MASTER’S THESIS

MEASUREMENTS FOR CALIBRATING PHASED ARRAY TRANSCEIVERS

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ABSTRACT

In future wireless communication standards, millimeter-wave frequencies will be used in order to provide enough bandwidth for the communication channels. In order to mitigate high path loss at very high frequencies, highly directive antenna arrays must be utilized. It is crucial to calibrate the phase and amplitude excitation non-idealities in antenna branches in order to achieve satisfactory performance. The objective of this thesis is to study how non-idealities in array excitation can be measured. Firstly, the effect of errors in excitation phases and amplitudes are studied by performing Monte Carlo simulations. For these simulations, a mathematical model of a uniform linear phased array is developed. The antenna elements in the model are given random, normally distributed phase and amplitude errors and the calculated distributions of defined performance parameters are obtained as an output. These simulation give insight into how accurate measurements must be performed so that the calibration of the array can be done accurately enough. In addition, a measurement system for measuring the phase and amplitude non-idealities of a 5G transceiver prototype is designed. Also a procedure for calibrating the measurement system itself is developed. Finally, Matlab software for controlling the measurement instruments is also developed. The developed measurement system can be used to determine phase and amplitude correction words that will be needed for calibrating the array.

Keywords: phased array, transceiver, calibration, measurement, non-idealities.
TIIVISTELMÄ


Avainsanat: antenniryhmä, lähettin-vastaanotin, kalibrointi, mittaus, epäideaalisuudet.
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FOREWORD

This thesis work was done at Radio Technologies research group at Centre for Wireless Communications, University of Oulu. The research work was done as a part of 5G Champion project. The objective of 5G Champion project is to implement various 5G technologies and demonstrate them at the 2018 Winter Olympics in PyeongChang, Korea. I would like to express my gratitude to RF Excellence in Oulu project that funded this thesis work.

I would like to thank Professor Aarno Pärssinen for offering me such an interesting thesis topic and for guiding me along the way. I would also like to thank Lic.Sc (Tech.) Risto Vuohtoniemi for all the guidance and help during my thesis work. Many thanks to doctoral student Nuutti Tervo for giving me a great amount of help during my thesis work and my traineeship. Thanks to doctoral student Alok Sethi for helping me to get started with the phased array simulation model.

I would also like to use this opportunity to thank other important people in my life. Thanks to my good friends with whom I have spent countless of memorable moments together. Last but not least, I would like to thank my parents for all the valuable support and encouragement.

Oulu, Finland, April 24, 2017

Ville Hevosmaa
# LIST OF ABBREVIATIONS AND SYMBOLS

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<td>5G</td>
<td>fifth generation</td>
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<tr>
<td>ADC</td>
<td>analog-to-digital converter</td>
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<td>AF</td>
<td>array factor</td>
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<td>ASIC</td>
<td>application-specific integrated circuit</td>
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<td>BIST</td>
<td>built-in self-test</td>
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<tr>
<td>DAC</td>
<td>digital-to-analog converter</td>
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<td>DC</td>
<td>direct current</td>
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<td>DOA</td>
<td>direction of arrival</td>
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<td>DSP</td>
<td>digital signal processing</td>
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<td>DUT</td>
<td>device under test</td>
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<td>EMF</td>
<td>electromotive force</td>
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<td>ESPRIT</td>
<td>estimation of signal parameters via rotational invariance technique</td>
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<td>FNBW</td>
<td>first-null beamwidth</td>
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<td>FPGA</td>
<td>field-programmable gate array</td>
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<td>GSG</td>
<td>ground-signal-ground</td>
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<td>HPBW</td>
<td>half-power beamwidth</td>
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<td>IF</td>
<td>intermediate frequency</td>
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<td>IFBW</td>
<td>intermediate frequency bandwidth</td>
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<td>I/Q</td>
<td>in-phase/quadrature</td>
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<td>IoT</td>
<td>internet of things</td>
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<td>LO</td>
<td>local oscillator</td>
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<tr>
<td>LTE-A</td>
<td>long-term evolution advanced</td>
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<td>MIMO</td>
<td>multiple-input-multiple-output</td>
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<td>MMSE</td>
<td>minimum mean square error</td>
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<td>MoM</td>
<td>method of moments</td>
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<td>MSNR</td>
<td>maximum signal-to-noise ratio</td>
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<td>MUSIC</td>
<td>multiple signal classification</td>
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<td>MV</td>
<td>minimum noise variance</td>
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<td>NEC</td>
<td>numerical electromagnetics code</td>
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<td>NF</td>
<td>noise figure</td>
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<td>PPF</td>
<td>poly-phase filter</td>
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<td>REV</td>
<td>rotational electric field vector</td>
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<td>RF</td>
<td>radio frequency</td>
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<td>RX</td>
<td>receiver</td>
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<td>SDMA</td>
<td>space-division multiple access</td>
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<td>SLL</td>
<td>sidelobe level</td>
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<tr>
<td>SNOI</td>
<td>signal not of interest</td>
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<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
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<tr>
<td>SOI</td>
<td>signal of interest</td>
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<tr>
<td>TX</td>
<td>transmitter</td>
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VCO  voltage-controlled oscillator
VGA  variable-gain amplifier
VNA  vector network analyzer

$AF$  array factor
$AF_{\text{max}}$  maximum value of array factor
$A_l$  in-phase amplitude
$a_{\text{err},n}$  amplitude error of the $n$th element
$a_m$  $m$th Fourier coefficient
$a_n$  excitation amplitude of the $n$th element
$A_{\text{ch0}}(\theta,\phi)$  element pattern matrix without mutual coupling
$A_{\text{ch1}}(\theta,\phi)$  element pattern matrix with mutual coupling
$A_Q$  quadrature amplitude
$C$  capacitance
$C_{\text{cal}}$  calibration coefficient
$C_{\text{tot}}$  total capacitance of a transmission line
$d$  element-spacing
$D_0$  error-free directivity
$D(\theta,\phi)$  antenna directivity in direction $(\theta,\phi)$
$E(r)$  electric field at point $r$
$E(r,\theta,\phi)$  electric field at distance $r$ in direction $(\theta,\phi)$
$E(x,y)$  total electric field at point $(x,y)$
$E_0(x,y)$  electric field of a single element at point $(x,y)$
e_c  conduction efficiency
e_d  dielectric efficiency
e_o  total efficiency
e_r  reflection efficiency
$f_c$  operating frequency
$f(r,\theta,\phi)$  element radiation pattern at distance $r$ in direction $(\theta,\phi)$
$F(z,A,n)$  synthesized Taylor $n$-bar pattern
$g_{av}(\theta,\phi)$  average active-element pattern in direction $(\theta,\phi)$
$g_{avn}(\theta,\phi)$  active-element pattern of the $n$th element in direction $(\theta,\phi)$
g_1(u)  isolated element pattern
g_n(u)  radiation pattern of the $n$th element
$G_e$  single-element gain
$G(\theta,\phi)$  antenna gain in direction $(\theta,\phi)$
$I_n$  excitation current of the $n$th element
$k$  wave number
$L$  inductance
$L_{\text{tot}}$  total inductance of a transmission line
$M$  coupling matrix
N number of antenna elements
Nmc number of Monte Carlo iterations
P power
Pi input power
Pmc incidence power
Pm power delivered to the mth element
Prad total radiated power
PQL power level of the first quantization lobe
RA antenna resistance
RL load resistance
R_{mnpq} mutual resistance between elements (m,n) and (p,q)
Ri radiation resistance
SLL sidelobe level
Snmn s-parameter from port n to port m
Sx array factor of a rectangular array in x-direction
Sy array factor of a rectangular array in y-direction
\tau time
T_m(z) mth order Chebyshev polynomial
U(\theta,\phi) radiation intensity in direction (\theta,\phi)
U_0 radiation intensity of an isotropic radiator
U_{\text{max}} maximum radiation intensity
V vector of element voltages
V_\infty vector of open-circuit element voltages
V_I induced voltage
XA antenna reactance
xi position of the ith element divided by interelement spacing
Z impedance matrix
Z_0 characteristic impedance
ZA antenna impedance
Zi input impedance

\beta progressive phase shift between antenna elements
\Delta^2 variance of beam direction deviations
\Delta\phi phase difference
\Gamma reflection coefficient
\theta_0 steering angle
\lambda wavelength
\sigma_a standard deviation of amplitude errors
\sigma_\phi standard deviation of phase errors
\psi quantity that characterizes the frequency, spacing, steering angle and progressive phase shift between antenna elements
\omega angular frequency
\phi phase shift
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<th>Symbol</th>
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<tr>
<td>$\phi_0$</td>
<td>phase separation between phase shifter states</td>
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<tr>
<td>$\phi_{\text{err},n}$</td>
<td>phase error of the $n$th element</td>
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<tr>
<td>$\phi_n$</td>
<td>phase of the $n$th element</td>
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1. INTRODUCTION

In the recent years, the growth of the number of wireless devices and wireless data traffic has been enormous, and this growth will continue even more in the near future. For example, people are constantly using more and more multimedia-rich mobile services, such as high-definition video streaming and sharing images and videos in social media. According to Ericsson [1], the mobile phone data traffic exceeded the total mobile data traffic in 2014. In the same publication, Ericsson forecast that there will be 9 billion mobile subscriptions by the year 2020. In the near future, wireless connectivity will also be utilized even more in Internet of Things (IoT) applications where any device or object can be connected in order to enable novel services. These services may include smart homes, fitness and healthcare applications and smart offices, as envisioned by Samsung [2]. In addition to human-centric use-cases, wireless connectivity will also be utilized in so-called “mission-critical services”, that may include for example autonomous vehicles, smart electricity distribution networks or “smart grids”, and industrial automation [3]. According to Nokia, there will be 10-100 times more of connected devices, over 10 Gbps peak data rates and 10 000 times more data traffic in the future [4].

The growing amount of data-traffic and new applications and use-cases set very high demands on data speeds, channel capacities and latencies. These demands cannot be satisfied with the current wireless communication standards, such as the Long-Term Evolution Advanced (LTE-A). Therefore, industry and research institutions have begun developing the fifth-generation (5G) wireless standard. According to Shannon-Hartley theorem, more bandwidth is needed in order to increase the network capacity. The 5G technology will therefore have to utilize much higher frequencies where huge bandwidths are available. The disadvantage of the utilization of higher frequencies is increased propagation loss. At higher frequencies, the propagation loss of electromagnetic waves increases because of atmospheric effects, such as absorption by water and oxygen molecules (Figure 1.1). One way to overcome this disadvantage is to use antenna arrays. The gain of an antenna array is much greater than that of a single antenna elements. The gain of an antenna array is \( N \) times greater than the single-element gain, where \( N \) is the number of antenna elements. The utilization of antenna arrays in 5G communication systems is possible because of higher frequencies. At higher frequencies, the sizes of antenna elements are small, and therefore antenna arrays can be integrated in compact devices, such as mobile phones.

Because the beams of antenna arrays are narrow, the beams of the transmitting and receiving devices must be aligned, i.e. the beam of the transmitting antenna must be directed towards the receiving antenna and vice versa. This alignment can be done by using a technique called beam steering which can be realized by progressively shifting the signal phases between successive antenna branches. When the phases between signals arriving to each antenna are different, the electric fields radiated by the antennas are interfered with each other in some distinct direction. The angle of this direction depends of the value of the progressive phase shift between elements. Also some other properties of the radiation pattern of an antenna array, in addition to beam direction, can be adjusted by setting specific amplitudes for the signals arriving to each antenna.
In order to obtain the desired radiation pattern, the phase and amplitude differences between antenna branches must be set very accurately. In practice, there exists several kinds of non-idealities in antenna array transceiver that affect the phases and amplitudes in different branches and therefore deteriorate the radiation pattern. These phase and amplitude errors can be either systematic or random. Systematic errors include for example the electromagnetic coupling between antenna branches. Systematic errors can be calibrated away. Random errors are caused by random variation of phase and amplitude of the RF components. It is impossible to calibrate the random errors away and therefore they set the ultimate limit for the phase and amplitude accuracy. In order to achieve the best possible performance for the antenna array radiation pattern,

1) the RF components must be designed so that the random phase and amplitude variations are as low as possible

2) the systematic errors must be calibrated away as accurately as possible.

In this thesis work it is studied, how phase and amplitude errors affect different characteristics of the radiation pattern of an antenna array. Theoretical and mathematical analysis of these effects has been conducted in various books and research papers, for example [5, 6, 7]. However, there has been only little simulation based research about the random phase and amplitude effects. Some of the few research papers are presented in [8, 9]. The simulations presented in these research papers are not comprehensive enough; in [8] it is only studied how the random errors affect the main lobe power, and in [9] the effect of random errors on the array pattern was studied mostly qualitatively. There is an interest to study how random errors

Figure 1.1. Atmospheric attenuation as a function of frequency (temperature 15°C, pressure 101.3 kPa, water vapor density 7.5 g/m³). Free-space path loss is not included in the curve.
affect other array pattern performance metrics as well. In this thesis work, the effect of random errors on various antenna array pattern characteristics is studied by performing Monte Carlo simulations with Matlab. In these simulation, a Matlab model for a uniform linear array is given random, Gaussian distributed phase and amplitude errors and the values of defined performance parameters of the resulting array factors are calculated. The simulations are performed with various combinations of amplitude and phase error variances, numbers of antenna elements and steering angles. The results are expected to give insight into how accurate phase and amplitude measurements must be performed so that an antenna array can be calibrated accurately enough.

In addition, a measurement system for phase and amplitude error measurements for a 5G transceiver prototype is designed. First, accuracy of different phase measurement methods is studied. This is important because measurement errors will lead to poor calibration and therefore to worse performance. The schematics of the measurement system for measurements in both transmit and receive modes are created and a measurement procedure is planned in detail. A procedure of calibrating the measurement system itself is also planned. In addition, control software for measurement instruments is developed.

