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RISK PARITY AND INVESTOR PORTFOLIO CHOICE

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In this study, we aimed to test the performance of risk parity portfolios against classically optimized Markowitz portfolios and conclude which technique leads to better performing portfolios in well developed, informationally efficient markets.

We constructed 5 risk parity portfolios, the Inverse Volatility portfolio, the Maximum Diversification portfolio, the Equal Risk Contribution portfolio, the Alpha Risk Parity portfolio and the Beta Risk Parity portfolio and 2 benchmark classical portfolios, the Equally Weighted (1/N) and the Minimum Variance portfolio to measure performance against. The data used is the 50 constituting stocks of the EuroSTOXX50 index. The index itself was used as the market benchmark that was included in the analysis.

The study is designed in a horse race style measuring performance using mean returns, mean excess returns, maximum drawdowns, Sharpe ratios as well as diversification and information ratios. The Sharpe ratio is used as a main comparison tool to determine the winner.

Our findings indicate that a risk parity portfolio will win the horse race against a classical portfolio in an all equity asset universe, however risk parity portfolios are very sensitive to the asset universe and to make the most out of these techniques, a wider asset scope, managerial skill as well as other resources are needed which might not make them easily available to the average investor.

**Keywords**
Risk parity – Portfolio optimization – Markowitz optimization – Market efficiency
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1 INTRODUCTION

In our modern world, investors are faced with various challenges when it comes to making the optimal investment choice. Information asymmetry, different risk aversion levels, different asset management styles and the large variety in investments tools and techniques can be confusing to individual investors and challenging to fund managers and corporate investors.

Everybody wants to make the right choices when it comes to investing to achieve a seemingly simple target: Highest possible return with lowest possible risk. To perform in a fast paced financial world where markets are supposed to be efficient, a number of questions need to be addressed like what is market efficiency? How would a market be described as efficient? Is the market efficient because it has reached an economic equilibrium or because of the adaptability of investors to the changing financial environments around them? What is the best asset allocation? What is the optimal portfolio? First, the question regarding market efficiency is addressed. Fama (1970) describes a market as efficient when the stock prices reflect all the information available to investors about a firm at every moment in time. Market Efficiency Hypothesis has been tested over the years and the literature available on the topic has contradicting results, however according to Fama (1997), the opposing hypothesis of “inefficient markets” is vague and does not make sense.

Market efficiency is discussed further by Fama (1997) where he states that market efficiency hypothesis has proven itself over the years by observing that over reactions to events are mostly equivalent to under reactions, hence offsetting the short run price shocks, returning prices to equilibrium every time and validating the theory of market efficiency. Rational investors aim to maximize their utility (Sharpe 1964) and subsequently, they will always sell overvalued stocks and buy undervalued ones bringing the market to equilibrium every time, Fama (1970). Lo (2004) introduces a parallel theory to the Efficient Market Hypothesis (EMH) and he calls it the “Adaptable Market Hypothesis”. He argues that behavioral finance literature finds that markets are directed by investors’ behavioral attributes such as greed and fear rather than their rational investing choices. His theory is presented in a
way that reconciles the EMH with the objections it faces from a behavioral point of view. This will be expanded on in the next chapter of this paper.

That would lead to the next question: What is the optimal portfolio that will generate the highest returns given a certain rate of risk? Ang (2013) discusses the concept of risk preference variation among different investors from a more psychological point of view. He argues that during bad times or market crashes, losses hurt an investor more than his/her happiness with gains in good times, hence a strategy that took risk aversion into account was necessary to satisfy different needs of risk averse investors.

According to Lee (2011) the financial crisis of 2008 imposed a challenge on all investors to comprehend losing money while holding what seemed to be well diversified portfolios. He explains that this dire need for a new way to address the diversification and risk allocation problem created a set of heuristic solutions, one of those is the modern Risk parity portfolios. Risk parity is a term widely used in Finance literature to define a portfolio constructed in a way that risk is allocated equally among all its constituting assets. In opposition to the classical optimization of portfolios and the Markowitz efficient frontier that assumes that the best portfolios are the ones that provide the highest returns for a given amount of risk or the portfolios that provide the lowest level of risk for a given level of return (Markowitz 1952), the risk parity portfolios does not consider returns as an important factor in constructing a portfolio, but rather takes risk factors as the main foundation of the portfolio. This is done in various ways by practitioners and discussed widely in modern financial research. The risk parity concept is new and till this moment is receiving a lot of interest in the financial literature world. This composes a significant factor in the motivation for this paper. The topic is interesting to test and discuss both theoretically and empirically to try and find out the reasons behind the seemingly superior performance of risk parity portfolios and whether or not they provide solutions to the empirical problems practitioners have with the Markowitz optimization objective function and the assumptions of the Modern Portfolio Theory.

In this paper, we will construct 5 risk parity portfolios and use 2 classically optimized portfolios as benchmarks, then compare performance using concrete
financial tools such as Sharpe ratio, mean returns, mean excess returns, volatility, maximum drawdowns, diversification ratio and information ratio.

The 5 risk parity constructed portfolios will be as follows:

- Inverse volatility portfolio
- Equal Risk Contribution portfolio
- Alpha Risk Parity portfolio
- Beta Risk Parity portfolio
- Maximum Diversification portfolio

The 2 benchmark portfolios are as follows:

- Equally weighted portfolio (1/N)
- Minimum Variance portfolio.

The data used is monthly data of individual stocks of the Euro STOXX 50 index since they represent 50 blue-chip\(^1\) stocks representing super sectors in 12 European countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The research period is from December 2000 till February 2016 totaling 185 observation for each stock. The data analysis is split to in the sample analysis from December 2000 to December 2010, totaling 123 observations for each stock, and out of sample analysis from January 2011 to February 2016 totaling 62 observations. The 1 month Euribor rates equivalent to the observations is used as the risk free rate in this analysis.

The aim of this paper is to test if the risk parity all equity portfolios will perform better in efficient, developed markets than the benchmark portfolios and if that is the case, will risk estimators replace return estimators, hence deeming superior information and price analysis to be unimportant in the modern financial world.

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\(^1\) Blue-chip stocks are a set of stocks representing market leading firms and usually reflect a market capital in billions.
In the next chapters we will expand on the topic from the literature point of view. Next we will carry out a horse race between different risk parity and classically optimized portfolios to provide empirical results. The chapters are organized as follows: Literature Review: revising what has been discussed relative to the topic in previous research. Data and Methods section: where the methodology and data used in conducting the test are further explained. Empirical work chapter: where the steps of the analysis are illustrated. Results: where the main findings and room for improving this research are discussed. And finally, Conclusion: summarizing the key points discussed in the paper.
2 LITERATURE REVIEW

This section is dedicated to introducing the main idea behind this research, previous discussions in financial literature, definitions to key elements in the research, providing the theoretical background of this paper and the motivation behind the topic choice. It shows that previous research has discussed the issues of market efficiency, optimal portfolio choice and risk parity separately and hence this paper is done in the attempt to introduce a link between those 3 important topics and this is what distinguishes this paper from previous research.

2.1 Classical Portfolio Optimization Problem

The choice of a sound and profitable investment portfolio is a risky business. This is one of the reasons to study if the markets are efficient. Market efficiency is one of the most researched financial topics of all times as the concept is important to all market participants. Investors would like to know if the market is efficient and all company’s information is reflected in its stock price or if there are opportunities for arbitrage and making risk free profits. The importance of this stems from the fact that prices are used as proxies for return. According to Markowitz (1952), a rational investor will always seek higher returns given a certain amount of risk. This makes understanding stock price movement of utmost importance to a rational investor. The investor seeking to maximize his profits and minimize his losses, needs to understand the market dynamics and reach the optimal level of asset allocation in a portfolio in order to be able to diversify any asset specific risk and bear only systematic risk in the portfolio which is by default rewarded by the market in the form of premiums on risky assets.

Fama (1970) addresses the market efficiency question from an informational point of view following an approach that states the basic assumption that market information is fully reflected in stock prices. According to Fama (1970), stock prices react to new information by over reacting, meaning: prices going up, or under reacting, meaning: prices going down almost immediately, hence validating the assumption of efficient markets. Should the stock prices fail to reflect new information, that would suggest that the market is not fully efficient. To expand on this idea, price shocks work as a
proxy for how efficient a certain market is. The ability of prices to adjust quickly to shocks and return to a state of equilibrium would mean an efficient market, where prices give a realistic idea to investors about the quality of a firm. Markets that crash in response to new information would be highly inefficient and hence there can be high chances for arbitrage and risk free profits, but also will be highly volatile and prices would not act as a good proxy for firms’ quality.

Singal (2006:1-2) clarifies how the market takes care of its own inefficiencies based on the transparency and availability of information as market participants will behave rationally to sell over valued stocks or hold on to undervalued stocks which will lead to price adjustments and reaching full efficiency. This, however, is applicable in developed markets where laws and regulations are ensuring transparency of information and fighting insider trading. Research have also addressed the question of return predictability in emerging markets. Harvey (1995) finds emerging markets to be highly volatile and lacking information which creates high profit opportunities but not without high risk. This might not be a suitable option for the average risk averse investor, however might provide a good opportunity for profiting for professional fund managers and investors with high level of skill and risk tolerance.

In this paper, top notch blue-chip stocks of the Euro STOXX 50 index are used to construct classical portfolios and risk parity portfolios, demonstrating full informational efficiency and transparent reporting. The analysis is conducted in and out of the sample, and the results should show how each portfolio is performing in an efficient market.

