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**THE UNCOVERED INTEREST PARITY PUZZLE AND TIME-VARYING
RISK PREMIUM**

Master's Thesis

Department of Finance

May 2016

Unit Department of Finance			
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Title THE UNCOVERED INTEREST PARITY PUZZLE AND TIME-VARYING RISK PREMIUM			
Subject Finance	Type of the degree Master's	Time of publication May 2016	Number of pages 115
<p>Abstract</p> <p>Uncovered interest parity puzzle is one of the most prominent puzzles in international finance that has remained unsolved for over 30 years. This theory stipulates that the currencies of the countries where risk-free interest rates are high should depreciate relative to the currencies associated with low interest rates. However, empirical findings of last three decades report the opposite effect, and none of the proposed theoretical models have been yet able to explain why. In the meantime, a popular trading strategy that exploits the puzzle and is commonly referred to as carry trade has been gaining popularity among traders and speculators. It on average generates significant positive excess returns that cannot be explained by any existing asset pricing models, but it also is susceptible to sudden crashes. Growing popularity of this strategy, and the persistence of the uncovered interest parity puzzle motivates us to analyze possible causes of the failure and to propose a model that corrects for the bias.</p> <p>Most of the literature in the field rely on the use of forward rates, but we argue that futures data provides the means to analyze evolution of the foreign exchange risk premium over time. In this thesis we use currency futures rates to perform the tests of the uncovered interest parity condition. We then test the hypothesis of futures rates unbiasedness and we link the two concepts together with a risk premium component. We find that both deviations from rational expectations and a presence of the risk premium term are possible causes for the failure of the UIP tests. Using futures rates at daily and weekly frequencies we are able to provide evidence that the risk premium varies over time. Incorporating this nature, we propose a modified UIP model that corrects for the bias. We find that the inclusion of the risk-premium component mitigates the puzzle. We also find that carry trade portfolios have high abnormal returns due to higher exposure to volatility and funding liquidity risks. During periods of market turmoils, the uncovered interest parity condition holds better because the trades that normally exploit the puzzle lose money. Overall, we conclude that there should be some non-traditional state variables that have their innovations related to the risk-premiums in a cross-section of currency returns.</p>			
<p>Keywords</p> <p>risk premium, foreign exchange, uncovered interest parity, forward premium, market efficiency, futures</p>			
Additional information			

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1. INTRODUCTION

1.1 Background and motivation

International financial markets are growing every decade to ever bigger sizes, and foreign exchange market has become the most liquid of all. According to the Bank of International Settlements (2013), in 2013 the daily trading volume in the FX market has reached \$5.3 trillions. Given these levels of liquidity, one can expect the market to be in a strong form of efficiency, especially when there are so many counterparties involved in the transactions. Central banks buy foreign currencies to fill up its foreign reserves, international investment banks and hedge funds act as arbitrageurs and multinational companies transfer their revenues from one currency to another. Understanding of the main laws that govern exchange rates fluctuations is crucial for the world economy, because exchange rate is a major risk for multinational corporations as well as an important variable affecting economic growth of every country through the values of imports and exports. Central banks, too, undertake monetary decisions based on the relative value of its currency. For all of these reasons, a good theoretical framework explaining the long-term movements in exchange rates is a necessity for financial decision-makers everywhere in the world.

Interest rates are closely linked to foreign exchange values through the so-called covered interest parity condition. It is one of the fundamental laws of macroeconomics and it states that the difference between a forward rate today for a delivery of a foreign exchange contract tomorrow and the spot price today should be equal to the difference between two countries interest rates on risk-free instruments, save for transaction costs. The reason why it should hold comes from the law of one price - two alternative investments should cost the same if they provide the same returns (all net of transaction costs). A US investor can invest today into risk-free securities, or, alternatively, she can buy some foreign currency, invest it in foreign risk-free bills, and immediately buy a forward contract to lock up the return in USD. These two alternatives should cost the same in a liquid market, otherwise an investor will be able to make a riskless profit - something that should not happen in efficient and liquid markets. According to the study of Akram, Rime, & Sarno (2008), this covered interest parity holds at daily frequencies and lower. However, a growing body of evidence over last three decades has indicated that the same condition does not work if instead of the forward contract, an investor would wait one period and exchange proceeds at that next period spot rate. This condition has been given a name of uncovered interest parity. More strikingly, Froot (1990) found that the condition works rather in an opposite direction. and two investment alternatives are not equal. Most of the empirical research found that investors can make abnormal profits if they choose one of the investment options based

on the interest rate differential between two countries. The problem that is associated with such findings is that no one yet has been able to explain why the uncovered interest parity condition does not work, and what drives the profits to investment strategies that exploit this failure.

Foreign exchange rates are determined by forces of supply and demand. Even for fixed rate currencies, a central bank needs to intervene and join either supply or demand side to keep the rate at a desired level. But if the failure of the uncovered interest parity condition remains unexplained, there is no reason why speculators should not borrow in their low-yielding home currencies and buy foreign-denominated riskless securities as long as foreign currency has higher interest rate. This type of trade has been more popular recently since last ten years has been marked as the time when the US Federal Reserve kept near zero interest rates. From the report of the Bank of International Settlements (2013), the Japanese Yen, another low interest currency, had the most substantial jump in trading activity - it rose by 63% in the period from 2010 to 2013. The Australian and New Zealand dollars are high interest currencies and their share in global FX trading increased significantly over last years as well. It is possible that some of the increases are attributed to the rising popularity of so-called carry trades - exactly the trading strategies that exploit the failure of the UIP to hold empirically. Brunnermeier, Nagel, & Pedersen (2009) finds that using the futures position data for noncommercial traders from the Commodity Futures Trading Commission, the carry trade speculators have higher positions in currencies with higher interest rates. He also finds that carry traders place even higher bets in currencies with high interest rates betting on appreciation of the currency. Such trades can potentially be destabilizing, because the build up of carry traders positions in a currency pair can in itself cause a sudden and violent depreciation at some point in future. As an example, there was no changes in fundamentals, or appearance of any news prior to the large depreciation of the USD against the Japanese yen on 7th October, 1998. If abnormal positive returns to carry portfolios are what the theory of asset pricing suggests it is - a compensation for bearing certain types of risks, - it is of high importance to find such risk factors so that traders can adjust their positions gradually on the changes in risk factors rather than cause sudden devaluations of the currencies. The latter effect is harmful to the world economies, because international trade and investments are highly dependent on the levels of exchange rates.

Our motivation to conduct a study on this topic is closely related to the problems outlined above. We would like to know if the failure of the uncovered interest parity indeed represents a puzzle, and if so, we want to know what drives such discrepancy between empirical findings and the theory. Carry trades look attractive up until the point when the traders incur big losses, and national economies suffer severe depreciation of the currency, possibly leading to defaults on foreign-denominated debt. Our

motivation is related to the fact that if we can offer a glimpse of explanation of why the forward premium puzzle exists, the FX traders could become more informant leading to more efficient FX market. Additionally, due to the failure of the UIP condition, the central banks around the world are faced with impossible trinity - a trilemma that states that it is impossible to have a stable FX rate, free capital movement, and an independent monetary policy at same time. We want to shed some light on the causes of the failure to give policymakers more factual information that they can use to make their policy decisions.

1.2 Relation to earlier research

In this Master's thesis, we use the work of Fama (1984) as a starting point. There have been an enormous amount of studies documenting similar to that paper findings - the UIP condition does not hold in practice. Froot (1990) provides a useful aggregation of 75 studies that had been conducted prior to 1990 that document on average economically significant negative coefficient to a UIP regression, while the theory suggests the coefficient should be unity. Subsequent research has found the same anomaly. The academia, however, is divided on the issue of what can cause such deviation from the theory. Some attribute the puzzle to measurement errors, as in Bekaert & Hodrick (1993), while some find the root problem in the existence of time-varying risk premium, as in Fama (1984), Cheung (1993), Canova & Takatoshi (1991), Cumby (1988), Menkhoff et al. (2012), Lustig & Verdelhan (2007), Lustig, Roussanov, & Verdelhan (2011) and others. The problem, however, is that consumption-based asset pricing models cannot explain such a high variability of the risk premium term found in the FX data. Consumption data is simply not variable enough compared to high variance of the risk premium term. Additionally, if market participants demand premium for bearing some risk, there should be risk factors exposure to which should be correlated with carry returns. Even though some of the factors were found, overall they can explain too small portion of abnormal returns.

Another complication in finding the causes of empirical failure arises due to the fact that the test of the UIP condition automatically implies that currency forward rate is an unbiased predictor of the future spot rate. However, a popular strain of literature attributes the forward premium puzzle to the failure of rational expectations hypothesis, as for example in the theory of rational learning of Lewis (1995) and Lewis (1989). Particularly in the model of Lewis (1989) it is assumed that market participants are rational in a sense that they make best possible predictions based on the information available to them. A consistent bias in their predictions appears when one of the fundamental variables is not observable, and traders have to make assumptions regarding the value of that variable. Monetary regime is one of the examples when it can happen that market participants cannot know if expansionary or contractionary regime is

currently in place. As long as investors put a non-zero probabilities on each of the potential regimes, while in fact only one is in action, the forecast errors will have a non-zero mean value. This could explain the puzzle of UIP, but Lewis (1989) found that forward rate errors are too persistent which is actually evidence against the rational learning model since market participants cannot forever learn about one regime change.

Lastly, some researchers found that the presence of Peso problem - a risk of severe losses not yet reflected in the data - can be the cause of the puzzle. When econometricians conduct tests of the uncovered interest parity, they operate with historical data, and the bias that can arise from the failure of rational expectations seems confounding given the data. However, back in history, investors on those trading days might have assigned very small, but still non-zero probabilities to possible severe crashes in exchange rate pairs. If no crashes happened, the econometrician's posterior probability distribution of exchange rates might be well different from the ex ante probabilities used by market participants. The wedge between two probability distributions can well be the cause of the UIP bias found in empirical research. Although this explanation is attractive, there is still one problem that makes the Peso problem explanation an unlikely candidate for the cause of the puzzle. The findings of wrong values of the coefficient in the UIP regression appeared in a majority of studies conducted for different currencies and different time periods. Peso problem is in itself a small-sample problem, and it cannot be present in so many different historical samples.

Alternatively, if the exchange rates exhibit explosive paths that deviate further and further away from economic fundamentals, it can happen due to so-called rational bubbles. Speculators may continue buying currencies even if they see that they are already overvalued. This can generate bias in expectations of future exchange rate, leading to puzzling results of UIP tests. However, the tests of rational bubbles cannot provide conclusive evidence in favor of this anomaly. First of all, tests based on comparing the variances between fundamentals and nominal exchange rates do not prove the existence of bubbles even if variances are different. Other explanations such as the Peso problem can cause such discrepancy. Second, other tests rely on assuming specific models of exchange rate determination, while all the models of exchange rates have very low predictive power and in fact the path of exchange rate changes is best approximated by a random walk. In short, to test for presence of rational bubbles, we need first to develop good models that relate fundamentals to exchange rates, and develop more advanced econometric techniques.

1.3 Research problem and research methods

In our work, we review all the relevant strains of the literature on these possible explanations of the puzzle. Our main goal of the research is to use futures data to find

evidence of the failure of uncovered interest parity in the sample of eighth exchange rate pairs and to find evidence that the failure is due to the time-varying risk premium. In particular, we can break down the research problem into smaller tasks:

- Perform robust tests of the hypothesis that the uncovered interest parity condition holds;
- find how risk premium or deviations from rational expectations can cause the failure;
- provide evidence that the risk premium is time-varying;
- test the UIP model corrected for omitted variable bias - namely by including risk premium term in the UIP regression;
- find risk factors that drive the risk premium.

Our starting point is the UIP regression of Fama (1984). However, unlike their study and the absolute majority of others, we use futures rate instead of forward rates to test for the uncovered interest parity null hypothesis. To our knowledge, only two papers used futures data to recreate the null hypothesis test¹. However, we argue that the use of futures rates provides the means to capture the time-varying nature of the risk premium. With forward contracts, different forward rates reflect the market's expectation of the value of the future spot rates at two different time periods (usually, one month apart). With futures rates, however, the two rates reflect the market's expectations of the same future spot rate on the same day. By studying the time-series variability in futures rates, we can look at the problem of the UIP failure from a different angle. First, the risk premium component should have different properties compared to what was found in the literature that used one month ahead forward rates. Each day, a futures rate encompasses the perceived risk of buying foreign currency for delivery at a fixed day. When we study two consecutive futures rates at frequencies finer than monthly, we can capture the risk premium variability in a way not available to us when we use forward rates. Two consecutive forward rates contain in themselves perceived risk components that may well be very different in size and in sign, because they pertain to contracts that promise delivery at entirely different future dates. Overall, we argue that if the risk is priced in the FX market, its properties should be better captured if we use futures rates, not forward rates. In addition to that, it is well known that most of the foreign exchange speculators trade futures contracts rather than forward contracts because of the availability of a common market place. Given that, our work has an advantage of providing evidence that is much more relevant to practitioners.

¹See Bernoth, Hagen, & Vries (2005) and Bernoth, Hagen, & Vries (2012)

Second, using futures rates we have an advantage in analyzing the failure of rational expectations hypothesis. When we use forward rates, at every monthly time period, we cannot possibly know what is the size and the sign of the rational expectations bias, if it is present². When we operate with futures data using weekly observations, we again can see the development of the bias over time, because the deviation from the rational expectations should follow a gradual development. The type of a process it follows can be approximated by existing general time-series processes. We do not have grounds to believe that the bias changes the sign over the lifetime of the future contract, at least not frequently. This fact leads us to build a theoretical framework around the process that the rational expectations bias follows, and we incorporate it into a general form of futures unbiasedness test. By construction, the test gives evidence of time-variability in risk-premium component if both biases are present or if only risk premium is there, but agents are rational. We also find that risk premium term is serially correlated.

Next, we test the UIP hypothesis using futures data. We employ panel regression model as in Bernoth, Hagen, & Vries (2012) with pooled OLS estimator of the slope coefficient and Beck & Katz (1995) standard errors. Unlike Bernoth, Hagen, & Vries (2012), we find presence of strong serial correlation at different cross-sections (where we treat futures maturities are cross-sections), and we refine testing procedure by either excluding some of the maturities, or by employing Cochrane & Orcutt (1949) transformation. Using the full sample, we were unable to reject the null that UIP holds in 7 currency pairs of total of 8. However, we find point estimates for the coefficient to be far off the unity for 5 pairs. Following this result, we test the UIP model with risk premium that is unique to a given futures contract but consecutive futures rates have possibly time-varying loading on the risk term. We use the common correlated effects pooled estimator of Pesaran (2006) for panel models to estimate the model, and we find that the model has a significantly better fit. The puzzle of wrong slope coefficient in the model remains in two currency pairs out of total 8, but the point estimates become closer to theoretically predicted value of unity than in the original model.

Motivated by this result, we then perform asset pricing modelling using excess returns to carry portfolio as a test asset. We construct multi-factor model and analyze the results. Similar to Menkhoff et al. (2012) we find that the measure of foreign exchange volatility is a significant predictor of carry returns, but we cannot find other significant factors. We then turn to cross-sectional asset pricing tests, and we confirm that volatility is priced across portfolios sorted on interest rate differential. This time, TED spread and VIX index are factors that have significance, suggesting that global funding conditions are important risk factors that can explain a portion of the uncovered interest

²It is possible to shed some light on the size of the bias using survey data on traders' expectations of future spot rates. However, Frankel & Froot (1987) points out the limitations of this type of data since using surveys as a measure of exchange rate expectations has a problem that survey-based expectations themselves are quite often not internally consistent.

parity puzzle. Additionally, the average excess returns to currency portfolio are found to be a significant factor explaining cross-section of currency returns, confirming the findings of Lustig, Roussanov, & Verdelhan (2011).

This thesis proceeds as follows. In chapter 2 we outline a theoretical framework necessary to understand in order to study uncovered interest parity condition. We discuss the findings from earlier literature regarding possible explanations for the bias. We also analyze the literature that provides evidence on empirical relationship between risk factors and uncovered interest parity condition. In chapter 3 we provide description of the data, and perform UIP tests using futures rates with robust transformations and the use of robust standard errors. We discuss the results and then in section 3.4 we build a theoretical model that states that if the risk premium is indeed present, it must be time-varying. We show that it holds also for a case when both risk premium and deviations from rational expectations are present. In section 3.4.1 we build a modified UIP model that corrects for omitted variable bias by including a risk premium term in the model. We test the model and discuss the results. In section 3.5 we outline the asset pricing tests that are commonly used in academia and we perform such tests on a set of candidate risk factors. We discuss the findings and we summarize the thesis with conclusions in the last chapter.

2. UNCOVERED INTEREST PARITY AND FORWARD PREMIUM BIAS

2.1 Violation of Uncovered Interest Parity

A Carry Trade is a strategy well known to many currency traders that consists of borrowing a low-yielding asset and investing in a high-yielding asset. Scientific research of the strategy focuses almost exclusively on Currency Carry Trade. If investors wish to implement this strategy, they choose a currency pair where one of the currencies is associated with high risk-free interest rate, while the other is low-yielding currency. The intuition is very simple: you borrow in a country with low key interest rate r_i and buy a risk-free bond in a country with high interest rates so that the bond promises annual return of r^* where $r^* > r_i$. The bond pays in a local currency, and now we assume that the exchange rate stays the same throughout investor's time-horizon. In this case, the investor is expected to earn annual return of $R = r^* - r_i$, abstracting from transaction costs and effect of leverage.

In this work we concentrate our attention on the economies that are well developed so that the condition of capital mobility among countries is not impeded. If we add negligible transaction costs and absence of credit and country risks to the equation, we can expect the law of one price to hold. The law of one price states that identical goods or goods with identical cash flows must have identical prices unless significant transaction costs exist or there limits to arbitrage take effect. This fundamental law is applied to all assets, be it bonds, commodities, company's stock etc. For currencies, the law of one price equalizes the rates of return on interest-bearing instruments in different countries. This is called interest rate parity. It postulates that for the investments made in interest-bearing security in foreign country, no matter what the difference between exchange rates is, the exchange rate adjusted return will be identical to the return on domestic security. If the exchange rate didn't adjust, then there would be arbitrage opportunities.

In the real world, exchange rate between currencies normally is not fixed. Moreover, Meese & Rogoff (1983) have found that spot exchange rate is well approximated by a martingale. Anyone who invests in foreign securities cannot predict what future spot exchange rate S_{t+1} will be. In case there is functioning and liquid futures market (or alternatively, well established forward OTC market), investors can "cover" their positions against uncertainty of future spot rates by using forward or futures contracts. Given all information available today, such as:

- spot rate today S_t denominated in units of domestic currency per a unit of foreign currency,
- one period forward rate $F_{t,t+1}$,

- interest rate in home country r_i ,
- interest rate in foreign country r^* ,

investor has two alternatives. First, she can simply invest into domestic money market instruments yielding risk-free rate of r_i per period. But instead of it, she can buy $1/S_t$ of foreign currency, invest it into foreign money market instruments that promise return of r^* . To eliminate exchange rate risk, she purchases forward contract that promises the exchange rate of $F_{t,t+1}$ at the end of the period. Both alternatives must obey the law of one price, because if not, arbitrageurs will excessively enter one or the other trade position until alternative payoffs equal. This is covered interest parity (CIP) condition, and it formally states:

$$(1 + r_i) = \frac{F_{t,t+1}(1 + r^*)}{S_t} \quad (1)$$

or equivalently

$$\frac{(1 + r_i)}{(1 + r^*)} = \frac{F_{t,t+1}}{S_t} \quad (2)$$

Note that if we want to express the formula in such terms that S_t means number of units of foreign currency per one unit of domestic currency, and similarly for $F_{t,t+1}$, then we simply need to switch their places in equations (1) and (2).

To help with understanding the equation (1), we introduce the notion of forward premium which is defined as:

$$\Phi = \frac{F_{t,t+1} - S_t}{S_t}$$

If $\Phi > 0$, then the foreign currency is said to be traded at forward premium against the domestic currency. We can rewrite equation (1) as:

$$r_i = \left[\frac{F_{t,t+1}}{S_t} - 1 + 1 \right] (1 + r^*) - 1$$

Because $\Phi = F_{t,t+1}/S_t - 1$, we can rewrite the previous formula as:

$$r_i = \left[\Phi + 1 \right] (1 + r^*) - 1$$

We note that the product of forward premium variable and interest rate r^* is a very small number usually, hence the last formula can be approximated as:

$$\Phi = r_i - r^* \quad (3)$$

In fact, more widely used version of CIP condition is obtained by taking the natural logarithms of both sides of the equation (2) and approximating the values of $\ln(1 + r)$ with r :

$$f_{t,t+1} - s_t = r_i - r^* \quad (4)$$

where $f_{t,t+1}$ and s_t are natural logarithms of forward and spot rates, respectively.

Fundamentally, covered interest parity condition implies that if the interest rate in foreign country is bigger than the domestic rate such that $r^* > r_i$, the foreign currency must be trading at forward discount, or in other words, it must depreciate relative to domestic currency. Intuitively, if CIP holds, then there are no grounds for positive excess returns from currency carry strategy.

There is another form of interest parity condition that makes use of the assumption of risk neutrality and rational expectations hypothesis. In these conditions, forward exchange rate $F_{t,t+1}$ is an unbiased predictor of future exchange rate S_{t+1} :

$$F_{t,t+1} = E_t[S_{t+1}] + \varepsilon_{t+1} \quad (5)$$

where ε_{t+1} is an error term. Because of that, one can omit entering into forward contract and use spot exchange rate at the end of investment horizon instead. In the long run this strategy should result in very similar returns as the one that uses forward contracts, due to the use of unbiasedness hypothesis described above. It lets us rewrite covered interest parity condition (1) as:

$$(1 + r_i) = \frac{E_t[S_{t+1}](1 + r^*)}{S_t} \quad (6)$$

The equation (6) is formally known as Uncovered Interest Parity (UIP) and it states that an investor must be indifferent about interest rates in two different countries because the exchange rate is expected to adjust in such a way that the gains on interest rates differentials are offset by losses on exchange rate fluctuations, and vice versa. Note that similarly to (1) we can use expected foreign currency appreciation term:

$$\Delta E_t[S_{t+1}] = \frac{E_t[S_{t+1}] - S_t}{S_t}$$

Applying similar approximation logic as in derivation of (4), we arrive at approximate formula for UIP condition:

$$\Delta E_t[S_{t+1}] = r_i - r^*$$

which intuitive interpretation states that positive difference between foreign and domestic interest rates must be offset by domestic currency expected appreciation. When analyzing exchange rates, it is common to use logarithms of spot and forward rates because it enables to be independent of what currency is the base and what is the quote currency. This way, UIP condition (6) under risk-neutral assumption transforms into

the following representation³:

$$E_t[s_{t+1}] - s_t = r_i - r^* \quad (7)$$

Note, that if both CIP and UIP hold, using (4) and (7) we obtain

$$f_{t,t+1} = E_t[s_{t+1}] \quad (8)$$

If we assume rational expectations hypothesis, then ex post exchange rate should differ from ex ante expected rate only by a rational expectations forecast error, that is:

$$s_{t+1} = E_t[s_{t+1}] + \varepsilon_{t+1}. \quad (9)$$

So, if agents rationally make decisions in the foreign exchange market, we can use the last formulation to rewrite eq.(8) as:

$$f_{t,t+1} = s_{t+1} - \varepsilon_{t+1}$$

Do these forms of interest parity hold in empirical work? With regard to covered interest parity condition, Frenkel & Levich (1975) found that only 15% of apparent deviations from CIP are not explained by the effect of transaction costs. For currency pairs not involving US dollar, almost all deviations are explained by the costs of transacting. In another study, Clinton (1988) introduce transaction costs from exchange market swaps and although they reject the hypothesis of the absence of profitable trading opportunities in covered interest arbitrage, they argue that those trading opportunities are empirically not large enough to cause a flow of excess returns over time.

With regard to Uncovered Interest Parity, the test of its validity is difficult to perform because expected exchange rates are hardly observable. To make UIP testable, the assumption of rational expectations needs to be made. Following this logic, Fama (1984) performs a joint test of rational expectations hypothesis and uncovered interest parity. The former one states that future realizations of exchange rate are an unbiased measure of the ex ante expected exchange rate (see eq. (9)). ε_{t+1} is a white-noise forecast error term and by definition it must be orthogonal to all the information known at time t . Further, unlike in (5), Fama (1984) represents forward exchange rate as a sum of expected future spot rate and a premium:

$$f_{1,t+1} = E[s_{t+1}] + P_t \quad (10)$$

³If we assume risk-averse world, the condition (6) doesn't transform into (7) due to the fact that $\ln(E_t[s_{t+1}]) \neq E_t[s_{t+1}]$. To see why, refer to Appendix 1. However, representation (7) is a suitable for risk-neutral framework as Bekaert & Hodrick (1993) find that omitting $\frac{1}{2}\sigma_t^2(s_{t+1})$ from UIP regression does not affect the result.

Based on it, Fama expresses the difference between forward and spot prices as:

$$f_{t,t+1} - s_t = P_t + E[s_{t+1} - s_t] \quad (11)$$

Note that last equation is in line with the formulas (4) and (7) derived above but it leaves some room for the premium term. Fama (1984) investigates whether forward rates contain any information on future changes in spot exchange rates and if the premium and the expected part are time-varying. The slope coefficients β_1 and β_2 must sum up to one in the following regressions from Fama (1984):

$$f_{t,t+1} - s_{t+1} = \alpha_1 + \beta_1(f_{t,t+1} - s_t) + \varepsilon_{1,t+1} \quad (12)$$

$$s_{t+1} - s_t = \alpha_2 + \beta_2(f_{t,t+1} - s_t) + \varepsilon_{2,t+1} \quad (13)$$

where $\varepsilon_{i,t+1}$ are the rational expectations forecast error terms which must be uncorrelated with any information Ω available at time t : $E[\varepsilon_{t+1}|\Omega_t] = 0$. This test of UIP is based on both rational expectations hypothesis and risk neutrality. By virtue of Covered Interest Rate Parity, the regressor $(f_{t,t+1} - s_t)$ can be replaced with difference between interest rates in two countries (see eq. (4)), thus any premium in the forward rate is due to the interest rate differential⁴. For the UIP to hold, the coefficient β_2 must be equal to one and α_2 be zero. Note, that if UIP holds, i.e. if $\beta_2 = 1$ and $\alpha_2 = 0$, then the regression (13) reduces to rational expectations definition (8), and it also implies that risk premium term P_t in Fama equation (10) is zero⁵, thus the hypothesis of uncovered interest parity simultaneously implies rational expectations with risk neutrality. Be risk premium non-zero and time-varying, the forward rate will no longer be an unbiased predictor of future spot rate casting doubt on the validity of rational expectations hypothesis. In other words, for the UIP to hold, both forward unbiasedness and risk neutrality are each assumed to hold, too. Thus the validity of the UIP condition indicates risk neutral efficient market.

Surprisingly, Fama (1984) finds that for all the sample, β_2 is negative (while β_1 is positive)⁶. On top of that, both the premium P_t and the expected change in the spot exchange rate $E[s_{t+1} - s_t]$ vary over time, and the variance of the premium component P_t is found to be reliably greater than the variance of expected spot rate change. It is also concluded that reliable negative estimates of β_2 are due to negative covariation

⁴Based on complete purchasing power parity and Fischer condition for nominal interest rates, Fama (1984) shows that the premium component P_t of forward exchange rate is merely the difference between the expected real returns on the nominal bonds of domestic and foreign countries.

⁵To be precise, P_t is the conditional bias in the forward forecast of future spot rate. There is a strand of research that shows that P_t in fact represents a risk premium. However, the models of risk averse agents have difficulties to link the conditional bias term with the risk premium conclusively.

⁶This finding has substantial support in the literature. Froot (1990) summarizes 75 published estimates of β_2 and finds the average value to be -0.88.

between premium component P_t and $E[s_{t+1} - s_t]$. This result suggests that when interest rate differential is negative so that the foreign currency is associated with higher interest rate compared to domestic currency, the negative β_2 in (13) causes positive foreign currency appreciation⁷. Such findings go against UIP condition. This anomaly in the literature was given a name of Forward Premium Puzzle. Much research has been conducted in order to find explanations for the bias with mixed results. First, we will discuss the possibility that the foreign exchange market generally cannot be considered efficient, and because of this fact we have a failure of a wrong sign for the slope in (13). Next, we will study explanations of the failure of the efficiency hypothesis.

2.2 Explanations for Forward Premium Puzzle

2.2.1 Is Foreign Exchange market efficient?

A stylized fact about exchange rates is that their time-series is nonstationary. A major finding by Meese & Rogoff (1983) states that exchange rates are best described as random walk processes. Once we take it into account, we see that β_2 in regression (13) has to have value of zero. It is simply not supported by empirical results of the research of last three decades. If random walk was indeed the process governing exchange rates, then combined with risk neutrality and rational expectations hypothesis we would have

$$f_{t,t+1} = E_t[s_{t+1}] = s_t$$

which in turn implies

$$f_{t,t+1} - s_t = 0$$

However, the difference between forward and spot rates are almost always non-zero. Given simple version of market efficiency, the last practical observation hints that the market might deviate from efficiency. In general, tests of efficiency in the foreign exchange market are often performed after making assumptions that investors are risk neutral, markets are competitive and frictionless and agents use information rationally. If the market is indeed efficient, then expected rate of return to speculative strategies in forward market must be zero. We can assume that the return to speculation is defined as $f_t - s_{t+1}$. So for efficient foreign exchange market, the return to speculators must

⁷We will see in the next sections that this relationship doesn't necessarily follow from the regression (13). Nonetheless, with respect to β_2 Bansal & Dahlquist (2000) point out that much of the empirical findings that confirm negative relationship come from the studies of developed economies. Motivated by the observation that emerging economies are economically different (they have lower income per capita, higher inflation and higher inflation uncertainty together with higher nominal interest rates), they performed a study of the correlation between exchange rate change and interest rate differential. Indeed, Bansal & Dahlquist (2000) found that for emerging and the lower-income developed economies there is no forward premium puzzle: a positive domestic interest rate differential predicts a depreciation of the domestic currency.

be:

$$f_{t,t+1} - E_t[s_{t+1}|\Omega_t] = 0$$

where $E_t[s_{t+1}|\Omega_t]$ indicates expected future exchange rate conditional on available information set at time t . The hypothesis above implies that the forward rate forecast error ($s_{t+1} - f_{t,t+1}$) is uncorrelated with information set available at time t ⁸. Hansen & Hodrick (1980) test this hypothesis using OLS regression. Choosing the most powerful variables from the set Ω_t helps with further tests of alternative hypothesis, and Hansen & Hodrick (1980) decided to use the most recent past forecast errors from all the exchange rates. The study discovers a problem of having a finer data sample than the maturities of the forward contracts. It results in serial correlations in forecast error term, thus Hansen & Hodrick (1980) assert that general least squares estimations cannot be used due to the failure of the requirement of strict exogeneity:

$$E[\varepsilon_{t+1}|\Omega_{t-1}, \Omega_t, \Omega_{t+1} \dots] = 0$$

It is intuitive that knowing the value of some variable in the information set Ω_{t+1} , such as lagged forward forecast error, one can refine her predictions on future spot exchange rate. Being unable to use GLS estimations, they perform OLS regression analysis with adjusted covariance matrix to test the hypothesis that both $\alpha = 0$ and $\beta = 0$ in:

$$s_{t+1} - f_{t,t+1} = \alpha + \beta\Omega_t + \varepsilon_{t+1}$$

The result of this regression analysis was the rejection of null hypothesis. In other words, they found that lagged forecast errors have explanatory power in predicting today's forecast errors. This failure of efficiency hypothesis is supported by high values of R^2 statistic, namely 0.402 and 0.508 for CAD and Deutsche Mark respectively.

Hansen & Hodrick (1980) point out, however, that the empirical rejection of this hypothesis should not be equated with an example of market failure. The hypothesis ignores any considerations on intertemporal allocation, risk aversion and agents' knowledge about stochastic properties of possible important monetary policy decisions. However, the failure of this efficiency hypothesis indicates that at least strong form of efficiency is rejected.

