Modeling galaxy interactions with Holmberg’s analog computer

Aku Venhola
Oulun Yliopisto
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1 Introduction

Galaxy dynamics using simulation models has been an important research field during the last 50 years. In the beginning of the 20th century there were already extensive catalogs of galaxies (for example NGC\[12\]) which showed the large diversity of the galaxy shapes, although there was no theory to explain those. Nowadays, galaxy dynamics can be extensively studied by simulating the dark matter, star, and gas cloud orbits numerically, but before the era of modern computers constructing even a simple model was a considerable challenge.

![Figure 1: An image of interacting galaxy pair Arp 87 (NGC 3808 on the right and NGC 3808A on the left) taken with the Hubble space telescope. Source: http://hubblesite.org/newscenter/archive/releases/hubble_heritage/2007/36/image/a/](image)

In the beginning of the 1940's Erik Holmberg (University of Lund) introduced his new integration procedure for galaxy simulations [16] which can be considered as a pioneering study in galaxy dynamics. In his work, Holmberg
replaced gravitation with light intensity, based on the fact that they both obey the same $1/r^2$ attenuation with distance. He modeled the interaction of two galaxies with light bulbs and came out with the conclusion that some features of the galaxies, for example tails and inter-galactic bridges, can be explained by gravitational tidal forces. Holmberg’s work was significant for being the first simulation of galaxy dynamics, which could be upgraded only 20 years later, when the first simulations with electronic computers were started.

Despite the significance of the Holmberg’s simulations and the large number of citations they have received, there is no marks that the simulations would have been repeated. During the summer of 2013 I reconstructed the Holmberg’s experimental setting in the University of Oulu. The experiment was made two times: once using exactly the same parameters that Holmberg used, and another time with small changes in the initial parameters. In this thesis I introduce the Holmberg’s experiments and the theory behind that.

In chapter 2, I give a brief overview of disk galaxies and methods used in galaxy simulations. In chapter 3, I present the theoretical basis of Holmberg’s analog simulation. Chapter 4 covers the hardware and the practical realization of the experiment. In chapter 4, I represent the results and compare them with the Holmberg’s results.
2 Galaxies

Galaxies are large gravitationally bound objects, consisting of stars, gas, dust and dark matter having masses of $10^8 - 10^{14}$ solar masses, and they can be considered as basic building blocks of the Universe. In Hubble’s classification galaxies are divided to: ellipticals, lenticulars, spirals and irregular galaxies. By definition, spiral and lenticular galaxies have large-scale rotating disks. Elliptical galaxies can also contain small central disks but their overall dynamics is dominated by strong random motions of stars. The low-mass irregular galaxies at the end of the Hubble sequence are also disk galaxies, but are not a topic of this study. In this thesis, I deal only with the massive disk galaxies because the integration method used in my experiment only applies for two dimensional systems. [29]

Figure 2: Hubble’s classification scheme presented as Hubble tuning fork diagram. (Image source: NASA)
2.1 Composition of disk galaxies

Disk galaxies are the most frequent kind of giant galaxies ($M_B \leq -19$) in the observed Universe, and for being strong star forming galaxies they produce most of the stellar light in it. Basic components of disk galaxies are the central bulge, disk, as well as baryonic and dark matter halo. Many disk galaxies probably have also a supermassive black hole in the bulge, and also other features like rings and lenses in the disk. Some disk galaxies have also extensive dust lanes in their disk. Finally, disk galaxies are divided into two main families depending on whether they have a central bar or not. [29]

The bulges in the field consist mostly of old stars [11], whereas in the clusters they can be rejuvenated. Generally bulges contain only a small amount of gas. There exists two types of bulges, classical and pseudobulges [17]. Classical bulges have formed by gravitational collapse or hierarchical merging of smaller objects in the early phase of the galaxy formation process [1]. Classical bulges have ellipsoidal shapes and photometric radial profiles similar to elliptical galaxies. Classical bulges are stabilized against gravity by the random motion of the bulge stars. Nevertheless, classical bulges may also possess some systematic rotation. Pseudobulges are suggested to be formed slowly via redistribution of disk material augmented by gas accretion and other secular processes [1]. They are generally suggested to have young stellar populations [9], and disk-like kinematics, but recent observations have shown that they can have also old stellar populations [11].

The most remarkable feature of disk galaxies is of the disk, which can be divided to vertically thin and thick disk components. Luminosity curves of stellar disks can be approximated with the function

$$I(R, z) = I(0) \exp\left(\frac{-R}{h_R}\right) \exp\left(-\frac{|z|}{h_z}\right),$$

(1)
where $I(0)$ is the central surface brightness of the disk, $h_R$ is the radial scale length in the disk plane and $h_z$ is the scale height in the direction perpendicular to the disk [29]. Disk galaxies have generally $h_R$ between 1 and 10 kpc and $h_Z$ about 1/10 of the $h_R$. However, it is also clear that the large scale disks in galaxies are rarely perfectly exponential, e.g. they have down bending and up bending truncations [24]. These are often connected to distinct structural features like rings or the outer edge of the spiral structure [19]. On most disk galaxies the luminosity of the disk consists of two stellar populations, associated to thin and thick disks. The thin disk consists of both of old and young stars, whereas the disk extending further out in the vertical direction consists mainly of old stars, with larger random velocities than in the thin disk [33].

If we measure the rotation curve of a disk galaxy, we commonly see that the curve rises steeply near the center of the galaxy, and then flattens out to a constant in the outer parts of the disk. Density profiles derived from the luminosity distributions of disk galaxies predict the velocity curve to start to decline as the radius increases. The deviation between the predicted and the observed velocity curves grows as the radius increases, which implies that increasing part of the velocity is caused by some other source than gravity of the luminous matter, e.g. dark matter halos [29]. As the gravitational effects of the dark matter can be observed in rotation curves of galaxies, but the dark halos have not been found to radiate any electromagnetic radiation, the dark matter is assumed to consist of particles with mass but no electromagnetic interaction. Rotation curves derived from galaxy decompositions have shown that the amount of dark matter varies widely between different galaxies, and that some rotation curves of disk galaxies can be produced even without massive dark matter halo, at least within the optical radius [18].
2.2 Spiral arms and tidal features in disk galaxies

Spiral arms are common and imposing features in the disk galaxies. The current theory of the spiral arms was introduced by P.O. Lindblad in 1960 and B. Lindblad in 1961, and further developed by C.C. Lin and F.H. Shu in 1964 [20]. According to this theory, spiral arms are density waves revolving in the disk with their own angular velocity which can differ from the orbital velocity of stars. Spiral arms originate from perturbations in the disk density caused by e.g. galaxy’s bar or some external disturbance. Spiral arms are also important sites of star formation in disk galaxies and are therefore relatively bright and blue. New stars are born rapidly in spiral arms as the density wave perturbs the interstellar gas/dust clouds and makes them collapse into new protostars [29].

Other features which have their origin in gravitational perturbations are galactic bridges and tails. These large-scale structures are caused by close and violent bypasses of galaxies. Tidal bridges and tails are most prominent in encounters of galaxies which have a massive disk component compared to their halo [10].
Figure 3: The Hubble space telescope view of the galaxy pair NGC 4038 and NGC 4039, commonly known as the "Antennae". The long tidal tails are produced during the encounter of the galaxies.