2.1.1 Return predictability and the CAPM

Sharpe (1964) and Linter (1965) are considered one of the earliest scholars to produce a viable tool to predict returns considering the risk involved, that tool being the Capital Asset Pricing Model or the CAPM as it is widely known. The CAPM, however is a controversial tool as it is widely used by finance professional and scholars but also widely criticized in Financial literature as it fails to capture some cross section return anomalies. (Jagannathan and Wang 1996).
The CAPM model is a simple regression formula that is used to price risky assets based on the risk free rate and the overall market compensation to risk as follows:

\[ E(R_i) = R_f + \beta_i (E(R_m) - R_f) \]  

(1)

where:

- \( E(R_i) \) = the expected return on the asset.
- \( R_f \) = the risk-free rate of interest.
- \( \beta_i \) = the sensitivity of the expected excess asset returns to the expected excess market returns, it is the riskiness of the \( i^{th} \) asset and is calculated using the formula:

\[ \beta_i = \frac{Cov(R_i,R_m)}{Var(R_m)} \]  

(2)

- \( E(R_m) \) = the expected return of the market.
- \( (E(R_m) - R_f) \) = is the market premium.
- \( E(R_i) - R_f \) = the risk premium.

The CAPM model however is challenging to work in real life as it has specific assumptions that cannot be found in a real world empirical setting. Perold (2004) states that for the CAPM model to work, four assumptions must be made. The first is assuming that all investors are risk averse and assess their portfolios only based on returns and volatility of those portfolios. The second is that all investors in a specific market can make the same investments according to their choices and preferences, i.e. all investment opportunities are available to all investors. The third CAPM assumption according to Perold (2004) is that the financial markets are 100% efficient in the sense that all information about all firms are available to all investors, there are no transaction costs or taxes, all investors can borrow and lend freely at the risk free rate, short selling is available to all investors without restriction and that there is enough of any asset to satisfy all desiring investors at any point of time. Fourth and final assumption is that all investors have the same evaluations of both expected returns and volatility of the same asset at the same time. (Perold 2004). These above mentioned assumptions provide a very unrealistic view of a financial world that does not exist and to simplify the world in such terms will not provide accurate or insightful analysis to investors.
The CAPM bases the return received by investors only on betas of portfolios or the sensitivity of the portfolio to the overall market risk, since asset specific risk should be diversifiable. But in practice, this rule is violated many times. It is observed that other factors can affect returns and not just betas. Accordingly, the CAPM was subjected to many adjustments and modifications over time. For example, The Fama and French (1993) three factor model adding the size, book-to-market equity value to the overall market risk factor in the analysis to obtain more relatable results and provide investors with better analysis. Including those 3 factors enriches the CAPM model and incorporate valuable information about firms in the analysis. This information can very well affect the stock price and gives intel about the quality of the firm and hence can affect the investor choice. Further adjustments were introduced by Carhart (1995) adding a one-year momentum factor to the analysis. Cochrane (1999) expresses how the CAPM that was once perceived as a useful tool in explaining difference in average returns for different assets, no longer work and various fluctuations could not be explained by the CAPM. He states it as an “old fact” that the CAPM was assumed to work because it referred any movement in an asset’s return to its beta. He, then expands that the “new fact” would be that the CAPM does work empirically provided that it is used as a multi-factor model because some assets return movement is explained by factors other than their beta.

This leads to analyzing various portfolio optimization techniques and how well they are performing in the real world. The following sections will be devoted to the theoretical framework and discussing various classical and more modern portfolio construction techniques. I begin with the classical Markowitz optimization.

2.2 Markowitz Optimization

In this section, a detailed explanation of the Markowitz optimization techniques, two classical examples of optimal portfolios, the equally weighted portfolio and the minimum variance portfolio. Also the criticism and shortcomings of the model are explained with more detail.
2.2.1 The Markowitz Optimization technique

The classical Markowitz optimization was first introduced by Harry Markowitz in 1952 and since then, it has been applied, tested and modified by finance researchers and portfolio managers over the years.

The Markowitz optimization bases portfolio selection on the preference of higher returns and lower volatility of those returns (Markowitz 1952). Assuming rationality of investors, Markowitz (1952) finds high returns to be a good measurement for portfolio performance accompanied by a low standard deviation. He also stresses on the importance of diversification in portfolio choices, and highlights the mean – variance rule which explains that the best portfolio is the one with the highest mean returns and lowest variance of those returns. The idea here is that investors only have one goal, and that is high returns, and they want to achieve that goal with a minimal amount of risk, hence we can assume accordingly that all investors are risk averse.

According to Donohue (2010), the Markowitz optimization is based on optimizing a quadratic objective function as follows:

\[ Z^* = \text{max} \ (\mu^T - \frac{\lambda}{2} x^T Q x) \]  

Where:
- \( Z^* \) is the optimal objective value to the mean-variance optimization problem
- \( \mu \) is the mean returns of the portfolio
- \( \sigma_i \) is the standard deviation of asset i
- \( Q \) is the covariance matrix between portfolio assets

Donohue (2010) stresses the importance of the model but acknowledges its shortcomings like sensitivity to inputs and obvious mismatches between inputs and outputs and hence introduces his own tweak on this model to try to solve these problems. This will be expanded on later in this paper when we discuss in more detail the criticism and shortcomings of the Markowitz optimization model.
For years, the Markowitz optimization framework and the insight behind it which is maximizing returns while simultaneously minimizing risk have been used by investors and fund managers. The general idea of diversification has resulted in two portfolios aiming to give maximum diversification benefits and according to the Markowitz rule, highest returns and lowest volatility. Those two portfolios are the Equally weighted portfolio and the minimum variance portfolio.

2.2.2 Equally Weighted Portfolio (1/N)

Equally weighted portfolios are constructed in a way that aims to maximize diversification by allocating equal weights to each asset in the portfolio. This technique is referred to as “naïve diversification” as per Benartzi and Thaher (2001). They also state that this technique is very old and go as far as saying that it was mentioned in the Talmud. They stress on the fact that is it very widely used and was used by Markowitz himself where he would put equal weights of various assets in his personal portfolios to “minimize his future regret”. This technique is supposed to provide good diversification, however in practice assigning equal weights to assets with different risk levels does not really provide diversification. A 50-50 equity and bond portfolio is not well diversified as most of the portfolio risk is concentrated in its risky asset, being the equity in this case. DeMiguel, Garlappi and Uppal (2009) perform a study using the 1/N technique applying fourteen models over seven different data set and conclude that it is very hard to beat the equally weighted portfolio consistently using the mean-variance optimization techniques. However, DeMiguel et al. (2009) stress that they do not endorse the empirical use of the 1/N but they state its importance as a benchmark to measure the performance of other portfolios against. This approach is also followed in this paper where the equally weighted portfolio is used as a benchmark portfolio to test the performance of risk parity portfolios which is the main objective of this study.

2.2.3 Minimum Variance Portfolio

Minimum variance portfolios are constructed in a way that assigns weights to assets in a portfolio based on the individual volatility of each asset in order to minimize the variance of the total portfolio. In other words, assets with higher volatilities are given
lower weights in the portfolio than those that have lower volatility. According to Clarke, De Silva and Thorley (2011) empirical research has found minimum variance portfolios to be appealing to investors as historically low volatility stocks have yielded returns that met or in some cases surpassed the market. Kempf and Memmel (2003) introduce a one-step method to estimate optimal weights for assets to construct a global minimum variance portfolio. Assuming there are N asset returns considered, the weighting goes as follows:

$$ W_{mv} = \frac{\Sigma^{-1}1}{1\Sigma^{-1}1} $$

Where:

- $W_{mv}$ is the weights of the minimum variance portfolio
- $1$ is a column vector of ones
- $\Sigma$ is the N x N matrix that includes the return variance and covariance $\sigma_{ij}$.

They show that their model provides lower variance levels than estimators used in literature.

### 2.2.4 Criticism and shortcomings

The finance literature world almost agrees that the Markowitz optimization is the pillar of financial analysis that many milestones of financial literature was based upon. The model however remains controversial in the sense that it has also been criticized and modified over the years. The model has been criticized repeatedly, but according to Donohue (2010) the criticism area focuses on two aspects: the first being the assumption of normal distribution of return forecasts and the type of inputs needed to reach that. The second being, the high sensitivity of outputs to any change in inputs. Michaud (1989) questions the Markowitz optimization from this direction as well, the sensitivity of output to slight changes in input, and states that it is justifiable why many real world financial practitioners are not using it because it is an “estimation error maximizer”. He also argues that “$MV$ optimizers have serious financial deficiencies that often lead to financially meaningless optimal portfolios”. “$MV$” here refers to mean-variance optimizers.
Various models have been introduced in an attempt to fix what was deemed to be problematic with the Markowitz MV model. Also many models were introduced to replace it altogether. Brodie, Daubechies, De Mol, Giannone and Loris (2009) also find the Markowitz framework to be overly sensitive to changes in data estimates, specifically returns and introduce a regulation method to the model that they believe will stabilize these errors. Their attempt is one of many to include other factors and to stabilize the seemingly empirically unstable model. Ehrgott, Klamroth and Schwehm (2004) also try to develop an equivalent to the Markowitz model as they find it problematic and time consuming to estimate the covariance matrix from historical data. Konno (1990) discusses several shortcomings of the Markowitz quadratic model and introduces a linear model to replace it. He states that his model is easier to construct and practical in use as quadratic programming is very difficult with bigger volumes of data. He also describes the model as unsuitable to many investors as it needs a lot of resources in terms of time and money to run such a portfolio which might not fit the needs or the capabilities of many. Škarica and Lukač (2012) address the Markowitz model’s weakness from its unrealistic assumption about the lack of transaction costs or taxes and consequently producing many long-short positions that could not be applicable in real life since there are high transaction costs in financial markets. They introduce their own model that includes transaction costs in the analysis.

