ANTTI NIKKANEN

PAIRS TRADING THE COMMODITY FUTURES CURVE

Master’s thesis
Finance Department
August 2012
I create a pairs trade on the commodity futures curve, which captures the roll returns of commodity futures and minimizes the standard deviation of the returns. The end result is a strategy that has an annualized arithmetic return of 6.04% and an annualized standard deviation of 2.01%. Transaction costs and liquidity are also accounted for.

The goal was to create and backtest a trading strategy that tries to capture the roll return component of commodity futures returns. In order to reduce the very high spot price volatility of commodity returns, a market neutral systematic arbitrage was introduced through a pairs trade. The pairs trade involves taking a counter position relative to the position that is designed to capture the roll return, with as small of a negative expected return as possible.

In practice capturing the roll return component means taking a long position into the largest dollar difference of a backwarded futures curve. And the pairs trade component is then a short position into the same curve, but with the smallest dollar difference. If the commodity futures curve was in contango, the procedure was reversed.

It can be concluded, that both of the targets of this research were reach; capturing the roll returns of commodity futures and minimizing volatility through a statistical arbitrage pairs trade.

The trades designed to capture the roll returns of commodity futures returned 5.55% annually (which was 91.9% of the portfolio’s total return), compared to the annualized return of the benchmark index of 0.5%

The pairs trade designed to minimize the standard deviation of these returns, lowered the annualized standard deviation of the portfolio’s returns from 6.37% to 2.01%, making it almost fully market neutral.
ACKNOWLEDGEMENTS

I would like to sincerely thank Kari Vatanen for instructing my thesis. Without his deep knowledge of the topic, and advice, this thesis would have not been possible in its current form. Kari’s input deserves special gratitude taking into account his parallel work load.

I would also like to thank Ph.D. Hanna Kahra for supervising the thesis.

A special mention also goes to my employer at the time, Varma, and more precisely Päivi Kalapuro and Mikko Koivusalo, for letting me write the thesis during the working hours.

Antti Nikkanen
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1 INTRODUCTION
In this thesis, I create a commodity futures trading strategy, which exploits the roll returns of commodity futures as its main driver of excess return. To minimize the volatility of the returns, I take it a step farther and introduce a pairs trading strategy on the commodity futures curve. The annualized arithmetic return of the strategy is 6.04% with the annualized standard deviation of just 2.01%. This leads to a Sharpe ratio of 3.01 and a Calmar ratio of 5.21. To make it as realistic as possible, and tradable in reality, I take into account the liquidity of each traded contract, and employ a conservative transaction cost, according to Locke and Venkatesh (1997), of 0.033 basis points per transaction.

According to Feurtes and Miffre (2010), commodity futures have become mainstream investment vehicles among traditional and alternative asset managers. The trend is likely to continue, not only because commodity futures can be used to successfully generate abnormal returns (Erb & Harvey 2006, Hoevenaars, Molneaar, Schotman & Steenkamp 2008), but also because traditional investment choices like equities continue to struggle (return of the S&P500 – index from the 14th of July 1999 to the 29th of June 2012: zero percent, Google Finance).

Figure 1 Commodity Index Funds (US$, bn) (Meir & Demler 2006)
In addition commodities have a number of attractive characteristics. Not only they work as an inflation hedge (appendix 2) (Bodie 1990) and in portfolio diversification (Jensen, Johnson & Mercer 2000), but they have a weak correlation with traditional risk factors (Erb & Harvey 2006), and also the return distribution of commodity returns is positively skewed (Gorton & Rouwenhorst 2005). Ilmanen (2011) considers this confluence of such traits, especially in oil, “too good to be true”. Ilmanen, as his personal opinion without empirical studies as a backing, states that “a fortuitously benign sample is part of the story – the increasing scarcity of oil and the growing demand from China and other emerging markets have boosted oil prices, resulting in persistent upside surprises.”

According to Gorton and Rouwenhorst (2005), even commodity futures have been traded in the U.S for over 100 years, they still remain somewhat unknown asset class. They consider up to four reasons of which one is as simple as scarcity of good data. The other three are more related to the characteristics of the actual commodity futures like, them being derivate securities, them being a short maturity claim on a real asset, and that they have pronounced seasonality (appendix 2) in price levels and volatility. I would also like to add from my personal experience that the coverage on commodities in my own University courses, and even more specifically on commodity futures, has been small compared to the time that spent covering the more traditional asset classes. So therefore the main reason for me choosing commodity futures as the topic for my thesis, a lot of new was to be learned.

Research in pairs trading has previously been mostly focused in equities. I take it into the commodity futures space. Research in commodity returns has had mostly a very aggregate focus. I take it to the level of a single commodity future, in creating a tradable strategy using the principles of pairs trading. I research that, is it possible to create an almost market neutral trading strategy in the commodity futures space, which abuses the single largest commodity futures excess return composition, roll return, as its main excess return driver. I find that this indeed is possible, and the concepts learned about pairs trading in equities are very well usable also in the commodity futures space.
2 LITERATURE REVIEW

Hong and Yogo (2012) investigate aggregate commodity returns. The most important finding of their paper relative to my thesis is that aggregate basis (ratio of the futures price to the spot price), is an important predictor of commodity returns, and that it is a local factor to the commodity market. They go as far as to suggest that even traditional predictors for bonds and stocks, like the short rate and the yield spread, do also predict commodity returns, but the aggregate basis can be considered the single most important determinant. The main driver behind the fluctuation of the aggregate basis is hedging pressure, i.e. how much producers short commodity futures to hedge their long positions in the underlying spot. Hence, a positive expected return is provided for the risk-averse speculator for taking the long side of the deal (Keynes 1923, Hicks 1939).

The main difference between my thesis and the paper of Hong and Yogo (2009) is that they consider the commodity market as a whole. This is understandable since Hong and Yogo (2009) use predictors which describe the situation in the economy at large, like the earlier mentioned yield spread. My focus simply is to deliver alpha, though which is based on mainly the same return driver as Hong and Yogos “aggregate basis”, and therefore it makes more sense for me to focus on the dynamics of each commodity as specifically as possible.

Hong and Yogo (2009) find an annualized average excess return of 7.09 percent with an annualized standard deviation of 14.13 percent. My own findings are very much in line with the findings of Hong and Yogo (2009). The main difference between the results of our studies is, that I account for the transaction costs (which leads to a lower excess return), but on the other hand my way of choosing the ‘best’ contract is more sophisticated and leads to a significantly lower standard deviation.

Erb and Harvey (2006) study historical commodity returns. They emphasize the importance not to naively extrapolate historical returns into the future, and to understand the balance between the different return drivers and how they might develop in the future. Gorton and Rouwenhorst (2006) is the starting point of their paper. Gorton and Rouwenhortst (2006) find that the annual excess return of individual commodity futures has been zero over the 1959-2004 period for 36 different commodity futures. It is
though very interesting to note that the characteristics of commodity futures contracts, high return volatility and low correlation between different commodities, make it possible to construct a rebalanced and equally weighted portfolio in which the average excess return is 4.5 percent. (Erb & Harvey 2006.)

The most important finding of Erb and Harvey (2006) concerning my study is that the roll return has explained 91.6 percent of the long-run cross-sectional variation of commodity futures returns over the period of 1982-2004. This is the return driver that my own empirical model tries to exploit. Exploiting specifically only this single return characteristic becomes slightly problematic, since most of the commodity futures excess return time-series variation is driven by the spot price movements, as Erb and Harvey (2005a) show. They go as far to conclude, that in explaining the excess return volatility of individual commodity futures, spot returns are more important than roll returns (average spot return standard deviation 26.76 percent and average roll return standard deviation 9.14 percent). To tackle this issue, my model creates a pair, which eliminates the risk of vertical movements in the futures curve, i.e. reduces the spot price movement factor of the returns significantly. This will leave me with almost just with the part of the commodity futures excess return which I want, the roll return. The process how I create the pair will be discussed in more detail at the later parts of this thesis. (Erb & Harvey 2006.)

The paper of Fuertes and Miffre (2010) take a more tactical view into the opportunities that commodity futures present than Erb and Harvey (2006) and Hong and Yogo (2009). For this reason it comes closest to my own thesis of the current academic research on the topic. They introduce the possibility of shorting the contangoed futures and not just taking a long position on backwardated contracts like many of the other academic studies do. In their study they also include momentum but since it is not relevant in the context of my thesis it will not be discussed farther here.

Gorton and Rouwenhorst (2005) study farther the simple properties of commodity futures in their paper “Facts and Fantasies about Commodity Futures”. They state as their findings that the returns of commodity futures are negatively correlated with those of equity and bond returns. The reasoning for this the fact, that these asset classes
behave differently depending on the business cycle. This supports strongly the earlier findings of Jensen, Johnson and Mercer (2000) who conclude that commodities are a good way to diversify a portfolio. But the financial crisis of 2008 proved that the negative correlation may disappear just the moment it is the most needed. Most, if not all, asset classes became strongly correlated to each other. Therefore it should be kept in mind that commodities work as a diversification tool the best in “normal” market conditions.