The structure of the thesis is as follows. In Chapter 2, theoretical background considering antenna elements and antenna arrays is presented. Some important terms and concepts are explained and the most important equations characterizing the operating principles are presented. Various concepts regarding whole phased array transceivers are presented in Chapter 3. Some of the most important phased array transceiver architectures are explained, and the advantages and disadvantages of these architectures are compared. Some phase shifter and feed network topologies are presented. Principles of adaptive arrays are discussed briefly. Some of the most important pattern synthesis methods are presented. In Chapter 4, theoretical and mathematical background of the effects of random phase and amplitude errors, quantization effects and mutual coupling are discussed. The effect of random phase and amplitude errors is studied by performing Monte Carlo simulations with Matlab. The simulation results are presented and analyzed. Chapter 5 contains a literature review of various measurement methods for phased array calibration. In Chapter 6, the most promising phase difference measurement methods are tested. Based on these tests, a suitable measurement method is selected for prototype measurements. In Chapter 7, a phase and amplitude measurement system and procedure for a 5G transceiver prototype are designed. The conclusions of the thesis work are discussed in Chapter 8.
2. ANTENNA ARRAYS

2.1. Fundamental antenna parameters

2.1.1. Radiation pattern and beamwidth

A radiation pattern is a graphical representation of the radiation intensity of an antenna as a function of angle. It typically shows the radiation properties in the far-field. The radiation pattern can be presented in various different ways in either polar, spherical or cartesian coordinate system. The pattern in the polar and spherical coordinates give insight into the manner in which the antenna actually emits radiation into the space, while the cartesian coordinate representation is useful for determining the levels and the positions of the peaks. When using polar coordinates, the antenna pattern can be shown in any plane, most typically in a horizontal plane as a function of azimuth angle. The radiation pattern can be determined based on either field strength or power and it can be shown in either linear or logarithmic scale [10].

The radiation patterns can be divided into three categories depending on their symmetrical properties. An isotropic radiator is a lossless antenna that radiates with the same intensity to all directions. Isotropic radiators are hypothetical but they are often used as a reference when characterizing radiation patterns of real antennas. In practice all antennas are directional, which means that they transmit or receive radiation more effectively in some specific directions. An omnidirectional antenna has a non-directional pattern in some plane but directional pattern in any orthogonal plane. An omnidirectional pattern can be considered as a special type of directional patterns [11].

The radiation pattern has distinct lobes that are illustrated in Figure 2.1. The main lobe contains the angle of maximum radiation. There can also be more lobes that have the same magnitude than the main lobe. These lobes are called grating lobes and they are usually undesired. All the lobes other than the main lobe are called minor lobes. Minor lobes can be subdivided into the back lobe and side lobes. Back lobe is located in the opposite direction compared to the main lobe. Any other minor lobes are side lobes [10, 11].

Beamwidth is a quantity that means the angle between two distinct points in which the main lobe power is above a certain threshold. The most common definitions for beamwidth are half-power beamwidth (HPBW) and first-null beamwidth (FNBW). Half-power beamwidth means the angle the points where the power is half of the maximum power. First-null beamwidth means the angle between the first nulls next to the main lobe [10].
2.1.2. Field regions

The electromagnetic field created by an antenna can be subdivided into three regions based on the electromagnetic properties of the field. These regions are reactive near-field region, radiating near field (Fresnel) region and far-field (Fraunhofer) region. The field regions are illustrated in Figure 2.2 [11].

The region closest to the antenna, reactive near-field region, is defined as the region where the reactive field predominates. That means that the majority of the energy is stored into the electromagnetic field and the rest of the energy is radiated away from the antenna. This region can be considered to be located at a distance $R_1 < 0.62\sqrt{D^3/\lambda}$, where $D$ is the largest dimension of the antenna and $\lambda$ is the wavelength. The radiating near-field region is located between reactive near-field region and far-field region, i.e. in the area $0.62\sqrt{D^3/\lambda} \leq R_2 < 2D^2/\lambda$. In this region the radiating field is predominant and the angular field distribution (the shape of the field) is also a function of the distance from the antenna [11].

In the far-field region the shape of the field pattern does not depend on the distance from the antenna. In this region, the electric and magnetic fields are in the same phase, and their field vectors are orthogonal to each other. Therefore, all of the energy in this region is radiated away from the antenna. The far-field region is considered to be located in the area where $R > 2D^2/\lambda$ [12, 13].
2.1.3. Input impedance and efficiency

The input impedance of an antenna is defined as

\[ Z_A = R_A + jX_A, \]  \hspace{1cm} (2.1)

where \( R_A \) and \( X_A \) are the resistance and reactance of the antenna, respectively [11]. The resistive part represents dissipation and it can be divided into two components:

\[ R_A = R_r + R_L, \]  \hspace{1cm} (2.2)

where \( R_r \) is the radiation resistance and \( R_L \) is the loss resistance [11]. Radiation resistance \( R_r \) represents the power that leaves the antenna as radiation, and it is one form of dissipation. Loss resistance \( R_L \) corresponds to another form of dissipation, namely the energy that is converted into heat in an antenna. Usually, the ohmic losses are significant only on electrically small antennas. The input reactance \( X_A \) is the power that is stored in the near field of an antenna. The equivalent circuit of an antenna in receiving mode is presented in Figure 2.3 [11]. In the figure, \( V_T \) is the voltage induced by an impinging wave and \( R_T \) and \( X_T \) are the load resistance and load reactance of the antenna, respectively [10, 11].

In an ideal antenna, all the power that is fed into the antenna is transformed into radiation. In reality, the efficiency of an antenna is limited by reflections and ohmic losses. The total efficiency of an antenna is defined as
\[ e_o = e_r e_c e_d , \]  

(2.3)

where \( e_r \) is the reflection efficiency, \( e_c \) is the conduction efficiency and \( e_d \) is the dielectric efficiency [11]. In Equation 2.3, \( 0 \leq e_r, e_c, e_d \leq 1 \), where 1 means maximum efficiency and 0 means zero efficiency. The reflection efficiency is defined as

\[ e_r = 1 - |\Gamma|^2 = 1 - \left| \frac{Z_i - Z_0}{Z_i + Z_0} \right|^2 , \]

(2.4)

where \( \Gamma \) is the reflection coefficient and \( Z_i \) and \( Z_0 \) are the antenna input impedance and the characteristic impedance of the transmission line, respectively [11]. To maximize the radiation efficiency, the antenna input impedance should be matched to the characteristic impedance of the transmission line, i.e. the impedances are real and \( Z_i = Z_0 \). Otherwise the reflection coefficient will get a non-zero value and radiation efficiency will be less than 1. The conduction efficiency and dielectric efficiency together are defined as

\[ e_{cd} = \frac{R_r}{R_L + R_r} \]

(2.5)

[11]. This means that the conduction-dielectric efficiency is maximized when the loss-resistance \( R_L \) is zero [11].

Figure 2.3. Equivalent circuit of an antenna in receiving mode.
2.1.4. Directivity and gain

The ability of an antenna to focus its radiation to a certain direction in space is characterized by *directivity*. Directivity of an antenna is defined as the ratio of the radiation intensity in a certain direction $U(\theta, \phi)$ to the radiation intensity of an isotropic source, or

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_0} = \frac{4\pi U(\theta, \phi)}{P_{rad}}, \quad (2.6)$$

where $P_{rad}$ is the total radiated power [11]. If the direction is not specified, directivity refers to the maximum directivity which is the ratio of the maximum radiation intensity $U_{\text{max}}$ to the radiation intensity of an isotropic source [10, 11]:

$$D(\theta, \phi) = \frac{4\pi U_{\text{max}}}{P_{rad}}. \quad (2.7)$$

Directivity of an antenna is solely determined by the radiation pattern of the antenna and it does not take into account the efficiency of an antenna when the antenna is used in a system. *Gain* of the antenna takes the efficiency into account. Mathematically it is defined as

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_i}, \quad (2.8)$$

where $P_i$ is the total input power [10]. If the direction is not specified, it is assumed to be that of the maximum radiation intensity:

$$G = \frac{4\pi U_{\text{max}}}{P_i}. \quad (2.9)$$

Gain can also be presented as a function of directivity as follows:

$$G = e_{cd} D, \quad (2.10)$$

where $e_{cd} = P/P_{in}$ is radiation efficiency. Here $P_{in}$ is the total input power and $P$ is the part of the input power that will not be transformed into radiation in the antenna [10].
2.2. Fundamentals of antenna arrays

An antenna array simply means a group of antennas that has been arranged to some geometrical configuration. An antenna array can have a fixed main beam direction or it can scan the main beam in space by changing the relative phases between antennas. In the latter case the antenna array is called a phased array. The advantage of antenna arrays is that they can produce highly directive beams and therefore increase the link budget of a communication system. The utilization of antenna arrays is a crucial concept in future 5G communication systems where high antenna gain is needed to overcome high propagation losses at high frequencies. Beam steering is needed in order to align the beams of a base station and user equipment when the user is on the move. Antenna arrays also enable such techniques as multiple-input-multiple-out (MIMO), in which information can be transmitted and received simultaneously with multiple antennas, and spatial filtering, which is a technique for reducing interference caused by ambient signals.

2.2.1. Linear arrays

The simplest geometrical configuration of antenna arrays is a linear array in which the individual antenna elements are located in a straight line [10]. Linear arrays are one-dimensional. In its simplest form, a linear array is uniformly excited, equally spaced array with an ideal, isotropic radiator as a single array element. Uniform excitation means that every element in an array is driven with uniform current (or voltage) signals with identical phases.

Figure 2.4. shows an arbitrary array of antenna elements [5]. The total radiation pattern of an antenna array can be calculated by summing the electric fields of all array elements. The far-field of the $i$th element at a distance $r$ in direction $(\theta, \phi)$ is

$$E_i(r, \theta, \phi) = f_i(\theta, \phi) \exp(-jkR_i/R_i), \quad (2.11)$$

where $f_i(\theta, \phi)$ is the radiation pattern of the $i$th element and the exponential term describes the effect of the distance of the $i$th element from the origin on the electric field of the corresponding element. If it is assumed that the pattern is measured on a sphere that has a constant radius, the normalization factor in Equation (2.11) can be removed so that it becomes

$$E_i(r, \theta, \phi) = f_i(\theta, \phi). \quad (2.12)$$

By summing the electric fields of all elements, the total electric field becomes

$$E(r) = \sum_{i=0}^{N-1} a_i f_i(\theta, \phi) \exp(jkr \cdot \hat{r}), \quad (2.13)$$
where $a_i$ is the excitation amplitude of the $i$th element, $r_i$ is the vector that points to the location of the $i$th element and $r$ with a circumflex is a unit vector directed away from the origin. Assuming that all elements have the same radiation pattern, which usually is the case, Equation (2.13) becomes

$$E(r) = f(\theta, \phi) \sum_{i=0}^{N-1} a_i \exp(jkr_i \cdot \hat{r}),$$

where $N$ is the number of elements [5].

Equation (2.14) shows an important principle considering antenna arrays: the total radiation pattern of an antenna array can be calculated by multiplying the pattern of a single element with a sum term that describes the excitation amplitudes and the locations of all elements. This is called the principle of pattern multiplication and it is valid for any array with identical elements that may have any amplitude and phase excitations and spacings between them. The sum term in Equation (2.14) is called the array factor. The array factor of a uniformly spaced linear array is usually expressed in the form

$$AF = \sum_{n=0}^{N-1} a_n e^{jn\psi},$$

where $\psi = kd \cos\theta + \beta$, where $k$ is the wave number, $d$ is the spacing between elements and $\beta$ is the progressive phase shift between elements [10].

Figure 2.4. General array configuration.
The parameters that affect the form of the radiation pattern of a linear array are the spacing between elements, the amplitude and phase excitation of each element and the radiation pattern of a single element. These parameters can be set in a suitable way in order to obtain the desired radiation pattern. The spacing and phase difference between the elements affect the way in which the electric fields of the elements add in all directions; they can add either constructively or destructively. By setting a same phase for each element and keeping the amplitudes constant, only the spacing between elements affects the shape of the array pattern. There exists two extreme cases: a broadside array and an end-fire array. In a broadside array the main lobe is directed perpendicularly to the axis of the array. In end-fire array the main-lobe is directed along the axis of the array. Figure 2.5 shows the array factor plots for a broadside and an end-fire array with 8 elements and half-wavelength and unity wavelength spacings, respectively. It can be seen that broadside array pattern is much more directive. Therefore, antenna arrays are usually utilized in the broadside mode [11]. In fact, usually the wide beams, e.g. the beams pointing at directions 90° and -90° in Figure 2.5(b) are considered harmful. These unwanted beams that have the same level as the main beam are called grating lobes. To avoid grating lobes, spacing between antennas in a full-scan array must be chosen so that [6]

\[
\frac{d}{\lambda} \leq \frac{1}{2}.
\] (2.16)

The technique for changing the direction of the main beam is called beam steering. This means that the direction of the main beam can be steered by setting a progressive phase shift between the elements, hence the name phased array. In Equation (2.15) the parameter \( \psi \) is defined mathematically as

\[
\psi = kd \cos \theta + \beta,
\] (2.17)

where \( \beta \) is the progressive phase shift between elements. If the main beam is to be scanned to direction \( \theta_0 \), the progressive phase shift between elements must be adjusted so that

\[
\beta = kd \cos \theta_0
\] (2.18)

[11]. Figure 2.6 shows an example of beam steering. When beam steering is used, the condition for avoiding grating lobes is changed into form

\[
\frac{d}{\lambda} \leq \frac{1}{1 + \sin |\theta_0|}
\] (2.19)

[14]. When steering angle is \( |\theta_0| = 90^\circ \), the above equation becomes the same as Equation (2.16).
Figure 2.5. Radiation patterns for (a) a broadside \((d = \lambda/2)\) and (b) an end-fire \((d = \lambda)\) array with 8 elements.

Figure 2.6. Array pattern of an 8-element linear array with spacing \(d = \lambda/2\) and steering angle (a) \(\theta_0 = 0^\circ\) and (b) \(\theta_0 = 30^\circ\).

### 2.2.2. Planar arrays

In planar arrays the elements have been arranged into a two-dimensional geometrical configuration. The simplest form of planar arrays is the uniformly spaced rectangular array, as shown in Figure 2.7. As in the case of linear arrays, the array factor of a planar array can also be calculated by summing the electric fields of all elements together. For a uniformly spaced rectangular array the array factor can be calculated by treating the rows and columns as separate linear arrays. The array factor is
\[
AF = S_{xm} S_{yn},
\]
(2.20)

where \(S_{xm}\) and \(S_{yn}\) are the array factors of a single row of elements and a single column of elements, respectively \([11]\). Mathematically they are defined as follows:

\[
S_{xm} = \sum_{m=1}^{M} a_{m1} \exp\left(j(m-1)(k d_x \sin \theta \cos \phi + \beta_x)\right)
\]
(2.21)

and

\[
S_{yn} = \sum_{n=1}^{N} a_{1n} \exp\left(j(n-1)(k d_y \sin \theta \sin \phi + \beta_y)\right),
\]
(2.22)

where \(a_{m1}\) and \(a_{1n}\) are the excitation amplitudes of the elements in x- and y-directions, respectively, \(d_x\) and \(d_y\) are the element spacings in x- and y-directions and \(\beta_x\) and \(\beta_y\) are the progressive phase shifts in x- and y-directions \([11]\). The parameters are illustrated in Figure 2.7 \([11]\).

Figure 2.7. General rectangular planar array configuration.

