Hamed Salehi

A MEAN-VARIANCE PORTFOLIO OPTIMIZATION BASED ON FIRM CHARACTERISTICS AND ITS PERFORMANCE EVALUATION

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Abstract

A flexible and financially sensible methodology that takes quantifiable firm’s characteristics into account when constructing a portfolio inspired by Brandt et al (2009) and Hjalmarsson and Manchev (2010) is described. The method imposes the weights to be a linear function of characteristics for investor that maximizes return and penalizes for amount of volatility and solves the optimization model with a statistical method suggested by Britten-Jones (1999). It is designed in a way to be dollar- and beta-neutral.

In order to exploit the information of some of the return-predictive factors with the described method, we form various single- and combined-factor strategies on a portfolio of 76 stocks out of FTSE100 in the period of January 2000 to October 2011 both in-sample and 60-rolling window out-of-sample. The results show that the designed strategies based on abnormal return, Jenson’s alpha and bootstrapped Sharpe-ratio lead to better performance in most of the designed strategies. Holding-based and expectation-based evaluation methods also support our results.

Keywords

Statistical portfolio optimization – Firm characteristics – Asset prices – Performance evaluation
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1 INTRODUCTION

Portfolio construction aims to combine assets in a way that maximizes wealth. In other words, the goal is to maximize expected return of the assets under investment relatively to the risk that they bear. Modern portfolio theory originated by Markowitz (1952), theoretically formulates the trade-off between risk and return based on assumptions which are assumed to hold. Mean-variance portfolio optimization maximizes expected mean and penalizes for bearing the risk. In spites of theoretical basis of mean-variance optimization, there are problem associated with this method. The method is called to be financially non-sense and error maximizer by Michaud (1989). In order to improve performance of mean-variance optimization and error estimation a lot of efforts has been done in the literature from Bayesian and non-Bayesian approaches to sophisticated statistical methods.

Returns are not unpredictable anymore as it was assumed to be before. There are broad discussions on dozens of factors that have predictability power over return. So called anomalies deviate from capital asset pricing model have been used as return-predictive factors. Factors such as size, value, momentum, dividend-price ratio, firm specific volatility and profitability along with many others are discussed to be among the predictive-factors of return.

This paper is devoted to introduce, discuss and evaluate a flexible portfolio weight estimation procedure that exploits information based on firm characteristics. Asset pricing literature supports predictability of some factors that might have of return. It is tried to be priced adjusted to the risk that they might bear. A trader with its own risk-return trade-off problem, try to form a strategy that exploit the factors that believes have highest predictability over return, must determine how much to buy or sell from each available security. This paper’s goal is to find a way to consider the investor’s problem and firm characteristics and to evaluate whether the method ends up with improvements in mean-variance optimization. The steps are first to determine how much each factor is important in determination of weight. After estimating this, the weight function is imposed to be a multiplication of the estimated importance of each factor over return and its value. Inspired by papers such as Brandt
(1999), Brandt and Santa-Clara (2006) and Brandt et al. (2009) and based on 
Hjalmarsson and Manchev (2010) we are seeking to examine the results of trading 
based on firm characteristics.

This study is unique in following terms comparing with previous research; More 
factors have been utilized in this paper in comparison with previous works. In terms 
of evaluation of the method, the robustness of difference in Sharpe-ratio of the 
portfolio that applies firm characteristics with the benchmarks has been examined 
with the same method as Ledoit and Wolf (2008). Furthermore, non-benchmarking 
methods to evaluate the performance are introduced.

The data is chosen to be monthly from stocks of FTSE 100 in the period of 2000 to 
2011. We exclude 24 stocks from our trading basket in order to have less missing 
values (reported in appendix). We apply single and combined-factor (3-factor, 5- 
factor and 7-factor strategy) strategy based on size, value, momentum, dividend-price 
ratio, firm specific volatility and profitability (see section 4.2.2). The same as Brandt 
et al. (2009) the factors are normalized to have mean equal to zero and standard 
deviation of 1. The same as Hjalmarsson and Manchev (2010) by formulating trade- 
off between risk and return (that we chose to be mean-variance), we set the weights 
to be a linear function of normalized characteristics of the firm that are likely to have 
predictability power over return, both in-sample and out-of sample. We impose two 
conditions in our problem to hold; First one is market neutrality and second is that 
there is no wealth to allocate (for each $1 going short, we long $1).

The methodology ties up the procedure of estimation of weight of the portfolio to 
firm specific characteristics. The method is straightforward, financially sensible and 
flexible. It is flexible because any quantifiable and favorable characteristics can be 
added easily. On the other hand, it is financially sensible because considers 
investor’s utility optimization problem and takes firm characteristics into account 
simultaneously. The point that should be highlighted is that we are not claiming 
factors that we are using are all that can capture the behavior of the return series or 
the combination is the best among others. Rather, we suppose manager X has 
preferences (beliefs) that Y quantifiable factors drive the return. How much from
each security is needed to buy or sell that considers those Y factors is the first technical question that the paper seeks to answer.

The results show that single-factor strategy bears high volatility comparing with benchmark. Trading based on all the single-factors beats the FTSE 100 while size-factor and momentum-factor strategies are not able to beat the equally-weighted portfolio of 76 stocks in a ten-year period (2001 to 2011) in terms of their Sharpe-ratio. In case of 5-year period (2006 to 2011) value-factor, volatility-factor, earning yield-factor and profitability-factor strategies yield higher Sharpe-ratio than the FTSE 100. This is robust only for trading based on volatility-factor and profitability-factor when using equally-weighted as the benchmark in the very same 5-year period.

Except for size effect where the method finds negative impact, positive effect of the other variables on future return in-sample and out-of-sample on average is captured. Volatility decreases as the number of factors in the combined-factor strategies increases. Trading based on 5-factor and 7-factor strategies earn higher excess return than the benchmarks, and the differences between Sharpe-ratio of 5-factor and 7-factor strategies with both FTSE 100 and equally weighted as the benchmark are positive and robust, both in-sample and out-of-sample. The positive and significant Jensen alpha is there for in-sample but not for out-of-sample. This is not the case for 3-factor strategy. The returns are negative, and extremely lower than the benchmarks. 3-factor strategy is not able to yield higher Sharpe-ratio than the benchmark. In case of holding-based evaluation method, weights for the 5-factor and 7-factor strategies have on average positive covariance with the next period's return. Inversely, the negative sample mean covariance is observable for the weights and the next period's return in 3-factor strategy. The results of expectation-based performance evaluation are more reasonable for 5-factor and 7-factor strategies comparing with 3-factor strategy.

We start with literature review of related subject that support or challenge our statements. Mean-variance optimization problem with some of its main advantages and disadvantages and methods to improve it is reviewed in section 2.1. Section 2 is devoted to discuss different views on findings related to some of the firm characteristics that have predictability power over return. Methods to evaluate
performance of a portfolio are discussed in 2.4. The methodology is described precisely in chapter 3. Our data definition, empirical design, how to apply the method and comprehensive discussion is provided in chapter 4. In the end, we report the empirical results and thereafter we conclude.
2 LITERATURE REVIEW

This chapter is devoted to provide a general and brief review of the main findings that is needed to support and argue this study. First, a brief background to mean-variance optimization problem and its pros and cons is discussed briefly. Thereafter, some of the anomalies of capital asset pricing model that they are characteristics of the firms as well are brought under discussion. Finally, a few simple methods to evaluate performance of a portfolio are mentioned.

2.1 Mean-Variance Optimization

How to allocate assets is one of the most important decisions that investors take in the very beginning of their investment process. This simply means how much to buy or to sell from each asset available to trade. The well-known theoretical modeling of this investor’s decision is formulated by Markowitz (1952). The approach is a trade-off between the first and second moment of the return series and the covariance matrix of them. Markowitz (1952) suggests the model in which the optimization problem maximizes the expected return while penalizes for amount of risk taken. The so called mean-variance optimization approach was the beginning of the concept of diversification and capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965).

In comparison with ad hoc asset allocation methods, from the advantages of mean-variance optimization method is that it proposes a theoretical framework that is flexible in terms of objectives and constrains and considers risks associated to the portfolio, market condition and investment information. In spite of benefits of mean-variance optimization method, Michaud (1989) calls mean-variance as a "non-financial optimizer" and also reports that the optimization is likely to maximize effect of errors. This phenomenon is because the mean-variance optimization estimates substantially high weight for the assets with high expected return, negative correlation and small variance. Similar story holds for the assets with low expected return, positive covariance and high variance where the method underweights those. These assets are those which are in higher exposure of estimation errors. The mentioned phenomenon is the main problem of mean-variance optimization strategy.
In another study, Fletcher and Hillier (2001) report improvements in Sharpe-ratio and abnormal return of resampled portfolio efficiency\(^1\) in comparison with mean-variance optimization method. More interestingly, they find little evidence of higher Sharpe-ratio and abnormal return generated from mean-variance and resampled strategies.

An excellent review of the methods to deal with the problems associated with mean-variance optimization problem in order to improve the performance and reduce the estimation error is examined by DeMiguel et al. (2007). In their study, DeMiguel et al. (2007) by out-of-sample evaluation of performance of 14 methods to reduce the estimation error such as Bayesian approaches and non-Bayesian approaches, find that none of them is consistently doing better than the naïve portfolio in terms of Sharpe-ratio.

The other approach to reduce the sensitivity of the weights of the portfolio is to set constrains to the mean based on asset pricing findings. As an example, see Pástor (2000) and Stambaugh (2000). This is the approach that we are going to take. With the goal to improve the performance of mean-variance optimization and using firm characteristics, we constrain mean by the information. This will be discussed more in section 2.3 and chapter 3.

### 2.2 Asset prices anomalies and Return Predictability

In contrast with traditional finance, the returns of the assets are not assumed to be unpredictable anymore, in modern finance. Evidences show that there are factors that explain some parts of changes in return, especially in longer periods. The factors that have predictability over return and cannot be explained by capital asset pricing model (CAPM) are called anomalies. Multifactor empirical models have been replaced by CAPM to explain the cross-section of return series such as, 3-factor

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\(^1\) Resampled portfolio efficiency is simply to simulate different vectors and excess return and covariance matrix based on true mean and true covariance matrix and then calculating the optimal portfolio among them. (See Michaud 1989).
model of Fama and French (1993) and 4-factor model suggested by Carhart (1997). (Cochrane 1999)

In the finance literature there is broad evidences of the factors that have predictability over return. This paper aims to exploits some of these factors in asset allocation procedure by constraining expected mean by the factors. In the following the relevant ones to this study are briefly brought under discussion;

2.2.1 Size

Banz (1981) shows that the size effect does exist in return series. Market value of the stock is used as a proxy of size of the firm and it is observed that on average firms with smaller size do generate higher risk adjusted return than the firms with bigger size. Besides reporting the existence of size effect, Banz (1981) mentions the effect might come from other correlated factors with size which is observable by size itself.