Galactic tails are elongated trails of stars and dust which are ripped from the galaxy by the tidal forces during an encounter with another galaxy [28]. Unlike spiral arms, the tidal tails are not density waves in the disk but part of the disk which can be elongated many times the diameter of the disk. Another difference between them is that spiral arms are typically trailing in respect of the rotation of the galaxy but tidal tails can be either trailing or leading. Their direction is determined by the relative motion of the interacting galaxies in a way that the tidal tails are trailing relative to the orbital motion [32].

Tidal bridges tend to be born between interacting galaxies if their peri-
center distance and relative velocities are small enough. Bridges, like tidal tails, consist of the stars and the interstellar gas and dust from the outer parts of the disks, and are prominent sources of star formation [28]. The subsequent evolution of tails and bridges is determined by their length, and the mass distribution of their parent galaxies [10]. If the tidal features are long enough they can tear away from their parent galaxies and produce new dwarf galaxies. In the other cases they fall back to their parents [32].

2.3 Dynamics

Orbital velocities of stars in galaxies are non-relativistic and are therefore well described by the Newtonian dynamics. If we look at a system of two particles, we know by the Newton’s law of gravity that a point mass $M$ attracts other mass $m$, at distance $\vec{r}$, giving it an acceleration $\ddot{\vec{r}}$ according to

$$m\ddot{\vec{r}} = -\frac{G M m}{|\vec{r}|^3} \vec{r}. \quad (2)$$

We can extend this law into cluster of $n$ stars with masses $m_\alpha$ and positions $\vec{r}_\alpha$

$$m_\alpha \ddot{\vec{r}}_\alpha = -\sum_{\beta \neq \alpha} \frac{G m_\alpha m_\beta}{|\vec{r}_\alpha - \vec{r}_\beta|^3} (\vec{r}_\alpha - \vec{r}_\beta). \quad (3)$$

If we now divide both sides with $m_\alpha$ and define gravitational potential $\Phi$ as

$$\Phi(\vec{r}) = -\sum_\alpha \frac{G m_\alpha}{|\vec{r} - \vec{r}_\alpha|}, \quad \text{for} \quad \vec{r} \neq \vec{r}_\alpha, \quad (4)$$

we can write:

$$\ddot{\vec{r}} = -\nabla \Phi. \quad (5)$$

If the number of stars is large, it is reasonable to write $\Phi$ into continuous form as a function of density $\rho$

$$\Phi(\vec{r}) = -\int \frac{G \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'. \quad (6)$$
If we apply $\nabla^2$ to both sides we get

$$\nabla^2 \Phi(\vec{r}) = -\int G\rho(\vec{r}') \nabla^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) d\vec{r}'^3. \quad (7)$$

By using

$$\nabla^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) = \nabla_{\vec{r}'}^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) = 0, \quad f o r \quad \vec{r} \neq \vec{r}', \quad (8)$$

where $\nabla_{\vec{r}'}^2$ means that the derivative is taken in respect to $\vec{r}'$, we know that the integrand on the right side in the Eq. 7 is zero outside a small sphere $S_\epsilon(\vec{r})$. If we choose the radius $\epsilon$ of this sphere to be very small, we can approximate the density to be constant inside the sphere.

$$\nabla^2 \Phi(\vec{r}) \approx \int_{S_{\epsilon}(\vec{r})} \nabla_{\vec{r}'}^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) dV' \quad (9)$$

Now we can use the divergence theorem to this integral and integrate over the surface of the sphere.

$$\int_{S_{\epsilon}(\vec{r})} \nabla_{\vec{r}'}^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) dV' = \oint_{S_{\epsilon}(\vec{r})} \nabla_{\vec{r}'} \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) \cdot d\vec{S} = -4\pi, \quad (10)$$

For the result we get Poisson’s equation:

$$\nabla^2 \Phi(\vec{r}) = 4\pi G\rho(\vec{r}) \quad (11)$$

Now we can calculate the gravitational potential for an arbitrary mass distribution or a cluster of point masses. The gravitational potential defines the paths of the particles but as the particles move the potential produced by the particles also changes. Solving the equations of motion for this kind of particle distribution is called the n-body problem. In general, the problem can not be analytically solved in cases where the particle number is more than two. Therefore we have to make numerical integrations to study the dynamics.
Convention in the galaxy dynamics is to use distribution function \( f(\vec{r}, \dot{\vec{r}}, t) \) which defines the number density of particles in the phase space, so that 

\[ f(\vec{r}_0, \dot{\vec{r}}_0, t) d\vec{r}^3 d\dot{\vec{r}}^3 \]

gives the number of stars which locate in a small volume element \( d\vec{r}^3 \), centered at \( \vec{r}_0 \), and have velocity components inside a velocity volume element \( d\dot{\vec{r}}^3 \), centered at \( \dot{\vec{r}}_0 \). Systems, in which the binary interactions of particles are insignificant, and the global gravitation field dominates the dynamics, are called collisionless systems. Distribution function of collisionless systems obeys the collisionless Boltzmann equation

\[
\frac{\partial f}{\partial t} + \dot{\vec{r}} \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial \dot{\vec{r}}} = 0,
\]

(12)

where \( \Phi \) is the gravitational potential defined by Poisson’s equation. Dynamics of a typical galaxy are determined by the total gravitational potential of the galaxy, and binary interactions between the stars are negligible. Therefore, collisionless Boltzmann equation can be used when considering dynamics of galaxies.

### 2.4 A short overview of history of galaxy simulations

Galaxy simulations have been an important tool in the development of the theories of galaxy dynamics. The theories and the simulations have developed in an interaction with each other as the simulations provide a possibility to test the theories, and the results of the simulations give rise to new problems and theories. [4]

History of galaxy simulations can be considered to begin from the analog computer simulations of Erik Holmberg in 1941 [16]. As Holmberg’s simulations are the main subject of this thesis, his method will be described in more detail later (section 3). From the historical point of view Holmberg’s research was important as it introduced a simple method for simulating galaxy dy-
namics. It also showed that during a close encounter, galaxies produce tidal features and may lose their orbital energy as a result of those deformations.

In Holmberg’s method, interactions between all of the simulation particles are taken into account when calculating the gravitational force which acts on a single particle. Equation of gravitational force, acting on a star $\alpha$ with mass $m_\alpha$, located at $\vec{r}_\alpha$, can be written

$$\vec{F}_\alpha = -\sum_{\beta=1}^{N} \frac{G m_\alpha m_\beta}{|\vec{r}_\alpha - \vec{r}_\beta|^3} (\vec{r}_\alpha - \vec{r}_\beta),$$

(13)

where $G$ is the gravitational constant and $N$ is number of particles. As we can see, the number of terms in the sum is proportional to $N$. To evaluate a time step in a simulation, we must calculate forces acting on every individual particle, which means that the former calculation must be done $N$ times. The total number of computations needed to calculate the forces, is hence proportional to $N^2$. This kind of procedure in force calculations is called direct summation method. When using Holmberg’s analog computer the number of computations is actually $\propto N$, as Eq. 13 can be calculated by making two illuminance measurements per dimension. When using a direct summation method with a digital computer, the speed of the processor limits the number of particles which can be used in a reasonable time. Because of this, direct summation methods were not optimal for the early computers, even though the method itself is quite simple.