The pairs trade, by reducing the exposure to vertical price movements of the curve, makes my strategy exceptionally strong in also these “abnormal” conditions. For example in November and December of 2008 my strategy returned 2.8% and 3.5% a month. This was in a time when the general commodity indexes plummeted.
3 THEORY

3.1 Commodities as an investment

3.1.1 Development of the futures markets

It can be said that the futures market was all about agricultural products to the end of the 1960’s. The first metal (silver) related commodity future was introduced by COMEX in 1969, 5 years after the U.S. Mint had discontinued its coining and the value of silver was left to the marketplace to determine. (Spurgin & Donohue, 2009.)

When the U.S. government stopped the use of the gold standard in 1971, it led to broader movements between the U.S. dollar and other currencies. This led to birth of the International Monetary Market (IMM) by Chicago Mercantile Exchange (CME) in 1972, and trading a range of international currency futures was now also possible. (Spurgin & Donohue, 2009.)

In 1977 Chicago Board of Trade (CBOT) introduced the U.S. 30-year Treasury bond contract and it quickly became the highest volume futures contract in the world. It was not until the 1983 when the first crude oil future was introduced. This happen on the New York Mercantile Exchange (NYMEX) in New York City after President Reagan wanted to lift the oil price and control allocation better. (Spurgin & Donohue, 2009.)

Even everything evolved around the commodity space, in today’s market, fixed income, equity indices and foreign exchange futures dominate up to 20 times that of the commodity futures in volume. This is natural following of a managed futures trader’s portfolio which often consist of only 15% to 25% of commodity futures. Hedging is still the mainstay of trading volume but the opportunity for speculators to profit has attracted new investors into the futures space in vast amounts already since the 1980s. Advances in information technology, leading to reasonable costs for real time futures price data, has sped the movement. (Spurgin & Donohue, 2009.)
3.1.2 Different commodities

There are about 30 tradable commodities that are usually sub-categorized into groups based on the specific attributes each commodity has. For example, the leading commodity index, S&P GSCI, which often serves as a benchmark for investments in the commodity markets, has the following subcategories: *Energy* (Crude Oil, Brent Crude Oil, Unleaded Gas, Heating Oil, Gas Oil, Natural Gas), *Industrial Metals* (Aluminum, Copper, Lead, Nickel, Zinc), *Precious Metals* (Gold, Silver), *Agriculture* (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, Cocoa) and *Livestock* (Live Cattle, Feeder Cattle, Lean Hogs). (S&P GSCI Index Methodology, 2012.)

![Figure 2 Top 20 commodity futures by turnover (NYMEX, ICE, DCE, LME, SHFE, CBOT, ZCE)](image)

3.1.3 Commodities as an alternative asset class

Commodities are usually considered as consumable or transformable assets. This makes them different from other asset classes. Meaning that, for example corn can be a
feedstock or a food stock, or crude oil can be transformed into gasoline and other petroleum products. They have economic value, but do not yield an ongoing stream of revenue. (Anson 2009, 309.)

Another distinguishing difference between commodities and other asset classes is the fact that, commodity futures prices are determined globally rather than regionally, and they are accounted for in US dollars. This is different from equities and bonds where local markets reflect economic developments within that region, and the local currency is used. (Anson 2009, 309.)

Already somewhat old research show, that commodity markets do not fit the Capital Asset Pricing Model (CAPM). The reason is twofold. First, it is difficult to make a distinction between systematic risk/return and unsystematic risk/return. And second, the price is dependent on supply and demand factors, not perceived adequate risk premiums. (Bodie & Rosansky 1980.)

### 3.1.4 Exposure to commodities

Even I talk about commodity futures mainly in this paper, it is important to note that there are multiple ways to gain exposure to commodities.

Firstly you can own the physical product. There are a few issues associated with the physical ownership of a commodity. It is not often considered as such, but in its simplest form, you drive to the petrol station and buy gas. Then, when the price rises, you sell it to your neighbor. The problem is that the price would have to rise very significantly to cover the trouble of doing this, with such small quantities. The next problem with physical ownership is that the smallest crude oil contract in the New York Mercantile Exchange is in the 1,000 barrel lots. One would have to find a storage tank to store the crude. Now the amount is very large for an individual, but very small in the space of the total oil markets, therefore storing the 1,000 barrels of crude would be very costly for you compared to a huge existing operator. Physical ownership thus exists, for example in India it is very common for people to have large parts of their wealth in physical gold, silver and platinum. (Anson 2009, 280.)
Second way is to own securities or bonds of a company that plays in the commodity space. For example the stock of the Finnish mining company Talvivaara follows closely the price of nickel, as they do not hedge for the nickel price exposure (Talvivaaran vuosikertomus 2011). But it is important to notice that many companies, especially the oil companies have hedged away its oil exposure. For example the Crude Oil beta for ExxonMobile is -0.04, and the stock market beta for the same stock is 0.67. (Schneeweis, Kazemi & Spurgin 2008.)

A third way is commodity mutual funds and exchange traded funds. Commodity mutual funds can usually include commodity indices, equities of commodities-based companies, or actual commodity futures (Schneeweis, Spurgin & Waksman 2006.) Commodity mutual funds usually have similar fees and costs structures as usual mutual funds (Burkart, 2006). With commodity ETF’s it is very important to note the large tracking error of some of the ETF’s. One of the most popular commodity ETF’s is for gold, ticker GLD, follows the spot price of gold very closely and is used by many of the industry professionals, like John Paulson and George Soros, to gain exposure to gold. On the other hand there are for example oil ETF’s, like the one behind the ticker USO, which has lagged massively the movements in oil price after the 2008 crash. This is mainly due to the rolling of the portfolio in times of negative roll returns. Roll returns are discussed more in debt in other chapters of this paper.

Fourth way to gain exposure to commodities is through commodity swaps and forward contracts. They perform the same function as commodity futures but have a few structural differences. Most importantly they are usually over the counter products, i.e. custom made for the individual investor. This leads to them being less liquid than futures contracts, that are standardized. The customization also leads to the fact that they are not traded on an exchange. This can also be sometimes considered a benefit, since a degree of privacy is possible which is not available in the public market place. (Anson 2009, 283.)

The fifth way is to invest in commodity-linked notes. In its simplest form, according to Anson (2009: 284), a commodity-linked note is “an intermediate term debt instrument
whose value at maturity will be a function of the value of an underlying commodity futures contract or basket of commodity futures contracts.”

There are also private commodity partnerships. They work similar to real estate investment trusts (REITs), but instead of owning buildings they can own for example pipelines, rail cars or extraction rights to oil fields. The structure of these partnerships is a pass-through entity, in which the partners earn a rental income on the underlying asset on which they do not need to pay a corporate level tax. The popularity of these partnerships has grown rapidly since the 2000. (Spurgin & Donohue 2009, 168.)

Lastly there are the commodity futures contracts. This will not be covered here, as it is discussed in debt in the others chapters of this thesis.

### 3.2 Relationship between futures prices and spot prices

Commodity futures contracts are transparent, are denominated in standard units, are exchange traded, are liquid, and depend on the spot price of the underlying commodity. In order to fully understand the dynamics of commodity futures markets, the last point, relationship between futures prices and spot prices, needs to be understood well. (Anson 2009, 288.)

Commodity futures contracts are standard deals that obligate the seller of the futures contract to deliver the underlying commodity at a pre-defined price and time. In case the buyer of the commodity futures contract does not wish to have the commodity delivered, he must close his long position by selling an offsetting contract. It is good to note that usually less than 1% of futures contracts result in a delivery of the underlying asset. (Anson 2009, 288.)

Gourton and Rouwenhorst (2005) have a good point to keep in mind when doing anything commodity futures related: “commodity futures do not represent direct exposures to actual commodities. Futures prices represent bets on the expected future spot price.”
The simplest case between the futures contract and the spot price is, when the asset pays no income.

\[ F = S e^{r(T - t)} \]  

(1)

Where:

- \( F \) = the price of the futures contract
- \( S \) = the spot price of the underlying asset
- \( e \) = the exponential operator, used to calculate continuous compounding
- \( r \) = the risk-free rate
- \( T - t \) = the time until maturity of the futures contract

As the formula shows, the price of the futures contract is dependent on the maturity, risk-free rate and the current spot price of the underlying asset. (Anson 2009, 288.)

Commodity futures have a few distinctions that will affect the pricing relationship from the simplest case described above. Physical commodities have a storage cost, and something called the convenience yield, both that needs to be factored in. (Anson 2009, 288.)

