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LOW RISK INVESTING AND RISK PARITY

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Low risk investing and risk parity

Abstract

This thesis finds evidence of the outperformance of the risk parity (RP) strategies in comparison to the traditional equal-weighted portfolios. The empirical study focuses on backtesting the portfolio strategies by using two datasets, a long sample and a broad sample. The long sample data consists of U.S. common stocks listed in NYSE, AMEX and NASDAQ as well as U.S. government bonds over January 1929 to December 2015. The broad sample consists of global multi-asset index data including stocks, bonds, credit, commodities, real estate and hedge funds over January 2002 to December 2015.

Risk parity refers to the asset allocation strategy that diversifies by risk, not by dollars. As stocks are much more volatile than bonds, traditionally diversified portfolios such as equal-weighted portfolio or market capitalization-weighted portfolio are most likely dominated by risks raising from equity markets. An optimal RP portfolio consists of equal risk contribution between and within asset classes. Put simply, a RP investor overweights low risk assets and underweights high risk assets.

The main objective of the thesis is to evaluate the performance of two RP strategies, the inverse volatility method and the equal risk contribution method, in comparison to the equal-weighted portfolios. As a RP portfolio typically has a heavy allocation in low risk assets, the strategy requires leverage to raise the expected return to desired levels. Hence, this thesis focuses on analysis of both strategies, leveraged and unleveraged RP portfolios. The main analysis is carried out in several phases including market friction adjusted and unadjusted analyses. In addition, the strategies are tested in different interest rate environments. The performance of the portfolios is measured by realized Sharpe ratios. The study also observes abnormal returns by using a simple regression model.

The key findings of the study are as follows: The RP strategies outperform the traditional equal-weighted portfolios on risk-adjusted basis after adjustments for market frictions. The unleveraged RP portfolios deliver higher Sharpe ratios than leveraged portfolios. However, the leveraged RP portfolio still achieves a higher Sharpe ration in comparison to the equal-weighted portfolio. The RP strategies underperform when interest rates are rising moderately or sharply. The equal risk contribution method outperforms the inverse volatility method.

Keywords
Risk parity, low risk anomaly, leverage, asset allocation
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Sincerely,
Teemu Blomqvist
M.Sc. in Economics and Business Administration as of January 2017
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INTRODUCTION

1.1 Background and motivation

This research shows that risk parity (RP) portfolios have outperformed traditional equal-weighted (EW) portfolios over the long-term on risk-adjusted basis. The standard asset allocation advice provided by the capital asset pricing model (CAPM) found by Sharpe (1964), Lintner (1965) and Mossin (1966) states that an investor should invest in the market portfolio, leveraged according to the preferred risk level. However, the recent trend towards risk-based asset allocation approaches has gathered popularity among investor community as these strategies are considered to be more diversified in terms of how risk is allocated across and within asset classes. As equities are much more volatile than bonds, the overall risk of the market portfolio is dominated by stock market fluctuations. Hence, the market portfolio is basically a pure equity portfolio when viewed from a risk perspective as the major proportion of the portfolio’s volatility is explained by the movements in equity markets. The risk-based asset allocation, also known as risk parity, suggests that investors should diversify by risk, not by dollars. Given that equities and bonds carry similar amount of risk in the portfolio, a RP investor generally needs to hold more low risk assets than high risk assets. The risk-balanced portfolio which overweights low risk assets and underweights high risk assets outperforms the traditionally diversified portfolios on a risk-adjusted basis, but achieves lower returns in absolute terms due to high allocation in low risk assets. This issue can be solved by levering up the portfolio risk to desired levels. However, it is not an option for all investors as they face leverage restrictions or are simply leverage averse. (Asness, Frazzini & Pedersen 2012).

Following the establishment of the CAPM, the research by Black, Jensen and Scholes (1972) already highlighted that low risk assets deliver higher risk-adjusted returns in comparison to high risk assets indicating that the security market line which describes the relation between expected returns and risk is too flat relative to the CAPM. Frazzini and Pedersen (2014), and Baker and Haugen (2012) recently argued that low risk assets do not only provide superior risk-adjusted returns, but also outperform high risk assets within several asset classes in absolute terms. According to Ilmanen (2011: 8 - 9), the empirical evidence on the relationship between expected returns and volatility is
inadequate. The author notes that even though volatility and average returns in the long run are positively related across asset classes, the most volatile assets within each asset class, such as stocks with high volatility, CCC-rated corporate bonds and 30-year treasury bonds, tend to provide low returns and even lower risk-adjusted returns in long-term. One reason for this anomaly according to Asness et al. (2012) is the leverage aversion among investors. Since some investors are averse or constrained to applying leverage, portfolio risk levels are typically raised by simply increasing exposure to high risk assets which drives the expected returns of the asset classes. Thus, assets with high risk will offer lower risk-adjusted returns and assets with low risk will offer higher risk-adjusted returns. Frazzini et al. (2014) find consistent evidence and argue that more leverage constrained investors tend to hold securities with higher beta than investors who are less leverage constrained. By applying leverage and overweighting safer assets while underweighting riskier assets an investor who is less leverage averse can earn higher risk-adjusted returns. In summary, leverage aversion breaks the standard CAPM (Asness et al. 2012).

1.2 Objectives and contribution

Main objectives of the thesis are to create a comprehensive overview of risk parity portfolio performance and find out whether this asset allocation approach can deliver superior risk-adjusted returns in comparison to portfolios that are not risk-balanced. In addition, this study evaluates the impact of leverage on portfolio performance. The aim is to rule in the impacts of market frictions such as trading costs and borrowing costs to truly reveal the outperforming strategy. Additionally, this study estimates the performance of the strategies during different interest rate environments to recognize whether the leveraged strategies can compete in rising rate environment.

This research contributes to the existing literature by providing comparative evidence of different risk parity methods whereas majority of earlier studies have focused on comparing only one of the risk parity methods with traditional diversification methods. Additionally, this research provides valuable evidence from practical and academic point of view as it expands the datasets by applying comprehensive multi-asset data.
1.3 Research questions

This research sets four main research questions and four research hypotheses. The hypotheses are presented in the third chapter. The key research questions of the study are as follows:

**Question 1:** Do risk parity strategies deliver higher risk-adjusted returns than a traditional equal-weighted portfolio?

**Question 2:** How does leverage affect to the performance of a risk parity portfolio?

**Question 3:** Can risk parity strategies outperform in a rising rate market environment?

**Question 4:** Does the Equal Risk Contribution (ERC) strategy outperform the Inverse Volatility (IV) strategy?

Conclusions regarding the research questions and the research hypotheses are discussed in the fourth chapter.

1.4 Scope and limitations

This research focuses on two datasets, a long sample and a broad sample. The long sample data consists of U.S. common stocks listed on the NYSE, AMEX and NASDAQ, as well as U.S. government bonds, all obtained from the CRSP database over the period from 1929 to 2015. The broad sample consists of global multi-asset data including stocks, bonds, credit, commodities, real estate and hedge funds over the period of 2002 – 2015. This thesis defines annualized excess returns of the different portfolio strategies over the both periods as well as standard deviations, skewness and kurtosis of monthly returns. No other risk measures are taken into account. Historical excess returns and their statistical significance are also evaluated by using simple regression analysis. Multi-factor regressions are not used in this study.
The focus of the study is to compare the performance of two risk parity strategies with the traditional equal-weighting portfolio strategy as well as to compare risk parity strategies with each other. The multi-asset data is relatively short due to the lack of suitable index data. Thus, some additional out-of-sample evidence would be optimal to make more reliable conclusions on multi-asset RP portfolio performance.

As the existing literature recognizes a number of different kinds of RP strategies, further research of differences of the strategies would be desirable. In addition, this thesis leaves the market-cap weighted portfolio out of the empirical research although it would also be suitable extension to the empirical study.

1.5 Structure of the thesis

This thesis includes five chapters. Introduction familiarizes the reader with the topic and the main objectives of the research. In addition, the first chapter presents the research questions and explains how this thesis contributes to the existing literature. Second chapter opens up the theoretical foundation of the study. The main goal of this chapter is to discuss the most relevant empirical evidence on risk parity performance and cover the main theories behind the risk parity strategies. After reading the chapter, the reader should be able to interpret empirical results of the study. Third chapter presents the data, sets research hypotheses and covers the research methodology. Note that the portfolio backtesting techniques are covered in the earlier chapter and this chapter only discusses the methods to evaluate portfolio performance. The final chapter discusses the conclusions of the study.
2 THEORETICAL FOUNDATION

This chapter discusses several topics such as the modern portfolio theory, traditional portfolio diversification methods and risk parity portfolio construction methods. In addition, the chapter covers constraints that investors face when making investment decision, low-risk anomaly in financial markets and characteristics of financial instruments with embedded leverage as they relate to the above-mentioned portfolio theories. The aim is to familiarize the reader to the fundamental concepts of portfolio theory and theoretical framework as well as demonstrate the techniques that are used in the empirical study. After reading the chapter, the reader should be able to interpret empirical results.

2.1 Modern portfolio theory

2.1.1 Return and risk

Before entering into the more complex issues related to the portfolio theory, it is highly crucial to understand probably the most fundamental terms in finance literature: return and risk. Returns of a portfolio need to be observed over some specific time period and by using some base currency. Moreover, the observations need to be accomplished by using one of the numerous averaging and compounding techniques. (Ilmanen 2011: 59.) Before the demonstration of alternative return calculation techniques, the reader should notice that this research uses monthly return data which is denominated U.S. dollars. Historical returns in every table are annualized and presented in geometric terms. The reason for this is discussed later in this section.

When carrying out empirical research, it is advisable to involve asset returns instead of asset prices. Campbell, Lo and MacKinlay (1997) listed two main reasons for this convention. Firstly, return of an asset is a scale-free and complete summary of the investment opportunity for an average investor. Secondly, return series are easier to handle since they have more attractive statistical properties.

Percentage (simple) returns or continuously compounded (logarithmic) returns are normally used to calculate asset returns. The result of the natural logarithm of the
simple gross return of an asset is called the continuously compounded return or log return. A linkage between simple returns and logarithmic returns can be mathematically demonstrated as follows:

\[ R_{i,t}^c = \ln \left( 1 + R_{i,t} \right) = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \]  

(1)

where \( R_{i,t}^c \) is continuously compounded return of the asset \( i \) while \( R_{i,t} \) marks simple return of the asset \( i \) and \( P_{i,t} \) denotes the price of the asset \( i \). As mentioned, logarithmic returns (or continuously compounded returns) are preferred by academics due to their statistical properties. For instance, cumulative returns of longer time periods are more simple as the procedure enables the returns of different time periods to be summed together. As the equation (1) demonstrates, the logarithmic return of one period is the remainder of two serial logarithmic prices. (Tsay 2005: 2 – 5.)

*When dealing with portfolio returns, the case is not the same when dealing with individual security returns.* The return of a portfolio is the weighted average of the simple returns of the individual assets in the portfolio which can be calculated as follows:

\[ R_{p,t} = \sum_{i=1}^{N} w_{i,t} R_{i,t} \]  

(2)

where \( w_{i,t} \) denotes the weight of the asset \( i \) in the portfolio. *In case of logarithmic returns, the same does not apply.* To be precise, logarithmic returns need to be converted back to the simple returns in order to calculated weighted sums. (Tsay 2005: 2 – 5; Rasmussen 2003: 9 – 11.)

Excess return of an asset or portfolio is the difference between the return of an asset and the return of the benchmark, such as an index or a risk-free rate. The benchmark asset is often assumed to be risk-free rate such as a short term U.S. T-bill return. In
this research, the portfolio returns are reported over a one-month T-bill rate if not stated otherwise.\(^1\) The excess return of the portfolio is calculated as follows:

\[
R_{p,t}^e = R_{p,t} - R_{b,t}
\]  

(3)

where \(R_{b,t}\) marks the simple return of the benchmark asset. (Tsay 2005: 6 – 7.)