The radiation pattern of a planar array differs from that of a linear array. The three-dimensional radiation pattern of a linear array is omnidirectional, i.e. it has a non-directional pattern in some plane and directional pattern in any perpendicular plane. The radiation pattern of a planar array, on the other hand, is not omnidirectional, and therefore it is much more directive. Another advantage of planar arrays over the linear arrays is that they can steer the beams three-dimensionally, i.e. in both the azimuth and the elevation directions. This can be understood by considering Equations (2.21) and (2.22) for a rectangular array. By changing the progressive phase shift between the elements in x-direction, the beam can be steered in any direction in respect to the x-direction. Similarly, by adjusting the progressive phase shifts between the elements in y-direction, the beam can be
steered in respect to the y-directions. An example of a three-dimensional array pattern for an 8-by-8 rectangular array with beam steering is presented in Figure 2.8.

![Three-dimensional array factor pattern](image)

Figure 2.8. Three-dimensional array factor pattern of an 8-by-8 planar array with an element spacing of \(d_x = d_y = \lambda/2\), steered to 30° in both azimuth and elevation angles.

### 2.2.3. Directivity of arrays

Directivity of an antenna array is greater than that of a single element. This is because the electric fields produced by the elements interfere constructively in some directions and destructively in some other directions, and thus the maximum level of the radiation pattern increases while the total radiated power remains the same. Directivity of a linear array of omnidirectional elements is the ratio of the array factor maximum and the total array factor, or

\[
D = \frac{4\pi |AF_{max}|^2}{\int_0^{2\pi} \int_0^\pi |AF|^2 \sin \theta \, d\theta \, d\phi}.
\]

(2.23)

A simplified equation for directivity can be derived for various specific cases. All the following equations yield the directivity as a dimensionless value. The value in
decibels can be calculated as $10 \cdot \log(D)$. For a linear, uniformly spaced array with uniform amplitude distribution, the directivity can be calculated from

$$D = \frac{N^2}{N + 2 \sum_{n=1}^{N-1} \sin(nkd) \cos(nkd \cos \theta_0)},$$  

(2.24)

where $N$ is the number of elements, $k$ is the wave number, $d$ is the spacing between elements and $\theta_0$ is the steering angle [5]. The directivity of a linear, broadside array ($d \ll \lambda$), with uniform spacings and uniform amplitude distribution, is given by

$$D \approx 2N \left( \frac{d}{\lambda} \right),$$  

(2.25)

which, when the element spacing is $d = \lambda/2$, simply becomes

$$D = N$$  

(2.26)

Equation (2.23) can also be used to derive equations for planar arrays with any geometrical configuration. For rectangular planar array, the directivity can also be approximated as

$$D \approx 2D_x D_y,$$  

(2.27)

where $D_x$ and $D_y$ are the linear array directivities of the rows and columns of a planar array [11].

Figure 2.9. illustrates how the element spacing and the number of elements affect the directivity of an antenna array. Figure 2.9.(a) shows the directivity plots for a linear, uniformly spaced and uniformly excited array. It can be seen that the directivity increases as the spacing increases. As the spacing approaches one wavelength, the first grating lobes start to form and the directivity decreases rapidly. After that the directivity again increases as a function of the element spacing. The same effect also happens with planar arrays, as shown in Figure 2.9.(b). From the figure it can been seen that increasing the number of elements also increases the directivity.
Figure 2.9. Directivity as a function of element spacing for different numbers of elements. (a) Uniform linear array. (b) Uniform planar (rectangular) array.
3. PHASED ARRAY TRANSCEIVERS

3.1. Antenna array transceivers

As it was shown in Chapter 2, the direction of the main beam of an antenna array can be changed by changing the spacing or the relative phases between elements. In *phased arrays* the main beam direction can be changed by changing the relative phases between array elements using phase shifters. Along with the main beam, also the nulls of the pattern can be steered. Additionally, the relative amplitudes of antenna branches can be adjusted with variable gain amplifiers (VGA), or alternatively with variable attenuators. By tuning the amplitudes, the level of different lobes can be adjusted. These techniques are beneficial for the future millimeter-wave communication systems for three main reasons. Firstly, with phased arrays highly directive beams can be formed in order to compensate high propagation losses, and, on the other hand, the beams must be steered in order to align the beams of the transmitter and receiver. Secondly, interference can be mitigated by steering the nulls of the radiation pattern towards interfering signals, i.e. a phased array can act as a *spatial filter*. Thirdly, highly directive beamforming enables *space-division multiple access* (SDMA), which means that the users connected to the same base station can be separated in space-domain.

Figure 3.1 shows a general structure of a phased array transceiver. The RF front-end generally consists of several branches, each of which has a variable phase shifter, a variable gain amplifier and an antenna element. Between the branches and the transceiver (intermediate frequency (IF) and baseband stages) there is a feed network that distributes signals to each branch. Finally, there is a computer or a microcontroller that sends digital words via digital control circuitry to the phase shifters and amplifiers in order to control their phase shift and gain values.

![Figure 3.1. General structure of a phased array transceiver.](image-url)
3.2. Transceiver architectures

There are various kinds of architectures for phased array transceivers and they can be
categorized in many ways. The most typical way to categorize phased array
transceivers is the implementation of phase shifting. Based on the type of phase
shifting, the architectures can be divided roughly into four categories: RF phase
shifting, LO phase shifting, analog baseband phase shifting and digital baseband
phase shifting architectures (Figure 3.2). Each of them have different trade-offs
regarding hardware, power consumption and complexity [15].

In RF phase shifting architecture (Figure 3.2 (a)) phase shifting is
implemented with analog phase shifters that are located in the RF path after the up-
converting mixer. The advantage of this architecture is that it requires the smallest
amount of components compared to other architectures. Therefore it features the
lowest cost and power consumption. RF phase shifting architecture is especially
suitable for discrete circuit implementations because of the routing challenges in
discrete circuits and the high cost of discrete components. In receivers this
architecture has an additional advantage; because the signals are combined before the
mixer, the signals coming from undesired directions are canceled out, which leads to
relaxed linearity and upper dynamic range requirements for the mixers [16, 17].

In LO phase shifting architecture (Figure 3.2 (b)) the analog phase shifters are
located in the LO path. The main advantage of the LO phase shifting architecture
over the RF phase shifting architecture is that because the phase shifters are not on
the signal path, the do not have as much impact on the loss, non-linearity and noise
in the signal path. Also, by applying the phase shift on the LO path, the phase shift is
constant across the whole RF frequency range. [18] Therefore, there is no
requirements for the bandwidth of the mixers. However, this architecture also has
disadvantages compared to the RF phase shifting architecture. The biggest
disadvantage is that a separate mixer is needed for each of the RF branches which
increases the cost of the transceiver. Luckily, the greater amount of mixers does not
increase the power consumption compared to the RF phase shifting architecture
because in LO phase shifting architecture one mixer drives only one power amplifier.
By power consumption’s point of view this corresponds driving all the power
amplifiers with one mixer as it is done in the RF phase shifting architecture. Another
drawback of the LO phase shifting architecture is the large LO routing network that
may be very power-consuming, may pose challenges in design and also makes the
circuit more sensitive to noise. The routing network can also cause I/Q imbalance
effect that must be calibrated away [16, 19].

One additional drawback of the RF and LO phase shift architectures is the
need for high frequency phase shifters. These phase shifters have losses that are not
constant over the frequency range and this gain variation need to be compensated.
This problem can be avoided by using the baseband phase shifting architecture. In
analog baseband phase shifting architecture (Figure 3.2 (c)) the phase shifts of the
RF branches are generated in the analog baseband-domain. Compared to RF and LO
phase shifting, at analog baseband the phase shift can be achieved with higher
resolution and lower power-consumption. Furthermore, because there are no phase
shifters in the RF domain in analog baseband phase shifting architecture, the noise
figure (NF) is also lower. One drawback of this architecture is the need for a
complex LO distribution network, as in the case of LO phase shifting architecture.
Also, the spatial filtering is performed in analog baseband stage which requires better linearity for the mixers [16].

In digital baseband phase shifting architecture phase shifts are implemented digitally in a digital signal processing (DSP) block (Figure 3.2 (d)). As the analog baseband phase shift architecture, the digital baseband phase shift architecture also lacks the need for phase shifters in the RF domain which leads to smaller noise figure. The DSP block also allows to use complex beamforming algorithms and Multiple-Input Multiple-Output (MIMO) algorithms that are necessary in practical phased array transceivers. However, one serious disadvantage of this architecture is that it requires a high resolution digital-to-analog (DAC) or an analog-to-digital (ADC) converter for each RF branch which raises the power-consumption significantly. Also, a separate RF chain is also needed for each branch which increases the chip area greatly. This may become a serious challenge in future 5G phased array transceivers where hundreds, or even thousands of antenna elements will be used [16].

![Beamforming architectures. (a) RF phase shifting, (b) LO phase shifting, (c) analog baseband phase shifting and (d) digital baseband phase shifting.](image-url)
In practical antenna array transceivers, the beamforming architecture will be a combination of the architectures described above. The Massive Multiple-Input-Multiple-Output (Massive MIMO) technology, that is going to be used in the future to improve the speed and capacity of wireless networks, will require hundreds or even thousands of antenna elements to be utilized in the transceiver. The transceivers will need to have digital parts in order to utilize complex signal processing algorithms, but on the other hand, a fully digital beamforming requires an AD/DA converter and a power amplifier for each of the antenna branch (Figure 3.2 (d)), which would increase the cost, power consumption and chip area greatly. Therefore, there is interest to use hybrid beamforming architectures, that have both digital and analog beamforming elements. By using hybrid beamforming architectures, the transceiver can be made more cost- and power-efficient while still retaining the advantages of digital beamforming. An example of a hybrid beamforming architecture is presented in Figure 3.3 [20]. In this example, the antennas are divided into a set of subarrays. There is a separate RF chain for each subarray. Each subarray can be controlled in the digital domain, and each antenna element can be controlled in the RF domain using phase shifters.

3.3. Phase shifters

Phase shifters are used to shift the signal phase of individual antenna branches. There exists several types of phase shifters. The choice of the correct phase shifter type depends on the specifications of the system under design. All phase shifter types have different trade-offs regarding die area, complexity of phase control, power consumption, bandwidth, insertion loss and variation of loss over the phase shift range. The phase shifter types can be subdivided into passive and active phase shifters [17, 19].

Figure 3.4 (a) shows a simple passive, digitally controllable switched delay phase shifter [16]. This phase shifter type consists of several transmission lines with
different lengths and two switches. The phase shift of each transmission line depends on its length. In this example there are three transmission lines and two single-pole triple-throw switches. By controlling the switches, one transmission line can be connected from the input to the output at a time. The phase shift of a transmission line is

\[ \phi = \omega \sqrt{L_{\text{tot}} C_{\text{tot}}}, \]

where \( \omega \) is the angular frequency and \( L_{\text{tot}} \) and \( C_{\text{tot}} \) are the total inductance and capacitance of the transmission line, respectively. The longer the transmission line is, the greater are the total inductance and capacitance, and consequently, the greater is the phase shift [16].

The disadvantage of the transmission line-based phase shifter is the trade-off between the phase resolution and number of the transmission lines. If a high resolution is required, a great number of transmission lines of different lengths should be used, which in turn increases the die area significantly. This problem can be overcome by using a synthesized transmission line (loaded line) structure that is illustrated in Figure 3.4 (b) [16]. In this structure, tunable inductors and capacitors are used. By tuning the inductance and capacitance, the corresponding phase shift can be adjusted according to Equation (3.1). Usually only variable capacitors are used as tunable elements because it is easy to implement them by using varactors and switched capacitors. However, this approach decreases the bandwidth. Therefore there is an interest in implementing variable inductors as well [16].

Figure 3.4 (c) shows a reflection-type phase shifter [19]. This type of phase shifter consists of a 4-port coupler with two of its terminals acting as the input and the output and the other two ports connected to reactive loads. In this structure, the phase shift is based on the reflection coefficients of the loads. By adjusting the inductance and the capacitance of the loads, the load impedance is changed according to equation

\[ Z_L = j \omega L + \frac{1}{j \omega C}. \]

[19]. This change in load impedance changes the reflection coefficient:

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \]

where \( Z_0 \) is the characteristic impedance [19]. The reflection coefficient causes a phase shift that is given by [19]

\[ \phi = \arg(\Gamma) = \pm \pi - 2 \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{Z_0}\right). \]
Figure 3.4. Passive phase shifters. (a) Switched-delay phase shifter, (b) loaded-line phase shifter and (c) reflective-type phase shifter.

Figure 3.5 shows a cartesian vector modulator which is an example of an active phase shifter [19]. The operating principle of this type of phase shifters is based on I/Q interpolation. In Figure 3.5, the upper branch corresponds to the I (in-phase) component and the lower amplifier corresponds the Q (quadrature) component. Adjusting the weights of the branches, i.e. the gain values of the amplifiers, and summing the resulting signals yields

\[ A_I \cos(\omega t) + A_Q \sin(\omega t) = \sqrt{A_I^2 + A_Q^2} \cos(\omega t + \phi), \]

(3.5)

where \( A_I \) and \( A_Q \) are the amplitudes of the I- and Q-branches, respectively, \( \omega \) is the angular frequency and \( \phi \) is the resulting phase shift. Solving for \( \phi \) yields

\[ \phi = -\arctan\left(\frac{A_Q}{A_I}\right) \]

(3.6)
The advantage of the cartesian vector modulator phase shifter compared to the passive phase shifters is that the use of several passive devices is avoided. Therefore the cartesian vector modulator phase shifter occupies much less die area and also has less loss. Another advantage of the vector modulator is that by using a differential topology for the variable gain amplifiers the polarity of the I and Q paths can be swapped. As a result, full 360° phase shift range can be achieved [16, 19].

![Cartesian vector modulator as a phase shifter.](Image)

3.4. Feed networks

A feed network is defined as hardware that connects all the elements in an array to each other and to the preceding stages. The purpose of the feed network is to combine the RF signals coming from each antenna element in receive mode and to distribute the RF signals to each element in transmit mode. In addition, feed networks can be used for beamforming. Typically feed networks comprise transmission lines, couplers and power splitters, amplifiers and impedance matching structures. In some definitions also the phase shifters are considered to be a part of a feed network.

There are four basic types of feed networks: parallel (corporate) feed, series feed, spatial feed and hybrid parallel-series feed (Figure 3.6). In series feed the antenna elements are connected in series with transmission lines. In this configuration there is a progressive frequency-dependent phase shift between elements because the signal has to travel different distances to different elements. As it has been shown in previous chapters, the main beam can be steered by setting a progressive phase shift between elements. Therefore a series feed networks can be used for frequency-scanning; the main beam can be steered by changing the frequency. However, series feed networks are not suitable for communication systems where the RF signals have a fixed frequency [10].