The next big steps were taken in the 1960’s as digital computers started to become available. In 1968 Miller and Prendergast [22] introduced a computer based particle-mesh integration algorithm which enabled simulations with a notably larger number of particles than before (120,000 was used). Miller and Prendergast simulated the evolution of two dimensional galaxies with
different initial conditions. The principle of the method was to discretize the phase space of the particles. The coordinate and the velocity spaces were divided into cells and the distribution of particles in those cells defined the dynamical state of the system. By counting the number of particles in the cells of the coordinate space, the density distribution could be found out. From the density distribution, the gravitational potential can be deduced by using Poisson's equation. The evolution of the system could then be numerically integrated.

In a particle-mesh simulation, forces between every cell must be calculated, and hence the number of computations is \( \propto M^2 \), where \( M \) is the total number of cells in the spatial mesh. If we consider a three dimensional particle-mesh simulation, where the mesh is a cube with a face length of \( M_x \) cells, the number of computations needed to evaluate the forces is \( \propto M_x^6 \). The amount of computations per step can be reduced by applying fast Fourier-transforms (FFT) to solve gravitational potential from Poisson's equation. By using FFT, the number of computations needed to solve the potential becomes \( \propto (M_x \log M_x)^3 \), for a cubic mesh [13]. Of course, after solving the potential, forces must be calculated for every particle individually. The total number of computations is then also \( \propto N \), where \( N \) is the number of particles.

Simulations with a method similar to the one Miller and Prendergast used, were made also by Hockeney and Hohl in 1969 [14], Miller, Prendergast and Quirk in 1970 [23], and Hohl in 1971 [15]. In the experiment of Hockeney and Hohl the stability of disks with different velocity dispersions were studied. In 1964 Alar Toomre presented his theory about gravitational stability of disks [30]. The theory proposed that smooth disks consisting of stars are very unstable against of tendency to form massive condensations,
unless they are stabilized by a random velocity dispersion in the plane of the disk. In the Hockeney’s and Hohl’s experiment the tendency of smooth disks to produce these condensations was confirmed. They also showed that adding a velocity dispersion into the disk reduces the production rate of condensations but does not manage to prevent them totally. In their study in 1970, Miller, Prendergast and Quirk added a gas component to the simulated galaxy and managed to produce spiral arms in gas. The gas component used in the simulations was produced by reducing the velocity dispersion of some particles (called "cooling") in each step. Also, particles from the gas component were allowed to change into the star component if the density was large enough. In the study of Hohl in 1971 [15] a disk of stars with a certain initial velocity dispersion was studied. He showed that the velocity dispersion cannot prevent the evolution of large scale disturbances in smooth axially symmetric disks. After two rotations the disk first produced a central bar, which then collapsed, leaving the disk into a nearly axisymmetric stable form with large velocity dispersion.

In 1972 Juri and Alar Toomre published their landmark study [32] of galaxy interactions. It was the first extensive study of galaxy encounters including several examples of encounters with different orbital parameters and size ratios of galaxies. Toomre and Toomre used a so called test-particle model, in which the galaxies are given some arbitrary potential, for example a softened gravity of a point mass, which does not change during the simulation. Massless test-particles are then placed in the potentials to study the dynamics during the galaxy encounters. Due to the use of these moving but rigid potentials, some unrealistic features can occur. For example, the simulated galaxies can not lose their orbital energy as a result of dynamical friction and tidal features which occur during the encounters cannot com-
press as a result of self-gravity. However, the test-particle model has some significant benefits compared to more sophisticated models. The potentials are easy to calculate, which saves time and makes it possible to perform large series of simulations in a reasonable time even with slow computers. In the test-particle method, the number of computations per time step is \( \propto N \), where \( N \) is the number of particles. Also, the stability of the disks is not a problem as there is no self-gravity to produce clumps in the disk. As Toomre and Toomre modeled encounters with a wide scale of initial parameters, their study provided a good view of the origins of the different tidal structures. They used the model also to produce the structure of some well known galaxies as a result of encounters [31].

In 1986, Barnes and Hut presented their new hierarchical tree code for modeling N-body problems [5]. In their model, the particle distribution is divided into cubic shells in a way that if there is more than one particle occupying a cell, the cell is divided into eight subcells. This algorithm produces a grid which is adapted to the density distribution and can be used for calculating the forces acting upon the particles. The calculation of the force is simplified in tree codes, in such a manner that if a cell with size \( s \) at distance \( d \) from the point where the force is calculated, has ratio \( \frac{s}{d} \) less than the tolerance parameter \( \theta \), its internal structure is ignored. The tolerance parameter is an arbitrary constant which adjusts the precision of the force calculations. When \( \theta \to 0 \) the code approaches the direct summation method. The hierarchical tree code can be used for larger variety of simulations than the codes with a static grid, as it requires no presumptions about the geometry of the modeled system. Also, in a hierarchical tree code the number of computations per time step for a system with \( N \) particles is proportional to \( N \log(N) \), which means a huge speed-up compared to \( N^2 \)
dependence of direct summation.

Summary of the different N-body methods are given for example by Hernquist in 1987 [13], Barnes and Hernquist in 1992 [4] and Binney and Tremaine in 2007 [6]. Models with direct summation of potential are the most accurate, but generally limited to the cases with small particle numbers. However, also N-body simulations with a large number of particles have been performed with direct summation by using hardware specially manufactured for this use. For example, with GRAPE-6, it is possible to perform galaxy simulations of $10^8$ particles [21]. The particle-mesh codes are typically faster than tree codes but their dynamical range is limited by the finite amount of cells covering the phase space. Therefore none of these techniques can be selected to be the superior simulation method for all systems, but the method used should be defined by the characteristics of the system.

As a summary we can say that only the direct summation method fulfills (within round-off accuracy of the computer) the conservation laws of energy, momentum and angular momentum when the time step approaches to zero. The particle-mesh methods can in principle be applied in a manner that conserves energy and linear momentum exactly, but angular momentum can vary during a simulation. As tree codes make some approximations during the force calculations so that none of the conservation laws are exactly valid. Conservation of these quantities can be improved by reducing $\theta$, as the model approaches the direct summation method. [4]
3 Theory of the Holmberg’s model

The Holmberg’s simulation method is basically a direct summation N-body method, which is based on brilliant usage of analogy between the gravitation and the light intensity. In the following chapters, I will introduce the structure of galaxy models and the integration method.

3.1 The construction of model galaxies

Galaxies in this simulation are divided into 37 particles or light bulbs (Fig. 4), which are initially placed in a plane on four circular orbits around the center of the galaxy with radii of 10, 20, 30 and 40 distance units. As all of the lamps are initially on circular orbits, there is no velocity dispersion in the system. The reason for the low resolution of the galaxy is that the integration method used is quite slow, and the amount of measurements needed is proportional to the number of particles. However, the used resolution is reasonable, as it makes possible to perform the simulations in a reasonable time, and enables us to study the large scale features, which occur during the encounters.
Figure 4: The distribution of lamps and the relative intensities of the rings used in the model galaxies.

By varying the luminosity and the distribution of the lamps we can produce almost arbitrary potentials. In this work, in a similar manner as in the original study, a Gaussian distribution is used for the luminosities of the lamps, giving to the central and to the two inner orbits a relative luminosity of 1.0, a luminosity of 0.7 to the third orbit and 0.3 to the outermost orbit. These values are taken from the original experiment. In the original
experiment, Holmberg chose these values as the simplest assumption, in lack of any reliable information about the density distribution of galaxies. Two years before Holmberg’s simulations, there was an article by H. W. Babcock [3], in which the rotation curve and mass-to-light ratio of the Andromeda galaxy were studied. The conclusion was that the rotation velocity of the Andromeda galaxy is unexpectedly high in the outer parts of the disk, in respect to the luminosity observed. In his paper Babcock proposed that the absorption of light becomes stronger in the outer parts of the disk, or alternatively that new dynamical considerations are required, which will permit a smaller relative mass in the outer parts of the disk [3].