Finally, the most crucial distinction must be made between arithmetic and geometric average returns. In this study, the historical returns are presented in geometric terms as they better capture wealth compounding. Put differently, they present investors’ realized buy-and-hold returns. It should be noted that geometric mean (GM, compounded average returns) is always lower than arithmetic mean (AM), and the difference between GM and AM widens when dealing with high-volatility assets.\(^2\) (Ilmanen 2011; 59 – 60.)

In contrast to the previous return calculation issues, determination of risk is slightly complex and contains bunch of variables and statistical methods. In practice, returns are commonly assumed to be symmetrically distributed (i.e. following normal distribution) meaning that there is equal probability to realize either positive or negative returns. Hence, return is most probably equal to the mean of the sample. Variance measures the spread between the numbers in the dataset. It simply measures the variability from the sample mean. Volatility (i.e. standard deviation), which is the most common measure of risk, can be calculated by taking square root of the variance. The variance (4) and the volatility (5) are calculated as follows:

\(^1\) When cumulating excess returns, the index returns and the risk-free asset returns are cumulated separately before subtracting the difference of the returns.

\(^2\) For instance, the arithmetic average of two successive returns of +25 \% and -25\% is 0\%, but the geometric average is \(\approx -3.20\% \approx ((1+0.25)*(1-0.25))^{1/2} = 0.968\), where \(n = \text{number of periods} = 2\). The most fundamental theories in finance literature such as mean variance optimization and the single-period capital asset pricing model (CAPM) require AMs as inputs even though the most of investors are interested in wealth compounding.
\[ \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (R_{i,t} - \bar{R}_i)^2 \]  

(4)

\[ \sigma = \sqrt{\sigma^2}, \]  

(5)

where \( \bar{R} \) denotes the mean return (i.e. expected value) while \( N \) marks the number of periods. (Tsay 2005: 7 – 10.)

When analyzing risk of a portfolio, correlations between asset returns have to be considered in addition to variances. Covariance is a measure of how much assets have moved in tandem over time. A positive covariance coefficient indicates that the asset returns tend to move together. A negative covariance coefficient indicates that the asset returns tend to move in opposite directions. If the covariance equals zero, there is no linear relationship between the assets. The equation of sample covariance is written as follows:

\[ Cov_{i,j} = \frac{1}{N-1} \sum_{i=1}^{N} \left[ (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) \right] \]  

(6)

The covariance of the returns of two assets can be standardized by dividing by the product of the standard deviations of the assets. Standardized covariance is also called correlation coefficient. The equation of the correlation coefficient is written as follows:

\[ Corr_{i,j} = \frac{Cov_{i,j}}{\sigma_i \sigma_j} \]  

(7)

Correlation coefficient is a pure measure of the co-movement of the asset returns and its values set between -1 and +1. (Elton, Gruber, Brown & Goetzmann 2011: 51 – 55.)
The total risk of the portfolio is commonly measured by using realized volatility of the portfolio. Variance of the portfolio is the weighted sum of the covariance coefficients. The equation of variance is written as follows (Markowitz 1952.):

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{Corr}_{i,j}$$

(8)

The chapter below discusses how the risk reduction (diversification) benefits can be reached by minimizing the correlations between the asset returns.

2.1.2 Diversification and the efficient frontier

Investors are generally assumed to be risk averse. Put simply, investors try to maximize the expected return of their portfolios and to minimize the risk of the portfolios. When the number of securities increases in the portfolio, the variance decreases as the correlation between the securities fall. Portfolio variance consist of two components. The first part is the variances of individual assets in the portfolio. This type of risk, also called as unsystematic risk, can be diversified away. However, the covariance terms, the second component of the portfolio variance remains. The remaining risk in the portfolio is called as systematic risk or market risk and it is not diversifiable. The risk in the portfolio is the average covariance which eventually equals the variance of the overall market in case the portfolio is efficiently diversified. (Elton et al. 2011: 56 – 62.)

For each level of expected return of the portfolio, an investor can choose the portfolio weights of the individual securities so that the portfolio has the lowest level of risk. Markowitz’s (1952) modern portfolio theory (MPT) provides a tool for investors to create such a portfolio. The portfolios that have the lowest volatility (standard deviation) of all portfolios with a given expected return are called as minimum-variance portfolios (MVP). As a combination, these portfolios form the minimum-variance frontier. On a risk versus return graph presented below, the portfolio laying farthest to the y-axis (i.e. having the lowest level of risk) is known as the global minimum-variance portfolio. Assuming risk averse investors, the portfolios that have
the highest expected returns are naturally preferred when choosing among portfolios that have the same volatility of returns. Those portfolios make up the efficient frontier. The efficient frontier contains the top portion of the minimum-variance frontier.

Figure 1. The efficient frontier and the capital market line (CML) (adapted from Markowitz 1952.)

The efficient frontier takes only risky assets into account. However, investors are also assumed to lend proportion of their savings at a risk free rate (e.g. a short-term government bill or savings account). Since the return is known and certain, the standard deviation of the returns on the risk free asset must be equal to zero. The two fund separation theorem states that every investor’s optimal portfolio is some combination of the optimal risky portfolio and the risk free asset. The capital allocation line (CAL) presents the combinations of the optimal risky portfolio and the risk free asset. The modern portfolio theory assumes investors having homogenous expectations and thus all investors face exactly the same efficient frontier of risky portfolios. Hence, every investor has the same optimal CAL as well as the risky portfolio. Under this assumption, the optimal CAL for all investors is the line that is just a tangent to the efficient frontier. Investors can simply choose the portfolio weights for the risky portfolio (tangency portfolio) and the risk free rate based on a preferred risk profile. The risky portfolio must be the market portfolio of all risky assets since investors that hold risky assets hold exactly the same kind of portfolio. (Elton et al. 2011: 81 – 85; Tobin 1958.)
Assuming homogenous expectations among the investor community, the optimal CAL for all investors is called as the capital market line (CML). Along this line, expected return of the portfolio is a linear function of the portfolio risk. The equation of the CML is written as follows:

\[
E(R_{p,t}) = R_{f,t} + \left( \frac{E(R_{M,t}) - R_{f,t}}{\sigma_{M,t}} \right) \sigma_P
\]  

(9)

where \( E(R_{M,t}) \) marks the expected return of the market portfolio while \( R_{f,t} \) denotes the risk free rate. As we can see from the equation (9), the ratio of expected excess return and the standard deviation of the risky portfolio is the slope of the CML. The slope is also known as the Sharpe ratio. The Sharpe ratio is probably the most fundamental measure of portfolio performance. The Sharpe ratio varies when moving along the efficient frontier. The minimum-variance portfolio has the highest Sharpe ratio and thus it is the most efficient portfolio. In other words, a rational investor should aim to maximize the Sharpe ratio of their portfolio. (Elton et al. 2011: 81 – 85; Sharpe 1966.) In this research, the realized Sharpe ratios are used to compare performance of the portfolios on risk-adjusted basis.

By decreasing risk of the portfolio to zero, an investor will naturally earn the risk free rate. When a proportion of the assets in the portfolio is allocated to the risky portfolio, expected return naturally rises. The difference between the expected return of the market portfolio and the risk free rate is termed the market risk premium. Every additional unit of market risk an investor accepts increases the expected return of the portfolio by one unit. Assuming that investors can borrow and lend money at the risk free rate, they can pick the portfolio to the right of the tangency portfolio (see figure 1.). In other words, investors can apply leverage to increase the expected return of their portfolio. The figure (1) demonstrates the relation between risk and return. The figure also presents the efficient frontier. (Elton et al. 2011: 81 – 85.)

Capital Asset Pricing Model (CAPM) found by Sharpe (1964), Lintner (1965) and Mossin (1966) is widely used among academics and practitioners to estimate the
required rate of return for risky assets. Consider the CAPM applied to asset or portfolio \( i \):

\[
E(R_{i,t}) - R_{f,t} = \beta_{i,t} [E(R_{M,t}) - R_{f,t}]
\]

(10)

where \( \beta_{i} \) marks the beta coefficient. The theory is derived from MPT and it assumes that the market portfolio is mean variance efficient. The CAPM takes into account only the sensitivity of the asset to the market risk which is measured by using a beta coefficient which is denoted as \( \beta_{i} \) in the formula (10). The beta coefficient describes whether the asset or the portfolio is more or less volatile than the overall market. In summary, according to MPT beta is the only factor that affects the risk of the investor’s portfolio.

The classical measure for a portfolio performance is Jensen’s Alpha (hereafter, alpha). The alpha measures average risk-adjusted returns of a portfolio above (or below) the benchmark portfolio. To be precise, it represents the abnormal return of a portfolio in excess of the theoretical expected return. The theoretical expected return can be derived from CAPM. In empirical finance, the abnormal excess return is a measure of the marginal return that derives from an investment strategy which is not explained by the return of the benchmark. The alpha can be measured by using the following CAPM regression model:

\[
R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_{i,t}(R_{j,t} - R_{f,j}) + \varepsilon_{i,t}
\]

(11)

where \( R_{i,t} \) marks the estimated value given \( R_{j,t} \), \( \alpha_{i} \) is the estimated intercept term (alpha) and \( \beta_{i,t} \) denotes the estimated slope coefficient (beta) and \( \varepsilon_{i,t} \) marks the error term. The estimated slope coefficient describes the change in the variable \( R_{i,t} \) for the single unit change in \( R_{j,t} \). The alpha is the regression line’s intersection with the y-axis when \( R_{j,t} \) equals zero. (Aragon & Ferson 2006; Jensen 1968; Ang 2014: 314 - 315).
Following the CAPM, academics in the 1970’s started to expand the model to identify other factors that determine expected returns. According to Merton (1973) and Long (1974), investors should also invest in so called “hedge portfolios” that capture other risks such as interest rate changes and commodity price inflation. Probably the hottest topic at the moment among equity portfolio managers are the style-based investment approaches that date back to the findings of Fama and French (1996) and Carhart (1997). The authors evaluated mutual fund performance using a four-factor model. In addition, the traditional market factor, they added the following style based factors to the model: value, size and momentum. As the number of factors increases in the regression model, there is always less scope for the return that is not explained by the systematic factors included in the model. Put simply, finding alpha becomes more challenging when analysis is conducted by using multifactor model. (Ilmanen 2011; 61 – 63.) However, this study does not focus on factor exposures but tries to identify whether the risk parity strategy achieves statistically significant alpha when the excess returns are regressed against the excess returns of the traditionally diversified portfolios using only the simple regression model described earlier.

2.2 Traditional diversification methods

2.2.1 Value-weighted portfolio

Investing in the market refers to an investing strategy that diversifies capital across all available investments. Since the overall market is value-weighted, investors who aim to achieve a broad exposure to the overall market are holding the market portfolio. (Utpal & Neal 2011.) As probably one of the most fundamental concepts in financial economics, the theory of the market portfolio derives from three seminal research papers written by Markowitz (1952), Tobin (1958) and Sharpe (1964). According to Markowitz, investors should only hold optimal portfolios. Tobin argues that investors need two optimal portfolios. Sharpe in turn concludes that one of these two portfolios is the market portfolio.

By assuming that the market portfolio is mean-variance efficient one also has to assume that the CAPM holds. Put simply, one has to assume that the market portfolio lies on the efficient frontier. The efficiency of the market portfolio has been continually
under a debate since the CAPM was introduced. Numerous studies (see Roll 1977; Ross 1977; Gibbons 1982; Gibbons, Ross & Shanken 1989; MacKinlay & Richardson 1991) argue that the market portfolio actually lies far from the efficient frontier. However, according to the recent study by Levy and Roll (2010), the market portfolio may be mean-variance efficient after all.

Mean-variance optimality, defined by Markowitz (1952), normally assumes an investment universe that includes a risk-free asset. According to Briere, Brut, Mignon, Oosterlinch and Szafarz (2013), the existence of a risk free asset is no longer a realistic assumption in the modern financial markets. The authors argue that the recent debt crisis has shown that even sovereign bonds issued by countries with stable credit rating are exposed to default risk. Hence, the mean-variance efficiency should be tested without assumption of a risk free asset. Their empirical application to the U.S. stock market indicates that the market portfolio is not mean-variance efficient. The authors end their research by arguing on behalf of skepticism on the validity of the CAPM despite of the recent rehabilitation attempts made by Levy and Roll (2010).