As an example, Vuolteenaho (2002) by decomposition of return series finds cash flow news as the main driver of stock returns and that has a high correlation with small size and zero correlation with big size firms. Let’s assume the hypothesis that size itself has no instinct value and there are other factors that drive changes in return which are highly correlated with size of the firms; This doesn’t reduce the importance of size effect while the fact is that the size effect has been observed and according to Daniel & Titman (1997) the characteristic is what determines the cross-section of returns and firms with the same characteristics can be sensitive to the same factors. Put it in another way, size is a characteristic of a firm, and firms with that common characteristic have similar factors affecting their return. Thus, it is useful to categorize firms to their size in order to form expectation of their returns.

Berk (1995) doesn’t name size as an anomaly of the market prices, instead claims that the size is a part of the asset pricing that is not explained by CAPM and recommends using size in the models of asset pricing as the effect has been observed. In three-factor model of Fama and French (1993) size is used as one of the proxies of risk factors or distress risk factor. They discuss size to be as one of the proxies of the systematic risk which has predictability power over future return. Size is also applied
in Carhart's (1997) four-factor model in order to explain the cross-section of return of mutual funds. Fama and French (2008) find the power of size effect to be stronger in microcap stocks.

To sum up size effect has been explored in return series. It doesn't matter whether size is the reason which makes differences in return in different firms. Or size is a proxy of the factor risk. What matters is that size effect exists - smaller firms generate higher return than the larger firms- and we will try to exploit this effect.

2.2.2 Value

Another anomaly of capital asset pricing is value. There is evidence in the finance literature that firms with higher book to market ratio (value firms) outperform the firms with low book to market ratio (growth firms). After controlling for liquidity and macroeconomic risks, Asness et al (2009) argue that the value assets are cheap and growth assets are expensive. See also, Fama and French (1992), DeBondt and Thaler (1985), Jegadeesh and Titman (1993) and Fama and French (2010).

Fama and French (1993) explain the anomaly to be a proxy for systematic risk and besides size effect apply this proxy in their three-factor model of return. The very same value factor is used in Carhart's (1997) four-factor model in order to explain the cross-section of return series. Daniel and Titman (1997) on the other hand, indicate the value premium is because of characteristic itself rather than factor loadings or covariance with the return series. In their paper, Daniel and Titman (1997) also discuss that the firms with the same range of characteristic have the same properties that make them to become distressed at the same time. Some other studies take behavioral approach to explain the anomaly. As an example, Lakonishok et al (1994) argue that investor's extrapolation of past earnings explains the profitability of value strategies.

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2 Microcap stocks are those one in which the market capitalization is bellow 20th percentile.
3 Based on asset pricing models, in equilibrium, covariance structure of return explains the future returns.
Either the value premium is a proxy of systematic risk or investors by their behavioral biases causes this premium, by categorizing firms by their book to market ratio as one of their characteristics, we do expect that firms with higher ratio are expected to earn higher return.

2.2.3 Momentum

Of course it is possible to classify a firm by its past performance. If the firm was doing good recently, name it winner and conversely if it was doing bad, name it looser in sense of their return. The question is whether the past return of a firm is informative for the future return? The past winners are likely to continue winning in the future and the past losers tend to loose. This phenomenon is called momentum. It has been a huge discussion in existence of momentum and profitability of momentum strategies\(^4\) in finance literature and it is accepted that it cannot be captured by capital asset pricing model. See Jegadeesh and Titman (1993) and Wu (2002).

Fama and French (2008) report pervasiveness of momentum. Fama and French (2010) finds momentum effect which decreases with size in all regions except Japan. Asness et al. (2009) report abnormal return generated by momentum in different asset classes and different markets. Carhart (1997) utilize momentum as a risk factor in its four-factor model of returns in order to explain the return of mutual funds. Some other studies by modeling behavioral biases of investors explain the momentum effect. As an example, Chan et al. (1999) suggests under-reaction of market to the information may cause the momentum effect.

Apart from explanation the reasons that momentum premium generates extra return, we tend to accept that momentum as a firm specific characteristics that exists and have predictability power over return. That's why we use momentum in our strategy.

\(^4\) Momentum strategies buy past winners and sell past losers.
2.2.4 Dividend-Price Ratio

The amount that firms pay dividend relative to their price – dividend-price ratio – is indeed one of the important time variant characteristics of a firm. The so called dividend yield is informative in examining a firm's behavior in corporate finance and also extremely controversial in asset pricing. Many studies evaluated effect of dividend yield in future stock return.


To conclude, since on different market stages the effect of dividend yield on return series might be different, a non-linear relationship between dividend yield and return is more likely to be hold. Regardless of mixed evidences of predictability power of dividend yield over future return, in this paper, we utilize dividend yield as one of the characteristics of the firms in our asset allocation procedure.

2.2.5 Firm Specific Volatility

In the modern finance, risk and return have been always interlinked. Since firm specific risk (idiosyncratic risk) has not been priced by the asset pricing models because it can be diversified away, standard deviation of the residual of the asset pricing model is used as a proxy for it. For example, in our case, we use standard deviation of capital asset pricing model to measure idiosyncratic volatility of a firm. However, investors investing in the firms with higher risk might demand higher return to compensate for holding it (Merton 1987). Idiosyncratic volatility can be proxy of many microeconomic variable of the firm that might be informative to
determine movements of future return. It is important to see how this is really working empirically.

There have been various evidences and results of testing the possible effect of idiosyncratic risk over return. For example, Lehmann (1990) and Tinic and West (1984) beside many others, indicate positive effect of the firm's specific risk over return. Malkiel and Xu (2006) report firms with high idiosyncratic risk, have higher average return. On the other hand, Bali and Cakici (2008), finds no robust effect of idiosyncratic volatility on future returns. Ang et al. (2006), totally come up with a different result. They report a negative relationship that is firms with higher unsystematic risk have lower average return. Moreover, in their recent study, Ang et al. (2009) conclude firm that had high unsystematic risk have generated future low average return.

Huang et al. (2007) explain the contradiction in Bali and Cakici (2008) and Ang et al. (2006) by the short-term reversal effect. They also claim premium for the strategies that sell (buy) high volatility firms and buy (sell) firms with low volatility is explained by "winner minus looser" factor (short-term reversal).

To conclude, even though the evidences of the relationship is mixed, firm specific volatility is known to be indeed one of the important characteristics of the firm that is likely to be informative to predict future return. The mixed evidences might be the results of modeling the behavior. The effect of firm specific volatility might be different in different market stages and conditional to exogenous factors, therefore a non-linear modeling and correcting for short term reversal might lead to improvement to discover the relationship. In our study, we will see the method is able to discover a time variant coefficient for the effect of idiosyncratic volatility.

2.2.6 Earning-Price Ratio

Earnings which is correlated with business condition, seems to be an informative characteristic for valuation of the firms and a reasonable predictor of future returns. (See Lamont (1998) and Ball (1978) Campbell and Shiller (1988) also indicate the power that earnings have on predictability of future dividends. Cochrane (2007)
counts price-earnings ratio and book to market ratio as similar variables that because of their effects on dividends have predictability power over return. Basu (1983) reports that firms with higher earning yield generate higher risk adjusted and absolute return than those with lower earning yields, even after controlling for size. Campbell and Yogo (2006) with a sophisticated statistical method, indicate the predictability power of earning yield over return. The significant effect of earning-price ratio has been also reported by Jaffe et al. (1989). See also Basu (1977).

Even though there are hypothesis that the earning yield effect can be explained by other factors such as Reinganum (1981) that observes the effect disappears after controlling for size, we tend to see the effect of this important factor in our study. As it was mentioned before, studies strongly suggest the existence of the earning yield effect. We are not concerned about the possibility that earning yield might be a proxy of other factors since if we are using the other factors that can capture the effect of the yield, we expect the effect of the earning yield to disappear and if we are not able to detect the factors, so this earning-price ratio can be used as a proxy for the unknown variable(s).

2.2.7 Profitability

Profitability which is defined as the net income relative to the total asset of a firm is another important characteristics of the firm. The amount of income that the firm has generated is likely to determine the business condition and other valuation ratio of a firm. More importantly, profitability seems to have direct and indirect (through other variables) effects on firm's future return (See Novy-Marx 2010) Fama and French (2008) report abnormal positive return achieved by profitable firm while they find no evidence that unprofitable firms generate low returns. Novy-Marx (2010) besides indicating predictability power of profitability over return indicates the improvements of the performance of value-based strategies. The author also shows that profitability explains the earning-related anomalies.

The obvious effects of profitability on other firm characteristics and consequently on firm's performance, and explanatory power that it has on many other factors,
convince us to apply this substantially important factor in the set of factors that have predictability over return.

2.2.8 Combination

Couple of return predictive factors briefly mentioned in this chapter. A high correlation among factors might be misleading; Effect of some factors on return might be capture by other ones, the factors are set in a way that the combination is not parsimonious or in the worst case, some factors cancel each other's effect out. I emphasize that the goal of this paper is not to find the best combination of firm characteristics to predict return; however the method that will be introduced later is expected to assign smaller (higher) weights to factors that have less (more) explanatory power on return. In another word, with a set of firm characteristics, we expect the method to determine the importance of each factor at each time period (in case of out-of-sample).

2.3 Portfolio Optimization with Information

As it was mentioned before, there obviously exist factors that have predictability over return. What does this fact add to asset allocation procedure? How is it possible to exploit this information? In contrary with the mean-variance portfolio optimization (a financially non-sense optimization) where the only important factors are first and second moments of return and the covariance between the returns of different assets, portfolio optimizations with information are constructed in a way that take also the effect of predictable variables into account. This is exactly what this paper is seeking to examine. The question is that is there a gain from utilizing characteristic in the trading strategy? And does it improve the problems associated with mean-variance optimization method?

Different approaches to this problem have been targeted through the literature. In the following we mention some of them briefly; as an example Aït-Sahalia and Brandt (2001) suggests a method in which the predictive variables of return moments determine the weights of the portfolio directly in an asset allocation procedure. They model the problem for investor that maximizes the utility function and also for
ambiguity aversion investor. See also Ferson and Siegel (2001) and Brandt and Santa-Clara (2006).

More specifically, with a nonparametric approach and emphasis on importance of investor horizon and rebalancing frequency, Brandt (1999) report changes in portfolio choice by setting return to be conditional to dividend yield, lagged excess return, default premium and term premium for investors with CRRA utility. In another study, Brandt et al. (2009) propose a nonparametric method that maximizes utility function of investor and set the weights to be a linear function of firm characteristics that have predictability power over return. They also empirically apply market capitalization, book-to-market ratio and lagged return and report gain for the portfolio. Similarly, Hjalmarsson and Manchev (2010) maximizes Mean-variance optimization by letting return to be conditional to value, momentum and dividend-price ratio reports improvements in the risk adjusted return.