One way to think of storage costs is to see them as negative income. A cash outflow is produced when holding a physical commodity. On the contrary, many other financial assets, like equities and bonds, have positive incomes from dividends and coupon payments. So now, an extra cost relating to the storage of the commodity is added to the equation. It comes next to the cost of financing (risk-free rate) the purchase of the physical commodity. The new formula can be expressed as

\[ F = S e^{(r + c)(T - t)} \]  

(2)

where the new term \( c \) is the storage cost associated with the ownership of the commodity. Rest of the terms, are as stated above. (Anson 2009, 288.)

Even the term storage cost refers more to things like the rental of storage facilities, insurance, inspections, transportation and maintenance costs; it also includes the spoilage cost and financing costs. Financing cost relates to the financing of the storage,
and it is assumed that the storage is fully financed. This cost is usually the cost of capital the firm applies to its working capital. The spoilage cost is related to the value loss that naturally happens in storage, like spillage. (Spurgin & Donohue, 2009.)

The second difference between a financial futures contract and a commodities futures contract is the convenience yield. It is considered as the benefit from holding the actual commodity instead of the futures contract. For different commodities it can mean a slightly different thing. For example in the case of oil, it could be the ability to profit from local supply or demand imbalances. Or like gold, which can be leased to jewelry manufacturers for a payment at a later date. Now the formula becomes

\[ F = S e^{(r + c - y)(T - t)} \]  

(3)

where \( y \), the convenience yield, is subtracted from the risk-free rate \( r \), and the storage costs, \( c \). It reduces the cost of ownership. (Anson 2009, 295.)

If the price of gold was $400 an ounce, the risk-free rate 6%, the cost of storage 2% of the purchasing price, and the lease rate was 1%, then the price of a 6 six-month futures contract can be calculated as following. (Anson 2009, 295.)

\[ F = $400e^{(0.06+0.02-0.1)(0.5)} = $429 \]
Figure 3 Futures versus spot returns (Gorton and Rouwenhorst 2007)

3.2.1 Economics of the commodity markets: backwardation versus contango

Commodity futures contracts have a similar term structure as interest rates. It is either upward sloping also known as contango (futures price is above the current spot price), or downward sloping also known as backwardation (futures price is below the current spot price). Speculators, hedgers, seasonal changes in demand and disruptions in supply all have a say in the shape of the curve. (Anson 2009, 296.)

The reason why backwardation is sometimes referred to as normal backwardation is that speculators are compensated for bearing the commodity price risk. Meaning that for the speculator to take a long position in the futures contract, he wants a price that is sufficiently discounted compared to the expected future spot price. On the other side of the contract is usually a petroleum producer (a hedger) who is inherently long crude oil exposure through oil exploration, developing, refining and marketing operations. (Anson 2009, 297.)
On the other hand, contango situations usually occur in commodities in which the hedger is inherently short to the exposure of a price change in the underlying commodity. An example of such could be an aircraft manufacturer that does not have its own aluminum mines. Therefore it is willing to purchase the futures contract of a future aluminum delivery, at an expected loss, in return for eliminating the price risk of the price rising. Thus it can be concluded that, profits for the speculator is determined by the amount the hedgers have interest for risk capital, not the long-term price trends of the commodity markets. (Anson 2009, 299.)

The theoretical background to backwardation first came from the rational expectations hypothesis which was introduced by Hicks (1939) to the forward commodity prices. The hypothesis states that “the price of an asset for delivery in the future must be the same as the market’s current forecast of the spot price of the asset on the future delivery date”, according to Hicks (1939). It is good to note that these rational expectation theories have not proven to be useful at explaining commodity forward curves in practice. The assumptions of perfect markets and risk neutrality are too strong according to Spurgin and Donohue (2009).
But instead different storage models have been the best explainers in practice. The storage models state that the relationship of the spot and forward price of a commodity depends largely on the relationship between storage levels and expected storage levels in the future. In a way it tells about the supply and demand levels at every specific time into the future. A commodity like natural gas which is difficult to store usually has a steep forward curve. As a rule of thumb, when the inventories are high relative to demand, the curve will be upward-sloping and in turn, when the inventories are tight, downward-sloping. (Till & Feldman, 2006.)

On a side note relating to this, Erb and Harvey (2006), find an interesting fact when studying the returns of different commodity future indexes. More specifically, the Goldman Sachs Commodity Index (GSCI). They find that the four commodities that Till and Feldman (2006) find difficult to store (heating oil, copper, live cattle and live hogs), have average excess return of 3,5 percent. Compared to the average excess return of those commodities that are not difficult to store, -4,3 percent.

![Figure 5 Term structure of commodity prices: crude oil and gold, 30 May 2004](image)
3.2.2 Commodity futures return composition

The returns of a commodity futures contract comes from three sources: a spot return, risk-free rate and the roll return.

According to Burkart (2006), the spot return, meaning the increase of spot price over time has historically remained under inflation. In the short term commodity markets are usually favorable for sudden spot price rises (Anson, 1998), but because of their mean-reverting tendency over longer periods, spot prices are not a source of positive returns (Till & Feldman, 2006).

![Figure 6 Stocks and commodity futures empirical distributions of monthly returns 1959/7 - 2004/12 (Gorton and Rouwenhorst 2007)](image)

Above the Gorton & Rouwenhorst’s picture shows well the commodities short term upside shock potential as a plateau near the 7 – 10 percent mark.

The second return factor comes from the cash collateral, which is posted when investing in commodity futures, usually the Treasury Bill rate in the U.S. This is positive even for longer periods. (Ilmanen 2009, 292.)
Roll return is the most important source of returns for commodity futures. When the markets are in backwardation simply buying the futures contract will earn a positive return. The return is earned because as time passes the future contract gets closer to maturity, and thus must convert closer to the spot price, since the futures price is the new spot price. The roll return is discussed more in depth in the other parts of this thesis; the importance is also discussed in multiple other papers like those of Erb & Harvey 2006 and Fuertes & Miffre 2010.

![Image](image_url)

**Figure 7 Annualised return vs average backwardation (Nash and Shrayer 2004)**

### 3.3 Commodity Trading Advisors, the CTAs

CTA is defined by the National Futures Association (NFA) website in 2008 as “an individual or organization which, for compensation or profit, advises others as to the value of or the advisability of buying or selling futures contracts or commodity options.” Schwager (2012, 129.) considers this official nomenclature as poorly chosen, since firstly Commodity Futures Advisors are managers, not advisors and secondly, the majority of trading they do is in the financial markets, meaning the stock indexes,
interest rates, and currencies and not commodities. As of the end of 2008 there were 902 CTA entities registered in the U.S.

Usually, the trading style of these so called managed futures accounts’, are systematic trading strategies used in the futures markets, and in the case of foreign exchange cash and forwards market. According to Fund and Hsiesh (1997) the most popular managed futures trading style is trend following. Trend following uses a systematic approach and takes a directional position once a trend is observed. This is in contrast with market timing strategies where statistical techniques are used to predict these trends before they become apparent. (Fund & Hsiesh, 1997.)

![Cumulative Returns Graph](image1)

**Figure 8 Returns of Managed Futures (hedgeindex.com)**

### 3.3.1 Managed Futures Strategies

To create a managed futures strategy, one needs either technical and/or fundamental information. After which, it is possibly to apply either a discretionary or a systematic trading strategy based on analysis of that information. Even though both strategies are discussed below shortly, the managed futures industry is dominated by using technical information systematically. (Kazemi & Ying, 2009.)

As an interesting side note, the world’s largest hedge fund Bridgewater uses a rare combination of fundamental based systematic approach. What this means in practice, for example in 2008 they spotted the possibility for either an inflationary or a deflationary
deleveraging. This they did through indicators like contractions in private credit growth, a declining stock market, and a widening credit spreads. Then they just adjusted their positions to do well in a deleveraging environment based on analysis of the 1920s German and 1980s Latin American inflationary deleveraging and the deflationary deleveraging’s of the Great Depression in the 1930s and Japan in the 1990s. (Schwager 2012, 65.)

A systematic trading strategy has strict rules in which the trader is left with little to decide on individual trades. What he can though do, is to develop the original model and how much risk or leverage to use. In systematic trading a quantitative method is used to analyze price and/or broader economic factors, and as earlier stated, the main focus on technical analysis. (Kazemi & Ying, 2009.)

On the other hand, with the discretionary strategy, the trader uses his own judgment on which trades to make. In discretionary trading, coming up with the trading idea is based on fundamental and/or technical analysis. (Kazemi & Ying, 2009.)

The idea of analyzing fundamental information is to come up with the true value of the underlying asset. In the case of managed futures the information is usually related to things like economic statistics, inflation, unemployment, and individual commodity supply and demand data. External shocks are also taken into account, like changes in monetary policy or terrorist attacks. (Kazemi & Ying, 2009.)

With technical analysis the focus is on price movements. A quantitative approach is taken to analyze historical prices to look for exploitable price patterns. One does not need to worry what causes the pattern, but can focus on identifying the pattern itself. Because of this focused approach multiple markets can be analyzed and traded simultaneously. (Kazemi & Ying, 2009.)