2.2.2 Tests of market efficiency in real terms

Tests of the regression (13) have appealing implications because such representation doesn't impose any restrictions on the environment of the agents and their preferences and yet it tests the efficiency of foreign exchange market. In essence, the null hypothe-

⁸Indeed, this is in line with the rational expectations hypothesis that states that forecast error term ε_{t+1} in (9) must be uncorrelated with any information available at time t .

sis that $\alpha_2 = 0$ and $\beta_2 = 1$ is associated with the equilibrium condition of the efficient markets and risk-neutral agents endowed with rational expectations. However, there might be a problem with such interpretation, because the agents value not nominal but real returns, therefore, the regression tests (13) could suffer from the omitted variable bias. As an example of such variable one can take price level or inflation rate in the agent's environment.

To test for foreign exchange market efficiency in the real terms, Engel (1996) states the condition of the absence of real profit opportunities by trading forward and spot contracts:

$$E_t\left[\frac{F_t - S_{t+1}}{P_{t+1}^\$}\right] = 0 \quad (14)$$

where $P_{t+1}^\$$ stands for the price level in the domestic market. Cumby (1988) tests the hypothesis that the mean value of the following variable conditional on the forward discount at time t is zero:

$$r_{t+1} = \frac{(S_{t+1} - F_t)P_t^\$}{S_t P_{t+1}^\$}$$

It represents real rate of return of the trade when an agent buys currency forward today and sells it on the spot market at maturity. It is not hard to see that if the hypothesis (14) holds, then the value of this variable should indeed be zero given the value of forward premium at time t . Cumby (1988) rejects the null for the pound, the mark, Swiss franc, French franc and Canadian dollar for the period from 1974 to 1986. He concludes that expressing returns in real terms rather than nominal terms doesn't change the result that the risk-neutral efficient market hypothesis is rejected.

2.2.3 A close look at Rational Expectation Hypothesis

One strand of research focuses on possible failure of rational expectation hypothesis as a reason for the bias. As we recall, the original UIP regression from Fama (1984) relies upon the assumption of unbiased forward rate:

$$s_{t+1} = f_{t,t+1} + u_{t+1} \quad (15)$$

that is, the forward rate is an unbiased predictor of future exchange rate. Thus there should be one-to-one relationship between future realized exchange rate and the forward rate and realized forecast error u_{t+1} must have expected value of zero. Engel (1996) outlines many studies that show that the spot exchange rate has a unit root, but it is difference stationary. In this case, it is easy to see that the forward premium term must be stationary, too. Under rational expectations hypothesis, $f_{t,t+1} = s_{t+1} - u_{t+1}$, and subtracting s_t from each of the sides yields:

$$f_{t,t+1} - s_t = s_{t+1} - s_t - u_{t+1} \quad (16)$$

If rational hypothesis holds, then the residual u_{t+1} must be stationary, because by definition the rational expectation forecast error should not be predictable from past values. Given all of these definitions, one can test for stationarity of forward premium $f_{t,t+1} - s_t$, with such implications that if it is rejected, then it would imply that forward rate unbiasedness is rejected, too. In other words, it would imply the failure of the rational expectations hypothesis which in turn could be the reason why UIP doesn't hold in practice.

Hai, Mark, & Wu (1997) reject the hypothesis that the forward premium has a unit root, consequently rejecting the null that it is I(1) integrated.

In contrary, Crowder (1994) uses monthly data for the exchange rates of the pound, Deutsche mark and Canadian dollar for the period 1974 - 1991, and rejects the null hypothesis of stationarity for the forward premium. Their tests indicate a strong unit root in the forward premium. Similarly, Baillie & Bollerslev (1994) can't reject a unit root in forward premium using Augmented Dickey-Fuller test. They reject the null hypothesis that forward discount is a I(0) process. However, they note that the results of Crowder (1994) can indicate fractional cointegration rather than a unit root. Interestingly, Baillie & Bollerslev (1994) find the orders of integration for forward discounts of the Canadian Dollar, the pound, and the mark to be between zero and the unity.

A question arises whether the risk-neutral efficient market hypothesis holds when forward premium is fractionally integrated. Given risk neutrality, it doesn't hold as we can recall that the error term u_{t+1} in (16) must be white noise. That is the reason why the forward premium $f_{t,t+1} - s_t$ must be of the same order of integration as the spot rate appreciation term $s_{t+1} - s_t$ (see eq. (16)). However, even if the forward premium is fractionally integrated, the efficient market hypothesis and rational expectations hypothesis can still hold if we allow for existence of a risk premium which would have to be fractionally integrated:

$$f_{t,t+1} - s_t = s_{t+1} - s_t - u_{t+1} + P_t$$

Hakkio & Rush (1989) note that if forward rate is an unbiased predictor of future spot rate, meaning that the forward rate doesn't overly predict or under predict the spot rate, then s_{t+1} and $f_{t,t+1}$ must be cointegrated with cointegration vector of [1, -1] because the difference $s_{t+1} - f_{t,t+1} = u_{t+1}$ is stationary due to the assumption of stationary forecast error⁹. Analyzing GBP/USD and DEM/USD rates from 1975 to 1986, Hakkio & Rush (1989) reject the null of no cointegration between the realized spot and forward rates. Hai, Mark, & Wu (1997) also perform a cointegration test for their 1976

⁹The cointegration between future realized spot and forward relies upon the assumption that each of the future spot rate and forward has a unit root.

- 1992 dataset of the following exchange rates: franc/dollar, pound/dollar, yen/dollar. They reject hypothesis that the forward premium contains a unit root. Further, they reject hypothesis that the spot and forward rates are not cointegrated. Similarly, Horvath & Watson (1995) find that s_{t+1} and $f_{t,t+1}$ have a cointegration vector of $[1, -1]$.

Engel (1996) notes that the requirement of cointegration between s_{t+1} and $f_{t,t+1}$ with a vector of $[1, -1]$ is a requirement for the rational expectations hypothesis to hold, the finding of such cointegration in itself is not a sufficient requirement for the hypothesis to hold true. As an example of the case when the hypothesis of rational expectation fails even though s_{t+1} and $f_{t,t+1}$ are stationary Engel (1996) considers main UIP regression (13). It can be rewritten as follows:

$$\begin{aligned}
 s_{t+1} - s_t &= \alpha_2 + \beta_2(f_{t,t+1} - s_t) + \varepsilon_{2,t+1} \\
 &\Downarrow \\
 s_{t+1} - f_{t,t+1} &= \alpha_2 + \beta_2(f_{t,t+1} - s_t) - (f_{t,t+1} - s_t) + \varepsilon_{2,t+1} \\
 &\Downarrow \\
 s_{t+1} - f_{t,t+1} &= \alpha_2 + (\beta_2 - 1)(f_{t,t+1} - s_t) + \varepsilon_{2,t+1}
 \end{aligned}$$

If the forward discount is stationary and the error term is stationary, $s_{t+1} - f_{t,t+1}$ is stationary, too. But if $\beta_2 \neq 1$ and $\alpha_2 \neq 0$, the null hypothesis of rational expectations (8) doesn't hold.

So far we have discussed econometric tests that we can perform to determine if the forward rate is indeed the unbiased predictor of future spot rate. The results are generally mixing even for the same set of currencies. It can be suggested that the tests are very sensitive to the sampling periods as well as different properties of test statistics can play a role in creating conflicting results. But we haven't yet discussed what likely reasons are for the detected failures of the rational expectations hypothesis. The forward rate can be biased because of the presence of forecast errors or the existence of time-varying risk-premium or both.

2.2.4 Rational Learning and Peso problems

As noted earlier, the rational expectations hypothesis assumes that the forecasts errors of future exchange rate must be orthogonal to the information set at time t and have a zero mean. However, agents might not know the distribution of forecast errors that they make, and in the end, their trades could cause autocorrelated rational expectations forecast errors with mean different than zero. Lewis (1995) suggests the following example: assume that there are two potential monetary regimes M_1 and M_2 each of which affects expected value of future exchange rate, i.e.: $E_t[s_{t+1}|M_1] \neq E_t[s_{t+1}|M_2]$.

This way, we can write unconditional expected value of the exchange rate at $t + 1$:

$$E_t[s_{t+1}] = (1 - p_t)E_t[s_{t+1}|M_2] + p_tE_t[s_{t+1}|M_1]$$

with p denoting the probability that the monetary policy M_1 is effective. Agents are assumed to follow rational learning process, and they update the probability p accordingly. Let's then assume that the change of monetary policy occurs at time τ prior to time t . That way, agents closely follow the behaviour of exchange rate trying to learn if the shift in monetary regime has occurred. They use Bayes' law to update their subjective probabilities of the regime switch:

$$p_t = \frac{p_{t-1}L(\Delta s_t, \Delta s_{t-1}, \dots, \Delta s_{\tau+1}|M_1)}{(1 - p_{t-1})L(\Delta s_t, \Delta s_{t-1}, \dots, \Delta s_{\tau+1}|M_2) + p_{t-1}L(\Delta s_t, \Delta s_{t-1}, \dots, \Delta s_{\tau+1}|M_1)}$$

where $L(\cdot|M_i)$ denotes the likelihood of the observation given the corresponding regime. In case the regime indeed changes at some time $\tau < t$, the actual observations of the exchange rate that agents see will tend to change accordingly, and therefore increase or decrease the likelihood of the new monetary regime being in place. As an example, assume that $E_t[\Delta s_{t+1}|M_1] > E_t[\Delta s_{t+1}|M_2]$ because M_2 represents a tightening of monetary policy while M_1 stands for expansionary money supply. Then, exchange rate will decrease over time and consequently agents will increase their subjective probabilities of contractionary monetary regime $1 - p_t$ relative to the probabilities of the old regime p_t . This posterior probability p_t then used as a prior probability in the next time frame when the new observation is available. In the end, after consecutive updates made by agents, $L(\cdot|M_1)$ shrinks as more and more observations becomes available - $p_t \rightarrow 0$ as $t \rightarrow \infty$. In other words, agents learn about the regime change over time.

An interesting feature of the Bayes' rule outlined above is that the agents during their learning attach non-zero probabilities to both regimes whereas there is actually only one in place. The implication of this is a serial correlation of forecast errors with non-zero mean. To see it, consider the situation that the monetary regime actually changes at time $\tau < t$ from M_1 to M_2 . This implies that the limit $\lim_{t \rightarrow \infty} p_t = 0$. Note that rational expectations forecast error is the difference between the observed actual exchange rate and the expected exchange rate which in turn is given as a weighted average of the exchange rates expected under each of the monetary regimes. To put it another way, the forecast errors are defined as following:

$$s_{t+1} - E_t[s_{t+1}] = \left(s_{t+1} - E_t[s_{t+1}|M_2] \right) - p_t \left(E_t[s_{t+1}|M_1] - E_t[s_{t+1}|M_2] \right)$$

As can be seen, the first two terms combined in the parenthesis indicate the rational expectations forecast error if agents knew with certainty that the new regime is in force.

From the equation, as long investors are learning, that is, as long as they believe $p_t \neq 0$, the mean of forecast errors will be non-zero, even if the agents are rational.

Lewis (1989) examine the data on exchange rates during the early 1980s when the US money markets experienced contractionary regime. She found that the model of rational learning can explain up to 50% of the changes in excess currency returns. However, as Lewis points out, the learning hypothesis cannot entirely explain UIP puzzle. As market participants learn about new regime, the probability of the old one must approach zero, thus implying that β coefficient in (13) must converge to one. Built on this intuition, the estimates of β coefficient at the end of 1980s must be closer to one than at the beginning of 1980s. Contrary to this, Lewis (1989) finds that estimates are significantly negative for the sample covering the end of the 1980s.

Despite these discouraging results, it can be possible that during the end of the 1980s agents actually were learning about another monetary regime shift. In general, they can act rationally and be in the process of learning about different regimes incorporating the possibility that the economic conditions can revert or simply change in the meantime. In other words, rational agents can be learning about the current regime, as well as learning about upcoming changes. Especially an interesting case arises when agents attach a relatively small probability to a large future move in economic fundamentals that hasn't yet been observed in the sample. This situation is commonly known as the peso problem.

In essence, the implications of the peso problem are the same as of the rational learning model - it creates the skew in the distribution of the forecast errors. Even a small probability of an extreme event can generate a large skew that would pose a problem for statistical tests. To see this, first assume the M_2 stands for a severely contractionary economic state, while M_1 denotes a stable state. Probability p_t represents the perceived probability that the economy enters the state M_2 during next period $t + 1$. Given all that, the expected future exchange rate can be written as:

$$E_t[s_{t+1}] = p_t E_t[s_{t+1}|M_2] + (1 - p_t) E_t[s_{t+1}|M_1]$$

This way we can express the forecast errors if the regime doesn't change from M_1 to M_2 :

$$s_{t+1} - E_t[s_{t+1}] = \left(s_{t+1} - E_t[s_{t+1}|M_1] \right) - p_t \left(E_t[s_{t+1}|M_2] - E_t[s_{t+1}|M_1] \right)$$

In this case, from the definition of the peso problem, the difference between expected values for the two regimes is large, thus providing a significant skew to the distribution of the rational forecasts expectation errors. Even when the shift indeed occurs, there

will be forecasts errors over and beyond the white noise:

$$s_{t+1}^{M_2} - E_t[s_{t+1}] = \left(s_{t+1}^{M_2} - E_t[s_{t+1}|M_2] \right) - (1 - p_t) \left(E_t[s_{t+1}|M_2] - E_t[s_{t+1}|M_1] \right)$$

However, given the monetary regime shift is announced publicly, abnormal errors should disappear after the period $t + 1$. Burnside et al. (2011) treat a peso event as a rare disaster event that sharply reduces the payoffs to an agent or sharply increases the value of the stochastic discount factor or both. They explain that when a currency is traded at a forward premium and an investor sells it forward, there might be a small chance of a very large appreciation that would result in severe losses on this trade. The compensation for such risk must give an apparent positive payoffs to investors who engage in carry trade in good times. The work of Burnside et al. (2011) provides valuable evidence against the peso problem as a source of expectation bias causing forward premium puzzle. They use currency options data to evaluate whether the payoffs to a hedged carry trade differ from the payoffs to naked carry trade. As the name suggests, the hedged version consists of selling the foreign currency forward and simultaneously buying a call option on that currency. It enables the agent to sell the currency forward at the strike price even if the exchange rate appreciates. Similarly, the downside risk can be hedged if the agent buys a put option while he enters a long forward contract on the currency. Using such trades eliminates the possibility of incurring large losses associated with a peso event.

Burnside et al. (2011) show that if there is indeed a small chance of a large shock to currency rates, then investors are expected to be compensated for the associated losses. That way, risk-adjusted payoffs in non-peso states should be positive, something that is also typical for carry trades. By comparing the risk-adjusted payoffs to hedged and unhedged carry trade strategies, Burnside et al. (2011) come to conclusion that peso events don't provide a rationale for the payoffs to the carry trade in non-peso states. They find that it's high values of stochastic discount factor in the peso state that better match the data, not the very large negative payoffs of the peso event. Based on their estimates, the average risk-adjusted payoffs of the hedged carry trade is not much different from the payoffs to the unhedged strategy in the non-peso state. Since the risk-adjusted payoffs in non-peso state must include a compensation for possible losses in the rare disaster state, the risk-adjusted losses of the two strategies in the peso state should also be of a similar level. By construction, the losses to the hedged version are small in the rare disaster event, which means that the losses to the unhedged carry trade must also be small in the peso event.

Burnside et al. (2011) argue that the peso even is likely to be associated with very high values of the stochastic discount factor. Even though, the losses to carry trade are moderate in that state, investors attach big importance to those losses. Further,

Burnside et al. (2011) estimate the value of the SDF in the peso state using carry trade returns. As a check, they construct a hedged and an unhedged stock portfolios that exhibit significantly different payoffs than the carry trade. Remarkably, the estimates of the SDF for the peso state calculated from these two contrasting strategies turned out to be both large and not statistically significantly different from each other.

2.2.5 Relaxing risk neutrality assumption: presence of Risk Premium

An alternative explanation for the Forward Premium Puzzle is based on the existence of the premium term in (10) which can be interpreted as the risk from foreign exchange fluctuations. But first, we must answer the question whether there should be the room for any risk premium in currency markets. To see the problem, consider the world with two countries only. Agents in their home country called A can buy short-term riskless bonds at home, or buy equivalent foreign bonds issued by the central bank of the country B. If they choose the latter option, they bear foreign exchange risk. The agents from the country B have exactly the same options, only that they will be exposed to the exchange rate risk if they hold the bonds issued by the country A, but they will bear no risk if they hold their home country bonds. The question is if the agents should be compensated for bearing foreign exchange risk if all of them can avoid bearing that risk by holding only the bonds from their own countries.

Agents from each country evaluate returns in the units of their home currency. We can rewrite uncovered interest parity condition (6) and allow non-zero excess return on foreign deposit held for one period by investor from country A and then changed back to his home currency:

$$1 + R_{Excess}^A = \frac{(1 + R^B) \cdot E_t[S_{t+1}]}{(1 + R^A) \cdot S_t}$$

Similar condition can be defined for the investor from country B:

$$1 + R_{Excess}^B = \frac{(1 + R^A) \cdot E_t\left[\frac{1}{S_{t+1}}\right]}{(1 + R^B) \cdot \frac{1}{S_t}}$$

We can rewrite both conditions in a logarithmic form:

$$r_{Excess}^A = r^B + \ln\left(E_t[S_{t+1}]\right) - r^A - s_t$$

and

$$r_{Excess}^B = r^A + \ln\left(E_t\left[\frac{1}{S_{t+1}}\right]\right) - r^B + s_t$$

Under the assumption of lognormally distributed exchange rates:

$$E_t[S_{t+1}] = \exp\left(E_t[s_{t+1}] + \frac{1}{2}\sigma_t^2[s_{t+1}]\right)$$

and

$$E_t\left[\frac{1}{S_{t+1}}\right] = \exp\left(-E_t[s_{t+1}] + \frac{1}{2}\sigma_t^2[s_{t+1}]\right)$$

Therefore, we have:

$$r_{Excess}^A = r^B + E_t[s_{t+1}] + \frac{1}{2}\sigma_t^2[s_{t+1}] - r^A - s_t$$

and

$$r_{Excess}^B = r^A - E_t[s_{t+1}] + \frac{1}{2}\sigma_t^2[s_{t+1}] - r^B + s_t$$

In other words, $r^A \neq r^B$ and $r^A \neq -r^B$ because of the convexity term. It is clear from this exercise that the risk premium demanded by the agent living in country A differs from the risk premium required by the agent from the country B. On the other hand, we can note that:

$$r_{Excess}^A - \frac{1}{2}\sigma_t^2[s_{t+1}] = -r_{Excess}^B + \frac{1}{2}\sigma_t^2[s_{t+1}]$$

What we showed in this example is simply that the risk premium, if exists, is different for every party involved in foreign exchange transactions. One might ask if it is possible to approach a study of the risk premium in foreign exchange markets on macro level without the need to specify the preferences of agents in each and every country. It appears that it is very well possible. To see this, take the simple average of risk premiums outlined above:

$$\frac{r_{Excess}^A - r_{Excess}^B}{2}$$

As a matter of fact, the average of the risk premiums in absolute value corresponds to the premium term (11) defined in Fama (1984). Therefore, when we study the presence of risk premium using macroeconomic data and abstracting away from the differences in tastes, risk aversion and other preferences between countries, we deal with average risk premium in foreign exchange market.

Now when we know that risk premium is theoretically plausible, let us continue with its properties. For a start, when we relax risk neutrality condition and allow for the exchange rate risk to be non-fully diversifiable, forward rate can no longer be seen as pure estimate of expected future spot exchange rate. As noted by Engel (1996), P_t in (10) can be viewed as a risk premium only if agents have rational expectations, forward rate doesn't consistently over or under predict future exchange rate and P_t is in

some way linked to economic variables. If we find the presence of the conditional bias in the forward forecasts of future exchange rate, then we can rewrite the hypothesis of unbiased forward rate (15) as:

$$s_{t+1} = f_t + P_t + u_{t+1}$$

To test for the existence of the time-varying risk premium, one needs to analyze time-series properties of P_t and to test if they are in line with the restrictions imposed by models of risk-averse behaviour.

Let us get back to Fama regressions (12) and (13). Fama (1984) specifies coefficients of the regressions under the assumption of rational expectations as:

$$\begin{aligned} \beta_1 &= \frac{\text{cov}(f_t - s_{t+1}, f_t - s_t)}{\sigma^2(f_t - s_t)} = \\ &= \frac{\sigma^2(P_t) + \text{cov}(P_t, E[s_{t+1} - s_t])}{\sigma^2(P_t) + \sigma^2(E[s_{t+1} - s_t]) + 2\text{cov}(P_t, E[s_{t+1} - s_t])}; \end{aligned} \quad (17)$$

$$\begin{aligned} \beta_2 &= \frac{\text{cov}(s_{t+1} - s_t, f_t - s_t)}{\sigma^2(f_t - s_t)} = \\ &= \frac{\sigma^2(E[s_{t+1} - s_t]) + \text{cov}(P_t, E[s_{t+1} - s_t])}{\sigma^2(P_t) + \sigma^2(E[s_{t+1} - s_t]) + 2\text{cov}(P_t, E[s_{t+1} - s_t])}. \end{aligned} \quad (18)$$

Fama (1984) notes that since $\sigma^2(E[s_{t+1} - s_t])$ in the definition of β_2 must be non-zero, the negative value of β_2 indicates that $\text{cov}(P_t, E[s_{t+1} - s_t])$ is negative and it is bigger in absolute terms than the variance of the expected exchange rate appreciation $\sigma^2(E[s_{t+1} - s_t])$. Further, since β_1 is found to be reliably positive, $\sigma^2(P_t)$ is bigger in absolute value than $\text{cov}(P_t, E[s_{t+1} - s_t])$, thus we have:

$$\sigma^2(P_t) > \sigma^2(E[s_{t+1} - s_t])$$

To proceed further, let us take the difference of betas:

$$\beta_1 - \beta_2 = \frac{\sigma^2(P_t) - \sigma^2(E[s_{t+1} - s_t])}{\sigma^2(f_t - s_t)}$$

Fama (1984) reports all the values of the differences between betas to be greater than 2 except for the case of Japan. Furthermore, estimates for β_1 and β_2 are perfectly negatively correlated because they sum to 1.0 and β_1 is found positive alongside with negative β_2 . Since the denominators in (17) and (18) are the same, the correlation between $\sigma^2(P_t)$ and $\sigma^2(E[s_{t+1} - s_t])$ must be negative. Additionally, negative β_2 indicates that $(s_{t+1} - s_t)$ is negatively correlated with $(f_t - s_t)$. Summing it all up we can conclude that:

- Risk premium term has negative covariance with expected exchange rate appreciation: $cov(P_t, E[s_{t+1} - s_t]) < 0$.
- Variance of the risk premium is reliably greater than the variance of the spot rate appreciation: $\sigma^2(P_t) > \sigma^2(E[s_{t+1} - s_t])$.
- Both the risk premium P_t and the expected future exchange rate appreciation $\sigma^2(E[s_{t+1} - s_t])$ are time-varying.

It is important to note that the models of the FX risk premium should be consistent with these inequalities. We will turn to them later in this section, but now let us outline the time-series properties of $(f_t - E[s_{t+1}])$ term. It is worth stressing it once again that the time-series behaviour of this term doesn't necessarily imply the same behaviour of the risk premium term, because first, (10) is applicable only under the assumption that the rational expectations hypothesis holds, and second, because behaviour of $(f_t - E[s_{t+1}])$ should be consistent with characteristics of risk premium predicted by the models of foreign exchange risk premium. Interestingly, if we find a strong evidence of the failure of the rational expectations, then the time-series study of the same term $(f_t - E[s_{t+1}])$ can be treated as a study of the time-series of deviations from rational expectations. Needless to say, in that case, models of irrational expectations must conform to the observed time-series properties of this term.

Cheung (1993) employs a Kalman filter to model $(f_t - E[s_{t+1}])$, thus treating the risk premium as an unobserved variable. He utilizes the data on dollar exchange rates for the pound, the mark and the yen from July 1973 to December 1987. Constructed measure of P_t is found to be significantly persistent and highly variable. He confirms the findings of Fama that the correlation between P_t and $(E[s_{t+1} - s_t])$ is negative.

Bekaert (1994) also confirms the finding that the measure of $(f_t - E[s_{t+1}])$ exhibits a large value of unconditional variance. As in Cheung's work, Bekaert obtains a result that $(f_t - E[s_{t+1}])$ is highly serially correlated, but he also finds that the conditional variance of the term exhibits heteroskedasticity due to volatility clustering.

Turning to finding the models of risk-averse behaviour that would explain the deviations from uncovered interest parity it is natural to start with a no-arbitrage condition that should hold for everyone:

$$1 = E_t[m_{t+1}R_{t+1}] \quad (19)$$

where R_{t+1} is the return on an asset and m_{t+1} is a random variable that is often called stochastic discount factor, marginal rate of substitution, or the pricing kernel. Cochrane (2005) explains that m_{t+1} is the rate at which an agent is willing to substitute his consumption at time $t + 1$ for consumption at time t . To capture investors' impatience

and aversion to risk, we can introduce a utility function:

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]$$

with c_t denoting consumption at time t and β representing subjective discount factor that captures investors impatience¹⁰. It is not important now which functional form is used for $u(c_t)$, it is just enough to state that the utility function is increasing, reflecting the notion that more consumption is better, and concave, reflecting the declining marginal value. Using such representation of investors' utility we are able to capture their fundamental desire for more consumption, as opposed to the desire to achieve intermediate goals such as specific values of mean and variance of their portfolios. By applying such utility function, it is possible to numerically adjust for the risk and the timing of the investment.

Cochrane (2005) shows how first-order condition is obtained for optimal consumption and choice of assets. Rearranging that condition, we arrive at the central asset pricing formula:

$$p_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]$$

where x_{t+1} is a payoff at time $t+1$ and p_t is the price of an asset. An important special case of this formula is expressed as:

$$1 = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1}^j \right] \quad (20)$$

where R_{t+1}^j represents real gross return on the asset j . This is a fundamental relationship and it should hold for any asset whether it is domestic or foreign. Denote $R_{t+1}^{\$}$ the real return on dollar-denominated assets, and R_{t+1}^{\pounds} as real return on Pound-denominated assets, then (20) turns into:

$$0 = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} (R_{t+1}^{\$} - R_{t+1}^{\pounds}) \right]$$

We can rewrite the difference in real returns in terms of spot and forward dollar rates of the pound:

$$0 = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{(F_t - S_{t+1})\Pi_t}{S_t\Pi_{t+1}} \right]$$

where spot and forward are GBP/USD rates, and Π is the dollar price level.

It is a useful approximation to assume that the logarithms of forward F_t , spot S_t rates as well as price level Π and the marginal rate of substitution all have multivariate

¹⁰Every investor has different value for β , but nothing stops us from averaging them out if we use macro data on the consumption in the economy.

normal distribution. It allows for the following representation of the last equation:

$$E_t \left[\frac{F_t \Pi_t M}{S_t \Pi_{t+1}} \right] = E_t \left[\frac{S_{t+1} \Pi_t M}{S_t \Pi_{t+1}} \right]$$

where M represents marginal rate of substitution. Computing expectations for multivariate lognormal distribution yields:

$$E_t(s_{t+1}) + \frac{1}{2} \sigma_t^2 (s_{t+1} + p_t + m - s_t - p_{t+1}) = f_t + \frac{1}{2} \sigma_t^2 (f_t + p_t + m - s_t - p_{t+1})$$

where lowercase letters stand for natural logarithm of the corresponding variables. With the use of the linearity of expectation and the bilinearity of covariance, the last equation can be simplified as:

$$E_t(s_{t+1}) = f_t - \frac{1}{2} \sigma_t^2 (s_{t+1}) + Cov_t(s_{t+1}, p_{t+1}) - Cov_t(s_{t+1}, m_{t+1})$$

Once utility function is represented in a power utility form, the last equation transforms into :

$$E_t(s_{t+1}) = f_t - \frac{1}{2} \sigma_t^2 (s_{t+1}) + Cov_t(s_{t+1}, p_{t+1}) + \gamma Cov_t(s_{t+1}, c_{t+1}) \quad (21)$$

given

$$u(c) = \frac{1}{1-\gamma} c_t^{1-\gamma}.$$

Given the risk premium definition (10), the last equation can be transformed to represent risk premium term as:

$$rp_t = \frac{1}{2} \sigma_t^2 (s_{t+1}) - Cov_t(s_{t+1}, p_{t+1}) - \gamma Cov_t(s_{t+1}, c_{t+1}) \quad (22)$$

The first two terms of the equation (22) are commonly referred to as the 'Jensen's Inequality Terms' (JIT). Cumby (1988) and many other studies have found that JIT is empirically a very small number, thus the value of risk premium largely depends on the last term of (22). It is only logical that when investors are risk neutral ($\gamma = 0$), the risk premium value is practically zero.

The model (22) and related asset pricing equations outlined above should hold for any utility-maximizing investor, irrelevant of location. Under assumption that investors have homogeneous preferences, the equations should also hold for aggregate consumption. This way it is possible to test this model using macroeconomic data on consumption such as consumption of nondurables, and consumption of non-durables and services per capita used in Mark (1985). He estimates the coefficient of relative risk aversion for the model (22) and finds the value above 40. Modjtahedi (1991) applies his analysis for the forward rates of maturities of one, three and six months and

estimates the values of risk aversion parameter to be around 54 with large standard errors of the values around 27 if the data on consumption of non-durables plus services is used. Such values of constant relative risk aversion parameter are well above acceptable limits.

For the most part, constant relative risk aversion utility function is a good approximation of investors' behaviour in the real-world setting, and it is especially appealing to assume the value of $\gamma = 1$ because then the utility function turns to logarithmic utility function $u(c) = \ln(c)$ which goes conveniently in line with a goal of maximization of consumption growth rate. However, many studies documented that most likely values of γ are higher than what is implied by logarithmic utility. Commonly accepted values of 2 to 4 generate too low values of rational expectations risk premium (22), but even relatively high plausible value of 29 suggested by Kandel & Stambaugh (1991) is still well below the values found by Mark (1985), Modjtahedi (1991) and others for risk premium to empirically explain forward premium bias under the assumptions of risk averse investors.

On the whole, the failure of consumption-based risk-averse models of foreign exchange rates is caused by inability to match the high degree of autocorrelation in the forward discount with the low variance in the stochastic discount factor. Given plausible values of investors' risk aversion parameter, consumption consumption must be highly correlated with exchange rate, but its data is simply not variable enough to explain high variance of forward premium.

2.2.6 First-order risk aversion for foreign exchange risk premium

The Arrow-Pratt measure of constant relative risk aversion used in the definition of expected utility function assumed in the model above is said to exhibit the risk aversion of the second order. It means that the risk premium is proportional to the variance of the change in consumption generated by an investment:

$$rp \approx \frac{1}{2}R(w_0)E[\sigma^2]$$

where rp stands for the risk premium, $R(w_0)$ is a coefficient of relative risk aversion and σ^2 refers to the variance of generated consumption. Such assumption implies that investors are risk-neutral with respect to small risks. However, as Rabin (2000) points out there is strong evidence in the literature that suggests that investors are averse to small risks. If we want the second-order risk aversion to accommodate this finding, it would as well result in extremely high aversion to larger gambles.

In contrast to utility functions that exhibit only second-order conditional risk aversion, Bekaert, Hodrick, & Marshall (1997) assumes investors' preferences that exhibit first-order risk aversion. For utility functions that exhibit first-order risk aversion, the

risk premium is proportional not to the variance, but to the standard deviation of the change in consumption. This type of models have a kink in the utility function at the initial level of wealth, just like assumed by prospect theory of Kahneman & Tversky (1979) or by a utility theory of disappointment aversion of Gul (1991). First-order risk aversion intuitively implies that the risk premium is larger in case of small risks because the standard deviation is significantly greater than the variance for small gambles. With respect to foreign exchange models that allow for risk premium, first-order risk aversion condition has an appealing feature - it can generate larger risk premiums for small investments and yet it maintains a realistic value of risk aversion.

Bekaert, Hodrick, & Marshall (1997) point out that thanks to first-order risk aversion among agents in their model, a small level of uncertainty in the exogenous environment has a potential to cause relatively large fluctuations in investors' intertemporal marginal rates of substitutions. This, in turn, can induce large variability of expected rates of returns in foreign exchange market, that is, the variance of the term $s_{t+1} - f_t$, as can be seen in eq. (21). The authors employ a discrete Markov chain model for the endowments and the money supply processes which are rendered as exogenous variables. The money supplies are quarterly M1 series, and the consumption data used are the nondurables and services for the US and Japan. They assume that consumers in different countries have the same preferences, and that US and Japanese consumers face the same transaction costs simply ignoring the effects of tariffs, shipping costs and risks associated with international payment system.