In parallel with the studies that introduced, we also aim to apply a strategy that exploits information achieved from firm special characteristics in an asset allocation procedure. The details of the methodology to do this and the approaches to evaluate the performance are discussed in chapter 3 and chapter 4.

2.4 Performance Evaluation

After designing a trading strategy, it is important to test the performance of it. One way is to compare the results with a benchmark. A benchmark should be available to trade, reliable, not easily beatable and trustworthy. Because of problems with benchmarking, suggested by Roll (1977) non-benchmarking methods of performance evaluation has been studied as well. In this part, we review two benchmarking measure and one non-benchmarking method. For a comprehensive review of portfolio performance see Aragon1 and Ferson (2006).

2.4.1 Benchmarking

One of the possible ways to evaluate the performance of a trading strategy, is to make a comparison; comparison with an available and reasonable benchmark. The
mostly applied benchmark is the market. Some of the market benchmarks are equally-weighted, value-weighted and price weighted of the available assets. However, Plyakha et al. (2012) report the outperformance of the equally-weighted in comparison with price- and value-weighted. Therefore, in our strategy, we choose equally-weighted portfolio of assets to be the main benchmark.

**Sharpe-ratio**

The classic measure of excess return (return of the portfolio substracted by the risk free rate) relative to its risk is suggested by Sharpe (1966). If we were able to allocate vertical axis to the excess return and horizontal axis to risk, the slope of the line would be Sharpe-ratio. Since volatility (standard deviation) of the portfolio is used as a proxy of the risk, Sharpe ratio of portfolio $p$ is defined as follow:

\[
SR_p = \frac{r_p - r_f}{\sigma_p}
\]  

(2.1)

Where return of the portfolio is $r_p$, standard deviation of it is $\sigma_p$ and risk free rate is $r_f$. The Sharpe-ratio of the portfolio and the market are compared afterwards. The higher the slope of the line leads to the higher excess return relative to the volatility. If the performance is evaluated in a time period, mostly the mean of the Sharpe-ratio is calculated. The problem with this is that if the returns are strongly skewed or non-normal, mean Sharpe-ratio would be misleading. That's why Leland (1999) suggests higher orders of moment to be used in order to evaluate the performance of a portfolio.

**Jensen's Alpha**

Another classical method to evaluate performance of the portfolio is suggested by Jensen (1968). In this method the evaluation is based on the deviation of the return of the portfolio form market (benchmark). Consider:

\[
r_{pt} = \alpha_{pt} + \beta_t r_{bt} + \varepsilon_t
\]

(2.2)
where the excess return of the portfolio at time $t$ is decomposed to an intercept, benchmark return and the error term with mean equal to zero and standard deviation of one. Based on CAPM$^3$, the expected $\alpha_p$ must be equal to zero. But the empirical results convey different story. If the intercept is positive, it means that the portfolio is underpriced and if it is negative it means that it is overpriced. In another word, portfolio with positive (negative) alpha, seems to be doing better (worse) than the market.

2.4.2 Weight-based performance

The weight-based performances aim to evaluate the performance of the portfolio without considering the benchmark since benchmarks might not be reliable (Roll’s (1977) critique). One of the non-benchmarking methods is suggested by Grinblatt and Titman (1993). In their paper they evaluate the performance of a fund by the covariance between the weights and the following return.

If we buy more units of an asset (allocating higher weight to that specific asset) and the return of holding that period become higher or in parallel, we sell more units of asset and the return of holding that asset decreases, we are happy from the trading. That is what covariance between weights and next period's return conveys. We will apply weight-based performance in our study as well.

---

$^3 E(r_p) = \beta Er_b$
3 METHODOLOGY

In this part the method is introduced and explained. First, within step by step of modeling investor’s problem the procedure of considering impact of characteristics and consequently estimation of weight is broken down. Thereafter, the intuition behind the method is discussed.

The methodology is inspired by Brandt et al. (2009) and based on Hjalmarsson and Manchev (2010) direct weight estimation without considering transaction and tax costs. First, we start with portfolio choice problem. We are seeking a model which maximizes return while penalizes for any unit of risk taken. This is in general what traders demand. The simple linear investor’s choice (3.1) is chosen to model this problem. Let \( w_t \) be the weight of the portfolio at time \( t \) and \( r_{t+1} \) to be actual \( n \times 1 \) (\( n \) is number of securities in portfolio) vector of excess return at \( t + 1 \). At the \( T \) time period the mean return of the portfolio is the first term in following model is the mean return and the second term is the variance of the return series multiplied by \( \frac{\gamma}{2} \) where \( \gamma \) is the risk aversion parameter. The bigger the risk aversion parameter, the more penalization is considered for bearing risk.

\[
\text{Max} \quad \frac{1}{T} \sum_{t=1}^{T}(w_t'r_{t+1}) - \frac{\gamma}{2T} \sum_{t=1}^{T}(w_t'r_{t+1})^2, \text{ respect to } w_t \tag{3.1}
\]

As it was mentioned before, the goal is to estimate weights in way that satisfies the model and be a function of firm characteristics. Considering this, we impose the weight to be a linear function of characteristics. By assuming \( X_t \) to be \( n \times k \) vector of normalized predictor variables (characteristics) where \( n \) is number of securities in the portfolio and \( k \) is number of characteristics in favor of investors to be considered. Normalizing here means each variable at a specific time to get mean of 0 and standard deviation of 1. We impose:

\[
w_t = \frac{1}{k} X_t \theta_p, \tag{3.2}
\]

Let also:
\[ \tilde{r}_{t+1} = X_t' \tilde{r}_{t+1}, \quad (3.3) \]

(3.3) is simply the sum of multiplication of each factor by its relevant excess return at each time. By substituting (3.2) and (3.3) in (3.1), the portfolio choice problem becomes:

\[ \max \frac{1}{T} \sum_{t=1}^{T} (\theta_p' \tilde{r}_{t+1}) - \frac{y}{2 T} \sum_{t=1}^{T} (\theta_p' \tilde{r}_{t+1})^2, \text{ respect to } \theta_p \quad (3.4) \]

Interestingly, (3.4) seems to be another form of (3.1) where the objective is to maximize mean of \( \tilde{r}_{t+1} \) by penalizing for its related variance that it might bear. To solve (3.4) we take two approaches. The first approach is to take derivatives respect to \( \theta_p \) in (3.4) and setting it equal to zero, thereafter calculating \( \theta_p \) from the achieved equation (see appendix 1 to see how (3.4) is optimized). We name calculated \( \theta_p \) from this method as \( \tilde{\theta}_p \) which is:

\[ \tilde{\theta}_p = \frac{1}{\gamma} \left( \frac{1}{T} \sum_{t=1}^{T} \tilde{r}_{t+1} \tilde{r}_{t+1}' \right)^{-1} \left( \frac{1}{T} \sum_{t=1}^{T} \tilde{r}_{t+1} \right), \quad (3.5) \]

Now that \( \theta_p \) is calculated and \( X_t \) is known beforehand, the weight function is estimated by (3.2). The second approach to estimate \( \theta_p \) from (3.4) is to use a statistical optimization method. Britten-Jones (1999) proves the proposition that regressing vector of ones over the matrix time series of excess return of different stocks without intercept by OLS, the coefficient is a proportion of tangency portfolio weights. Consider following regression;

\[ 1 = \tilde{r}_{t+1} * b + \mu, \quad (3.6) \]

where 1 is a vector of ones, \( \tilde{r}_{t+1} \) is the same as defined before, \( b \) is the vector of coefficients and \( \mu \) is the error term. Name the estimated \( b \) from (3.6) \( \tilde{b} \). Then, following would be the tangency portfolio solution that estimates \( \theta_p \);
\[ \tilde{\theta}_p = \frac{\tilde{b}}{1 + b}, \quad (3.7) \]

This enables us to estimate \( \theta_p \) and consequently \( w_t \). In our work we prefer statistical based optimization which seems to be performing better because of reasons that have been discussed in literature review.

One of the major advantages of this method is that it makes good intuitive sense. The main assumption that is imposed is the weight to be a linear function of its characteristics (3.2). To estimate to what extent each characteristic should be considered in weight formation (\( \theta_p \)), the trick is to consider portfolio choice (3.1) as a lever that enhance us to solve the problem. (3.5) is the theoretical optimization and (3.7) is the statistical optimization of the problem that accomplish our weight estimation. Both (3.5) and (3.7) estimate \( \theta_p \) in ways that capture proportion of each factor in return determination. This is more explained in next chapter with more details by an example.

To sum up, here we provided a method that estimated the weight that we are going to allocate to each asset in our portfolio, by forcing the weight to be a linear function of characteristics and then solving the problem, one by direct optimization method and another by a statistical technic. The method is straightforward, simple and flexible.
4 EMPIRICAL WORK

We break down the procedure of our empirical work to subchapters in order to provide more precise explanations. The stock has been chosen as asset class to be traded. The firm characteristics are chosen to be size, value, momentum, dividend-price ratio, firm specific volatility, earning price ratio and profitability.

This chapter is devoted to define and explain step by step of definition and formation of each variable, procedure of applying method (both in-sample and out-of-sample) and evaluation of the performance with different approaches.

4.1 Data & Variables

The data is the monthly London Stock Exchange - FTSE 100 - and retrieved from Thomson Reuters Datastream database. In order to have a neater data series and less missing values, I have excluded 24 stocks out of 100 stocks listed in appendix 2 and I have used period of Jan. 2000 to Oct. 2011. All data are from beginning of each month (6th of each month). The whole period is used to calculate all variable and thereafter data of the year 2000 (12 month) is excluded from the data because of missing values that it might bears. For example, for calculating momentum, past 12 month return is needed. Therefore, the momentum for year 2000 cannot be calculated. In the following definition and the way that the variables are calculated is clarified.

4.1.1 Risk free rate and Market Return

For risk free rate 1-Month London Interbank Offered Rate (LIBOR) is used. We believe this might be the most relevant one, since the data is monthly and as we will see later, our rebalancing happens to be on a monthly basis. The market return is FTSE 100 index return calculated monthly. This is reasonable while we are trading on 76 out of 100 of the very same index.
4.1.2 Normalizing

We are aiming to let weight to be a linear function of characteristics. These characteristics are in different scales, therefore, in order to have homogenous factors we bring them to the same scale by normalizing them. That is transforming each characteristic at each specific time to have zero mean with standard deviation equal to 1. For normalizing at each specific time $t$ the following function is applied\(^6\):

$$Norm(X_{it}) = \frac{X_{it} - \mu(X_t)}{\sigma(X_t)},$$

(4.1)

Where $X_{it}$ is the actual value of characteristic of the $i$th firm, $\mu$ is the arithmetic mean and $\sigma$ is standard deviation function that is applied for the characteristic of all the firms at each specific time $t$.