3.3.2 Systematic trading styles
The most popular categories for systematic trading styles are trend following, non-trend following and relative value. I will only discuss relative value strategies here since the strategy presented in this thesis is closes to it. (Fund & Hsiesh, 1997.)
The idea of relative value strategy is to profit on price divergences between two correlated futures contracts. These contracts can involve different products, like for example oil and gasoline, or the same underlying with different expirations at a market neutral ratio. The assumption behind the trade is that the prices of individual futures contracts can be inefficient as they correspond to new information slightly differently and/or there is a liquidity imbalance. The idea is to analyze the correlation structure between the futures contracts and to exploit price deviations from the “equilibrium.” (Fund & Hsiesh, 1997.)

3.4 Commodity futures in the portfolio context
Portfolio theory establishes that an investor can improve portfolio performance by investing in multiple asset classes. Usually this leads to reduced portfolio risk without reducing expected returns due to imperfect correlation between the assets classes. The traditional approach has been to create a portfolio consisting of stocks, bonds and cash equivalents to demonstrate the advantages of adding asset classes to portfolios. (Jensen, Johnson & Mercer, 2000.) So therefore it is important to look at the correlation and return characteristics between commodities and other asset classes.

Commodity futures peak at a different part of an economic cycle than stocks and bonds. This counter-cyclical nature of commodities is a good characteristic for potential portfolio diversification. For this reason it is better to analyze commodity futures in a portfolio context, this way their diversification potential can be achieved. The below table shows that the four different commodity indexes (GSCI, DJ-AIGCI, MLMI and CPCI) are all negatively correlated with the U.S. equity markets (S&P500), global equity markets excluding U.S. and Canada (EAFE) and U.S. Treasury bonds. (Anson 2009, 228.)
### 3.4.1 The efficient investment frontier

The efficient frontier is part of modern portfolio theory, Markowitz one of the main inventors of it, won the Nobel Prize for his work on it. A portfolio is considered to be on the efficient frontier when it has the best possible expected level of return for its level of risk. In other words it can be considered a graphical illustration of the tradeoff between risk and return. Usually the standard deviation of the different assets returns in the portfolio is used as the proxy for the risk. (Elton & Gruber, 2011.)

In the figure below the Eff. Frontier is what can be considered the classical presentation of the efficient frontier. Only stocks and bonds are used, it is calculated from the returns of the S&P500 and U.S. Treasury bonds from the period of 1973-2000. At the highest return point, the portfolio consists of only stock. And in the lowest return point, the portfolio consists of only U.S. Treasury bonds. In the space between, the portfolio changes from 90% of stocks and 10% of bonds to 10% of stocks and 90% of bonds. (Anson 2009, 229.)

---

### Table 1 Correlation of commodities, bonds and stocks 1973-2000 (Anson 2009)

<table>
<thead>
<tr>
<th></th>
<th>GSCI</th>
<th>DJ-AIGCI</th>
<th>CPCI</th>
<th>MLMI</th>
<th>S&amp;P 500</th>
<th>US Treas</th>
<th>EAFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSCI</td>
<td>1.0000</td>
<td>0.9486</td>
<td>0.9190</td>
<td>0.3699</td>
<td>-0.3777</td>
<td>-0.2480</td>
<td>-0.2235</td>
</tr>
<tr>
<td>DJ-AIGCI</td>
<td>0.9486</td>
<td>1.0000</td>
<td>0.9589</td>
<td>-0.0439</td>
<td>-0.4343</td>
<td>-0.3424</td>
<td>-0.2797</td>
</tr>
<tr>
<td>CPCI</td>
<td>0.9190</td>
<td>0.9589</td>
<td>1.0000</td>
<td>0.3646</td>
<td>-0.2543</td>
<td>-0.2936</td>
<td>-0.1484</td>
</tr>
<tr>
<td>MLMI</td>
<td>0.3699</td>
<td>-0.0439</td>
<td>0.3646</td>
<td>1.0000</td>
<td>-0.5343</td>
<td>-0.0092</td>
<td>-0.3973</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-0.3777</td>
<td>-0.4343</td>
<td>-0.2543</td>
<td>-0.5343</td>
<td>1.0000</td>
<td>0.3869</td>
<td>0.4513</td>
</tr>
<tr>
<td>US Treas</td>
<td>-0.2480</td>
<td>-0.3424</td>
<td>-0.2936</td>
<td>-0.0092</td>
<td>0.3869</td>
<td>1.0000</td>
<td>0.0803</td>
</tr>
<tr>
<td>EAFE</td>
<td>-0.2235</td>
<td>-0.2797</td>
<td>-0.1484</td>
<td>-0.3973</td>
<td>0.4513</td>
<td>0.0803</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
In the same figure, the other line “with GSCI” is the GSCI (Goldman Sachs Commodity Index) added to the “classical” efficient frontier. The two endpoints are the same as in the Eff. Frontier, 100% stocks at the highest return level and 100% bonds at the lowest return level. But in the space between, there is a constant allocation of 10% to commodity futures (GSCI), with a 5% reduction in both bonds and stocks. So the data points become 85% stocks, 5% bonds, and 10% commodity futures, 75% stocks, 15% bonds, and 10% commodity futures etc. (Anson 2009, 230.)

The efficient frontier that includes commodity futures is clearly above and to the left of the frontier with just stocks and bonds. This area has a more efficient risk and return characteristics, and thus one can conclude that commodity futures as part of a portfolio improves the risk and return tradeoff. (Anson 2009, 230.)

Baltas and Kosowksi (2012) also find that commodity futures strategies have low correlation with other futures strategies. Commodity futures strategies also tend to take advantage of both up and down markets. This leads them to be good in portfolio
diversification even though the relative returns tend to be on the low side. (Baltas & Kosowski, 2012.)

3.4.2 A hedge against inflation

Overall inflation rate is dependent on prices of physical commodities. This leads to indexes of commodity futures to being good inflation proxies. Just this reason leads many to argue that commodities should be a part of a plain stock and bond portfolio because of their ability to hedge against inflation (supporting evidence Bodie 1983, Edwards & Park 1996, Irwin & Landa 1987.) In inflationary periods, usually long commodity future positions benefit and stock returns are negatively impacted. (Jensen, Johnson & Mercer, 2000.)

Bonds are a senior claim on revenue earned by a corporation, and stocks are a claim on the future earning potential of the corporation. Bonds perform poorly in inflationary scenarios because the purchasing power of money declines and stock decline in value because of the earning power of the corporation is eroded due to inflationary concerns. Real assets, like commodity futures, can hedge this decline in value due to inflation. Below table represents the correlation of different asset classes to inflation, GSCI, DJ-AIGCI, CPCI and MLMI are different commodity indexes. (Anson 226, 2009.)

![Figure 10 Correlation with Inflation 1990-2000 (Anson 2009)]
3.5 Pairs trading

Pairs trading is a form of statistical arbitrage. The idea is to create a trade in the nature of a long-short position, in a way that the position itself has a negligible beta and thus becomes almost uncorrelated to market returns and therefore is often considered as a market neutral strategy. Quant Nunzio Tartaglia in the mid-1980s was the first to practice statistical pairs trading while working at Morgan Stanley. It involved trading securities in pairs. First you identify a pair of securities whose prices moves together, and once an anomaly in this relationship is noticed, the pair would be traded in such a way that correction of the anomaly would be profitable. This lead to the name “pairs trading.” Since then pairs trading has become a common strategy used by institutional investors and hedge funds. (Vidyamurthy, 2004: 73.)

The requirement for two assets to be pairs tradable in the first place, is that the two assets (or more) are cointegrated, meaning that the asset prices are tied together in the long term by some common stochastic trend. When an anomaly in a relationship of the assets prices is noticed, it can be considered to correct sometime in to the future. For testing cointegration, econometrics has a number of tests. In this thesis, because the futures curve has multiple observations for every point of time, I will mainly discuss the Johansen test for its ability to test multiple time series at the same time.

One key theme in pairs trading is the need not to know and care about the absolute value of the assets in real terms. This is an advantage since the valuation process is usually very difficult. Instead, with pairs trading, we only focus on the relative movements in price between the cointegrated assets. This way the specific price of each asset becomes an unimportant factor. With stocks, it is more common that just one of the assets is over / underpriced. (Gatev, Goetzmann & Rouwenhorst 2006.)

The same applies to the futures curve, but with the difference, that even the underpriced contracts when in contango, usually have a negative expected return. Thus when pairs trading the futures curve, because of the roll effect, the other trade of the pairs trade (the long trade when in contango and the short trade when in backwardation), has a slightly
negative expected return (proof of this in the results section). Therefore the main reason
to pairs trade the future curve is to hedge price movement risk and only capture the part
of the commodity futures return I want, the roll return. By capturing only this small part,
a very small volatility of returns is gained.

The pairs trade could be made much more efficient by dynamically adjusting the trading
patterns according to the spread or deviation between the assets. In my study I simply
trade once a month on a predetermined date. Dynamically adjusting could ultimately
lead to both of the short and long positions to have a positive expected return, making
the strategy more profitable.