Levy and Roll (2010) find divergent evidence compared to the existent literature by showing that small variations of the sample parameters can actually make the market portfolio mean-variance efficient. The authors note that the variations in the sample parameters are well within the range of estimation error. The results are achieved by solving so called “reverse optimization” problem that looks for return parameters that make the portfolio mean-variance efficient and are also as close as possible to their sample counterparts. A lack of similar results seems to indicate that such parameter sets may be very rare and thus it is very unlikely to reach them by coincidence. However, the reverse optimization problem provides a direct path to those parameter sets. In summary, the authors argue that the CAPM is consistent with the empirically observed return parameters and the market portfolio weights. Thus, the sample betas provide the superior estimates of the expected returns of the assets.

2.2.2 Equal-weighted portfolio

The equal-weighted portfolio (EW or 1/N) refers to the strategy of holding a portfolio that involves weight $w_t = 1/N$ in each of the $N$ portfolio components. Being probably
the most straightforward investment strategy, it completely ignores the data and does not involve any estimations or optimizations. Despite of being a somewhat plain strategy, it has delivered relatively high Sharpe ratios in comparison to more sophisticated optimized strategies. The EW strategy has a long history in asset allocation. The strategy was actually recommended in the Talmud. In the fourth century, a Rabbi Issac bar Aha gave the following recommendation: “A man should always place his money, a third into land, a third into merchandise, and keep a third at hand”. This simple, but evidently effective investment strategy is still widely used. (Benartzi & Thaler 2001).

According to Schoenfeld (2004: 348 – 349), the main benefit of the equal-weighted strategy is the elimination of country concentration issues in asset allocation. On the other hand, the major downside is that by applying the equal-weighted strategy, an investor makes relatively large allocations on illiquid and small markets. Schoenfeld notes that this is a significant issue for large pension funds as they make sizable investments to the emerging markets. In addition, these allocations to smaller markets would require significant ongoing costs for rebalancing which may reduce the total benefits of the strategy. An exclusion of these smaller markets may reduce the level of ongoing costs but would however do it at the expense of diversification benefits. This strategy also makes unintended regional and sector bets in comparison to the market capitalization-weighted strategy even though if may minimize country concentration risks. Paradoxically, an equal-weighted portfolio may be exposed to security concentration as a few securities dominate very often dominate small markets.

DeMiguel, Garlappi and Uppla (2007) compare the performance of 14 optimization models to that of EW strategy. The authors apply seven different datasets and simulated data in the research and document that the out-of-sample Sharpe ratio of the sample-based mean-variance strategy is significantly lower than that of EW strategy. The findings indicate that the errors in estimating means and co-variances dilute the gains from the optimized portfolio relative to the EW portfolio. In addition, the authors show that making various extensions to the sample-based mean-variance model to process the problem of estimation error normally leads to underperformance relative to the EW portfolio. In summary, there is no single optimization model presented in
the academic literature that consistently delivers superior risk-adjusted returns than that of the EW portfolio, which also seems to have a remarkably low turnover ratio.

Relatively poor performance of the optimizing portfolio construction models derives from the length of the estimation period. Before the sample-based mean-variance investment strategy can be expected to outperform the EW portfolio, the estimation window should be as long as 250 years with a portfolio of only 25 assets. A typical estimation period in the literature is around five to ten years. In case of simulated data, the various extensions to the sample-based mean-variance model reduce only moderately the estimation window required for the model to produce higher risk-adjusted returns than that of the EW portfolio. (DeMiguel et al. 2007).

2.3 Risk parity

2.3.1 Defining risk parity

According to Qian (2012), a true risk parity strategy (RP) simply targets risk across assets and should have a balanced, but not necessarily equal, risk contribution for risks raising from equities, interest rates and inflation. Roots of RP portfolio construction date back to Qian (2005), when the author examined the unbalanced risk allocation of traditional 60/40 portfolio of stocks and bonds, respectively. The study proposed equal-risk allocation portfolios which led to the concept of risk parity. Qian (2006) studied the relation of risk and return contribution and established the financial interpretation of risk contribution. Afterwards quantitative risk budgeting approaches have evolved and are nowadays well established among traditional portfolio construction methodologies.

Before defining RP more precisely, one should first understand what is not at risk parity. For instance, a traditional 60/40 allocation of stocks and bonds is not at risk parity. The invested capital may be well balanced, but in terms of risk allocation, the portfolio’s equity risk exposure is approximately 90 to 95 % making it basically a pure equity portfolio. On the other hand, the use of risk budgeting techniques does not necessarily mean that the portfolio is at risk parity. In case that portfolio manager picks three equity index products and two bond index products to the portfolio and uses an
equal risk contribution technique, 60 % of its risk rises from equities and 40 % rises from bonds. In terms of risk allocation, the portfolio is perhaps more effectively diversified in comparison to 60/40 portfolio, but definitely not at risk parity. Hence, risk parity strategies should be treated with consideration. (Qian 2011, 2012).

Another challenge with RP is that asset classes may carry some degree of risk exposure rising from another asset classes. For instance, high yield credit is mostly a pure equity investment when viewed from the risk perspective. This leads to the risk allocation which is skewed to equity risk and thus risk parity is not achieved. The key point here is that portfolio at risk parity should hold balanced risk allocation between the economic risk dimensions such as growth and inflation risk that are the major drivers of portfolio returns. These risks have a direct effect on risk premiums provided by different asset classes. Equity risk premium and interest rate risk premium hedge each other when economic growth fluctuates. When the expectation of upcoming inflation changes, real-return premium and nominal-return premium provided by the inflation risk dimension move to opposite directions. When the portfolio budgets these risk premiums in the portfolio to achieve a balanced exposure to both growth and inflation risk, it should hold a balanced risk contribution from three sources of risk: equity, interest rate and inflation. (Qian 2012, 2013; Chaves, Hsu, Feifei & Omid 2011).

2.3.2 Inverse volatility

The inverse volatility (IV) method enables an equal risk allocation across the asset classes. The IV method focuses on risk allocation whereas the traditional diversification focuses on dollar allocation. Due to the disproportionate risk in equities, the overall risk of a traditional portfolio is most likely dominated by the equity risk. Hence, a RP investor who applies the IV method as an investment strategy aims to achieve a risk diversified portfolio. (Asness et al. 2012).

The IV method, also known as a simple risk parity or naïve risk parity, is the most simple version of risk parity methods as it only deals with volatilities without taking correlations into account. The portfolio construction starts by estimating the volatilities $\tilde{\sigma}_t$ of all assets in the dataset at the end of the estimation period. This study estimates a three-year rolling volatility of monthly returns as follows:
\[ E(\sigma_i) = \text{std}(R_{i,t-36}, \ldots, R_{i,t-1}). \]  

(12)

The RP portfolios are rebalanced annually to target an equal risk allocation across the assets in the dataset. The weights of the portfolio components \( w_{i,t} \) are set as follows:

\[ w_{i,t}^{IV} = k_t \sigma_{i,t}^{-1} \]  

(13)

where,

\[ k_t = \frac{1}{\sum_{i=1}^{n} \sigma_{i,t}^{-1}} \]  

(14)

The variable \( k_t \) controls the leverage of the portfolio and it is the same for all of the constituents. In order to reach the equal risk allocation across the assets, the inverse volatility method overweights the assets with lower volatility and underweights the assets with higher volatility. (Asness et al. 2012; Anderson, Bianchi & Goldberg 2012).

Since the expected return of the unlevered RP portfolio is excessively moderate for most of the investors, RP investor applies leverage to shift the expected return to desired levels while keeping the risk allocation constant. In accordance with Anderson et al. (2012), the levered RP portfolio in this research is constructed by equalizing \textit{ex ante}\(^3\) volatilities across asset classes. The leverage ratio is set so that the \textit{ex post}\(^4\) volatility of unlevered RP portfolio \( \sigma_u \) equals the \textit{ex post} volatility of the equal-weighted portfolio \( \sigma_{EW} \) at each rebalancing. Unlike Asness et al. (2012), this method matches the trailing realized volatilities of the portfolios to find the required leverage ratio. Since this study aims to achieve as realistic investment strategy as possible, the method presented by Anderson et al. is chosen. The required leverage ratio is set as follows:

---

\(^3\) Ex ante is Latin for “before the event” and refers to the expected values of, for instance, returns or volatilities. (Ilmanen 2011: 63).

\(^4\) Ex post is Latin for “after the event” and refers to the realized values. (Ilmanen 2011: 63).
\[ \ell_t = \frac{E(\sigma)^{EW}}{E(\sigma)^{URP}}, \]  

(15)

U.C. \[ \ell_t \geq 1 \]

Asness et al. (2012) implemented an unconditional strategy in the risk parity portfolio by setting \( k_t \) so that the excess returns estimated over the time period matches the realized value-weighted portfolio’s volatility. The asset class weights in the strategy are dependent on a time-independent scale factor \( k_t \). Put simply, their target \( \sigma \) is not known until the end of the entire period making the unconditional levered RP strategy not investable in practice (Anderson et al. 2012). Finally, the excess return of the RP portfolio is calculated as follows:

\[ R_{p,t}^{RP} = \sum_{i=1}^{N} w_{i,t}(R_{i,t} - R_{f,t}) \]  

(16)

Note that the trading costs and borrowing costs are also taken into account in the later stages of portfolio backtesting. The methodology for borrowing and trading costs are introduced later in this section.

The existing evidence on performance of inverse volatility strategy is mixed. According to Anderson et al. (2012), the unleveraged IV strategy outperforms both the market portfolio and 60/40 portfolio of stocks and bonds on risk-adjusted basis after adjustments for market frictions. However, according to the same research, the leveraged IV strategy underperforms the same benchmark over the same sample period on risk-adjusted basis after adjustments for market frictions. Both of the strategies led to the higher Sharpe ratios before adjustments for turnover and the three-month Eurodollar deposit rate as the borrowing rate. The documented Sharpe ratios for unadjusted risk parity strategies are 0.52 and 0.42, former being the unleveraged and later being the leveraged strategy. P-values for the excess returns in the both cases were 0.00 indicating statistically significant results. The Sharpe ratios delivered by the market friction adjusted strategies are 0.50 and 0.25 with the p-values of 0.00 and 0.01, respectively. The sample in the research is over the period of 1926 – 2010. The authors
apply the three-month T-bill rate as a borrowing rate in their unadjusted sample and a three-month Eurodollar deposit rate in the market friction adjusted sample. The data consists of CRSP stocks and CRSP bonds. The portfolios are rebalanced monthly.

Asness et al. (2012) reported similar results by using the same dataset over the same period. The Sharpe ratio for unleveraged RP portfolio is 0.52 and for leveraged 0.53 with the t-values of 4.67 and 4.78, which indicates statistically significant results. The higher Sharpe ratio of the leveraged RP portfolio in the main long sample in comparison to the results of Anders et al. (2012) is the probably achieved due to the lower borrowing rate as the authors apply the one-month T-bill rate as a risk free rate instead of a three-month T-bill rate. Asness et al. (2012) also documented positive returns over the value-weighted portfolio when the leveraged RP portfolio is adjusted with the one-month LIBOR rate. Note that, the authors ignore trading costs in the research.

Asness et al. (2012) also constructed a broad multi-asset sample in their research which consists of stocks, bonds, credit and commodities over 1973 – 2010. The authors find similar results compared to their long sample. The unleveraged RP portfolio achieved the Sharpe ratio of 0.62 with the t-value of 3.65 whereas the leveraged strategy reached the Sharpe ratio of 0.61 with the t-value of 3.57 when the portfolio consisted of multiple assets.