Velocity of a particle on a circular orbit \( v_{\text{circ}} \), depends on the radial force \( \vec{F}_R \), acting towards the center of the galaxy, and radius \( r \), as follows

\[
v_{\text{circ}} = \sqrt{r \left| \vec{F}_R \right|}.
\]

(14)

The rotation curve obtained, by the lamps and the associated luminosities at different orbits, is shown in Fig. 5. For comparison, a theoretical rotation curve of a disk with a Gaussian surface density distribution

\[
\Sigma(r) = \Sigma_0 \exp \left( - \left( \frac{r}{r_{\text{Gauss}}} \right)^2 \right),
\]

(15)

and the same total mass as the model galaxy, is plotted over the rotation curve used in the simulations. In Eq. 15, \( \Sigma_0 \) is the surface density in the center of the system, \( r \) is the distance from the center, and \( r_{\text{Gauss}} \) is a parameter which defines the extent of the disk. By fitting Eq. 15 to the velocity curve of a galaxy used in Holmberg’s model, we obtain \( r_{\text{Gauss}} = 25 \text{ cm} \).
Figure 5: **Plot 1:** The smooth line: Gaussian surface density with $r_{Gauss} = 25$. The histogram: The surface density of an galaxy used in the analog computer as a function of radius. **Plot 2:** Surface density of the same Gaussian disk, which was fitted to Holmberg’s galaxy model in plot 1. **Plot 3:** Rotation curve of the Gaussian disk (lines) compared to velocity curve of Holmberg’s galaxy model (stars). The red dashed line corresponds to a Gaussian disk truncated at radius of 40 distance units. **Plot 4:** Angular velocity as a function of radius. The symbols are defined as in the plot 3.

The disk with a Gaussian surface density distribution fitted to Holmberg’s galaxy model (Fig 5), has a slightly lower rotation curve as the model galaxy due to its continuous mass distribution. The scatter in the orbital velocities of Holmberg’s galaxy are due to its imperfect symmetry. Also, rotation
curve of a Gaussian disk truncated at 40 distance units, is considered in Fig 5. Rotation curve of the truncated model does not differ significantly from the untruncated one. The untruncated disk is used in the simulations where Gaussian disks are applied.

As mentioned before, disk galaxies tend to have a steeply rising rotation curve near the center, and a flat curve in the outer parts of the disk. A thin disk with a Gaussian density distribution produces a rotation curve which acts in a similar manner near the center, but starts to decline after $r > r_{Gauss}$. Anyway, it is not essential to make an exact reproduction of any individual galaxy, as the aim of the simulation is only to study the general principles of the encounters.

In the experiment, two galaxies are initially in zero-energy parabolic relative orbit (orbital kinetic energy of galaxies is as large as potential energy between them) which is shown in Fig. 6. At the pericentre of the orbit the galaxies are located next to each other in a way that the edges of the galaxy disks just cross. This corresponds to the pericenter distance of 80 distance units measured from the center of the galaxies. Magnitude of tidal perturbations is inversely proportional to the distance between the encountering galaxies and their velocities. The pericentre velocities are slower for elliptical orbits than for the parabolic ones but if we want to study an encounter of two unperturbed disks it is reasonable to choose the parabolic orbit: elliptical orbits are periodic which would mean that either the galaxies have encountered earlier and the disks would have been perturbed, or the galaxies were formed on an elliptical orbit. By choosing the parabolic orbit we do not have to analyze the earlier encounters of the galaxies as the parabolic orbit is not periodic.

Parabolic orbits are also an interesting borderline case between bound
and unbound systems: If the galaxies lose any of their orbital kinetic energy during the encounter the system will become bound. Gravitation is a conservative force which means that the total energy of the system can not change. However, the kinetic energy of the relative motion between the galaxies may change into their internal kinetic energy [8]. This phenomenon is called dynamical friction.

The direction of rotation of the galaxies is chosen to be prograde with respect to the orbital motion of the galaxies. It has been shown that prograde encounters produce more distinct tidal features than the retrograde ones [32]. As the experiment has a low resolution and a chaotic nature, a set up which is expected to lead to clearly defined tidal arms is favored.
3.2 Stability of the model disks

In simulations with a present-day technology, galaxies of up to $10^8$ particles can be simulated \[21\]. Generally, galaxy simulations have less particles than the real galaxies, and hence the particles in the simulations represent a cloud of stars or gas in the real galaxy. To simulate the physical processes which take place in the galaxies realistically the reduced resolution of the simulation has to be taken in account.

When the particle number decreases, every particle has a bigger fraction of the total mass of the galaxy. As the total potential of a galaxy consists
of smaller number of particles than the real galaxies, the potential becomes more grainy than in reality. The graininess of the potential causes the mass distribution to produce clumps more easily than the real galaxies (see Fig. 8). Also, the rate of growth of the random velocities of particles is increased by the grainy potential. This phenomenon is called heating of the galaxy and its dependence of the particle number and shape of the galaxy is discussed for example by Sellwood [27].

Also, in simulations with a small number of particles, the encounters of particles become more important, and there are encounters in which particles accelerate each other, in such a way that the particle slings out from the galaxy which thus loses mass. This kind of effect is extremely rare in real galaxies and is therefore an error in the simulation.

If we look at a system which consists of \( N \) gravitationally interacting particles, we can calculate a characteristic time in which the magnitude of velocity perturbations by encounters of stars, become comparable to their initial velocities. This time scale is called the relaxation time \( T_{\text{relax}} \). The relaxation time for a spherical system consisting of \( N \) particles, can be estimated as follows

\[
T_{\text{relax}} \approx \frac{N}{6 \log(N)} T_{\text{dyn}}, \quad \text{with} \quad T_{\text{dyn}} = \frac{R_e}{v}, \tag{16}
\]

where \( R_e \) is the half-mass radius of the system and \( v \) is a typical speed for a star in the system [27]. For a galaxy with \( 10^{12} \) stars the relaxation time would be \( \approx 10^9 \) years, which is more than the age of the universe, and they can thus be considered as collisionless systems. For a system of 37 particles, as used in Holmberg’s experiment, the relaxation time is about one \( T_{\text{dyn}} \). In this case, the encounters have a significant effect on the dynamics of the system, and the system cannot be considered as a collisionless system anymore. The galaxy composition used in experiment will lose its initial
shape in a quite short time due to the heating of the disk.

To reduce the impact of binary encounters in N-body simulations, softening parameter $\epsilon$ is generally used [4]. The softening parameter is a constant added to the gravitational potentials of particles to smooth the total potential of the galaxy.

$$\vec{F}_{1,G} = -\frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) \Rightarrow \vec{F}_{1,Soft,G} = -\frac{G m_1 m_2}{(|\Delta \vec{r}|^2 + \epsilon^2)^{3/2}} (\vec{r}_1 - \vec{r}_2).$$

(17)

In a two-dimensional system the softening parameter can also be interpreted to introduce a thickness correction to the simulation [26].

