the indoor case, by using \( 8 \times 8 \) multi-stream beamspace MIMO compared to single-stream. The average capacity gain of the links with link distance below 60 m is 200% for the indoor case and 150% for the outdoor case.
This study focuses on the ergodic capacity and rank of beamspace MIMO channels under different antenna specifications and environmental conditions. The beamspace MIMO channel shows a median (med.) rank of 3.0 and 1.7 for the indoor and outdoor environments, respectively. These results are comparable with the 60 GHz channels and significantly lower than the 5 GHz channels as summarized in Table IV. The rank estimates for the 60 GHz and the street scenario at 5 GHz are approximated from Fig. 2(e) of [10] and Fig. 2 of [11], respectively. The lower rank in the outdoor site compared to the indoor site, as observed in Fig. 2 for our 142 GHz measurements, can be explained by the indoor site having walls on all sides and ceilings, generating MPCs in more directions than the outdoor site. For both environments, the rank itself does not strongly vary with the link distance, while the capacity decreases due to the reduced SNR.

### B. Varying Antenna Size, Beam Density, and Threshold

The median rank across all the links under different Tx and Rx antenna sizes, given in terms of UCA radius $r_{T/R}$ in $\lambda$ are summarized in Table V. The corresponding UCA specifications are listed in Table III. For both environments, the rank increases as the antenna size increases due to the decrease in beamwidth and increase in SNR. Increasing the Tx size, while the Rx size is fixed, shows a limited rank improvement as the Rx’s relatively wide beamwidth limits its capability to resolve. The subsequent increase of Rx antenna size to $5.1\lambda$ shows a significant increase in rank. The median rank for the large antenna case is 4.9 and 2.4 for the indoor and outdoor sites. In all cases, the rank is approximately higher by 1.8 in our indoor site than the outdoor site.

The rank under different beam densities is listed in Table V. Reducing the beam density by 50% results only in a 20% decrease in rank for the indoor case and a 15% decrease for the outdoor case. Fig. 3 shows the median and quartiles of the rank under different thresholds. The rank in the indoor site increases and disperses more with the increasing threshold than in the outdoor site. Using a higher rank threshold than the typical 20 dB reveals the presence of weak spatial clusters, especially in the indoor site.

### V. Conclusion

In this paper, we evaluated the capacity and rank of the beamspace MIMO channels using double-directional multipath estimates in indoor and outdoor scenarios at 142 GHz. The capacity gain at link distances below 60 m is 200% and 150% for the indoor and outdoor when spatial multiplexing with Tx and Rx UCA radii of $5.1\lambda$ and $1.3\lambda$. The box’s bottom, middle, and top lines represent the first, second (median), and third quartiles, respectively.

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**TABLE IV. Rank at Different Frequencies and Environments**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>5 GHz</th>
<th>60 GHz</th>
<th>142 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
<td>7.5</td>
<td>7.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Med. rank</td>
<td>[12]</td>
<td>[11]</td>
<td>[10]</td>
</tr>
</tbody>
</table>

**TABLE V. Rank under Different Antenna Specifications**

<table>
<thead>
<tr>
<th>Beam density</th>
<th>Indoor</th>
<th>Outdoor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx UCA rad. ($\lambda$)</td>
<td>5.1</td>
<td>1.3</td>
</tr>
<tr>
<td>Rx UCA rad. ($\lambda$)</td>
<td>10.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Med. rank</td>
<td>Indoor</td>
<td>Outdoor</td>
</tr>
<tr>
<td>20.4</td>
<td>6.1</td>
<td>4.9</td>
</tr>
</tbody>
</table>

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![Fig. 2](image-url)  
Fig. 2. Ergodic capacity of each link using single-stream ($M_{T/R} = 1$) and multi-stream ($M_{T/R} = 8$) beamspace MIMO, and beamspace rank in (a) indoor and (b) outdoor scenarios. The total Tx power is set to 20 dBm, the noise floor is set to $-84$ dBm, and the rank threshold is set to 20 dB.

![Fig. 3](image-url)  
Fig. 3. Rank under varying thresholds with Tx and Rx UCA radii of $5.1\lambda$ and $1.3\lambda$. The box’s bottom, middle, and top lines represent the first, second (median), and third quartiles, respectively.
REFERENCES