As a result of applying first-order risk aversion in their model, the variance of risk premium is increased dramatically. However, the model is unable to reproduce the variability observed in the data - the standard deviation of the fitted value of risk premium, or simply $(s_{t+1} - f_t)$, is 12.4% while the model can generate the largest value of standard deviation of 0.356%. In other words, their model is unable to reproduce the variability observed in the data. Moreover, the model cannot replicate the values of β estimated in UIP regression (12). As Bekaert, Hodrick, & Marshall (1997) point out, substantial time-variation in risk premium is needed if the model is to match observed behaviour of UIP puzzle. As a degree of first-order risk aversion increases, the variance of ex ante risk premium increases in the model, but it doesn't result in comparable increase in the slope coefficient, because the slope coefficient also depends on the variances of the expected spot appreciation and on the covariances between expected spot changes and the risk premium. These moments also change when the first-order risk aversion increases. Overall, Bekaert, Hodrick, & Marshall (1997) conclude that the predictability of excess returns in foreign exchange is unlikely to be explained simply by modifying preference assumptions alone.

2.2.7 External habit preferences model

We have shown so far that it is hardly possible to explain UIP puzzle with standard risk aversion models. Nevertheless, some success has been achieved in the literature that employs the representative agent framework with non-standard preferences. A good example is the paper of Verdelhan (2010). It introduces the model of two endowment economies with one good only and the same initial wealth level. The model relies on a specification of external habit persistence developed in Campbell & Cochrane (1999). The agent is set to maximize his utility function which is defined as:

$$U(C, H) = E \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}$$

where γ is a risk aversion coefficient, C_t is consumption, β captures impatience, and H_t denotes the external habit level, or practically a subsistence level. Additionally, the surplus consumption ratio is defined as $K_t = [C_t - H_t]/C_t$. The relationship between consumption and external habit level is defined through the following autoregressive process:

$$k_{t+1} = (1 - \phi)\bar{k} + \phi k_t + \lambda(k_t)(c_{t+1} - c_t - g)$$

where $0 < \phi < 1$ and \bar{k} are parameters, g is the average consumption growth rate and $\lambda(k_t)$ denotes the sensitivity of the surplus to consumption growth which describes how habits are formed given past consumption. It in essence governs the changes in surplus consumption. c follows simple random walk process with a drift equal to the average growth rate:

$$c_{t+1} = g + c_t + u_{t+1}$$

where u_{t+1} is the idiosyncratic shock to consumption growth and it is i.i.d and lognormally distributed. Verdelhan (2010) defines "bad" times as the times of low surplus consumption, while "negative shocks" are referred to negative values of u_{t+1} . Country's habit level in the model depends on domestic aggregate consumption. When consumption is low so that the surplus consumption is close to the subsistence level, the agent becomes very risk averse. Next, Verdelhan (2010) outlines that the pricing kernel is equal to:

$$M_{t+1} = \beta \frac{U_c(C_{t+1}, H_{t+1})}{U_c(C_t, H_t)} = \beta \left(\frac{K_{t+1} C_{t+1}}{K_t C_t} \right)^{-\gamma}$$

In logarithmic form the SDF is transformed as follows:

$$m_{t+1} = \ln(\beta) - \gamma \left[g + (1 - \phi)(k_t - \bar{k}) + (1 + \lambda(k_t))(c_{t+1} - c_t - g) \right]$$

After making assumptions about the values of long-run stable gap between con-

sumption and the habit level, \bar{K} , and about maximum value of K , Verdelhan (2010) obtains the time-varying risk-free rate. He found that interest rates in their model need to be set low in bad times and high in good times in order to match UIP condition requirements. Overall, the model dictates that market price of risk is a linear function of the sensitivity function $\lambda(k_t)$.

Verdelhan (2010) shows that the expected log currency return is equal to $E_t(r_{t+1}^E) = 0.5\sigma^2(m_{t+1}) - 0.5\sigma^2(m_{t+1}^*)$. Under assumptions of the same risk-aversion coefficients for domestic and foreign agents, the same persistence $\phi = \phi^*$, the same long-run values of the surplus consumption coefficients, i.e. $\bar{k} = \bar{k}^*$, the same mean value and volatility for consumption growth rates, i.e. $g = g^*$, $\sigma = \sigma^*$, the variance of the pricing kernel can be rewritten as:

$$\sigma_t^2(m_{t+1}) = \frac{\phi^2\sigma^2}{K^2} [1 - 2(k_t - \bar{k})]$$

Putting together the last two equations, the expected currency excess return transforms to:

$$E_t(r_{t+1}^E) = E_t(\Delta q_{t+1}) + r_t^* - r_t = \frac{\phi^2\sigma^2}{K^2} (k_t^* - k_t)$$

where Δq_{t+1} is referred to as real exchange rate appreciation. Accordingly, the model of Verdelhan (2010) finds for the real interest rates $r_t - r_t^* = E_t[q_{t+1}] - r_{pt}$. This result suggests that $cov(s_{t+1} - s_t, r_t - r_t^*) < 0$ which matches the empirical findings for UIP puzzle, save for the use of real interest rates instead of the nominal rates implied in the UIP condition. Given the fact that in advanced low-inflation countries changes in real and nominal interest rates are closely correlated¹¹, the model of Verdelhan (2010) is capable to explain the negative slope in UIP regression. The model also delivers the result that $\sigma^2(P_t) > \sigma^2(E[s_{t+1} - s_t])$ which is according to Fama (1984) a necessary condition to explain the UIP puzzle. However promising the results are, Verdelhan himself states that the model needs to be refined. One of the undesirable results of the model is that it implies strong and positive correlation between exchange rate appreciation and consumption growth rates. While power utility models of risk averse agents discussed above are characterised with higher correlation than the habit preferences model, the later one still fails to match empirical findings of negative correlation¹². The second drawback of the model is the admission of negative real interest rates in the economies.

¹¹See Engel (2014)

¹²see Backus & Smith (1993)

2.3 Empirical evidence of the failure of Uncovered Interest Parity

2.3.1 Abnormal positive returns to carry-trade strategies

Foreign exchange market is deemed to be inefficient if there exist trading strategies that generate consistent positive risk-adjusted returns. For this part, numerous researchers concentrated their attention on the analysis of the returns to the carry trade. After all, if returns to this strategy are merely a compensation for bearing risks, then it is unlikely that the market is inefficient. As we discussed in the previous part, the mere existence of risk premium in currency trading is hard to justify using traditional economic models. From the broad perspective, the risk premium should arise because of the comovement of exchange rates with stochastic discount factor. Refer to (21) to see that if the forward rate is higher than the expected future spot rate, it is because of the negative covariance between the spot rate and consumption. The risk premium is high for the currency that has positive covariance with consumption simply because such currency delivers low returns (defined as $f_{t,t+1} - s_{t+1}$) during the times of low consumption. The use of consumption data as a factor predicting future spot rate is common, but there could be other factors that we can try to find.

In general, returns to trading strategies must reflect compensation for the exposure to one or multiple risk factors, otherwise arbitrage opportunities exist. The presence of arbitrage opportunities in itself doesn't indicate a problem, the problem arises when these opportunities persist. In that case we can talk about market inefficiencies. One starting point is the finding of Fama (1984) that the risk premium term is time-varying. It is only natural to try to search for risk factors that cause this time-variability.

Burnside, Eichenbaum, & Rebelo (2008) study payoffs to the carry trade portfolios in the period from January 1976 to June 2007. They consider three strategies: equally-weighted carry-trade, where all the currencies that exhibit either forward premium or forward discount are given an equal dollar position compared to other currencies. The second portfolio is just a single currency carry trade, while the last "high-low carry trade" is implemented by taking position in a currency with the highest forward premium and the opposite position in a currency with the lowest discount. Equally-weighted carry-trade strategy exhibits the Sharpe ratio of 0.83 with even greater ratio of 1.23 for the sample starting in 1999. Currency-specific carry-trade strategy generates the value of 0.54 of the Sharpe ratio. Overall, the rolling Sharpe ratios for the equally weighted carry-trade strategy are positive and large for the last decade. Moreover, the excess returns are positive for 22 out of 23 currency pairs, and it is 5.4% for the equally-weighted carry portfolio.

Lustig & Verdelhan (2007) build eight currency portfolios grouped by the interest rates differential. They use data on 81 foreign exchange rates and the interest rates on three-month government securities covering the period from 1953 to 2002. Only the

countries that did not default during the sample and that had sufficient capital account liberalization are chosen for the study. They found that the spread between the portfolio that contains low interest rate currencies and the one that contains high interest rate currencies is over 5% for the whole sample period, and it is close to 7% for some shorter time periods. With the exception for very large inflation currencies, the returns in the sample increase monotonically with the increase in interest rate differential. Lustig & Verdelhan (2007) test how well different consumption-based factor models explain the actual returns to the eight currency portfolios. They found that the models with durable consumption growth produce little pricing errors. The finding is that the expected returns on currency portfolio is basically governed by the covariance of the returns with durables consumption growth. Another finding is related to the fact that with the increase in the interest rate differential the exposure of US investors to consumption risk increases as well. However as they choose the value for the risk aversion parameter to minimize the mean squared pricing error for the Euler equation, the success of the model is valid only for very high levels of risk aversion. Their estimated risk aversion parameter is around 113, well above all the conventional levels. In fact, Lustig & Verdelhan (2007) point out that almost the same value for the risk aversion parameter is obtained if currency portfolios are replaced with 25 Fama-French equity portfolios. Suspecting that consumption-based asset pricing models in general behave poorly in practice, we can still make an observation that Epstein-Zin Durables CAPM is capable to explain the currency returns as accurate as consumption based models possibly can.

2.3.2 In the search for risk factors

Now that we know that the aggregate consumption is not the factor capable to explain excess returns to carry-trade under realistic assumptions, the task is to find alternatives to it. Many researchers turn their attention to the search of economic fundamentals that would explain currency returns. The task seems as a logical step, albeit there exist a set of puzzles in international economics that hinder the work. Obstfeld & Rogoff (2001) uses the term "exchange-rate disconnect" to show that there is a very weak relationship between the exchange rate and any of the macroeconomic variables in short to medium terms. Meese & Rogoff (1983) shows that a random walk model does a better job at forecasting exchange rates than any model that incorporates data on fundamentals even if it is ex post data. Engel (2014) offers an explanation for this puzzle through the presence of news beyond the current economic fundamentals. He finds that the variance of the real exchange rate gets bigger once his model defining real exchange rate allows for the presence of news. Taking into account the fact that the covariance of the real exchange rate with monetary shocks remains the same for both cases, the correlation of the real exchange rate with economic fundamentals is reduced

due to the increased unconditional variance of the real exchange rate. However, there exists an alternative explanation for the "disconnect puzzle". It states that the exchange rate trading is not driven by changes in fundamentals as much as by noise trading. Imperfect expectations about other traders' actions can lead to fluctuations in exchange rates.

Nonetheless, Jorda & Taylor (2012) show that deviations from "fundamental" equilibrium exchange rate, or potentially any other frictionless equilibrium condition, have predictive power over the future change of exchange rate. Their trading strategy extends simple carry trade and includes the rules based on the nonlinear threshold model that gives certain weights to the signals based on the degree of deviation from equilibrium UIP and purchasing power parity conditions. Departures from the equilibrium can be triggered by noise trading, heterogeneous beliefs, behavioural biases or any other type of limits to arbitrage. Jorda & Taylor (2012) assumes that the real exchange rate is mean reverting around fundamental equilibrium exchange rate. Deviations from that mean level are a cointegrating vector. Jorda & Taylor (2012) report naive carry-trade exhibits a Sharpe ratio of 1.59 with 54 basis points returns per month in the period from 2002 to 2006. However, the same portfolio losses money in 2007 and 2008 at a rate of 24 basis points per month and -0.47 Sharpe ratio. The second model includes the lagged real exchange rate deviation from fundamental equilibrium as a regressor. The estimated coefficient on the deviation term is -0.02 indicating that the reversion occurs at a rate of 2% on average per month. This portfolio exhibits enhanced performance in the period from 2002 to 2006 with 62 basis points of returns per month. During financial crisis, the profits turn to losses similar to the results of the naive model, although negative returns for augmented model are -0.12 basis points per month. Nonetheless, the augmented model mitigates a negative skewness to some extent.

Next, Jorda & Taylor (2012) include another model where a dependent variable as in previous models is an appreciation of log nominal exchange rate, but independent variables are the first lag of the change in nominal exchange rate, inflation, interest rate differential and the logarithmic change in the real exchange rate. The inclusion of the last term proves to yield increase in the profitability of the model throughout 2007 and 2008. But Jorda & Taylor (2012) considers a non-linear model as well. There is a strand of research that suggests that deviations from UIP are corrected in non-linear manner depending on the degree of the deviation. The non-linear correction of deviations from the PPP condition, captured by the difference in real exchange rates, can be another option. Statistical tests reject linear relationship between the log change of the nominal interest rate and the lagged nominal rate, inflation, interest rate differential and real exchange rate change. Thus, Jorda & Taylor (2012) develop a threshold four-regime model where the regimes are:

1	$ i_t^* - i_t < I$	$ q_t - \bar{q} < Q$
2	$ i_t^* - i_t > I$	$ q_t - \bar{q} < Q$
3	$ i_t^* - i_t < I$	$ q_t - \bar{q} > Q$
4	$ i_t^* - i_t > I$	$ q_t - \bar{q} > Q$

I denotes the median of the absolute value of the interest rate differential, and Q stands for the the median of the absolute value of the real exchange rate deviation. The null hypothesis of no differences across the regimes is rejected. The coefficients on the terms are significant. The statistical tests prefer this model over linear versions considered by Jorda & Taylor (2012). Contrary to naive carry-trade model, this one provides 30 basis points of returns per month during the period of 2007 and 2008 with 33 basis points for the preceding periods. Interestingly, the skewness of this model remains close to zero throughout the sample. Overall, this nonlinear model exhibits a better predictive power for large movements in exchange rate.

The work of Jorda & Taylor (2012) tries to link the degree of divergence from the fundamental real exchange rate to the risk factors for the currency traders. However, it is worth noting that Jorda & Taylor (2012) don't compare the forecasting accuracy of the model with alternative models such as the random walk. Instead, they show in the profitability analysis that the deviation from the equilibrium real exchange rate is a significant factor improving directional forecasting.

Menkhoff et al. (2012) sort the currency portfolios by their forward discount at the end of each month and invest in the quintile with highest interest rate and short the quintile with lowest interest rate. This strategy yields significant excess return of more than 5% per annum even after accounting for the transaction costs. However, Menkhoff et al. (2012) argue that these returns do not represent a free lunch. There could be a negative volatility risk premium involved explaining abnormal positive returns. Portfolios that covary positively with market volatility innovations are demanded to a greater degree than the ones that have negative covariance. The reason for it comes from the fact that when returns are high during the time of increased market volatility, such asset provides a hedge against low consumption. As a result, the asset price is high, and expected returns are low. Menkhoff et al. (2012) suggest using aggregate volatility innovations as a risk factor given the fact that the volatility is known to have strong persistence.

Supporting the general view on the presence of the crash risk in carry trade portfolios, the findings are that the skewness gets more negative with the increase in interest rate differential. The risk factor suggested by this work is volatility innovations. First, Menkhoff et al. (2012) calculate absolute log returns for each currency at the end of each trading day. Then they are averaged out. The global volatility proxy is calculated

as:

$$\sigma_t^{FX} = \frac{1}{T_t} \sum_{\tau \in T_t} \left[\sum_{k \in K_\tau} \left(\frac{|r_\tau^k|}{K_\tau} \right) \right]$$

where K_τ is the total number of available currencies on a day τ , and T_t is the total amount of days in month t . Further, the volatility is fit to AR(1) model where the residuals represent volatility innovations. Exactly these residuals are used as a risk factor for the currency portfolios. Menkhoff et al. (2012) find that the times of high volatility changes correspond to the times when the carry-trade portfolios exhibit poor returns. In contrast, low interest rate countries perform well during the periods of high volatility increases. Volatility innovations is a significant risk factor helping explain cross-section of currency returns. About 90% of the spread in returns between investment and funding currencies is explained by global volatility innovations risk factor. However, there is a possibility that the volatility innovations encompass changes to a set of other state variables that actually themselves cause the movements in exchange rates.

Clarida, Davis, & Pedersen (2009) examine weekly returns on long/short portfolios that invest in 10 high-yielding currencies by borrowing low yielding currencies. For the portfolios that contain long (and equivalently short) positions in more than one currency, the weights among currencies are equal. They analyze the time-series of returns on 3v3 portfolio - three high-yield currencies are bought, and three low-yield currencies are sold short, - together with time-series variation in realized carry return volatility. Both returns and realized volatility are measured as exponentially weighted moving averages, and transformed to Z-scores. Then the inverse of the scaled volatility series is taken. They found correlation between the two series of 0.65 which means that during the times carry trade has high returns, return volatilities are have low values, and vice versa. Also, during the time when volatility is in its lowest 25%, subsequent carry returns are significantly higher than they are during the times that follow the high volatility regime. Even stronger observation is that the returns to carry portfolios are actually negative following the time periods that exhibit volatility levels above 75% threshold in the distribution.

Additionally, Clarida, Davis, & Pedersen (2009) study the relationship between implied exchange rate volatility from options and carry trade returns. They found that increases in Black-Scholes implied volatility lead to lower carry returns, on average. Interestingly, declines in implied volatility are followed by appreciation of investment currencies against the lending currencies. Last, but not least, they found that β coefficient in Fama regression (13) increases in value for the times of high volatility. The intuition here is that high currency volatility is the period of time when UIP condition actually works better, and carry trade portfolios lose money.

Lustig, Roussanov, & Verdelhan (2011) sort currencies according to their forward

discount and allocate them to portfolios. The portfolios are rebalanced at the end of each month. The returns for 35 currencies are computed at the end of each month using spot and forward exchange rates for the period from November 1983 to December 2009. They use data on bid-ask quotes to calculate returns net of transaction costs. Their carry portfolio is defined as long positions in all the currency portfolios but short position in the portfolio with lowest interest rate.

Lustig, Roussanov, & Verdelhan (2011) perform a principal component analysis on all currency portfolios in the study, and found that two first factors explain 80% of the variation in the returns. The first principal component can account for 70% of the variation in returns, and the loadings on this factor are almost the same across all currency portfolios. Interesting result appears for the second principal component that accounts for about 12% of the total variation, and the loadings on this factor increase monotonously from the portfolio with highest interest rate to the portfolio with lowest interest rate. It is only logical to conclude that the second principal component is the only component associated with carry risk factor because it is the only one that explains a cross-section of the returns of the currency portfolios sorted on forward discount.

To find plausible risk factors explaining abnormal returns to the carry trade strategies, Lustig, Roussanov, & Verdelhan (2011) take the difference between the net return on the highest-yielding portfolio and the net return on the lowest-yielding portfolio. In other words, it represents the return on a zero-cost strategy that is implemented by going long in the highest interest rate currencies and going short in the lowest interest rate currencies. Surprisingly, this risk factor, named as HML_{FX} factor, has a 0.94 coefficient of correlation with the second principal component. Using GMM estimation and a two-stage OLS estimation, Lustig, Roussanov, & Verdelhan (2011) run first a time-series regression of returns on HML_{FX} factor, and then a cross-sectional regression of average returns on the regression coefficients on the factors. They achieved a high-value of the adjusted R^2 of 0.70 and small pricing errors. It is worth noting that the HML_{FX} factor is basically a return, thus the risk price of the factor is the same as the average excess return of the high-minus-low carry trade strategy. The result is that if beta coefficient is one, the risk premium is found to be 5.5% annualized. This risk price is significantly different from zero.

Lustig, Roussanov, & Verdelhan (2011) find that β coefficient for the HML_{FX} factor increases monotonically from negative values to 0.61 when we go from the portfolio with low interest rate differential to the high-yielding portfolio. But still there might be a possibility that the relationship between betas and the portfolios is driven by interest rates changes, not the exchange rates changes. They found that the covariance between the risk factor betas and the returns are driven by exchange rate changes. Thus, it can be said that high interest rate currencies have higher exposure to HML_{FX} risk factor because of the depreciation of these currencies, not because the

decline of the interest rates on these currencies.

Sorting foreign exchange rate portfolios according to the forward discount (or, equivalently, on the interest rates) builds an interesting picture. The more is the difference between the US and foreign interest rates, the higher is the excess return and the higher is the exposure to the HML_{FX} factor. Lustig, Roussanov, & Verdelhan (2011) also examine the effect of volatility innovations measured by from stock returns in each country on the currency portfolios returns. This volatility risk factor exhibits an apparent pattern - the coefficient value decreases monotonically from 0.37 to -0.81 with the increase of interest rate differential. Intuitively it indicates that high interest rate currencies offer low returns if equity volatility increases. The opposite is true for the low volatility countries. Stock market volatility innovations explain the cross-section of the currency returns. and this is a statistically significant result. However, Lustig, Roussanov, & Verdelhan (2011) note that since the stock volatility factor is not directly observable, HML_{FX} risk factor wins a horse race in explanatory power for the cross-section of the excess returns.

In another paper Brunnermeier, Nagel, & Pedersen (2009) suggest that the risk in currency carry trade portfolios is created by carry trade itself. They find that higher interest rate differential predicts positive speculators position in such portfolios. First of all, their data confirms the UIP bias. Second, carry portfolios have negative conditional skewness. Brunnermeier, Nagel, & Pedersen (2009) regress skewness on time t variables such as interest rate differential, currency gains, skewness at time t and average net futures position on that currency (a proxy for carry trade activity). Futures position data is taken from the Commodity Futures Trading Commission, where net position means long minus short aggregate position of noncommercial traders in the currency. Noncommercial traders are often referred to as speculators that use futures market. The findings are such that futures position is strongly positively correlated with positive currency excess returns in the period. Another substantial finding states that past currency returns predict future negative skewness. This effect shows the tendency of speculators to increase their position once the investment currency appreciates. There is a clear possibility that the negative skewness of carry trade returns, i.e. the crash risk, can be endogenously created by carry trade activity itself. The build-up of carry positions in a given currency obviously has a bigger impact on the exchange rate when potential market-wide unwinding of carry positions occurs. Brunnermeier, Nagel, & Pedersen (2009) argue that this is the reason behind negative skewness of carry portfolios.

In the same regression they study effect of risk reversal variable on skewness. Risk reversal is measured as implied volatility of an out-of-the-money call option on that currency pair minus the implied volatility of an out-of-the-money put option. If this difference in prices, or simply risk reversal, is negative, then it means that the skewness

of the risk-neutral distribution of the exchange rate is negative. Risk reversal measures both effects combined - expected skewness and a risk premium on expected skewness. Interestingly, the implied skewness measured from risk reversal is found to be negatively correlated with interest rate differential. This confirms that implied risk-neutral skewness on carry portfolios is negative as well. Additionally, a higher risk reversal is associated with lower future skewness of the portfolio. At the same time, for a given interest rate differential, current risk reversal and current skewness have positive correlation. In light of last two conditions, one can conclude that after the crash event the price of the options-based insurance increases, even though the crash risk decreases.

Additional findings of Brunnermeier, Nagel, & Pedersen (2009) include the fact that after the interest rate differential experiences a positive shock, the exchange rate does not appreciate suddenly and gradually depreciate afterwards, a somewhat expected result if UIP was to hold. Instead, the interest rate of the investment currency appreciates gradually and doesn't depreciate in 15 quarters following the shock. Overall, using VAR analysis they find that the exchange rate initially underreacts to the change in interest rates, while later it overreacts because of too many speculators unwinding their positions. In essence, carry trade activity can create "bubbles" because each trader faces a synchronization risk (she does not know when to start unwinding positions, and it is irrational to do it before the market) which in turn prevents the exchange rate to revert to its "fundamental" value.

Brunnermeier, Nagel, & Pedersen (2009) also examine global risk tolerance as a factor. They use the VIX index as a proxy. Regression analysis confirms that when VIX increases, carry trades are unwound. Increase in VIX also increases the price reversal, that is, the price of buying option-based insurance against currency crash risk. On average, carry trades are found to be losing money in times of increasing VIX. To sum up these findings, carry trades experience negative returns because of the unwinding of speculators positions triggered by higher values of VIX index, that is, lower risk tolerance. Brunnermeier, Nagel, & Pedersen (2009) suggests that currency crashes can occur endogenously because reduced risk appetite can cause premature liquidations of the positions.

Lastly, Brunnermeier, Nagel, & Pedersen (2009) study the effect of TED spread as a proxy for liquidity risk factor for carry trade excess returns. The TED spread is defined as the difference between the 3-month LIBOR rate and the 3-month T-Bill rate. They do not find statistical significant coefficients related to TED spread variable. However, the sign of the coefficients suggest that TED and VIX are common risk factors for exchange rates. It is also the reason why a diversified carry portfolio is also exposed to a crash risk (negative skewness). It suggests that TED and VIX are related to a systematic risk factor that is connected to a "flight to quality" phenomenon. What is important for our topic is that the change in VIX index has a strong negative con-

temporaneous correlation, while the change in TED spread has a negative correlation with the next week return. Finally, both increase in VIX and in TED predict increase in carry trade returns some quarters onwards, albeit the immediate effect is, as it was noted already, is related to the decrease in carry returns.

Traditional macro risks are not very helpful in explaining carry trade returns according to Kojien et al. (2015). They consider carry strategies across different asset classes including the foreign exchange. They find that carry returns, however, tend to be smaller during the times of global recessions. Not only carry portfolios do poorly during these times, they actually do worse than passive portfolios. This suggests that carry excess returns could simply be a compensation for poor performance in bad times. The question is what these bad times are. They could be caused by macroeconomic risks, by reduction in global risk aversion, or the times of limited arbitrage due to the liquidity squeezes.

Findings of Kojien et al. (2015) are concentrated around the idea that part of the returns attributed to carry effect is a strong predictor of future returns. With respect to well-known risk factors, the paper does not find any explanatory power of value, momentum and time-series momentum factors, and there is a consistently positive alpha with respect to them. Kojien et al. (2015) suggest that the component of the return associated with carry is the risk factor itself, and is capable to explain total excess returns to the asset. Liquidity and volatility risk premiums is also confirmed in their work, although these two factors don't explain all the return premium to carry strategies.

3. EMPIRICAL WORK

3.1 Description of the Data

The data on spot and futures exchange rates against USD is obtained from Bloomberg terminal for the Australian Dollar, Canadian Dollar, Russian Ruble, Swiss Franc, New Zealand Dollar, South African Rand and Japanese Yen. We also obtain the data on the GBP/USD exchange rate from US Federal Reserve Data Releases database on Quandl.com. The data sample ranges from 01-Jan-1990 to 01-Jan-2015. USD/RUB spot rates are available from 12-Jul-1993, but the futures rates are available only from 22-Apr-1998. The future rate for the South African Rand is available from 08-May-1997 and the NZD has the same starting date. Our data sample covers 5 currencies (AUD, CAD, CHF, GBP, JPY) from 01-Jan-1990 until 08-May-1997, 7 currencies from 08-May-1997 until 22-Apr-1998 (with NZD and ZAR added) and 8 currencies for the rest of the sample. We use first generic futures contract for each currency pair. These contracts have 4 expiration dates around each year - 15th of March, June, September and December unless the 15th of the month is a public holiday. If that's the case, the expiration date is the next business date. Up until expiration date, futures contract refers to the closest future, and the day after expiration the rate refers to the new contract that expires in approximately 3 months.

We also collect data to use in our asset pricing analysis, and for that reason we obtain the following. We retrieve from Bloomberg terminal prices for Standard and Poor's 500 Index which is a capitalization-weighted index of 500 US stocks. We obtain data on The Chicago Board Options Exchange Volatility Index (VIX Index henceforth). From Bloomberg website¹³, the index reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of strikes. First and second month expirations are used until 8 days from expiration, then the second and third are used. We also obtain the data on the TED spread which is the difference between the three-month LIBOR USD rate and the yield on 3-months US zero-coupon T-bill. The TED spread is expressed in basis points, but we use decimal points in later analysis. From Bloomberg we also use the data on Standard and Poor's GSCI Index expressed in the USD, and it is used to represent a measure of general commodity price movements.

We obtain the data on risk-free rate from FRED database, Federal Reserve Bank of St. Louis. We define it as the yield on 3-month Treasury Bill with constant maturity. The rate from FRED database is a market rate, so we transform it to annual yields. We also use the data on 10-year Treasury constant maturity rates from the FRED database, and on 2-year Treasury to form the proxies of interest rate and yield curve risk factors.

¹³refer to <http://bloomberg.com> for further description of the data.

All the data for risk factors covers the full sample period from 01-Jan-1990 to 01-Jan-2015.

3.2 Descriptive statistics of the data and unit root tests

First of all, all the exchange rates in the data have to be transformed to a logarithmic form. Siegel (1972) discovered a paradox that the market expectation of an exchange rate doesn't have the same value as a reciprocal of the market expectation of the same exchange rate but quoted in the other currency:

$$1/E_t(S_{t+1}) \neq E_t(1/S_{t+1})$$

where S_{t+1} stands for the future exchange rate expressed in the units of domestic currency. In fact, this inequality is a special case of Jensen's inequality which dictates that the transformation of the function and taking its average might give a different result depending on what operation is performed first. In general, if the transformation is linear, then for a function $f(x)$ it does not matter what you do first. But in case it is not linear, such as the case with taking a reciprocal, the two operations performed at different sequence will give different results. Taking reciprocal and then averaging will in fact give a greater value as a result than averaging first and then taking reciprocal, so $1/E_t(S_{t+1}) < E_t(1/S_{t+1})$.

To circumvent this problem, we take natural logarithm of all the exchange rates used in the study. This way, if we want to change the base currency, we simply need to multiply a logarithm of exchange rate by minus unity. Unlike taking a reciprocal, it is a linear transformation, thus Jensen's inequality does not apply in this case. Instead, the following relationship holds:

$$E_t[\ln(1/S_{t+1})] = E_t[-\ln(S_{t+1})]$$

Expressed in logarithm of the exchange rate, it is equivalent to:

$$E_t(-s_{t+1}) = -E_t(s_{t+1})$$

The descriptive statistic of the forward premium for all the currencies are shown in table 1. As it can be seen, the Australian Dollar, New Zealand Dollar, Russian Ruble and South African Rand have negative forward premium throughout most of the time period. For example, the AUD was traded on average at 0.3% of futures discount for the sample period, while for Russian Ruble the futures discount is substantial 7.7%. The latter can be explained by the fact that the sample period includes the events of August 1998 when Russian Central Bank defaulted on its debt and devalued the currency. The fact that the calculation of the mean value for Ruble is affected by a relatively

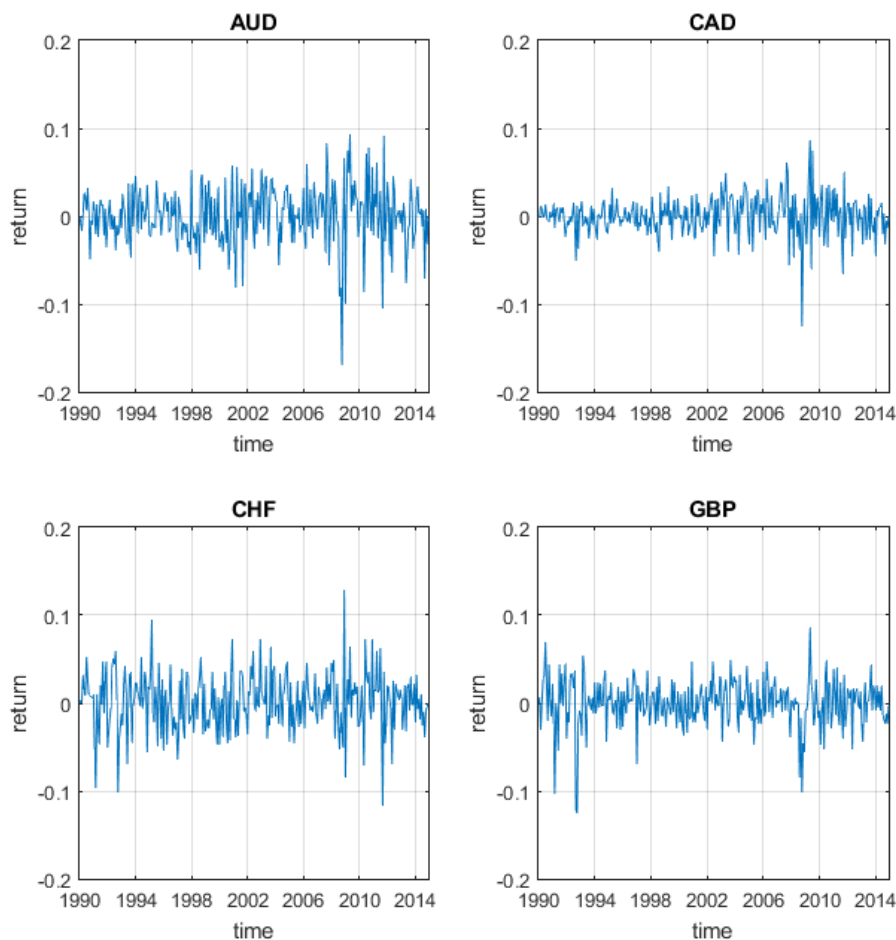


Figure 1: Monthly returns on currency pairs AUD-GBP

small number of extreme observations is supported by much more moderate value of 4.3% for the sample median. But even then the Ruble exhibits strong futures discount for the sample period, with the ninth decile having a value of -0.3% .