Scherer (2002) on the other hand, refers the error maximization error argument to the fact that the data itself is flawed and suggests a refined method to input data into the model rather than modifying the model itself. He refers to this process as” resampled efficiency”. Donohue (2010) works in the same direction where he states that the model in its classical form produces complex and almost inapplicable solutions and confirms that the framework needs to be modified in some way. However, he addresses the estimation error issue with a different perspective through adjusting the choice of estimates rather than modifying the mean-variance model itself. He builds a framework that consider assets weights in the scope of optimality rather than the general approach of optimizing the quadratic objective function.

The fact that fund managers and finance practitioners are not using the Markowitz optimization more often or the situation described as the “Markowitz enigma” by
Michaud (1989), the rapidly changing financial world, the financial crises that have happened, the many investors losing money even though they thought their portfolios were well diversified and that they were protected have created a new need for understanding risk and practicing diversification in a better way. New heuristic methods are introduced to solve portfolio choice and market prediction problems, one of which has risen to popularity among scholars and practitioners; that new method being risk parity portfolios.

In the next chapter, the Adaptable Market Hypothesis is introduced and discussed in the light of more heuristic and practical points of view as risk parity solutions are not based on optimizing an objective function but rather on maximizing diversification benefits and providing better risk allocation among portfolio constituents. This theory provides a link from the world of theoretical approaches to practical and behavioral ones that led to the risk parity mind set and practical uses.

2.3 Adaptable Market Hypothesis

Lo (2004) starts his argument by stating what has been discussed in a previous section of this paper about the Efficient Market Hypothesis (EMH) and how the EMH is one of the most studied topics in the history of finance. He points out the fact that after over thirty years of research, the economic and financial community has not come to a consensus on whether or not the markets are efficient as traditionally stated by the EMH. The theory of Adaptable Market Hypothesis (APH) is heavily based on the concepts of “evolutionary psychology”. The theory is meant to utilize behavioral concepts to analyze and explain why investors do what they do and choose specific things in specific times. Lo (2004) states that in studying behavioral variables such as kindness, fairness, natural selection and other human attributes, a theory that harmonize the EMH with all its behavioral counterparts to reach the new AMH which he describes as “revolutionary”.

Lo (2004) begins by analyzing the concept of “bounded rationality” introduced by Simon (1955). The concept discusses the actuality versus the economic expectation of investor behavior. Simon (1955) argues that investors are bound by certain limitations such as their individual specific abilities and amount of wealth available
to them and consequently, many times it is hard for them to rise up to the optimization level imposed by general economic theories. Instead, he explains, they tend to make “satisfying” rather than “optimal” choices according to their capabilities. In other words, humans are limited by their individual rationality and mental capacities, and so they might not make the best or the “optimal” choice in every situation, but the one that personally satisfies them the most. This contradicts one of the core assumptions of the EMH where investors’ unlimited rationality is assumed and defined in the sense of them wanting to achieve the highest return for any given amount of risk they are taking (Markowitz 1952). According to Lo (2004) the most important question addressed to Simon was about the method that can be used to know the point at which an investor is “satisfying” and not “optimizing”. He then provides the answer that this happens through a natural selection process. Investors will continue to make mistakes and learn from them in order to reach the most satisfying level they can reach at a specific point of time and mean while these choices will be adapting to the negative or positive feedback the investor is receiving from the market and this process will eventually lead to almost optimal choices as investors are learning to make better choices through trial and error (Lo 2004). Lo (2004) further explains that the AMH is basically the same as EMH but from an evolutionary perspective, where prices reflect all information available but only as allowed by the market participants’ collective behaviors along with the nature or the state of the economy in which they are functioning. The investment strategies based on AMH are heuristic approaches based on the amount of competition, the current business cycle as well as the amount of profitable opportunities available for the time (Lo 2004).

The AMH is starting to obtain momentum in economic and financial research. Empirical studies have been done to test whether or not investors’ behaviors and investment choices can be explained by the AMH. Kim, Shamsuddin and Lim (2011) study the AMH by analyzing a century long United States data, from 1900 to 2009, of Dow Jones average industry indices. They perform their analysis through two types of autocorrelation tests, those being the variance ratio and portmanteau methods. Their empirical findings are in favor of the AMH as they find a strong link between changes in market conditions and return predictability. Todea, Ulici and Silaghi (2009) perform a Moving Average (MA) analysis to six Asia-Pacific markets
for the period from 1997 to 2008. Their main finding is that the profitability of the MA strategies is time varying and cyclical in nature and this is in support of the AMH theory. Neely, Weller and Ulrich (2009) also find evidence supporting the AMH, however this time their study is conducted on the intertemporal stability of excess returns in foreign exchange markets. They find that profitability opportunities that existed in the 1970s and 1980s have disappeared in the 1990s, hence supporting the adaptability of investor behavior explained in the AMH.

These heuristic attempts, the trial and error method and investors’ adaptability to the investment opportunity set available to them in a certain market at a certain time can provide a good link between the strict theoretical Markowitz model and the practical approach of risk parity and the mentality of risk allocation. The mindset behind both types of thinking is different. One being classically normative, focusing on how the world should work and found to provide little practical success. The other being heuristic, based on trial and error and human evolving in response to feedback received on right or wrong choices, business cycles and changing market conditions.

In the next section, we expand on the idea of risk parity from the definition to the history and empirics behind it.

2.4 Risk Parity

As discussed previously in this paper, in the light the failure of the Markowitz optimization to provide solid empirical success in the portfolio choice problem, new heuristic models have been introduced. Risk is divided into asset specific risk factors and distributed among the constituents of a portfolio equally to maximize diversification. Risk budgeting as a means of assigning weights to assets in a portfolio is widely discussed in the recent financial literature. In this section, the risk parity definition, benefits, and criticism are discussed in more detail.
2.4.1 Risk parity: Definition

According to the Merriam Webster dictionary, the word parity means: “the quality or state of being equal or equivalent. The name is a direct way of identifying the main characteristic of risk parity portfolios. They are portfolios that are built in a way that assigns equal risk to each building block/asset. As discussed previously, risk is divided between systematic risk or a portfolio beta that measures the tendency of the portfolio to move with the market and asset specific risk. The market or systematic risk is undiversifiable, but the asset specific risk is. The diversifying of the asset specific risk is the goal of theoretical and practical approaches in order to maximize investors’ gains and minimize their losses. In this section, a brief definition and explanation for the risk parity portfolios and how they work is discussed.

According to Clarke, De Silva and Thorley (2013) the concept of risk parity has come a long way since it was first introduced in the 1990s. They discuss that the term was originally used to describe portfolios where weights are assigned to assets according to their class inverse volatility. In other words, the higher the volatility of an asset, the less weight it is assigned in the portfolio. Qian (2006) was the first to introduce the more modern concept of risk parity defining it as assigning weights to assets in a portfolio such that the risk contribution is equal among all portfolio constituents. Bruder and Roncalli (2012) argue that the increasing risk aversion levels of individual investors after the financial crisis of 2008 has led to the new investment models based on risk budgeting and better diversification, rather than return forecasting. They emphasize that the new process of investors’ choice is based entirely on what level of risk they are willing to take, not on what amount of return they are anticipating putting risk allocation “at the heart of the investment process”. Hurst, Johnson and Ooi (2010) explain that many times portfolios are assumed to be diversified since they contain many equities however true risk diversification can only be achieved through diversifying asset classes in a portfolio between equities, bonds and commodities depending on the state of the economy. They make a note that equities perform well in times when the economy is thriving, while fixed income assets perform better in periods when the economy is not doing well. They further confirm that portfolios heavily depending on equities, which is typically riskier than other asset classes, for example bonds, are in fact not diversified at all, but rather
putting a concentrated risk on the investor that can be avoided by widening the asset classes used to construct a portfolio leading to better performance in the long run. According to Lee (2011) the theoretical framework of why risk parity portfolios outperform classical portfolios is yet to be found. He explains further that should such a theory be found it will impose a question on the importance of market informational efficiency since the whole approach is based on risk forecasts and not return forecasts. The investor who does not have an estimate of returns is expected to perform better than the one who actively seek return forecasts according to the risk parity theory (Lee 2011).

Maillard, Roncalli and Teiletche (2008) address the same point of view saying that equal risk contribution approaches have shown ex-ante success in maximizing diversification but without an appropriate theoretical framework that can be generalized. This implies that investors who will stop seeking superior information regarding asset prices and obtain better risk estimators can still outperform even though that is an obvious contradiction with the Modern Portfolio Theory and the Market Efficiency Hypothesis. Chaves, Hsu, Li and Shakeria (2011) find that the outperformance of risk parity portfolios in comparison with classically optimized ones is highly dependent on the investment universe and as a result it is very important to act with absolute care. They consider risk parity strategies to be “more of an art than a science for the time being”.