If we soften the gravitational force in our simulation, the relaxation time of the system increases. Anyway, as shown by Hohl in 1970’s [15], self-gravitating axisymmetric disks are still unstable against large-scale perturbations. In Fig. 7 it is shown that by softening the gravity of the disk consisting of 37 particles, we prevent the disk from scattering by the encounters between the disk particles. Even with softened gravity some of the disk particles are scattered outside the galaxy and a central bar is formed. The formation of bar takes place also in a more accurate simulation, although the properties of the bar may be quite different. As shown by e.g. Combes and Sanders in 1981 [7], axisymmetric disk galaxies tend to form a bar, which strength first increases to a certain maximum, and then starts to decrease.

Considering these phenomena, occurring in simulations due to a small particle number, it is important to study stability of the galaxies used in Holmberg’s simulations. Also, a proper length for the softening parameter is important to find to achieve realistic simulations.

I performed series of simulations with a digital computer to study the above-mentioned parameters. The simulations were made with the leapfrog integration method (see Eq. 25) and using direct summation for calculating...
the interactions. The time evolution of a galaxy used in Holmberg’s simulation is represented in Fig. 7. In these simulations, the galaxy rotates freely without external perturbations. Time unit in the simulations is one rotation, which corresponds to the rotation time of a particle initially in the second ring of the galaxy, counted from the center. In a simulation with a step size of 2 cm in the outermost ring, one rotation corresponds to 90 steps.

Figure 7: Time evolution of a self-gravitating disk. The top row: A disk of 37 particles without softening. The middle row: A disk of 37 particles with \( \epsilon = 0.2 \, r_{Gauss} \). The bottom row: A disk of 10,000 particles with \( \epsilon = 0.2 \, r_{Gauss} \).

The model galaxy is extremely unstable if gravity is not softened, as it can be seen from the first row of Fig. 7. Almost all particles are scattered and the galaxy does not exist anymore. The second row shows the time evolution, when softening is used. In this simulation, \( \epsilon = 0.2 \, r_{Gauss} \) is used, which corresponds to the scaling parameter, which is used in the simulation.
with the analog computer.

To study the effects of the small number of particles in the Holmberg’s type simulation, I performed also simulations, similar to the above-mentioned, but using 10,000 particles. The number of particles is still small, but the simulations will provide at least some point of comparison. The Gaussian disk used in this simulation has the same mass and the density distribution as the galaxies in Holmberg’s simulation. To study which would be a proper value for $\epsilon$, I first used $0.2 \, r_{Gauss}$, which is near the optimal softening parameter for a Plummer-sphere of the same size as the disk used in the simulation, according to Athanassoula et al. [2]. The time evolution of this disk is represented in Fig. 7.

Simulations of the same disk consisting of 10,000 particles were also made with other values of $\epsilon$ to see its effect on the evolution of the galaxy. The time evolution of the galaxy using $\epsilon = 0.02 \, r_{Gauss}$ and $\epsilon = 0.4 \, r_{Gauss}$ is shown in Figs. 8 and Fig. 9, respectively. The disk in the simulation with $\epsilon = 0.02 \, r_{Gauss}$ is unstable, and produces strong clumps, which further perturb the disk. After two rotations the velocity dispersion of the disk has increased strongly and the disk particles are spread much further than in the initial state. On the other hand, in the simulation where $\epsilon = 0.4 \, r_{Gauss}$ is used, the formed structures are quite weak compared to the fact that the cold disks with systematic rotation are quite unstable systems.
3.3 The analogy between gravitational force and intensity of light

The main idea of the Holmberg’s model is that mathematically the force of gravity and light intensity drop similarly with respect to the distance from the source. If we measure the intensity of light $I$ at a distance $r$ from the light source, which radiates with a constant power $P$, or measure the radial acceleration $\ddot{r}$, caused by a mass $m$ at the same distance, we get two similarly behaving functions:

\[ I = \frac{P}{4\pi r^2} \Leftrightarrow \ddot{r} = -\nabla \Phi = -\frac{Gm}{r^2}. \quad (18) \]

In Eq. 18, $G$ is the gravitational constant. If we can keep the power of the light source constant, we have an analogy between the gravitational acceleration caused by a point mass and the intensity of light of the point source,
which is utilized in this simulation method.

By using this analogy it is possible to choose a certain radiation power to correspond to a certain mass. We can scale the light intensity and the acceleration due to gravity as follows:

$$\ddot{r}(r) = -a \cdot I(r), \quad \text{where} \quad a = 4\pi G \frac{m}{P}. \quad (19)$$

After choosing $a$ which defines the proportionality between the mass and the radiation power, we can transform any system of point masses into the corresponding system of point-like light sources. For example, if we want to resolve accelerations caused by a gravitational potential, produced by some distribution of massive particles, we can perform the calculations by replacing the massive particles with an analogical distribution of light bulbs, and then measuring the intensity of light.

In the experiment intensity of light was measured with a photo cell. The photo cell measures illuminance $E_v$ ($[E_v]=1 \text{ lm/m}^2=1 \text{ lux}$). The reading of the photo cell is directly proportional to the solid angle in which the cell is seen from the light source (Eq. 20 and Fig. 10). If the area of the cell is constant and small in respect to the second power of distance from the light source, then the reading of the photo cell is inversely proportional to the second power of distance from the light source

$$E_v = \frac{\phi_v}{A} = \frac{1}{A} I_v \Omega \approx \frac{1}{A} I_v \frac{A}{r^2} = \frac{I_v}{r^2}. \quad (20)$$

In Eq. 20, $\phi_v$ is the luminous flux of the light source, $I_v$ is its luminous intensity, $A$ is the area of the photo cell, $\Omega$ is the solid angle in which the cell is seen from the light source and $r$ is the distance between the detector and the light source.

In a two-dimensional system, in addition to the magnitude of the force, the direction must also be quantified. If $\vec{F}$ is a force acting in a plane with
a magnitude of \( F \), we can define \( \theta \), which is the angle between the direction of \( \vec{F} \) and the x-axis, measured counterclockwise from the x-axis. Now the components of \( \vec{F} \) can be written as

\[
\begin{align*}
F_x &= F \cos(\theta) \\
F_y &= F \sin(\theta)
\end{align*}
\] (21)

Let us consider a single lamp, located at \( \vec{r} \), and define \( \theta \) in a similar way as before. When we make a light measurement in the direction of x-axis, the measuring area of the photo cell is seen in angle \( \theta \) from the direction of the lamp, in respect to the normal of the photo cell. Therefore the solid angle in which the photo cell is seen from the lamp is \( \approx \frac{A}{r^2} \cos(\theta) \), and we can write the component of force in the direction of x-axis as \( -a \cdot I(r) \cos(\theta) \). A similar method can be used to obtain the component in the direction of y-axis, which is \( -a \cdot I(r) \sin(\theta) \). To calculate the components of the total force from all of the lamps, with the analog computer, we must make four light measurements in the directions of the positive and negative x- and y-axis. If we name these measurements as \( I_{-x} \), \( I_x \), \( I_{-y} \) and \( I_y \), we can write the components of force as

\[
\begin{align*}
F_x &= -a (I_x - I_{-x}) \\
F_y &= -a (I_y - I_{-y})
\end{align*}
\] (22)
Figure 10: Left picture: The photo cell is placed in the same plane with the lamps. Right picture: The photo cell is lifted up from the plane of the lamps to obtain softening to the illuminance measurements.