3.5.1 Cointegration
Econometricians Engle and Granger found that even if two time series are non-
stationary, there still could be a linear combination of the two that is stationary;
meaning, that the two series move together over time. Engle and Granger termed this
phenomena cointegration, and also received the economic Nobel Prize in 2003 for it.
Formally this means that if $y_t$ and $x_t$ are two non-stationary time series, and for a
certain value of $\gamma$, the series $y_t - \gamma x_t$ is stationary, then the two time series are
cointegrated. (Vidyamurthy, 2004: 76.)

For the two time series to move together there needs to be something called the error
correction. Its ‘job’ is to correct the error in price deviation. The error correction can be
represented as (when $\varepsilon_{xt}$ is the white noise process corresponding to the time series $\{x_t\}$
and $\varepsilon_{yt}$ for the time series $\{y_t\}$):

\[
y_t - y_{t-1} = \alpha_y (y_{t-1} - \gamma x_{t-1}) + \varepsilon_{yt}
\]
\[
x_t - x_{t-1} = \alpha_x (y_{t-1} - \gamma x_{t-1}) + \varepsilon_{xt}
\]

(4)

So for example in the first equation the term $y_{t-1} - \gamma x_{t-1}$ represents the deviation from
the long-run equilibrium. $Y$ is the coefficient of cointegration, and $\alpha_y$ indicates the
speed in which the time series corrects itself closer to the long-run equilibrium. In
simple terms, the white noise process creates the deviation from the long-run
equilibrium, which is then corrected in future time steps by the error corrector
\[ \alpha y(y_{t-1} - y\alpha_{t-1}). \] (Engle & Granger, 1987.)

3.5.2 Dickey-Fuller

Usually the order of integration is first determined with a unit root test before running an actual cointegration tests. There are two main motives behind an unit root testing. Firstly knowing the order of integration is necessary for setting up the actual model, and making inference. Secondly, with random walk or a martingale processes, the variables should be integrated. So in a sense, unit root tests are a descriptive tool to describe the series of being stationary or non-stationary. With tests of integration, common sense with a combination of graphics is crucial, no matter how complex or famous your test is. (Sjö, 2008.)

Here the Dickey-Fuller test is considered because of its uniformly good characteristics. Though usually it is good to compare results with other types of unit root tests, giving one a feel for the sensitivity of the results. More formally, if you assume that \( x_t \) is a random walk process, \( x_t = x_{t-1} + \epsilon_t \), then the regression model is \( x_{t-1} = \rho x_{t-1} + \epsilon_t \). When I subtract \( x_{t-1} \) from either side we get the Dickey-Fuller model:

\[ \Delta x_t = \pi x_{t-1} + \epsilon_t \] (5)

\( \Delta x_t \) is the variable of interest (\( \Delta \) is the difference operator), \( t \) is the time index, \( \pi \) is the coefficient, and \( \epsilon_t \) is the error term. Here the test is done over the residual term, so a specific distribution known as the Dickey-Fuller table needs to be used for the critical values. (Sjö, 2008.)

It is possibly to setup the Dickey-Fuller test in three different ways. Still the null hypothesis is always that \( x_t \) is a random walk without a drift. The alternative hypothesis then are 1) we just test for a unit root so that \( x_t \) is stationary (above), 2) \( x_t \) is driven by a deterministic trend (t)

\[ \Delta x_t = \alpha + \pi x_{t-1} + \epsilon_t \] (6)
or 3) \( x_t \) is driven by a deterministic quadratic trend \( (t^2) \)

\[
\Delta x_t = \alpha + \beta t + \pi x_{t-1} + \epsilon_t
\]  

(7)

The problem often is that the empirical distribution is assumed to be a random walk with a white noise residual, but in reality it often has autoregressive structure in it (or even worse, a moving average process). Luckily this can be dealt by augmentation of the regression equation with lagged \( \Delta x_t \) variables. \( \epsilon_t \) becomes white noise and the different Dickey-Fuller distributions stay valid. Formal representation of the Augmented Dickey-Fuller in the case of autocorrelation:

\[
\Delta x_t = \pi x_{t-1} + \sum_{l=1}^{k} \gamma x_{t-1} + \epsilon_t
\]  

(8)

It is also possible to take seasonal deterministic factors found in the \( x_t \) into consideration, or impulse dummies that take out extreme values. But they are of no interest in my case and therefore will not be discussed farther. (Sjö, 2008.)

### 3.5.3 Johansen

According to Sjö (2008) the Johansen test is superior test for cointegration. The only weakness in his mind is that it relies on asymptotic properties, which leads to possible specification errors in limited samples. Also for the fact that, it is the only cointegration test that allows for multiple variables, it is the one used in this paper. Here I will discuss the VAR-based cointegration tests developed by Johansen (1991 & 1995).

Consider a VAR of order \( p \):

\[
\gamma_t = A_1 \gamma_{t-1} + \cdots + A_p \gamma_{t-p} + B x_t + \epsilon_t
\]  

(9)

Here the \( \gamma_t \) is a \( k \)-vector of non-stationary \( l(1) \) variables, \( x_t \) is a \( d \)-vector of deterministic variables, and \( \epsilon_t \) is a vector of innovations. It can be rewritten as:
\[ \Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + Bx_t + \varepsilon_t \]

(10)

Where:

\[ \Pi = \sum_{i=1}^{p} A_i - I, \] \quad \[ \Gamma_i = - \sum_{j=i+1}^{p} A_j \]

The logic of Johansen’s method is to first estimate the \( \Pi \) matrix from an unrestricted VAR and then test, if rejecting the restrictions implied by the reduced rank of \( \Pi \) is possible. It is based on the Granger’s representation theorem which states that if the coefficient matrix \( \Pi \) has a reduced rank \( r < k \), then there exists \( k \times r \) matrices \( \alpha \) and \( \beta \) each with rank \( r \). This, the cointegration rank, determines the number of cointegrated relations. (Juselius, 2009: 95.)

Depending on the characteristics of the data to be tested, up to five different versions of the Johansen test can be deployed. The formal presentations of the five different trend assumptions can be found below, in the cases, \( y_t \) describes the data. (Juselius, 2009:100.)

1. No deterministic trends in the data and the cointegrating equations do not have intercepts:

\[ H_2(r): \Pi y_{t-1} + Bx_t = \alpha \beta' y_{t-1} \]

(11)

2. No deterministic trends in the data and the cointegrating equations have intercepts:

\[ H_1^*(r): \Pi y_{t-1} + Bx_t = \alpha (\beta' y_{t-1} + \rho_0) \]

(12)

3. Linear trends in the data but the cointegrating equations have only intercepts:
4. Linear trends in the data and the cointegrating equations also have linear trends.

\[ H_1(r): \Pi y_{t-1} + Bx_t = \alpha (\beta' y_{t-1} + \rho_0) + \alpha \gamma_0 \]  (13)

5. Quadratic trends in the data and the cointegrating equations have linear trends:

\[ H^*(r): \Pi y_{t-1} + Bx_t = \alpha (\beta' y_{t-1} + \rho_0 + \rho_1 t) + \alpha \gamma_0 \]  (14)

\[ H(r): \Pi y_{t-1} + Bx_t = \alpha (\beta' y_{t-1} + \rho_0 + \rho_1 t) + \alpha (\gamma_0 + \gamma_1 t) \]  (15)

Exogenous variables, like usually used seasonal dummy variables will not be discussed here as they are not used in the thesis. Lag intervals and critical values will be discussed in the empirical part of the thesis along with the actual calculations.

### 3.6 Performance measures: Sharpe and Calmar ratios

The two main measurements of performance used are the Sharpe Ratio and the Calmar Ratio. The Sharpe Ratio was first introduced by William Sharpe in a paper published in the 1966, for a measure of performance for mutual funds. Sharpe proposed the term *reward-to-variability ratio* to describe it, but other authors have preferred using other names like the Sharpe Measure (Bodie, Kane and Marcus 1993), Sharpe Index (Radcliff 1990) and most commonly the Sharpe Ratio (Morningstar 1993). (Sharpe, 1994.)

Today Sharpe recommends using the Information Ratio over the Sharpe Ratio (R Documentation). I have used the Sharpe Ratio as my main measure because of its mainstream status as a performance measure of funds and portfolios, but I have also included the Information Ratio in the results.