The set of currencies that exhibit positive or near zero futures premium includes the Swiss Franc, the Japanese Yen and the Canadian Dollar. The British Pound was traded on average at 0.16% futures discount, with close to zero median value. In most of the sample the Swiss Franc and the yen traded at futures premium.

3.3 The Fama regression

A natural place to start the analysis of the carry trade models is to perform a regression analysis to test the joint hypothesis of the forward rate unbiasedness and risk neutrality. We rewrite the UIP regression (13) here for convenience:

$$s_{t+1} - s_t = \alpha_2 + \beta_2(f_{t,t+1} - s_t) + \varepsilon_{2,t+1}$$

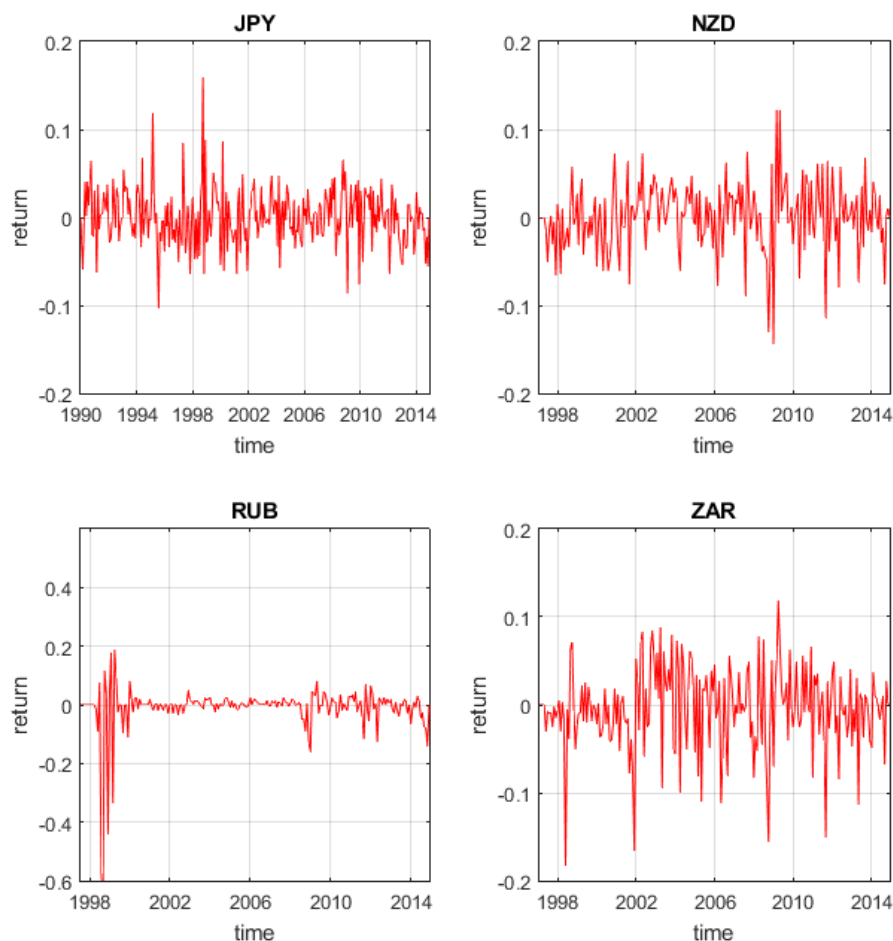


Figure 2: Monthly returns on currency pairs JPY-ZAR

Table 1: Descriptive Statistics for Forward Premium

	AUD	CAD	CHF	GBP	JPY	NZD	RUR	ZAR
1st Decile	-7.73	-3.91	-2.15	-5.84	-0.84	-8.30	-216.62	-17.40
1st Quartile	-5.01	-1.91	-0.28	-2.93	-0.28	-5.73	-111.74	-12.01
Median	-2.37	-0.49	1.03	-0.91	1.87	-3.17	-43.14	-7.21
3rd Quartile	-0.60	0.78	3.37	0.34	5.17	-0.79	-12.52	-3.41
9st Decile	0.94	2.03	6.36	1.59	9.33	1.50	-3.24	-1.01
Mean	-2.96	-0.75	1.48	-1.63	3.01	-3.28	-77.64	-8.27
St. dev.	3.93	2.63	3.55	3.60	4.09	4.20	96.00	7.21

Notes: The table reports measures of central tendency and dispersion of the futures premiums for eight currencies for the entire sample period. However, as discussed in the description of the data, the sample for Russian Ruble starts from 22 April 1998, for South African Rand and New Zealand Dollar from 8 May 1997 due to the the lack of the futures data for the earlier time period. All the estimates are multiplied by 10^3 for better readability. The futures premium is defined as $\Phi = \frac{F_{t,m}^k - S_t}{S_t}$. Essentially, with the representation used in the table, numbers mean tenths of a percent. Positive values are associated with higher futures price compared to the spot which means that the price of a currency is expected to appreciate relative to USD. Negative values mean that the market expects the given currency to depreciate in relation to US dollar.

If we find the hypothesis that $\beta_2 = 1$ and $\alpha_2 = 0$ to hold true, then we should not expect any stable long-term profits from carry trade.

The reason s_t is subtracted from both sides of the equation (13) is because most of the times the exchange rates are believed to have a unit-root. If that is the case, then we cannot rely on the estimates of the coefficients of the regression. For unit-root time series, the effect of a shock never dies out, and thus the estimates of the coefficients of the regression equation could be overstated. The sample variance for a non-stationary time-series tends to be too high because the ordinary least squares estimator minimizes the variance of the residuals in the regression. We conduct the Augmented Dickey-Fuller test suggested by Said & Dickey (1984). The seemingly intuitive way of testing for unit-root can be by testing the null hypothesis that θ_1 coefficient is equal to one in the following regression:

$$s_t = \alpha + \theta_1 s_{t-1} + \sum_{i=1}^h \theta_i s_{t-i} + \varepsilon_t$$

However, if the null is in fact true, meaning that θ_1 is unity, our OLS estimators of the coefficients will be biased due to the non-stationarity of the both sides of the regression. To overcome this problem, Dickey & Fuller (1979) suggested transforming this regression into a form that utilizes the differences of the time-series variables:

$$\Delta s_t = \gamma + \delta s_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta s_{t-i} + \varepsilon_t \quad (23)$$

where p is a maximum lag, $\delta = \sum_{i=1}^p \theta_i - 1$ and $\beta_i = -\sum_{j=2}^p \theta_j$. This is a regression for Augmented Dickey-Fuller test with the null hypothesis of $\delta = 0$ versus the alternative hypothesis that $\delta < 0$. According to Shumway & Stoffer (2011), the choice of lag p is crucial. If the chosen lag is too small, then the regression model will not capture important variability in the data resulting in serially correlated errors. If it is too big, then the power of the test will suffer. To determine the maximum lag, we employ a rule of thumb proposed by Schwert (1989) based on Monte Carlo simulations:

$$p_{max} = \left\lceil 12 \left(\frac{T}{100} \right)^{1/4} \right\rceil$$

where T is the sample size. We perform ADF tests for the lags $p = 1 \dots p_{max}$ and for each of the tests we look at the t-statistic of the last lag included in the regression (23). For the set of regressions (23) each of which is a $AR(p)$ process, we also compute Bayesian Information Criterion and Akaike Information Criterion. We pick one $AR(p)$ model for which the last lag has significant coefficient and the model has the lowest values of AIC and BIC. Selected model undergoes the Ljung-Box test of its residuals.

Table 2: Unit root tests for spot exchange rates

	AUD	CAD	CHF	GBP	JPY	NZD	RUR	ZAR
Model	<i>AR</i> (1)	<i>AR</i> (9)	<i>AR</i> (15)	<i>ARD</i> (1)	<i>ARD</i> (10)	<i>AR</i> (3)	<i>ARD</i> (14)	<i>AR</i> (2)
Lag p-value*	0.00068	0.000011	0.0137	0.00635	0.000042	0.0506**	0.000004	0.0052
AIC	-45338	-51025	-45930	-48585	-46464	-45585	-34807	-42564
BIC	-45325	-50957	-45821	-48565	-46382	-45558	-34701	-42543
Ljung-Box lags	9	9	9	9	9	9	9	9
p-value	0.197	0.99	1	0.611	1.0	0.689	0.99	0.362
Test Statistic	12.304	0.446	0.058	7.25	0.062	6.496	1.449	9.858
Critical value	16.92	16.92	16.92	16.91	16.92	16.92	16.92	16.92
ADF p-value	0.321	0.4	0.138	0.104	0.184	0.319	0.001	0.983
Test Statistic	-0.903	-0.68	-1.449	-2.552	-2.265	-0.910	-4.320	1.81
Critical Value	-1.942	-1.94	-1.94	-2.862	-2.862	-1.942	-2.862	-1.94

Notes:

* - it represents the p-value of the significance test for the coefficient in the regression model where the coefficient is for the last lagged variable.

** - For the NZD the lag of 3 was chosen even though the p-value is higher than significance level. The reason for this is that the only lag that has a p-value smaller than alpha is 29. Using AR(3) proved to be appropriate since no serial correlation in residuals was found. Using AR(29), instead, would potentially decrease the power of the ADF test.

Note: all tests were performed at $\alpha = 0.05$ significance level.

Since the log spot exchange rate does not normally exhibit seasonal trends, we do not need to choose the number of lags for the Ljung-Box test as a multiple of the seasonal lags. However, we determine the lags m by a rule recommended by Tsay (2005):

$$m = \ln(T)$$

where T represents the sample size. In parallel with Ljung-Box test, we investigate the plots of the sample autocorrelation function and sample partial autocorrelation function. In the end, the null hypothesis of the unit-root in the logarithmic spot exchange for a each currency is conducted for the selected lag p that satisfies aforementioned criteria and for which there is no serial correlation of residuals. For robustness, we repeat the tests for the variations of the Augmented Dickey-Fuller test that assume the autoregressive model with drift and the trend-stationary model. The results of the tests for each currency are presented in the table 2.

As results in the table 2 indicate, the null hypothesis of a unit-root in the time-series cannot be rejected for all the currency pairs but one. Augmented Dickey Fuller test confirms that only logarithm of the spot exchange rate of the Russian Ruble does not exhibit a unit root. To circumvent the problem of biased OLS estimators for non-stationary variables, we take the difference of the time-series the same way as other studies have done including Fama (1984). However, we cannot use the same regression model (13) to test for UIP due to the fact that we use futures data instead of forward contracts. Our futures data is daily, but all observations tied to a specific year refer to

Table 3: Unique futures contracts per currency pair

	AUD	CAD	CHF	GBP	JPY	NZD	RUR	ZAR
number of contracts	100	100	100	104	100	72	68	72

one out of four futures contract. If we denote a specific expiry date for futures contracts as k , today's observations as t , then we can rewrite the Fama regression (13) in futures terms:

$$s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + \varepsilon_{t+m} \quad (24)$$

where m stands for a number of days to maturity of the contract k . For each contract the maturities range from zero to 91 days. Daily data can be too noisy, so we decide to change it to weekly frequency. We take exchange rates at the end of every week. If there is missing data, the nearest data point preceding the end-of-week date is used. After making such change, m refers to the number of weeks to maturity. It is logical to limit the minimum value for the variable m to at least one week, since when the prices of futures and spot rates converge, the value of futures premium will be close to zero, and thus the estimate of β is not of much interest for us for such a time frame. In total for all the currencies under the analysis there are 716 distinctive contracts with the most contracts available for the British Pound, and the least for the Russian Ruble. For the GBP/USD pair, $k = 1, \dots, 105$. The rest can be found in the table 3.

One way to perform a UIP hypothesis test that $\beta = 1$ and $\alpha = 0$ for (24) is by applying an OLS estimator. The problem, however, is that along with different contracts, we have different maturities for each contract. Note that for each contract, since we have only one delivery date, we have to fix the s_{t+m}^k term because we have only one expiration date for the corresponding contract $f_{t,t+m}^k$. So for each regression, the spot price s_{t+m}^k has to be the same. Thus, this creates a situation when we need to run 716 regression, each of which will estimate parameters based on a maximum of 13 weekly observations. In addition, the futures and spot prices are known to be correlated, which will result in error terms to be correlated across maturities m . This leads us to treat the regression (24) as a panel regression, instead. The sequence of individual contracts we assume to be a time-series while the maturities $m \in 1, \dots, 13$ will represent the cross-sectional dimension.

At this stage an important question we have to ask - does the maturity itself pose a fixed effect on the exchange rate returns. Saying it differently, we would like to know if for some maturities the gains or losses on exchange rate differ from the ones that are typical for other maturity. Maybe, over the period of 12 weeks to maturity, an exchange rate regularly appreciates by a certain rate, which is statistically different from how much the same exchange rate usually appreciates over the period of 4 weeks to maturity. We can reject such assumptions outright due to the fact that Meese & Rogoff

(1983) found that an exchange rate is best approximated by a random walk process, and hence, we cannot have a persistently different rate appreciation or depreciation over certain time frames. Otherwise, this persistency would mean predictability of exchange rate returns. Of course, it is possible that the variance for larger maturity units is higher than for the smaller ones, but the mean and median values are likely to be random and not statistically different from each other. To confirm this fact, we report box plots for all the exchange rates in A.1 in Appendix 2. Since there is no significant difference in response variable distribution over different maturities, we confirm that a fixed effect model based on maturities as the fixed variables is inadequate. In fact, if the joint hypothesis of rational expectations and risk neutrality holds, we should not have any individual error component μ_i in the following general representation of the panel OLS:

$$Y_{i,t} = \alpha + \beta^T X_{i,t} + \mu_i + \varepsilon_{i,t}$$

where Y and X are dependent and independent variables, respectively. For the purpose of having our model as close to the uncovered interest parity regression, we use a pooled OLS regression model which is also a common model to apply for the Time-Series Cross-Sectional data. However, we first need to test the poolability of futures rates with different times to expiration, i.e. we need to be sure that our model (24) can be tested using pooled OLS. As noted by Baltagi (2005), a Chow test is popular for this. It essentially checks the equality of the regression coefficients $\beta_i = \beta_j$ where i and j are different time-to-maturity periods, and $i \neq j$. In practice, however, the Chow test is not necessary for the cases when time dimension is much bigger than cross-sectional dimension, $T \gg N$, which is the case with our model. Another caveat of applying this test is due to the condition of a spherical error terms. This condition is unlikely to be met in our model due to the presence of temporal serial correlation. However, Baltagi (2005) notes, that the failure to meet this condition makes Chow test be more biased towards rejection of the null hypothesis that the regression coefficients are equal. From our estimates of the F-statistic reported in table 4, we conclude that we cannot reject the null for all the currency pairs at 95% significance level. So, given the plausible bias towards rejection of the null, we still conclude that our pooled version of Fama model is valid.

Table 4: Chow test of equality of regression coefficients

	AUD	CAD	CHF	GBP	JPY	NZD	RUB	ZAR
F test statistic	0.513	0.405	0.447	0.484	0.421	0.754	1.470	0.754
p-value	0.980	0.997	0.993	0.987	0.996	0.808	0.062	0.808

Note: the test is performed at 95% confidence level.

For TSCS regressions, or simply panel regression where the size of time dimension is higher than number of cross-sectional units, the generalized least squares (GLS)

estimator is commonly used. Beck & Katz (1995) argue that the method can result in seriously inaccurate standard errors. While applying GLS to panel data, a problem arises because GLS assumes we have the knowledge of the error process. It is most often not possible, and we have to resort to a trick of estimating the process first, and then assuming we know the error process based on our estimations. This variation of GLS is commonly known as feasible GLS, or simply FGLS. Unfortunately, for panel data the error process has more parameters to be estimated than for simpler cases of time-series or cross-sectional data only. As a result, true variability of the estimated coefficients gets understated and accordingly the estimates of the standard errors are often smaller than they should be. If we use FGLS to our problem, it can produce a false high confidence in the estimates. A Monte Carlo experiment ran by Beck & Katz (1995) show that the FGLS estimator can underestimate the variability of the coefficients by some 50% - 300%. Our work applies suggested by Beck & Katz (1995) method to use OLS for parameter estimation but use panel-correction for OLS standard errors. This will fix the problem of highly inaccurate OLS standard errors for panel regressions. At the same time, OLS estimates for this type of models in general are quite adequate.

For panel regressions like ours, OLS would be optimal, that is, it would be best linear unbiased estimator, if errors were generated in a spherical manner. It means that the error processes are independent of each other (i.e. no serial or spatial correlation) and they are homoscedastic. Most of the time, for the panel data, the assumption of nonspherical errors does not hold and OLS is no longer the most optimal estimator because the standard errors could be wrong. Having standard errors as correct as possible is a necessary condition for the accurate statistical tests.

According to Beck & Katz (1995), any regression model might suffer from spherical errors, but the panel regression most often is associated with this problem. For this type of models, one can expect to have contemporaneous correlation of errors. On top of that, a notorious panel heteroscedasticity of errors is a common phenomena for panel regressions. More precisely, the Time-Series Cross-Sectional data often is equipped with the following complications:

1. large errors for one unit at time t is accompanied by a large error for another unit at time t (contemporaneous correlation of errors);
2. the variances of the error process are constant over time, but vary for different units at time t (panel heteroskedasticity of errors);
3. errors can be serially correlated with the same degree of autocorrelation across units, or alternatively, with different degrees of autocorrelation (temporal correlation of errors);

4. for some units, errors are contemporaneously correlated more among these units and less correlated with other units (spatial correlation of errors).

Given these characteristics, the OLS estimates of regression coefficients will be consistent but inefficient. However, contrasting this with high inaccuracy of standard errors provided by FGLS method for panel regressions, the OLS is still preferred especially after correcting the OLS standard errors.

Beck & Katz (1995) warns to correct for any unit-specific serial correlation of errors before performing robust panel errors estimation. After that, the contemporaneous correlation of errors and panel heteroskedasticity are corrected. The errors estimated this way are often called panel-corrected standard errors.

To see the need of the correction, first, note that for the panel or TSCS data¹⁴, we have the model:

$$Y = X\beta + \varepsilon \quad (25)$$

where all the variables are presented in a matrix form. X is a $NT \times (p + 1)$ matrix, with N indicating the length of the cross-sectional dimension, and T standing for the length of the time dimension. β is a $(p + 1) \times 1$ vector, ε is a $NT \times 1$ vector, and Y 's dimensions are $NT \times 1$ as well. For the UIP hypothesis testing we are concerned with variance-covariance matrix of the OLS estimates because the p-values depend on the variability of the coefficients. The formula for the variance-covariance matrix of $\hat{\beta}$ is well-known as:

$$Cov(\hat{\beta}) = (X'X)^{-1}X'\Omega X(X'X)^{-1} \quad (26)$$

In the case that the errors follow the Gauss-Markov Assumptions, that is they are spherical errors, omega becomes:

$$\Omega = \sigma^2 I \quad (27)$$

where I is a $NT \times NT$ identity matrix and σ^2 stands for the variance of the standard errors.

The problem, however, arises from the fact that for the panel models errors do not typically obey spherical errors assumption. Instead, such models exhibit properties described by the elements 1 - 3 of the list above. This is why we cannot use eq. (27). However, we can still use (26), but the Ω must be estimated. For TSCS models, Ω is a

¹⁴TSCS stands for time-series cross-sectional data which is a very similar concept to a panel structure, but often the TSCS name is used to indicate that the time dimension is much larger than cross-sectional

$NT \times NT$ block diagonal matrix:

$$\Omega = \begin{bmatrix} \Sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \Sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \Sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \Sigma_T \end{bmatrix} \quad (28)$$

That is, Ω is a $T \times T$ matrix with blocks of $N \times N$ size as elements. Each Σ_t represents a $N \times N$ matrix of contemporaneous covariances of residuals for that particular time t :

$$\Sigma_t = \begin{bmatrix} \omega_1^2 & \omega_{1,2} & \omega_{1,3} & \dots & \omega_{1,N} \\ \omega_{2,1} & \omega_2^2 & \omega_{2,3} & \dots & \omega_{2,N} \\ \omega_{3,1} & \omega_{3,2} & \omega_3^2 & \dots & \omega_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{N,1} & \omega_{N,2} & \omega_{N,3} & \dots & \omega_N^2 \end{bmatrix} \quad (29)$$

so for the time t

$$\omega_{i,j} = Cov(u_{i,t}, u_{j,t}) \quad (i = 1, \dots, N; j = 1, \dots, N)$$

Overall, this representation accommodates the properties of TSCS data very well since:

1. Error for a unit i will be correlated with the error for the unit j at the same time t : $\omega_{i,j} = Cov(u_{i,t}, u_{j,t}) \neq 0$; it represents contemporaneous correlation;
2. error for a unit i at time t is not correlated with the error for other units at other times - it is represented by blocks of zeroes in Ω matrix in eq. (28).
3. errors have unique variances for each time period t : $\omega_i^2 \neq \omega_j^2$; it represents panel heteroskedasticity of errors.

Now, we need to estimate $\hat{\Sigma}$ to be able to use the equation (26). Even though without using panel-errors robust estimator for β in (25), we cannot reliably perform hypothesis tests, the OLS estimates for (25) by themselves are consistent¹⁵. A consistent estimate of Σ is obtained if we use OLS estimates of residuals. We denote by $\hat{u}_{i,t}$ the OLS estimated residual for a unit i at time t . Then, an element of Σ_t can be estimated according to this:

$$\hat{\omega}_{i,j} = \frac{\sum_{t=1}^T \hat{u}_{i,t} \hat{u}_{j,t}}{T} \quad \hat{\omega}_i^2 = \frac{\sum_{t=1}^T \hat{u}_{i,t}^2}{T} \quad (30)$$

¹⁵They are biased because of the non-spherical errors, but consistent.

If we write down a matrix E that is a $T \times N$ matrix of OLS estimated residuals, then we will be able to write an expression for the estimate $\hat{\Sigma}$.

$$E = \begin{bmatrix} \hat{u}_{1,1} & \hat{u}_{1,2} & \hat{u}_{1,3} & \dots & \hat{u}_{1,N} \\ \hat{u}_{2,1} & \hat{u}_{2,2} & \hat{u}_{2,3} & \dots & \hat{u}_{2,N} \\ \hat{u}_{3,1} & \hat{u}_{3,2} & \hat{u}_{3,3} & \dots & \hat{u}_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{T,1} & \hat{u}_{T,2} & \hat{u}_{T,3} & \dots & \hat{u}_{T,N} \end{bmatrix}$$

Then the estimate $\hat{\Sigma}$ is expressed as

$$\hat{\Sigma} = \frac{E'E}{T} \quad (31)$$

which yields the estimate for $\hat{\Omega}$:

$$\hat{\Omega} = \hat{\Sigma} \otimes I_T \quad (32)$$

where \otimes indicates the Kronecker product.

In our panel regression, another complication is present due to the fact that our data set is unbalanced. What it means is that there are missing observations for some of the dimensions. For example, not for all contracts we have exchange rates that are 13 weeks away from the maturity date. In this case, for some of Σ_t in eq. (29) we cannot compute variance-covariance $\omega_{N,j}$ for the missing N . The complication, then, will be in the fact that we would not be able to use eq. (31). For that reason, we use a pairwise computation of correlations between two units based on a common for them time frame, not on the full time period. We fill in the missing observations for X and $E'E$ with zeros, but divide each element of $E'E$ by the number of the pairs of the non-missing observations. In order to compute the corrected variance-covariance matrix for the parameters using this pairwise approach, we create an additional matrix M of the size $T \times M$ and fill it with ones if the corresponding observation in E is non-missing, and with zeros otherwise. Then, instead of (26) we use this modified form including the changes made to $E'E$ and X :

$$Cov(\hat{\beta}) = (X'X)^{-1}X' \left(\frac{E'E}{M'M} \otimes I_T \right) X(X'X)^{-1}$$

Before we apply the panel version (see eq. (24)) of the Fama model, we need to test the poolability of futures rates with different times to expiration. A Chow test is popular in the literature. It essentially checks the equality of the regression coefficients $\beta_i = \beta_j$ where i and j are different time-to-maturity periods, and $i \neq j$. In practice, however, the Chow test is not necessary for the cases when time dimension is much

bigger than cross-sectional dimension, $T \gg N$, which is the case with our model. Another caveat of applying this test is due to the condition of a spherical error terms. This condition is unlikely to be met in our model due to the presence of temporal serial correlation. However, notes, that the failure to meet this condition makes Chow test be more biased towards rejection of the null hypothesis that the regression coefficients are equal. From our estimates of the F-statistic reported in table 4, we conclude that we cannot reject the null for all the currency pairs at 95% significance level. So, given the plausible bias towards rejection of the null, we still conclude that our pooled version of Fama model is valid.

To obtain panel-corrected standard errors we apply just outlined procedure of Beck & Katz (1995). In a nutshell, our regression becomes a so-called pooled OLS regression. To be able to rely on any of its results, the first and the foremost condition of stationarity must be met. In effect, we have $N = 1 \dots 12$ weeks to maturity as the cross-sectional dimension, and $T = 1 \dots 107$ as the time dimension in our panel. Before we start with Beck & Katz (1995) robust estimation, we need to make sure there is no temporal serial correlation. However, we first perform Augmented Dickey-Fuller test from Said & Dickey (1984). We combine visual inspection of the plots of autocorrelations of residuals and partial autocorrelations of residuals with an inspection of Bayesian Information Criterion and Akaike Informaton Criterion to choose the best model fit for each of the Augmented Dickey-Fuller tests. Dickey-Fuller test statistics, its critical values, and the null hypothesis p-values for the test are reported for each currency in table 4 in Appendix 4.

For the Canadian Dollar, Australian Dollar and British Pound, both the dependent variable and the regressor are stationary according to the tests. For the Swiss Franc, the independent variable has unit root in all but first three and the seventh cross-sectional units. A similar but a little more severe case is found for the Japanese Yen, where only for first two weeks to maturity we can reject the unit root in the time-series. For the New Zealand Dollar only for the weeks to maturity of 1 and 3 we can reject unit root in independent variable, while for the South African Rand we reject it for maturities 1,4,7,8,9,11. For all of these currency pairs, the dependent variable $s_{t+m}^k - s_t$ seems stationary, however, that is not the case with the Russian Ruble. We cannot reject the null of unit root in dependent variable for most of the maturities, save for 2, 5 and 11 weeks. Interestingly, the regressor $f_{t,t+m}^k - s_t$ appears stationary in all maturities.

The presence of unit root in time series is a serious problem that prevents us from using OLS estimators for such time series. The only situation where we can still use OLS is when the left-hand side and right-hand side terms, $s_{t+m}^k - s_t$ and $f_{t,t+m}^k - s_t$ respectively, are cointegrated. However, for it to be the case, both variable must be integrated of the same order. For all currency pairs, there is unit-root in either a response variable (Russian Ruble), or in a regressor, but not in both. Given all of

that, the only way to run non-spurious regressions is to take the difference of each non-stationary variable. However, by transforming (24) in this fashion, we will have difficulties interpreting the results of the estimations. Hence, for further analysis, we select those cross-sectional units for which we reject the null of unit root.

The next step consists of carrying out OLS regression (24) and inspecting the residuals on a subject of serial correlation. The results for each currency can be found in table 5 in Appendix 5. We report Durbin-Watson test statistic values for the test proposed by Durbin & Watson (1971) with the null that there is no serial correlation against the alternative that the autocorrelation is of the order 1. We also report the test statistic for LM serial correlation test also known as Breusch-Godfrey test. We test the null of no serial correlation against the alternative that the series is an AR(1) process. Interestingly, we reject the null for the residuals of the Swiss Franc at all stationary cross-sectional units. For the AUD/USD pair, there seven cross-sectional units for which we cannot reject the null, thus we include them into estimation of panel corrected errors and estimation of regression coefficients. For the Canadian Dollar, 1,2,4,5,6,7 and 9 weeks to maturity series seem to have no serial autocorrelation¹⁶. All maturities of the GBP/USD pair are included into our panel regression, whereas for the JPY we use 2 weeks to maturity for further analysis, essentially making it simple time-series regression. Maturities of 1 and 3 weeks are chosen for the New Zealand Dollar, 2 and 11 for the Ruble and 1,4,7,8,9,11 for the South African Rand. The table 5 in Appendix 5 also report Shapiro-Wilk test statistic with corresponding p-values for 95% significance level. For many cross-sectional units of the currency pairs we reject the null that the residuals follow normal distribution. However, the Gauss-Markov theorem does not require the residuals to be normally distributed for the OLS estimator to be the best linear unbiased estimator. Moreover, we find that the rejection of the null of the normally distributed errors is mostly due to large kurtosis, whereas the mean value is reasonably close to zero.

After determining cross-sectional units for which we cannot reject the null of no serial correlation in residuals, we proceed to the next step of performing panel regression (24). For the pairs AUD/USD, CAD/USD, GBP/USD, NZD/USD, RUB/USD and ZAR/USD we simply select maturities that comply with condition of no temporal correlation. We also perform simple OLS regression for the JPY/USD pair because only one cross-sectional unit fulfills the criteria. The results of estimation of the parameters and the standard errors are reported in the table 5. We want to pay special attention to the CHF/USD pair. For the Swiss Franc, we do not have cross-sectional units to choose from due to the presence of serial correlation in errors in all maturities that

¹⁶We understand that there is still a possibility of higher order autocorrelation in residuals to exist even if we cannot reject the hypothesis of no autocorrelation against the alternative of AR(1) process. However, the visual inspection of residuals autocorrelation function plots doesn't show significant correlation at lags higher than 1.

we determined as stationary. Due to this complication, we perform a Cochrane-Orcutt transformation for the corresponding maturity series in order to eliminate serial dependence of errors and to enable us to run the panel regression (24) for the CHF/USD pair. A quick explanation of the procedure follows.

Table 5: Estimated coefficients for UIP regression

	α	β	R^2	N. Obs
AUD/USD	-0.0034 (0.0050)	-0.9802 (0.9967)	0.0054	700
CAD/USD	0.0014 (0.0021)	0.8923 (0.7477)	0.0069	700
CHF/USD	NA NA	1.0819 (1.3223)	0.0022	400
GBP/USD	-0.0015 (0.0027)	0.5466 (0.6803)	0.0030	1243
JPY/USD	0.0088 (0.0050)	-1.5162 (2.3591)	0.0042	100
NZD/USD	2.60E-05 (0.0032)	0.1852 (0.9659)	0.0003	142
RUB/USD	-0.0085 (0.0092)	0.7844* (0.1605)	0.1732	130
ZAR/USD	-0.0131 (0.0071)	0.0300 (0.6632)	1.21E-05	422

Notes: the table presents pooled OLS estimation results of the model $s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + \varepsilon_{t+m}$ with Beck & Katz (1995) standard errors. α is a model's intercept, and β is a slope coefficient. We report adjusted R-squared measure as R^2 in the table. Additionally, we report the number of observations for each test. All tests are performed at 95% confidence level.

* - the estimate is significantly different from zero at 95% confidence level.