If the photo cell is lifted up from the plane of lamps (Fig. 10) we get a softening-like effect to illuminance. Now the solid angle of the photo cell’s measurement surface, as seen from the light source, can be written as follows

$$\Omega \approx \frac{A}{R^2} \cos(\beta) = \frac{A}{r^2 + h^2} \cos(\beta) = \frac{A}{r^2 + h^2} \frac{r}{R} = \frac{A}{r^2 + h^2} \frac{r}{(r^2 + h^2)^{\frac{3}{2}}} = \frac{Ar}{(r^2 + h^2)^{\frac{3}{2}}}.$$  

Then we can substitute the expression of the solid angle in to the equation 20.

$$E_v = \frac{1}{A} I_v \Omega \approx \frac{1}{A} I_v \frac{Ar}{(r^2 + h^2)^{\frac{3}{2}}} = I_v \frac{r}{(r^2 + h^2)^{\frac{3}{2}}} \Leftrightarrow -Gm_1m_2 \frac{r}{(r^2 + \epsilon^2)^{\frac{3}{2}}} \quad (24)$$

$I_v, G, m_1$ and $m_2$ are constants so we get a mathematically similar equations for the softened illuminance (Eq. 24) and the softened gravitation (Eq. 17). In the equation of the softened illuminance, $h$ (which tells how much the photo cell is lifted from the plane) corresponds to the softening parameter $\epsilon$ in the softened gravity.
3.4 Integration method

As mentioned before, in systems where a large number of particles interact gravitationally with each other, we cannot analytically solve their trajectories, and the solution is always a numerical approximation. To approximate the motion of some particle after a certain interval of time $\Delta t$, we need to know the speed and the acceleration of the particle. In this experiment, leapfrog integration is used to calculate trajectories of the particles. Leapfrog is a second order integration method in which the positions and the velocities are calculated in different points, in a way that if the positions are calculated at integer time steps $i$, the velocities are calculated at integer plus half time steps $i + \frac{1}{2}$. In the leapfrog method, the motion of a particle is approximated as follows

\[
\begin{align*}
    x_i &= x_{i-1} + (v_x)_{i-\frac{1}{2}} \Delta t \\
    y_i &= y_{i-1} + (v_y)_{i-\frac{1}{2}} \Delta t \\
    (v_x)_{i+\frac{1}{2}} &= (v_x)_{i-\frac{1}{2}} + (a_x)_i \Delta t \\
    (v_y)_{i+\frac{1}{2}} &= (v_y)_{i-\frac{1}{2}} + (a_y)_i \Delta t
\end{align*}
\]  

(25)

In Eq. 25, $v_x$ and $v_y$ are the components of the velocity in the directions of the x- and y-axis, and $a_x$ and $a_y$ are components of the acceleration. Applying the fact that the velocity changes linearly between the steps, the former equations can also be written in form of integer time steps

\[
\begin{align*}
x_{i+1} &= x_i + (vx)_i \Delta t + \frac{1}{2} (ax)_i \Delta t^2 \\
y_{i+1} &= y_i + (vy)_i \Delta t + \frac{1}{2} (ay)_i \Delta t^2 \\
(vx)_{i+1} &= (vx)_i + \frac{1}{2} [(ax)_i + (ax)_{i+1}] \Delta t \\
(vy)_{i+1} &= (vy)_i + \frac{1}{2} [(ay)_i + (ay)_{i+1}] \Delta t
\end{align*}
\]  

(26)
3.5 Limitations of Holmberg’s model

Holmberg’s method is suitable for this kind of simulation where the amount of particles is small. If the amount of particles grows, the amount of measurements and moving of the lamps rises rapidly and the simulations cannot be performed in practice.

Also a test-particle model could have been used in this experiment to simplify the experiment. If we leave only the two light bulbs in the centers of the galaxies on and switch off the other lamps we get an analogical experiment. Both models have their pros and cons. In the test-particle model the galaxy disks will not produce unrealistic density clumps, but if there is an intergalactic bridge or a tail produced by the interaction, it does not affect the potential of the galaxy, and is therefore not affecting the movement of the test-particles. In the Holmberg’s self-gravitating model the potentials of galaxies are dynamic but grainy and unstable.

One limitation of the Holmberg’s integration method is also its two-dimensionality. In reality, galaxies in groups are not located on a fixed plane. The orbits and the positions of galaxies usually must be expressed in three dimensions and galaxy encounters where the disks of interacting galaxies and their relative orbits are fixed into a plane are very rare. This limitation does not invalidate the results of our simulation. It only reduces the kinds of situations which can be simulated with this model. In principle, a three-dimensional model of the Holmberg’s experiment could also be build if the lamps were hanged into roof with ropes.

Also, the lack of gas and dust is a weakness of this simple model. Interstellar clouds account only for a small fraction of total mass of a galaxy, but they affect significantly to the dynamics and the appearance of galaxies, as new stars are born in them, and long dust tails are produced quite often.
as a result of an interaction of galaxies (for example M51 or the Magellanic stream). As well as birth of stars, the death of stars is neglected in this model. Supernovae affect significantly to the dynamics of nearby gas clouds and stars, but could not be taken into account in this model.
4 The Holmberg’s simulation

The simulation experiment was made in University of Oulu during July and August of 2013. The experiments took place in the student laboratory during the summer holiday, when there were no other experiments going on. All the test simulations, building and programming were done in May and June to make sure that there is enough time to successfully complete the simulations.

![Figure 11: Overview of the simulation.](image)

4.1 The experimental arrangements

The base of the experiment was made of two layers of cardboard which were joined to each other with black adhesive. On top of the cardboard base I placed a layer of black paperboard to reduce the reflection of light from the base. To the black paperboard I draw a grid to make the moving of the lamps
easier and faster. The grid's resolution was 1 cm, but the computer program which stored the coordinates of the lamps had more precise accuracy, so when placing the lamps I tried to place them as accurately as possible with an accuracy of about 1-2 mm. It would have been possible to draw a grid with better precision but it would have taken too much time compared to its benefits. In digital computer simulations mimicking the analog model I verified that with an error of less than 1 cm in the moving of the lamps the large scale features remained the same. During the experiment the cardboard base was located on tables in the middle of the room, in a way that I was able to move around the table. The size of the grid was chosen to be 2 m x 2 m which was tested to be large enough with computer simulations. If the grid would have been larger it would have become difficult to measure and move the lamps in the middle of the grid. In Holmberg's experiment a grid of 3m x 4m was used. Holmberg probably was forced to step on the grid or have some instruments to perform the measurements.

The photo cell used in the experiment was Airam UVM-8. There were several scales in the photo cell from which I used two in the experiment. Measured illuminances varied from 0 to 200 lux. The scales I used in the experiments were 0-100 lux with accuracy of 1 lux and 0-300 lux with accuracy of 5 lux, so values under 100 lux were measured with the first scale and others with the larger scale. There were also scales with a better accuracy and smaller ranges than those used, but since the typical luminosities varied between 30-150 lux, they could not generally be used. Before the experiment I measured the luminosity of one lamp with the different scales to make sure that there will be no error when switching between the different scales. I also made measurements from different radii to produce a illuminance curve. The measured curve, a fitted theoretical acceleration curve and a fitted theoreti-
The acceleration curve with softening are plotted in Fig. 13. It shows that we can produce the softened gravity law quite accurately with our instruments. To make the metering with the photo cell easier and steadier I constructed a stand for it. The stand was built of a half of a dark plastic bottle. When measuring the luminosity for a certain lamp, the stand covered the lamp and prevented it to produce error to the measurement. The height of the stand was chosen in a way that the middle of the photo cell was 5 cm above the plane of the lamps giving the illuminance measurement a softening parameter of 5 cm.