2.3.3 Equal risk contribution

A scope of the Equal Risk Contribution (ERC) method is to equalize the risk contribution for all assets in the portfolio. In comparison to the IV method, the ERC takes the correlations into account. According to Maillard, Roncalli and Teiletche (2010), the ERC portfolio is an appealing alternative to MV and EW portfolios since the volatility of the ERC portfolio is located between those of MV and EW portfolios. The risk contribution of a portfolio constituent is the proportion of total portfolio risk attributable to that component. The risk contribution is calculated by taking the product of the weight in the constituent $i$ with its marginal risk contribution (MRC). MRC is the change in the total risk of the portfolio induced by an infinitesimal increase in the
weight in the constituent \(i\). Consider the portfolio \(x = (x_1 \ldots x_n)\) of risky assets. Let \(\Sigma_{j \neq i}\) be the covariance matrix. Mathematically the MRC is defined as follows:

\[
\frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i} x_j \text{Cov}_{i,j}}{\sigma(x)}
\]  

(17)

Put simply, the equation above demonstrates the shift of the portfolio volatility when the weight of the one component is increased by a small amount. Given that \(\sigma(x) = w_i \times \partial x_i \sigma(x)\) stands for the risk contribution of the constituent \(i\), then the total risk of the portfolio can be written as the sum of those risk contributions:

\[
\sigma(x) = \sum_{i=1}^{n} \sigma_i(x)
\]  

(18)

Note that the above-mentioned equation (18) can also be written as the equation (8) is presented in the earlier chapter.

In order to interpret the results of the empirical study, a significant consideration has to be clear between a case of two asset portfolio and a multi-asset portfolio when comparing the ERC strategy to the IV strategy. The both methods may look exactly the same after studying them for the first time. The reason is that in a two asset case both IV and ERC techniques have the same mathematical solution. Nevertheless, in case of multi-asset portfolio, the solution differs due to the asset correlations. Hence, it is highly crucial to understand the difference to avoid improper application or interpretation. Maillard et al. (2010) demonstrated the difference by first assuming the equal correlations between the assets in the portfolio. The authors find that the asset weights in the portfolio are equal to the ratio of the inverse of their volatilities with the harmonic average of the volatilities. In other words, it leads to exactly the same mathematical solution that is achieved with the IV risk parity method. In the second phase, the authors assume the volatilities to be equal, but the correlations to differ. The authors conclude that the weight of each asset is equal to the ratio between the inverse
of its beta coefficient and the harmonic mean of the asset betas. Thus, the lower (higher) the beta coefficient, the higher (lower) the weight in the portfolio meaning that assets with high volatility or high correlation with other assets will be penalized. In general case, there is no closed-form solution for the problem and thus a numerical algorithm is required. The problem can be solved by applying a SQP (Sequential Quadratic Programming) algorithm:

\[
 w_{i}^{ERC} = \arg \min_{w} \sum_{i=1}^{N} \sum_{j=1}^{N} [w_i \text{Cov}_{i,j} - w_j \text{Cov}_{i,j}]^2
\]

\[
 \text{U.C.} \quad \sum_{i=1}^{N} w_i = 1
\]

\[
 0 \leq w_i \leq 1
\]

The program minimized the variance of the risk contributions. As noted earlier, the volatility of the ERC portfolio sets between that of MV and that of EW portfolio. In addition, the portfolio weights of ERC and MV are ranked in the same order. However, the ERC approach offers more risk-balanced solution.

According to Maillard et al. (2010), the globally diversified ERC portfolio outperformed both EW and MV portfolios on risk-adjusted basis over the period of January 1995 to December 2008. The ERC strategy provided the Sharpe ratio of 0.67 whereas EW strategy and MV strategy delivered the Sharpe ratios of 0.27 and 0.49, respectively. The ERC strategy also provided an annualized return of 7.58% that dominates the returns of EW (7.17 %) and MV strategy (5.84 %).

Maillard et al. (2010) also backtest the ERC strategy using the U.S. sector index data over the period of January 1973 up to December 2008 and find evidence of superior risk-adjusted returns of the MV portfolio in comparison to the ERC and the EW portfolio. The MV portfolio provided the Sharpe ratio of 0.77 whereas the ERC strategy delivered the ratio of 0.65. The annualized returns of the sector samples were 9.54 % for the MV portfolio and 10.01 % for the ERC portfolio. The EW portfolio resulted to the Sharpe ratio of 0.62 with annualized return of 10.03 %. Despite the dominance of the MV portfolio, the authors note that the real advantage of the ERC
In summary, Maillard et al. (2010) note that the EW strategy is inferior in terms performance and any measure of risk. The MV portfolio may deliver superior risk-adjusted returns due to lower volatility, but can cause higher drawdowns in the short period. In addition, the MV strategy is more concentrated in terms of diversification and less efficient in terms of turnover leading to higher trading costs.

2.4 Leverage and portfolio constraints

2.4.1 Investor’s constraints

As discussed, an RP investor can apply appropriate degree of leverage to raise the expected return and risk to the desired levels. However, this assumption is normally presented only on paper and not possible in reality for most of the investors. Thus the management of a RP portfolio is typically outsourced to an external portfolio manager due to the restriction on using leverage. In addition to leverage constraints, number of other restrictions are also normally taken into account in finance theories. (Lee 2014).

Chow, Kose and Li study (2016) the impact of constraints on minimum-variance portfolios and find evidence of increased volatility caused by each additional portfolio constraint. The constraints improve investability of the portfolio, but shift portfolio characteristics toward those of the market capitalization weighted portfolio. The authors examine MV strategies on the U.S. markets, global developed markets and emerging markets. The portfolios are tested by applying commonly used constraints such as minimum weight\(^5\), maximum weight\(^6\), capacity\(^7\), sector concentration\(^8\),

---

\(^5\) If an asset gets weight less than 0.05 %, it is changed to 0.00 %.

\(^6\) Individual asset weights are capped at 5.00 %.

\(^7\) The weight of an asset is capped at the lower of 1.5 % or 20 times its weight in the corresponding capitalization-weighted portfolio. This constraint dominates the maximum weight constraint.

\(^8\) Sector weights are not allowed to deviate more than +5.00 % or -5.00 % from the corresponding market capitalization weighted portfolio.
regional concentration\(^9\) and turnover\(^{10}\). Their aim is to determine the effects of those constraints on simulated portfolio characteristics, performance and trading costs.

According to Chow, Kose and Li (2016), the simulated minimum-variance portfolios outperformed traditional passive investment strategies on risk-adjusted basis. The simulated portfolios provided higher Sharpe ratios in all markets in comparison to their market-capitalization weighted benchmarks. However, the tested strategies are difficult to implement effectively in practice due to their tilt toward small-cap companies, high turnover rates and concentration on individual stocks, sectors or countries. These portfolio characteristics lead to lower investment capacity and increased transaction costs. The authors also argue that minimum-variance index providers succeed in controlling transaction costs and improving the investability of the strategies by applying constraints at the security and portfolio levels. On the other hand, these constraints entail greater-than-minimal volatility and shift the portfolios toward their market capitalization-weighted benchmarks. In summary, even though the constrained minimum-variance portfolios might deliver superior returns compared to the market capitalization weighted benchmarks, one should be aware of these trade-offs when choosing between options to implement a low-volatility strategy.

Lee (2014) studies optimality of the RP portfolios and argues that in the special case where Sharpe ratios and correlations among all assets are the same, the RP portfolio is actually mean-variance optimal portfolio. The author argues that even though Sharpe ratios and correlations deviate over shorter periods, they have been more comparable over the longer periods. If this assumption holds, the RP portfolio is indeed a more efficient portfolio compared to, for instance, 60/40 portfolio of stocks and bonds which is far more bullish on stocks. As noted, regardless of the more effective diversification, the expected return of the RP portfolio may prove to be too low given its higher allocation to low-risk assets. Investors who are not constrained on using leverage can

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\(^9\) If the capitalization-weighted region weights are less than 2.50 %, the weight in minimum variance portfolio is capped at three times its weight in the capitalization weighted portfolio. Otherwise, it is not allowed to deviate more than -5.00 % or +5.00 % from the corresponding capitalization-weighted region weights.

\(^{10}\) The maximum allowed one-way index turnover is limited to 20.00%.
easily solve this problem by raising the expected return to desired levels by applying leverage.

In conclusion, the superiority of the RP investing is largely based on the finding that a constrained efficient frontier is in fact less efficient than an unconstrained efficient frontier, if all else equal. The RP strategy is definitely not a magic bullet, but it may provide improvements in portfolio efficiency by removing the frictions caused by leverage constraints and short-selling constraints that may prevent the investor from reaching the efficient frontier. (Lee 2014).

2.4.2 Leverage aversion and low risk anomaly

The intuition behind the risk allocation strategies relies on an implicit assumption about expected returns of asset classes. Given that the expected returns of equities versus fixed income were high enough, an investor should hold a portfolio whose risk is dominated by equities. An argument blaming the investor of bearing too much risk by weighting portfolio components based on the market capitalizations is accurate only if the equity risk premium is not high enough compared to fixed income to compensate such a heavy risk allocation. On the other hand, an argument in favor of an equal risk allocation between assets is accurate only if every asset is believed to offer equal risk-adjusted returns. A strategy of equal risk allocation between assets is not optimal only due to the better diversification. Moreover, an investor needs to believe that equities do not deliver enough returns to get a disproportionate part of our risk budget. An investor also need to prove the CAPM wrong since it assumes that the risk premiums present levels where the market capitalization-weighted portfolio is optimal. (Asness et al. 2012).

According to Black (1972) and Frazzini and Pedersen (2014), leverage aversion among some investors has an impact on asset class returns. Since some investors are averse to applying leverage, assets with high beta will offer lower risk-adjusted returns and assets with low beta will offer higher risk-adjusted returns. Hence, by applying leverage and overweighting safer assets while underweighting riskier assets an investor who is less leverage averse can earn the highest risk-adjusted returns. Put differently, leverage aversion breaks the standard CAPM.
Frazzini et al. (2014) document that stocks with low beta have higher Sharpe ratios in 20 international equity markets. The authors also find evidence of the similar effect within other asset classes. According to the research, corporate bonds with lower risk offer better risk-adjusted returns than high yields do. The same anomaly holds with safer short-maturity treasuries in comparison to long-term government bonds. Asness et al. (2012) argue that the empirical evidence of the anomaly described above is a reason for RP portfolios’ convincing historical performance. The theory of leverage aversion emphasizes how additional out-of-sample evidence can be found when the risk-adjusted returns of major asset classes are being compared.

Many investors, such as pension funds, mutual funds and individual investors are very often constrained to apply leverage in their investment strategy. Therefore, those investors bid up the prices of riskier assets since they need to maintain the required level of portfolio’s return potential. Investment behavior of tilting the portfolios toward high-beta assets claims indicates that those assets require lower risk-adjusted returns in comparison to low-beta assets. (Frazzini et al. 2014). According to Black, Jensen and Scholes (1972), the SML for U.S. stocks is too flat relative to the CAPM. The authors argue that the SML is better explained by the CAPM when borrowing is restricted.

Frazzini et al. (2014) study the low risk anomaly by constructing a betting against beta (BAB) factor which goes long on leveraged low-beta assets and short on high-beta assets. The BAB factors are constructed by raking each security within an asset class in ascending order based on their beta coefficients. The two portfolios are constructed by dividing the securities into two portfolios. The high- (low-) beta portfolio includes the stocks with a beta above (below) its asset class median. The both portfolios weight the assets by the ranked betas. Put simply, lower-beta assets get a bigger weight in the low-beta portfolio and higher-beta assets get a bigger weight in the high-beta portfolio. The authors find evidence of significant positive risk-adjusted returns. The BAB factor provides lower returns in case of constraints for funding.

Investors’ portfolio selection seems to be driven by the degree of leverage constraints. Frazzini et al. (2014) document that more leverage constrained investors buy riskier securities than investors with more availability to exploit leverage. The investors with
constraints for leverage hold portfolios with betas above one on average. On the contrary, highly leveraged funds such as leveraged buyout funds and the famous Berkshire Harhaway funds hold stocks with betas below one on average. In summary, the investors with access to leverage can benefit of the BAB effect by leveraging up the weights of low-beta assets and being rewarded by investors who do not have the same access.