4.1.3 Size

Market capitalization is used as a proxy of size. At each time, market capitalization is calculated as multiplication of number of shares outstanding and natural logarithm of price for each stock. After calculating size of each stock, it is normalized by (4.1) at each time among all 76 stocks.

4.1.4 Value

The value of a stock is defined by the ratio of its book value of equity over market value of equity. We use the difference between total asset and total liabilities in order to calculate book value of equity of a firm at each time. The market value of a firm, on the other hand is the price of that firm multiplied by numbers of shares outstanding at each time. After dividing book value by market value of each firm at each specific time, the so called value is achieved. The procedure of normalization of firm’s value is done by applying (4.1) at each time among all 76 stocks.

\(^6\) This is proved to hold by central limit theorem that the discussion can be found on most of basic statistical book.
4.1.5 Momentum

In line with many other studies, momentum of a specific stock at time $t$ is defined as cumulative return of the stock from $t - 12$ to $t - 2$. The One month is skipped because possibility of occurrence of short-term reversal (see Jegadeesh (1990)). Therefore, momentum of stock $i$ at time $t$ is defined as:

$$MOM_{i,t} = (1 + r_{i,t-12})(1 + r_{i,t-11}) \ldots (1 + r_{i,t-2}),$$

where $R_{i,t}$ is the excess return of stock $i$ at time $t$. After calculating $MOM_{i,t}$, normalization is done by (4.1) among all stocks at each time.

4.1.6 Dividend-Price Ratio

Simply after dividing total dividend paid by number of shares outstanding at each time, dividend per share is determined. The dividend-price ratio then can be achieved from dividing dividend per share by price of the firm. Finally, the series are normalized by (4.1) among all 76 firms at each time.

4.1.7 Firm Specific Volatility

First, we run a regression of firm’s excess returns over market’s excess return. Then, obviously the residuals are achieved from the regression. Each firm’s specific volatility is the deviation from the estimated model. Thus, by taking standard deviation of residual of the model firm’s specific volatility is built. Again, with the same function (4.1), the procedure of normalization is done among all firms at each time.
4.1.8 Earning-Price Ratio

After dividing total earnings\(^7\) by number of shares outstanding, for each firm at each time, earning per share is achieved. By dividing earning per share of each firm by its price at each time the earning-price ratio is calculated. Using (4.1) we are able to normalize the earning-price ratio among firms at each time.

4.1.9 Profitability

By dividing net income of a firm by its book value of equity (defined before in section 4.1.4) at each time profitability is calculated. Then, normalizing is done by (4.1) among all stocks at each time.

4.2 Empirical Design and Discussion

So far, from Jan. 2000 to Oct. 2011 seven firm characteristics, their return series, market return and risk free rate were defined and sorted. We skip first year’s (12 initial) observations to avoid missing values of beginning of the period. Therefore, with a monthly data in a period of about 10 years, we estimate weights of a portfolio of 76 stocks with 1) single-factor strategy and 2) combined-factor strategy (in-sample and out-of-sample). Finally, the results are evaluated by three different approaches; comparing with a benchmark (in sense of their Sharpe-ratio and Jenson alpha), holding-based and expectations-based method.

The point that must be noticed is that, we are neither claiming combination of factors applied in this paper lead to a higher asset pricing predictability, nor they are the best combinations that yield to the better estimation of weights. As it was mentioned before, this paper seeks to find an efficient method that brings firm’s characteristics into account and exploits them in constructing portfolio’s weight. The perception and prospective of various investors might differ from each other in forming their

\(^7\) For firms 43 and 47 there are 3 and 50 missing values on earnings, respectively. Instead of estimating missing values, for sake of simplicity we replace the missing values with zero. This might not have significant effect on the results while when calculating weight function, we skip first 12 observations.
combinations. It might be a difficult task to find the optimal combination of firm’s characteristics that have best predictability power over return and this is not the issue of this paper. The advantage of our combined-factor strategies is that it includes more factors in comparison with the related previous studies and all of them have been discussed widely in asset pricing literature that might have predictability power over return. During evaluation the method’s improvement would become clearer.

4.2.1 Single-factor Strategy

The single factor strategy is simply the pure trading based on one factor, solely. That is, picking one of the Normalized factors and imposing the weight to be a linear function of that factor. From equation (3.2) and (3.7) we will have $\theta_p = 1$. Therefore, the weigh becomes the very same as normalized characteristics. The outcome of single factor strategy might be very risky while the procedure of weight estimation is not adjusted to volatility of the factors. The point that we can take advantage of is to use these as a benchmark. Based on the 7 factors, we report the results of seven separate single factor strategies. The weights are calculated in two periods\(^8\); from Feb. 2001 to Sep. 2011 and Jan. 2006 to Sep. 2011 with a monthly rebalancing. This is obviously the case of out-of-sample.

4.2.2 Combined-factor Strategy

When it comes to considering more than one factor in weight estimation procedure, the question becomes to what extent each factor should be taken into account. This is the interesting part that the method does i.e. estimation of $\theta_p$ by (3.7) that is the importance rate of each characteristic. It highlights the effect of each factor on weight estimation.

Let’s assume there are three different investors with different perceptions about the factors that might have significant predictability over stock’s future return. The first investor wants to build a strategy based on three factors that are known as risk factor

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\(^8\) The reason that two periods are chosen is to match the two periods that are considered for in-sample and out of sample combined strategies. This is discussed in the next part.
in asset pricing literature; *value, size and momentum*. The second investor, beside these three factors demands to bring two more financial variables into account; *dividend yield and firm specific volatility*. The third investor adds two more accounting valuation variables to 5-factor strategy; *earning per share and profitability*. Table 1 summarizes the factors under consideration in each strategy. With the method explained before, the portfolio weight estimation procedure which is tailored by these three investor’s prospective (three-, five- and seven-factor strategies) is reported, in-and out of sample. Using three different strategies described, is also informative to observe the changes in importance rate of each characteristic when a new factor is added. This might be important in asset pricing view.

The case of evaluating portfolio’s performance might be out-of-sample one, but at the same time analysis of whole sample period reveals more comprehensive information. In the following the details about the in- and out-of-sample estimation is provided.

**Table 1 Combined-factor strategies**

<table>
<thead>
<tr>
<th>3-Factor</th>
<th>5-Factor</th>
<th>7-Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Size</td>
<td>Size</td>
</tr>
<tr>
<td>Value</td>
<td>Value</td>
<td>Value</td>
</tr>
<tr>
<td>Momentum</td>
<td>Momentum</td>
<td>Momentum</td>
</tr>
<tr>
<td></td>
<td>D/P</td>
<td>D/P</td>
</tr>
<tr>
<td></td>
<td>Volatility</td>
<td>Volatility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E/P</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Profitability</td>
</tr>
</tbody>
</table>

*In Sample*

Estimation of $\tilde{\theta}_p$ or in another word the amount that each factor should be considered in formation of weight over whole data period is the case of in-sample estimation. That is calculating $\tilde{r}_t$ for the whole data period and running regression (3.6). With this estimation, at each time, the final weights are calculated by (3.2). To avoid confusion, the step by step procedure of weight estimation for 3-factor strategy is explained in following. Others are an extension of this especial case;
Consider 3-factor strategy; the vector of characteristics at time $t$ is

$$X_{t-1} = \begin{bmatrix} \text{Size}_{1 \times n} \\ \text{Value}_{1 \times n} \\ \text{Mom}_{1 \times n} \end{bmatrix}_{t-1},$$

and vector of return of each stock at time $t$ is

$$\begin{bmatrix} r_{1 \times n} \end{bmatrix}_t,$$

where $t = 1, \ldots, T$ and $T = 0$ hints “Jan. 2001”. From (3.3) and by letting $\otimes$ to stand for the Kronecker Product, $\tilde{r}_t$ at each time for whole sample period is calculated by:

$$\tilde{r}_t = \begin{bmatrix} \text{Size}_{1 \times n} \\ \text{Value}_{1 \times n} \\ \text{Mom}_{1 \times n} \end{bmatrix}_t' \otimes \begin{bmatrix} r_{1 \times n} \end{bmatrix}_t.$$

By running regression (3.6) and standardizing coefficients with (3.7), estimated $\theta_p$ is achieved. Multiplication of vector of characteristics and estimated $\theta_p$ at time $t$ leads to estimation of vector of weights of the portfolio. Notice that, the first vector of weight is formed at Feb. 2001 and the last at Sep.2011. In order to calculate return of the portfolio it’s just needed to multiply the estimated weight by the following month’s return (Mar. 2001 to Oct.2011).

The in-sample estimation enables us to observe behavior and effect of each characteristic on portfolio’s weight in the whole period.

**Out of Sample**

The case of out-of-sample estimation is quite similar to in-sample one in all aspects but the period in which $\theta_p$ is estimated. Estimation of $\theta_p$ is chosen to be at a 60-

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9 First row is the normalized size, second row is normalized value and third line is normalized momentum calculated at each time where $n$ is number of firms in the portfolio.
month rolling window instead of whole sample period. More closely, the out-of-sample estimation procedure is explained in following;

First, \( \hat{\bar{r}}_t \) for the whole period with the same procedure explained before is calculated. In order to estimate 60-moth rolling window of \( \theta_{p,t} \) which is the amount that each factor should be considered in formation of weight at time \( t \), \( \hat{\bar{r}}_t \) for \( t = -60, ..., -1 \) is regressed by (3.6). By this, we have to skip 60 first observations i.e. the first weight is estimated on Jan. 2012 and it will continue every month to Sep. 2011. Thus, the return of the portfolio is achieved from multiplication of the weights and their relevant next month return (Feb 2006 to Oct. 2011).

The out-of-sample estimation makes more sense in evaluating performance of a portfolio. In fact, out-of-sample estimation is similar to simulate a procedure of trading in the data from a point that enough information is available. According to Robotti (2003) the out-of-sample 60-month rolling window in international stocks and out-of-sample 120-month rolling window in aggregate domestic stocks benefits from information. In our case, because of the length of the data, 60-month rolling window is applied. In other words, we are interested to observe the behavior of importance rate of each characteristic determined by the method.