First, according to Cochrane & Orcutt (1949), we make sure that residuals are in fact stationary. We perform Augmented Dickey-Fuller test with the lag of 5 and 95% significance level. For all the cross-sections under analysis (these include maturities of 1,2,3 and 7 weeks) we reject the null of unit root in residuals. Next, based on our earlier tests of autocorrelation (see table 5 in Appendix 5), we can write the residual term as:

$$\varepsilon_t = \rho\varepsilon_{t-1} + e_t$$

where the condition that $|\rho| < 1$ and that e_t is a white noise. Let us denote $s_{t+m}^k - s_t$ as y $f_{t,t+m}^k - s_t$ as x , then, according to Cochrane & Orcutt (1949), the new transformed values can be obtained by:

$$y_t^* = y_t - \rho y_{t-1}$$

and

$$x_t^* = x_t - \rho x_{t-1}$$

To obtain ρ , we simply perform OLS regression for each cross-sectional unit, then carry out an OLS regression of $\hat{\varepsilon}_t$ on $\hat{\varepsilon}_{t-1}$. We estimate ρ for 1 week to maturity series to be equal to 0.4564, for 2 weeks to be equal to 0.4571, for 3 it is 0.4154 and for 7

it is 0.4081. To see how the Cochrane-Orcutt transformation reduces serial correlation of errors for the Swiss France, see figure A.3 in Appendix 6.

One particularly painful caveat of applying Cochrane-Orcutt transformation is associated with inability to estimate α for our version of Fama regression. According to Cochrane & Orcutt (1949), we can obtain the value of alpha for pre-transformed regression using the following calculation:

$$\hat{\alpha} = \frac{\hat{\alpha}^*}{1 - \hat{\rho}}$$

It is a straightforward task if we estimated one OLS regression per each maturity cross-section. However, since we use TSCS OLS regression, the estimated alpha in our modified regression for the Swiss Franc is related to all the cross sectional units. But because we transformed alpha for each cross section using its specific value of ρ , we cannot perform reverse transformation from the alpha we obtain as a result of the pooled OLS regression. Thus, we sacrifice our ability to estimate the constant parameter from the regression (24) for the benefit of having a bigger sample. Additionally, in the first approximation, we would like to have an estimate of β for the Swiss franc in general, and only then to inspect whether it varies significantly with its maturities.

As opposed to obtaining an estimate of the pre-transformed alpha, an estimated $\hat{\beta}^*$ from the transformed model directly corresponds to the estimate of the original $\hat{\beta}$, and so do its standard errors. This enables us to apply Beck & Katz (1995) standard error correction for the Swiss Franc and to obtain estimate of $\hat{\beta}$ parameter in the model (24) together with robust standard errors. The result can be found in table 5.

If the uncovered interest parity is to hold, we expect β coefficient to be unity, and the constant term to be equal to zero. As reported in table 6, α is not statistically different from zero for all currencies in the study. But by looking at point estimates for β in table 5, one can cast doubt on whether the UIP condition holds in the data. For the Canadian Dollar, Swiss Franc and the Ruble, the estimated value of β is not far from unity. However, the point estimates are negative for the Japanese Yen and the AUD. For the Rand and the New Zealand Dollar, the estimate is closer to zero than to the unity.

Ultimately, we want to test the null hypothesis of $\beta = 1$ versus an alternative hypothesis that it is not equal to unity. If the uncovered interest parity holds, the result should be that we would not be able to reject the null. Interestingly, it is exactly what we find in our parameters estimation. For all the currency pairs save for the Australian Dollar, we cannot reject the null at 95% significance level. We also test the null using a two-tailed t-test with 90% confidence interval, but the results of the test do not change - we reject the null only for the AUD/USD pair. Summing up, using futures panel data and weekly observations, we find that we cannot reject the null of the uncovered

interest parity for all currencies except for one. However, we note that point estimates only in the regressions of the CAD, CHF and RUB currencies are close to the unity. We also note very large standard errors for all the estimated coefficients. It is not a surprise, because the Beck & Katz (1995) procedure has been known for yielding larger standard errors than the values one could achieve applying time-series OLS. Lastly, we note extremely small values of R^2 for all the panel regressions. This might hint that the model is not well-specified.

It is interesting to compare our findings with the results of other papers. Fama (1984) finds point estimates for CAD, CHF, GBP and JPY of -0.87, -1.14, -0.9 and -0.29, respectively. He uses the sample ranging from 31/08/1973 to 10/12/1982. For all these currencies, he rejects the null of $\beta = 1$. In another paper, Sarno, Schneider, & Wagner (2012) estimate α and β on weekly horizons for the AUD, CAD, CHF, GBP and JPY. The intercept term is not statistically significantly different from zero. The beta coefficient was estimated as being equal to -5.673 , -3.444 , -1.419 , 0.250 and -1.942 for the AUD, CAD, CHF, GBP and JPY, respectively. For all of these pair except the GBP/USD, they reject the null at 95% significance. Additionally, the coefficients of determination R^2 are small in their regressions as well - the smallest is 0.0001 for the GBP, and the highest is 0.0166 for AUD.

Table 6: Results of significance test and null hypothesis test

		Significance test		The null $H_0 : \beta = 1$	D.fr.	N. Obs
		α	β	vs. $H_A : \beta \neq 1$		
AUD/USD	t-value	-0.683	-0.983	-1.987	698	700
	p-value	0.495	0.326	0.047		
CAD/USD	t-value	0.678	1.193	-0.144	698	700
	p-value	0.498	0.233	0.886		
CHF/USD	t-value	NA	0.818	0.062	398	400
	p-value	NA	0.414	0.951		
GBP/USD	t-value	-0.545	0.803	-0.666	1241	1243
	p-value	0.586	0.422	0.505		
JPY/USD	t-value	1.777	-0.649	-1.077	98	100
	p-value	0.079	0.518	0.284		
NZD/USD	t-value	0.008	0.192	-0.844	140	142
	p-value	0.994	0.848	0.400		
RUB/USD	t-value	-0.924	4.888	-1.343	128	130
	p-value	0.357	3.00E-06	0.182		
ZAR/USD	t-value	-1.828	0.045	-1.463	420	422
	p-value	0.068	0.964	0.144		

Notes: the table presents the results of significance tests for the parameters of the model $s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + \varepsilon_{t+m}$ with Beck & Katz (1995) standard errors. α denotes an intercept and β is the slope. We also report the results of hypothesis testing where the null is $H_0 : \beta = 1$ versus the alternative hypothesis $H_A : \beta \neq 1$. We additionally report degrees of freedom used in the test along with total number of observations for each test. The tests are performed at 95% confidence level.

To analyze the variability of β with maturity, we plot its estimates and standard errors by maturity in Appendix 3. We essentially use time-series OLS estimator for

each maturity series where both the response variable and the regressor in eq. (24) are stationary. These maturities are present in table 4. We employ Newey-West corrected standard errors, proposed by Newey & West (1987), to account for serial correlation in residuals. What we can see in figure A.2 in Appendix 3, is that for the Australian Dollar the slope coefficient is dominantly negative. All the point estimates and most of the confidence intervals are below zero. However, none of the other currency pairs exhibit as strong evidence of the forward premium bias as the AUD/USD. We can only notice a somewhat weaker evidence of the violation of the UIP condition in the ZAR/USD pair. Unity is within a small range from all the point estimates but one for the Canadian Dollar. For the British Pound, the estimates and confidence intervals are mostly positive, and they are all above zero and close to 1 for the Ruble. At the same time, the South African Rand, a currency that is associated with rather high interest rates, exhibits puzzling negative estimates. Overall, we find that for some currencies the forward premium puzzle is evident, for others the results are mixing (i.e. GBP), and for the Russian Ruble, the UIP seems to generally hold. Additionally, it would be useful to point out that despite the variability we see in the estimates of β for different maturities, we can note that they are not significantly different for most of the currency pairs¹⁷. It is also confirmed by Chow test that we performed earlier. Given all that, we assume that panel-corrected estimates and standard errors in table 5 are good representation of the average β value over all maturities.

3.4 Evidence of the presence of the risk premium

One of the advantages of using futures data on foreign exchange rates rather than forward prices is that for a given contract we have a sequence of expectations of the spot exchange rate for a fixed date. This wealth of observations make it possible to test for the presence of the risk premium in futures prices. Hodrick & Srivastava (1987) show that you can decompose a future price into a today's expectation of the futures price tomorrow and a one-day risk premium:

$$f_{t,m}^k = E_t[f_{t+1,m-1}^k] + rp_{t,1} \quad (33)$$

where $rp_{t,1}$ stands for one-day the risk premium at time t which is essentially the expected one-day profit on a short position in the currency futures market. Thanks to using futures data, we can test the presence of this one-day risk premium term by testing if the futures price is an unbiased predictor of the futures price on the next day.

¹⁷The estimates for the Japanese Yen seem to be significantly different for 1 and 2 weeks to maturity. However, the importance of this finding can be questioned since we have only two β estimates for JPY and they are for the short maturities. In general, short-maturity estimates seem to be different from the estimates for other maturities. For instance, short maturity β estimates for CHF and ZAR are far from the average estimates.

Naturally, if the premium term is zero, i.e. $rp_{t,1} = 0$, then the futures price is the unbiased predictor.

We have to comment on the results of Samuelson (1965) who argued that for an efficient market, futures prices are martingales:

$$E_t[f_{t+1,m-1}^k] = f_{t,m}^k \quad (34)$$

Extending this formulation by the use of the law of the iterated expectations, we can see that:

$$f_{t,m}^k = E_t[E_{t+1}(f_{t+2,m-2}^k)] = E_t[f_{t+2,m-2}^k]$$

and thus, in the limit:

$$f_{t,m}^k = E_t[f_{t+m,0}^k]$$

Given that the condition $f_{t+m,0}^k = s_{t+m}$ must hold due to the absence of arbitrage, the previous formula can be rewritten as:

$$f_{t,m}^k = E_t[s_{t+m}]$$

which is in fact a fundamental condition (8) that is to hold if UIP holds. In the previous section we obtained the values for the estimates of the slope coefficients that contradict with the notion of uncovered interest parity. There is no evidence that would suggest that these relationships would be different for futures contracts. Cornell & Reinganum (1981) found that forward rates and futures prices for delivery on the same day are indistinguishable. All of it indicates that the finding of Samuelson (1965) is based on the null that uncovered interest parity holds, or, in other words, that forward and futures prices are unbiased predictors of future exchange rate. When the null of futures unbiasedness does not hold, we can use condition (33) as a starting point in the search of risk premium as a cause for the failure.

The presence of the risk premium in futures price can explain the finding of $\beta \neq 1$ in our main regression (24). If the risk premium term is non-zero, the results must be distorted by omitted variable bias. More specifically, let us assume that true relationship between the spot price change and the futures premium is:

$$s_{t+m}^k - s_t = \alpha + \beta_1^*(f_{t,m}^k - s_t) + \beta_2^*\Pi_{t,m} + \varepsilon_{t+m} \quad (35)$$

If this variable $\Pi_{t,m}$ and the futures premium are negatively correlated, then the OLS estimate of β_1 is downward biased which would explain our findings. This downward bias appears from the fact that the estimate of β in the model with only one explanatory variable takes a portion of the value β_2 . In fact, Dougherty (2011) shows that the degree

of bias can be computed as:

$$plim(\hat{\beta}) = \beta_1^* + \beta_2^* \frac{Cov(f_{t,m}^k - s_t; \Pi_{t,m})}{\sigma^2(f_{t,m}^k - s_t)}$$

where coefficients β_i^* are coefficients from the model (35). If we include $\Pi_{t,m}$ in the model, we can expect to obtain the value of β_1^* that is free from the bias inherent in β from (24) and more close to the unity.

If we refer to the theory on the forward premium bias, we notice that there is another possibility of why the estimate of the slope coefficient can be biased downwards. Many researchers pointed out that along with risk-premium there is a room for the bias caused by the failure of the rational expectations. In fact, it is quite possible that both effects are present and they drive down the result of the estimation of β . When we test the null hypothesis of forward premium unbiasedness, we unfortunately cannot test both the presence of risk premium and the deviation from the rational expectations. If we test for the presence of risk premium, we must assume that rational expectations hold. If we test for the failure of rational expectations, we must assume the risk premium is zero. This is why the test of uncovered interest parity (13) is also called a test of joint hypothesis. Consequently when we work with forward rates, it is virtually impossible to answer a question if it is the risk premium or deviations from rational expectations that causes the negative slope coefficient. However, we argue that the use of futures data can provide a way of testing the extent to which the risk premium clouds the estimation of the slope coefficient.

To see how we come to this conclusion, we use as a starting point the finding (33) of Hodrick & Srivastava (1987):

$$f_{t,m}^k = E_t[f_{t+1,m-1}^k] + rp_{t,1}$$

Then we assume that even though agents make no systematic errors in predicting next futures value based on all the information that they think is true at time t , which we denote as I_t^A , they in fact make biased forecasts because the true information is different from what agents believe, $I_t \neq I_t^A$. This is not hard to imagine, because it is perfectly possible investors might not know the true monetary regime at the moment, and they can put different weights on the probabilities of what regime is taking place, as we discuss in section. Based on this intuition, we rewrite the expression for futures price as:

$$f_{t,m}^k = f_{t+1,m-1}^k + \Omega_{t,1} + \varepsilon_{t,1} + rp_{t,1} \quad (36)$$

where ε_{t+1} is a white noise, a forecast error resulted from making rational predictions, and $\Omega_{t+1,1}$ a bias in the forecast caused by the use of incorrect information I_t^A instead

of I_t . In essence, the agent's predictions seem to be rational to him:

$$f_{t+1,m-1}^k = E_t[f_{t+1,m-1}^k | I_t^A] + \varepsilon_{t+1,1}$$

However, if we take into account all the information actually exists at time t (which would imply a strong-form market efficiency), his forecasts are not rational:

$$f_{t+1,m-1}^k = E_t[f_{t+1,m-1}^k | I_t] + \Omega_{t+1,1} + \varepsilon_{t+1,1} \quad (37)$$

The conditional expectation of the forecast bias is zero:

$$E_t[\Omega_{t+1,1} | I_t] = E_t[f_{t+1,m-1}^k | I_t] - E_t[f_{t+1,m-1}^k | I_t] - E_t[\varepsilon_{t+1,1} | I_t] = 0 \quad (38)$$

It follows directly from the fact that at time t the conditional expectation $E_t[f_{t+1,m-1}^k | I_t]$ is the conditional expectation itself, and that the expectation of the white noise is zero.

We start with general assumption that the futures price today is the price of the spot at maturity plus risk premium:

$$\begin{aligned} f_{t,m}^k &= E_t[s_k] + rp_{t,m} \\ f_{t+1,m-1}^k &= E_{t+1}[s_k] + rp_{t+1,m-1} \end{aligned} \quad (39)$$

where $rp_{t,m}$ means the risk premium covering a period from t to maturity, with the length of m .

Now, if we take the expectation of the futures price tomorrow, we will get:

$$E_t[f_{t+1,m-1}^k] = E_t \left[E_{t+1}[s_k] + rp_{t+1,m-1} \right]$$

Applying the law of iterated expectations, we obtain:

$$E_t[f_{t+1,m-1}^k] = E_t[s_k] + E_t[rp_{t+1,m-1}]$$

Substituting $E_t[s_k]$ with $f_t^k - rp_{t,m}$ from (39) transforms the previous expression into:

$$f_{t,m}^k = E_t[f_{t+1,m-1}^k] - E_t[rp_{t+1,m-1}] + rp_{t,m} \quad (40)$$

which is equivalent to

$$f_{t,m}^k = E_t[f_{t+1,m-1}^k] - rp_{t,1} \quad (41)$$

To save space, we use expectation notation $E_t[x]$ for time t to represent expectations conditional on true information available at that moment. For simplicity, let us

assume that k is 3 days away from t :

$$\begin{aligned} f_{t,m}^k &= E_t[f_{t+1,m-1}^k] + rp_{t,1} = E_t[E_{t+1}(f_{t+2,m-2}^k) + rp_{t+1,1}] + rp_{t,1} = \\ &= E_t[E_{t+1}(E_{t+2}(s_k) + rp_{t+2,1}) + rp_{t+1,1}] + rp_{t,1} \end{aligned} \quad (42)$$

Taking into account the formula for biased expectations (37), we rewrite the last expression as:

$$f_{t,m}^k = E_t[E_{t+1}(s_k - \Omega_{t+2,1} - \varepsilon_{t+2,1} + rp_{t+2,1}) + rp_{t+1,1}] + rp_{t,1} \quad (43)$$

Note the result (38) that the conditional expectation of the bias is zero, thus, the last expression can be rewritten as:

$$\begin{aligned} f_{t,m}^k &= E_t[E_{t+1}(s_k) + E_{t+1}(rp_{t+2,1}) + rp_{t+1,1}] + rp_{t,1} = \\ &= E_t[s_k - \Omega_{t+1,1} - \varepsilon_{t+1,1} + E_{t+1}(rp_{t+2,1}) + rp_{t+1,1}] + rp_{t,1} = \\ &= E_t[s_k] + E_t[E_{t+1}(rp_{t+2,1})] + E_t[rp_{t+1,1}] + rp_{t,1} = \\ &= E_t[s_k] + E_t[rp_{t+2,1}] + E_t[rp_{t+1,1}] + rp_{t,1} \end{aligned} \quad (44)$$

This result can be generalized in the following form:

$$f_{t,m}^k = E_t[s_k] + E_t\left[\sum_{i=1}^{m-1} rp_{t+i,1}\right] + rp_{t,1} \quad (45)$$

which under assumption of biased expectations transforms into a solution under biased expectations:

$$f_{t,m}^k = s_k + \Omega_{t,m} + \varepsilon_{t,m} + E_t\left[\sum_{i=1}^{m-1} rp_{t+i,1}\right] + rp_{t,1} \quad (46)$$

So now we see that the risk premium for t is the sum of one period risk premium (for the period between two futures prices) and expected risk premiums for the rest of the periods:

$$rp_{t,m} = rp_{t,1} + E_t\left[\sum_{i=1}^{m-1} rp_{t+i,1}\right]$$

When it comes to testing the hypothesis that futures price is an unbiased predictor of future rate, we need to construct test in a way that the null would imply the relationship (34), while alternative would imply (45). McCurdy & Morgan (1987) propose the following regression model to test the null:

$$f_t^k - f_{t-1}^k = \alpha + \beta(f_{t-1}^k - f_{t-2}^k) + \varepsilon_t \quad (47)$$

Under the null, the coefficients α and β are equal to zero while the residual is uncorrelated and has the mean value of zero. This null, if holds, implies so-called weak-form market efficiency where past rates do not have any predictive power.

At this point, we would like to emphasize that tests of forward rate unbiasedness are found to fail due to the failure of rational expectations or the presence of time-varying risk premium. Some papers, including Fama (1984), find that the premium must be time-varying but only after assuming that rational expectation hypothesis holds true. With the wealth of data futures contracts provide, we argue, we are able to prove the time-varying nature of foreign exchange risk premium without imposing such a restriction as full rationality of expectations agents form. We use the same test (47) as McCurdy & Morgan (1987), but we allow for both deviations to take place.

First and the foremost, we need to make some assumptions regarding the form of deviations from rational hypothesis. Lewis (1989) find that systematic errors during 1980s' had on average the same sign. For example, monthly forward forecast errors from 1979 to 1984 in their sample were -1.1% for Deutsche Mark. The presence of systematic errors in itself is evidence of rational expectations violation. It's worth noting that if agents form their expectations based on the set of underlying fundamentals, there might be some of the fundamentals that are unknown to the agents, such as the current monetary regime. In this case, agents can be in the process of learning about the regime while making their "bets" about future spot rate. This situation is often called rational learning and it is discussed in section 2.2.4.

When we operate with data that consists of monthly forward rates, we cannot see the progression of the learning process as we can see in futures data. With daily futures data, what we essentially have is daily predictions of the same future spot rate. If agents make systematic prediction errors, and they are in the process of learning about true underlying fundamental values, one can reasonably expect the size of the error to diminish, on average, as we get closer to the delivery date. Lewis (1989) show that as the market estimate of a hidden fundamental value such as monetary regime converges to the true value, the expected value of the forecast error also converges to zero. Otherwise, it has persistent sign (either positive or negative) and it converges to zero over time.

Based on the logic that forecast errors gradually diminish every day because investors update their probabilities of the values for hidden fundamental variables, we can make an assumption that forecast error follows AR(1) process with non-zero initial value:

$$y_t = \rho y_{t-1} + u_t$$

where $|\rho| < 1$ and u_t is a white noise. As an example, we show in figure 3 the form such process can take when $\rho = 0.9$, initial value is 20 basis points of the error and

$\sigma = 1$.

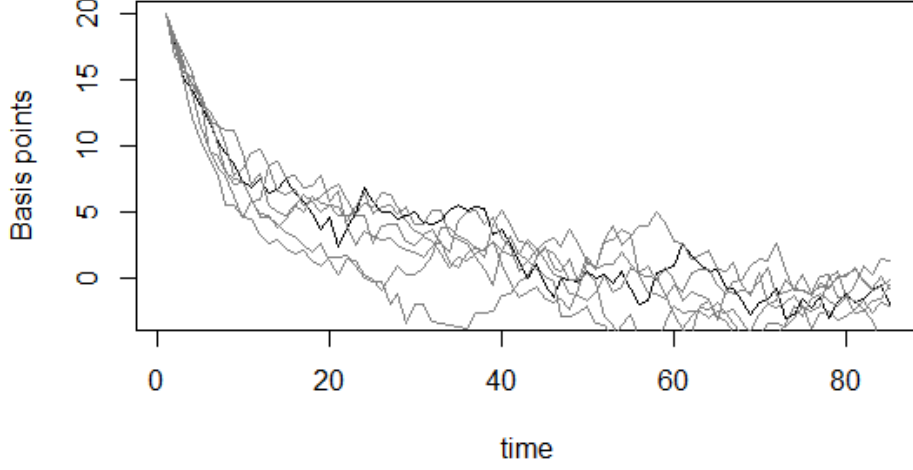


Figure 3: Development of simulated AR(1) process
with parameters $\rho = 0.9$, $y_1 = 20$ and $\sigma = 1$

We find that by testing relationship (47) we are able to prove that risk-premium is time-varying under the assumption of AR(1) systematic deviations from rational expectations. To see this, first let us rewrite (47) as

$$f_{t+2}^k - f_{t+1}^k = \alpha + \beta(f_{t+1}^k - f_t^k) + \varepsilon_{t+2}$$

Note also that if we assume that both risk premium and systematic forecast errors affect the futures rate, then using (46) we can represent the regressor and the regressand as:

$$f_{t+2}^k - f_{t+1}^k = -(\Omega_{t+1,1} + rp_{t+1,1})$$

and

$$f_{t+1}^k - f_t^k = -(\Omega_{t,1} + rp_{t,1})$$

Thus, by testing the model (47), we test:

$$-(\Omega_{t+1,1} + rp_{t+1,1}) = \alpha - \beta(\Omega_{t,1} + rp_{t,1}) + \varepsilon_{t+2} \quad (48)$$

Under an assumption that the deviation from rational expectation takes form of AR(1) process, we can write:

$$\Omega_{t,1} = \rho\Omega_{t-1,1} + u_t$$

where u_t is a white noise process. Taking into account this relationship, we can rewrite

(48) as follows:

$$\begin{aligned}
-(\rho\Omega_{t,1} + rp_{t+1,1} + u_{t+1}) &= \alpha - \beta(\Omega_{t,1} + rp_{t,1}) + \varepsilon_{t+2} \\
-rp_{t+1,1} &= \alpha + \rho\Omega_{t,1} - \beta\Omega_{t,1} - \beta rp_{t,1} + \varepsilon_{t+2} + u_{t+1} \\
rp_{t+1,1} &= -\alpha - \rho\Omega_{t,1} + \beta\Omega_{t,1} + \beta rp_{t,1} - \varepsilon_{t+2} - u_{t+1} \\
rp_{t+1,1} &= -\alpha + \Omega_{t,1}(\beta - \rho) + \beta rp_{t,1} - \varepsilon_{t+2} - u_{t+1}
\end{aligned} \tag{49}$$

The result has some interesting implications for our analysis. First of all, note that the difference between two white noise processes must be white noise itself. Second, it would be reasonable to assume that the size of one day currency risk premium should not depend on the size of forecast error in last period. This condition implies that $\beta - \rho = 0$. As a result, the null hypothesis of futures unbiasedness which implies the absence of risk premium and perfectly rational expectations, requires α and β estimates to be not statistically different from zero. If they are different from zero, not only it will be evidence of existence of the risk premium, but it will also indicate that risk premium is serially correlated, thus it is time-varying.

Alternatively, if we assume that the individuals are risk neutral, but the deviations from the unbiasedness hypothesis take place, we can see that by estimating the model (47) we end up with the test of serial correlation for the rational expectations errors:

$$\Omega_{t+1,1} = -\alpha + \beta\Omega_{t,1} + \varepsilon_{t+2} \tag{50}$$

In this case, the null of no bias in the foreign exchange market would indicate that the deviations from rational expectations are not of autoregressive nature. If we find β different from zero, we can infer that the deviations from the null are due to systematic errors resulted from rational learning process.

A third possibility is to have agents forming rational expectations of future values, but demanding risk premium on purchases in the foreign exchange market. This would imply that the null of the model (47) tests the following relationship:

$$rp_{t+1,1} = -\alpha + \beta rp_{t,1} - \varepsilon_{t+2}$$

It is easy to notice that the estimate of β different from zero implies that the risk premium is time-varying. Overall, after outlining all three possible states of the problem, we can define how we will interpret results of estimating (47) based on the broad review of the puzzle we conducted in section 2.1. It is unlikely that the hypothesis of rational expectations indeed holds. However, it has been found that currency returns can to some extent be explained by certain risk factors such as TED spread or foreign exchange market volatility. Thus, we assert that both irrational expectations and risk premium cause the deviations from the null hypothesis. In this case, rejection of the

null in (47) will indicate that both deviations are present, and that the risk premium is actually time-varying.

Having these considerations in mind, we estimate pooled OLS model (47) for all the currencies with panel corrected standard errors proposed by Beck & Katz (1995). We use daily futures data, and treat different contracts as a cross-sectional units, while the days to the contract delivery as time dimension. We test for the presence of cross-sectional heteroskedasticity of errors with Breusch-Pagan test which revealed that for some of the maturity days, we can reject the null of homoskedastic errors at 95% confidence. We restrict our estimation to the range of maturities from 4 to 86 days. The results of estimation can be seen in table 7.

Table 7: Results of pooled OLS estimation of the futures unbiasedness model

	α	β	R^2
AUD/USD	0.0001 (0.0001)	-0.057* (0.029)	0.0033
CAD/USD	0 (0.0001)	-0.021 (0.027)	0.0005
CHF/USD	0 (0.0001)	-0.015 (0.021)	0.0002
GBP/USD	0 (0.0001)	0.007 (0.022)	0.0001
JPY/USD	0 (0.0001)	-0.006 (0.022)	0
NZD/USD	0.0001 (0.0001)	0.010 (0.026)	0.0001
RUB/USD	0 (0.0002)	0.216* (0.088)	0.0471
ZAR/USD	0.0001 (0.0002)	-0.008 (0.027)	0.0001

Notes: the table displays the results of pooled OLS estimation of the model $f_{t+2}^k - f_{t+1}^k = \alpha + \beta(f_{t+1}^k - f_t^k) + \varepsilon_{t+2}$ with Beck & Katz (1995) panel corrected standard errors. We report estimates of the intercept, the slope as well as adjusted R^2 measure. The significance tests are performed at 95% confidence level.

* - significantly different from zero at 95% confidence level.

To our surprise, for 6 out of 8 currencies we cannot reject the null hypothesis of futures rate unbiasedness. The estimate of β is practically zero for the British Pound, the Japanese Yen and South African Rand. Only for the Australian Dollar and the Ruble the slope is statistically different from zero. For the rest of the currencies, the point estimate is small negative or positive, but not statistically different from zero.

If we go back to the results of the null hypothesis test for UIP regression in the table 6, we will note that the Australian dollar is the only currency pair for which we reject the null of UIP. The results of futures unbiasedness hypothesis suggest that for the AUD/USD and the RUB/USD pairs, we have evidence of time-varying risk premium. Are the estimates consistent with our assumption of autoregressive deviations from rational expectations? Not very much. With coefficient values of $\rho = 0.216$ and

$\rho = -0.057$ for the Ruble and the AUD respectively, the AR(1) process is unlikely a good approximation for the true process representing expectational errors. First of all, with such values of the parameter, the deviations quickly converge to zero - in a matter of a few days. Then, since the AR(1) process with these parameters is stationary, it more resembles the case where agents are equipped with rational expectations, because the error is stationary and circulates around 0.

Three conclusions we can make from this result. First, the deviations from rational expectations probably follow different type of process. Second, the nature of bias can be systematic, but could be unrelated to the concept of rational learning. Alternatively, the rational expectations hypothesis can actually hold true, while the deviations from the futures unbiasedness are explained by risk premium alone. Without a proxy that would depict well the deviations from rational hypothesis, and without survey data on market expectations, we cannot accommodate the failure of rational hypothesis into our results. In any way, since non-zero coefficient β in (47) appears to be attributed to the serially correlated risk premium, we shall assume that market participants require compensation for bearing foreign exchange risk, and that their expectations are approximately rational. With this intuition, we dismiss the model (50).

3.4.1 Building and testing modified UIP model

One additional theory can be tested in the framework of the regression (35). Since we know the risk-premium is time-varying, we can break it down into time-varying component and contract-specific exposure to it:

$$\Pi_{t,m} = \phi_m g_k + u_{m,k} \quad (51)$$

In this form, ϕ represents the cross-sectional loading on the risk-factor. In our panel model m is a cross-section, although when we move along this dimension, the variation in m in fact represent the time-variation in its exposure to risk. g represents the risk factor, and it is assumed fixed for each contract and does not vary with maturity. Since exposure to g evolves with maturity, the representation (51) has a time-varying nature, since maturities m in fact is a movement along the time. We simply use them as cross-sections in our analysis to be able to apply panel regressions and utilize a wealth of observations it provides. $u_{m,k}$ is a stochastic part of the premium term.

The term g can represent the general riskiness of the contract for investors. Since g_k refers to uncertainty about spot rate at time k , - the time of expiration of the futures contract, this risk is driven by risk factors that threaten to affect the value of the spot s_k . One can raise a valid concern that such representation (51) ignores innovations in risk factors between the contract delivery dates k and $k + 1$. We argue, that it is not the case, because investors account for all changes in underlying risks at time t

along the maturity timeline m up until k . Indeed, any shocks to risk factor g before k alters perceived conditional probability distribution of the future value of g_k . That time-varying conditional probability is the reason why different cross-sections m will have different loadings ϕ . Overall, even though ϕ_m changes only along cross-section, and g_k along time dimension, the combination of them in fact is driven only by time variation in latent risk factors, thus the term $\Pi_{t,m}$ represents the time-varying risk premium.

The use of representation (51) is not only motivated by economic sense, but also is well in line with empirical findings. First off, when we ran regression (24), we automatically assumed that effects of omitted variables such as risk factors, are independently distributed across maturities. However, the results in table 7 as well as results from Hodrick & Srivastava (1987) indicate that futures rates have non-zero serial correlation. It is likely that shocks to a future rate at time t has some effect at consecutive periods as well, but it fades out at some later date. If this shock comes from an omitted variable, then, given the outlined logic, the effect of it is absorbed by observations at different maturities to a different extent. In other words, cross-sectional units are likely affected differently by omitted variable, and there is some relationship that ties up close maturities together while leaving others independent of the effect. Cross-sections are cannot be called independent in this case. The implication of this for our panel model (24) is such that our estimates could be inconsistent in situations when agents demand non-zero risk premiums. In table 8 we present estimates of a CD test statistic for cross-sectional dependence in panel data proposed by Pesaran (2004).

Table 8: Pesaran Cross-Sectional Dependence Test Statistic

	AUD	CAD	CHF	GBP	JPY*	NZD	RUB	ZAR
CD statistic	38.505	33.472	19.946	52.945	70.885	5.5852	45.184	46.183
p-value	0	0	0	0	0	2E-08	0	0

Notes:

* - for this currency pair the test is performed on all cross-sections.

The test is performed at 95% confidence level.

As it can be seen, the null of no cross-sectional dependence is rejected for all currencies. When we perform an OLS estimation of (24) in a presence of cross-sectional dependence, then even asymptotically $\sqrt{NT}(\hat{\beta} - \beta)$ is biased. According to Hsiao (2014), the bias is in a form of:

$$\left(\frac{T}{N}\right)^{\frac{1}{2}} C$$

where C represents the bias coming from cross-sectional dependence of errors.