In parallel feed networks a signal is distributed to $2^n$ (n = 1, 2,...) antenna elements by using parallel transmission lines. Parallel feed networks are general and versatile. In parallel feeds it is very easy to control the amplitude and phase of each element and therefore they are ideal for phase-scanned antenna arrays that are used in communication systems. The phase of each element can be adjusted using phase...
shifters and the amplitudes can be adjusted using either variable amplifiers or variable attenuators [11]. The disadvantage of parallel feed networks is their large size, especially in arrays with a large number of elements. This problem can be solved by using hybrid parallel-series feed networks where series-connected subarrays are connected via a parallel feed network. The element in each subarray have the same amplitude and/or phase [10].

The fourth basic type of feed networks is the spatial feed network. In spatial feeding approach the primary antenna radiates the signal which is picked up by several pick-up antennas. The amplitude distribution of the array depends on the radiation pattern of the primary antenna and the distances between the primary antenna and the pick-up-antennas. The phase deviation between the pick-up antennas depends on the relative distances between the primary antenna and the pick-up antennas. These phase deviations can be compensated using phase shifters [10].

In addition to the four basic feed network types presented above, there are more complex feed networks that are used for beamforming. These are called beamforming matrices. These are multiple beam switched beam networks, i.e. they can form multiple beams from a set of predefined beam directions. By combining
such networks with phase shifters the beams can also be steered. Some examples of beamforming matrices are Butler matrix and Blass matrix.

Butler matrix (Figure 3.7) consists of transmission lines, 3-dB couplers and phase shifters with fixed phase shift values. In receiver mode Butler matrix transforms the spatial signals arriving to the antenna elements to samples in angular space which correspond to main beam peaks of array factors. In transmitter mode it radiates an orthogonal set of beams with uniform angular distribution [5]. A Butler matrix with \( N = 2^M \) \((M = 1, 2, \ldots)\) antenna elements can form \( N \) beams with uniform angular separation between each beam [5]. The array factor of an \( N \)-element linear array fed by a Butler matrix with 90°-couplers is

\[
AF(\theta) = \frac{\sin N(\frac{\pi}{\lambda} d \sin \theta - \frac{1}{2} \phi)}{N \sin(\frac{\pi}{\lambda} d \sin \theta - \frac{1}{2} \phi)}, \tag{3.7}
\]

where

\[
\phi = (2m - 1) \frac{\pi}{N}, \quad m \in \mathbb{Z} \cup [1 - \frac{N}{2}, \frac{N}{2}]
\tag{3.8}
\]

is the phase difference between antenna elements [21]. For example, for a 4-element array \( m \) would get four values (-1, 0, 1, 2) and therefore four different values for the progressive phase shift, each of which produces a radiation pattern with a distinct direction.

Blass matrix (Figure 3.8) consists of input ports, antenna elements, transmission lines, matched loads and couplers. The couplers are equally spaced along the transmission line in such a way that they create a constant phase shift between elements which enables steering of the beam. The produced radiation patterns are orthogonal because of directional couplers and matching of antenna element impedances [22].

The disadvantage of Butler matrices is the complexity of their interconnection which becomes a problem in large arrays. For large arrays Blass matrix is more suitable. The disadvantage of Blass matrices is that the phase increment depends on wavelength. Therefore Blass matrices are not suitable transmitters where the bandwidth of the transmitted signals is very wide [22].
Figure 3.7. Butler matrix for four antennas.

Figure 3.8. General structure of a Blass matrix.
3.5. Adaptive antenna systems

Adaptive antennas are defined as antenna array transceivers that can adapt to the prevailing signal environment by utilizing digital signal processing hardware, i.e. application-specific integrated circuits (ASIC) or field-programmable gate arrays (FPGA), and software-based signal processing algorithms. The functionality of an adaptive antenna system depends on whether the transceiver is in transmitting or receiving mode. In transmitting mode, the purpose of the adaptive antenna is to steer the main beam towards the wanted user (or multiple beams towards multiple users). In receiving mode, the adaptive antenna calculates the direction of arrival (DOA) of the desired users. It also calculates the DOAs of unwanted signals and steers the nulls towards these directions or attenuates the sidelobes in these directions in order to mitigate interference. All of these functions are realized by using an adaptive algorithm that takes into account some performance criteria and the DOAs of the desired signal and the interfering signals and sets the phase and amplitude weights in order to create an optimal antenna pattern. The adaptive antennas also enable space-division multiple-access (SDMA); the users can be separated from each other in space domain. This technology increases the capacity of a communication system significantly. The general structure of an adaptive antenna system in receiving mode is presented in Figure 3.9 [11].

One of the inputs for the adaptive algorithm is the DOAs of the desired signal and the interfering signals. There exists several methods for the DOA estimation. In conventional methods the DOAs are found by scanning a beam in all direction in space and producing a power spectrum. The spatial angles corresponding the peaks in this spectrum are considered as the DOAs. The disadvantage of these methods is poor resolution. A better resolution can be obtained with subspace methods, for example the Multiple Signal Classification (MUSIC) algorithm, that takes the advantage of the eigen-structure of the input covariance matrix. The Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) is also based on the same mathematical principle, but it is much faster and requires much less memory than the MUSIC algorithm, and therefore it is preeminently more popular [11].
The other input for the adaptive algorithm is the output of the antenna array, as shown in Figure 3.9. The adaptive algorithm analyzes the output of the array and adjusts the weight vector in order to minimize a cost function with certain criteria. The cost function describes the performance of the antenna array and it is inversely proportional to the quality of the signal at the array output. In other words, by minimizing the cost function the quality of the output signal is maximized. The most common criteria of minimizing the cost function include the Minimum Mean Square Error (MMSE), Maximum Signal-to-Noise Ratio (MSNR) and Minimum noise Variance (MV) [11].

The outputs of an adaptive algorithm are the amplitude and phase weight vectors that produce the optimal array pattern for the prevailing signal environment. The main purpose of the phase weights is to steer the main beam towards the signal of interest (SOI), in the similar manner as explained in Section 2.2.1., and to steer the nulls towards signals not of interest (SNOI). The amplitude weights mostly affect the levels of the sidelobes. The purpose of the amplitude distributions is to reduce the level of the sidelobes in the directions of the interfering signals.
3.6. Pattern synthesis

As it was discussed in Section 3.5, the level of the sidelobes can be reduced by using a suitable amplitude distribution. Furthermore, by adjusting the amplitude weights one can also form nulls in desired directions, shape the envelope of the radiation pattern and adjust the beamwidth of the main beam. Usually an antenna system has some specifications to be fulfilled, considering e.g. the shape of the radiation pattern, beamwidth, null-directions and sidelobe-levels. The method of designing the amplitude distribution of an array so that it fulfills such criteria, is called pattern synthesis. There exists several mathematical methods for designing amplitude distributions for optimizing certain parameters. The pattern synthesis methods can be divided into low-sidelobe, beam-shaping and null-forming methods.

One example of low-sidelobe methods is the Dolph-Chebyshev synthesis. The purpose of Dolph-Chebyshev synthesis is to produce an array pattern with low sidelobe-levels with equal levels. In this method the sidelobe-levels can be set to a desired value by calculating suitable Chebyshev polynomials. The calculation of the Chebyshev polynomials starts by considering the array factor. In Equation (2.15), the array factor is given in exponential form. By using Euler’s formula, the array factor can also be presented mathematically as

\[
AF_{2M} = \sum_{n=1}^{M} a_n \cos[(2n-1)u], \quad \text{even } M
\]

\[
AF_{2M+1} = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u], \quad \text{odd } M
\]

where \(M\) is the number of antenna element and \(u = (\pi d/\lambda)\cos\theta\) [11]. The cosine terms in the array factors can be presented as series of cosine functions as

\[
m = 0: \cos(mu) = 1
\]

\[
m = 1: \cos(mu) = \cos u
\]

\[
m = 2: \cos(mu) = 2\cos^2 u - 1
\]

\[
m = 3: \cos(mu) = 4\cos^3 u - 3\cos u
\]

\[\vdots\]

[11]. By replacing \(\cos u\) with \(z\), Equation (3.10) can be presented as

\[
m = 0: \cos(mu) = 1 = T_0(z)
\]

\[
m = 1: \cos(mu) = z = T_1(z)
\]

\[
m = 2: \cos(mu) = 2z^2 - 1 = T_2(z)
\]

\[
m = 3: \cos(mu) = 4z^3 - 3z = T_0(z)
\]

\[\vdots\]

[11]. These polynomials \(T_m(z)\) are the Chebyshev polynomials. The recursive equation for these polynomials is [11]

\[
T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z).
\]
The design of a Dolph-Chebyshev array starts by selecting the correct array factor depending on whether the array is to have an even or an odd number of elements and by expanding the array factor as shown in Equations (3.9) to (3.12). After that, the point \( z = z_0 \) is calculated so that \( T_m(z) = R_0 \), where \( R_0 \) is the desired level of the sidelobes as a linear number. Next, \( \cos(u) \) is substituted with \( z/z_0 \). This term is fed to the Chebyshev polynomial and finally the amplitude weights are calculated [11]. Figure 3.10 shows an example of an array factor for an 8-element Dolph-Chebyshev array with 30-dB sidelobe attenuation.

Another example of low-sidelobe level methods is the Taylor n-bar synthesis. The Taylor n-bar distribution is a combination of the Dolph-Chebyshev distribution with its constant-level sidelobes and the envelope fall-off of the \( \sin(x)/x \) pattern. In Taylor n-bar distribution a specified amount of the first sidelobes next to the main lobe are attenuated to the desired level, and the rest of the side lobe peaks lie on the \( 1/u \) envelope [23]. The synthesized pattern normalized to unity is given by

\[
F(z, A, \bar{n}) = \frac{\sin \pi u}{\pi z} \prod_{n=1}^{n-1} \frac{1 - u^2/z_0^2}{1 - u^2/n^2}
\]  

(3.13)

[23]. Figure 3.11 shows an example of an array factor for an 8-element Taylor nbar array with 30-dB sidelobe attenuation and nbar = 3.
Figure 3.11. Array pattern of an 8-element Taylor array with 30-dB sidelobe attenuation and nbar = 3 and spacing \( d = 0.5\lambda \).

The Dolph-Chebyshev and Taylor methods presented above aim to reducing the level of the sidelobes to a specified level. In beam shaping methods, on the other hand, an array factor pattern with an arbitrary shape can be formed. An example of these methods is the Fourier transform method. In the Fourier transform method, the determination of the amplitude weights is based on calculating the Fourier coefficients for the desired pattern shape. The Fourier coefficients are given by

\[
a_m = \frac{1}{T} \int_{-T/2}^{T/2} AF(\psi)e^{-jm\psi} \, d\psi = \frac{1}{2\pi} \int_{-\pi}^{\pi} AF(\psi)e^{-jm\psi} \, d\psi, \quad -M \leq m \leq M \quad (3.14)
\]

for odd number of elements, and
\[ a_m = \frac{1}{T} \int_{-T/2}^{T/2} A_F(\psi)e^{-j[(2m+1)/2]\psi} \, d\psi \]

\[ \Rightarrow a_m = \frac{1}{T} \int_{-T/2}^{T/2} A_F(\psi)e^{-j[(2m+1)/2]\psi} \, d\psi, \quad -M \leq m \leq -1 \]

\[ a_m = \frac{1}{T} \int_{-T/2}^{T/2} A_F(\psi)e^{-j[(2m-1)/2]\psi} \, d\psi \]

\[ \Rightarrow a_m = \frac{1}{T} \int_{-T/2}^{T/2} A_F(\psi)e^{-j[(2m-1)/2]\psi} \, d\psi, \quad 1 \leq m \leq M \]

for even number of elements [11]. In the equations above, \( A_F(\psi) \) is the desired array factor. These coefficients can be extended to \( 2M - 1 \) elements by mirroring the coefficients in respect with the first coefficient, i.e. \( a_{-m} = a_m \) [11]. Figure 3.12. shows an example of a square-shape array factor produced by the Fourier method.

![Figure 3.12. Approximately square-shaped array factor of a 21-element array produced by the Fourier method.](image)

The third class of pattern synthesis methods are the null forming methods. One example of these methods is the Schelkunov method. In this method, based on the desired number of nulls and their locations, the number of elements and their amplitude weights can be calculated. The mathematical formulation of the Schelkunov methods starts from the equation of array factor

\[ AF = \sum_{n=1}^{N} a_n e^{j(n-1)\psi}. \]
By using a substitution $z = \exp(j\psi)$, the array factor can be presented in form

$$AF = \sum_{n=1}^{N} a_n z^{n-1} = a_1 + a_2 z + a_3 z^2 + \cdots + a_N z^{N-1}.$$  \hspace{1cm} (3.17)

This polynomial can be changed into form

$$AF = a_n(z - z_1)(z - z_2)(z - z_3)\cdots(z - z_{N-1}).$$ \hspace{1cm} (3.18)

The design of a Schelkunov array starts by determining the terms $z_m$. These correspond to complex values with the desired null locations as the angle values and absolute values of unity. These complex numbers (in cartesian form) are fed to Equation (3.17). The terms are then multiplied which results in a polynomial of the same form as Equation (3.18). The number of term in this polynomial corresponds the number of antenna elements and their magnitudes correspond the amplitude weights of the array [11]. Figure 3.13 shows an array factor of a 4-element array with spacing $d = \lambda/4$ and nulls at directions $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$.

![Figure 3.13](image_url)

Figure 3.13. A 4-element Schelkunov array with spacing $d = \lambda/4$ and nulls at directions $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$. 

4. ARRAY EXCITATION NON-IDEALITIES

4.1. Introduction

Phased arrays always have errors due to manufacturing tolerances, element failures, aging, quantization and errors caused by environmental effects such as temperature fluctuations. These errors reduce the precision of the array excitation which leads to deterioration of the array performance. The most noticeable effect of these non-idealities is the rise in sidelobe levels. The errors may raise the average sidelobe-level or the levels of some distinct sidelobes. Some of the other problems are beam pointing error and loss in directivity. These errors should be calibrated or taken into account by some other means [5].

Errors can be either systematic (static) or random (dynamic), or a combination of both. Systematic, i.e. correlated errors are troublesome because they can raise the levels of some distinct sidelobes significantly. Systematic errors can be caused by various different mechanisms and thus their effect on the array performance is difficult to predict. Therefore the aim is usually to remove all the systematic errors in the design phase or by measuring the phase and amplitude errors and calibrating them away after manufacturing. Once the systematic errors have been removed, all that remains are the random errors that are caused by the tolerances of electronic components. They are independent from element to element and they can be modeled statistically. Random errors can be assumed to be normally distributed with zero mean. In comparison with systematic errors, random errors are less troublesome because their effect on the radiation pattern of an array effectively gets averaged out, provided that the number of antenna elements is sufficiently large [5].