4.3 Performance Evaluation

The method to construct weights of the portfolio as a function of characteristics which considers portfolio choice was tailored, and on the sample data three strategies designed. Now the important part is to check to what extent the performance of the method and strategies are plausible. In order to evaluate the performance, we take three approaches; first is to compare the result of the portfolio with a reliable benchmark; second, to check whether the weights and following period’s returns move to the same direction; and the last to test whether the asset allocation was in the line of asset pricing expectations.
4.3.1 Benchmark

One of the regular ways to compare the result of a portfolio is to compare it with a reliable, tradable and not easily beatable benchmark. Since we are trading on FTSE 100, it is tenable to use the index as one of the benchmarks. The other option for a proper benchmark would be equally-weighted portfolio of the 76 chosen stocks while Plyakha et al. (2012) show that equally-weighted portfolio with a monthly rebalancing outperforms value and price weighted portfolios in many criteria such as mean return, Sharpe-ratio and four factor alpha. We compare the result of portfolio and the benchmark by their:

**Summary statistics of return**

Mean, standard deviation, skewness and kurtosis, summarize the statistical behavior of the achieved return achieved from each trading strategy is reported in a monthly basis in order to examine and compare the absolute performance of the returns.

**Sharpe-Ratio**

Sharpe-ratio defined before in section 2.4 that is the excess return of the portfolio (from risk free rate) divided by its standard deviation. The higher Sharpe-ratio, the better the performance of the portfolio, since for each unit of volatility, higher excess return is achieved. Moreover, after taking the difference of the Sharpe ratio of the portfolios and the benchmark, we test whether the mean difference is greater than zero. This test is based on bootstrapping and the method is from Ledoit and Wolf (2008).

**Alpha**

That is the same as Jensen alpha, defined in section 2.4 is the estimated intercept of the regression runs from portfolio excess return over benchmark’s excess return. If the alpha is positive and significant, it means that, the portfolio generates higher return than the benchmark or in another word; the portfolio is underpriced relative to
benchmark. On the other hand, if alpha is statistically non-zero and negative, it means that the portfolio is overpriced relative to the benchmark.

4.3.2 Holding-based

Regarding to the problems with the performance evaluation by benchmarking that discussed by Roll (1977), holding-based evaluation of performance suggested as an alternative. The holding-based evaluation, examines co-movements of the weights of the portfolio that were choses in each specific period and the return achieved from this asset allocation procedure.

Assume that firm characteristics chosen in a strategy explain 100 percent of the asset allocation procedure (weight selection). If the combination of factors have predictability\(^{10}\) over returns, it is tenable to expect that the mean covariance between weights at time \(t\) and returns at time \(t+1\) \((\text{cov}(w_t, r_{t+1}))\) is positive. But what if the factors chosen in a strategy explain 100 percent of the weights formation and at the same time mean \(\text{cov}(w_t, r_{t+1})\) is not positive? That might be explained in two ways. Either that combination of factors has no predictability over return (at least on the time period chosen to test the strategy), or the method is not able to bring that combination’s return predictability into account of estimating weights. To check the argument above we test two following hypothesizes.

- Hypothesis 1: Whether estimated weights can be completely explained by the characteristics.
- Hypothesis 2: Whether the covariance between weights of the portfolio and following return is positive.

\(^{10}\) By predictability, we mean predictability over the specific sample data which estimates \(\theta_p\). For example, in our out-of-sample case, if we claim a combination of factors has predictability over return series, it means that on each 60-month data period that combination of factors provides information for the next month’s return. Inversely, if we say that a combination of factors have no predictability over return series, we mean on 60-month data period of those factors, there is no return predictability for the next month’s return.
To support the hypothesis 1, we run a regression from weights to all factors. High $R^2$ (which we will see in our case hypothesis 1 holds since weights are defined to be linear function of characteristics) supports the hypothesis. For testing second hypothesis we calculate $\text{cov}(w_t, r_{t+1})$ and perform sample t-statistics on that with the null that the mean covariance is equal to or smaller than zero versus the alternative that it is greater than zero. Therefore, rejection of the null supports hypothesis 2. Another feature of holding-based performance evaluation is that it provides a comparison criterion for the strategies besides the available ones (alpha and Sharpe-ratio).

To sum up, if the first and second hypotheses are proved to hold, we claim the method is sensible and is likely to exploit the predictability of the factors that they have over return, on average. Otherwise, if the second hypothesis does not hold, it might be likely that the combined factor is not informative in future determination of future return.

4.3.3 Expectation-based

As it discussed before, some of the firm characteristics have predictability power over return series. For example, the cross-section of return among firms with bigger size is expected to be lower than the firms with smaller size. Therefore, if the method is able to bring this information into account and the size effect does exist, it is more likely that the method to give higher weights to the firms with smaller size and lower to the bigger size.

To evaluate whether the method’s estimation goes along with asset pricing and firm valuation expectations, we take the covariance between weights at time $t$ and characteristics at time $t-1$ ($\text{cov}(w_t, x_{t-1})$). In the same example of size case, by increasing of the size of a firm, it might be expected that the method to allocate lower weights. We test the hypothesis whether the mean of this covariance for each factor and following period’s return is as it was expected (One-tailed t-test). The intensity of the covariance might differ among factors since each factor’s effect over return is different in different time. Expected sign of the covariance of the returns and factors are summarized in table 2. If the covariance for each characteristic on average has
the same sign as it was predicted, we claim that the method is able to capture in what direction the factor has predictability over return and brings this impact into weight formation process. On the other hand, if this doesn’t hold, there are different possible explanations;

1) The effect is exploited by the other factors in the combination.
2) Time period of effect has not been chosen in a way to catch the effect of each variable.
3) The effect of the factor on return is time-variant and non-linear.
4) There is no predictability of that factor over return.
5) The method is not able to grasp the effect (methods flaw).

As it was discussed before, the important point that must be noticed is that, literature offers mixed results for the side that each factor has predictability over return. This phenomenon discussed to be because effect of one factor might disappear when checking for another related factor, the relation might be time variant and non-linear, endogenity problem or many other unknown factors. What expectation-based evaluation conveys is informative at least in a way that enables us to observe the way that each factor is affected in weight selection procedure.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Expected Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>-</td>
</tr>
<tr>
<td>Value</td>
<td>+</td>
</tr>
<tr>
<td>Momentum</td>
<td>+</td>
</tr>
<tr>
<td>D/P</td>
<td>+</td>
</tr>
<tr>
<td>Volatility</td>
<td>+</td>
</tr>
<tr>
<td>E/P</td>
<td>+</td>
</tr>
<tr>
<td>Profitability</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2 The expected sign of the covariance between estimated weights and their relevant factors (\(\text{cov}(w_t, x_{t-1})\))

To conclude, expectation-based evaluation of a method is informative in an asset pricing view. Expectation-based evaluation, out-of-sample tells us how the effect of each factor in each period is affected in asset allocation procedure (weight selection)
procedure. This is important to see if the imposed impact is in line with the asset pricing expectations.
5 EMPIRICAL RESULTS

In the section 3.2 single-factor and combined-factor strategy design and discussed. Section 3.3 argues how the methods are being evaluated. In this chapter the empirical results of trading based on the strategies designed in 3.2 are reported. Moreover, their performance is evaluated by the suggested methods of 3.3. It should be pointed out that all the reported returns are in excess of risk free rate (Libor). Also, transaction costs and taxes are not considered in this study.

Table 3 illustrates summary statistics of the excess return generated by trading based on single-factor strategies for 10-year period (panel A) and 5-year period (panel B). As it was discussed before, the benchmarks are chosen to be portfolio of monthly rebalanced equally-weighted of the 76 stocks under trade and the index which is FTSE 100. Both equally weighted and FTSE 100 reported in tables are in excess of risk free rate. In each panel, row 5 and row 7 are the estimation of the intercept of the regression running from each portfolios excess return to the benchmarks.

The main problem with single-factor strategy is that there is no estimation of $\theta$, since $\theta$ is 1 and consequently weights are the same as the normalized value of each factor. This means that single-factor strategy applies each firm characteristic by assuming positive effect on return which is not always true. As it was discussed before, effect of some factors are likely to be negative, time variant and/or non-linear. As an example, the effect of size seems to be negative on return, that is firms with smaller size are tend to generated higher return. The high negative return in table 3 as a result of size-factor strategy admits our argument. This is not the case of combined-factor strategy since in estimation of $\theta$ (importance rate of each factor) each variable's impact on return is likely to be captured by (3.6). The good side is that, single factor strategy enables us to observe the performance of each factor solely before the combination-strategy formation. Moreover, single-factor strategies provide a benchmark for the combined-factor strategies as well.
Table 3 Single-Factor strategy