2.4.2 Risk Parity: Empirics

Lee (2010) questions the measuring of the success of risk parity portfolios and the implied superior diversification they are claimed to have by discussing the diversification measures used to test it. He stresses on the idea that in the absence of a theoretical framework and a constant objective function, the measuring is done through old tools like risk adjusted returns or Sharpe ratios which make the researchers still revolve in the Modern Portfolio theory circle even when they claim they are not. Lohre, Opfer and Orszag (2014) discusses the same point which is the very definition of diversification and what it means. They conclude that in agreement with Meucci (2009) diversification is the equal distribution of portfolio volatility on its constituents, given that the risk factors of those constituents are uncorrelated.
Roncalli (2014) addresses the criticism that risk parity techniques received in literature because it focuses on risk budgeting rather than portfolio performance and this makes them less actively managed and he includes excess returns into constructing the portfolios in order to integrate equal risk allocation and performance indicators such as excess returns and volatility.

Hurst et al. (2010) on the other hand discusses risk parity portfolios as the “*truly diversified portfolios*” that minimize investors risk. They design a strategy that involves various assets that are less risky than equities and rebalance their portfolio weights according to the desired level of volatility set in the beginning. They call their method the “Simple Risk Parity Strategy” and confirm it outperforms the traditional portfolios using 40 years of historical data in the analysis. They, however stress that their portfolio is not promised to hold that superior performance in any other medium of assets or different investment environments.

Alvarez, Luo, Cahan, Jussa and Chen (2013) first discuss the correlation between different indices in different countries, and while they are in favor of including other non traditional asset classes such as commodity futures or oil related stocks, they make the observation that since some indices contain oil related stocks, this means there is a correlation between the index performance and oil as a commodity and conclude that asset assigning should be handled carefully to avoid such correlations between asset classes to ensure maximum diversification. In their empirical study, they stress the outperformance of risk parity portfolios over the traditional 60-40 portfolio -that includes 60% equity and 40% bonds- and encourage investors to use risk budgeting in setting up their portfolios. They also differentiate between risk parity portfolios and maximum diversification (MD) portfolio mechanics in their analysis, conversly in this paper the MD portfolio is considered one of the risk parity portfolios. They also discuss the Alpha Risk Parity (ARP) portfolio used in this paper as a way of integrating excess returns into the construction of risk parity portfolios. According to Qian (2011), risk parity portfolios provide higher Sharpe ratios and mean returns than classical 60-40 (stocks to bonds ratio) portfolios.

Chavez et al. (2011) reported their findings that indicate that risk parity portfolios might not cosistently beat an equally weighted portfolio accross all mediums,
however they do outperform Markowitz efficient portfolios incessantly. Roncalli and Weisang (2012) state that risk parity is becoming popular in financial spheres and they discuss previous research while introducing their own model that they call “risk factor parity” where risk factors are chosen as a basis of asset allocation in a portfolio. Their reasoning behind focusing on risk factors is that portfolios in many cases carry hidden risks and they introduce their model in an attempt to overcome that problem. They stress on the importance of the asset universe when studying which risk an investor should diversify and break down risk into risk components or factors that their model diversifies effectively according to their findings.

2.4.3 Criticism and shortcomings

As the risk parity technique is still relatively new, financial literature is handling it with a mix of deep interest and care. The sound scientific theory explaining the apparent success of the risk parity based portfolios has not been introduced fully. However, risk parity techniques have been found by various researchers to be heavily dependent on the asset class universe, for example: Chavez et al (2011), Roncalli and Weisang (2012) and Alvarez, Luo, Cahan, Jussa and Chen (2011). This indicates that the asset medium is important for the success of such risk based portfolios and hence empirical results cannot be generalized to a different medium of assets. The findings of the previous research papers mentioned focus on the diversification based on asset classes as portfolio constituents and it is considered common knowledge that equities are the source of risk in a mixed asset portfolio, hence most techniques are designed in a way to diversify away the risk of the portfolio by focusing more on investing in non-equity investment tools such as fixed income or commodities for example. If this is proven true, the risk parity portfolio techniques will only be valid for mixed asset portfolios and there is no conclusive evidence that this is the case. In this paper, we use only equity data to test whether or not an all-equity risk parity portfolio beat the classical portfolios.

Expanding on the idea of widening the asset base of portfolio constituents, this might be very hard for regular investors in real life who might only have access to a limited number of popular or well-known assets such as stocks and bonds. The problem with the apparent success of risk parity portfolio is the same problem with the
assumptions of the Modern Portfolio Theory; that being no transaction costs and availability of resources to all investors equally. Risk parity is still not a confirmed and well established science, hence it relies heavily on the investor’s skill and ability to understand and carry out such techniques. Also, the constant rebalancing required to keep the weights and the amount of short selling that might be necessary again makes this a difficult choice for the average investor.

On the other hand, Institutional investors and high net worth individuals who have wider knowledge and funding base might be more able to use these techniques and make full use of the potential that they provide. This can be considered a downside or not, since most investment tools are designed for institutional investors and even simple equity and fixed income investments are best made use of by such investors who have the means and resources to best utilize them.
3 RESEARCH PROBLEM AND HYPOTHESES

In this paper the concept of risk parity portfolio performance in developed markets is discussed. The aim is to analyze the performance of portfolios constructed using risk parity techniques and compare that to the performance of portfolios constructed using classical Markowitz optimization and determine if the risk parity portfolios outperform consistently in developed, informationally efficient markets with all-equity portfolios. Performance will be tested using different financial tools such as Sharpe ratios, mean returns, volatility, maximum drawdowns, diversification ratio and information ratio of each portfolio in a horse race manner to evaluate which is the best performing portfolio constructed using different techniques.

In case of risk parity outperformance, this will raise the question of the importance of informational efficiency and whether investors should still seek superior information in financial markets to make more profit. Since risk parity portfolio construction is based entirely on risk factors, a risk estimator will be the necessary factor investors seek to better diversify their portfolios instead of stock prices that are currently used as a proxy to estimate returns as maximum returns is the base of Markowitz optimization.

The hypothesis tested will be:

Risk parity portfolios have higher Sharpe ratios than classically optimized portfolios such as equally weighted portfolio and minimum variance portfolio assuming market efficiency.
4 DATA AND METHODS

4.1 Data used

As mentioned in the previous sections in this paper, this research aims to reach the optimum portfolio construction technique and whether in the post financial crisis world, information about stock prices and return estimates will still hold the same status when it comes to portfolio choice or will this be replaced by risk estimators. The data used in the analysis needed to reflect market informational efficiency in order to be able to fairly evaluate risk parity portfolio performance and compare it to classically constructed ones.

The data used is monthly data of individual stocks of the Euro STOXX 50 index since they represent 50 blue-chip stocks representing super sectors in 12 European countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The reasoning behind this choice is that they represent fairly well established companies in developed markets. These stock prices should reflect by default the informational efficiency of the market. Blue-chip stocks are for large well established firms that have been operating for a long time and usually have market capitalization in billions. A blue-chip stock company is a market leader, usually among the top 3 in the industry and this reflects the assumption of transparency and information availability about the firm and the stock to investors. Also the data is well organized and readily available for all the index constituents. The stock prices data is obtained from Thomson Reuters DataStream data base.

The research period is from December 2000 till February 2016 totaling 185 observation for each stock. The data analysis is split to a sample period analysis from December 2000 to December 2010, totaling 123 observations for each stock, and an out of sample analysis from January 2011 to February 2016 totaling 62 observations. The 1 month Euribor rates equivalent to the observations is used as the risk free rate in the analysis as it seems like the most fitting rate to use to match the data. The data is monthly returns of a European index and so the 1 month Euribor rate seems suitable. The Euribor rates data is downloaded from Europe Central bank historical
data warehouse website. Market returns are the returns of the Euro STOXX 50 index itself calculated on a monthly basis to fit the data. This seems logical since, all 50 stocks of the index are used to construct the portfolios. Since the data covers over 25 years of monthly returns, this should give an idea about the performance of the risk parity portfolios over a relatively long period of time.

4.2 Methodology

I aim in this research to construct a horse race between 5 risk parity portfolios and 2 classical portfolios to measure and compare performance in order to test for the hypothesis which is whether or not risk parity portfolios can consistently outperform classically optimized portfolios in an efficient market. Performance will be tested using widely accepted financial tools like Sharpe ratios, mean excess returns, variances, information ratio, diversification ratio and maximum drawdowns. The portfolios will be constructed using R financial program and all the financial analysis will also be done in R.

First, I will code all the risk parity portfolio functions into R and build the portfolios. Next, I will construct an equally weighted portfolio (1/50) and a minimum variance portfolio to use as benchmarks. After constructing the portfolios, I will start the analysis and calculate the Sharpe ratios and arrange the portfolios in a descending order as the higher the Sharpe ratio the better the performance and this will answer the first question of this paper which is: Do risk parity portfolios outperform classically optimized portfolios consistently in efficient markets?

According to the results of the horse race, I will try to answer the consequent question which is: Does the outperformance of risk parity portfolios lead to the replacement of market informational efficiency with analysis of risk estimators? Based on this, should investors stop seeking superior information for purposes of return estimation and try to obtain better estimations of risk factors?

The horse race analysis will follow the analytics provided by Chaves, Hsu, Li and Shakernia (2012) and Clarke, De Silva and Thorley (2013) as they provide a step by step technical analysis of risk parity portfolio construction and the reason behind it.
According to Clarke, De Silva and Thorley (2013) the minimum variance portfolio unifies the marginal risk contributions (MRC) of each asset in a portfolio in contrast to the risk parity portfolios that equalize the total risk contribution (TRC) of each asset. This puts the risk parity portfolios inside the efficient frontier curve rather than on it as illustrated in figure (1) below:

![Figure (1) Risk Parity and efficient frontier curve.](image)

According to Markowitz (1952), the efficient frontier is the combination of all the possible “efficient” portfolios given a specific level of risk. That being given a set amount of assets, no portfolio can achieve higher returns at the same level of risk and no other portfolio can achieve a lower level of risk at a certain level of return.