In this thesis work the effects of random errors on specified performance parameters were studied with Matlab by performing Monte Carlo simulations. The purpose of these simulations is to gain insight into how accurately the phases and amplitudes must be measured for calibration. Even though the purpose of calibration is to remove systematic errors, the analysis of the effects of random errors shows the worst-case scenarios that could happen in the presence of systematic errors. These simulations will be presented later in Section 4.5. Firstly, theory of the effects on random phase and amplitude errors, quantization and mutual coupling are discussed.

4.2. Random phase and amplitude errors

The inaccuracies in the manufacturing process lead to random errors in phased array transceivers. It is difficult to predict, or to present mathematically, how certain errors in phase or amplitude affect the properties of the array factor. Therefore the effect of random errors must be modelled statistically. In this section, statistical equations for different performance parameters are presented. Apart from these, the effects of random errors can also be modeled by performing Monte Carlo simulations.

The effects of random errors can be modeled by modifying the equation of array factor to take the random errors into account. The error-free array factor can be presented mathematically as follows:
\[ AF = \sum_{n=1}^{N} a_n \exp(j\phi_n) \exp(j(n-1)\psi), \] (4.1)

where \( a_n \) and \( \phi_n \) are the amplitude excitation and the phase excitation of the \( n \)-th array element, respectively. The random phase and amplitude errors can be taken into account in the array factor by adding the phase and amplitude errors to the corresponding nominal values:

\[ AF = \sum_{n=1}^{N} (a_n + a_{\text{err},n}) \exp(j(\phi_n + \phi_{\text{err},n})) \exp(j(n-1)\psi), \] (4.2)

where \( a_{\text{err},n} \) and \( \phi_{\text{err},n} \) are the amplitude and phase errors of the \( n \)-th array element, respectively.

In all of the following statistical equations for different performance parameters, it is assumed that the phase and amplitude errors are Gaussian distributed. Both phase and amplitude errors reduce the directivity of the array. The reduction in directivity due to these errors for an array of omnidirectional elements is approximately

\[ \frac{D}{D_0} \approx \exp(-\left(\sigma_a^2 + \sigma_\phi^2\right)) \approx \frac{1}{1 + \sigma_a^2 + \sigma_\phi^2}, \] (4.3)

where \( D \) is the directivity of the array with errors, \( D_0 \) is the directivity of the error-free array and \( \sigma_a \) and \( \sigma_\phi \) are the standard deviations of amplitude and phase errors, respectively [5]. Usually reduced directivity due to random phase and amplitude errors is not the greatest concern in the array performance because most often the rise in sidelobe levels becomes severe before there is any significant loss in directivity [5].

Beam pointing error can be quantified by calculating the variance of the beam direction deviation. The variance of beam direction deviation due to phase and amplitude errors for a symmetrical array excitation is

\[ \bar{\Delta}^2 = \sigma_\phi^2 \frac{\sum a_i^2 x_i^2}{(\sum a_i x_i)^2}, \] (4.4)

where \( \sigma_\phi \) is the variance of phase errors, \( a_i \) is the excitation amplitude of the \( i \)-th element and \( x_i \) is the element position of the \( i \)-th element divided by interelement spacing [5]. For an array with a uniform amplitude distribution, the beam pointing error becomes

\[ \bar{\Delta}^2 = \frac{12}{N^3} \sigma_\phi^2 \] (4.5)
From Equation (4.5) it can be seen that when the amplitude distribution is uniform, the amplitude errors do not affect the variance of beam pointing error.

Both phase and amplitude errors also contribute to the rise in sidelobe level. Assuming that all elements in an array have identical radiation patterns, the average sidelobe level can be approximated as

\[
SLL \approx G_e (\sigma_a^2 + \sigma_\phi^2),
\]

where \( G_e \) is the gain of a single element [5]. This equation yields the \( SLL \) in power domain. From Equation (4.6) it can be seen that the plot of \( \sigma_\phi \) versus \( \sigma_a \) is a circle whose radius is determined by \( SLL \) and \( G_e \). From such a plot it is easy to determine combinations of phase and amplitude errors that result into a certain \( SLL \). As an example, constant average sidelobe level circles for values -10 dBi, -15 dBi and -20 dBi are illustrated in Figure 4.1 [5].

![Figure 4.1. Constant average sidelobe level circles caused by phase and amplitude errors. Element gain is assumed to be \( G_e = \pi \).](image)

Usually, the peak sidelobe level is a more critical parameter for the array performance than the average sidelobe level. The level of the peak sidelobe relative to the mainbeam can be expressed mathematically as [6]

\[
SLL < 10\log(\sigma_\phi^2 + \sigma_a^2) - 10\log(N) + 10 \text{ [dB]}.
\]

\[(4.7)\]
4.3. Quantization errors

In modern phased array transceivers, adjustable components are controlled digitally. The advantage of digital components is high speed and controllability. On the other hand, discretization of component parameters result in quantization. For example, the phase shift values of digitally controllable phase shifters are discrete and cause quantization errors. These errors in phase affect the radiation pattern in the same manner as phase errors. Compared to random phase errors, the quantization errors are usually larger and therefore they result in greater errors in the radiation pattern. Quantization errors may raise sidelobe levels significantly. The sidelobes caused by quantization errors are called quantization lobes.

The phase separation between phase shifter states depends on the number of bits:

\[ \phi_0 = \frac{2 \pi}{2^N}, \]  

(4.8)

where \( N \) is the number of bits. The quantization of phase shift allows only a staircase approximation of the ideal, continuous, progressive phase shift. The quantization results in periodic, triangular phase error that causes quantization lobes. If the array current distribution is assumed to be a continuous function, which is a valid assumption when the number of array elements is sufficiently high, the level of the first quantization lobe is

\[ P_{QL} = \frac{1}{2^{2N}} \]  

(4.9)

in watts [W] and

\[ P_{QL}(dB) = -6N \]  

(4.10)

in decibels [dB] [5]. Figure 4.2 shows an example of array factors for an array with a certain number of elements and steering angle with three different numbers of phase shifter bits. From the figure it can be seen that the greater the number of bits is the smaller is the highest sidelobe next to the mainlobe. In other regions, the effect of the number of bits on the sidelobe levels is more arbitrary. The array factors in the figure do not fully comply with Equation (4.10) because the number of elements is rather small.
Figure 4.2. Array factors when there are 5-, 4- and 3-bit phase shifters. Number of elements is $N = 8$, element spacing is $d = 0.5 \lambda$ and steering angle is $20^\circ$.

### 4.4. Mutual coupling

#### 4.4.1. Characterization of mutual coupling

The elements in an array are never isolated from each other electrically. The elements are always coupled with each other through their electric and magnetic fields. This phenomenon is called *mutual coupling*. The exchange of energy between antenna elements due to mutual coupling changes the current distribution and the input impedance of each element which in turn alters the radiation pattern to errors. The strength of the coupling depends on various factors, for example the radiation characteristics of the antennas, the relative position and orientation of each element and steering angle. In addition to the antenna elements, the feed network is also affected by mutual coupling. Because the nature and strength of mutual coupling depends on many parameters, compensating the coupling effects is challenging. In communication phased array transceivers mutual coupling can be a significant issue because the array elements lie in close proximity to each other and on the other hand the demands for the performance parameters are strict [5, 11].

The effect of mutual coupling to the radiation pattern of an array can be characterized in various ways. One approach to account for all coupling effects is the *active-element pattern* method. The active element patterns are obtained by exciting each element at a time and loading all other elements with the generator impedance $Z_g$. The active-element pattern consists of the direct radiation from the $n$th element
and all the fields radiated from the other elements that receive their power through coupling from the \( n \)th element. The radiation pattern is

\[
F(\theta, \phi) = \sum_{n=1}^{N} g_{ae}^{n}(\theta, \phi) I_n e^{j\psi},
\]

(4.11)

where \( g_{ae}^{n}(0,\phi) \) is the active-element pattern of the \( n \)th element [10]. This approach requires that the radiation pattern of each element is measured which is very costly and time-consuming. However, if the array consists of a large number identical, equally spaced elements, the environment for each element is approximately the same, excluding the edge elements. In this case, the total radiation pattern can be approximated using an average active-element pattern, which can be considered to be the radiation pattern of the center element. The radiation pattern becomes

\[
F(\theta, \phi) = g_{ae}(\theta, \phi) \sum_{n=1}^{N} I_n e^{j\psi},
\]

(4.12)

where \( g_{ae}(0,\phi) \) is the average active-element pattern [10]. The average active-element pattern method is accurate for arrays with a large number of equally spaced elements but does not apply for arrays with a small number of elements where there is significant variation in the active impedances between elements [24].

Another way to characterize the effect of mutual coupling is to use a coupling matrix. A coupling matrix describes the coupling effect between all array elements. The relationship between active element patterns and stand-alone element patterns is

\[
A_{true}(\theta, \phi) = M A_{theo}(\theta, \phi),
\]

(4.13)

where \( M \) is the coupling matrix, and \( A_{true} \) and \( A_{theo} \) are the element patterns with the coupling effect and without the coupling effect, respectively [25].

The coupling matrix can be either a \( Z \)-matrix, where the coupling effect is modeled using self and mutual impedances, or an \( C \)-matrix, where the coupling effect is modeled using complex coefficients. The \( Z \)-matrix can be determined by viewing the antenna array of \( N \) elements as an \( N \)-port microwave network. Circuit analysis yields

\[
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1N} \\
Z_{21} & Z_{22} & \cdots & Z_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N1} & Z_{N2} & \cdots & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix} =
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix},
\]

(4.14)

where \( Z_{ii} \) is the self-impedance of the \( i \)th element, \( Z_{ij} \) is the mutual impedance between the \( i \)th and the \( j \)th element, and \( I_n \) and \( V_n \) are the current and the voltage of the \( n \)th element, respectively [10]. Equation 4.14 transforms the excitation current into voltages. If the antenna elements are excited with voltage signals instead of current signals, the real voltages can be calculated from the ideal voltages as follows:
where $Z_{ij}$ is the mutual impedance between the $i$th and the $j$th elements, $Z_{il}$ is the load impedance of the $i$th element, $V_i$ is the true voltage of the $i$th element due to coupling and $V_{i,oc}$ is the open-circuit voltage of the $i$th element. Equation (4.15) can be written in the form

$$V = Z^{-1}V_{oc}$$

(4.16)

and therefore the mutual coupling matrix is

$$M = Z^{-1}.$$  

(4.17)

The $Z$ approach is mathematically rather simple but it gives accurate results only in certain situations [25].

With an $C$-matrix the real excitation amplitudes can be calculated from the ideal excitation amplitudes as follows:

$$\begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1N} \\
C_{21} & C_{22} & \cdots & C_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
C_{N1} & C_{N2} & \cdots & C_{NN}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix} = \begin{bmatrix}
V_{c1} \\
V_{c2} \\
\vdots \\
V_{cn}
\end{bmatrix},$$

(4.18)

where $C_{ii}$ is the reflection coefficient of the $i$th element, $C_{ij}$ is the transmission coefficient from the $j$th element to the $i$th element, and $V_i$ and $V_{cn}$ are the ideal and real voltages, respectively. Equation (4.18) is valid for currents as well.

The $Z$- and $C$-matrices must be calculated using computational electromagnetic methods that are quite complex. The most common of these methods is the Method of Moments (MoM). In the method of moments the current in an antenna element is represented by basis functions where the needed coefficients are computed with integral equations. Other computational electromagnetic methods include the Numerical Electromagnetics Code (NEC) and the Induced Electromotive Force (EMF) method [11].
4.4.2. Effects of mutual coupling

Mutual coupling in an array changes the relative amplitudes and phases between antennas and therefore changes the shape of the radiation pattern and the direction of the main beam. However, these error effects are static and they can be calibrated away. There are also some methods to reduce the effect of mutual coupling in the design phase of the antenna array transceiver.

As it has been shown in Section 4.2, errors in phase and amplitude reduces the directivity of an array. Mutual coupling causes some variations in phase and amplitude between elements, and therefore mutual coupling reduces the directivity of an array. For example, directivity of a broadside, planar array with small elements can be expressed as a function of self- and mutual impedances as

\[
D = \frac{120 \left( \sum_m \sum_n a_{mn} \right)^2}{\sum_m \sum_n \sum_p \sum_q a_{mn} a_{pq} R_{mnpq}} \tag{4.19}
\]

[5]. This equation has been obtained by giving each element two numbers that correspond its location in the lattice. In other words, \((m,n)\) means the element in the \(m\)th row, in the \(n\)th column. \(a_{mn}\) and \(a_{pq}\) are the excitation amplitudes of elements \((m,n)\) and \((p,q)\), respectively, and \(R_{mnpq}\) is the mutual resistance between elements \((m,n)\) and \((p,q)\).

One special case of defects caused by mutual coupling is scan blindness which means a great reduction of radiated power for certain scan angles called “blind angles”. This phenomenon is caused by the changes in impedances due to mutual coupling. Usually the impedance matching in an array is done for broadside direction, i.e. the reflection coefficients of antenna elements are zero when the beam is not steered. When the beam is steered, the impedances are changed and the reflection coefficients get non-zero values. If the reflection coefficient of the \(m\)th element in an array is denoted by \(\Gamma_m(\theta_o, \phi_o)\), the power delivered to the \(m\)th element is

\[
P_m = P_{inc} \left[ 1 - |\Gamma_m(\theta_o, \phi_o)|^2 \right], \tag{4.20}
\]

where \(P_{inc}\) is the incident power. The average active-element pattern can therefore be expressed as

\[
g_{ae}(\theta_o, \phi_o) = g_i(\theta_o, \phi_o) \left[ 1 - |\Gamma_m(\theta_o, \phi_o)|^2 \right]. \tag{4.21}
\]

Scan blindness occurs when the reflection occurs when the reflection coefficient is \(\Gamma_m(\theta_o, \phi_o) = 1\). Then, according to Equation 4.20, no power is delivered to the element [10].
4.5. Modeling of error effects

In this thesis work, a model for the most important error sources was designed. The schematic of the error model is presented in Figure 4.3. In the figure, the phase shift value of each phase shifter consists of the nominal phase of the phase shifter and a random phase error. Correspondingly, the amplitude of the variable gain amplifier consists of the nominal amplitude of the amplifier and a random amplitude error.

\[
\phi_1 + \phi_{err,1} \quad a_1 + a_{err,1}
\]

\[
\phi_2 + \phi_{err,2} \quad a_2 + a_{err,2}
\]

\[
\phi_n + \phi_{err,n} \quad a_n + a_{err,n}
\]

Figure 4.3. Schematic of the designed error model.