The Information Ratio measures the risk-adjusted return of a financial security (or portfolio). It can be defined as expected active return divided by tracking error. Active return means the difference between the return of the security and the return of a selected benchmark index. Tracking error in turn is the standard deviation of this active return. (Sharpe, 1994.)
For the Sharpe Ratio, the return of the portfolio is $R_{Pt}$ in period $t$, $R_{Rt}$ is the risk free rate, which in the case of commodity futures is zero, as the risk free rate has already been accounted for in the price of the futures contract.

\[
D_t \equiv R_{Pt} - R_{Rt}
\]  

(16)

It is good to note that the return of the portfolio is the logarithmic return. And thus is calculated (Hull 2008: 282):

\[
R_{Pt} = \ln \left( \frac{P_i}{P_{i-1}} \right) \text{ for } i = 1,2,\ldots,n
\]  

(17)

Where

\[
n+1 = \text{Number of observations}
\]

\[
P_i = \text{Price of the portfolio at the end of } I \text{ th interval, with } i = 0,1,\ldots,n
\]

$D$-bar is the average value of $D_t$ over the period of $t=1$ through $T$:

\[
\bar{D} \equiv \frac{1}{T} \sum_{t=1}^{T} D_t
\]  

(18)

And then $\sigma_D$ is the standard deviation over the period:

\[
\sigma_D \equiv \sqrt{\frac{\sum_{t=1}^{T} (D_t - \bar{D})^2}{T - 1}}
\]  

(19)

And thus we get the historical Sharpe Ratio ($S_h$):

\[
S_h \equiv \frac{\bar{D}}{\sigma_D}
\]
The second performance measure used is the Calmar Ratio. It was first introduced by Terry W. Young in the trade journal *Futures*, in 1991. It has since become a popular measure to determine return relative to drawdown risk with hedge funds (Eling & Schuhmacher, 2006).

\[
Calmar(T) = \frac{\text{Return over } (0,T)}{\text{MDD over } (0,T)}
\]

(21)

MDD is the absolute value of maximum drawdown in percentages. (Young, 1991.)

### 3.7 Practical example: Petroleum markets

#### 3.7.1 A basic hedge

A crude-oil trader is going to buy a cargo of crude oil at Ras Tanura, Saudi Arabia, with the idea of selling it on the spot market in Rotterdam. For him to forego with this trade he obviously needs to believe that he can sell the cargo at a much higher price, to cover the costs of transportation and lock in some profits. For the ship to arrive to Rotterdam from Ras Tanura it takes 35 to 45 days, much can happen in that period. The problem for this trader is time, it is a risk. The risk can be eliminated if he can sell his crude at the price he expects to receive in Rotterdam as soon as he takes delivery in Ras Tanura. This is what a forward and a futures contract allows him to do. (Tippee, 1993, 217.)

In practice this means that the trader needs to find a speculator who believes that the spot price of crude is higher, than what it will cost him to purchase the crude and ship it to Rotterdam. If he happens to find a speculator willing to take the risk, they can work out a forward deal covering the trader’s cargo. (Tippee, 1993, 218.)

The day when the oil is loaded at Ras Tanura, the trader pays $100/bbl in cash to Saudi Aramco, and at the same time sells the same cargo at $110/bbl to the speculator for delivery in Rotterdam on the day when the tanker is expected to arrive. The trader has locked in a sure profit of $8/bbl (it costs $2/bbl to charter a tanker for the voyage), but at the same time he has left the potential additional profits to the speculator, if the spot
price of crude happens to rises above the $110/bbl in Rotterdam 35 to 45 days forward. (Tippee, 1993, 218.)

In other words the trader wants to reduce price risks associated with ownership of a physical commodity and the speculator takes a bet on the price of crude in hopes of profits without the interest in physical ownership. (Tippee, 1993, 218.)

3.7.2 Getting a perspective

In 1992 more than 7,000 ships were active carrying crude oil, products, chemicals, liquid petroleum gas, and liquefied natural gas in the world (The Shell Review, 1992). Of these roughly 3,000 are larger tankers designed to carry crude oil and refined products between nations. Average steaming speed for these ships is about 15 knots. Largest of these are called ultra large crude carriers (ULCCs) which can have a deadweight tonnage of 500,000 (the measurement is the difference in weight of water displaced by a vessel when fully loaded and the water displaced when it is empty.) leading to a massive capacity of 3.5 million barrels of crude. One oil barrel consists of around 35 imperial gallons leading to 159 liters. A ship this size is around 400 meters long. (Tippee, 1993: 136.)

To get this into perspective, an A380 flying from London to Kuala Lumpur consumes 1,181 barrels of fuel for its 494-passangers (Bloomberg BusinessWeek, June 2012), leading to 2,39 barrels per passenger. The price for jet fuel was 131.8$/barrel on the 21st of August (www.iata.org), meaning that for an A380 to fly from London to Kuala Lumpur the price of fuel for each passenger is 315$.

In other words, one ULCC can supply an A380 for roughly 1482 round trips from London to Kuala Lumpur (3,500,000 / (1,181*2) = 1481,79). Or about 6,5 million round trips from Helsinki to Äkäslompolo with a V50 e-drive. (959km*2=1918km, 4,5l / 100km leads to 86,31 liters for the round trip. ULCC carries 3,500,000 barrels * 159 liters = 556,500,000 liters. This leads to 556,500,000 liters / 86,31 liters = 6,447,688 liters).

Figure 9 shows how oil is consumed around the world and figure 10 highlights the fact that the usage of oil has steadily risen and is expected to do so also into the future. For
the price of oil to remain steady, as the current supply and demand theories state, the supply needs to continue to grow steadily. Tippee (1993) argues that even there are periods (like the boom leading to the 2008 crisis), when the supply cannot fully meet with the demand, in longer periods the supply always catches the demand. This is both because of more drilling becomes economically viable due to an increased price (larger supply), and less usage of oil from some market participants because of increased price (lower demand).
4 EMPIRICAL WORK

4.1 Data

The dataset is from Bloomberg and spans the period April, 1 1991 to May, 1 2012. In consist of daily closing prices of the twelve nearest contracts of 20 commodities: Gold (GC), Natural Gas (NG), Silver (SI), Crude Oil Brent (CO), Gasoil (QS), Heating Oil (HO), Gasoline RBOB (XB), Wheat (W), Winter Wheat (KW), Soybean (S), Sugar (SB), Coffee (KC), Corn (C), Cotton (CT), Crude Oil Light Sweet (CL), Cocoa (CC), Lean Hogs (LH), Live Cattle (LC), Feeder Cattle (FC), Crude Oil WTI, Copper (HG). Also for each contract and date, the open interest is included. For calculating different performance measures, in which a benchmark was needed, the values of the S&P GSCI excess return index were used. This data was also downloaded from Bloomberg.

Two historical commodity futures price series are used. The other one is the ‘real’ prices, without any adjustments. This is where the trading signal is gotten from. The other series is a ratio adjusted price series based on which the returns are calculated from. Bloomberg defines the two series in the following way a) not adjusted prices “are actual futures prices without any adjustment. The disadvantage is that the generic may jump because of the spread when it rolls from one month to the next. In this case, the changes are not always from moves in the market. The advantage is that all the prices are from actual futures trades and are not altered in any way. This method gives a reasonable view of price levels at any point in time” and b) ratio adjusted prices “are calculated as front month price / expiration month price. (Front month and expired month prices are derived from settlement price from the roll date of both contracts.) Then the ratio is multiplied by the entire historical pricing, so each month the ticker rolls, the entire historical pricing adjusts. Additionally, a new ratio is multiplied every time a new contract rolls (since it needs to readjust the entire time series for new data).” So for example if there are 30 days between the rolls, on the first day the generic is entirely weighted towards the front month. On the 10th day, the generic is 2/3 front month and 1/3 second month; on the 20th day, the generic is 2/3 second month and 1/3 front month. After each roll date, the averages are reset. Looking back historically on
averages, the date range should never matter; once the ticker rolls, the percentages remain the same (and never get readjusted).

E-views, Excel and R were used to produce the material presented in this thesis.

4.1.1 Transaction costs and liquidity
One could argue that when back testing a strategy like this; it is possible to over fit the model so that the returns and Sharpe become what one wants and that they are not reproducible into the future. To account for this I have taken two purely technical steps to improve the possibility of the model to also work in real trading.

First is to include a conservative estimate of Locke and Venkatesh (1997) concerning the transaction costs in the futures market. Their study finds the transaction costs to be between 0,0004% and 0,033% in the futures markets, which is significantly lower than that of the equity markets (according to Jegadeesh and Titman (1993) 0,5%). I use the transaction cost of 0,033%. Equities can also be subject to short-selling restrictions, but with futures taking a short position is as straightforward as taking a long position.

Second is to include the liquidity of each traded contract. The model only trades contracts with open interest exceeding 20’000.

It should also be mentioned that the model has not been altered after running the first tests with data, meaning that the model is based on prior research and logical reasoning, which was then tested on the data. These results and the model are presented in this thesis.

4.2 Methodology
I determine the shape of the curve, i.e. is it in contango or backwardation by taking the difference of the first five contracts, sum the differences and then take an average of the sum. \( f_n \) indicates the closing price of the \( n \)th contract.

\[
\sum_{i=1}^{5} f_i = \frac{(f_1 - f_2) + (f_2 - f_3) + \cdots + (f_4 - f_5)}{5}
\]
If the result is positive, then the curve is in backwardation and if negative, then the curve is in contango. It is very important to know what shape the futures curve is, since it makes exploiting the roll return possible.