### 4.2.1 Risk Parity mathematical analysis

Following the Chaves et al. (2012) model in mathematically defining risk parity the model goes as follows:
Let $r_i$ represent returns of asset $i$ and $x_i$ represent the weight of asset $i$ in the portfolio. The returns of the portfolio will be calculated with the simple formula of:

$$r_p = \sum_{i=1}^{N} x_i \cdot r_i$$  \hspace{1cm} (5)

And given that $\sigma_{ij}$ represent the covariance between asset $i$ and asset $j$ the standard deviation of the portfolio will be calculated by:

$$\sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{ij}}$$  \hspace{1cm} (6)

Marginal Risk Contributions (MRC) are calculated using the following formula:

$$MRC_i = \frac{\partial \sigma_p}{\partial x_i} = \sum_{j=1}^{N} x_j \cdot \sigma_{ij} = \text{cov}(r_i, r_p)$$  \hspace{1cm} (7)

And Total Risk Contributions (TRC) are calculated using the following formula:

$$TRC_i = x_i \cdot \frac{\partial \sigma_p}{\partial x_i} = \sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{ij} = x_i \cdot \text{cov}(r_i, r_p)$$  \hspace{1cm} (8)

Equations (7) and (8) will be discussed further in the constructing portfolios subsection of this paper to show how they will be used in the portfolio construction process and to empirically show the difference between utilizing the risk component in risk parity portfolios and the classic minimum variance portfolio as explained by Chavez et al. (2011).

**Constructing Portfolios**

**Minimum Variance Portfolio (MV):**

Using the above mentioned equations, particularly equation (7), constructing an MV portfolio is simply equalizing all marginal contribution to risk (MRC) of all assets. If one asset has higher MRC, the weight of that asset will be reduced in the portfolio to achieve that goal. This has the potential problem of
excluding certain assets just because they have high volatility at the specific
time of constructing the portfolio.

- **Equally Weighted portfolio (EW):**
  Using the straightforward formula $1/50$ assigning equal weights to all stocks
  in the portfolio as there are 50 individual stocks in the index used. The naïve
diversification method is an astounding benchmark in financial research and
provides insight on the performance of other portfolios by comparison. Also,
the equally weighted portfolio has been found to perform well within certain
asset classes in previous research discussed in the literature review part of
this paper.

- **Risk Parity Portfolios:**
  The idea behind equalizing risk contribution can easily and straightforwardly
  be applied by equalizing the total risk contribution (TRC) of all assets
calculated in equation (8).
  In this paper 5 different risk parity portfolios are constructed and their
  performance is measured in a horse race style against classical two
  benchmark portfolios. The 5 portfolios are explained in more details below:

- **Inverse Volatility Portfolio (IV)**

  This portfolio is considered to be a simple application of risk parity techniques. It
  relies on assigning the same volatility weight to each constituent of the portfolio in
  inverse proportion as follows:

  $$W_{i}^{IV} = \frac{1}{\sigma_i} \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i}}$$  \hspace{1cm} (9)

  This portfolio is one of the earliest risk parity portfolios and it has a shortfall of
  overlooking correlations between assets and blindly excluding assets with high
  volatility by definition. The over simplification of the underlying assumptions of this
type of portfolio construction technique, such as overlooking correlations between
assets, might not make it the best choice, however for the sake of diversifying and testing various risk parity techniques, it was included in this paper.

- **Equal Risk Contribution Portfolio (ERC)**

In a way to overcome the problem of overlooking correlations and focusing only on volatility levels in the previous IV portfolio, the ERC is constructed in a way that unifies the risk budget allocated to each of the portfolio constituents. In other words, the Marginal Risk Contribution (MCR) must be equal across all assets used in constructing the portfolio as follows:

$$W_{i}^{ERC} = \arg \min (w) \sum_{i=1}^{N} \sum_{j=1}^{N} \left( w_{i} \cdot \text{cov}(r_i, r_p) - w_{j} \cdot \text{cov}(r_j, r_p) \right)^2$$

MCR is calculated as the product of the weights of the assets, the standard deviation of the portfolio and the correlation between the asset and the portfolio.

This way both volatility and correlations are considered when constructing the ERC portfolio.

- **Alpha Risk Parity Portfolio (ARP)**

To further include more important factors into the portfolio construction techniques, this portfolio is constructed based on budgeting risk among assets in a portfolio according to their "Alpha" or excess returns as follows:

$$W_{i}^{ARP} = \arg \min (w) \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{w_{i} \cdot \text{cov}(r_i, r_p)}{\alpha_i} - \frac{w_{j} \cdot \text{cov}(r_j, r_p)}{\alpha_j} \right)^2$$

This portfolio construction technique was introduced by Alvarez et al. (2011) as a way to include a portfolio’s alpha into the analysis and provide even more insight on the risk parity portfolio performance testing.
- **Beta Risk Parity Portfolio (BRP)**

Another variation on the ARP method, however assigning risk budgets to each portfolio constituent according to its Beta or systematic risk as follows:

\[
W_{i}^{BRP} = \arg \min(w) \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{w_i \text{cov}(r_i, r_p)}{\beta_i^2} - \frac{w_j \text{cov}(r_j, r_p)}{\beta_j^2} \right)^2
\]  

(12)

Beta measures the sensitivity of a specific asset to the market movement as a whole and hence is a measure of the systematic risk within a portfolio. The less sensitive an asset to the market shocks, the lower its beta estimator will be.

- **Maximum Diversification Portfolio (MD)**

This portfolio is constructed in a way that aims to increase the distance between the weighted average volatilities of the assets in the portfolio and the total volatility on the portfolio as follows:

\[
W_{i}^{MD} = \arg \max(w) \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w^T \Sigma w}}
\]  

(13)

This means that the lower the correlation between particular assets, the larger the weight they will be assigned in the portfolio.

4.3 **Empirical work**

This section is dedicated to discussing the process of constructing and analyzing portfolio performance. The constructed portfolios will be done using R, programming the previously discussed formulas and obtaining performance analytics and arrange portfolios in a horse race style according to which one has the best performance ratios.

The logic behind the horse race is testing if risk parity portfolios will have higher excess returns, Sharpe ratios, information ratios as well as lower variances and
maximum drawdowns than the benchmark portfolios using the data set of informationally efficient stocks. Diversification ratios are calculated for all portfolios to test if risk parity provides higher diversification benefit or not. Traditional performance indicators such as Mean returns and volatility will be tested for, and this section will provide detailed explanation of the important variables tested for and how they are beneficial to the analysis. The main estimator to conclude the best performing portfolio will be the Sharpe ratio, the higher it is the better the portfolio is performing.

4.3.1 Data set:

EuroSTOXX 50 index constituents are used to construct all portfolios and the reasoning behind this data set choice is the need to make sure that all stocks are informationally efficient and all data about the firms are available to a relatively excellent extent. This serves the research purpose of testing which type of portfolio will perform better in an efficient market and give a better idea on the current importance of information availability and its reflection on stock prices. The historical data is monthly price data starting from December 2000 and that is due to the missing data of earlier dates as well as the changes that took place in the index components over the years. The data used reflect historical prices of the latest 50 participants of the index.

The historical returns of the index itself is used as the market returns benchmark, which makes sense as all 50 constituting stocks are used to construct the portfolios in question and so it is consistent with the data set used.

The risk free rate is set to be the Euribor one-month rate as it also is the logical choice to match the data used in the analysis.

Below is a brief description to the main performance indicators used and their significance to the analysis:
4.3.2 Mean Returns, Mean Excess Returns and Volatility

The 3 performance indicators mean returns, mean excess returns and volatility of a portfolio are tightly tied together in financial analysis. According to the Modern Portfolio Theory, a rational investor is seeking the highest possible returns at the lowest possible risk level (Markowitz 1952). The excess returns represent how successful was the portfolio against a certain benchmark or even against the expected value of excess returns. The mean excess returns are often referred to as Alpha. Mean returns and volatility often go hand in hand since it can only make sense to analyze the rate of returns generated by a certain portfolio or asset combination if the risk the investor took to achieve that rate of return is also considered. All numbers representing those three factors are in percentages.

4.3.3 Sharpe Ratio

Sharpe ratio is the tool introduced by financial scientist William Sharpe in the 1960s and since then has been widely used in finance literature to analyze portfolio performance. The Sharpe ratio is a straightforward formula to measure risk adjusted returns by subtracting the risk free rate from the mean portfolio returns and dividing the result with the standard deviation of the portfolio returns as follows:

\[
S = \frac{\bar{r}_p - r_f}{\sigma_p}
\]

Where:
\(\bar{r}_p\) = expected portfolio return
\(r_f\) = risk free rate
\(\sigma_p\) = portfolio standard deviation

4.3.4 Information Ratio

This tool measures the level of activeness the management of a portfolio is. The higher the information ratio, the more consistent the portfolio manager has beaten the market and generated excess returns. The formula is fairly simple:
Where $R_i - R_p$ is the difference between the Portfolio returns and the market return, i.e. the excess return generated by the portfolio over the benchmark index/market return. While $\sigma_i - \mu$ refers to the standard deviation of that difference. Information ratio is a way to include the market risk and return into testing a portfolio’s performance, rather than just evaluating excess returns.