Based on the error model, a simulation model was created with Matlab. The simulation model utilizes Monte Carlo method to analyze statistically how the random errors affect different performance parameters of an array. The general structure of the simulation program is presented in Figure 4.4. The definitions of the performance parameters are illustrated in Figure 4.5. The desired type of array is defined by giving the number of elements \(N\), operating frequency \(f_c\), spacing between elements \(d\) and steering angle \(\theta\) as inputs. The phase and amplitude errors are set by defining variances for them. Based on the given variance values, the Monte Carlo loop will assign each antenna element a random phase error value and a random amplitude error value. The random phase and amplitude errors are normally distributed so that the mean value of phase errors is 0 and the mean value of amplitude errors is 1. All antenna branches have their own error values, and they are independent of each other.
Figure 4.4. General structure of the developed Matlab software.
In the Monte Carlo loop, the phase and amplitude errors are given random values as shown in Figure 4.4. Based on these error values and the given array parameters, the array factor is calculated with equation

\[ AF = \sum_{n=0}^{N} (a_n + a_{\text{err},n}) \exp(j(\phi_n + \phi_{\text{err},n})), \]  

(4.22)

where \(a_n\) and \(a_{\text{err},n}\) are the ideal amplitude and the amplitude error of the \(n\)th element, respectively, and \(\phi_n\) and \(\phi_{\text{err},n}\) are the ideal phase and phase error, respectively. In each of the \(N_{\text{mc}}\) iterations of the Monte Carlo loop, new random error values are assigned and the resulting array factor is saved to a \(N_{\text{mc}} \cdot M\) variable, where \(M\) is the number of data points in a single array factor. Once the loop has been iterated \(N_{\text{mc}}\) times, the performance parameters (the outputs illustrated in Figure 4.4) are calculated with a separate Matlab function for each of the \(N_{\text{mc}}\) array factors. As a result, a \(1 \cdot N_{\text{mc}}\) array of values is obtained for each performance parameter.

The effect of random errors was simulated using the Monte Carlo method described in the beginning of Section 4.5. Firstly, the Monte Carlo simulation was performed with the parameters listed in Table 1.
Table 1. Parameters for the 1st Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements $N$</td>
<td>[4, 16, 64]</td>
</tr>
<tr>
<td>Steering angle $\theta_0$</td>
<td>0°</td>
</tr>
<tr>
<td>Operating frequency $f_c$</td>
<td>28 GHz</td>
</tr>
<tr>
<td>Element spacing $d$</td>
<td>$\lambda/2$</td>
</tr>
<tr>
<td>Standard deviation of amplitude errors $\sigma_a$</td>
<td>[0, 5, 10, 15] %</td>
</tr>
<tr>
<td>Standard deviation of phase errors $\sigma_\phi$</td>
<td>[0, 5, 10, 15, 30]°</td>
</tr>
<tr>
<td>Number of Monte Carlo iterations $N_{mc}$</td>
<td>10 000</td>
</tr>
</tbody>
</table>

The Monte Carlo simulation with the above-mentioned parameters was run with all combinations of numbers of antenna elements and standard deviations of phase and amplitude errors, i.e. $3 \times 4 \times 5 = 60$ different combinations. At each of the 60 simulations, the array factors and the resulting performance parameter values were calculated 10 000 times. The resulting 10 000 values for each performance parameter were saved to a mat-file. The performance parameters that were calculated are

1) mainlobe gain
2) 3-dB beamwidth
3) mainlobe direction
4) sidelobe-level relative to the mainlobe (e.g. -6 dBc means that the highest sidelobe is 6 dB less than the mainlobe level.)

The goal was to study how the phase and amplitude errors separately affect various performance parameters and also what is the effect of amplitude and phase errors combined. In order to do so, the distributions of the obtained performance parameters were plotted. For each performance parameter, 3 figures were created, each of which corresponds to the number of elements. Each of these figures contain three plots; one with different phase errors variances while amplitude error variance remains zero, one with different amplitude error variances while phase error variance remain zero, and one with both phase and amplitude errors. From these plots one can analyze the effect of phase and amplitude errors separately, the combined effect of phase and amplitude errors and also the effect of the number of elements, on the array performance parameters. Figures 4.6, 4.7 and 4.8 show the plots for mainlobe gain. Figures 4.9, 4.10 and 4.11 show the plots for 3-dB beamwidth. Figures 4.12, 4.13 and 4.14 show the plots for mainlobe direction. Figures 4.15, 4.16 and 4.17 show the plots for sidelobe-level.

From Figure 4.6 it can be seen that both phase and amplitude errors affect the mainlobe gain. The effect of phase errors is always degrading; the distribution spreads when the phase error variance increases but the mean value of the distribution decreases so that the phase errors never raise the mainlobe level above the nominal level. The amplitude errors, on the other hand, can either decrease or increase the mainlobe level. The variance of the mainlobe level distribution increases as the amplitude error variance increases but the mean value always remains the same. The effect on increasing the number of elements can be seen by comparing Figures 4.6, 4.7 and 4.8. It can be seen that when the number of elements is increased, the phase errors have a greater impact on the mainlobe level, while the effect of the amplitude errors decreases.
It can be seen from Figures 4.9, 4.10 and 4.11 that the phase errors have an insignificant effect on the 3-dB beamwidth. For example, when \( N = 4 \), \( \sigma_\phi = 30^\circ \) and \( \sigma_\Lambda = 0 \% \), the total variation range of 3-dB beamwidth is roughly \([18.50, 19.25]^\circ\). When \( N \) is increased, the variation is decreased even more. The effect of amplitude errors is much greater. When \( N = 4 \), \( \sigma_\phi = 30^\circ \) and \( \sigma_\Lambda = 15 \% \), the total variation range of the 3-dB is approximately \([17, 21]^\circ\). However, when \( N \) increases, the effect of amplitude errors decreases greatly, and it can be neglected, as in the case of phase errors. The discrete nature of the plots with \( N = 16 \) and \( N = 64 \) is a result of the angular resolution used by the Matlab routine.

The effect of errors on the mainlobe direction can be seen in Figures 4.12, 4.13 and 4.14. The phase errors have quite a significant effect on the mainlobe direction. When \( N = 4 \) and \( \sigma_\Lambda = 0 \% \), the mainlobe direction varies quite much even with low phase error variance values. When \( \sigma_\phi = 5^\circ \), the mainlobe direction varies roughly \( 5^\circ \). However, with greater numbers of elements, the effect of phase errors can be neglected. It can also been seen from the figures that the amplitude errors have no effect on the mainlobe direction. As in the 3-dB beamwidth simulation, the roughness of the distribution curves is caused by the resolution of the Matlab routine.

From Figures 4.15, 4.16 and 4.17 the effects of phase and amplitude errors on the sidelobe-level can be seen. It can be seen from Figure 4.15 that when \( N = 4 \), the phase errors always deteriorate the sidelobe level; as the phase errors variance increases, the sidelobe-level is more likely to increase. On the other hand, when \( N \) increases, the effect on increasing the phase error variance is unexpected. With greater number of elements, increasing the phase error variance may sometimes decrease the sidelobe-level. This can be seen well in Figure 4.17. The amplitude errors, on the other hand, are as likely to increase as to decrease the sidelobe-level; increasing the amplitude error variance increases the sidelobe-level variance but the mean value always remains the same. Increasing the number of elements decreases the effect of amplitude errors on the sidelobe-level.

As a summary, the amplitude errors are always as likely to decrease as to increase each performance parameter (expect the mainlobe direction, on which the amplitude errors have no effect). The phase errors generally deteriorate the gain-domain performance parameters (mainlobe gain and sidelobe-level). An exception is the sidelobe-level which can actually be improved as an effect of phase errors when \( N \) is sufficiently large. When the number of elements is large, the effect of phase and amplitude errors on any performance parameter is insignificant. This is because when the number of elements is large, the average of the errors at any given time instance approaches zero.

The results of the simulations give important insight into the effects of random phase and amplitude errors on different performance parameters of antenna arrays. The general conclusion is that when the number of antenna elements is sufficiently large, the effect and phase and amplitude errors on any of the performance parameters is insignificant. The Matlab programs for calculating the array factor and the performance parameters were carefully tested before the simulations were performed, and so there is a great confidence in that the software works properly. The simulated beam direction variation results (Figures 4.12, 4.13 and 4.14) qualitatively agree with Equation (4.5) presented in Section 4.2.; a uniform amplitude distribution is used in the simulations, and therefore only phase errors affect the beam direction errors. However, the rest of the simulation results do not fully comply with the equations presented in Section 4.2. According to Equation
(4.3), both phase and amplitude errors should reduce the directivity. The simulated
gain values do not behave this way; the amplitude errors are as likely to increase as
to decrease the gain. The explanation for the contradiction between the equation and
the simulated results may be the definition of gain used in the simulations. In the
simulations, the absolute level of the main lobe is considered the gain of the array.
On the other hand, directivity is defined as the level of the main lobe divided by the
total radiated power. Furthermore, Equation (4.7) does not explain why phase errors
can only increase the sidelobe-level (except in some rare cases) but amplitude errors
are as likely to decrease as to increase the sidelobe-level. More work is needed to
fully understand the difference between theory and simulated results.
Figure 4.6. Main lobe gain distributions with 4 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.7. Main lobe gain distributions with 16 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.8. Main lobe gain distributions with 64 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.9. 3-dB beamwidth distributions with 4 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.10. 3-dB beamwidth distributions with 16 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.11. 3-dB beamwidth distributions with 64 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.12. Main lobe distributions with 4 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.13. Main lobe distributions with 16 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.14. Main lobe distributions with 64 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.15. Sidelobe-level distributions with 4 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.16. Sidelobe-level distributions with 16 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Figure 4.17. Sidelobe-level distributions with 64 elements and (a) only phase errors, (b) only amplitude errors and (c) both phase and amplitude errors.
Next, it was studied how the steering angle affects the susceptibility of arrays to performance degradation caused by random phase and amplitude errors. The Monte Carlo simulations were run with different combinations of steering angles and numbers of elements. The variances of phase and amplitude errors were kept constant. Table 2 summarizes the parameter values used in the simulations.

Table 2. Parameters for the 2nd Monte Carlo simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements $N$</td>
<td>[4, 16, 64]</td>
</tr>
<tr>
<td>Steering angle $\theta_0$</td>
<td>[0, 10, 20, 30, 40, 50, 60]$^\circ$</td>
</tr>
<tr>
<td>Operating frequency $f_c$</td>
<td>28 GHz</td>
</tr>
<tr>
<td>Element spacing $d$</td>
<td>$\lambda/2$</td>
</tr>
<tr>
<td>Standard deviation of amplitude errors $\sigma_A$</td>
<td>15 %</td>
</tr>
<tr>
<td>Standard deviation of phase errors $\sigma_\phi$</td>
<td>15$^\circ$</td>
</tr>
<tr>
<td>Number of Monte Carlo iterations $N_{mc}$</td>
<td>10 000</td>
</tr>
</tbody>
</table>

Based on the simulation results, four figures were created corresponding to each of the four performance parameters. Each of the four figures contains three plots (corresponding the numbers of antenna elements), where the performance parameter distributions for each steering angle setting is plotted.

From Figure 4.18, it can be seen that the increasing the steering angle does not have any effect on the mainlobe gain with any number of elements. The 3-dB beamwidth, on the other hand, is significantly affected by the steering angle as it can be seen from Figure 4.19. The smaller the number of elements is, the more the beamwidth is affected by the increase in steering angle. An interesting detail is that when the number of elements is 4, the distributions with $\theta_0 = 50^\circ$ and $\theta_0 = 60^\circ$ have two separate peaks, as it can be seen in Figure 4.19. (a). This is because when then steering angle is too large, the main lobe merges with the back lobe. Then the level of the main lobe does may not fall 3 decibels before the level starts to rise again as the back lobe is approached. The result is that one of the 3-dB points is on the other side of the back lobe. The larger the steering angle is, the more likely the random phase errors are to shift the main beam across the steering angle threshold after which the merging effect happens. This effect is illustrated in Figure 4.22. This merging effect may be trivial because in practical array transceivers such antenna elements are used that as a result of pattern multiplication (Equation (2.14)) the back lobe is removed, and also because a larger number of antenna elements is used. As in the previous Monte Carlo simulations, the discrete behaviour of the distributions in Figure 4.19. (c) is because of the insufficient angular resolution used in the simulations.

Figure 4.20 illustrates the effect of increasing the steering angle on the main lobe direction. As the steering angle increases, the variation of the main beam direction increases. At greater numbers of elements, this effect becomes more insignificant.

Figure 4.21 shows how the increase in steering angle affect the sidelobe-level. When $N = 4$, the mean value of the distributions rise very rapidly as a function of steering angle after a certain threshold of steering angle has been crossed. This is due to the same beam merging effect as described in the context of the beamwidth. It
has been empirically observed that when the main lobe and the back lobe start to merge with each other, the power levels between these lobes increase, and the power levels in the opposite direction to the direction between the main lobe and the back lobe also increases heavily. This causes a large sidelobe in the corresponding direction. This effect is illustrated in Figure 4.22. With a greater number of elements, the increase in steering angle does not affect the level of the sidelobes.
Figure 4.18. Mainlobe gain distributions with different steering angles for (a) 4 elements, (b) 16 elements and (c) 64 elements. Phase and amplitude error variances are $\sigma_\phi = 15^\circ$ and $\sigma_A = 15\%$. 

\[\text{Mainlobe gain distribution, } N = 4\]

\[\text{Mainlobe gain distribution, } N = 16\]

\[\text{Mainlobe gain distribution, } N = 64\]
Figure 4.19. 3-dB beamwidth distributions with different steering angles for (a) 4 elements, (b) 16 elements and (c) 64 elements. Phase and amplitude error variances are $\sigma_\phi = 15^\circ$ and $\sigma_A = 15 \%$. 
Figure 4.20. Mainlobe direction distributions with different steering angles for (a) 4 elements, (b) 16 elements and (c) 64 elements. Phase and amplitude error variances are $\sigma_\phi = 15^\circ$ and $\sigma_A = 15\%$. 
Figure 4.21. Sidelobe-level distributions with different steering angles for (a) 4 elements, (b) 16 elements and (c) 64 elements. Phase and amplitude error variances are $\sigma_\phi = 15^\circ$ and $\sigma_A = 15\%$. 
Figure 4.22. The effect of the main lobe and the back lobe merging. The plot in the figure is an arbitrary array factor with a steering angle of 60° and some random phase and amplitude errors ($\sigma_\phi = 15^\circ$ and $\sigma_A = 15\%$).
5. MEASUREMENT METHODS FOR ARRAY CALIBRATION

As it was shown in the previous chapter, antenna array systems always contain non-idealities that alter the phase and amplitude distributions of the array to errors. These errors can be subdivided into systematic and random errors. Systematic errors (for example phase and amplitude offsets of components, mutual coupling-related errors and environmental effects) are deterministic, and they can be measured and calibrated away [26]. The simplest way to calibrate an array is to measure the amplitude and phase response of the array at different steering angles, calculating the amplitude and phase deviations with respect to the desired values and adjusting the digital amplitude and phase control words to compensate the deviations. When the effect of systematic errors are calibrated away, ideally only the random errors are remaining. These errors set the ultimate limit for the array performance. In practical cases, there exists a vast amount of different calibration algorithms that are usually based on rather complex matrix algebra. In this thesis work the calibration algorithms are not discussed in detail, but the discussion in focused on phase and amplitude measurement methods.