To model the relationship between spot exchange returns and futures premium, and

correct the bias, we shall assume the error term from (24) follows a linear factor model:

$$\varepsilon_t = b'_i f_t + u_{it}$$

where f_t is a vector of random factors, and b_i is factor loading coefficients, and u_{it} denotes stochastic error term. Notice, that it is the same representation as (51). Now we can rewrite our main panel model (24) as follows:

$$s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + b'_i f_t + u_{it} \quad (52)$$

Pesaran (2006) suggests augmenting (52) with the average of dependent variable, the average of independent variables and individual intercepts:

$$s_{t+m}^k - s_t = \alpha_m + \beta(f_{t,t+m}^k - s_t) + \overline{(s^k - s_t)}_k C_m + \overline{(f_t^k - s_t)}_k D_m + u_{mk} \quad (53)$$

where m denotes time-to-maturity, or individual dimension, and k marks observations in time dimension. Averages are taken over the cross-section for each contract. Pesaran (2006) calls pooled OLS estimator of this model as common correlated effects (PCCE) pooled estimator, and it is robust to cross-sectional dependence. We present results of estimation of (53) in table 9.

Table 9: The results of estimating the PCCE model

	AUD	CAD	CHF	GBP	NZD	RUB	ZAR
β	0.175	0.765	0.831*	0.601*	1.149*	-1.017*	0.417
std. err.	(0.264)	(0.438)	(0.387)	(0.299)	(0.000)	(0.051)	(0.876)
p-value	0.508	0.081	0.032	0.044	0.000	0.000	0.634
R^2	0.881	0.794	0.856	0.798	0.855	0.902	0.854

Notes: the table reports results of estimation of the model $s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + b'_i f_t + u_{it}$ using approach of Pesaran (2006). α denotes an intercept, and β denotes the slope coefficient. f_t is a vector of random risk factors, and b_i is factor loading coefficient. Since in our model each day of futures contract is used as cross-sectional unit, b_i stands for evolution in time of exposure to risk factors f_t . α is not reported because the technique of Pesaran (2006) does not enable us to estimate the intercept. For convenience, we present the p-value of significance test for β that is conducted at 95% confidence level. We use adjusted R-squared as R^2 .

* - denotes statistically significant coefficient at 95% confidence level.

Unfortunately, we had to exclude the Japanese Yen from the model estimation, since it has only one cross-section that is neither non-stationary nor has autocorrelated errors. But the results for most of the currencies are promising. First, notice that the estimates of β for AUD, GBP, NZD and ZAR have now the values closer to unity than in the original model estimation (see table 5 for reference). Not only the point estimates are now closer to what the theory predicts, but also for most of the currencies they are statistically significant. In our original analysis, only estimated coefficient is

statistically significant, where is now it is four of them. Additionally, by allowing for the time-varying and the contract-specific components of the risk premium (see equation 51) in the model, the coefficient of determination has improved from negligible values of less than 1% to 80–90%. We can conclude that strong cross-sectional dependence in UIP model based on futures data appears to be in line with two-component representation of the time-varying risk premium. Accounting for such structure of the premium significantly improves explanatory power of the model.

Now we turn to testing the null hypothesis that $\beta = 1$ versus the alternative that $\beta \neq 1$. We exclude condition of $\alpha = 0$ from the null because the intercept is allowed to vary over the cross-section, as it is shown in 53. Test statistics are presented in table 10. For the AUD, as in the case with the original regression (24) we performed for our panel, we reject the null, but we also reject it for the NZD and RUB. For all the rest currencies in our tests we cannot reject that the slope coefficient is unity.

Table 10: Results of the null hypothesis for PCCE model

	AUD	CAD	CHF	GBP	NZD	RUB	ZAR
t-value	-3.124	-0.536	-0.438	-1.336	1.2E+14	-39.792	-0.666
p-value	0.002	0.592	0.662	0.182	0	0	0.506

Notes: the table reports results of a test of the null hypothesis that $\beta = 1$ versus alternative of $\beta \neq 1$ for the model $s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + b_i'f_t + u_{it}$
P-values are reported for 95% confidence level.

To see the full picture, we have to also notice that for the Russian Ruble rate, estimation of the original model (24) gave a statistically significant slope coefficient with the point estimate of 0.78, while in the model corrected for cross-sectional dependence, the estimate is basically a minus unity. To investigate further, we study a plot of spot rate appreciation versus futures premium for the estimated model (53) for the Ruble. On both plots in figure 4 you can see also the line drawn from the OLS estimated slope coefficient of the model. We can notice, that a handful of extreme values affect the alignment of the line on the left plot - the plot that refers to the entire data period.

To see from where these extreme values may come from, we draw a plot of the forward premium for the RUB/USD pair for the full data period (see figure 5). We notice that during the period from 1998 to 2000, the volatility of the forward premium is very high compared with the rest of the time frame. It comes as no surprise because in August 1998 Russia defaulted on its debt. Up until the event, the Russian Central Bank was actively supporting the exchange rate by buying the Rubles and selling its foreign currency reserves. A sequence of failed reforms prompted foreign investors to sell Russian assets. To prop up the economy, the interest rate on short-term zero-coupon bills was increased to 150% which resulted in high interest payments one month before the default - the payments exceeded monthly tax collections by 40%. A quick erosion of foreign exchange reserves that were used to support the floating peg prompted the gov-

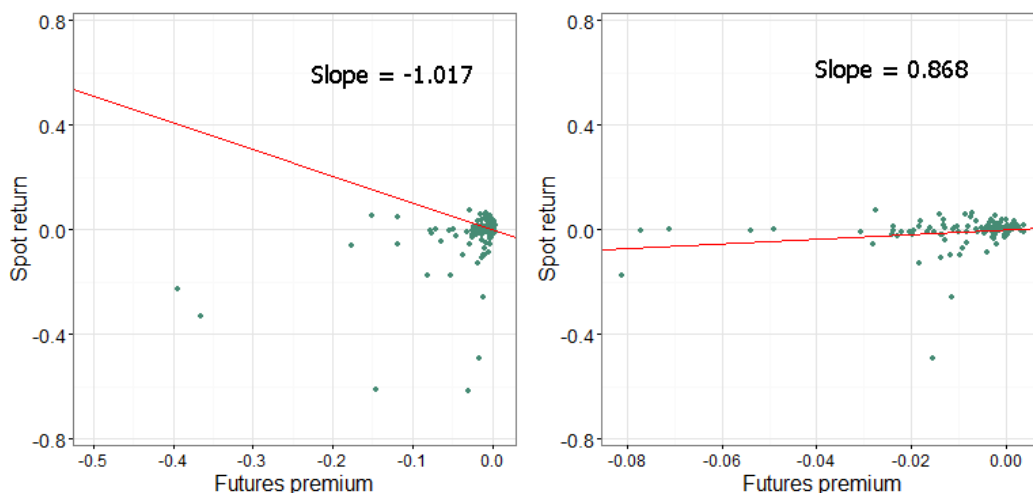


Figure 4: Spot return vs futures premium for the RUB
for full period (left) and for post-default period (right)

ernment to default on its domestic debt and impose a moratorium on its Eurobonds. As a result, the exchange rate went from 6.43 USD/RUB to 21 USD/RUB in one month.

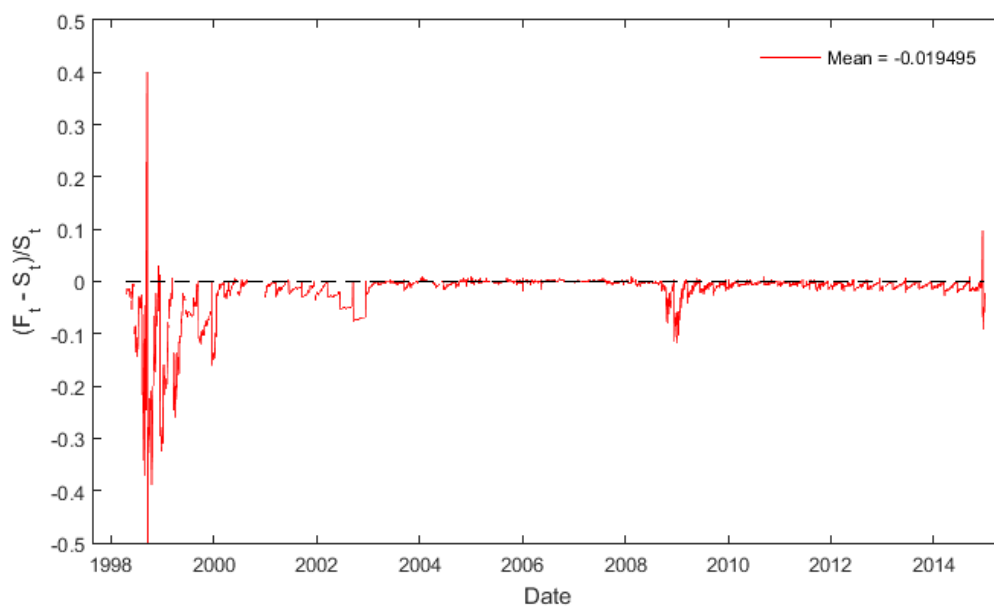


Figure 5: Forward premium of RUB/USD

The period of time from the August of 1998 until the end of 1999 was associated with very high volatility of the RUB/USD exchange rate. Political uncertainty and security risks in 1999 did not let the pair to stabilize until 2000. For our analysis, we are interested to see if excluding the period of such extreme variation would give us a different estimates for the slope coefficient in the model (53). For that reason, we build a new sample ranging from March 2000 until 28th November 2014. We test the model for this sample and present results in table 11.

In table 11 we see that the slope coefficient is now close to unity. Indeed, the null of $\beta = 1$ cannot be rejected at 95% level. Right side plot in figure 4 shows now that for the

Table 11: Estimation of the PCCE model for the Ruble and the null hypothesis test for smaller sample

	β	std. err.	p-value	R^2	$H_0 : \beta = 1$ t-value	vs. $H_A : \beta \neq 1$ p-value
RUB	0.868	0.223	0.0001	0.951	-0.591	0.556

Notes: the table reports results of estimation of the model $s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + b'_i f_t + u_{it}$ using Pesaran (2006) approach for the RUB/USD currency pair using a post-default data sample that covers the period of 03/2000-12/2014. It also reports results of the null hypothesis test where $H_0 : \beta = 1$ is tested against alternative $H_A : \beta \neq 1$.

The tests are performed at 95% confidence level.

post-default period, the slope is positive, and in fact, it better matches the data. Indeed, the data points on the left side plot associated with very low futures premiums are not present in the post-default period - the lowest values of $(f_{t,t+m}^k - s_t)$ are now around -0.08 while we had several points below that level in the first plot. These extreme observations dragged the slope line in OLS estimation downwards. After refining the data period, we find ourselves with higher R^2 and the slope coefficient that is close to what the theory dictates.

Overall, the results for all currencies suggest that accounting for the time-varying and contract-specific risk premiums improves the model fit and makes it more in line with uncovered interest parity condition. The question of what particular risk sources are captured by the term (51) remains an open one, and we will attempt to shed some light on this side of the problem.

3.5 Carry Risk Factors

3.5.1 Outline of asset pricing tests and portfolio construction

When it comes to searching for risk factors that are associated with abnormal currency returns, one common approach is used in the academia. In its simple form, it constitutes a construction of carry portfolio and using its excess returns series as a dependent variable in a time-series regression analysis. In the role of regressors, several traded and non-traded factors are tested. However, using the factors that are not represented by excess returns poses some challenges to estimation of the model, and the meaning of the test becomes much less intuitive. Such test cannot be interpreted as asset pricing test, and you can only answer a question if some non-traded factor has some impact on the portfolio returns or not, but you cannot answer precisely the question "how much?". To use non-traded risk factors, we need to perform a cross-sectional regression of excess returns on those risk factors. Campbell & Cochrane (1999) underlines the test based on stochastic discount model that can be estimated using Fama & MacBeth (1973) procedure. No matter whether we apply this type of test, or time-series test based on traded risk factors, if the model exhibits high explanatory power, and the

coefficients on these factors are both statistically and economically significant, such test results become a good proof that high returns to a carry trade strategy are simply a compensation to bearing some particular risks. They also would give support to a risk-based explanation to the failure of the uncovered interest parity.

First off, we construct a portfolio that invests in each currency based on its forward discount. To do this, we have to determine weights for each currency for every period. We use monthly periods, so at the end of every month we calculate the weight for each currency as follows:

$$w_{n,t} = \frac{\text{sign}(s_t - f_t)}{N_t} \quad (54)$$

where $n = 1 \dots N$ with N denoting a number of currencies at time t for which we have futures discount data. At its maximum it is 8, as we have 8 currency pairs in our analysis, but not for every month we have the data for all currencies. This weighting scheme gives equal weights for the currencies that at moment t have the same sign for futures discount. Since it ignores the size of the futures discount, the portfolio constructed using this scheme we call a naive carry portfolio. To make this type of portfolio better fit with reality of investment practices, we use 5-days exponential moving average for the futures discount instead of the simple value. First, this way we want to eliminate the situation when a one day sudden change in spot or in futures rate doesn't solely affect the decision to invest in the currency for one month. Second, we believe many practitioners compare an asset performance at day t with how it got there, that is, with how a few preceding days the asset was trading. If you know the value of futures discount only for one day, you probably would want to know how it was developing a few days prior to this to see if it is widening or shrinking. An exception in this case appears when we have a value of futures discount for the first trading day of a new futures contract. In this case, we simply do not have valid moving average for it, thus we use simple futures discount. We refine our sample not only to the month ends for each currency, but we also retain data on rates for expiration days and for next day that stands for the first trading day of the new contract. These specifics are important when we calculate returns.

We calculate monthly excess log returns based on futures data as:

$$R_t = f_t - f_{t-1}$$

However, for the months of March, June, September and December (the months when contracts expire), we calculate the monthly excess return as:

$$R_t = (s_k - f_{t-1}) + (f_t - f_{k+1})$$

where s_k denotes spot rate at expiration, and f_{k+1} denotes the futures rate on first day

of the next contract. Once we calculate monthly excess returns, we multiply them by the weights as $R_t^{naive} = \sum_{t=2}^T \left(\sum_{n=1}^N w_{n,t-1} R_{n,t} \right)$ which gives the returns series for the naive carry portfolio.

Ignoring the size of futures discount might pose a serious shortcomings if we want to replicate carry investment strategy. You can see in figures 6 and 7 that the futures discount varies significantly across time with some currencies having a much bigger discount than others. To exploit such variability, we calculate the weights for a so-called HML portfolio, which stands for High-minus-low portfolio. It is another version of carry trade portfolio, and, in fact, we argue that it is more applicable among practitioners. The nature of the weighting scheme is such that we assign a weight of 1 for a currency pair that had the biggest futures premium in the last month, and a weight of -1 to a portfolio with the smallest futures premium. It is indeed a classic version of carry trade portfolio, because in it, a trader borrows money in low interest rate currency (hence, the weight is -1), and invests in a currency with highest interest rate (futures discount). We present a descriptive statistic for this portfolio returns in table 12.

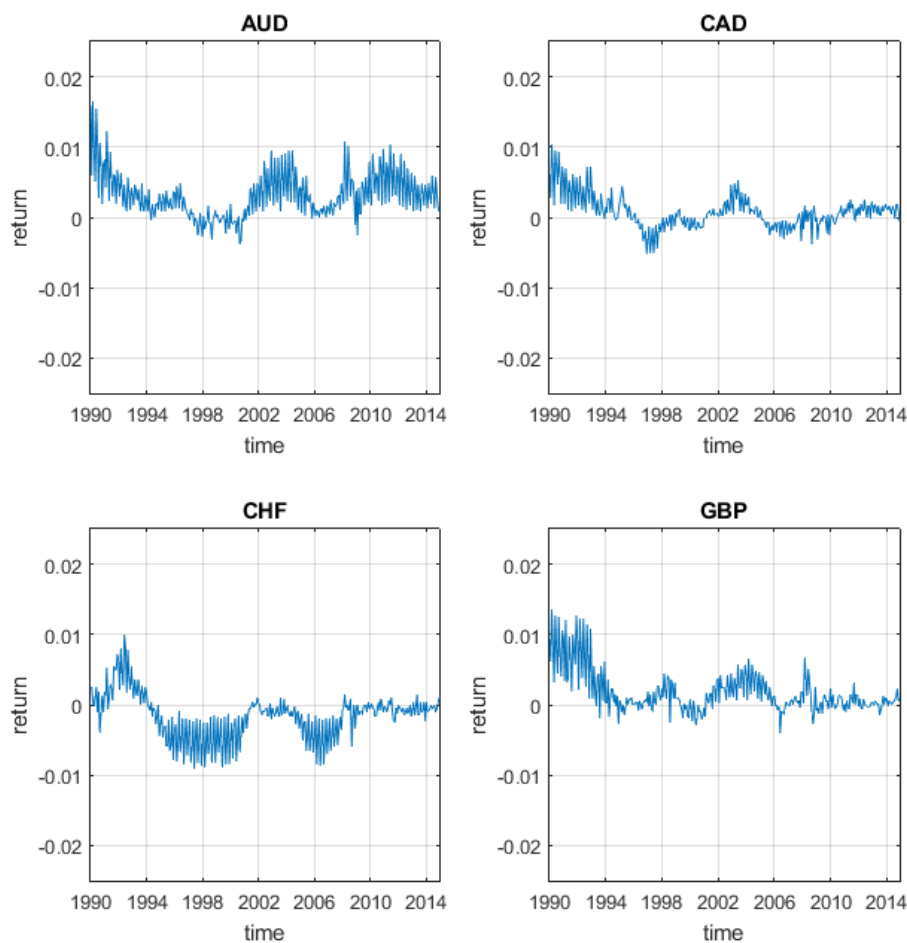


Figure 6: Futures discount across time for AUD, CAD, CHF and GBP

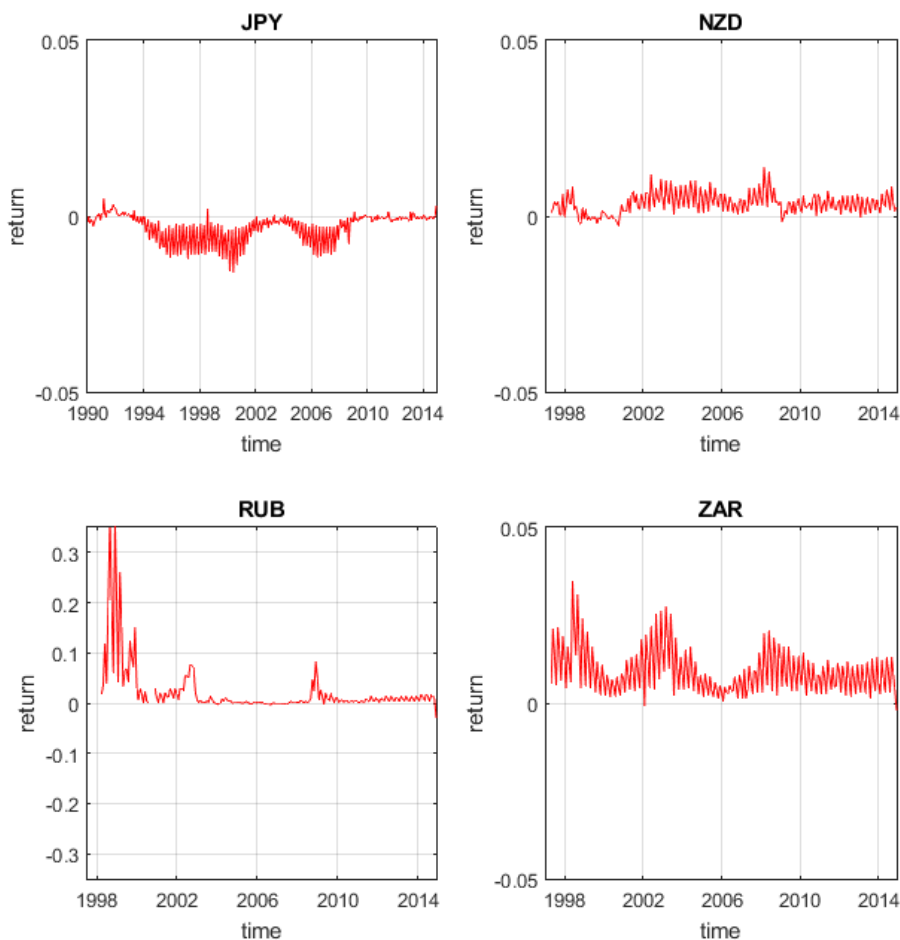


Figure 7: Futures discount across time for JPY, NZD, RUB and ZAR

Lastly, we construct a portfolio of average returns for all the available currencies. It is simple a portfolio with equal weights of

$$w_{n,t} = \frac{1}{N_t}$$

We call such portfolio as DOL portfolio, as in Menkhoff et al. (2012). It is simply the average excess return of all the currency pairs available to us. It represents the excess returns to an American investor who invests in a diversified bucket of foreign currencies. We present its descriptive statistics in table 12.

We can note in table 12 that currencies that are traditionally considered to be investment currencies (AUD, NZD, RUB, ZAR), have on average higher annualized returns than the funding currencies. Also, interestingly, skewness is more negative for investment currencies. The only currency that has positive skewness is a traditional funding currency, the Japanese Yen. Further, if we look into the table 12 and examine the moments for Naive Carry portfolio, we notice, that it also has low annualized excess

returns (lower than for the Russian Ruble), but even more pronounced negative skewness. As in Brunnermeier, Nagel, & Pedersen (2009), we notice that the currencies associated with higher interest rates have more pronounced negative skewness, and thus their positive excess returns could be a compensation for a crash risk. Negative skewness is associated with frequent small positive returns, but rare crashes. In a support of this view, we report that the monthly skewness on the HML Carry portfolio has a significantly negative value of -2.84 .

Table 12: Descriptive statistic for currency portfolios excess returns

(a) Individual currency portfolios								
	AUD	CAD	CHF	GBP	JPY	NZD	RUB	ZAR
Mean	0.025	0.008	0.001	0.005	-0.017	0.022	0.053	0.001
Median	0.035	0.013	-0.004	0.004	-0.039	0.050	0.048	0.000
Std. dev.	0.114	0.077	0.108	0.090	0.110	0.132	0.205	0.167
Skewness	-0.615	-0.492	-0.141	-0.607	0.431	-0.449	-4.323	-0.422
Kurtosis	5.572	6.849	3.462	5.521	5.029	4.813	50.146	3.939
Sharpe Ratio	-0.040	-0.283	-0.267	-0.271	-0.423	-0.059	0.116	-0.172
coskew (MKT)	-0.494	-0.565	0.032	-0.107	0.294	-0.440	-0.689	-0.330
coskew (DOL)	-0.497	-0.360	-0.219	-0.288	0.334	-0.421	-0.669	-0.322

(b) Carry and DOL portfolios			
	Naive Carry	HML Carry	DOL
Mean	0.026	0.093	0.009
Median	0.034	0.113	0.012
Std. dev.	0.058	0.219	0.079
Skewness	-0.749	-2.760	-0.499
Kurtosis	5.001	31.003	5.357
Sharpe Ratio	-0.065	0.290	-0.266
coskew (MKT)	-0.380	-0.643	-0.427
coskew (DOL)	-0.539	-0.543	-

Notes: the table reports descriptive statistics for currency portfolios. For excess kurtosis, subtract 3 from the values presented in the table. The mean, median and standard deviations are presented as annualized values, so is Sharpe ratio. The rest of the statistics are based on monthly frequencies. Coskewness measures a comovement of portfolio returns with volatility in a benchmark. As benchmarks we use volatilities of excess returns on S&P500 index for coskew(MKT) and the average FX excess returns to a US investor for coskew(DOL).

We also report value of coskewness for each currency and each portfolio. Coskewness is measured as:

$$\text{coskew} = \frac{E[(r_t^i - \mu_i)(r_t^M - \mu_M)^2]}{\sigma(r^i)\sigma^2(r^M)} \quad (55)$$

where r_t^i denotes the return at time t on portfolio i , r_t^M denotes return on a benchmark, μ stands for the mean value, and σ and σ^2 are the standard deviation and the variance, respectively. We use excess returns on equally-weighted portfolio of currencies as one market measure, and the excess return on S&P500 index as the other. High coskewness measure for a portfolio indicates that its returns are high when the

market volatility is high. Such portfolios serve as hedge against market turmoil, and thus, on average, they should earn smaller returns. As we can see from the table 12, a classical lending currency the JPY has the highest and the only positive values for both coskewness estimates. The second highest value is obtained for the CHF which is another common funding currency. Both carry portfolios have negative values, with HML Carry exhibiting one of the lowest coskewness with markets. It gives us a hint that the returns on carry trade portfolios might be exposed to market volatilities, and be a compensation for bearing that type of risk.

In our asset-pricing models that we build in the next section, we use HML portfolio as a proxy for carry trade abnormal returns. We can see that Naive carry portfolio does not compare with HML portfolio in terms of cumulative returns which is evident in figures 9 and 8. So high excess returns on HML portfolio must be explained by exposure to some of the risk factors, and we perform the multi-factor asset-pricing modeling in the following section.

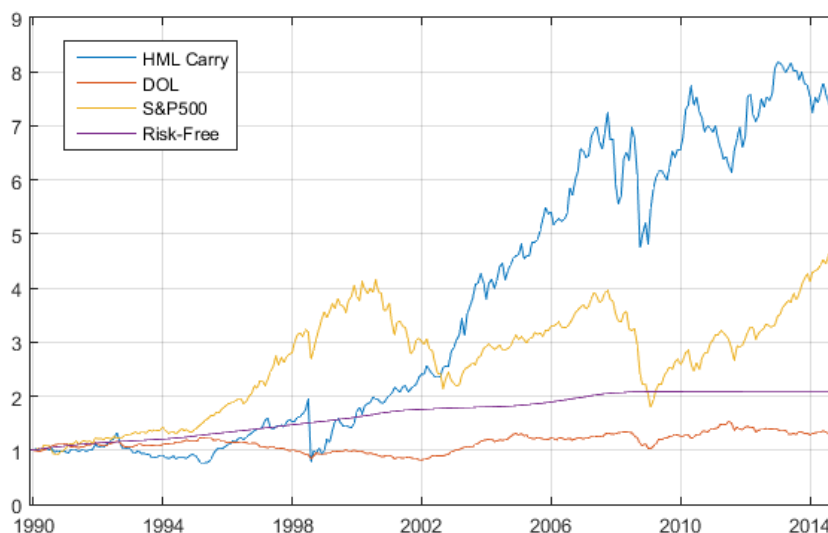


Figure 8: Cumulative return of High-minus-low carry portfolio relative to other portfolios

3.5.2 Time-series test

Up until now, we established the fact that the HML portfolio has on average economically significant positive abnormal returns. To analyze what drives such results, we start with Euler equation for an American investor:

$$E_t[m_{t+1}R_{t+1}^i] = 0 \quad (56)$$

where m is a stochastic discount factor, and R^i is the excess return on portfolio i . For any portfolio, and for any currency, this equation should hold. Campbell & Cochrane

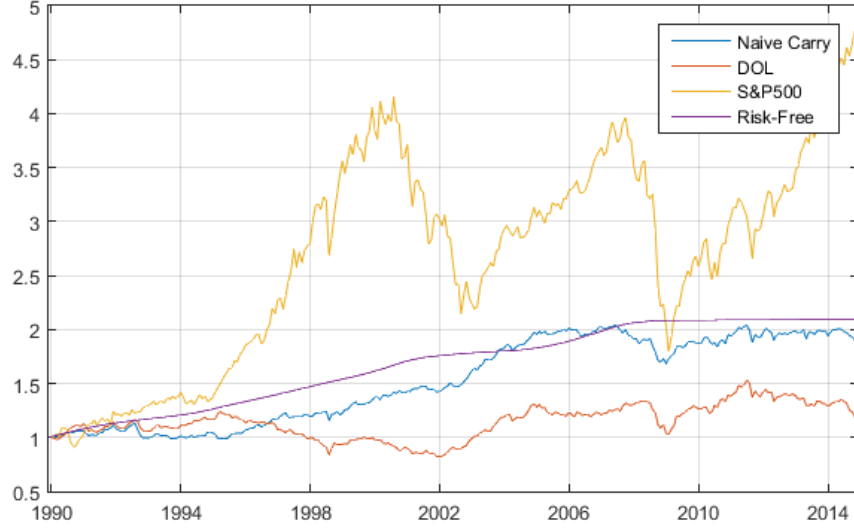


Figure 9: Cumulative return of Naive carry portfolio relative to other portfolios

(1999) shows that this equation can be rewritten as:

$$E_t[m_{t+1}R_{t+1}^i] = E_t[m_{t+1}]E_t[R_{t+1}^i] + cov(m_{t+1}R_{t+1}^i) = 0$$

and hence:

$$E_t(R_{t+1}^i) = -\frac{cov(m_{t+1}R_{t+1}^i)}{E_t(m_{t+1})}$$

The value of the SDF cannot be negative, thus, the most likely explanation for positive excess returns is negative covariance between the stochastic discount factor and the abnormal returns. We can replace the terms from the previous equation as:

$$\beta_i = \frac{cov(m_{t+1}R_{t+1}^i)}{\sigma^2(M_{t+1})}$$

and

$$\lambda = -\frac{\sigma^2(m_{t+1})}{E_t(m_{t+1})}$$

and we obtain the following beta representation:

$$E_t(R_{t+1}^i) = \beta_i\lambda \quad (57)$$

As in Menkhoff et al. (2012) and in Lustig, Roussanov, & Verdelhan (2011) we assume that the SDF follows a linear form:

$$m_{t+1} = 1 - (h_t - \mu)'b \quad (58)$$

where h denotes a vector of risk factors, b are the factor loadings and μ represents the

factor means. Campbell & Cochrane (1999) shows that there exist λ such that the two models (57) and (58) become equivalent. Such λ is:

$$\lambda = -\Sigma_{hh}b$$

where Σ_{hh} denotes variance-covariance matrix of the factor.

Normally, the estimation of exposures to risk factors comes as a two-stage approach where we first run a time-series regression, and at the second step we run a cross-sectional regression. However, in its simplest case, the tests of (57) can be performed as a test of zero intercept in the following time-series regression form:

$$R_t^i = \alpha_i + \beta_i f_t + \varepsilon_t^i$$

where R_t^i are excess returns on portfolio i , and f_t is the price of risk expressed as excess returns on a portfolio that exploits the exposure to the factor. This form is easily expandable to a multi-factor specification. Only in the case of all f being excess returns, we can test the model (57) using time-series OLS regression, and it is because the estimate of the factor risk premium $\hat{\lambda}$ is the sample mean of the factor f .

The first step of our attempts to explain abnormal carry returns, we apply OLS regression analysis on a set of traded risk factors. We employ the return on the S&P500 index over the risk-free rate for the stock market risk proxy. Next, we include the excess returns on the GSCI commodity index as a measure of commodity-linked risk. We do this simply because some of the investment currencies are related to commodity-exporting economies, and the swings in commodity prices could have potential to explain abnormal carry returns.

As the third factor we use the DOL portfolio itself. Based on work of Lustig, Roussanov, & Verdelhan (2011), factors that are derived from currency portfolios themselves have shown significant explanatory power in cross-sectional tests. As the fourth factor, we want to employ excess returns of a portfolio exposed to the volatility on the equity market. We cannot use VIX index directly in our time-series OLS analysis, but we can construct an excess return on a variance swap as a risk factor. We calculate it as follows:

$$R_{VS,t+1} = \left[\sum_{\tau=2}^{T_{t+1}} \left(p_{t+1,\tau} - p_{t+1,\tau-1} \right)^2 \right] \left(\frac{365}{T_{t+1}} \right) - \left(\frac{VIX_t}{100} \right)^2 \quad (59)$$

where $p_{t+1,\tau}$ is the log price of the S&P500 index at day τ of the month $t+1$, and T_{t+1} is the number of trading days in the month $t+1$. The VIX index is the annualized volatility expressed in percentage points, i.e. $VIX = 100\sqrt{\sigma_{annual}^2}$, so in order to obtain the value for the variance strike based on VIX, we need to divide the index

value by 100 and raise to the power of 2. The VIX index is a measure of global risk appetite for investors, not only in equity markets. It is a well-accepted proxy for global risk aversion. Brunnermeier, Nagel, & Pedersen (2009) notes that the index is related to corporate credit market and a variation in sovereign credit default swaps. They also find that the increase in VIX coincides with carry trade losses.