4.3.5 Maximum Drawdowns

Maximum drawdown measures the maximum drop that can happen to a portfolio from its peak value before a new peak is achieved. This is an important indicator of portfolio risk as it indicates how much can a portfolio lose before it starts recovering. The maximum drawdowns are usually downward pointing and sometimes accompanied with a minus sign for that reason. Maximum drawdowns level is a percentage change.

4.3.6 Diversification Ratio

According to Schoen (2012) diversification ratio is the “proper” way to measure the performance of risk allocation based portfolios. The Diversification ratio is a measurement of a portfolio’s weighted average asset volatility to the portfolio’s volatility in total. The formula for calculating the DR goes as follows:

$$DR = \frac{\sum_{i=1}^{n} W_i \sigma_i}{\sigma_p}$$

(16)

Where:

- $DR$ = the Diversification ratio.
- $W_i$ = the weight of asset $i$
- $\sigma_i$ = the volatility of asset $i$
- $\sigma_p$ = the volatility of the portfolio

The portfolio’s overall risk should be less than the sum of its constituents’ volatilities in a well-diversified portfolio, hence this ratio is typically greater than 1.
5 RESULTS

As discussed in previous chapters of this paper we have 5 risk parity portfolios constructed as well as an equally weighted portfolio and a minimum variance portfolio as benchmarks for performance analysis. Performance indicators used are mean returns, mean excess returns (alphas), volatility, Sharpe ratio, maximum drawdowns and information ratios for each portfolio. Also diversification ratios are calculated for the portfolios to test the effect of diversification on portfolio performance. The same performance indicators -except the diversification ratio- are calculated for the benchmark index which is the EuroSTOXX50 as all the constituents of the index are used to construct the different portfolios. The 1 month Euribor rate is used as the risk free rate in the calculations.

This section is dedicated to discussing the main empirical findings of the analysis performed using the methods described in the previous sections of this paper. Neither transaction nor taxes costs were included in this analysis. In real life this might very well affect the portfolio choice for an investor. It is also worth mentioning that as established in previous sections of this paper, the risk parity portfolio performance is sensitive to the asset universe, and since all the data used in this paper is stocks data, the findings might not give a generalizable idea about whether or not risk parity will out or underperform the benchmark portfolios using a different set of assets. None the less, it should shed some light on the performance of risk parity portfolios in equity based portfolios since the data used covers 26 years of historical monthly returns and this could be sufficient to draw conclusions about the performance in an all-equity asset universe.

The results are illustrated in two main tables, table (1) and table (2) that summarize the values of the performance indicators for the sample period and the out of the sample period respectively. The diversification ratios are illustrated separately in table (3).
In Table (1) and (3) below, the results of the horse race of the sample period from December 2000 to December 2010 are shown.

The Alpha Risk Parity (ARP) portfolio has the highest Sharpe ratio of 0.65 and the highest mean returns of 63.23%. This is an impressive number compared to all the other portfolios in the set. However, the volatility of the portfolio is also extremely high reaching 92.51% and the maximum drawdowns level is 39.67% which might make it an unlikely choice to risk averse investors. This very high volatility can be explained by the testing period analyzed and the way the portfolio is constructed. The 2008 financial crisis happened within this sample period and all of its underlying causing factors as well as its consequences and recovering period are also within the sample period. ARP portfolios are constructed in a way that factors excess returns or alphas in the portfolio construction and since this was a period of big fluctuations in prices and hence returns, this might have been the reason for this really high volatility rate within this specific portfolio. This portfolio, however have generated a much higher mean returns than any other during the same period which means it could be a winning choice to those who are willing to take such a risk. The maximum drawdown of the portfolio is 39.67% which is not much higher than the rest of the portfolios. The portfolio had an information ratio of 0.19 which means it has beaten the benchmark index, however the EuroSTOXX50 index did not perform very well during this period as illustrated in the table and as will be explained below.

Surprisingly, the ARP portfolio has a very low diversification ratio of 0.07. Usually, for the portfolio to be well diversified, it has to have a lower volatility than the sum of the volatilities of its constituents, hence it is normally greater than one. In this case, the portfolio is not considered well diversified at all which is consistent with the very high volatility. Again, this could be explained by the period of the analysis and the way the portfolio is constructed. It is established that this is a highly risky portfolio, but rewarding to those who would be willing to take such a risk with mean returns of approximately 64% which is very high.
The Minimum Variance portfolio (MV) as illustrated in Table (1) comes second to the ARP with a Sharpe ratio of 0.44, with mean returns of 5.72% and a very low volatility level of 6.95%. This performance indicates that this could be a perfect choice to a risk averse investor who would prefer a steady stream of relatively low returns but with a higher level of certainty that comes with that low volatility level. This benchmark portfolio has performed relatively well during the unstable period of the sample analysis that included the financial crisis. The portfolio had maximum drawdowns of 10.89% and an information ratio of 0.14 which might indicate a relatively low impact of active management on the performance. The MV has a diversification ratio of 2.51 which means that it is well diversified in this context and this data set and this could be the reason for this superior performance. This portfolio is considered to be performing well in the sample period.

The maximum diversification portfolio (MD) comes third to the ARP and MV portfolios with a Sharpe ratio of 0.38, mean returns of 7.73% and a volatility level of 13.14%. This portfolio is considered to have performed quite well in a troubled period of high market fluctuations and financial distress. The MD portfolio has a maximum drawdown level of 30.27% and an information ratio of 0.14 same as the minimum variance portfolio and again this could indicate a relatively low impact of active management on the performance but indeed both portfolios have beaten the index and performed positively in comparison with the indexed benchmark. The diversification ratio of the MD portfolio is 2.04 which is also relatively high but surprisingly, not higher than the MV in the sample period. The superior diversification for the MV could explain its second place in the race.

The Equal Risk Contribution (ERC), The Inverse Volatility (IV) and the Equally Weighted (EW) portfolios come next with close Sharpe ratios of 0.17, 0.15 and 0.13, mean returns of 5.26%, 5.05% and 5.17% and volatility levels of 15.05%, 16.25% and 19.10% respectively. Their Sharpe ratios are relatively low in comparison to the 3 winning portfolios and hence, we can conclude that these portfolios have not performed as well as the others and did not represent the best stocks combination possible during that period. The ERC, IV, EW portfolios had maximum drawdowns of 32.94%, 36.66% and 43.27% and information ratios of 0.10, 0.10 and 0.09 in the same order. The performance of the portfolios is very close which makes them close
substitutes to each other. The diversification ratio for the ERC, IV and EW portfolios is 1.72, 1.64 and 1.57 respectively. The three portfolio have close numbers and close diversification ratio. This confirms the above mentioned conclusion that they act as close substitutes to each other.

The Beta Risk Parity portfolio (BRP) comes last in this race with a Sharpe ratio of 0.03, mean returns of 3.31% and volatility level of 19.53%. The portfolio scored a maximum drawdown level of 45.05% and an information ratio of 0.06. This indicates that this portfolio might not be the best choice in financial distress periods. The diversification ratio of the BRP is 0.58 which is smaller than 1 meaning that the total volatility of the portfolio is not smaller than the sum of its constituents’ volatilities. This is due to the nature of this portfolio that does not provide diversification benefits and basically work with the market as the stocks are allocated weights according to their betas as explained in a previous section of this paper.

The EuroSTOXX50 index have performed poorly during the period in question with a Sharpe ratio of -0.26. The index mean returns are -2.45% with a volatility of 20.15%. Maximum drawdowns level was 59.90%. This indicates that the market was down and as stated above, this is due to the testing period and the financial distress that accompanied that period.

All constructed portfolios however have beaten the market and had positive information ratios as well as positive returns and Sharpe ratios. Most portfolios have provided diversification benefits and the Sharpe ratios are consistent with the diversification ratios except for the ARP portfolio that is considered a special case in this race.
Table (1): Sample analysis results:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Mean Returns (%)</th>
<th>Mean Excess Returns (alpha) (%)</th>
<th>Volatility (%)</th>
<th>Sharpe Ratio</th>
<th>Max.DD (%)</th>
<th>Information Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>5.17</td>
<td>2.47</td>
<td>19.10</td>
<td>0.13</td>
<td>43.27</td>
<td>0.09</td>
</tr>
<tr>
<td>MV</td>
<td>5.72</td>
<td>3.02</td>
<td>6.95</td>
<td>0.44</td>
<td>10.89</td>
<td>0.14</td>
</tr>
<tr>
<td>EuroStoxx</td>
<td>-2.45</td>
<td>-5.20</td>
<td>20.15</td>
<td>-0.26</td>
<td>59.90</td>
<td>-</td>
</tr>
<tr>
<td>IV</td>
<td>5.05</td>
<td>2.35</td>
<td>16.25</td>
<td>0.15</td>
<td>36.66</td>
<td>0.10</td>
</tr>
<tr>
<td>MD</td>
<td>7.73</td>
<td>5.03</td>
<td>13.14</td>
<td>0.38</td>
<td>30.27</td>
<td>0.14</td>
</tr>
<tr>
<td>ERC</td>
<td>5.26</td>
<td>2.56</td>
<td>15.05</td>
<td>0.17</td>
<td>32.94</td>
<td>0.10</td>
</tr>
<tr>
<td>ARP</td>
<td>63.23</td>
<td>60.53</td>
<td>92.51</td>
<td>0.65</td>
<td>39.67</td>
<td>0.19</td>
</tr>
<tr>
<td>BRP</td>
<td>3.31</td>
<td>0.61</td>
<td>19.53</td>
<td>0.03</td>
<td>45.05</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Analysis period: December 2000 to December 2010

Table illustrates the analysis results for the sample period between December 2000 and December 2010. The first two rows represent the mean returns, mean excess returns, volatility, Sharpe ratio, maximum drawdowns (Max.DD) and information ratio for the Equally weighted portfolio (EW) and the minimum variance portfolio (MV) respectively. The third row shows the results for the EuroSTOXX50 index. Row 4 through 8 show the performance indicators for the Inverse Volatility (IV), Maximum Diversification (MD), Equal Risk Contribution (ERC), Alpha Risk Parity (ARP) and Beta Risk Parity (BRP) portfolios respectively. The Max.MDD for the EuroSTOXX index is obtained from the Deutche Bank analytics website (https://index.db.com/dbiqweb2/servlet/indexsummary)

Table (2) and (3) below, illustrates the horse race performance analysis figures for the Out of the sample period which is from January 2011 till February 2016.