2.4.3 Embedded leverage

Even though many investors face constraints for applying leverage in their investment strategies, it does not directly mean that they cannot increase their risk exposure through the financial instruments that are designed to alleviate those constraints. Nowadays many financial instruments such as options and leveraged exchange traded funds (ETF)\textsuperscript{11} carry so called embedded leverage. Embedded leverage refers to the amount of market exposure per unit of committed capital. More precisely, embedded leverage is measured with percentual change in price of a security for a one percent change in the underlying asset. A simple example of a security with embedded leverage would be equity investment in firms with outstanding debt. (Frazzini & Pedersen 2012).

Demand for the financial instruments with embedded leverage derives from investors’ inability or unwillingness to apply enough outright leverage to increase the risk exposure to the desired levels. As mentioned, that inability arises from many reasons, such as margin requirements or regulatory capital constraints. Investors can simply raise their risk exposure without violating their leverage constraints by buying the financial instruments that are designed with embedded leverage. In addition, by applying the above-mentioned strategy investors can also avoid the possibility of losing more than 100% of the invested capital. Therefore, the intermediaries who supply those financial instruments need to be compensated for the risks arising from

\textsuperscript{11} Exchange Traded Funds (ETF) are marketable securities that track indices, such as stock, bond, commodity or real estate indices. ETFs are traded like a common stock on an exchange. ETFs are normally passively managed funds. (Bodie, Kane & Marcus 2014: 49).
embedded leverage. Conversely, investors are willing to pay a premium for those instruments. (Frazzini et al. 2012).

Frazzini et al. (2012) hypothesize that embedded leverage lowers the required returns of the instruments that carry embedded leverage. The authors find evidence that the financial instruments across asset classes with embedded leverage deliver lower risk-adjusted returns. In addition, the authors find securities with lower embedded leverage to deliver higher risk-adjusted returns in each of the cross-sections of leveraged ETFs, index options and stock options. By constructing BAB portfolios, the authors find large and statistically significant abnormal returns. The BAB portfolios go long on securities with low-embedded leverage and short on securities with high-embedded leverage. The portfolios are designed to be market neutral as well as neutral to each of the underlying securities. The portfolios delivered Sharpe ratios in excess of one with high t-statistic indicating statistically robust results.

2.4.4 Market frictions

In order to achieve a realistic and practical overview of the performance of the investment strategies, financing costs for the leverage must be taken into account in case of leveraged strategies. The performance of leveraged risk parity portfolios is reduced with the borrowing rate. The return of the levered risk parity portfolio is calculated as follows:

$$R_{p,t}^{LRP} = \sum_{i=1}^{n} w_{i,t}^{URP} R_{i,t} + \sum_{i=1}^{n} (w_{i,t}^{LRP} - w_{i,t}^{URP}) (R_{i,t} - b_t)$$

(20)

where $w_{i,t}^{URP}$ and $w_{i,t}^{LRP}$ are the weights of the asset in the unlevered and levered portfolios while $b_t$ denotes the borrowing rate. At the first phase of this research, the one-month T-bill rate is used as a borrowing rate which is, according to Asness et al. (2012), consistent with the academic literature, but according to Anderson et al. (2012) not an applicable assumption in practice. Therefore, similarly to Asness et al. and Anderson et al. this research observes robustness of the results also by applying one
month LIBOR, Fed funds and three-month Eurodollar deposit rate as the alternative borrowing rates.

The research also observes the effect of trading costs on the portfolio performance which is a consistent approach with Anderson et al. (2012). The turnover required to rebalance the portfolio to maintain an investment strategy:

\[
T_t = \sum_{i=1}^{N} |w_{i,t} - w_{i,t-1}|
\]  

(21)

The effect of leverage to the turnover of the portfolio is written as follows:

\[
T_t = \sum_{i=1}^{N} |w_{i,t}\ell_t - w_{i,t-1}\ell_t|
\]

(22)

At the next stage, the trading cost at the time \( t \) is calculated as follows:

\[
c_t = T_t z_t
\]  

(23)

where \( z_t \) is assumed trading cost percent. Finally, trading cost-adjusted returns can be calculated as follows:

\[
R_{i,t}^{TCA} = R_{i,t} - c_t
\]

(24)

In accordance with Anderson et al. (2012), the trading costs are set (by assumption) as follows: \( z_t \) equals 1.0 % for 1929 – 1955, 0.5 % for 1956 – 1970 and 0.1 % for 1971 – 2015.
3 DATA AND METHODOLOGY

This chapter revises the research questions, sets the hypotheses and introduces the data that is used in the empirical research. In addition, the research methodology is presented.

3.1 Hypotheses

This study focuses on the RP strategies’ performance in comparison the EW strategy. On general level, the aim of this research is to specify the RP strategies, compare their historical performance to the EW strategy and evaluate the impact of market frictions on portfolio performance. In addition, the aim is to analyze leverage as a tool to enhance expected returns and how this strategy performs in a rising rate market environment. Lastly, this research also compares two RP strategies, the Inverse Volatility (IV) and the Equal Risk Contribution (ERC), with each other to observe whether the more sophisticated optimized ERC strategy outperforms the simple IV strategy.

The research questions are as follows:

**Question 1**: Do risk parity strategies deliver higher risk-adjusted returns than a traditional equal-weighted portfolio?

**Question 2**: How does leverage affect to the performance of a risk parity portfolio?

**Question 3**: Can risk parity strategies outperform in a rising rate market environment?

**Question 4**: Does the Equal Risk Contribution (ERC) strategy outperform the Inverse Volatility (IV) strategy?

To answer the following questions, this research sets one hypothesis for each of the research questions. Hypotheses are based on the key findings from existing literature.
As noted, the existing evidence on performance of risk parity portfolio strategies is mixed. According to Maillard et al. (2010) and Asness et al. (2012), the risk parity strategies outperform the traditional diversification strategies such as the EW strategy and the MV strategy. However, according to Anderson et al. (2012), the risk parity strategy underperforms when its performance is adjusted for market frictions. Note that Anderson et al. and Asness et al. study the performance of the IV strategy whereas Maillard et al. study the performance of the ERC strategy.

Since the existing empirical evidence of the risk parity strategies seems to be slightly skewed towards the superiority of the RP strategies instead of the traditional strategies, the first research hypothesis is as follows:

**Hypothesis 1:** The risk parity strategies outperform the equal-weighted strategy on risk-adjusted basis after adjustments for market frictions.

As the RP strategies require heavy allocation to low risk assets, expected returns of the RP portfolios are naturally lower than those of traditional approaches. Hence, leverage is relatively remarkable tool for the RP investors to raise the expected returns on desired levels. According to Asness et al. (2012), the leverage enhances the returns significantly and provides better risk-adjusted returns when compared to the value weighted portfolio. The authors find that the leveraged IV strategy provides the Sharpe ratio of 0.53 while unleveraged strategy achieved the Sharpe ratio of 0.52 over the period of 1926 – 2010. The portfolio consisted of the U.S. stock and bonds. When the backtest was made over the period of 1973 – 2010 and the portfolio was constructed by using multi-asset data, the leveraged IV strategy provided the Sharpe ratio of 0.61 while unleveraged delivered the ratio of 0.62. However, when the authors examine robustness of the results by using alternative borrowing rates, the Sharpe ratio of the leveraged IV strategy decreased significantly, but still provided higher risk-adjusted returns than the market portfolio.

Anderson et al. (2012) report significantly lower Sharpe ratio for the leveraged IV strategy when the returns are adjusted for market frictions. The authors adjusted the performance by using three-month Eurodollar deposit rate as a borrowing rate and
reducing trading costs at every rebalancing. The leveraged IV strategy provided the Sharpe ratio of 0.25 with the p-value of 0.01 and the unleveraged strategy delivered the ratio of 0.50 with the p-value of 0.00.

Since Asness et al. (2012) documented a relatively slight difference between the Sharpe ratios of the leveraged and the unleveraged strategies and Anderson et al. (2012) reported weaker performance for the leveraged strategies, the second research hypothesis is as follows:

**Hypothesis 2:** Leveraged risk parity strategies deliver lower risk-adjusted returns than unleveraged strategies.

As a RP investor allocates a high proportion of assets into fixed income products and applies leverage at the same time, interest rate environment has a significant impact on the portfolio’s performance. According to Hurst, Mendelson and Ooi (2013), a RP strategy may perform well even during moderately rising interest rates. The authors also note that the strategy tends to suffer during periods where interest rates rise fast and unexpectedly. The third research hypothesis assumes that the RP strategies can outperform in a rising rate environment.

**Hypothesis 3:** The risk parity portfolios can outperform in a rising rate market environment.

After going through existing literature on RP strategies, I did not find any research comparing different RP strategies with each other. Hence, I decided to focus on comparing two different RP strategies. The strategies where mainly chosen due to their level of sophistication. The IV method is the most straightforward and simplest risk parity strategy and extremely easy to apply in practice. The ERC strategy requires more sophisticated optimization methods and thus is a relatively demanding approach in practice. The main motive however for the chosen strategies is to find out whether the optimized ERC strategy that takes the correlations into account provides better risk-adjusted returns than the IV methods that only penalize assets with high realized volatility. Correlations between asset classes are definitely one of the most significant factors within the modern portfolio theory. Thus, it is fascinating to see whether the
strategy that totally ignores the co-movement of the asset classes can compete with the 
optimized approach. Fourth and the last research hypothesis is as follows:

**Hypothesis 4:** The Equal Risk Contribution (ERC) strategy delivers 
higher risk-adjusted returns than the Inverse Volatility (IV) strategy.

The next section familiarizes the reader with the data and the methodology used.

### 3.2 Measuring portfolio performance

This research applies a simple linear regression model to examine whether the risk 
parity portfolios generate statistically significant alpha over the sample periods. The 
aim of the simple linear regression model is to explain the variation in a dependent 
variable in terms of the variation in an independent variable. The dependent variable 
refers simply to the variable whose variation is explained by the independent variable. 
The dependent variable explains the variation in the independent variable. (Hansen 
2014: 35; Ang 2014: 314 - 315). It is better recognized as the alpha that has been 
walked through in the theory section. In the following sections, the excess returns of 
the RP strategies are regressed against the excess returns of EW portfolio.

Before making any conclusions regarding the importance of the beta or the alpha, 
variables require determination of their statistical significance. The research applies a 
t-test to examine whether the results are statistically significant. The decision rule in 
this research is based on the 95% confidence level.

Another main portfolio performance measure in this research is the Sharpe ratio. 
Performance analysis of a single asset or an investment strategy is naturally carried 
out on returns that are adjusted for the taken risk. Most commonly this adjustment is 
made by using volatility as a measure of risk. As discussed in the theory section, the 
Sharpe ratio is the average excess return over the risk free rate divided by the volatility 
of the excess returns. (Sharpe 1966). However, using only the Sharpe ratio may be 
misleading in case of undiversified portfolio as the Sharpe ratio totally ignores the 
distinction between systematic and unsystematic risk. In addition, as a risk measure 
volatility does not distinguish between unexpected downside or upside surprises. This
study also presents the higher moments of the return distribution that represent the
distribution’s asymmetry and fat tails. In other words, tail risk and skewness are
presented in addition to volatility.

### 3.3 Descriptive statistics

The table below describes the indices that are used in the research to construct the
portfolios. The empirical research consists of two samples: a long sample and a broad
sample. The long sample consists of U.S. stocks and government bonds over January
1929 to December 2015 and the broad sample consists of multiple indices from
multiple asset classes from January 2002 to December 2015.

The long sample data is obtained from the CRSP database. The CRSP stock index is
a value-weighted index consisting of monthly returns on the NYSE, AMEX and
NASDAQ markets including dividends. Similarly, the CRSP bond index is a value-
weighted index consisting of average monthly unadjusted holding period return for
each bond in the CRSP monthly U.S. Treasury database. The bonds are weighted by
their outstanding face value. In other words, the long sample data is equivalent to the
data used by Asness et al. (2012) and Anderson et al. (2012), but slightly longer.