In the case of backwardation I take a long position into the contract ‘most’ backwarded, which could be also considered the most out of its path regarding its cointegration with the other data points on the curve. I do it by taking the largest value of differenced contracts.

\[
\text{MAX } (|f_n - f_{n+1}|)
\]

Once the contracts have been determined with the largest difference in price, in the case of backwardation a long position is taken onto the further contract. So for example if we look at the twelve data points for CL on the April, 4\textsuperscript{th} 1991

<table>
<thead>
<tr>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
<th>$f_9$</th>
<th>$f_{10}$</th>
<th>$f_{11}$</th>
<th>$f_{12}$</th>
</tr>
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<tbody>
<tr>
<td>20,94</td>
<td>20,75</td>
<td>20,55</td>
<td>20,4</td>
<td>20,28</td>
<td>20,19</td>
<td>20,12</td>
<td>20,03</td>
<td>19,95</td>
<td>19,89</td>
<td>19,83</td>
<td>19,78</td>
</tr>
</tbody>
</table>

The largest difference here is between the second and third contracts (20,75 – 20,55 = 0,2). So a long position is entered into the 3\textsuperscript{rd} contract for the price of 20,55.

The short position is determined by taking the smallest value of differenced contracts.

\[
\text{MIN } (|f_n - f_{n+1}|)
\]

And in this example that is the difference between the 6\textsuperscript{th} and 7\textsuperscript{th} contract (20,19 – 20,12 = 0,07). So the short position is taken into the 7\textsuperscript{th} contract with the closing price of 20,12.

It is important to note that the contracts get only chosen as a part of these equations if the open interest exceeds 20,000 units. This is to make sure that the contracts possibly to be traded are liquid enough to be actually tradable in reality at decent cost.
When the curve is in contango the process is pretty much the same but reversed. I take a short position into the largest difference and a long position into the smallest difference.

At the start of every month the portfolio is set up for the next 30 days. This means entering any, depending on the liquidity, (most of the time all) of the twenty commodity futures in the data with a both a short and a long position with the bet size of 1/20th of the whole portfolio. Meaning that, each commodity gets a 1/20th of the portfolio.

4.2.1 Testing for cointegration

As the Johansen test is for cointegrations the test is only valid when the series is known to be non-stationary, since the critical values used for maximum eigenvalue and trace test statistics are based on a pure unit-root assumption. Hjalmarsson and Österholm (2007) argue that variables such as inflation, interest rates, real exchange rates and unemployment are often modeled as being a unit root process. When in fact they have a root close to unity and often show signs of mean reversion in long enough samples. This leads to a problem as most unit-root tests have limited ability to distinguish between a unit-root and a close alternative.

As with the commodity futures curve, I suspect the unit root process to be “closer” to a, root close to unity process, than a pure unit-root process. The augmented Dickey-Fuller test (appendix 4) states the series’ to be non-stationary at 1% level of risk and the first difference to be stationary at 1% level of risk.
Figure 14 Daily differenced price series of CC (1991-2012)

Luckily Johansen (1995) states that having stationary variables in the system is theoretically not an issue and thus pre-testing the variables is not needed. He continues to explain “if a single variable is I(0) instead of I(1), this will reveal itself through a cointegrating vector whose space is spanned by the only station variable in the model.” In practice this can be done by testing $\beta = (0 \ 1)'$ (In case where $\gamma_{1,t}$ is I(1) and $\gamma_{2,t}$ is I(0)) to rule out spurious relationships that the usual maximum eigenvalue or trace test do not notice.

In my tests for cointegration I use one lag in the level series and the trend assumption of no deterministic trends and the cointegrating equations have intercepts. The results are presented in the appendix (3). All of the commodity futures curves are found to be cointegrated.
4.3 Results

I find the empirical results to be in line with what one could expect from the theoretical background presented in the Theory section of the thesis, concerning topics like pairs trading cointegrated assets and the return components of commodity futures.

Table 3 Annualized return statistics

<table>
<thead>
<tr>
<th>Observations</th>
<th>21,0000</th>
<th>Sharpe Ratio</th>
<th>3.082</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAs</td>
<td>0.0000</td>
<td>InformationRatio</td>
<td>0.371</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0080</td>
<td>TrackingError</td>
<td>0.214</td>
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<tr>
<td>Quartile 1</td>
<td>0.0370</td>
<td>ActivePremium</td>
<td>0.079</td>
</tr>
<tr>
<td>Median</td>
<td>0.0548</td>
<td>SortinoRatio</td>
<td>2.083</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>0.0629</td>
<td>CAPM.beta</td>
<td>-0.012</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>0.0621</td>
<td>CAPM.beta.bull</td>
<td>0.011</td>
</tr>
<tr>
<td>Quartile 3</td>
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<td>CAPM.beta.bear</td>
<td>-0.063</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1876</td>
<td>TimingRatio</td>
<td>-0.186</td>
</tr>
<tr>
<td>SE Mean</td>
<td>0.0096</td>
<td>CAPM.alpha</td>
<td>0.005</td>
</tr>
<tr>
<td>LCL Mean (0.95)</td>
<td>0.0430</td>
<td>Skewness</td>
<td>1.4990</td>
</tr>
<tr>
<td>UCL Mean (0.95)</td>
<td>0.0829</td>
<td>Kurtosis</td>
<td>1.8121</td>
</tr>
<tr>
<td>Variance</td>
<td>0.1417</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stdev</td>
<td>0.0201</td>
<td>Return over risk free rate (U.S. treasury 3-month bill rate)</td>
<td>0.0366</td>
</tr>
</tbody>
</table>

What comes to these measures I would like to single out the Information Ratio, which does not fully capture the ‘goodness’ of this strategy in my opinion. It misses the point that commodity futures can easily be leveraged and more importantly that the volatility of the returns is so low. With the somewhat large variance of the benchmark index, the ratio fails to grasp the fact that the portfolio return is almost purely alpha, and thus “underperforms” the index due to the index’s larger variance at times. To show what I mean, I ran the same strategy with 5x leverage and received an Information Ratio of 1.46, as where the Sharpe Ratio only increased to 3.4. Active Premium also jumps to 0.36.
Figure 13 demonstrates how the returns of the portfolio have evolved through time. Two phases stand out, the first few years of the period and the financial crisis of 2008. Liquidity explains the years 1991 and 1992, there was less of it and/or reliable data was not available. 2008 was the financial crisis when it was common to have miss pricings in different assets due to mass selling, and one-sided liquidity. This made the strategy very profitable during those times.

![Figure 15 Portfolio returns (1991-2012)](image)

To show the importance of the roll returns as the driver of excess returns it is possible to decompose the different parts of the return. Figure 14 below counts only the long trades of backwarded curves and the short trades of the contangoed curves. In a way this could be considered the alpha generating position.
And in the figure 15 I have only counted the short trades of the backwarded curves and the long trades of the contangoed curves. In a way this could be considered the pairs trade, or “the hedge position”, to minimize the variance of the overall portfolio.

In figure 16 I have decomposed the returns of the portfolio to a single commodity level. The percentages are calculated as if the whole portfolio was invested in just the single asset in question for the whole period. All components ended up in the positive territory. Even though both NG and CL ended up showing annualized returns of about 13%, the volatility of the returns was just slightly over 10% for CL and 16% for NG.
Figure 18: Returns of portfolio components when whole portfolio invested in a single component

Due to the liquidity of the contracts it was not possible to trade all of the commodities all the time. Table 3 presents statistics for how much of the time there was a position in the different components of the portfolio. So for example there were liquid enough contracts for CL (Light Sweet Crude Oil) in every period but on the contrary FC (Feeder Cattle) was traded just in 3% of the time. In practice this means that having such a non-liquid asset in the portfolio just drags the rest of the portfolios results and is not recommended.

Table 4: Percentage of time invested

<table>
<thead>
<tr>
<th></th>
<th>GC</th>
<th>NG</th>
<th>SI</th>
<th>CO</th>
<th>QS</th>
<th>HO</th>
<th>XB</th>
<th>W</th>
<th>KW</th>
<th>S</th>
<th>SB</th>
<th>KC</th>
<th>C</th>
<th>CT</th>
<th>CC</th>
<th>LH</th>
<th>LC</th>
<th>FC</th>
<th>CL</th>
<th>HG</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>89</td>
<td>85</td>
<td>70</td>
<td>90</td>
<td>70</td>
<td>93</td>
<td>28</td>
<td>81</td>
<td>58</td>
<td>92</td>
<td>89</td>
<td>57</td>
<td>88</td>
<td>71</td>
<td>77</td>
<td>44</td>
<td>57</td>
<td>3</td>
<td>100</td>
<td>56</td>
</tr>
</tbody>
</table>

And figure 17 shows the percentage of the portfolio invested at each time period. It is worth mentioning that when there were no liquid contracts available the extra cash was not invested in anyway. In reality the idle cash would at least produce the risk free rate, which in this study was not calculated towards the results. From the figure it can be concluded that the liquidity of the followed commodities futures contracts has gone up through time. For the time period between 1991 and 1992, the case for liquidity might not actually have been as bad as the picture shows; there was scarce availability of reliable liquidity data from that period.
Figure 19 Percentage of portfolio invested at each time period

Figure 18 presents the components of the portfolio as a factor of volatility and returns. The worst performers were LC and C. For all the other assets the return volatility tradeoff was very good. (FC was not traded enough to be certain.)