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Mean</th>
<th>SD</th>
<th>skewness</th>
<th>kurtosis</th>
<th>α EW</th>
<th>β EW</th>
<th>α Index</th>
<th>β Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>-0.19</td>
<td>0.97</td>
<td>0.78</td>
<td>3.24</td>
<td>-0.17</td>
<td>1.08</td>
<td>-0.10</td>
<td>3.24</td>
</tr>
<tr>
<td>Value</td>
<td>0.19</td>
<td>2.11</td>
<td>5.28</td>
<td>42.71</td>
<td>0.49</td>
<td>19.19</td>
<td>0.64</td>
<td>17.35</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.13</td>
<td>2.27</td>
<td>-2.34</td>
<td>12.58</td>
<td>-0.17</td>
<td>-19.20</td>
<td>-0.34</td>
<td>-17.88</td>
</tr>
<tr>
<td>D/P</td>
<td>0.11</td>
<td>0.96</td>
<td>0.17</td>
<td>-0.13</td>
<td>0.05</td>
<td>-2.80</td>
<td>0.49</td>
<td>-5.74</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.31</td>
<td>2.03</td>
<td>2.12</td>
<td>8.90</td>
<td>0.66</td>
<td>10.35</td>
<td>2.45</td>
<td>8.34</td>
</tr>
<tr>
<td>E/P</td>
<td>0.20</td>
<td>2.06</td>
<td>5.28</td>
<td>43.31</td>
<td>0.44</td>
<td>21.83</td>
<td>2.41</td>
<td>8.34</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.08</td>
<td>0.77</td>
<td>2.63</td>
<td>5.69</td>
<td>0.04</td>
<td>15.24</td>
<td>0.53</td>
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</tr>
<tr>
<td>E.W. Index</td>
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<td>0.07</td>
<td>0.74</td>
<td>-4.24</td>
<td>0.22</td>
<td>-4.54</td>
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</tr>
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<td></td>
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<tr>
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<td>0.97</td>
<td>0.78</td>
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<td>-0.10</td>
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<td>19.19</td>
<td>0.64</td>
<td>17.35</td>
</tr>
<tr>
<td>skewness</td>
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<td>2.12</td>
<td>3.24</td>
<td>-0.17</td>
<td>-19.20</td>
<td>-0.34</td>
<td>-17.88</td>
</tr>
<tr>
<td>kurtosis</td>
<td>0.20</td>
<td>0.77</td>
<td>2.63</td>
<td>5.69</td>
<td>0.04</td>
<td>15.24</td>
<td>0.53</td>
<td>12.71</td>
</tr>
<tr>
<td>α EW</td>
<td>-0.08</td>
<td>0.33</td>
<td>-0.10</td>
<td>-0.10</td>
<td>0.64</td>
<td>19.21</td>
<td>0.64</td>
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</tr>
<tr>
<td>β EW</td>
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<td>0.77</td>
<td>-0.10</td>
<td>10.35</td>
<td>-0.10</td>
<td>22.53</td>
<td>0.49</td>
<td>21.83</td>
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<tr>
<td>α Index</td>
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<td>0.64</td>
<td>0.16</td>
<td>8.34</td>
<td>0.22</td>
<td>15.81</td>
<td>0.16</td>
<td>12.71</td>
</tr>
<tr>
<td>β Index</td>
<td>3.24</td>
<td>2.11</td>
<td>0.55</td>
<td>18.83</td>
<td>-0.01</td>
<td>4.92</td>
<td>0.55</td>
<td>14.96</td>
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<tr>
<td>Panel B. Period 2006 – 2011</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Mean</td>
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<td>1.16</td>
<td>1.16</td>
<td>4.37</td>
<td>-0.08</td>
<td>3.86</td>
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<td>0.33</td>
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<td>-0.49</td>
<td>2.45</td>
<td>-1.15</td>
<td>-4.92</td>
<td>-1.15</td>
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<td>α EW</td>
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<td>0.11</td>
<td>8.89</td>
<td>-1.42</td>
<td>19.21</td>
<td>1.16</td>
<td>18.83</td>
</tr>
<tr>
<td>β EW</td>
<td>3.86</td>
<td>1.99</td>
<td>-0.10</td>
<td>2.06</td>
<td>0.17</td>
<td>15.81</td>
<td>0.55</td>
<td>14.96</td>
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<tr>
<td>α Index</td>
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<td>0.49</td>
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<td>19.21</td>
<td>0.55</td>
<td>18.83</td>
</tr>
<tr>
<td>β Index</td>
<td>5.17</td>
<td>1.97</td>
<td>2.45</td>
<td>10.35</td>
<td>0.30</td>
<td>14.96</td>
<td>1.37</td>
<td>14.96</td>
</tr>
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</table>

Table illustrates the result of the single-factor portfolios where the weights are the same as the normalized characteristics and the returns are multiplication of weights by the following year’s relevant excess return. Panel A. is the results from Mar. 2001 to Oct. 2011 and panel B. from Feb. 2006 to Oct. 2011. The first 4 rows at each panel are the mean, standard deviation, skewness and kurtosis of the excess return of the portfolios, achieved by single characteristics strategies. α EW and α Index are the intercept and β EW and β Index are the estimated coefficient from regression of each strategy’s excess return over equally weighted portfolio and FTSE100 index excess return, respectively. Two last columns are equally weighted portfolio and FTSE100 index excess return, respectively. The bold values are statistically significant in 10% level.

In 10-year period trading based on single-factor strategy, except size-factor, positive mean excess return seems to be generated by all the strategies. On the other hand, by trading on 5-year single strategy the mean excess return of single-factor strategy is negative for momentum- and dividend-price ratio-factor strategy as well as size-factor strategy. The very high standard deviations show how volatile the strategies are especially in comparison with the benchmarks. Most of the single-factor strategies are skewed to the right, conveying that most of the values are located in the
left of the mean, except for momentum- and profitability-factor in 10-year period and momentum- and dividend-price-factor in 5-year period trading which are skewed to the left. Kurtosis of the excess return of the strategies, indicate that most of the values are concentrated around the mean for all factor-strategies except for dividend-price ratio-factor strategy and profitability in 10-year period trading.

Table 3 also indicates that the alphas for trading on 10-year period are significant and positive for value- volatility- and earning-price ratio-factor strategies using both-equally weighted and FTSE100 as the benchmark. Size loses to the equally weighted (as it was expected since the strategy doesn't take the negative effect of size on return). The null that the alpha is equal to zero cannot be rejected for the rest. In case of trading single-factor strategies in 5-year based on volatility and profitability on equally-weighted generate significant and positive alpha while using FTSE100 as the benchmark, positive alpha is there for strategies trading on value and volatility and negative for dividend-price ratio! The huge βs indicate the high systematic risk that the strategies bear. The results support the importance of time period in the impact that each factor might have on return.

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
<th>Value</th>
<th>Momentum</th>
<th>D/P</th>
<th>Volatility</th>
<th>E/P</th>
<th>Profit.</th>
<th>E.W.</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Period 2001 – 2011</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.192</td>
<td>0.091</td>
<td>0.056</td>
<td>0.111</td>
<td>0.154</td>
<td>0.099</td>
<td>0.108</td>
<td>-0.253</td>
<td>-0.466</td>
</tr>
<tr>
<td>Median</td>
<td>-0.258</td>
<td>0.006</td>
<td>0.139</td>
<td>0.082</td>
<td>0.066</td>
<td>-0.016</td>
<td>0.141</td>
<td>-0.153</td>
<td>-0.337</td>
</tr>
<tr>
<td>Diff.EW</td>
<td>0.061</td>
<td><strong>0.344</strong></td>
<td>0.309</td>
<td><strong>0.364</strong></td>
<td>0.407</td>
<td><strong>0.352</strong></td>
<td><strong>0.362</strong></td>
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</tr>
<tr>
<td>Boot.EW</td>
<td>0.647</td>
<td>0.002</td>
<td>0.156</td>
<td>0.022</td>
<td>0.001</td>
<td>0.001</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff.MI</td>
<td><strong>0.274</strong></td>
<td><strong>0.557</strong></td>
<td><strong>0.522</strong></td>
<td><strong>0.577</strong></td>
<td><strong>0.619</strong></td>
<td><strong>0.565</strong></td>
<td><strong>0.574</strong></td>
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</tr>
<tr>
<td>Boot.MI</td>
<td>0.053</td>
<td>0.053</td>
<td>0.026</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B. Period 2006 – 2011</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.113</td>
<td>0.002</td>
<td>-0.040</td>
<td>-0.099</td>
<td>0.188</td>
<td>-0.022</td>
<td>0.295</td>
<td>-0.187</td>
<td>-0.359</td>
</tr>
<tr>
<td>Median</td>
<td>-0.286</td>
<td>-0.140</td>
<td>0.083</td>
<td>0.024</td>
<td>0.055</td>
<td>-0.127</td>
<td>0.251</td>
<td>-0.097</td>
<td>-0.262</td>
</tr>
<tr>
<td>Diff.EW</td>
<td>0.075</td>
<td>0.189</td>
<td>0.147</td>
<td>0.089</td>
<td><strong>0.375</strong></td>
<td>0.165</td>
<td><strong>0.483</strong></td>
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<tr>
<td>Boot.EW</td>
<td>0.646</td>
<td>0.178</td>
<td>0.530</td>
<td>0.732</td>
<td>0.008</td>
<td>0.169</td>
<td>0.054</td>
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<tr>
<td>Diff.MI</td>
<td><strong>0.246</strong></td>
<td><strong>0.361</strong></td>
<td>0.319</td>
<td>0.260</td>
<td><strong>0.547</strong></td>
<td><strong>0.337</strong></td>
<td><strong>0.654</strong></td>
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</tr>
<tr>
<td>Boot.MI</td>
<td>0.151</td>
<td>0.033</td>
<td>0.221</td>
<td>0.299</td>
<td>0.001</td>
<td>0.026</td>
<td>0.016</td>
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</tr>
</tbody>
</table>

The first two rows are the mean and median of the Sharpe-ratio of each portfolio. The mean and median Sharpe-ratio is reported is the first and second row. Diff.EW is the difference between Sharpe-ratio of each strategy from equally-weighted portfolio Diff.MI is the mean difference between Sharpe-ratio of each strategy and FTSE 100. The P-value of testing difference in Sharpe-ratio to be equal to zero in each strategy from equally-weighted (Boot.EW) and FTSE 100 (Boot.MI) are shown in third and fourth rows, respectively. The bold values are statistically significant in 10% level.
Table 4 reports the mean and median Sharpe-ratio of the trading based on single-factor strategies for 10-year period (panel A) and 5-year period (panel B). P-value of testing null that the difference in Sharpe-ratio with the benchmark is equal to zero is tested by bootstrapping method suggested by Ledoit and Wolf (2008) is also reported in table 4. The results of testing the difference of Sharpe-ratios suggest that for trading single-factor beats the FTSE 100 for all the factors in a 10-year trading horizon while the null cannot be rejected for trading in the 5-year based on size, momentum and dividend yield factor. Except single-factor strategies based on size and momentum, all the rest beat the equally weighted in the 10-year horizon and in the 5-year period the difference becomes un-significant for trading based on value, earning yield and dividend yield.

To summarize results of table 3 and table 4, we can conclude, the single-factor strategies are not able to grasp which direction the effect of each predictive variable is on return over time. They are very much risky and volatile. In some cases they generate significant and positive alpha using equally-weighted and the FTSE 100 as the benchmark. In many cases, they beat equally-weighted and FTSE 100 in sense of their Sharpe-ratio. Trading based on volatility has the highest alpha and volatility- and profitability-factor strategies are among the strategies with the mean Sharpe-ratio.

After examining the outcomes of trading based on single-factor strategies and evaluating their performance, we turn into combined factor strategies which were designed before. $\theta$ the rate of importance of each factor, determines how much to consider effect of each strategy in weight formation procedure that is standardized for of $b$, estimated by (3.6). Estimated $b$ is reported in table 5 for each combined factor strategy. Panel A. shows the in-sample estimated $b$ and its relevant t-value of testing the null that it is equal to zero and panel B shows the out-of-sample mean of estimated $b$ and their relevant average of t-statistics.

The important point that can be extracted from table is that, in in-sample combined strategy, by adding more predictive variables, the level of their importance changes. This is also observable in their significancy level. For example, panel A shows that, in the 3-factor strategy significant variables are size and value, but this changes in 5-
factor strategy when dividend yield and firm specific volatility are added. The effect of value becomes negligible. This can be interpreted in a way that the value effect is captured by the new factors. This is also the case of 7-factor strategy. In the case of out-of-sample, the most important feature is that $b$ is calculated to be time varying. That is, the factors are time dependent and their impact might change during the time.