Figure 12: The photo cell used in the experiment. The stand of the detector covers the lamp for which the illuminance measurements are being done.
Figure 13: On the top: The theoretical and the measured illuminances as a function of the distance from the lamp. On the bottom: Measured illuminances and a cosine function (solid line) fitted over them.

The lamps chosen to the experiment, had power of 1.2 Watts and were working with voltage of 12 V. The basis for choosing the lamps was to achieve constant luminosity in respect to a rotation in the horizontal plane. This is mainly affected by the shape of the lamp’s filament and the smoothness of
the bulb. The best shape for a filament would probably have been a straight line, which would be positioned in the vertical direction, but no such lamp was available. The lamps I chose had filaments of a shape of an isosceles triangle without a base. The maximum proportional error in the lamps’ luminosity from different angles were 5%. A comparison of different lamps’ light intensity from varying angles is found in Fig. 14.

Figure 14: The relation between the illuminance and the azimuth angle of some lamps I tested. For the best analogy with the gravity, a flat as possible luminosity curve is wanted. The lamp 1 was chosen for the experiment as it has the smallest relative error. The absolute luminosity of the lamps is irrelevant as it can be scaled with an arbitrary mass in the experiment.
Holmberg used lamps with a vertical filament and a roughened surface to diffuse the light as smooth as possible. In Holmberg’s lamps the proportional error was 2.1%. I also tried to roughen the surface of the lamps but noticed that the amount of work needed to roughen all the lamps would have been too large in respect to the advantage gained to the constancy of light.

The lamps were fastened to sockets to make it easier to do the electric circuits, and to make the lamps more stable. The sockets were put on wine bottle corks to lift the plane of the lamps from the grid. The lifting had to be done to prevent the electric chords from covering the lamps, and to reduce the reflection of light from the grid. To the bottom of the lamp bases I glued pins to make it easy to move and attach them on the grid and to reduce the error in moving lamps. Every base was also painted black to reduce the amount of light reflected from them. Every lamp’s base was numbered with a silver marker. There were no risk to mix the lamps with each other when the galaxies moved undisturbed, but after the pericentre when the tidal structures begin to form the lamps pass each other nearby, and a mixing of the lamps is possible. Mixing the lamps with each other does not yet affect the results unless lamps with different light powers are mixed with each other.
Figure 15: Every lamp was attached to a mobile base.

The electric circuits of the experiment are presented in the figure 16. There was a 2-3 m long electric chord attached to every lamp to make it possible to move the lamps individually. The chords of individual lamps were gathered to larger packs which were attached to a proper resistor to obtain the wanted luminosity for every brightness group. Every lamp was connected parallel to the electric circuit to make it easier to find out which lamp is defective if something goes wrong. The electricity to the circuit was produced with an adjustable power supply. The voltage in the circuit was set to 11.4 V and the current to 6.8 A, to make sure that the lamps will endure the whole simulation.
Figure 16: The lamps were divided into three parallel groups with different resistors. The number of electric chords of lamps is reduced in picture to make it clearer.

Storing of positions and calculation of new positions and velocities based on measured accelerations were made by a digital computer program. In original experiment there were no electrical computers available, so most of the computations were made by hand. The article of H.J. Rood (1987)[25] mentions that Holmberg used a mechanical computer for the calculations during the experiment, but I did not consider that it was important to exactly follow Holmberg in that area.

During the experiment I took a photograph of the lamps after every step. I used Canon 450D digital camera which was linked to a laptop. The camera with a stand was tied to an air conditioning pipe in the ceiling of the room. Unfortunately the angle of view of the lens was not wide enough to place the camera straight over the experiment, so the camera had to be placed little
inclined from the simulation table. That somewhat distorts the size ratio of the galaxies, making the further objects to look smaller. To make it easier to follow the progression of the experiment I took two photographs of every step: one with the the lights on and one with the lights off.

4.2 Experiment in practice

I executed the whole experiment twice: The first time I used a smaller step and no fixed center. I made this experiment to see if Holmberg’s relatively large step size of 12 cm at the outermost ring effected the results. I also used no fixed center to see how that influences to the dynamics. Holmberg mentioned in his article about using a fixed center in his experiment, but he did not mention how many of the inner rings were fixed or what was the reason for stabilizing the center. From the pictures in the Holmberg’s article I interpreted that he kept the first ring of galaxy rigid and others were freely moving. I studied the case with some simple computer simulations and noticed that with a large step size the center of the galaxy is quite unstable and the innermost ring will deform rapidly. In the second experiment, I used exactly the same parameters as Holmberg did. Both times I used a parabolic prograde orbit and the same mass distribution as Holmberg did.

Execution of a one step proceeded in as follows: First I read the coordinates of the lamps from the computer and placed the lamps in the right positions. After that I took two photos of every step. One in which the lights of the room were switched on, and one with lights off. Next I measured the illuminances, from the directions of axes, at every lamp’s position and registered them to the computer program. The illuminances were only measured from one galaxy to save time. The galaxies were symmetric so theoretically the values of the illuminances should be the same for both of the galaxies.
Making the measurements for both galaxies, and then using a mean, would have probably decreased error in measurements but would have doubled the time used for the measurements. When all the illuminances were measured the program calculated the new coordinates and velocities for the lamps, and stored the old ones in to a table which was saved for a further analysis.

One step took about 1.5-2 hours to complete. Most of the time were spent for measuring the luminosities of the lamps. At the beginning there was a problem with the pins detaching from the lamp bases which made the first steps quite slow. But after changing the glue and gaining some experience, steps started to flow faster. On usual day I finished 2-4 steps so the first experiment lasted about a month. The second experiment in which I used the original parameters took about 5 days to complete.
5 Results

Photo sequences of both of the simulations are presented in the figures 18 and 20. For a comparison, an illustration of a prograde passage from the original paper of Holmberg is presented in the figure 17. Clearly we can see tidal features appearing during the encounters, which supports the assumption that we can study the principles of these complex large-scale processes with quite a simple model.

Figure 17: An illustration of a prograde encounter from the Holmberg's article [16].

In the first experiment, which was made with step size of 2 cm for the outermost lamps and without a rigid center, the galaxies produced long precisely defined trailing tidal tails and a bridge which connects the galaxies. The shapes and the directions of the tails are similar to those produced in
the original simulation. The feature which clearly differs between these sim-
ulations is the central bar which begins to occur after the galaxies pass the
pericenter of their trajectory.
Figure 18: Time evolution of the first simulation represented at intervals of 7 steps. Step size in this simulation was 2 cm for the outermost lamps, and the innermost ring was allowed to move freely.
Also the second simulation, which was performed with step size of 10 cm for the outermost lamps, and a rigid innermost ring, resulted in tidal tails and a bridge being formed. In my simulation, the centers of the galaxies are a bit more disordered than in the Holmberg’s simulation. An exact match in the results would be almost impossible to obtain due to the chaotic nature of the system and the large number of error sources.