Figure 20 Yearly return relative to volatility of portfolio components

4.3.1 Robustness analysis
To check my code I also create a portfolio which is traded daily. The results should be in line with the monthly traded portfolio, though smaller because of transaction costs
associated with more frequent trading. And this is found to be the case. Over the 20 year period, the overall profit of the daily traded portfolio is 83%, compared to the 132% of the monthly traded portfolio. The daily portfolio’s profit should have been even smaller, if it was simulated from the average returns of the monthly portfolio. But it seems like the daily traded portfolio better captures the fat positive return tails of the strategy, and thus does not lose as much to the monthly traded portfolio as it ‘should’. These facts support’s the premise, that the trading should be spread based, and/or have a human input, and not just a mechanical monthly operation. In appendix (1) the profit graphs of the daily traded portfolio can be found.

4.3.2 Improvements
The strategy I formed is quite stupid concerning when to trade. A very nice improvement, which could improve the results significantly, would be to trade based on the price deviation (spread) from the equilibrium value. Meaning that when the price deviates $\Delta$ to either direction of the long-run equilibrium $\mu$, that is when the position is opened, and once the spread between the assets unwinds, the position is closed, resulting in:

$$\log(P^A_t) - \gamma \log(P^B_t) = \mu - \Delta$$

$$\log(P^A_{t+1}) - \gamma \log(P^B_{t+1}) = \mu + \Delta$$

(25)

Also on being pickier on what to trade, could improve the results. Instead of trading all the 20 commodities every month, trading on the best 5 opportunities would be smarter.

One purely technical improvement into my code, which would improve the strategy, would be to choose the “hedging” pair of the pairs trade from the real difference of the futures prices and not the absolute difference. This would capture the, though rare, instances where the futures curve has elements of both backwardation and contango.

Since liquidity issues the strategy was very little time deployed in the period from 1991 to 1992 so therefore the returns of this particular period are especially poor compared to the rest of the period (0.8% and 2% annually). Using different constraints for periods of illiquidity might yield overall better results.
5 CONCLUSION

I find it safe to conclude that it is possibly to create a commodity future trading strategy, which captures the roll returns of commodity futures and minimizes the volatility of the returns through a pairs trade on the futures curve. Figure 19 illustrates how profitable such strategy actually would be in practice, 1 euro invested in the mid-1991 (and its profits thereafter) would have turned into 3.6 euros in a period of 21 years.

![Figure 21 Cumulative return of a 1 eur invested in the portfolio](image)

The results confirm earlier studies that commodity futures are a space where profitable opportunities exist even today. They provide a low correlating, low volatility addition to almost any portfolio. This study also confirms that the theoretical return component, roll return, is also possible to capture in practice, and thus also entails, that such component truly exists.

Putting these results into practice would be pretty straight forward. One would need a futures account, and up to date futures contract prices and open interest information.

For further study or practical experiment, it would be nice to test a portfolio consisting of only the best performing commodities found in this study, and to trade only the best spread opportunities found in these commodities.
References


Appendices

Appendix 1 – Returns of the daily traded portfolio

Figure 22 Long backwarded largest difference, short backwarded smaller difference + short contangoed largest difference, long smallest difference returns

Figure 23 Long backwarded largest difference + short contangoed largest difference returns
Appendix 2 – Characteristics of commodity returns

Figure 24 Short backwaded smallest difference + long contangoed smaller difference

Figure 25 Seasonality function in the convenience yield of corn
Figure 26 GSCI excess return and unexpected inflation (1969-2003) (Erb and Harvey 2006)
Appendix 3 – Johansen’s test results

I have not included all the Johansen’s test results to save space, but the results are in line with the ones presented here.

HG:

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob. **</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.321777</td>
<td>5461.395</td>
<td>251.2650</td>
<td>1.0000</td>
</tr>
<tr>
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<td>0.304699</td>
<td>3402.741</td>
<td>208.4374</td>
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</tr>
<tr>
<td>At most 2 *</td>
<td>0.060704</td>
<td>1475.841</td>
<td>169.5981</td>
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<td>At most 3 *</td>
<td>0.063546</td>
<td>1143.906</td>
<td>134.6789</td>
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<td>At most 4 *</td>
<td>0.051949</td>
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</table>

Trace test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

HO:

Max-eigenvalue test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
Trend assumption: No deterministic trend (restricted constant)
Series: SERIES01 SERIES02 SERIES03 SERIES04 SERIES05 SERIES06 SERIES07 SERIES08 SERIES09 SERIES10
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

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<th>Prob.**</th>
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Trace test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**McKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

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Max-eigenvalue test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**McKinnon-Haug-Michelis (1999) p-values
Trend assumption: No deterministic trend (restricted constant)
Series: SERIES01 SERIES02 SERIES03 SERIES04 SERIES05 SERIES06 SERIES07 SERIES08 SERIES09 SERIES10
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

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<td>0.0395</td>
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<tr>
<td>At most 9</td>
<td>0.000802</td>
<td>4.252709</td>
<td>9.164546</td>
<td>0.3759</td>
</tr>
</tbody>
</table>

Trace test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvale</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
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<tr>
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<td>At most 2 *</td>
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<td>53.16784</td>
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<tr>
<td>At most 3 *</td>
<td>0.034100</td>
<td>183.9958</td>
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<td>0.0000</td>
</tr>
<tr>
<td>At most 4 *</td>
<td>0.027596</td>
<td>148.3733</td>
<td>40.05680</td>
<td>0.0000</td>
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<tr>
<td>At most 5 *</td>
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<tr>
<td>At most 6 *</td>
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</tr>
<tr>
<td>At most 7 *</td>
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<td>9.164546</td>
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</tr>
</tbody>
</table>

Max-eigenvalue test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
CL:

Trend assumption: No deterministic trend (restricted constant)
Series: SERIES01 SERIES02 SERIES03 SERIES04 SERIES05 SERIES06 SERIES07 SERIES08 SERIES09 SERIES10
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob **</th>
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<tbody>
<tr>
<td>Nons *</td>
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<tr>
<td>At most 1 *</td>
<td>0.332819</td>
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<tr>
<td>At most 2 *</td>
<td>0.271270</td>
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<td>At most 3 *</td>
<td>0.195720</td>
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<td>At most 4 *</td>
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<td>At most 5 *</td>
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<td>At most 6 *</td>
<td>0.037701</td>
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<tr>
<td>At most 7 *</td>
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<tr>
<td>At most 8 *</td>
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</tr>
<tr>
<td>At most 9</td>
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<td>1.731915</td>
<td>0.164646</td>
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</table>

Trace test indicates 9 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

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<tr>
<th>Hypothesized No. of CE(s)</th>
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<th>Max-Eigen Statistic</th>
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<td>0.164646</td>
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</tbody>
</table>

Max-eigenvalue test indicates 9 cointegrating eqn(s) at the 0.05 level
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**MacKinnon-Haug-Michelis (1999) p-values
CO:

Trend assumption: No deterministic trend (restricted constant)
Series: SERIE01 SERIE02 SERIE03 SERIE04 SERIE05 SERIE06 SERIE07 SERIE08 SERIE09 SERIE10
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob. **</th>
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<tbody>
<tr>
<td>None *</td>
<td>0.331981</td>
<td>7612.711</td>
<td>251.2660</td>
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<tr>
<td>At most 1 *</td>
<td>0.327435</td>
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</tbody>
</table>

Trace test indicates 8 cointegrating eqn(s) at the 0.05 level
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**McKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob. **</th>
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<tbody>
<tr>
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<tr>
<td>At most 2 *</td>
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<td>2013.350</td>
<td>53.18784</td>
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<td>At most 3 *</td>
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<td>9.164545</td>
<td>0.7008</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 8 cointegrating eqn(s) at the 0.05 level
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**McKinnon-Haug-Michelis (1999) p-values
Appendix 4 – augmented Dickey-Fuller test results

I have not included all the augmented Dickey-Fuller test results to save space, but the results are in line with the ones presented here.

CC:

| Null Hypothesis: SERIES01 has a unit root |  |
| Exogenous: Constant |  |
| Lag Length: 16 (Automatic - based on AIC, maxlag=32) |  |

| Augmented Dickey-Fuller test statistic | -1,736 | 0,4142 |
| Test critical values: |  |
| 1% level | -3,43141 |
| 5% level | -2,86189 |
| 10% level | -2,567 |