The Maximum Diversification Portfolio (MD) comes first with a Sharpe ratio of 0.95, mean returns of 10.44% and a volatility level of 10.68%. This Sharpe ratio is considered relatively high compared to both the other portfolios and the MD’s previous Sharpe ratio in the sample period analysis which was 0.38. This recovery could be due to the overall improvement of the financial market conditions and the recovery from the implications of the 2008 financial crisis. The portfolio has a maximum drawdown of 8.49% which is low relative to other portfolios and indicates a stable performance as the downward trough is not very deep. The information ratio
for this portfolio is 0.16 which means it has beaten the underlying index which is the EuroSTOXX50 and generated positive excess returns, this could be due to active management. The diversification ratio of the portfolio despite the fact that it dropped from its previous 2.04 to 1.83, it has achieved the highest DR among all other portfolios and came first. This is consistent with the assumption of diversification benefits and the claim that diversification pays.

The Alpha Risk Parity portfolio (ARP) comes down from first position in the sample period analysis to the second position despite the fact that its performance indicators have decreased but perhaps it is heading for a more stable performance with a much lower volatility level of 68.69% falling from 92.51%. This volatility level is still considered high in absolute terms and in comparison to other portfolios constructed using the same data. The portfolio generated mean returns of 53.57% which is still very high compared to the other portfolios but also lower than its previous level of returns in the sample period, and still combined with a relatively very high volatility levels. The Sharpe ratio of the portfolio have increased however to 0.78 from its previous 0.65 which shows that even though the mean returns level has decreased, the portfolio over all relative performance has improved. The maximum drawdowns level of the ARP is 33.68% and the information ratio have decreased from 0.19 to 0.12 which could suggest that the improvement in performance is not due to active management but rather to the overall improvement of market conditions. This drop can also be explained by the decrease in the mean excess returns of the portfolio from 60.53% to 53.25%. The diversification ratio of the ARP has dropped even further to a -0.05 which is again approximately zero. As explained above, this portfolio provides high returns due to the technique it was constructed with and focuses mainly on alphas or excess return as a weight assigning criteria that does not benefit from diversification and is highly risky.

The Minimum Variance portfolio (MV) and the Equal Risk Contribution (ERC) portfolios perform relatively well in this period analysis with very close Sharpe ratios of 0.56 and 0.55 respectively. The ERC portfolio’s Sharpe ratio has elevated from 0.17 during the sample period to 0.55 showing a big improvement in performance. This is accompanied by an increase in the mean returns from 5.26% to 7.02% and a decrease in the volatility from 15.05% to 12.28%. The ERC scored a maximum
drawdowns level of 15.45%, again showing improvement from its previous 32.94% and an increase in the information ratio from 0.10 to 0.11. The small increase might not give a clear idea about the active management impact on performance. The MV portfolio also achieved an increase in the mean returns from 5.72% to 7.36% but accompanied with almost a double increase in the volatility level from 6.95% to 12.53%. The maximum drawdown level has fallen from 10.89% to 8.27% which is a slight improvement in the performance. The information ratio has fallen from 0.14 to 0.11. In this out of sample period analysis, those two portfolios can be considered close substitutes to each other due to the closeness in performance analytics. The diversification ratio for the MV and ERC is 1.39 and 1.63 in the same order. The ERC is better diversified but the MV wins as it has a 0.01 higher Sharpe ratio. The MV portfolio comes third and the ERC comes fourth in this horse race.

The Inverse volatility portfolio (IV) even though it comes at the fifth place out of seven but it has achieved a relatively large increase in Sharpe ratio from 0.15 to 0.46 showing an enhancement in performance. The mean returns of the IV have increased from 5.05% to 6.3% while its volatility decreased from 16.25% to 13.07%. The maximum drawdowns made a drastic fall almost by half from 36.66% to 18.13% while the information ratio remains constant at 0.10. This indicates that the better performance could be justified by the overall improvement of the market conditions or other factors rather than the involvement of active management and rebalancing.

The diversification ratio of the IV has dropped slightly from 1.64 to 1.58 in the out of sample period, however the small drop did not impact the performance negatively.

The Equally weighted portfolio (EW) also shows an improvement in its performance with a rise in Sharpe ratio from 0.13 to 0.36. The mean returns of the portfolio increased slightly from 5.17% to 5.57% while the volatility have fallen from 19.10% to 14.50%. The maximum drawdowns have fallen from 43.27% to 22.98% while the information ratio is still almost constant with a very small fall from 0.09 to 0.08. The EW finishes in the sixth place falling from its status in the sample period analysis. The diversification ratio makes a very small drop from 1.57 to 1.54 in this period analysis, so it is inconclusive whether it had an impact on the performance or not.
The Beta Risk Parity Portfolio (BRP) again finishes last in spite of showing improvement in performance from the previous period analysis with a relatively large rise in the Sharpe ratio from 0.03 to 0.17. This, however is accompanied with a slight fall in the mean returns from 3.31% to 3.02% while the volatility level falls from 19.53% to 15.81%. The maximum drawdowns have fallen from 45.05% to 25.51%. The diversification ratio has increased from 0.58 to 1.09 and from this we can say that once the market was moving up or recovering from the impacts of the crisis, the BRP started performing better as well and it did provide some diversification benefits.

The poor performance of the Beta Risk Parity Portfolio could be due to the link between the beta of the portfolio and the overall market systematic risk. This period was characterized by high fluctuations and financial distress as well as a global financial crisis. The obvious instability of the market has indeed affected the performance of this portfolio. This portfolio construction technique might need to be tested using a different set of assets or a different time period in order to fairly judge its success or failure.

The EuroSTOXX50 index shows an improvement in performance. In spite of the fact that the index is still generating negative returns, but it has improved from a previous -2.45% to -0.09% and less volatility than its previous level reaching 17.25%. Sharpe ratio of the index reached -0.02 after its previous -0.26. The maximum drawdowns level has also fallen to 0.26 from the previous 0.6. This shows that the overall market performance has improved from the previous period.
### Table (2): Out of sample analysis results

<table>
<thead>
<tr>
<th>Analysis Period: January 2011 to February 2016</th>
<th>Mean Returns (%)</th>
<th>Mean Excess Returns (alpha) (%)</th>
<th>Volatility (%)</th>
<th>Sharpe Ratio</th>
<th>Max.DD (%)</th>
<th>Information Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>5.57</td>
<td>5.24</td>
<td>14.50</td>
<td>0.36</td>
<td>22.98</td>
<td>0.08</td>
</tr>
<tr>
<td>MV</td>
<td>7.36</td>
<td>7.04</td>
<td>12.53</td>
<td>0.56</td>
<td>8.27</td>
<td>0.11</td>
</tr>
<tr>
<td>EuroStoxx</td>
<td>-0.09</td>
<td>-0.41</td>
<td>17.25</td>
<td>-0.02</td>
<td>25.90</td>
<td>-</td>
</tr>
<tr>
<td>IV</td>
<td>6.30</td>
<td>5.98</td>
<td>13.07</td>
<td>0.46</td>
<td>18.13</td>
<td>0.10</td>
</tr>
<tr>
<td>MD</td>
<td>10.44</td>
<td>10.12</td>
<td>10.68</td>
<td>0.95</td>
<td>8.49</td>
<td>0.16</td>
</tr>
<tr>
<td>ERC</td>
<td>7.02</td>
<td>6.70</td>
<td>12.28</td>
<td>0.55</td>
<td>15.45</td>
<td>0.11</td>
</tr>
<tr>
<td>ARP</td>
<td>53.57</td>
<td>53.25</td>
<td>68.69</td>
<td>0.78</td>
<td>33.68</td>
<td>0.12</td>
</tr>
<tr>
<td>BRP</td>
<td>3.02</td>
<td>2.36</td>
<td>15.81</td>
<td>0.17</td>
<td>25.51</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table illustrates the analysis results for the sample period between January 2001 and February 2016. The first two rows represent the mean returns, mean excess returns, volatility, Sharpe ratio, maximum drawdowns and information ratio for the Equally weighted portfolio and the minimum variance portfolio respectively. The third row shows the results for the EuroSTOXX50 index. Row 4 through 8 show the performance indicators for the Inverse Volatility (IV), Maximum Diversification (MD), Equal Risk Contribution (ERC), Alpha Risk Parity (ARP) and Beta Risk Parity (BRP) portfolios respectively. The Max.MDD for the EuroSTOXX index is obtained from the Deutche Bank analytics website (https://index.db.com/dbiqweb2/servlet/indexsummary).