Table 5 Estimated $b$ from (3.6)

<table>
<thead>
<tr>
<th></th>
<th>3-factor</th>
<th>5-factor</th>
<th>7-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. In-sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>b-Size</strong></td>
<td><strong>-0.268</strong></td>
<td>-0.144</td>
<td><strong>-0.177</strong></td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(-2.751)</td>
<td>(-1.349)</td>
<td>(-1.688)</td>
</tr>
<tr>
<td><strong>b-Value</strong></td>
<td>0.153</td>
<td>-0.005</td>
<td>0.023</td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(2.651)</td>
<td>(-0.057)</td>
<td>(0.199)</td>
</tr>
<tr>
<td><strong>b-Momentum</strong></td>
<td>0.077</td>
<td><strong>0.147</strong></td>
<td><strong>0.179</strong></td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(1.454)</td>
<td>(2.591)</td>
<td>(2.937)</td>
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<tr>
<td><strong>b-D/P</strong></td>
<td>0.281</td>
<td>0.237</td>
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<tr>
<td><strong>t-Value</strong></td>
<td>(2.149)</td>
<td>(1.738)</td>
<td></td>
</tr>
<tr>
<td><strong>b-Volatility</strong></td>
<td>0.247</td>
<td><strong>0.259</strong></td>
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</tr>
<tr>
<td><strong>t-Value</strong></td>
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<td>(2.937)</td>
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<td><strong>b-E/P</strong></td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(0.358)</td>
<td></td>
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<tr>
<td><strong>b-Profitability</strong></td>
<td>0.308</td>
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<tr>
<td><strong>t-Value</strong></td>
<td>(2.649)</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>3-factor</th>
<th>5-factor</th>
<th>7-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B. Out-of-sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>b-Size</strong></td>
<td>-0.204</td>
<td>-0.125</td>
<td>-0.228</td>
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<tr>
<td><strong>t-Value</strong></td>
<td>(-1.210)</td>
<td>(-0.676)</td>
<td>(-1.108)</td>
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<tr>
<td><strong>b-Value</strong></td>
<td>0.235</td>
<td>0.080</td>
<td>0.201</td>
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<tr>
<td><strong>t-Value</strong></td>
<td>(1.617)</td>
<td>(0.323)</td>
<td>(0.875)</td>
</tr>
<tr>
<td><strong>b-Momentum</strong></td>
<td>0.082</td>
<td>0.079</td>
<td>0.070</td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(0.877)</td>
<td>(0.819)</td>
<td>(0.625)</td>
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<tr>
<td><strong>b-D/P</strong></td>
<td>0.150</td>
<td>0.190</td>
<td>0.190</td>
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<tr>
<td><strong>t-Value</strong></td>
<td>(0.657)</td>
<td>(0.726)</td>
<td>(0.726)</td>
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<tr>
<td><strong>b-Volatility</strong></td>
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<td>0.226</td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(1.345)</td>
<td>(1.651)</td>
<td>(1.651)</td>
</tr>
<tr>
<td><strong>b-E/P</strong></td>
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<td></td>
<td>0.136</td>
</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(-0.404)</td>
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<td>(0.120)</td>
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<tr>
<td><strong>b-Profitability</strong></td>
<td>0.512</td>
<td></td>
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</tr>
<tr>
<td><strong>t-Value</strong></td>
<td>(2.252)</td>
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</tbody>
</table>

In panel A, the estimated $b$ and their t-statistics in parentheses are shown. In panel B, mean of estimated $b$ and their standard deviation (SD) besides the mean of their relevant t-statistics is reported. In panel A the significant values in 10% level are bolded.
After estimating $b$ from (2.6) and standardizing it by (3.7), it is the time to form weights. The excess return achieved from combined factor strategies beside the Jensen alpha and relevant beta is reported in table 6 in-sample (panel A) and out-of-sample (panel B). The benchmark again is chosen to be equally-weighted and FTSE 100. As it can be seen, the mean excess return of 5- and 7-factor strategy is positive while the negative excess mean is observed for 3-factor strategy. The results also suggest that volatility of the return decreases as the number of predictive factors increases.

<table>
<thead>
<tr>
<th>Table 6 Combined-factor strategy</th>
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</thead>
<tbody>
<tr>
<td>3-factor</td>
</tr>
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<td>Panel A. In-sample</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>SD</td>
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<tr>
<td>kurtosis</td>
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<tr>
<td>$\alpha$ EW</td>
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<td>t-Value</td>
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<td>$\beta$ Index</td>
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<tr>
<td>t-Value</td>
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<tr>
<td>Panel B. Out-of-sample</td>
</tr>
<tr>
<td>Mean</td>
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<tr>
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<tr>
<td>$\beta$ EW</td>
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</tr>
<tr>
<td>$\alpha$ Index</td>
</tr>
<tr>
<td>t-Value</td>
</tr>
<tr>
<td>$\beta$ Index</td>
</tr>
<tr>
<td>t-Value</td>
</tr>
</tbody>
</table>

Table illustrates summary statistics of the excess returns achieved from combined factor strategy for in-sample (panel A) and out-of-sample (panel B) estimation of $b$. E.W. is the portfolio of equally weighted and Index is FTSE 100. The significant values at 10% level are bolded.

Another important message that that table 6 conveys is that 5- and 7-factor strategy generate positive and significant alpha in-sample while the alpha for 3-factor strategy
is negative using both equally-weighted and FTSE 100 as the benchmark. Moreover, in case of out-of-sample, the only strategy generating positive alpha is the 7-factor strategy with benchmark to be portfolio of equally-weighted. It is important to notice that the betas are not statistically different from zero and that is because the method itself is beta neutral since it is designed in a way to be an active portfolio.

The same evaluation that we did for the single-factor strategies, applies here that is comparing Sharpe-ratio of each strategy with the benchmarks. Table 7 shows that both 5- and 7-factor strategy, beat the equally-weighted portfolio and FTSE 100 while there is no such evidence for the 3-factor strategy. In order to compare the strategies with each other, on the basis of alpha and Sharpe-ratio it is easy to conclude that 7-factor strategy beats the others in terms of mean Sharpe-ratio.

The negative excess return on average along with negative alpha and difference in Sharpe-ratio for 3-factor strategy seems to be explained by the extreme negative excess returns achieved by the strategy. Furthermore, the better performance of in-sample in comparison with out-of-sample is likely to be because of the time period (60-rolling window). Trying longer periods might improve the performance suggested by in-sample results.

More clearly, figure 1 and figure 2 demonstrate the difference between Sharpe-ratio of each factor-strategy and the Sharpe-ratio of equally weighted portfolio, in-sample and out-of-sample, respectively. The illustration of figure 1 and figure 2 trivially supports the out-performance of 7-factor strategy in comparison with other factor-strategies and benchmark.

To sum up with combined factor strategies performance in terms of their excess returns, Jensen alpha and Sharpe-ratio, it is observable that in contrast with 3-factor strategy, that generates negative excess return and alpha, 5-factor and 7-factor strategies generate positive mean excess return and positive and robust Sharpe-ratio which is extremely higher than those ones of the benchmarks, both in-sample and out-of-sample. Moreover, positive and significant alpha is visible for 5-factor and 7-factor strategies in case of in-sample but not out-of-sample.
In the following we continue with interpretation of the other two performance evaluation methods; holding-based and expectation-based evaluation methods described before.

Table 7 Sharpe-ratio analysis of combined-factor strategy

<table>
<thead>
<tr>
<th></th>
<th>3-factor</th>
<th>5-factor</th>
<th>7-factor</th>
<th>E.W.</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. In-sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.270</td>
<td>0.303</td>
<td>0.413</td>
<td>-0.253</td>
<td>-0.466</td>
</tr>
<tr>
<td>Median</td>
<td>-0.338</td>
<td>0.355</td>
<td>0.414</td>
<td>-0.153</td>
<td>-0.337</td>
</tr>
<tr>
<td>Diff.EW</td>
<td>-0.017</td>
<td><strong>0.556</strong></td>
<td><strong>0.666</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boot.EW</td>
<td>0.911</td>
<td>0.002</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff.MI</td>
<td>0.196</td>
<td><strong>0.769</strong></td>
<td><strong>0.879</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boot.MI</td>
<td>0.207</td>
<td>0.001</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B. Out-of-sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.030</td>
<td>0.125</td>
<td>0.243</td>
<td>-0.187</td>
<td>-0.359</td>
</tr>
<tr>
<td>Median</td>
<td>-0.009</td>
<td>-0.009</td>
<td>0.111</td>
<td>-0.097</td>
<td>-0.262</td>
</tr>
<tr>
<td>Diff.EW</td>
<td>0.157</td>
<td><strong>0.313</strong></td>
<td><strong>0.430</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boot.EW</td>
<td>0.338</td>
<td>0.028</td>
<td>0.031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff.MI</td>
<td>0.329</td>
<td><strong>0.484</strong></td>
<td><strong>0.602</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boot.MI</td>
<td>0.073</td>
<td>0.005</td>
<td>0.009</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first two rows are the mean and median of the Sharpe-ratio of each portfolio. The mean and median Sharpe-ratio is reported is the first and second row. Diff.EW is the difference between Sharpe-ratio of each strategy from equally-weighted portfolio Diff.MI is the mean difference between Sharpe-ratio of each strategy and FTSE 100. The P-value of testing difference in Sharpe-ratio to be equal to zero in each strategy from equally-weighted (Boot.EW) and FTSE 100 (Boot.MI) are shown in third and fourth rows, respectively. The bold values are statistically significant in 10% level.
Figure 1 In-sample difference between Sharpe-ratio of each factor-strategy and the Sharpe-ratio of equally weighted portfolio

Figure 2 Out-of-sample difference between Sharpe-ratio of each factor-strategy and the Sharpe-ratio of equally weighted portfolio
The next method of performance evaluation discussed before is holding-based. That is to see how the weights co-move with the next period's return. First of all to support hypothesis 1 in section 3.3.2 one can consider a regression where the dependent variable is weight and explanatory variables are the factors chosen to form the weights. Obviously, since weights are linear function of those factors the r-squared becomes 1 that is the explanatory variables completely explain the weights that is weights are 100 percent explained by characteristics chosen to form the strategy. Table 8 shows the mean covariance between weight chosen and the next periods return for in-sample (panel A) and out-of-sample (panel B) performance that is examining hypothesis 2 is section 3.3.3. Moreover, in the second row of each panel the p-value of testing the null that the sample mean of mentioned covariance is equal to or smaller than zero. The rejection of the null in 5- and 7-factor, in-sample and 7-factor out-of sample suggests higher asset allocation done by the strategy. More specifically, figure 1 and figure 2, draw the histogram of the covariance for each strategy, in-sample and out of sample, respectively. Compatible with alpha and Sharpe-ratio results, higher the covariance, better the performance and this is observable in seven-factor strategy, especially case of in-sample. The surprise rises for the 3-factor strategy, where the covariance seems to be negative. The explanation for this again seems to be the extreme few losses experienced by 3-factor strategy, both in- and out-of-sample.