The broad sample data is obtained from the Datastream by Thomson Reuters,
Bloomberg and from the MSCI index providers website. To include multi-asset class
data into the research, I needed to concentrate on the more recent period due to the
lack of suitable data. Hence, the broad sample portfolios are constructed over the
period of January 2002 and December 2015. The broad sample data consists of
monthly index returns. Note that every index in the broad sample is also investable in
practice by using ETF.

The interest rate data is obtained from Bloomberg, Ibbotson Associates and the FRED
database. The table below reports the excess returns over the one-month T-bill rate
obtained from Ibbotson Associates. According to Asness et al. (2012), the one-month
T-bill rate is a proper risk free rate and thus a proper borrowing rate as well. However,
this research considers it to be a bit naïve in terms of realistic borrowing costs faced
by the investors. Hence, the robustness check applies alternative borrowing rates to
estimate the impact of the different borrowing rates on performance of leveraged portfolio strategies. According to Hull (2012: 104), participants in derivatives markets are normally assumed to borrow and lend at LIBOR rather than Treasury rates. As a RP investor can construct the portfolio by using index future contracts, LIBOR is a more suitable assumption of the borrowing rate and thus used in this research. The main conclusions are based on the results achieved by using LIBOR as the borrowing rate.

Table 1. Descriptive statistics

<table>
<thead>
<tr>
<th>Index</th>
<th>Excess Return</th>
<th>T-Stat. ER</th>
<th>Volatility</th>
<th>Sharpe</th>
<th>Skew</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Long sample (1929 – 2015)</strong></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>CRSP stocks*</td>
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<td>CRSP bonds*</td>
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<td>4.58</td>
<td>3.31</td>
<td>0.49</td>
<td>1.21</td>
<td>6.93</td>
</tr>
<tr>
<td><strong>B. Broad sample (2002 – 2015)</strong></td>
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<tr>
<td>Stocks</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI Emerging Markets</td>
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<td>1.44</td>
<td>22.26</td>
<td>0.39</td>
<td>-0.60</td>
<td>2.01</td>
</tr>
<tr>
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<td>19.40</td>
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<td>1.54</td>
</tr>
<tr>
<td>MSCI Japan</td>
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<td>0.99</td>
<td>15.95</td>
<td>0.26</td>
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<td>0.60</td>
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<td>MSCI Nordic Countries</td>
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<td>1.12</td>
<td>23.81</td>
<td>0.30</td>
<td>-0.33</td>
<td>1.88</td>
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<td>1.37</td>
<td>15.13</td>
<td>0.37</td>
<td>-0.72</td>
<td>1.87</td>
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<tr>
<td>Barclays U.S. Corporate IG*</td>
<td>4.60</td>
<td>2.90</td>
<td>5.94</td>
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<td>-1.02</td>
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<td>9.86</td>
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<tr>
<td>Barclays U.S. Treasury*</td>
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<td>2.62</td>
<td>4.71</td>
<td>0.70</td>
<td>0.45</td>
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<tr>
<td>Barclays Euro Aggregate Corporate</td>
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<td>11.64</td>
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<td>1.55</td>
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<td>3.93</td>
<td>3.72</td>
<td>3.95</td>
<td>1.00</td>
<td>-0.03</td>
<td>0.36</td>
</tr>
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<td>J.P Morgan EMBI*</td>
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<td>-1.51</td>
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<tr>
<td>S&amp;P GSCI</td>
<td>-3.16</td>
<td>-0.50</td>
<td>23.80</td>
<td>-0.13</td>
<td>-0.50</td>
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<tr>
<td>S&amp;P Global REIT</td>
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<td>0.69</td>
<td>19.57</td>
<td>0.18</td>
<td>-1.39</td>
<td>6.53</td>
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<tr>
<td>HFRI Fund Weighted Hedge Fund*</td>
<td>4.11</td>
<td>2.59</td>
<td>5.93</td>
<td>0.69</td>
<td>-0.89</td>
<td>2.37</td>
</tr>
</tbody>
</table>

Notes: The table provides calendar-time index returns. Panel A reports returns of stocks and bonds only. Panel B reports returns of all the available asset classes. Excess returns are above the U.S. one-month T-bill rate. Excess Kurtosis equals to the kurtosis of monthly excess returns minus three. Volatilities and returns are annual percentages. All returns are nominated in U.S. dollars. Excess returns, volatilities and Sharpe ratios are annualized.

*Significant at the 5 percent level.
Table 2. A correlation matrix of the broad sample indices

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<td>0.11</td>
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<td>0.58</td>
<td>0.67</td>
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<td>0.41</td>
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<td>0.17</td>
<td>-0.08</td>
<td>0.43</td>
<td>-0.07</td>
<td>0.28</td>
<td>0.41</td>
<td>0.62</td>
<td>0.34</td>
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<tr>
<td>Japan Stocks</td>
<td>1.00</td>
<td>0.55</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.08</td>
<td>0.58</td>
<td>-0.18</td>
<td>0.42</td>
<td>0.56</td>
<td>0.80</td>
<td>0.37</td>
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<tr>
<td>Nordic Stocks</td>
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<td>0.04</td>
<td>0.06</td>
<td>-0.04</td>
<td>0.58</td>
<td>-0.18</td>
<td>0.42</td>
<td>0.56</td>
<td>0.80</td>
<td>0.37</td>
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<tr>
<td>EM Debt</td>
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<td>0.59</td>
<td>0.13</td>
<td>0.34</td>
<td>-0.07</td>
<td>0.02</td>
<td>0.17</td>
<td>0.20</td>
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<tr>
<td>US Inv. Grade</td>
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<td></td>
<td>1.00</td>
<td>0.09</td>
<td>0.20</td>
<td>0.09</td>
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<td>US High Yield</td>
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<td></td>
<td>1.00</td>
<td>-0.07</td>
<td>0.42</td>
<td>0.66</td>
<td>0.65</td>
<td>0.30</td>
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<tr>
<td>Europe Treasury</td>
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<td></td>
<td>1.00</td>
<td>0.19</td>
<td>0.04</td>
<td>-0.17</td>
<td>-0.21</td>
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<tr>
<td>Europe Credit</td>
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<td>S&amp;P REIT</td>
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<td>1.00</td>
<td>0.52</td>
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<tr>
<td>S&amp;P GSCI</td>
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<td></td>
<td></td>
<td>1.00</td>
<td></td>
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</tbody>
</table>

Notes: This table presents historical correlations of the indices in the broad sample over January 2002 to December 2015.
4 RESULTS OF THE EMPIRICAL STUDY

This chapter discusses the results of the empirical study and compares achieved results with the findings in the existing literature. In addition, the chapter critically examines possible issues regarding the research methods and aims to find potential targets for additional research.

4.1 Portfolio backtesting and regression analysis

This research finds consistent results with the existing literature. However, the results do not support explicitly either one of the benchmark studies published by Asness et al. (2012) and Anderson et al. (2012). Nevertheless, the research partly supports the findings of the reference studies as well as argues against both of them.

The differing findings might result from the several differences in the empirical methods used. Firstly, this research rebalances portfolios annually whereas both of the benchmark studies use monthly rebalance frequency. Divergent rebalance frequency is chosen due to the characteristics of the asset classes in the dataset. The broad data sample of this study includes hedge funds that are not tradable in practice. According to Macey (2008: 242), the standard lockdown period\footnote{Lockdown period refers to the period when a hedge fund investor is not able to redeem the assets from the fund.} in the hedge fund industry is one year. Thus, I decided to rebalance portfolios annually to achieve more realistic backtesting results. Note that there is an ETF provided by ProShares tracking the HFRI Fund Weighted Fund Index which technically makes hedge funds tradable. However, due to the lack of tradable hedge fund instruments I find annual rebalance frequency more realistic from practical perspective.

In the panels A and B, portfolio strategies are backtested without adjustments for trading costs. The leveraged portfolios are adjusted for one-month T-bill rate as a borrowing rate. Note that in the panel A, the IV strategy and the ERC strategy lead to
exactly the same results. The reason for the same mathematical solution of the strategies in case of two assets is discussed in the theory section.

In the panels C and D, the borrowing rate for the leveraged strategies is replaced with the one-month LIBOR rate. The more conservative LIBOR reduced annualized performance quite dramatically in the long sample, but the effect on the broad sample is limited.

In the panels E and F, the portfolio performance is adjusted for LIBOR and trading costs. The trading costs are assumed to be 1% over 1929 – 1955, 0.5% over 1955 – 1970 and 0.1% over 1970 – 2015. (Anderson et al. 2012). Since the portfolios are rebalanced annually, the turnover remains relatively low and trading costs have only a minor effect on the performance.

Figure 2. Risk parity vs. equal-weighted portfolio: cumulative returns of the long sample (1929 – 2015).
Table 3. Historical performance of risk parity and equal-weighted portfolios.

<table>
<thead>
<tr>
<th>Base Case</th>
<th>Excess Return</th>
<th>T-Stat. ER</th>
<th>Volatility</th>
<th>Sharpe</th>
<th>Skew</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Long sample, 1929 – 2015</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>4.19</td>
<td>4.16</td>
<td>9.33</td>
<td>0.45</td>
<td>-0.09</td>
<td>5.88</td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>2.40</td>
<td>5.67</td>
<td>3.93</td>
<td>0.61</td>
<td>0.71</td>
<td>4.65</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>6.16</td>
<td>5.81</td>
<td>9.84</td>
<td>0.63</td>
<td>-0.34</td>
<td>4.59</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>2.40</td>
<td>5.67</td>
<td>3.94</td>
<td>0.61</td>
<td>0.71</td>
<td>4.65</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>6.16</td>
<td>5.81</td>
<td>9.84</td>
<td>0.63</td>
<td>-0.34</td>
<td>4.59</td>
</tr>
<tr>
<td><strong>B. Broad sample, 2002 – 2015</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>5.69</td>
<td>2.20</td>
<td>9.67</td>
<td>0.59</td>
<td>-1.04</td>
<td>4.56</td>
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<tr>
<td><strong>C. Long sample, 1929 – 2015, adjusted for LIBOR</strong></td>
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</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>4.19</td>
<td>4.16</td>
<td>9.33</td>
<td>0.45</td>
<td>-0.09</td>
<td>5.88</td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>2.40</td>
<td>5.67</td>
<td>3.93</td>
<td>0.61</td>
<td>0.71</td>
<td>4.65</td>
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<tr>
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<td>3.94</td>
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<tr>
<td>Equal Risk Contribution – leveraged</td>
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<td>4.53</td>
<td>9.62</td>
<td>0.49</td>
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<td>5.04</td>
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<td><strong>D. Broad sample, 2002 – 2015, adjusted for LIBOR</strong></td>
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<tr>
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<td>0.59</td>
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<tr>
<td>Inverse Volatility – unleveraged</td>
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<td>3.18</td>
<td>5.88</td>
<td>0.85</td>
<td>-1.23</td>
<td>6.08</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>8.65</td>
<td>3.33</td>
<td>9.71</td>
<td>0.89</td>
<td>-1.15</td>
<td>5.26</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>5.01</td>
<td>3.19</td>
<td>5.87</td>
<td>0.85</td>
<td>-1.22</td>
<td>6.02</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>8.74</td>
<td>3.40</td>
<td>9.62</td>
<td>0.91</td>
<td>-1.07</td>
<td>4.74</td>
</tr>
<tr>
<td><strong>E. Long sample, 1929 – 2015, adjusted for LIBOR and trading costs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>4.18</td>
<td>4.18</td>
<td>9.33</td>
<td>0.45</td>
<td>-0.09</td>
<td>5.88</td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>2.46</td>
<td>5.81</td>
<td>3.93</td>
<td>0.63</td>
<td>0.71</td>
<td>4.65</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>4.50</td>
<td>4.34</td>
<td>9.62</td>
<td>0.47</td>
<td>-0.39</td>
<td>5.11</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>2.46</td>
<td>5.81</td>
<td>3.93</td>
<td>0.63</td>
<td>0.71</td>
<td>4.65</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>4.50</td>
<td>4.34</td>
<td>9.62</td>
<td>0.47</td>
<td>-0.39</td>
<td>5.11</td>
</tr>
<tr>
<td><strong>F. Broad sample, 2002 – 2015, adjusted for LIBOR and trading costs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>5.69</td>
<td>2.20</td>
<td>9.68</td>
<td>0.59</td>
<td>-1.04</td>
<td>4.56</td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>5.00</td>
<td>3.18</td>
<td>5.88</td>
<td>0.85</td>
<td>-1.23</td>
<td>6.08</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>8.63</td>
<td>3.33</td>
<td>9.71</td>
<td>0.89</td>
<td>-1.15</td>
<td>5.26</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>5.00</td>
<td>3.19</td>
<td>5.87</td>
<td>0.85</td>
<td>-1.22</td>
<td>6.01</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>8.72</td>
<td>3.39</td>
<td>9.62</td>
<td>0.91</td>
<td>-1.07</td>
<td>4.74</td>
</tr>
</tbody>
</table>