In order to include a bond market proxy, we construct two variables. First, we compute excess returns on the 10-year Treasury over the risk-free rate. This measure represents the risk of changing interest rates in the US. We denote this excess return as $R^{B,rate}$. The second proxy is the difference in returns on 10-year and 2-year Treasuries which is associated with a risk arising from the changes in the US yield curve. We denote such return difference as $R^{B,curve}$. For an American investor who is long in carry portfolio, both of these bond returns represent risk for his portfolio.

At this point, we fit different models with HML Carry excess returns as dependent variable and a various combination of the excess returns on risk factors as regressors. We test each model for the absence of the serial correlation the heteroskedasticity in residuals. We employ a formal selection criteria based on three factors:

- overall model fit measured by adjusted R-squared;
- AIC and BIC information criteria;
- significance of the included variable measured as p-value.

We try different lags for each variable to find the best fit. The results of our OLS regression model selection has yielded the model of the form:

$$R_t^{HML} = \alpha + \beta_1 R_t^{DOL} + \beta_2 R_t^{VS} + \beta_3 R_t^{B,curve} + \varepsilon_t \quad (60)$$

The inclusion of the US yield curve risk proxy does improve the fit of the model as measured by AIC and BIC criteria, although the coefficient is not statistically significant. We test the assumptions of the Gauss-Markov theorem for the model, and find that the linearity assumption is satisfied, as well as the assumption of the absence of the perfect multi-collinearity among regressors. We investigate the latter by studying variance-inflation and generalized variance-inflation factors. The assumption of zero mean for the residuals is also met, although the distribution of errors does not conform to normal Gaussian distribution. We find the residuals to be serially correlated of order 1 and we also reject the null of the studentized Breusch-Pagan test indicating that the error term exhibits heteroskedasticity. In light of these findings, we calculate Newey and West robust standard errors and we report the estimates for the parameters in table 13.

Although we report the estimated coefficient on DOL factor to be statistically insignificant, this was not the case when we estimated the standard OLS regression with-

Table 13: Time-series asset pricing results

	α	DOL	VS	Yield Curve	R^2
HML portfolio	0.015** (0.005)	0.563 (0.371)	-0.198** (0.040)	-0.586* (0.324)	0.104

Notes: the table reports the estimates and the Newey and West standard errors for the OLS model where excess returns on HML carry portfolio are regressed on excess returns on DOL portfolio (also called the dollar risk factor, or simply the return on equally-weighted portfolio of foreign currencies), on the returns to Variance Swap measured as the difference between a realized volatility on S&P500 index and the VIX implied volatility and on the returns to a portfolio that is long in 10-year constant maturity Treasury and short in 2-year Treasury which measures the exposure to the changes in the US yield curve. α denotes an intercept of the model, and it is interpreted as the portion of the excess returns on the HML portfolio not explained by the model. The model is correct if α is not statistically significant from zero. The R^2 reported value is adjusted R^2 .

Significance codes:

** - significant at 99% confidence level;

* - significant at 90% confidence level;

out robust errors. We reject the null hypothesis that $\alpha = 0$ at 99% significance. We note that the value of alpha is roughly translated to 1.5% of unexplained monthly returns. It is a staggering finding given the fact that the median monthly excess return on the HML Carry portfolio is 0.94%.

The coefficient on the US yield curve steepness has a negative value. The result suggests that when the long term yield increases, or when the short term yield decreases, the carry strategy loses money. The spread can widen when the Federal Reserve lowers the rate. In this case we would actually anticipate the increase of carry trade returns. But the spread can widen also due to the increase in the long-term rate, and it can be caused by rise in anticipated inflation or prospects of higher economic growth. To investigate changes in the spread, we study the plot provided in figure 10. We can notice that until 1999 the two yields moved very closely, but after the Asian crisis and the ".com" crisis, the risk-free rate dropped, causing the 2-year Treasury yield to fall as well. In the time period before the global financial crisis, the short rate picked up mainly due to the rise in the Fed rate, but the long-term bonds did not pick up. The finding of the negative coefficient on the yield curve steepness is a somewhat puzzling, because most of the increases in the spread during the sample period were caused by sudden drops in shorter-term yields. Contrary to our finding, such decreases would induce the carry trade activity since the decrease in the US interest rates makes investment in high interest rate currencies more attractive to an American investor. With all of this in mind, we do however admit that the result could be caused by carry trade activity itself. Once the short rates fall in the US, American investors can start loading up on carry trade portfolios. Subsequent increases in positions drive up FX rates of high interest currencies, producing high abnormal returns for HML portfolio. With the increase in global volatility and always accompanying it reduction in global funding liquidity, carry trade lose money, hence the returns exhibit strongly negative skew-

ness. If this scenario indeed takes place, the model (60) rewritten with lagged $R_t^{B,curve}$ should have negative and statistically significant loading on the factor. Indeed, the version of the model with $R_{t-9}^{B,curve}$ has a statistically significant coefficient of -0.6345 and a Newey-West standard error of 0.343. The rest of the estimated coefficients are roughly identical to the values we estimated for the original model (see table 13). The lagged model has smaller value for AIC and BIC criteria compared to the original. Nonetheless, we do not know why for the original model, the contemporaneous yield curve risk factor has almost the same negative coefficient.

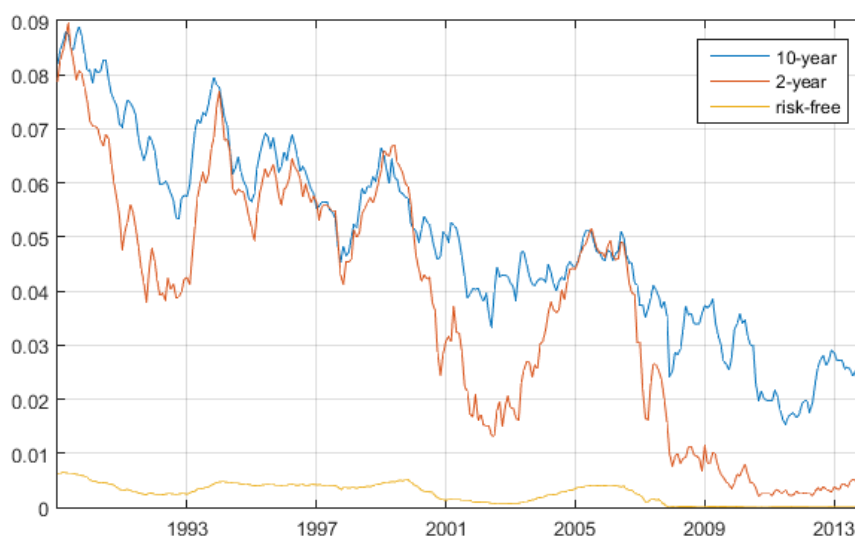


Figure 10: The time-series plot of the yields on US Treasuries
10-year Treasury, 2-year Treasury and the risk-free rate

As far as the market volatility is concerned, we notice a negative and statistically significant coefficient on the excess returns on the volatility swap. As in Menkhoff et al. (2012), we document the finding that carry trade tends to lose money when the stock market volatility increases.

3.5.3 Portfolio sorting and cross-sectional asset pricing

The asset pricing steps just performed did not lead to a discovery of the risk factors that would explain abnormal returns to carry trade. Even though we found a strong negative relationship between carry returns and returns on volatility swap, only small variation if the overall variation was explained. This is not by any means a surprising result in this line of research. Indeed, much of the most prominent literature on the topic could find variables that explained only small part in the variation of abnormal returns. This way, it is useful to perform a cross-sectional regression analysis to filter out common to all currency portfolios factors and to pick out factors that explain cross-sectional variability. To use this approach, we can sort the portfolios of currencies by

their futures discount, or equivalently, by their interest rate differential relative to the US Dollar¹⁸. As Lustig & Verdelhan (2007) point out, by sorting the portfolios in this way, we are able to filter out the innovations in currency returns that are orthogonal to changes in interest rates. By sorting on the forward discount, we are able to operate on the returns that are conditional on the interest rates. For each portfolio, the change of the FX rate will on average reflect mostly the risk premium component. Unlike Lustig & Verdelhan (2007), we form just 3 portfolios for this type of analysis due to the fact that our sample covers only a total of 8 currency pairs, and for some time periods the rate futures discounts have similar values which makes it hard to separate them in more than three portfolios. We provide descriptive statistic on these portfolios in table 14.

Table 14: Descriptive statistic for the portfolios sorted on the basis of futures discount

Portfolios	1	2	3
Mean	-0.047	0.048	0.049
Median	-0.053	0.065	0.019
Std. dev.	0.111	0.111	0.163
Skewness	-0.959	-0.377	-0.744
Kurtosis	6.505	5.150	4.960
Sharpe Ratio	-0.556	0.302	0.210
coskew (MKT)	0.073	-0.324	-0.331
coskew (DOL)	-0.256	-0.453	-0.575

Notes: the table reports descriptive statistics for currency portfolios sorted on interest rates differential. The portfolio 1 is the portfolio where an investor goes long the currency with the smallest futures discount of all the currencies available for him in the period t . Portfolio 3 reflects the returns on the investment in the currency with the highest futures rate. After each month, portfolios are rebalanced. Note that the sample period covers time from 28-Feb-2001 to 31-Dec-2014 due to the fact that we have to reduce our sample to match the sample period of a new risk factor - TED spread. Also note, that the values for the mean, median, standard deviation and Sharpe ratio are annualized while the others are expressed at monthly frequencies. To get excess kurtosis, subtract 3 from the values of kurtosis presented in the table. Coskewness measures comovenet of portfolio returns with volatility of returns of a benchmark. We use excess returns on S&P500 as MKT benchmark and excess returns on equally-weighted FX portfolio as DOL benchmark.

Upon analysis of the table, we notice that the high-interest portfolio has the highest mean return, albeit it is not much higher than the middle portfolio. Average returns for the low futures discount portfolio are negative with negative Sharpe ratio. As expected, low interest differential portfolio number 1 provides a hedge against volatility on stock market, which is evident in the sign of the coskewness estimate. The fact that portfolio 1 offers low returns but also a hedge for stock market investors provides evidence that the two markets, US stock market and the FX market, are linked. The link could be associated with the general risk aversion of investors or with funding liquidity dry-ups. It has been found in various research papers that both risk aversion and liquidity constraints spill over to different markets.

¹⁸Sorting portfolios of currencies based on their interest rate differential is also done in Lustig, Rousanov, & Verdelhan (2011), Burnside et al. (2011), Lustig & Verdelhan (2007), Menkhoff et al. (2012), Verdelhan (2010) and others.

To illustrate the difference among these three sorted portfolios, we plot cumulative returns on the investment of \$1 at the beginning of the sample period. In figure 11 we see that the highest interest rate portfolio earns higher returns than the other two. We also notice, that the portfolio with highest futures discount is more susceptible to currency crashes, as you can see sudden drops in value during the Asian crisis/Russian default and at the beginning of the subprime mortgage crisis. Both periods are associated with heightened volatility of the stock market (see shaded areas in figure 12). This fact suggests that carry returns, too, lose money when the risk aversion proxied by the VIX index increases.

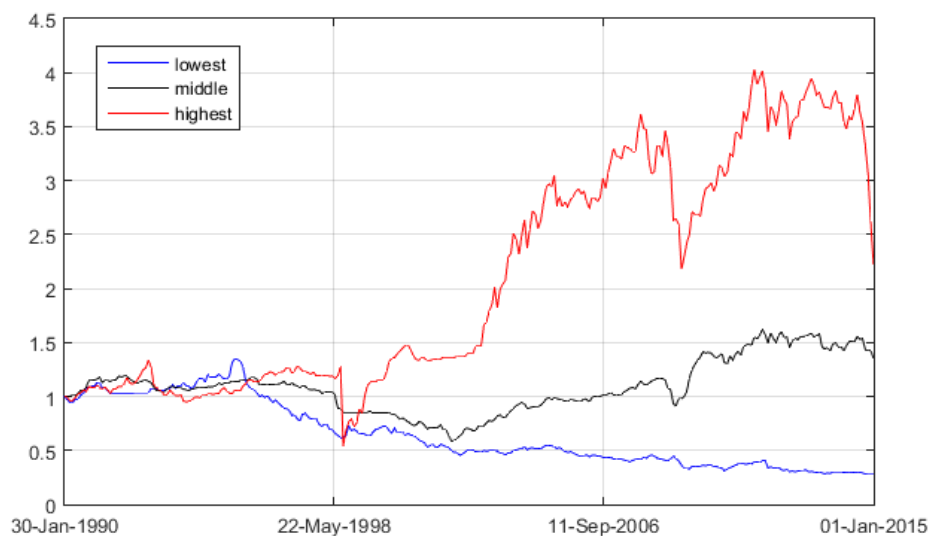


Figure 11: Cumulative returns of portfolios sorted on interest rate differentials

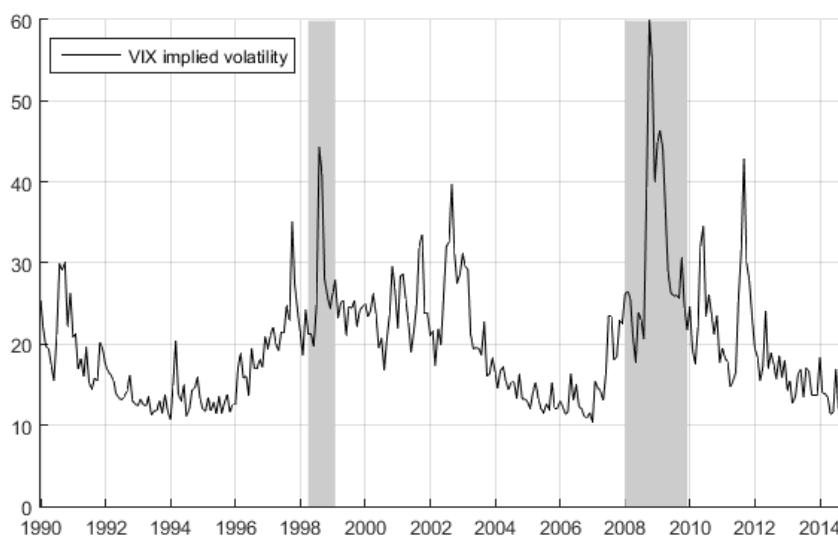


Figure 12: The time-series plot of the CBOE Volatility Index

Motivated by the work of Menkhoff et al. (2012), we construct a measure of foreign exchange volatility for each time period as:

$$\sigma_t^{FX} = \frac{1}{T_t} \sum_{\tau=1}^{T_t} \left[\sum_{k=1}^{K_\tau} \left(\frac{|r_\tau^k|}{K_\tau} \right) \right] \quad (61)$$

where K_τ denotes the number of the currencies available in the sample on a day τ , and T_t is the total number of trading days in the month t . The absolute daily log return $|r_\tau^k|$ for each currency is defined as $|f_\tau - f_{\tau-1}|$ or as $|s_\tau - f_{\tau-1}|$ if τ is the expiry day for a futures contract. The use of absolute returns is preferred over the squared returns to minimize the impact of outliers on the measure. The plot of the estimated foreign exchange volatility measure based on our 8 currencies is presented in figure 13. We can notice the spikes in volatility during Asian crisis of late 90s and Russian default, as well as during the beginning of the global financial crisis of 2008.

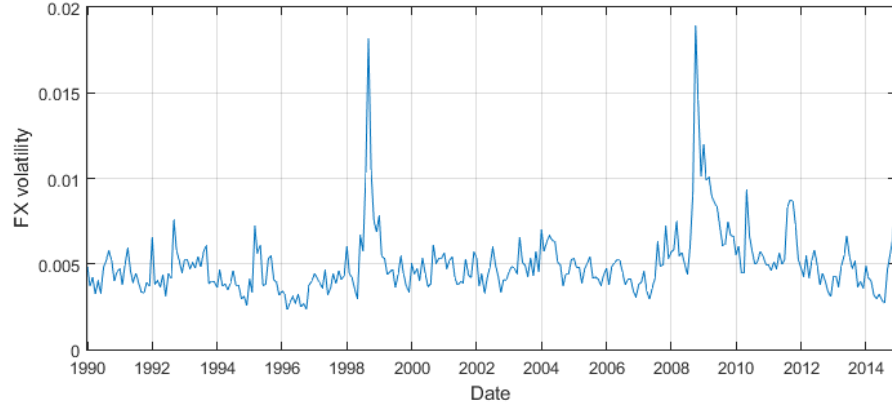


Figure 13: Estimated foreign exchange market volatility

For our empirical work, we would be more interested in FX volatility innovations. Taking the first differences of the measure σ_t^{FX} is not a suitable option since we find the first differences to be serially correlated with a coefficient -0.169 on the lagged value. Thus, we estimate a simple AR(1) model for the volatility measure σ_t^{FX} and use its residuals as the measure of volatility innovations $\Delta\sigma_t^{FX}$. We plot the estimated volatility innovations in figure 14.

In addition to the use of σ_t^{FX} in our cross-sectional analysis, we would like to add another risk factor that measures liquidity. Mancini, Rinaldo, & Wrampelmeyer (2013) find that high interest rate currencies are exposed to liquidity risk, while low interest rate currencies have negative loading on this factor. They argue this effect comes from the fact that low interest currencies are in general more traded on FX market, hence they are more liquid compared to the high interest currencies. Favorable funding liquidity conditions can provide a boost to a trading liquidity for high interest currencies, therefore they are expected to appreciate during that period of time, while

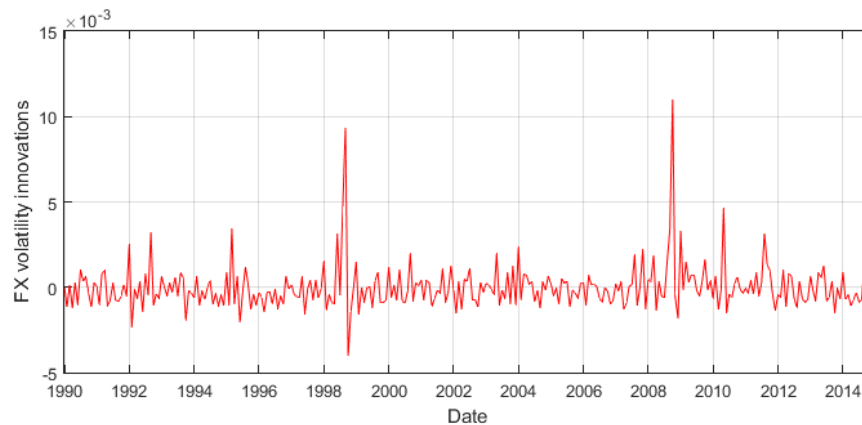


Figure 14: Foreign exchange volatility innovations
measured as residuals to AR(1) process

the funding currencies would depreciate. This would drive the UIP condition away from its theoretically predicted values and create positive abnormal returns for carry trade strategies. However, liquidity constraints would be a negative factor for high interest currencies, therefore we would expect carry portfolios to lose money. Based on the intuition that carry returns can be driven by liquidity risk, we employ a funding liquidity risk factor into our analysis, proxied by TED spread. We calculate TED spread as the difference between 3 month LIBOR USD rate and 3-month yield on US T-bill. It is set in the interbank market and measures the perceived level of credit risk and funding liquidity risk. TED spread was used as a proxy for funding liquidity risk in Menkhoff et al. (2012) and Brunnermeier, Nagel, & Pedersen (2009).

Employing the volatility innovations measure, HML and DOL excess returns, as well as TED spread factor, we perform estimation of the beta-pricing model (57) under the assumption that the SDF follows linear factor form 58. In this case we use our three sorted portfolios as test assets, and we perform a two-step Fama-MacBeth procedure. This approach enables us to estimate first β coefficients using time-series regressions of each portfolio's returns on risk factors. Second, we perform cross-sectional regressions for each month where the dependent variable is a vector of portfolio returns for that month, and β coefficients are independent variables. Fama & MacBeth (1973) uses time-series regression in the first step based on five-year time windows. We, due to the scarcity of available observations, perform that regression on the whole sample. That way, in the second step, we estimate 167 cross-sectional regressions with the same β vectors¹⁹ for each λ in each of these regressions. Overall, for each risk factor, we

¹⁹Since we use only three sorted portfolios, for each risk-factor's λ we have three values of β - each estimated from one of three sorted portfolios.

estimate a risk premium measure λ as follows:

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_t$$

where T is the total number of cross-sectional tests (in our case, it is 167 - the amount of monthly observations). We report the values of risk premiums λ_i and risk quantities β_i estimated from Fama-MacBeth procedure along with standard errors in table 15. We also perform Sargan-Hansen J-test that checks whether model's moment conditions match the data well or not. For all the possible combinations of risk factors, the null that the model is valid is rejected at 95% significance. The model that we choose to estimate is thus has the form:

$$E_t(R_{t+1}^i) = \beta_{DOL}\lambda_{DOL} + \beta_{HML}\lambda_{HML} + \beta_{VOL}\lambda_{VOL} \quad (62)$$

and the estimated parameters are present in table 15. During our model fitting step, we noticed several interesting facts:

- TED factor and VOL (volatility innovations) factors are largely interchangeable in a sense that the model is not improved by the inclusion of both, and that the estimated risk premiums on other factors do not change significantly when changing TED to VOL and vice versa;
- Both TED and VOL have beta coefficients positive for portfolio 1 and portfolio 2. The coefficient is higher than 2. When volatility increases, or when funding liquidity condition worsens, the model estimates that low interest and high interest portfolios make money. This result can be taken with a scepticism since non of the estimated betas are significant at 95% level;
- DOL factors is the only factor that has significant values of β for each portfolio. All loadings on this factor are positive.
- HML factor is significant for both low interest portfolio (Portfolio 1) and high interest portfolio. The price of risk is positive. Both the low interest portfolio and portfolio 2 have negative betas - meaning these portfolios lose money when Carry portfolio gains positive returns. High interest portfolio has positive beta.
- The price of commodity risk proxied by GSCI excess returns is positive and higher than unity. All portfolios have negative loadings on it, albeit none of the estimated betas are significant.
- S&P500 excess returns used as a risk factor has a positive risk price (of 0.4 when used together with DOL and HML risk factors) but negative loadings in portfolio

1 and 3. However, none of the loadings are statistically significant.

- Bond market risks have all insignificant betas. Overall, after including bond market risk factors, their estimated risk premiums take some of the value from DOL factor. However, DOL factors have always significant estimates of quantity of risk for each portfolio, while bond risk factors do not.

Table 15: Fama-MacBeth Estimation of cross-sectional asset pricing multi-factor model

Factors	DOL	HML	VOL	J-stat	p-value
Factor λ	0.031* (0.007)	0.095* (0.014)	-0.016* (0.002)	10.067	0.997
Portfolio 1 β	0.980* (0.090)	-0.385* (0.043)	2.542 (3.003)		
Portfolio 2 β	1.050* (0.082)	-0.033 (0.034)	-1.120 (1.716)		
Portfolio 3 β	1.085* (0.087)	0.579* (0.046)	2.463 (3.023)		

Notes: portfolio 1 represent currency portfolio where an investor goes long the lowest futures discount currency. Portfolio 3 is investment in highest futures discount currencies. Standard errors are reported in parentheses. We also report the result of the null hypothesis SarganâHansen J-test that states that H_0 : the model is valid and the alternative is H_A : the model is invalid. The test is a test for over-identifying restrictions. For convenience, we report the p-value of the J-test using 95% as a confidence level. λ denotes the price of risk, and β denotes the amount of risk (risk exposure).

* - indicates significant value at 95% confidence level.

Overall, as in Lustig, Roussanov, & Verdelhan (2011) we find that two factors DOL, the dollar risk factor, and HML, the high-minus-low risk factor, have good explanatory power for the model. Foreign exchange volatility represents risk, but we could not obtain significant quantities for risk exposures for any portfolio on this factor using cross-sectional asset pricing test. Lastly, the J-test rejects the model, leaving unanswered the question of what risk factors are associated with foreign exchange risk premium.

4. CONCLUSIONS

In this thesis we pursued the goal of connecting the failure of the uncovered interest parity condition with existence of time-varying risk premium. Unlike the absolute majority of other studies on the topic, we used futures rates instead of forward rates in testing the condition. Indeed, we found that the UIP condition does not hold well for more than half of the exchange rate pairs when we use futures rates. The results of a panel version of the UIP regression that we undertake are robust to cross-sectional correlation in errors and to panel heteroskedasticity. We make sure that all the panel data conforms to Gauss-Markov assumptions, that is why we exclude maturities that exhibit serial correlation and perform Cochrane-Orcutt transformation for currencies where all cross-sectional units exhibit temporal correlation.

The use of futures rates provides additional testable settings that enable us to find evidence of time-variability of the risk-premium in foreign exchange returns. We find that under a reasonable assumption that both risk premium and rational expectations bias are present, the risk-premium term must be time-varying. Our result assumes that the problem of rational learning is behind the bias in rational expectations hypothesis, but the model can be extended to the Peso problem and to some extent, to the problem of rational bubbles. The latter will be applicable only if the bubbles do not burst during the time period of a futures contract. The use of futures data also enables us to modify the UIP regression to accommodate the risk premium factor and estimate the model coefficients using common correlated effects pooled estimator of Pesaran (2006). New UIP model has significantly better fit, and the point estimates of the model coefficient got closer to unity, indicating that the coefficient in the original UIP model is biased due to omitting the important risk variable.

We found as well that the FX volatility is a risk factor priced in carry trade returns, even though our time-series asset pricing model has a significant alpha. High interest currencies have lower values of coskewness with stock market and foreign exchange market indicating that when stock or FX market volatility increases, the high-interest currencies earn smaller returns. The opposite is found for low interest currencies signaling that they act as a hedge against market turmoils. Moreover, the cross-section of returns to portfolios sorted on interest rate differential is found to be partially explained by risk factors associated with funding liquidity (VIX implied volatility index and TED spread) and the "dollar risk factor", confirming the findings of Menkhoff et al. (2012), Christiansen, Rinaldo, & Soderlind (2011), Brunnermeier, Nagel, & Pedersen (2009) and Lustig, Roussanov, & Verdelhan (2011). The funding liquidity risk is negatively related to returns on portfolios so that when the global risk aversion increases or when the TED spread increases, currency pairs with higher interest rate differential

lose money. These findings support the view that higher values of VIX index or TED spread are related to tighter funding liquidity, which forces a reduction in speculating positions on currencies with high interest rates. Such reductions can lead to massive sell offs in high interest currencies, which would explain negative skewness of carry returns.

The economic importance of the results is significant. First of all, our findings indicate that interest rates play an important role in explaining cross-section of currency returns. Secondly, we find evidence that returns in high-interest currencies are driven by exposure to market volatility and funding liquidity systemic risk factors. In this case, the failure of the uncovered interest parity can well be attributed to the omitted variable bias. The null hypothesis that is commonly tested in the literature only includes interest rate differential as a regressor, whereas true model seems to have also the risk premium as a second regressor. Lastly, this thesis provides evidence on the use of futures rates in testing the UIP condition and performing asset pricing tests.

The finding of the relationship between volatility shocks and excess returns to carry trade portfolios might in fact capture a broader array of innovations in state variables that are important to the risk-return profiles of investors. That is why we hope that future research can shed light on what particular state variables are associated with these risk premiums. Moreover, since there is still no asset pricing model explaining abnormal currency carry returns, we expect more work in this area to be done. The resolution of the uncovered interest parity will require modifications to the UIP theoretical model to account for risk premium term, and we are looking forward to seeing more research in this domain. Lastly, we expect to see studies similar to ours that use the data on futures rates, but that include a broader set of currencies. That will give a more general result to the phenomenon of futures premium puzzle that we documented. To conclude the thesis, we will state once again that the failure of uncovered interest parity can be attributed most likely to a misspecification of the UIP model, and there is still a lot of work needs to be done to specify the correct model.

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Appendix 1**DERIVATION OF UIP CONDITION IN LOGARITHMIC FORM**

A well-known Uncovered Interest Parity condition postulates that a currency appreciation should happen only if the interest rate of the country is lower than in the foreign country:

$$\frac{(1 + r_i)}{(1 + r^*)} = \frac{E_t[S_{t+1}]}{S_t}$$

Taking the logs of both sides of the equation yields:

$$\ln \left[\frac{(1 + r_i)}{(1 + r^*)} \right] = \ln \left[\frac{E_t[S_{t+1}]}{S_t} \right] \Rightarrow \ln(1 + r_i) - \ln(1 + r^*) = \ln(E_t[S_{t+1}]) - \ln(S_t)$$

Natural logarithm of the sum of one and the interest rate expressed in decimal form is close in value to the interest rate, that is $\ln(1 + r) = r$.

Next, using basic information about lognormal distributions, we can rewrite the expected value of the next period exchange rate assuming the exchange rate is lognormally distributed:

$$E_t[S_{t+1}] = \exp \left(E_t[s_{t+1}] + \frac{1}{2} \sigma_t^2(s_{t+1}) \right)$$

where $s_{t+1} = \ln(S_{t+1})$. Incorporating it result into the UIP equation results in the following logarithmic representation:

$$r_i - r^* = E_t[s_{t+1}] - s_t + \frac{1}{2} \sigma_t^2(s_{t+1})$$

Appendix 2

MATURITY-SPECIFIC FIXED EFFECTS

To check whether there are any maturity-specific fixed effects in the data, we construct the following box-plots.

Response variable is the left-hand side of the main regression model we use in our analysis:

$$s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + \varepsilon_t$$

where subscript t refers to a current date, m stands for the number of weeks to maturity of the future contract and k refers to a specific future contract.

Upon looking at the figure A.1, we can note, that different maturities do not carry any specific effect on the spot exchange rate differential. This finding supports the application of pooled OLS regression model with the futures data, as well as facilitates estimation of model parameters at currency pair levels, and not at maturity levels. Thus, we are interested in estimating β and α for each currency across all available maturities, and are not interested in estimating these parameters for each maturity.

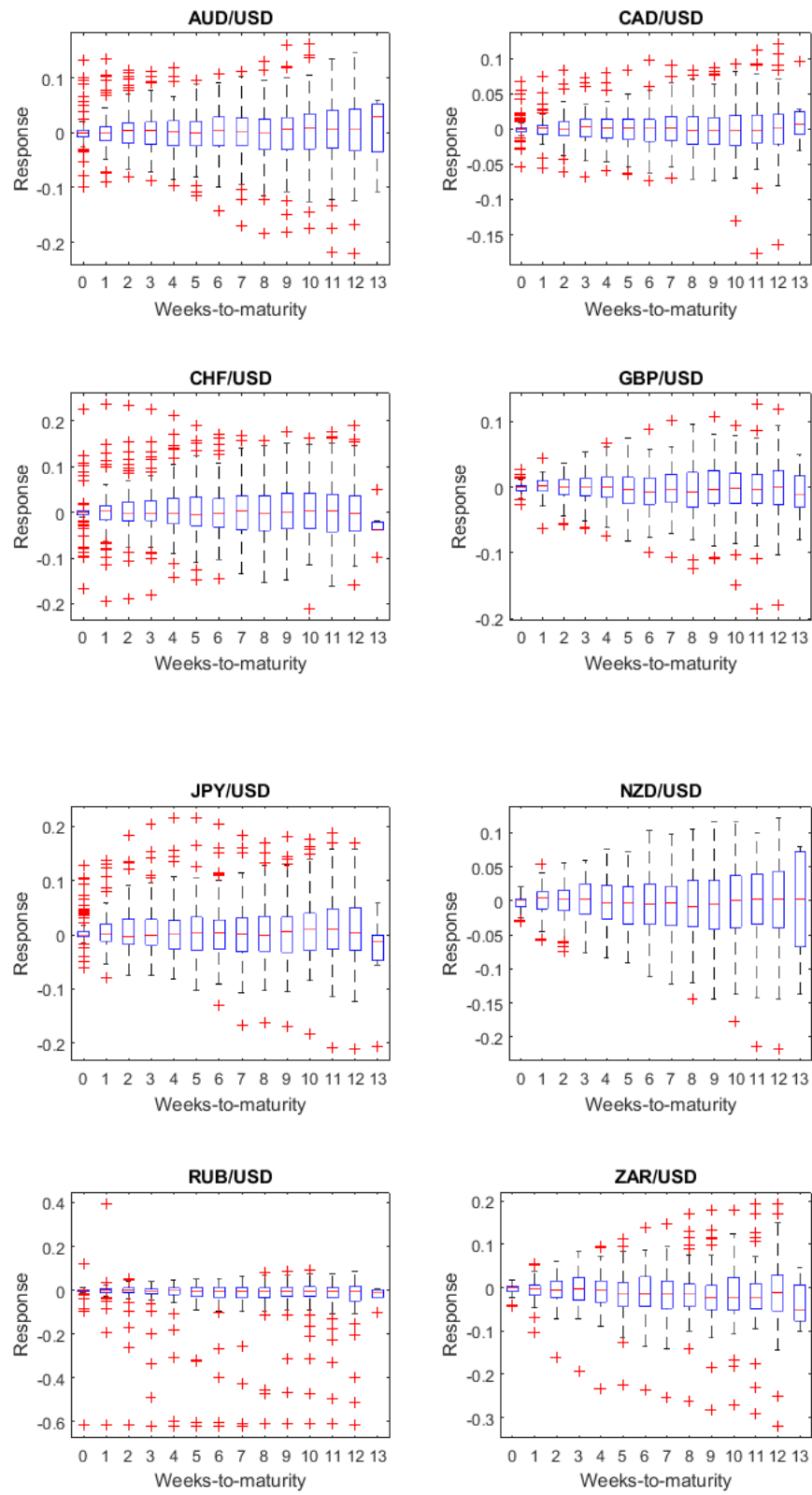


Figure A.1: Spot returns grouped by maturity

Appendix 3

ILLUSTRATION OF SLOPE ESTIMATES BY MATURITY

In this thesis we apply the following panel version of Fama regression:

$$s_{t+m}^k - s_t = \alpha + \beta(f_{t,t+m}^k - s_t) + \varepsilon_t$$

where f refers to a future contract on a given currency pair and m stands for a number of days to maturity of the contract k . To see how the slope coefficient β varies with maturity, we perform time-series OLS regressions for each maturity, and estimate the slope coefficient together with its standard errors. Following the procedure of Newey & West (1987), we compute standard errors that are robust to serial correlation in residuals. To illustrate the variability of the slope coefficient, we provide figure A.2.