Table 8 Testing the null that the sample mean of covariance between weight of each stock and following period's return $\text{cov}(w_t, r_{t+1})$ is smaller than or equal to zero

<table>
<thead>
<tr>
<th></th>
<th>3-factor</th>
<th>5-factor</th>
<th>7-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0089</td>
<td>0.0006</td>
<td>0.0004</td>
</tr>
<tr>
<td>Panel A. In-sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.9986</td>
<td>0.0004</td>
<td>0.0000</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0021</td>
<td>0.0020</td>
<td>0.0004</td>
</tr>
<tr>
<td>Panel B. Out-of-sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.5993</td>
<td>0.1510</td>
<td>0.0239</td>
</tr>
</tbody>
</table>

The bold values are statistically significant in 10% level.
Figure 3 In sample covariance between weights of 3-5 and 7 factor and next period's return.

Figure 4 Out-of-sample covariance between weights of 3-5 and 7 factor and next period's return.

Expectation based is the last tactic to evaluate the method. It should be noticed that the expectation-based performance evaluation method is free from evaluating the returns. Instead, the way that the method brings each characteristic into account in weight formation procedure is examined by the expectation-based approach.
By taking the covariance between each factor and the estimated weight in the next period we are aiming to see how the method allocates each factor to construct the weight. To do this, we test the one-tailed sample mean of the covariance between the weights in each period and the characteristics in the previous period. Table 9 is the suggested sign of the sample mean covariance between weight and characteristics in each of the combined strategies in-sample (panel A) and out-of-sample (panel B). Figure 3 to Figure 8 more clearly draw the histogram of the mentioned covariance.

The results of the expectation-based method might be compared with asset pricing expectations for each characteristic. Adding the estimated importance rate for each factor $b$ in each strategy, which is reported in table 5 assists for a better interpretation of table 9. From table 5, for both in-sample and out-of sample cases, the variables mainly have positive effect (importance rate) in weight determination except for size factor which has negative estimated size. This is in line with the expectations from table 2. What happens after is that weights in each period become the sum of the multiplication of each normalized factor by its relevant estimated standardized importance rate ($b$).

Figures 5 to figure 10, shows that the covariance between the weights and the factors is time variant and different in sign during the time. In case of in-sample estimation of weights for 5-factor strategy, the results are the same as it was expected. The covariance has the negative sample mean for value factor in 7-factor strategy, that might be explained by adding new variable that might capture the effect of value.

<table>
<thead>
<tr>
<th></th>
<th>Panel A. In-sample</th>
<th>5-factor</th>
<th>7-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size</strong></td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Value</strong></td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td><strong>D/P</strong></td>
<td></td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td><strong>E/P</strong></td>
<td></td>
<td></td>
<td>+</td>
</tr>
</tbody>
</table>

*Table 9 Suggested sign of covariance between each factor and allocated weight ($\text{cov}(w_t, x_{t-1})$) in each strategy by one-side sample t-test over mean*
### Panel B. Out-of-sample

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Momentum</th>
<th>D/P</th>
<th>Volatility</th>
<th>E/P</th>
<th>Profitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>0</td>
<td>+</td>
<td></td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D/P</td>
<td>0</td>
<td></td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>+</td>
<td></td>
<td></td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E/P</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profitability</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*“+”: The null that sample mean is less than or equal to zero is rejected at 5% level.*

*“-“: The null that sample mean is greater than or equal to zero is rejected at 5% level.*

*“0”: The null that true mean is equal to zero cannot be rejected at 5% level.*

---

**Figure 5** In-sample covariance between factors and the weights chosen next period 3-factor strategy.
Figure 6 In-sample covariance between factors and the weights chosen next period 5-factor strategy.

Figure 7 In-sample covariance between factors and the weights chosen next period 7-factor strategy.
Figure 8 Out-of-sample covariance between factors and the weights chosen next period 3-factor strategy.

Figure 9 Out-of-sample covariance between factors and the weights chosen next period 5-factor strategy.
The main surprise is caused by 3-factor strategy. Even though the effect of variable are estimated to be negative for size factor and positive for value and momentum affected in weight estimation (b), the weights covary completely on the other direction. This problem arises from a technical problem in equation (3.7). To elaborate, remember estimated value of b in table 5. Estimated values for size, value and momentum are −0.268, 0.153 and 0.077, respectively. Therefore, denominator of (3.7) becomes −0.038 which is negative. This is not the case in 5- and 7-factor strategy. However, this technical problem might be improve by using absolute value in the denominator or using estimated b instead of θ, we rather not to address this issue, since this is out of the scope of this study and is suggested for further studies.

In the case of out of sample, since the effect of each variable is captured in a 60-rolling window, the results might differ from in-sample. For 3- and 5-factor strategy, except those which the sample mean is zero, the others are in line with expectations. Suggested by figures 5 to figure 10, time-variant effect of each variable is visible. The negative sample mean covariance suggested for earning yield in 7-factor strategy again might cause from problem associated with (3.7) or the possibility that the effect is captured by other variables is the system.
Expectation-based evaluation method reveals that the method bears a technical problem that might affect the results and be misleading. In most of the cases, the results are in line with the expectations.
6 CONCLUSION

In order to improve performance of mean-variance optimization and exploiting the facts that there are return-predictive factors, a methodology that sets the weights to be a linear function of characteristics and considers investor's portfolio policy is discussed and evaluated. Market-neutrality and dollar-neutrality are imposed to hold as well. The method is financially sensible and flexible while it is able to bring every quantifiable firm characteristic into account easily. On the 76 stocks out of FTSE 100 during 2000 to 2011 with monthly rebalancing single-factor (in a 5-year and 10-year period) and combined-factor strategies (in-sample and out-of-sample) defined in this paper has been examined. The performance is evaluated based on benchmarking and non-benchmarking methods.

We start with a brief literature review of the mean-variance optimization, its advantages and disadvantages and some methods to improve the performance. Besides a review of the return-predictive factor which of have been used in this paper, including size, value, momentum, dividend-price ratio, firm specific volatility, earning-price ratio and profitability is provided. Thereafter, we continue with introducing the methodology in details and designing some trading strategies based on that.

By designing factor based strategies, we find that adding return-predictive variables to the mean-variance optimization and by applying a statistical method leads to better performance in some cases. 5-factor and 7-factor strategies, designed in this paper, yield higher excess return and Sharpe-ratio in comparison with the benchmarks. Alpha is positive and significant and the results are robust for both in- and out-of-sample estimations. The volatility of the return decreases by adding more return-predictive factors. On the other hand, 3-factor strategy yields very poor results comparing with the benchmarks. Non-benchmarking evaluation methods also support performance of 5-factor and 7-factor strategy. In the case of single-factor strategy, results are very much risky and volatile. However, in some cases there are still factors that beat the FTSE 100 and portfolio of 76 equally-weighted stocks (volatility-factor and profitability-factor).
The results might be interesting for investment companies and traders. There are technical problems associated with the methodology that is possible to be improved by more constrains and/or more sophisticated statistical methods.

Further research might apply the same method in a wider dataset in order to check for the possible biases. Also, focus on setting variance conditional to information as well as considering conditional mean might lead to improvements in estimations. Furthermore, the other possible way to check the performance of the method is to generate dozen sets of random rates of importance for each characteristic and evaluate whether the results are on average doing better than the estimated values from the method.
REFERENCES


Solving optimization problem equation 3.4

The optimization problem defined in equation 3.4 is solved in the following. The objective is to maximize following function respect to $\theta'_p$:

$$
\frac{1}{T} \sum_{t=1}^{T} (\theta'_p \hat{r}_{t+1}) - \frac{\gamma}{2T} \sum_{t=1}^{T} (\theta'_p \hat{r}_{t+1})^2
$$

Therefore, we set the derivatives respect to $\theta'_p$ equal to zero. Thus:

$$
\frac{1}{T} \sum_{t=1}^{T} (\hat{r}_{t+1}) - 2 \times \frac{\gamma}{2T} \sum_{t=1}^{T} \theta'_p (\hat{r}_{t+1} \hat{r}'_{t+1}) = 0
$$

By rearranging:

$$
\frac{1}{\gamma} \frac{1}{T} \sum_{t=1}^{T} (\hat{r}_{t+1}) = \frac{1}{T} \sum_{t=1}^{T} \theta'_p (\hat{r}_{t+1} \hat{r}'_{t+1})
$$

Multiplying both sides to the inverse of $\frac{1}{T} \sum_{t=1}^{T} \hat{r}_{t+1} \hat{r}'_{t+1}$, we have $\theta'_p$:

$$
\bar{\theta}_p = \frac{1}{\gamma} \left( \frac{1}{T} \sum_{t=1}^{T} \hat{r}_{t+1} \hat{r}'_{t+1} \right)^{-1} \left( \frac{1}{T} \sum_{t=1}^{T} \hat{r}_{t+1} \right)
$$
APPENDIX 2

List of 24 excluded firms from FTSE 100 in this paper

In this paper, in order to have a neater data set and dealing with less missing values, 24 following stocks have been eliminated from the whole stocks. The reason was that the amount of missing value for these stocks on the applied time period is high.

<table>
<thead>
<tr>
<th>Excluded firm's Name</th>
<th>Datastream Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADMIRAL GROUP PLC</td>
<td>29549F</td>
</tr>
<tr>
<td>BURBERRY GROUP</td>
<td>25968K</td>
</tr>
<tr>
<td>CARNIVAL PLC</td>
<td>265148</td>
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<tr>
<td>COMPASS GROUP</td>
<td>255049</td>
</tr>
<tr>
<td>EURASIAN NATRES.CORP.</td>
<td>51385N</td>
</tr>
<tr>
<td>EVRAZ</td>
<td>77863Q</td>
</tr>
<tr>
<td>EXPERIAN</td>
<td>410124</td>
</tr>
<tr>
<td>FRESNILLO</td>
<td>53414Q</td>
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<tr>
<td>G4S</td>
<td>871674</td>
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<tr>
<td>GLENCORE INTERNATIONAL</td>
<td>77128V</td>
</tr>
<tr>
<td>HARGREAVES LANSDOWN</td>
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<tr>
<td>ICTL.HTLS.GP.</td>
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</tr>
<tr>
<td>INTL.CONS.AIRL.GP.(CDI)</td>
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</tr>
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<tr>
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<td>POLYMETAL INTERNATIONAL</td>
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<tr>
<td>RESOLUTION</td>
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<td>STANDARD LIFE</td>
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<tr>
<td>VEDANTARE SOURCES</td>
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<tr>
<td>WOOD GROUP(JOHN)</td>
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<tr>
<td>XSTRATA</td>
<td>15322M</td>
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