There exists various ways to measure the vector response of the array. Traditionally, the vector response has been determined by electromagnetic field measurements. The drawback of the field measurement-based methods is that they are easily influenced by the environment. They are also expensive and time-consuming.

Another way to measure the vector response of an array is to inject test signals to one RF branch at a time and measure the output signal going to an antenna element by placing a probe on a test pin right before the antenna element. However, in arrays with a large number of elements, this procedure is very expensive and time-consuming. The current trend is to utilize built-in self-test (BIST) systems that are integrated on the chip. The BIST systems greatly reduce the testing cost and also allow the phased array to be tested and calibrated in-situ [27].

This chapter contains a literature review of the basic principles of the measurement methods mentioned above. The calibration algorithms are beyond the scope of this thesis. Some calibration algorithms are presented for example in [7, 28, 29, 30, 31].

5.1. Field measurement methods

Far-field measurement is perhaps the most popular method of measuring phase and amplitude errors for phased array calibration. These measurements can be done either in transmit mode, where a probe antenna measures the electric field radiated by the Device Under Test (DUT) phased array antenna, or in receive mode, where the DUT phased array antenna measures the electric field of an external transmitting antenna.

The most common way to perform the far-field measurement is to excite one antenna element at a time and to measure its far-field electric field with a probe antenna. Usually some modulation technique, such as amplitude modulation or phase modulation, is used to isolate the amplitude and phase of a single element. In the amplitude modulation technique, two measurements are made: one with the entire
array deactivated and one with a single element excited with desired amplitude and phase values. The results of the measurement can be presented as shown in Figure 5.1. (a). The vector $A$ represents noise, and the vector $A'$ represents the measurement where a single element has been activated. Thus, the amplitude and phase of the $n$th element can be obtained from

$$A_n \exp(j \phi_n) = A' \exp(j \phi') - A \exp(j \phi).$$

(5.1)

With the amplitude modulation technique it is easy to determine the phase and amplitude of each element. On the other hand, this method requires $2 \cdot N \cdot M$ measurements, where $N$ is the number of elements and $M$ is the number of unique element states, and this can be a very time-consuming process. Furthermore, this technique does not account for mutual coupling effects [32].

A much more popular technique is the phase modulation technique which is more commonly known as the rotating element electric field vector (REV) method. In the REV method, multiple measurements are performed while rotating the phase of a single element [32]. The complex electric field of a single element can be obtained from three factors of the measured power variation: the maximum power, the minimum power and the phase shift corresponding to the maximum power [33]. An example of the process is shown in Figure 5.1. (b) [32].

In both of the above methods, the noise vector $A$ contains both systematic offset and random noise. Therefore, two subsequent measurement do not correlate each other. To obtain accurate results, one must measure the noise vector several times and calculate the average. The accuracy depends on the number of averaging points and also the signal-to-noise ratio (SNR).

![Figure 5.1. (a) Amplitude modulation technique. (b) Phase modulation technique.](image)

There are several methods for performing the calibration based on the measured data in the far-field. A commonly used technique is the Fourier transform technique, in which the calibration matrix elements are derived as Fourier coefficients of the measured element patterns as follows:
\[ c_{nm} = \frac{d}{\lambda} \int_{\lambda/2d}^{2\lambda/2d} \frac{g_n(u)}{g_i(u)} \exp(-jkmd\mu) du, \]  

(5.2)

where \( g_n(u) \) is the radiation pattern of the \( n \)th element and \( g_i(u) \) is the isolated element pattern. Other methods include for example the beamspace technique [34].

Calibration procedures based on far-field measurements have some drawbacks. Firstly, far-field measurements are prone to influences by the environment because of ambient radiation that arrives to the probe antenna. Secondly, if the operating frequency is too low, the measurements have to be done outdoors which does not meet the standards for security and military applications [35]. Therefore, there is an interest for calibration methods based on near-field measurements. One of such methods is presented in [35]. In this method the electric field is measured in the near-field in two cases: 1) a single element is activated and 2) all the elements are activated. These electric field are denoted by \( E_0(x,y) \) and \( E(x,y) \), respectively. The calibration parameters are obtained by dividing the Fourier transform of \( E(x,y) \) by the Fourier transform of \( E_0(x,y) \) and by taking the inverse Fourier transform from the result of the division:

\[ \text{IFT} \left( \frac{FT(E(x,y))}{FT(E_0(x,y))} \right) = \sum_{i=1}^{n} a_i \delta(x - (i - 1)d), \]  

(5.3)

where \( a_i \) signifies the actual phase and amplitude excitation of the \( i \)th element and \( d \) is the inter-element spacing. The actual excitation coefficients can be calculated from the equation above. After that, the calibration coefficients can be calculated based on the loss and phase shift coefficients \( c_i \) that are obtained from

\[ a_i = b_i \cdot c_i, \]  

(5.4)

where \( b_i \) is the desired amplitude and phase coefficient of the \( i \)th element.

5.2. Element input signal measurement methods

The disadvantage of field measurements is that they are easily influenced by the environment. They also require bulky measurement systems and the measurements must usually be done in anechoic chambers, which reduces the flexibility of these methods. The measurements can be made in a much more compact scale by measuring the signals that enter the antenna elements. The drawback of these methods is that they do not take the radiation patterns of the antenna elements into account. However, these methods are sufficient in the scope of this thesis because it concentrates on the array factors that essentially are functions of only the excitation phases and amplitudes. Also the errors in element locations affect the array factor, but these effects can be neglected.

One way to measure the amplitude and phase differences between two signals is to measure the waveforms with an oscilloscope and to determine the phases and amplitudes of the waveforms with the internal functions of the oscilloscope. Oscilloscopes are intended for time-domain measurements and therefore they are
very well suited for the phase and amplitude measurements. The waveforms could also be processed afterwards with a computer, e.g. with Matlab. The amplitude difference can simply be calculated by subtracting the amplitude of the first signal from the amplitude of the second signal and by taking the absolute value of the result. For phase difference calculation there exists three common methods: cross-correlation method, Fourier transform method and Hilbert transform method.

Another relatively simple way to measure the amplitude and phase differences of two signals with a vector network analyzer is presented in [36]. With this method the amplitude and phase differences can be calculated with only power measurements and s-parameter measurements. In the measurement setup, two signals are coupled with a coupler and the coupled signal is fed to the network analyzer, as shown in Figure 5.2. The measurement has three phases:

1) S-parameter measurement of the coupler;
2) Power measurement to both input signals separately;
3) Power measurement to the combined signal.

Once the above-mentioned measurement have been performed, the amplitude and phase differences can be calculated. The amplitude difference can be calculated from

\[ \Delta A = \left| \frac{S_{31}}{S_{32}} \right| \sqrt{\frac{P_2}{P_1}} - 1, \]  

where \( S_{31} \) and \( S_{32} \) are the transmission coefficients from port 1 to port 3 and from port 2 to port 3, respectively, and \( P_1 \) and \( P_2 \) the powers of signals 1 and 2 when measured separately, respectively. The phase difference can be calculated from

\[ \Delta \varphi = \arccos \left( \frac{P_S - P_1 - P_2}{2 \sqrt{P_1 \cdot P_2}} \right) - \varphi_c, \]  

where \( P_S \) is the measured power of the combined signal and \( \varphi_c \) is the phase difference of the signal paths of the combiner.

\[ \text{Figure 5.2. Setup for s-parameter and power measurements.} \]
5.3. Measurement methods based on built-in self-test systems

Typically the element input signal measurements described previously have been done by using ground-signal-ground (GSG) probes. Although signal measurement methods are suitable for array factor based calibration, these methods are still rather expensive and time-consuming. In fact, the cost of testing a phased array is much higher than that of the chip itself. To address this problem, built-in self-test (BIST) systems for phased array chips have been developed [37].

One approach for implementing BIST systems is presented in [38]. In this method, the amplitude and phase response of the RF paths are determined using an integrated I/Q receiver. The structure of the BIST system is shown in Figure 5.3. One RF path is switched on at a time. The local oscillator (LO) signal is used as a BIST signal and it is fed to the RF path. The LO signal is also fed to a poly-phase filter (PPF) that divides the LO signal into I and Q components. The output signal of the RF path is multiplied with the I and Q components of the LO signal in the I/Q mixer. Because the frequencies of the input signals of the I/Q mixer are the same, the resulting I and Q outputs are at baseband. The amplitude and phase response of the RF path in question can be calculated with equations

\[
A = \sqrt{A_I^2 + A_Q^2}, \\
\phi = \arctan\left(\frac{A_Q}{A_I}\right),
\]

(5.7)

where \(A_I\) and \(A_Q\) are the baseband output voltages of the I/Q mixer.

Figure 5.3. Phased array BIST system with I/Q receiver.
A method similar to the BIST system with an I/Q receiver explained above is presented in [39]. Instead of an I/Q mixer, a symmetric mixer structure (Figure 5.4.) is used for phase and power measurements. The mixer structure consists of two mixers and two phase delay elements with the phase delay value of $\phi$. The first signal with phase $\theta_A$ is connected to the unit mixer A and the upper phase delay element. The second signal with phase $\theta_B$ is connected to the unit mixer B and the lower phase delay element. The outputs of unit mixers A and B are denoted by $A$ and $B$, respectively. The following proportionalities are valid for the mixer structure:

$$
A + B = \cos \theta \cdot \cos \phi \\
A - B = \sin \theta \cdot \sin \phi,
$$

(5.8)

where $\theta = \theta_A - \theta_B$ is the phase difference. Thus, the phase difference can be calculated if the amplitudes $A$ and $B$ and the value of the phase delay elements $\phi$ are known.

In the phase difference measurement both input signals are connected to the symmetric mixer simultaneously. The power difference measurement, on the other hand, can be made by connecting the input signals to the symmetric mixer at different times. Firstly, the input signal A is connected to the symmetric mixer and its power level is detected as a DC output level using square-law detection. After that, the same is done for the input signal B [39].

![Figure 5.4. Symmetric mixer structure.](image-url)
6. TESTING THE MEASUREMENT METHODS

In this chapter the measurement methods for calibration presented in Chapter 5 are compared briefly. The most promising methods for the phased array phase and amplitude difference measurements will be selected. The selected methods will be tested in laboratory by measuring the phase and amplitude differences of two generated sinusoidal signals. Based on the measurement error performance of these methods along with some practical considerations, one of these methods is selected for measuring the phase and amplitude differences of a 5G phased array transceiver prototype.

6.1. Comparison of measurement methods

The only way to measure the actual radiation pattern of a phased array is to perform field measurements. The basic principles of field measurements are explained in Section 5.1. The advantage of this method when measuring the radiation pattern of an antenna array is that it takes the radiation pattern of a single element into account. However, this measurement method has many drawbacks. Field measurement systems are very complicated. For example, to be able to measure the radiation patterns, a robot system with servo motors would be needed to rotate the antennas. Building and programming such a robot system would be very time-consuming. In addition, field measurements are easily influenced by the environment, especially when an anechoic chamber with suitable frequency range is not available. Besides, in this thesis work the main focus is in the array factor rather than the element patterns. For the above reasons, the field measurement methods were ruled out at an early phase of the thesis work.

It was decided that the output RF signals would be measured from the antenna element inputs. The principles of phase and amplitude measurement used in the IQ-receiver-based method and the symmetric mixer-based method presented in 5.3. were considered. The principle of the IQ-mixer-based method is mathematically rather simple. However, it was decided that this method would be too complex in practice because it would require a separate IQ-mixer. An IQ-mixer-based system is also potentially difficult to calibrate. The symmetric mixer-based method, on the other hand, is theoretically suitable for the measurement of the prototype circuit; it would be possible to probe two output RF signals at a time and to measure the phase and amplitude differences with the symmetric mixer. One drawback of this method is that phase shifters would have needed to be purchased because there were none available in the RF laboratory. Furthermore, just like the IQ-mixer structure, the symmetric mixer circuit is rather complex which may make the calibration of the measurement system challenging. Both the IQ-mixer-based methods and the symmetric mixer-based method were ruled out.

The VNA power and s-parameter measurement-based method and the oscilloscope measurement-based method were chosen as the most promising methods for phase and amplitude difference measurements. The advantages of these methods are the relative simplicity of implementation and the availability of the necessary measurement equipment and RF components in the RF laboratory of Centre for Wireless Communications.
6.2. Testing the chosen measurement methods

The accuracies of the methods based on VNA and oscilloscope measurements were tested by measuring phase differences with both methods and comparing the results. Because the RF frequency of the prototype circuit is approximately 28 GHz and the maximum input frequency of the oscilloscope is 14 GHz, there was a need to test the phase difference measurement with the oscilloscope using down-conversion. For this reason, the measurement methods were tested in two cases: using RF frequency of 10 GHz and using RF frequency of 15 GHz. The measurements could not be tested at the frequency of 28 GHz because, at the time of writing, there were no mixers available with high enough frequency range. In the former case the oscilloscope measurement was performed by measuring two RF tones directly, and in the latter case the oscilloscope measurement was performed by down-converting the 15 GHz input signals and measuring the resulting down-converted signals.

6.2.1. VNA power and s-parameter measurement method

Firstly, the phase difference measurement was tested with VNA power and s-parameter measurements. The measurement setup is illustrated in Figure 6.1. In the measurements, Keysight N5247A PNA-X network analyzer with frequency range 10 MHz – 67 GHz was used. The effect of the cables and the combiner on the amplitude and phase measurements were calibrated out. The calibration was performed between the calibration planes shown in Figure 6.1. Both s-parameter calibration and source power calibration were performed. With Differential IQ option of the VNA the source power calibration was possible to be performed only for one source port. The source power calibration was performed for port 3 because it was experimentally noticed that this results in more accurate results. After calibration, two RF signals were generated to ports 1 and 3 with Differential IQ option. Three distinct measurements were made:

1) Only port 1 was turned ON and the output power was measured with port 2.
2) Only port 3 was turned ON and the output power was measured with port 2.
3) Both ports 1 and 3 were turned ON and the output power was measured with port 2 while sweeping the phase difference between ports 1 and 3 from 0° to 180°.