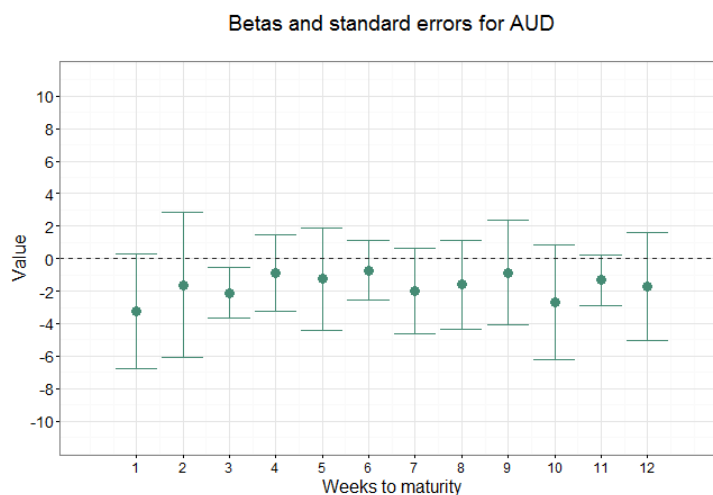


Figure A.2 (a) AUD/USD

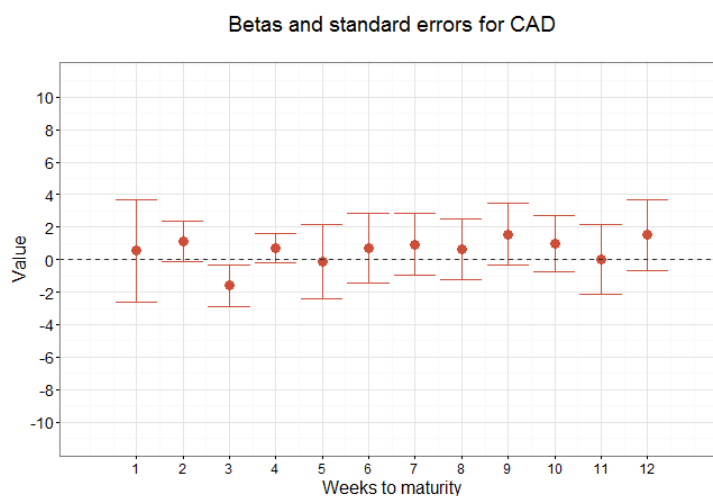


Figure A.2 (b) CAD/USD

Betas and standard errors for CHF

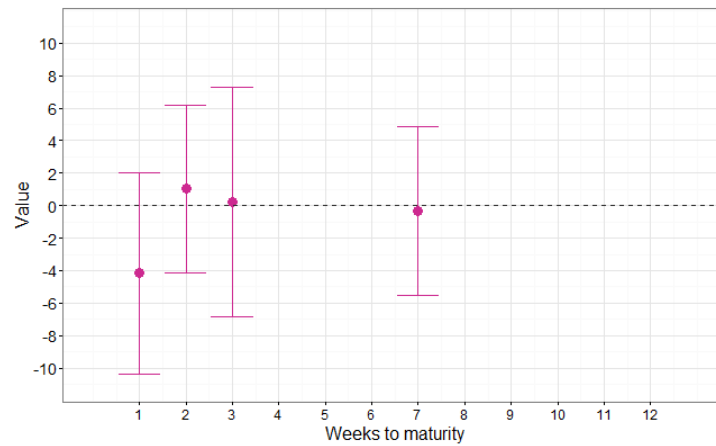


Figure A.2 (c) CHF/USD

Betas and standard errors for GBP

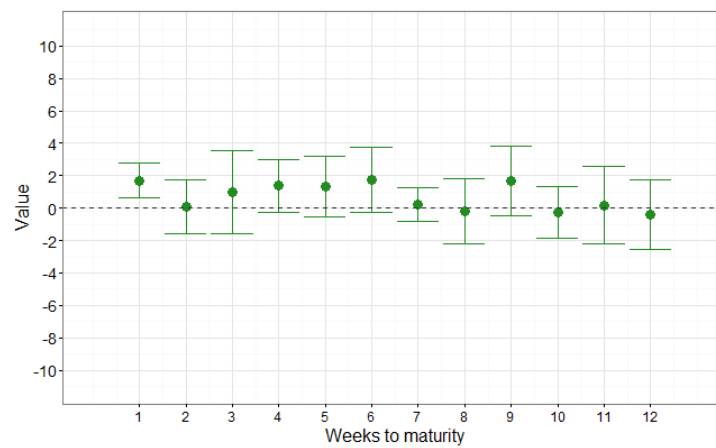


Figure A.2 (d) GBP/USD

Betas and standard errors for JPY

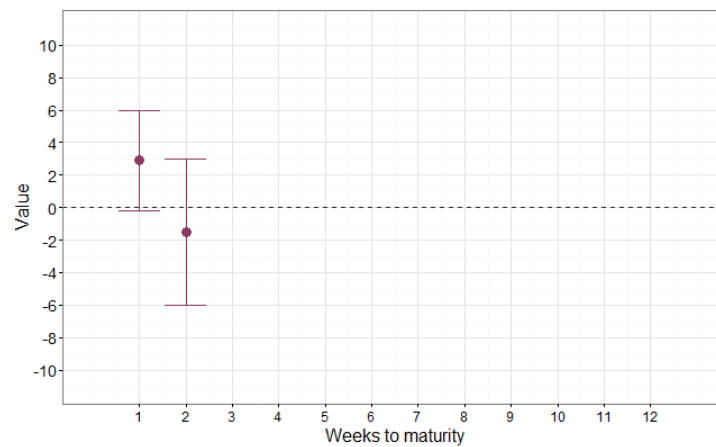


Figure A.2 (e) JPY/USD

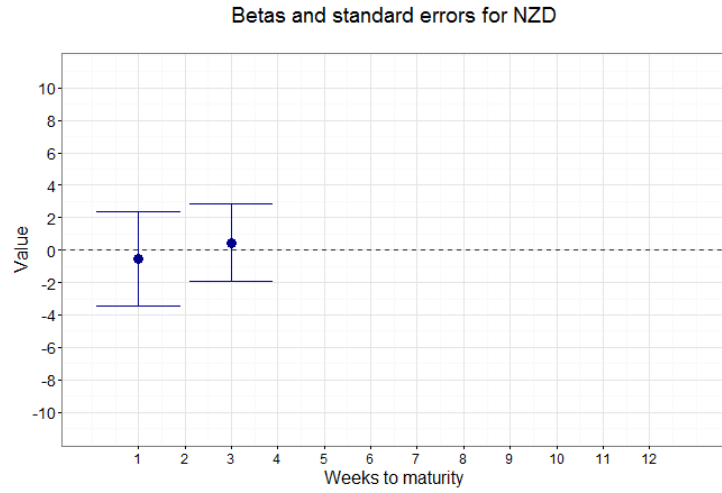


Figure A.2 (f) NZD/USD

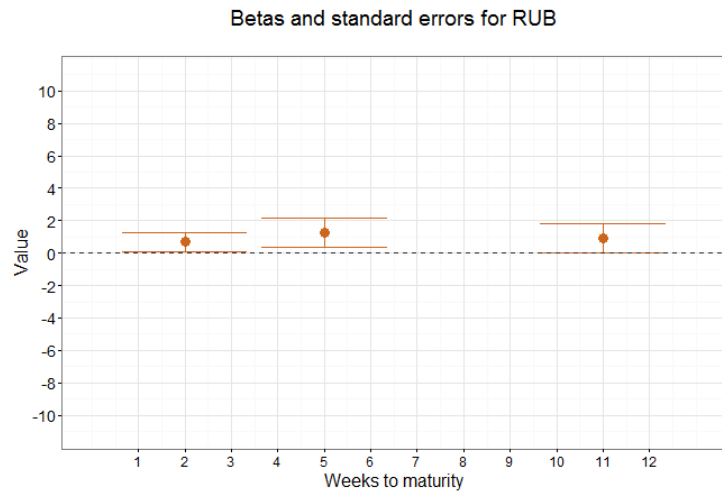


Figure A.2 (g) RUB/USD

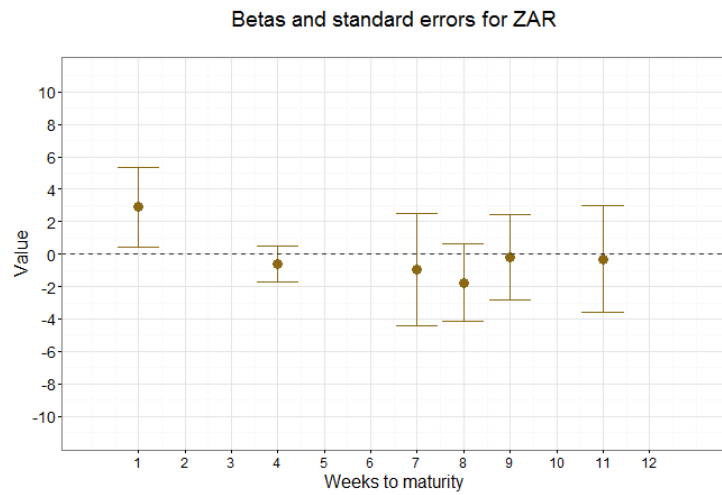


Figure A.2 (h) ZAR/USD

Figure A.2: Estimated slope coefficients with autocorrelation-robust standard errors

Appendix 4**UNIT ROOT TESTS**

The following table display results of Augmented Dickey-Fuller tests for each currency pair. We test the spot returns which are defined as a spot rate at day k of futures contract expiration, minus the spot rate at a current day. We also test if futures premium is stationary, where futures premium is defined as today's futures price for a contract that expires on day k minus the same day's spot rate. Description of the table follows.

As a note for the table, all tests are performed at 95% confidence level. Maturity refers to weeks-to-maturity until futures contract expiration. The model abbreviations are as follows: AR(x) denotes autoregressive process of order x , ARD(x) is autoregressive process with drift, TS(x) is a trend stationary process of order x . We fit one of these models into Augmented-Dickey Fuller test based on AIC and BIC criteria. Then we report p-values of ADF test. together with Dickey-Fuller test-statistic and its critical value for a given number of degrees of freedom.

Table A.1: Augmented Dickey-Fuller unit root tests for spot returns and futures premiums

		(a) AUD/USD											
Maturity		1	2	3	4	5	6	7	8	9	10	11	12
$s_{t+m}^k - s_t$	Model	AR(3)	AR(7)	AR(6)	AR(1)	AR(1)	AR(1)	AR(4)	AR(4)	AR(4)	AR(6)	AR(6)	AR(6)
	Test p-value	0.001	0.003	0.016	0.001	0.001	0.001	0.001	0.001	0.004	0.001	0.001	0.001
	DF test statistic	-4.312	-3.149	-2.425	-4.606	-4.497	-4.851	-3.324	-3.011	-3.497	-3.459	-3.447	-3.498
	DF critical value	-1.945	-1.945	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.945	-1.945
$f_{t,t+m}^k - s_t$	Model	AR(2)	ARD(3)	ARD(2)	ARD(4)	AR(4)	TS(1)	AR(2)	AR(3)	ARD(2)	AR(2)	AR(3)	ARD(5)
	Test p-value	0.001	0.006	0.002	0.001	0.041	0.001	0.027	0.009	0.017	0.027	0.007	0.049
	DF test statistic	-3.659	-3.672	-4.094	-4.391	-2.032	-5.351	-2.212	-2.653	-3.316	-2.204	-2.734	-2.899
	DF critical value	-1.944	-2.891	-2.891	-2.892	-1.944	-3.457	-1.944	-1.944	-2.890	-1.944	-1.944	-2.893
Observations	100	100	100	100	100	100	99	99	99	102	99	99	97
		(b) CAD/USD											
Maturity		1	2	3	4	5	6	7	8	9	10	11	12
$s_{t+m}^k - s_t$	Model	AR(2)	AR(2)	AR(2)	AR(2)	AR(1)	AR(2)	AR(4)	AR(4)	AR(4)	AR(1)	AR(1)	AR(2)
	Test p-value	0.001	0.001	0.001	0.001	0.001	0.001	0.003	0.002	0.001	0.001	0.001	0.001
	DF test statistic	-4.322	-4.360	-4.022	-3.920	-5.439	-4.032	-3.042	-3.172	-3.403	-7.080	-6.764	-4.863
	DF critical value	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944
$f_{t,t+m}^k - s_t$	Model	ARD(3)	AR(8)	AR(3)	AR(3)	AR(1)	AR(3)	AR(2)	AR(2)	AR(7)	AR(2)	AR(2)	AR(12)
	Test p-value	0.001	0.005	0.001	0.001	0.001	0.003	0.001	0.004	0.013	0.005	0.004	0.006
	DF test statistic	-4.920	-2.868	-3.506	-4.555	-3.337	-3.157	-3.364	-3.012	-2.489	-2.894	-2.950	-2.781
	DF critical value	-2.891	-1.945	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.945
Observations	100	100	100	100	100	100	100	100	100	100	99	99	97

(c) CHF/USD

	1	2	3	4	5	6	7	8	9	10	11	12
Maturity												
$s_{t+m}^k - s_t$	AR(3)	AR(5)	AR(2)	AR(2)	AR(1)	AR(1)	AR(2)	AR(3)	AR(2)	AR(1)	AR(1)	AR(7)
Test p-value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
DF test statistic	-5.411	-3.529	-5.032	-4.688	-5.093	-5.491	-5.088	-4.724	-4.721	-5.987	-5.995	-4.283
DF critical value	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.945
$f_{t,t+m}^k - s_t$	AR(4)	AR(5)	AR(2)	AR(5)	ARD(5)	AR(4)	AR(1)	AR(3)	AR(1)	AR(2)	AR(1)	AR(2)
Test p-value	0.004	0.017	0.011	0.090	0.134	0.088	0.026	0.132	0.063	0.154	0.071	0.116
DF test statistic	-2.918	-2.396	-2.549	-1.634	-2.439	-1.677	-2.216	-1.467	-1.837	-1.382	-1.785	-1.539
DF critical value	-1.944	-1.944	-1.944	-1.944	-2.892	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944
Observations	100	100	100	100	100	100	100	100	100	99	99	97

(d) GBP/USD

	1	2	3	4	5	6	7	8	9	10	11	12
Maturity												
$s_{t+m}^k - s_t$	AR(1)	AR(5)	AR(1)	AR(1)	AR(1)	AR(2)	AR(1)	AR(5)	AR(1)	AR(2)	AR(1)	AR(2)
Test p-value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.003	0.001	0.001	0.001	0.001
DF test statistic	-6.929	-4.303	-6.835	-7.277	-7.787	-6.352	-7.646	-3.101	-7.582	-6.550	-8.321	-7.019
DF critical value	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944
$f_{t,t+m}^k - s_t$	AR(6)	AR(3)	AR(2)	AR(3)	AR(2)	AR(4)	AR(4)	AR(1)	AR(9)	AR(1)	AR(1)	AR(2)
Test p-value	0.001	0.001	0.001	0.005	0.002	0.005	0.003	0.001	0.001	0.001	0.004	0.002
DF test statistic	-6.274	-4.079	-4.608	-2.832	-3.209	-2.833	-3.050	-3.475	-3.872	-3.378	-2.980	-3.101
DF critical value	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944
Observations	104	104	104	101	107	104	103	103	106	103	103	101

(e) JPY/USD

Maturity	1	2	3	4	5	6	7	8	9	10	11	12
$s_{t+m}^k - s_t$	AR(9)	AR(2)	AR(2)	AR(4)	AR(5)	AR(4)	AR(10)	AR(5)	AR(5)	AR(5)	AR(9)	AR(9)
Test p-value	0.012	0.001	0.001	0.001	0.002	0.004	0.043	0.002	0.002	0.001	0.005	0.005
DF test statistic	-2.533	-3.993	-3.828	-3.309	-3.171	-2.993	-2.016	-3.255	-3.233	-3.429	-2.814	-2.832
DF critical value	-1.945	-1.944	-1.944	-1.944	-1.944	-1.944	-1.945	-1.944	-1.944	-1.944	-1.945	-1.945
$f_{t,t+m}^k - s_t$	AR(2)	AR(2)	AR(4)	AR(1)	ARD(9)	AR(2)	AR(4)	AR(2)	AR(5)	AR(4)	AR(4)	AR(1)
Test p-value	0.001	0.034	0.121	0.062	0.146	0.155	0.164	0.119	0.247	0.123	0.193	0.225
DF test statistic	-4.589	-2.110	-1.515	-1.847	-2.392	-1.377	-1.348	-1.523	-1.098	-1.505	-1.249	-1.160
DF critical value	-1.944	-1.944	-1.944	-1.944	-2.894	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944	-1.944
Observations	100	100	100	100	100	100	100	100	100	99	99	97

(f) NZD/USD

Maturity	1	2	3	4	5	6	7	8	9	10	11	12
$s_{t+m}^k - s_t$	AR(1)	AR(1)	AR(1)	AR(1)	AR(1)	AR(6)	AR(1)	AR(6)	AR(5)	AR(6)	AR(1)	AR(6)
Test p-value	0.001	0.001	0.001	0.001	0.001	0.003	0.001	0.007	0.012	0.004	0.001	0.003
DF test statistic	-5.123	-4.926	-5.348	-4.742	-4.847	-3.057	-4.913	-2.757	-2.517	-2.983	-4.921	-3.008
DF critical value	-1.945	-2.905	-1.945	-1.945	-1.945	-1.946	-1.945	-1.946	-1.946	-1.946	-1.945	-1.946
$f_{t,t+m}^k - s_t$	AR(2)	AR(4)	AR(2)	AR(5)	AR(4)	AR(4)	AR(2)	AR(5)	AR(7)	AR(5)	AR(3)	AR(1)
Test p-value	0.007	0.137	0.017	0.084	0.204	0.424	0.212	0.404	0.529	0.332	0.257	0.119
DF test statistic	-2.721	-1.444	-2.400	-1.697	-1.215	-0.612	-1.191	-0.667	-0.324	0.864	-1.069	-1.522
DF critical value	-1.945	-1.945	-1.945	-1.945	-1.945	-1.945	-1.945	-1.946	-1.946	-1.945	-1.945	-1.945
Observations	71	71	71	71	71	70	70	70	70	70	70	68

(g) RUB/USD

	1	2	3	4	5	6	7	8	9	10	11	12
Maturity												
$s_{t+m}^k - s_t$	AR(4)	AR(1)	AR(4)	TS(9)	AR(2)	AR(4)	AR(6)	AR(11)	AR(10)	AR(8)	AR(2)	AR(7)
Test p-value	0.279	0.048	0.820	0.970	0.001	0.085	0.428	0.659	0.804	0.690	0.001	0.549
DF test statistic	-1.009	-1.962	0.497	-0.610	-5.441	-1.694	-0.600	0.033	0.434	0.116	-3.997	-0.268
DF critical value	-1.946	-1.946	-1.946	-3.488	-1.946	-1.946	-1.946	-1.946	-1.946	-1.946	-1.946	-1.946
$f_{t,t+m}^k - s_t$	AR(1)	AR(3)	AR(4)	AR(5)	ARD(4)	AR(2)	ARD(4)	ARD(3)	AR(5)	AR(3)	AR(5)	AR(5)
Test p-value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.006	0.007
DF test statistic	-7.376	-3.433	-5.346	-3.843	-6.669	-4.396	-7.322	-6.047	-4.862	-9.180	-2.809	-2.755
DF critical value	-1.946	-1.945	-1.946	-1.946	-2.912	-1.946	-2.912	-2.912	-1.946	-1.946	-1.946	-1.946
Observations	65	65	66	66	66	66	66	65	65	65	65	63

(e) ZAR/USD

	1	2	3	4	5	6	7	8	9	10	11	12
Maturity												
$s_{t+m}^k - s_t$	AR(3)	AR(10)	AR(5)	AR(3)	AR(9)	AR(9)	AR(9)	AR(10)	AR(1)	AR(1)	ARD(1)	AR(2)
Test p-value	0.001	0.042	0.004	0.001	0.004	0.005	0.007	0.047	0.001	0.001	0.001	0.001
DF test statistic	-4.099	-2.020	-2.931	-3.730	-2.991	-2.844	-2.710	-1.967	-5.561	-5.945	-6.082	-3.486
DF critical value	-1.945	-1.946	-1.945	-1.945	-1.946	-1.946	-1.946	-1.946	-1.945	-1.945	-2.906	-1.945
$f_{t,t+m}^k - s_t$	AR(1)	AR(3)	AR(3)	AR(1)	AR(2)	AR(1)	ARD(7)	ARD(4)	ARD(1)	ARD(2)	ARD(1)	ARD(3)
Test p-value	0.001	0.090	0.143	0.020	0.164	0.093	0.014	0.004	0.043	0.107	0.034	0.085
DF test statistic	-3.795	-1.638	-1.421	-2.221	-1.345	-1.648	-3.415	-3.871	-2.973	-2.560	-3.066	-2.669
DF critical value	-1.945	-1.945	-1.945	-1.945	-1.945	-1.945	-2.912	-2.909	-2.905	-2.907	-2.906	-2.910
Observations	71	71	71	71	71	70	69	70	71	70	70	68

Appendix 5

TESTS OF SERIAL AUTOCORRELATION IN RESIDUALS

Table A.2: Tests of serial correlation in residuals

	WTM	DW stat.	DW p-value	LM stat.	BG p-value	W stat.	SW p-value
CHF	1	1.133	0	19.759	0	19.759	0
	2	1.082	0	20.926	0	0.892	0
	3	1.215	0	15.472	0	15.472	0
	7	1.345	0.002	11.032	0.001	0.922	0
JPY	1	1.556	0.038	4.813	0.028	0.896	0
	2	1.755	0.198	1.353	0.245	0.896	0
NZD	1	1.779	0.370	0.872	0.350	0.977	0.204
	3	2.108	0.628	0.223	0.637	0.968	0.064
RUB	2	1.459	0.074	2.530	0.112	0.393	0
	5	0.497	0	22.947	0	0.514	0
	11	0.986	0	2.572	0.109	0.618	0
ZAR	1	1.688	0.164	1.494	0.222	0.979	0.270
	4	1.592	0.062	2.782	0.095	0.901	0
	7	1.654	0.122	1.966	0.161	0.961	0.027
	8	1.898	0.610	0.156	0.693	0.946	0.004
	9	1.896	0.560	0.174	0.677	0.923	0
	11	1.874	0.536	0.265	0.606	0.911	0

Notes: all tests are performed at 95% confidence level. WTM stands for weeks-to-maturity - the number of weeks until futures contract matures. They represent cross-section of the UIP panel model with futures rates. DW stat denotes a Durbin-Watson test statistic for the Durbin-Watson test with the null H_0 that there is no correlation among residuals and H_A that residuals are autocorrelated. LM stat denotes test statistic for Breusch-Godfrey LM test for serial correlation of order 1. The null H_0 is that there is no serial correlation of order 1. BG p-value represents the p-value of the Breusch-Godfrey LM test statistic. W stat is the W statistic of Shapiro-Wilk normality test, whereas SW p-value is its p-value. The SW test has the null H_0 that states that the sample is drawn from normal distribution.

Table A.3: Tests of serial correlation in residuals for AUD, CAD and GBP

(a) AUD/USD												
Maturity	1	2	3	4	5	6	7	8	9	10	11	12
DW test statistic	1.184	1.675	1.605	1.708	1.693	1.643	1.832	1.733	1.930	1.594	1.493	1.396
DW test p-value	0	0.106	0.046	0.108	0.116	0.064	0.330	0.154	0.662	0.036	0.012	0.004
LM test statistic	17.061	2.417	3.551	1.988	2.155	2.804	0.422	1.513	0.077	3.962	7.237	8.530
BG test p-value	3.62E-05	0.120	0.060	0.159	0.142	0.094	0.516	0.219	0.781	0.047	0.007	0.003
W statistic	0.886	0.948	0.959	0.977	0.988	0.980	0.981	0.975	0.978	0.985	0.964	0.969
SW p-value	3.43E-07	0.001	0.003	0.077	0.512	0.144	0.153	0.054	0.086	0.345	0.008	0.023
(b) CAD/USD												
Maturity	1	2	3	4	5	6	7	8	9	10	11	12
DW test statistic	1.847	1.777	1.559	1.817	1.618	1.610	1.805	1.533	1.647	1.539	1.407	1.551
DW test p-value	0.438	0.222	0.022	0.384	0.054	0.052	0.308	0.008	0.064	0.018	0.002	0.010
LM test statistic	0.433	1.049	4.205	0.564	3.381	3.493	0.765	5.132	2.899	5.082	8.409	4.375
BG test p-value	0.511	0.306	0.040	0.453	0.066	0.062	0.382	0.023	0.089	0.024	0.004	0.036
W statistic	0.988	0.936	0.952	0.970	0.976	0.983	0.986	0.987	0.987	0.957	0.931	0.946
SW p-value	0.512	1.18E-04	0.001	0.024	0.068	0.218	0.346	0.422	0.422	0.003	6.06E-05	0.001

(C) GBP/USD

Maturity	1	2	3	4	5	6	7	8	9	10	11	12
DW test statistic	2.142	2.124	2.237	2.309	2.292	2.021	1.921	1.836	1.903	1.892	1.811	1.807
DW test p-value	0.476	0.512	0.23	0.132	0.162	0.902	0.706	0.378	0.546	0.524	0.286	0.334
LM test statistic	0.632	1.125	1.962	2.955	2.405	0.02	0.158	0.621	0.233	0.247	0.843	0.761
BG test p-value	0.427	0.289	0.161	0.086	0.121	0.888	0.691	0.431	0.630	0.619	0.359	0.383
W statistic	0.949	0.969	0.991	0.995	0.990	0.991	0.991	0.988	0.993	0.981	0.961	0.974
SW p-value	0.001	0.015	0.694	0.97	0.633	0.719	0.716	0.488	0.877	0.152	0.004	0.040

Notes: all tests are performed at 95% confidence level. Maturity stands for weeks-to-maturity - the number of weeks until futures contract matures. They represent cross-section of the UIP panel model with futures rates. DW test statistic denotes a Durbin-Watson test statistic for the Durbin-Watson test with the null H_0 that there is no correlation among residuals and H_A that residuals are autocorrelated. LM test statistic denotes test statistic for Breusch-Godfrey LM test for serial correlation of order 1. The null H_0 is that there is no serial correlation of order 1. BG test p-value represents the p-value of the Breusch-Godfrey LM test statistic. W statistic is the W statistic of Shapiro-Wilk normality test, whereas SW p-value is its p-value. The SW test has the null H_0 that states that the sample is drawn from normal distribution.

Appendix 6

COCHRANE-ORCUTT TRANSFORMATION FOR THE SWISS FRANC

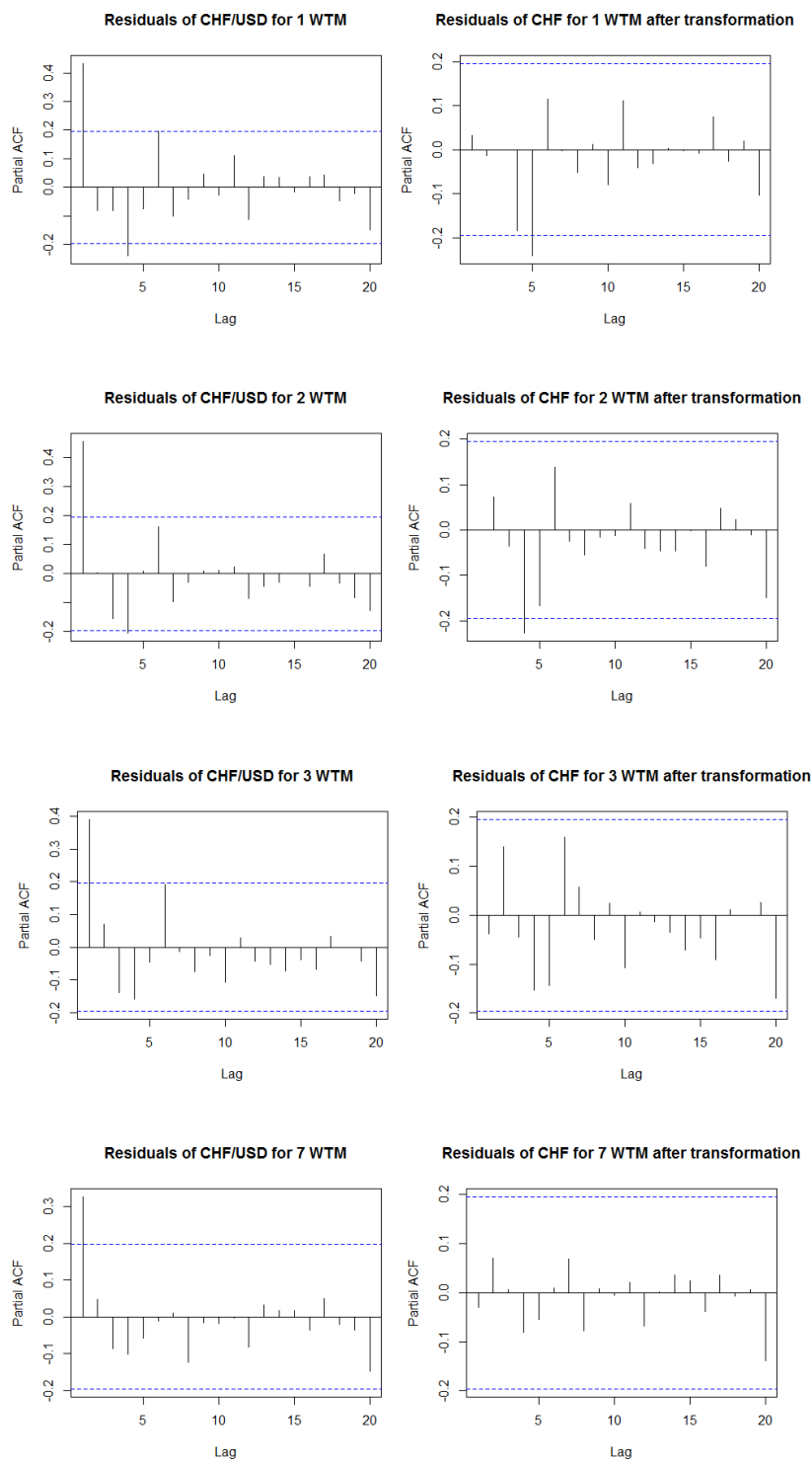


Figure A.3: The effect of Cochrane-Orcutt transformation