Next, we show a graphical representation to the horse race results according to portfolios’ Sharpe ratios both in and out of the sample period. Figure (2) represents the horse race results for the sample period analysis and figure (3) illustrates the results for the out of the sample period analysis.
Figure (2): Sharpe ratios (annualized) for sample period

Figure (3): Sharpe ratios (annualized) for Out of the sample period

Next, we try to discover if the superior Sharpe ratios achieved by certain portfolios are due to a superior diversification. Below the diversification ratios of all portfolios in the sample period analysis and out of sample period analysis is presented in table (3) and under it figure (4) where ratios are presented graphically.
Table (3): Diversification ratio horse race results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>1.57</td>
<td>1.54</td>
</tr>
<tr>
<td>MV</td>
<td>2.51</td>
<td>1.39</td>
</tr>
<tr>
<td>IV</td>
<td>1.64</td>
<td>1.58</td>
</tr>
<tr>
<td>MD</td>
<td>2.04</td>
<td>1.83</td>
</tr>
<tr>
<td>ERC</td>
<td>1.72</td>
<td>1.63</td>
</tr>
<tr>
<td>ARP</td>
<td>0.07</td>
<td>-0.05</td>
</tr>
<tr>
<td>BRP</td>
<td>0.58</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table (3) illustrates the results of the Diversification ratio for each portfolio in and Out of the sample.

Figure (4): Graphical representation of diversification ratio in and out of sample periods.

According to the discussed analysis, both in and out of the sample, a risk parity portfolio has come in the 1st place in almost every performance measure used. However, the minimum variance portfolio has also shown good performance compared to the other portfolios analyzed. To present conclusive evidence that risk parity portfolios will consistently outperform classically optimized portfolios given any set of assets, more asset classes have to be incorporated in the study and
performance tested accordingly. Risk parity portfolios have also shown relatively higher volatility levels which might not be suitable to many risk averse investors. The frequent rebalancing required to keep the weights consistent with equal risk contribution is costly in the real world as there are high transaction costs and this further decreases the suitability of this type of portfolios to the general public. In this specific asset universe which is stocks, and using this specific data set, according to the out of sample analysis, the maximum diversification portfolio has won the horse race showing the best performance among all other studied portfolios with the highest Sharpe ratio of 0.95 and the highest information ratio of 0.16 accompanied by the lowest maximum drawdowns level of 0.09 and a mean returns percentage of 10.44. The Alpha Risk Parity (ARP) portfolio came second with a Sharpe ratio of 0.78 and information ratio of 0.12, but accompanied with a very high volatility level of 68.69%. The ARP portfolio mean returns were very high compared to all the other portfolios at 53.57% and its maximum drawdowns level was almost 34%. This makes it a highly risky investment that generates high returns but at a very high level of risk, making it less appealing to the average risk averse investor. The minimum variance portfolio came third with a Sharpe ratio of 0.56 and a close fourth was the Equal Risk Contribution portfolio with a Sharpe ratio of 0.55. All the constructed portfolios have efficiently beaten the market and outperformed the EuroSTOXX50 index both in and out of the sample.

The most obvious implication of this study is that risk parity portfolios perform well, and sometimes very well, however they might not be everybody’s cup of tea when it comes to the general masses of investors. In a pure stocks investment plane, they can be costly, require frequent adjustments and rebalancing and can have very high volatility levels as was shown by the Alpha Risk Parity portfolio. They also can be closely linked to the market, which might not be very good in times of distress as we have seen from the poor performance of the Beta Risk Parity portfolio as an example of that. The level of skill, money and risk tolerance that those portfolios mostly require might not be available to the ordinary investor. On the other hand, risk parity portfolios can be a mine of gold to institutional investors and fund managers who have higher risk tolerance and the required skill level to manage such portfolios, since they on average generate high returns and achieve high Sharpe ratios compared to the equally weighted portfolio or the market benchmark index.
There is no irrefutable conclusion that could be drawn from those results without further testing to the portfolios using a different set of stocks/assets. However, to answer the question of this paper and given this specific set of data, risk parity portfolios have outperformed the two benchmark portfolios and the market index. But in order to stand on a more solid ground, the research plan has to be expanded in a way to include more asset classes and diversify in the asset universe used as well as the portfolio construction techniques. The question still remains partly unanswered on whether superior risk estimators will for sure replace returns proxies and informational superiority in efficient markets. None the less, risk estimators have been proven to be an important tool in diversification and building portfolios that are better diversified than classical portfolios, but whether it will be the sole estimator needed in the future or not is still to be found out.
6 CONCLUSION

The modern financial world is very complex and this complicates the investors’ decision making process with the wide set of available assets and trading strategies presented in today’s markets. This makes the investor choice and portfolio optimization problem an interesting topic in financial research. Since the Modern Portfolio Theory was introduced by Markowitz in 1952, it has been a pillar for the modern financial research. Optimizing a portfolio is done according to Markowitz (1952) based on an assumption of rationality in investors. This rationality dictates that they demand the highest possible returns with the lowest possible level of risk. For a portfolio to be “efficient” it has to provide the highest return given a certain amount of risk, or the lowest risk given a specific level of return. A combination of all such portfolios given a specific set of assets is the Markowitz efficient frontier.

However, the Markowitz’s efficient frontier and his objective optimization of a quadratic function have been criticized by practitioners claiming it is too complicated to apply and the underlying theory has unrealistic assumptions. The model contains no transaction costs and assumes rationality and skillfulness of investors which might not be true in every case. The model also assumes the infinite availability of resources and information equally to all investors which is not the case in real life. The Adaptive Market hypothesis (AMH) by Lo (2004) provides an alternative way of looking at market efficiency and investors’ rationality in the sense that it does not assume that investors are rational but rather looks at the whole choice process from an evolutionary approach. The AMH utilizes the biological theories of natural selection to provide a heuristic explanation to investors choices that bases such choices on trial and error and learning from the feedback given by the market to the investor on every choice in the form of profits or losses. This trial and error and the adaptability of the market participants to the changing market conditions and the results of their own choices leads them to almost optimal choices in the long term, however not through the traditional frame work of the Efficient Market Hypothesis where everyone is rational and optimizing an objective function, usually related to an investor’s utility, that in many cases does not hold in empirical terms.
The financial crises that happened in recent years imposed new heuristic approaches to the portfolio optimization and diversification problems. Among these heuristic approaches came the risk parity techniques that is based on a seemingly simple reasoning which is focusing on risk budgeting and studying risk factors rather than forecasting returns. This approach is applied through the unification of the total risk contributions of a portfolio’s constituents to the portfolio for the purpose of better diversification rather than equalizing the marginal risk contributions as in the minimum variance portfolio. Risk parity portfolios are heuristic approaches that are unrelated to the investor’s utility and are not based on optimizing an objective function and hence they do not lie on the Markowitz efficient frontier but somewhere inside it as illustrated in figure (1) in this paper. The risk parity is unrelated to the investor’s utility, but its main focus is on risk budgeting and providing superior diversification benefits to investors.

This study was conducted in a horse race style calculating fundamental performance indicators such as Sharpe ratios, mean returns, mean excess returns, volatility, maximum drawdowns as well as information and diversification ratios for each of the constructed portfolios to see if risk parity portfolios will out or underperform the benchmarks. The Sharpe ratio is used as the main comparison tool and the portfolio with the highest Sharpe ratio will be the winner. We have constructed 5 risk parity portfolios: The Inverse Volatility, the Equal Risk Contribution, the Maximum Diversification, the Alpha Risk Parity and the Beta Risk Parity portfolios along with 2 benchmark classical portfolios: The Equally Weighted and the Minimum Variance portfolios. The data set used is the data for 50 blue chip companies that are the constituents of the EuroSTOXX 50 index. Blue chip stocks are market leaders and giant firms and so they best represent informationally efficient stocks in developed markets. The risk free rate used in conducting the analysis is the 1 month Euribor rate as it seems to best fit the data than any other rate. The EuroSTOXX50 index itself was used as the benchmark index in this analysis.

The results of the horse race were divided to a sample period analysis and an out of sample period analysis. The winning portfolio in the sample period analysis was the Alpha Risk Parity portfolio with a Sharpe ratio of 0.65. The Winning portfolio in the
out of sample period analysis was the Maximum Diversification portfolio with a Sharpe ratio of 0.95.

This paper aimed to answer the question of whether or not risk parity portfolios consistently outperform classically optimized portfolios and whether that will have an impact on the type of information investors should seek before engaging in a specific investment. The findings show that both in and out of the sample analysis, a risk parity portfolio did outperform the other portfolios achieving higher Sharpe ratios and in most cases higher returns and less volatility, however not without a price. Risk Parity portfolios are “high maintenance” portfolios that require constant rebalancing and portfolio management skills that might not be available to all investors. This can be costly in a real world setting as transaction costs are high in financial markets and short selling is not allowed in some markets. On the other hand, this makes risk parity an attractive choice to finance professionals and institutional investors who possess the skills and the resources that allows them to maintain a balanced risk parity portfolio.

This study also has shown that risk parity portfolios are sensitive to the asset universe and the data set used, hence the results obtained cannot be generalized to other asset classes or investment mediums at this point. A room for further research would be testing the same portfolio constructing techniques and apply them in a more generalizable asset universe using data of a more versatile nature like adding bonds, commodities ETFs and Real Estate Investment Trusts (REITs). This will take the study to another level of academic accuracy and perhaps better answer the question that have been asked frequently in financial research but have not been answered conclusively till this point of time when this study was done.

The topic of risk parity and its comparison to classically optimized portfolios, remains interesting in the financial literature world. Financial research is yet to find the theory behind risk parity and its apparent practical success but as mentioned before in this paper’s literature review section: “Risk parity remains more of an art than a science” for the time being.
7 REFERENCES