Notes: The table provides historical performance of five investment strategies over the periods of 1929 – 2015 and 2002 – 2015. In panels A and B, the leveraged risk parity portfolios are financed at U.S. one-month T-bill rate. The data is obtained from Ibbotson Associates. Rest of the panels are adjusted for one-month LIBOR rate obtained from the FRED. In panels E and F portfolios are adjusted for turnover. Trading costs are 1.0% over 1929 – 1955, 0.5% over 1955 – 1970 and 0.1% over 1970 – 2015. Portfolios are rebalanced annually. The long sample consists of US stocks and government bonds from CRSP database. The broad sample consists of 14 indices of all the available asset classes including stocks, bonds, credit, commodities, real estate and hedge funds obtained from Bloomberg and Datastream by Thomson Reuters. All indices are reported in the table 1. Returns, volatilities and Sharpe ratios are annualized. The excess returns are above the T-bill rate. The excess kurtosis is equal to the kurtosis of the monthly returns minus three. All returns are in U.S. dollars.
Results of the regression analysis are consistent with the existing literature. Asness et al. (2012) reported statistically significant alpha for unleveraged and leveraged RP strategies when the monthly excess returns are regressed against the value-weighted portfolio’s excess returns over the period of 1926 - 2010. The authors document 1.39 % positive alpha for unleveraged strategy and 5.50 % for leveraged strategy whereas this research finds 1.33 % and 3.04 % for the same strategies over the period of 1929 – 2015. The coefficient of determination is relatively low for both regressions in this research indicating that the regression model does not explain much of the variability of the response data around its mean. After adjustments for LIBOR, the alpha of leveraged strategy drops significantly to 1.69 %. However, the alpha remains statistically significant with t-value of 2.21. Anderson et al. (2012) find equivalent results for the unadjusted strategies over the period of 1926 - 2010. The authors document 1.36 % alpha for unleveraged strategy and 3.53 % for leveraged strategy with p-values of 0.00 for both strategies. After adjustments for the three-month Eurodollar deposit rate and trading costs, the alpha decreases to 1.29 % for unleveraged strategy and 0.79 % for leveraged strategy both still being statistically significant results.

Table 4. The regression analysis

<table>
<thead>
<tr>
<th>Base case</th>
<th>Excess Return</th>
<th>Alpha</th>
<th>t-Stat for Alpha</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Long sample 1929 – 2015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Parity – Unleveraged</td>
<td>2.40</td>
<td>1.33</td>
<td>3.92</td>
<td>0.382</td>
</tr>
<tr>
<td>Risk Parity – Leveraged</td>
<td>6.16</td>
<td>3.04</td>
<td>3.98</td>
<td>0.492</td>
</tr>
<tr>
<td>B. Broad sample 2002 – 2015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>5.00</td>
<td>1.45</td>
<td>3.70</td>
<td>0.937</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>8.88</td>
<td>3.17</td>
<td>4.26</td>
<td>0.921</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>5.01</td>
<td>1.57</td>
<td>3.73</td>
<td>0.937</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>8.96</td>
<td>3.29</td>
<td>4.41</td>
<td>0.920</td>
</tr>
<tr>
<td>C. Long sample 1929 – 2015, adjusted for LIBOR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Parity – Unleveraged</td>
<td>2.40</td>
<td>1.33</td>
<td>3.92</td>
<td>0.382</td>
</tr>
<tr>
<td>Risk Parity – Leveraged</td>
<td>4.69</td>
<td>1.69</td>
<td>2.21</td>
<td>0.491</td>
</tr>
<tr>
<td>D. Broad sample 2002 – 2015, adjusted for LIBOR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverse Volatility – unleveraged</td>
<td>5.00</td>
<td>1.45</td>
<td>3.70</td>
<td>0.937</td>
</tr>
<tr>
<td>Inverse Volatility – leveraged</td>
<td>8.65</td>
<td>3.04</td>
<td>4.13</td>
<td>0.921</td>
</tr>
<tr>
<td>Equal Risk Contribution – unleveraged</td>
<td>5.01</td>
<td>1.57</td>
<td>3.73</td>
<td>0.937</td>
</tr>
<tr>
<td>Equal Risk Contribution – leveraged</td>
<td>8.74</td>
<td>3.17</td>
<td>3.95</td>
<td>0.920</td>
</tr>
</tbody>
</table>

Notes: This table provides the results of the regression analysis. RP portfolios’ excess returns are regressed against the excess returns of the equal-weighted portfolios. As the equal risk contribution and the inverse volatility method are identical portfolios in case of two assets, risk parity refers to the both strategies in panels A and C.
According to Asness et al. (2012), the multi-asset RP strategy that includes global stocks, bonds, credit and commodities delivers positive and statistically significant alpha over the period of 1973 – 2010. The unlevered multi-asset IV strategy provided alpha of 1.68 % with a t-value of 2.65 whereas the leveraged strategy delivered positive alpha of 3.03 % with a t-value of 2.52. The results in this research are equivalent over the period of 2002 - 2015. The ERC strategy outperformed the IV strategy by providing the alpha of 1.57 % for unleveraged strategy whereas the IV strategy led to the alpha of 1.45 %. The leveraged ERC multi-asset strategy provided the alpha of 3.29 % whereas the leveraged IV strategy delivered the alpha of 3.17 %. After the strategies are adjusted for more conservative LIBOR and trading costs, the alphas remain as high as 3.17 % for the leveraged ERC and 3.04 % for the leveraged IV. The coefficient of determination is high for all the broad sample regressions indicating that the regression model explains almost all of the variability of the response data around its mean.
Figure 3. Risk parity portfolios vs. equal-weighted portfolio, 2002 - 2015

Figure 4. Inverse volatility – the broad sample asset class weights over 2002 - 2015

Figure 5. Equal risk contribution – the broad sample asset class weights over 2002 - 2015

Figure 6. Risk parity – the long sample asset class weights over 1929 - 2015
4.2 Risk parity strategy in a rising rate environment

The following table illustrates historical performance of the strategies during the different interest rate environments. Table A reports gross returns, annualized volatilities and Sharpe ratios for the portfolio strategies over 1947 – 1979 when interest rates rose moderately. Panel B reports the same figures over 1979 – 1981 when interest rates rose sharply. Panel C presents performance over 1981 – 2015 when interest rates have been falling.

Table 5. Performance analysis across three sub-periods

<table>
<thead>
<tr>
<th>Base case</th>
<th>Gross Return</th>
<th>Volatility</th>
<th>Sharpe</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Moderately Rising Rates, August 1947 – September 1979</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>7.56</td>
<td>6.88</td>
<td>0.56</td>
</tr>
<tr>
<td>Risk Parity – Unleveraged</td>
<td>4.66</td>
<td>2.54</td>
<td>0.37</td>
</tr>
<tr>
<td>Risk Parity - Leveraged</td>
<td>7.07</td>
<td>7.88</td>
<td>0.43</td>
</tr>
<tr>
<td>Risk Parity - Leveraged, adjusted for LIBOR</td>
<td>5.15</td>
<td>7.61</td>
<td>0.19</td>
</tr>
<tr>
<td>T-bill</td>
<td>3.71</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td><strong>B. Sharply Rising Rates, October 1979 - September 1981</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>7.92</td>
<td>9.48</td>
<td>-0.51</td>
</tr>
<tr>
<td>Risk Parity – Unleveraged</td>
<td>6.10</td>
<td>4.98</td>
<td>-1.34</td>
</tr>
<tr>
<td>Risk Parity – Leveraged</td>
<td>0.68</td>
<td>15.49</td>
<td>-0.78</td>
</tr>
<tr>
<td>Risk Parity - Leveraged, adjusted for LIBOR</td>
<td>1.57</td>
<td>15.17</td>
<td>-0.74</td>
</tr>
<tr>
<td>T-bill</td>
<td>12.79</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td><strong>C. Falling Rates, October 1981 - December 2015</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal-weighted portfolio</td>
<td>9.81</td>
<td>7.91</td>
<td>0.71</td>
</tr>
<tr>
<td>Risk Parity – Unleveraged</td>
<td>8.75</td>
<td>4.98</td>
<td>0.92</td>
</tr>
<tr>
<td>Risk Parity – Leveraged</td>
<td>11.82</td>
<td>7.90</td>
<td>0.97</td>
</tr>
<tr>
<td>Risk Parity - Leveraged, adjusted for LIBOR</td>
<td>11.14</td>
<td>7.62</td>
<td>0.91</td>
</tr>
<tr>
<td>T-bill</td>
<td>4.18</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

Notes: As the equal risk contribution and the inverse volatility method are identical portfolios in case of two assets, risk parity refers to the both strategies.

According to Hurst et al. (2013), traditional allocations tend to suffer during the periods of rapidly increasing rates, especially when it happens unexpectedly. Obviously sudden yield hikes hurt directly fixed income investments (both nominal and inflation-linked). Hurst et al. report results in favor of RP strategies despite the heavy allocation to fixed income assets that tend to be vulnerable for interest rate fluctuations. The authors find that the RP strategies have tended to perform well in
environments where interest rates are either steady or falling or rising moderately. For instance, from 1947 to 1979 the 10-year treasury note’s yield rose approximately from 1.8 % to 9.4 %. Hurst et al. find that the RP strategies outperformed both 60/40 strategy of global stocks and bonds as well as 60/40 strategy of US stocks and bonds. However, over the period of 1979 to 1981, the 10-year treasury note rose additional 6.4 % leading to the underperformance of every strategy compared to short-term treasury notes. The RP strategy suffered the most over the period of sharply rising rates.

This research finds deviating results compared to the findings of Hurst et al. (2012) over the same time periods. During the period of moderately rising rates the equal-weighted portfolio outperformed both leveraged and unleveraged RP strategies even before adjustments for more conservative LIBOR as the borrowing rate. After adjustments for LIBOR, the leveraged RP strategy suffered significantly and led to the Sharpe ratio of 0.19 while the equal-weighted portfolio delivered the Sharpe ratio of 0.56. Hurst et al. reported the Sharpe ratio of 0.81 for the RP portfolio over the same period.

During the period of sharply rising rates, the results are consistent with Hurst et al. (2012). The RP portfolio underperformed the equal-weighted strategy. The Sharpe ratio of the RP strategy was -1.35 whereas global and U.S. 60/40 portfolios delivered Sharpe ratios of -0.82 and -0.98, respectively. The volatility of the RP strategy also rose to 14.4 % laying significantly over the target risk level. This research demonstrates how borrowing costs significantly reduce the leveraged RP portfolio’s performance during a rising interest rate environment. The excess returns over the T-bill rate drop down to -12.11 % and -11.22 %. Note that the LIBOR adjusted RP portfolio actually outperformed the T-bill adjusted portfolio during the two-year period when the rates were peaking. The equal-weighted portfolio outperformed the RP strategies over the period with the gross return of 7.92 %. Note that the Sharpe ratio gives counterintuitive results when the excess return is negative and thus it is not adequate to rank portfolio performance according to Sharpe ratios in those situations.

According to Hurst et al. (2013), RP strategies outperformed traditional 60/40 portfolios over 1981 – 2013, the time period when interest rates were falling. This research finds similar results. Even after adjustments for LIBOR, the leveraged RP
strategy delivered the Sharpe ratio of 0.91 whereas the equal-weighted strategy delivered the Sharpe ratio of 0.72.