Figure 20: Time evolution of the second simulation represented step by step. In this simulation exactly the same parameters as in analog simulation, were used.
Figure 19: A comparison between simulations performed with the analog and a digital computer. The used parameters of are the same in both of the methods.
Figure 21: A comparison between simulations performed with the analog and a digital computer. The rigid centers of the galaxies are colored.

One way to approximate reliability of the results is to run the simulations with a computer using the same parameters as in the analog simulations. A comparison between the simulations is represented in the figure 22.
major features in both of the simulations are the same, but the tidal tails and the central bars are more condensed in the analog simulation. One explanation for the difference could be the approximation which was made for the solid angle ($\Omega \approx A/R^2$), during the derivation of the softened illuminance (Eq. 24). The approximation starts to fail when $R^2$ is small compared to $A$. Also, errors in the light measurements or increased illuminance due reflections could have influenced the results.
Figure 22: A comparison between the analog simulations and simulations made with a digital computer. Step size of 0.03 distance units (corresponding to 3 mm in analog simulation) is used in the digital simulations to ensure the conservation of total energy.

Again, to see how the low resolution used in the simulations affected the formed tidal features, we compare the results with a simulation with a better resolution. Time evolution of an encounter of two disks both consisting of 10,000 particles is plotted in the figure 23. The disks have the same mass and the orbital parameters (step size of 2 cm) as in the former simulations, and
the same Gaussian surface density as the disk in the simulation studying the
stability of disks (figure 7). The density distribution of the disks have also
a component in the direction of z-axis (the direction perpendicular to the
plane of the disks). Z-component of the density distribution follows initially
Gaussian distribution with a scale height of $0.04 \sigma_{Gauss}$. The galaxies are not
exactly identical as before, because the initial positions of the disk particles
are defined by a random number generator following a Gaussian distribution.
$\epsilon = 0, 2 \sigma_{Gauss}$ is used in the simulation.
Figure 23: An encounter between two disks both consisting of 10,000 particles. For a comparison, the time evolution of a similar unperturbed disk is plotted also.

In the simulation of the encounter between two galaxies both consisting of 10,000 particles, the major tidal features produced during the encounter
are similar to those which were formed in the analog simulations. Again, the galaxies tend to form a central bar during the interaction as it was the case in the analog simulation without a rigid center. The results show us that with a proper softening Holmberg’s model can be used quite accurately to study the major tidal features.

As discussed in section 3.1, the galaxies used in these simulations tend to form structures in the disk even without external perturbations. Therefore, the internal evolution of an unperturbed disk is also considered in Fig. 23. A similar central bar as in the encounter is formed also without external triggering. Also, spiral arms are formed, although they remain more tightly wrapped around the galaxy than those produced during the encounter. As the bar and the spiral arms tend to form into this kind of disks by internal evolution, we must conclude that the impressive features formed during the encounter are partly triggered by the internal evolution.

To study the energy conservation of the simulations, we can calculate the total energy of the system

\[
E_{tot} = \frac{1}{2} \sum_{i=1}^{N} m_i v_i^2 - \frac{1}{2} \sum_{i,j = 1, i \neq j}^{N} G \frac{m_i m_j}{\sqrt{|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2}},
\]

where \(\vec{r}\) is the position of a particle, \(v\) is the speed of the particle, \(G\) is the gravitational constant, \(N\) is the number of particles and \(m\) is mass of an individual particle. In a realistic simulation \(E\) should be conserved during the whole simulation. Evolution of the total energy in different simulations is plotted in Fig. 24. As it can be seen, the energy is conserved only in the digital computer simulation without a rigid center. The other digital simulation, does not retain energy due to its rigid center, even though it is performed with the same step size and the integration method as the other.
Total energy is not conserved in the analog simulations due to the larger step size used in them, and the errors in the illuminance measurements.

Figure 24: The evolution of total energy in different simulations. The red lines correspond to the simulations with rigid galaxy centers and the blue lines to the freely moving ones. The dashed lines correspond to simulations made with a digital computer and the solid lines to analog simulations. Different step sizes are scaled to the step size of 2 cm for the outermost particles.

The effects of dynamical friction during the encounter can be studied by approximating the galaxies as a point mass and calculating their orbital energy

$$E = \frac{1}{2} V_{rel}^2 - \frac{G(M_1 + M_2)}{R_{rel}},$$

(28)

where $V_{rel}$ is the relative velocity of the galaxies, $G$ is the gravitational con-
stant, \( M_1 \) and \( M_2 \) are the masses of the galaxies and \( R_{\text{rel}} \) is the distance between the galaxy centers. Coordinates of the galaxy centers are defined as a median of the initially innermost particles. In the galaxies consisting of 37 particles, the seven initially innermost particles are used for the determination, and the hundred innermost in the galaxies consisting of 10,000 particles. The systematic velocities of the galaxies are defined as the mean velocity of the particles used in the determination of the galaxy center. Evolution of the orbital energy in different simulations is plotted in Fig. 25.

![Evolution of orbital energy](image)

Figure 25: Orbital energies of the galaxy pair in different simulations. The dashed lines correspond to the computer simulations and solid lines to the analog simulations. The red lines correspond to simulations with a solid center and and the blue lines correspond to the simulations without a rigid center. The black dashed line is the simulation with 20,000 particles.
Orbital energy of the analog simulations act contrary to expectations as the orbital energy is increased during the encounter, and the orbits alter from parabolic to hyperbolic. In the simulations made with a digital computer, the galaxies consisting of 37 particles lose their orbital energy, and the parabolic orbits are turned into elliptical orbits. In the simulations where the galaxies consist of 37 particles, it is difficult to determine where the center of a galaxy is, as the galaxy spreads during the encounter due to the tidal forces. Reliable results are therefore hard to obtain. In the simulation of 20,000 particles, orbital energy is slightly decreased during the encounter and the orbit is turned into a strongly eccentric elliptical orbit. As the resolution is better in this simulation, the center of the galaxy is easier to determine, and the evolution of orbital energy is more realistic. As a conclusion, we can say that using Holmberg’s analog simulation method it is hard to get reliable results of the effect of the dynamical friction to the orbital energy.
6 Summary

In this thesis I constructed the analog computer, which was presented by Erik Holmberg in 1941. Since no one has apparently repeated the simulations of Holmberg, one of the interests of this study was to find out if his simulations could be repeated. I made two simulations of galaxy encounters using the analog computer, one similar to the original simulation of Holmberg, and the other with slightly different parameters. To study the stability of the galaxies used in the analog simulations, I modeled them with a digital computer using different values of softening parameter and different number of particles. I also modeled galaxy encounters with similar parameters, as used in the analog simulations to study the accuracy of the analog method.

The results of my simulation, which was made with the same parameters as Holmberg, resemble his results to a great extent. In both of the simulations tidal tails and inter-galactic bridges were formed in a similar manner as in the original experiment. A new feature occurred in the simulation without rigid galaxy centers, as the central bars were formed to the galaxies. The simulations made with a digital computer showed that the galaxies used in the simulation will evolve also without external perturbations. This implies that the tidal features formed during the encounter could also be partly a consequence of the internal evolution of the galaxies.

This study has shown that the Holmberg’s simulations can be repeated and used for studying the galaxy encounters. It has also confirmed that the tidal tails and bridges are formed in the encounters, as stated by Holmberg.
Figure 26: A sequence of the first simulation without lights.
7 References

References