Hurst et al. (2013) set an annualized target volatility to 10% for the RP portfolio based on the historical 12-month ex-post volatility. The risk levels of the portfolios are equivalent to the strategies that are backtested in this research. However, the research by Hurst et al. does not adjust the portfolios for market frictions such as borrowing costs and trading costs. The authors argue that the market frictions would not materially change the backtest results. This research shows the significant impact of market frictions on portfolio performance especially during turbulent market environments. Even though the RP strategy may be more effectively diversified in terms of risk allocation viewpoint, the strategy is relatively weak for rising interest rate trends.
4.3 Robustness check

This section depicts portfolio returns of RP portfolio minus an equal-weighted portfolio over the long sample and the broad sample. However, the RP strategies are tested using alternative risk free rates as borrowing rates for leveraged strategies. The analysis is carried out by subtracting the monthly returns of an equal-weighted portfolio from the monthly returns of the RP portfolio.

Table 6. Robustness check: the risk parity portfolios minus the equal-weighted portfolio – adjusted for alternative borrowing rates

<table>
<thead>
<tr>
<th></th>
<th>Average Spread over T-bill (bps)</th>
<th>Excess Return</th>
<th>T-Stat. ER</th>
<th>Volatility</th>
<th>Sharpe</th>
<th>Skew</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Long sample 1929 - 2015 - Risk Parity minus Equal-Weighted Portfolio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-bill</td>
<td>0</td>
<td>1.62</td>
<td>2.02</td>
<td>7.45</td>
<td>0.22</td>
<td>-0.06</td>
<td>6.72</td>
</tr>
<tr>
<td>Fed funds</td>
<td>56</td>
<td>0.34</td>
<td>0.43</td>
<td>7.38</td>
<td>0.05</td>
<td>-0.07</td>
<td>7.14</td>
</tr>
<tr>
<td>LIBOR</td>
<td>63</td>
<td>0.14</td>
<td>0.18</td>
<td>7.36</td>
<td>0.02</td>
<td>-0.06</td>
<td>7.22</td>
</tr>
<tr>
<td>3-M Eurodollar DR</td>
<td>105</td>
<td>-0.79</td>
<td>-1.00</td>
<td>7.37</td>
<td>-0.11</td>
<td>-0.11</td>
<td>7.24</td>
</tr>
<tr>
<td><strong>B. Broad sample 2002 - 2015 - Inverse Volatility minus Equal-Weighted Portfolio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-bill</td>
<td>0</td>
<td>2.95</td>
<td>4.00</td>
<td>2.76</td>
<td>1.07</td>
<td>0.41</td>
<td>0.96</td>
</tr>
<tr>
<td>Fed funds</td>
<td>15</td>
<td>2.86</td>
<td>3.88</td>
<td>2.75</td>
<td>1.04</td>
<td>0.41</td>
<td>0.97</td>
</tr>
<tr>
<td>LIBOR</td>
<td>33</td>
<td>2.74</td>
<td>3.71</td>
<td>2.76</td>
<td>0.99</td>
<td>0.42</td>
<td>0.96</td>
</tr>
<tr>
<td>3-M Eurodollar DR</td>
<td>50</td>
<td>2.62</td>
<td>3.55</td>
<td>2.76</td>
<td>0.95</td>
<td>0.40</td>
<td>0.90</td>
</tr>
<tr>
<td><strong>C. Broad sample 2002 - 2015 - Equal Risk Contribution minus Equal-Weighted portfolio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-bill</td>
<td>0</td>
<td>3.02</td>
<td>4.08</td>
<td>2.77</td>
<td>1.09</td>
<td>0.44</td>
<td>1.13</td>
</tr>
<tr>
<td>Fed funds</td>
<td>15</td>
<td>2.93</td>
<td>3.95</td>
<td>2.77</td>
<td>1.06</td>
<td>0.44</td>
<td>1.13</td>
</tr>
<tr>
<td>LIBOR</td>
<td>33</td>
<td>2.81</td>
<td>3.80</td>
<td>2.77</td>
<td>1.02</td>
<td>0.45</td>
<td>1.13</td>
</tr>
<tr>
<td>3-M Eurodollar DR</td>
<td>50</td>
<td>2.70</td>
<td>3.64</td>
<td>2.77</td>
<td>0.97</td>
<td>0.44</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Notes: This table presents portfolio returns of the levered RP portfolio returns minus the returns of an equal-weighted portfolio. For comparison purposes alternative risk-free rates are used as a borrowing cost for the levered RP portfolios. T-bill is the one-month Treasury bill rate. Fed funds rate is the effective federal funds rate obtained from Bloomberg. LIBOR is the one-month London Interbank Offered Rate. 3-M Eurodollar DR is the three month Eurodollar deposit rate from the FRED. If the interest rate was unavailable, I used the one-month T-bill rate added with the average spread over the sample period. As the equal risk contribution and the inverse volatility method are identical portfolios in case of two assets, risk parity refers to the both strategies in panel A.
The results show statistically weak outperformance for the RP strategy over the long sample backtest. The RP portfolio underperforms over 1929 - 2015 only when the leveraged RP portfolio is financed with the most conservative three-month Eurodollar deposit rate.

The broad sample shows more robust results of the outperformance of the RP strategies against the equal-weighted portfolio. The RP portfolio delivered statistically significant results even after adjustments for the most conservative three-month Eurodollar deposit rate.
Figure 7. Inverse volatility portfolio’s leverage ratio - broad sample 2002 - 2015

Figure 8. Equal risk contribution portfolio’s leverage ratio - broad sample 2002 - 2015

Figure 9. Risk parity portfolio’s leverage ratio – long sample 1929 - 2015
4.4 Summary and discussion of the results

This study finds evidence for the outperformance of the RP strategies in comparison to equal-weighted portfolios over both backtesting periods. However, the outcome of the RP strategy is heavily dependent on the investor’s ability to apply leverage with fair borrowing costs as a part of the investment strategy. Indeed, an investor may exploit the unlevered RP strategy which also delivers superior risk-adjusted returns that will however remain relatively low in absolute terms. Relatively low expected returns of unlevered RP strategies basically force RP investors to seek additional returns by applying leverage which naturally increases the risk level but maintains equal risk allocations across the portfolio components. As discussed, many investors face constraints in outright leverage or financial instruments with embedded leverage. Thus, they are not able to apply this strategy in practice.

The leveraged and unleveraged RP portfolios outperformed the equal-weighted portfolio over 1929 – 2015 on risk-adjusted basis when the portfolios consisted of U.S. stocks and government bonds. When the portfolio consists of multiple assets from different asset classes, such as stocks, bonds, credit, commodities, real estate and hedge funds over 2002 - 2015, the results are in the line with the long sample. The results are statistically significant at the 5 % level. The RP strategies delivered higher risk-adjusted returns in the both samples after the performance of the portfolios was adjusted for market frictions. In addition, when the portfolio excess returns were regressed against the excess returns of the equal-weighted portfolio, the RP strategies delivered statistically significant alpha. According to the robustness check, the leveraged RP strategies outperformed the equal-weighted portfolio when the strategies were adjusted for alternative borrowing rates. Based on these findings, we accept the first hypothesis of this study and conclude that the RP strategies outperform the equal-weighted strategy.

As RP investors are exposed to risks that derive from financial leverage as well as higher portfolio turnover, costs of leverage and trading costs are crucial factors having a direct impact on the outcome of the strategies. Hence, this study examines the performance of the portfolios gradually by taking the market frictions into account in order to evaluate their significance to the overall portfolio. As one may assume, the
costs deriving from financial leverage must decrease the portfolio performance as the investor naturally has to carry the borrowing costs. According to the results, the leveraged RP strategy underperformed the unleveraged strategy over 1929 – 2015 after adjustments for turnover and the more conservative borrowing rate. However, the leveraged multi-asset portfolio outperformed the unleveraged strategy over 2002 – 2015 even after the adjustments for LIBOR and portfolio turnover. Both results are statistically significant. Since there is a conflict between these findings, this research uses the longer sample to make a conclusion to the second research hypothesis as it captures the impact of different interest rate environments more effectively. As the leveraged RP portfolios deliver lower risk-adjusted returns than the unleveraged RP strategies, we accept the second research hypothesis.

To achieve a comprehensive overview of the RP strategy performance, this research splits the longer backtesting sample to shorter periods based on the interest rate environments. As the RP strategy typically consists of large allocation to low risk assets that are highly vulnerable to unexpected changes in interest rate movements, I analyzed the performance of every portfolio in the long sample during moderately rising rates, sharply rising rates and falling rates. According to existing literature, the RP strategies have performed relatively well even during the rising rates. Hence, third hypothesis was set in the line with these findings. However, this study finds deviating evidence on the RP portfolio performance during the rising rate market environment. The equal-weighted portfolio outperformed the RP strategy when interest rates rose moderately over August 1947 – September 1979. Based on these findings we reject third hypothesis and conclude that the RP strategy does not outperform in the rising rate environment.

The last hypothesis was set based on the assumption of an optimized ERC portfolio outperforming the IV portfolio. The ERC method takes correlations between asset classes into account when calculating risk contributions whereas the IV method only relies on volatilities of the assets. As correlations act as a fundamental basis for the portfolio diversification and the benefits rising from it, the IV method appears to be an inadequate method. The assumption appeared to be correct as the ERC portfolio delivered higher statistically significant Sharpe ratio than the IV portfolio. The regression analysis also supports the hypothesis as the ERC portfolio provided higher
alpha coefficient in comparison to the IV portfolio. Hence, we accept fourth research hypothesis and conclude that the ERC strategy outperforms the IV strategy.
CONCLUSIONS

The most significant finding of the research is the outperformance of the RP strategies in comparison to the equal-weighted portfolios over the long sample and the broad sample. This finding is consistent with the empirical evidence of the existing literature. This research evaluates the risk parity performance over the long sample which clearly indicates long and persistent outperformance of the strategy as well as highlights low risk anomaly as a source of superior portfolio returns. Nevertheless, the study also highlights the importance of unconstraint portfolio construction methods to take full advantage of the risk parity strategy. Leverage constraints and investors’ unwillingness to apply financial leverage into the portfolios drive prices of riskier asset classes upwards as constrained investors are seeking for greater expected returns from riskier securities. The investors who do not share the same constraints benefit directly as they can keep the main exposures in low risk assets and increase the expected return of the portfolio simply by applying leverage. Even though the RP strategies outperformed the traditional equal-weighted portfolios on risk-adjusted basis, leverage also presents a risk that investors require to be compensated for carrying.

An ongoing interest rate environment affects the RP strategy performance significantly. The RP strategy underperformed the equal-weighted portfolio over the periods of August 1947 and September 1979 when interest rates rose moderately and October 1979 – September 1981 when interest rates rose sharply. During the falling rates, the RP strategy outperformed the equal-weighted strategy. One should notice that the multi-asset data used in this research consists of monthly returns over the extremely abnormal market environment period of January 2002 – December 2015. The RP portfolios delivered superior returns over the above-mentioned period in comparison to the equal-weighted strategy. However, we would need some out-of-sample evidence to make final conclusions on the multi-asset portfolio performance as the interest rate trend has been mainly bearish over 2002 – 2015.

Lastly, when choosing between risk parity methods, investors should choose an equal risk contribution method over an inverse volatility method. The optimized ERC method delivered higher returns with lower portfolio volatility. The differences between the annualized returns of the strategies were surprisingly low, but still high
enough to conclude in favor of the ERC method. This finding is probably the main contribution to the existing academic literature as there is a relatively small number of studies comparing different RP methods.

Indeed, a risk parity strategy is not a magic bullet, but definitely an effective tool to enhance the performance of the portfolio. It is also a rational way to evaluate portfolio holdings from the risk perspective as a true diversification is much more than just dollars invested in stocks and bonds. That said, by applying a risk-based approach in asset allocation one will most certainly end up holding the truly diversified portfolio.
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