The three measurements explained above were performed separately at frequencies 10 GHz and 15 GHz with IF bandwidths 10 kHz and 100 kHz. The measurement data was saved to a computer and the phase differences were calculated as explained in Section 5.2. At each phase difference point the deviation of the measured phase difference was calculated. The absolute value of this deviation is referred to as measurement error. The measurement errors of measurements at RF frequencies 10 GHz and 15 GHz are illustrated in Figures 6.2 and 6.3, respectively.

From Figure 6.2 it can be seen that at 10 GHz the phase measurement errors with IF bandwidths 10 kHz and 100 kHz are almost the same. The maximum phase measurement errors are 14.7° and 13.5° for IF bandwidths 10 kHz and 100 kHz, respectively. The respective average phase measurement errors are 1.4° and 1.2°. Thus, the measurement with a higher IF bandwidth gives slightly more accurate results.
At 15 GHz, the choice of IF bandwidth has a greater impact on the measurement accuracy, as it can be seen from Figure 6.3. The maximum phase measurement errors are 6.4° and 7.5° for IF bandwidths 10 kHz and 100 kHz, respectively. The respective average phase measurement errors are 1.6° and 1.8°. The difference in maximum measurement error is not significant considering that the measurement error decreases very rapidly when the phase difference increases from zero. The average measurement error is lower when a lower IFBW is used. However, it is important to note that when the spacing between an antenna array is λ/2, which usually is the case, the progressive phase shift β is

\[
\beta = kd \cos \theta_0 = \frac{2\pi \lambda}{\lambda} \cos \theta_0 = \pi \cos \theta_0,
\]

where \( \theta_0 \) is the steering angle. This means that the phase difference between two successive elements in an antenna array is 90° at most. Thus, even though the average measurement error is lower with IFBW = 10 kHz, the average phase difference measurement error is lower with IFBW = 100 kHz when the phase difference is less than 90°. Therefore, in a λ/2-spaced antenna array phase difference measurements higher IFBW yields more accurate results.

In each case the phase difference measurement errors are at their highest at low phase differences. This is probably due to the accuracy of calibration. One potential problem is that the source calibration could be performed for one source port only. The problems related to calibrating the VNA with Differential IQ option were left unsolved.

![Figure 6.1. Measurement setup for the VNA phase difference measurement.](image)
Figure 6.2. Measurement error as a function of phase difference for the VNA measurement at 10 GHz.

Figure 6.3. Measurement error as a function of phase difference for the VNA measurement at 15 GHz.
6.2.2. Oscilloscope method

After the VNA measurements, basic oscilloscope phase difference measurement was tested. In these measurements, Agilent Infinium DSO81204A oscilloscope with maximum frequency of 14 GHz and maximum sample rate of 40 GSa/s was used. At first, this method was tested by generating two RF signals at 10 GHz that were fed to channels 1 and 2 in the oscilloscope. The measurement setup for this direct measurement is presented in Figure 6.4. The phase difference of the RF signals was varied from 0° to 180° with 10° steps and the phase differences were manually measured with the oscilloscope. There were some phase offset between the signal arriving to the channels of the oscilloscope due to the cables. This offset was calibrated by setting zero phase difference between the signals, measuring the phase difference with the oscilloscope and subtracting this measured value from all the measured phase differences later on. The absolute values of the measured deviations between the real phase differences and the measured phase differences are plotted in Figure 6.5. The results are very accurate; the maximum measurement error is only 0.25°.
Next, the method was tested by down-converting the RF signals before feeding them to the oscilloscope. The measurement setup for the down-conversion-based measurement is presented in Figure 6.6. The 15 GHz RF signals were fed to the RF ports of the mixers. The local oscillator signal was generated with Agilent E8257C PSG analog signal generator. Again, the phase difference of the RF signals was varied from 0° to 180° with 10° steps and the phase differences were measured. This was done by separately by down-converting the RF signals to 100 MHz and 1 GHz in order to see, how the frequency of the mixers’ output signals affect the measurement results. The phase offset caused by the cables was calibrated as in the direct oscilloscope measurement. The measured absolute values of the measurement errors are presented in Figure 6.7. As it can be seen, the accuracy of the phase difference measurement is significantly better when using lower IF frequency. At 1 GHz IF frequency the maximum measurement error is approximately 6.5° while at 100 MHz IF frequency the maximum measurement errors is approximately 2.2°.
Figure 6.6. Measurement setup for phase difference measurement with an oscilloscope using down-conversion.

Figure 6.7. Measurement error as a function of phase difference for the oscilloscope measurement at IF frequencies of 1 GHz and 100 MHz.
6.2.3. Comparison of the methods

The accuracies of phase difference measurement of the two methods are compared at RF frequencies 10 GHz and 15 GHz in Figures 6.8 and 6.9, respectively. Figure 6.8 shows that, at 10 GHz, the VNA measurement yields very inaccurate results at low phase difference values. This is probably due to calibration inaccuracy. In this case, the IF bandwidth of the network analyzer has only a little effect on the measurement results. The measurement results obtained with the direct oscilloscope measurement are preeminently more accurate than the VNA measurement results.

In Figure 6.9 the VNA measurements are compared to the oscilloscope measurement with down-conversion at 15 GHz. This case is more important than the comparison between VNA measurements and direct oscilloscope measurement because when measuring the prototype circuit, down-conversion will be needed due to high operating RF frequency. As it can be seen in Figure 6.9, the average measurement errors of the VNA measurement with both IFBW values are approximately the same, but below 90° real phase difference, the higher IFBW values give more accurate results. The oscilloscope measurement with 1 GHz IF frequency yields the most inaccurate results. On the other hand, when IF frequency of 100 MHz is used, the oscilloscope measurement yields the most accurate results.

![Graph](image)

Figure 6.8. Comparison of the phase difference measurement accuracies between the VNA measurement with two IFBW values and the direct oscilloscope measurement at 10 GHz RF frequency.
Figure 6.9. Comparison of the phase difference measurement accuracies between the VNA measurement with two IFBW values and the oscilloscope measurement with down-conversion with two IF frequencies. RF frequency is 15 GHz.

As a summary, the oscilloscope measurements lead to the most accurate results provided that a suitable IF frequency is selected. This is not surprising because phase difference measurement is essentially a time-domain measurement and the purpose of oscilloscopes is exactly to study waveforms in time-domain, while network analyzers are intended for s-parameter and power measurements. Theoretically, the s-parameter and power measurements could result in exact results but accurate enough calibration of the measurement setup is very difficult. In contrast, the oscilloscope measurement setup can simply be calibrated by measuring the phase offset of both signal paths, i.e. the measured phase difference when the real phase difference is zero, and subtracting the offset from all measured values. Therefore, oscilloscope is a more suitable equipment for phase difference measurements. Furthermore, oscilloscope can measure amplitudes directly, while VNA can only measure power values that would be needed to convert to voltages. As a conclusion, the oscilloscope measurement method was chosen for both phase and amplitude difference measurements.
7. PROTOTYPE MEASUREMENT SYSTEM

In the final phase of this thesis work, a measurement system for the phase and amplitude error measurements was designed for initial characterization of the prototype circuit. The measurements in TX mode and RX mode differ from each other slightly, so measurement setups for TX mode- and RX mode-measurements were designed separately. In addition to the measurement setups, setups for calibrating the measurement system were also designed. Furthermore, flowcharts explaining the measurements and measurement system calibration were created. In the near future, a software for controlling both the transceiver and measurement instruments and for harvesting and analyzing measurement data needs to be developed. Most of this software development is beyond the scope of this thesis, except for some instrument control function. However, the calibration and measurement flowchart are expected to help in the future software development. Finally, some initial phase and amplitude measurements were performed.

7.1. Transceiver prototype structure

The simplified structure of the transceiver prototype is shown in Figure 7.1. The only input pin of the transceiver is the IF signal input/output. A local oscillator signal produced by a voltage-controlled oscillator (VCO) is used to up-convert the IF signal in transmitter mode and to down-convert the RF signal in receiver mode. The upper branches with power amplifiers are the transmitter mode paths and the low branches with low-noise amplifiers are the receiver mode paths. When either TX or RX mode is selected, the transceiver control software connects the corresponding paths to the signal path by controlling the switches. The control software also controls the phase shifters, variable gain amplifiers and the RX path attenuator. Next to the antenna elements there is a feedback line that couples the output RF signal to two directions.

The RF signal lines connected to the phase shifters have no cable connectors. This causes some difficulties in the measurement system design; the phase and amplitude response of each RF branch cannot simply be measured with VNA s-parameter measurement because the input and the output are at different frequencies. The IF signal port is the only port that can be used as a reference. For measuring the RF output, there are two options. The measurement cables can be connected directly to the connectors that are located next to the antenna elements. Another method is to use a feedback line, which would enable a higher degree of automation of the measurement system. However, measuring the signals directly from the RF output ports is a sufficient method for initial characterization of the transceiver prototype.
7.2. Measurement set-up

Figure 7.2 shows the measurement set-up for the TX mode. The measurement system comprises a PC and three measurement instruments: Keysight PNA-X N5247A network analyzer, Keysight Infiniium DSOS104A oscilloscope and Agilent E8267D signal generator. The network analyzer is used to create a sinusoidal signal that is fed to the IF input port of the transceiver. Measurement cables are connected to two RF output ports at a time, one of which is always at the first (reference) port. These RF signals are down-converted with high-frequency mixer to low enough IF frequency, in order to enable the measurement with the oscilloscope. The signal generator is used to create the LO signal for down-conversion.

The corresponding measurement flowchart is presented in Appendix 1. During the measurements, the transceiver and measurement instruments are controlled by software. The measurements are performed for all combinations of predefined RF frequency points, progressive phase shift settings for all RF branches. This is done by connecting the RF cables to two RF ports at a time; one to the first port and one to the $n$th port, $n = 2, 3, \ldots, 8$. When the cables are connected, the phase and amplitude are measured at all combinations of RF frequencies and progressive phase shifts. The phase of the $n$th branch is measured relative to the first branch, and the phase of the first branch is assumed to be $0^\circ$. The measurement results are saved to the PC with descriptive file names. The measurements are performed for each branch in turn.
Figure 7.3 shows the measurement set-up for the RX mode. The same measurement instruments are used in RF mode measurement as in TX mode measurements. The network analyzer is used to create two RF signals with equal amplitudes and 0° phase difference. These signals are fed to two RF input ports at a time, one of which always is the first port that is considered as a reference. The approximately 6 GHz IF output has to be down-converted before feeding it to the oscilloscope because the maximum input frequency of the oscilloscope is only 1 GHz. The signal generator is used to create the LO signal for down-conversion. Once all cables are connected, the amplitudes are measured in three phases. Firstly, only the first port is switched ON and the IF output amplitude is measured. Secondly, only the nth port is switched ON and the IF output amplitude is measured. This is done at all frequency points for all RF branches. Finally, both the first and the nth branches are switched on at the same time and the amplitude of the resulting sum signal is measured. These measurements are repeated for all frequency points and all branches. The phase difference of the first and the nth branch can be calculated from the measured amplitudes as follows:

\[ \Delta \phi = \arctan \left( \frac{A_{\text{comb}}^2 - A_1^2 - A_n^2}{2 A_1 A_n} \right), \]  

(7.1)

where \( A_{\text{comb}} \), \( A_1 \) and \( A_n \) are the amplitudes of the sum signal, the single signal received from the first branch and the single signal received from the nth branch, respectively.
Figure 7.2. Measurement set-up for TX mode measurements.
7.3. Calibration set-ups

Before performing the measurements, the measurement systems must be calibrated. The input and output of the TX mode measurement system (Figure 7.2) must be calibrated with separate calibration measurement set-ups. The calibration measurement set-ups for input and output calibration are presented in Figures 7.4 and
7.5, respectively. In the measurement system shown in Figure 7.2, the input cable between the network analyzer and the transceiver prototype causes some power loss. The loss of the cable is measured with the network analyzer in a defined frequency range, and the results are saved to a file. These values can be used for correcting the actual measurement values when the measurements have been performed. The cables and the mixers between the prototype output and oscilloscope cause some power loss and phase deviation. These effects can be calibrated with the calibration measurement system shown in Figure 7.5. Two RF signals with equal phases and amplitudes are generated with the network analyzer, and the phase and amplitude deviation caused by the mixers and the cables can be measured with the oscilloscope. These deviation values can be used to correct the measurement values.

Figure 7.4. Measurement set-up for TX mode measurement system input calibration.
Similar input and output calibration should be performed for the RX mode measurement set-up of Figure 7.3 as well. At the input, the purpose is to feed two signals with equal phase and amplitude to the prototype input ports. Therefore the loss and the phase shift of the two input cables must be measured. This can be done with the calibration measurement set-up presented in Figure 7.6. The loss and the phase shift of the cables can be measured simply by measuring $S_{21}$ and $S_{43}$ parameters with the network analyzer. The measured values are stored to the computer and they can be used in the measurements to give initial phase and amplitude offset to the RF signal generated by the network analyzer so that the signals arriving to the antenna ports of the transceiver have equal phases and amplitudes. The calibration set-up for output calibration is presented in Figure 7.7. The loss caused by the cables and the mixer can be measured by measuring the $S_{23}$ parameter. In the RX mode, the phase differences are calculated by the measured...
amplitude values, as explained in Section 7.2, so in this case it is not necessary to perform phase calibration in the output. The measured values are stored to the computer and they can be used to correct the measured amplitude values.

Figure 7.6. Measurement set-up for RX mode measurement system input calibration.

Figure 7.7. Measurement set-up for RX mode measurement system output calibration.
7.4. Measurement software

Matlab software was written for controlling the measurement instruments. The basic principle of the software is explained here. The software comprises five functions:

- Main control
- Instrument connection function
- Network analysis control function
- Signal generator control function
- Oscilloscope control function

The main control code is used to call all other functions. In the beginning of the main control code, all the necessary measurement parameters are defined. The main control code then calls the instrument connection function that establishes a connection from the PC to all measurement instruments via LAN interface. Next, the instrument control function calls the measurement instrument control functions one at a time. In each of the instrument control function, the instrument is controlled based on the parameters given in the main control code.
8. CONCLUSIONS

This thesis work had two objectives. The first objective was to study how random phase and amplitude errors affect various performance characteristics of phased array antennas. The analysis of these effects was expected to give insight into how accurate phase and amplitude measurement should be performed for a phased array so that adequate performance is achieved after calibration. The second objective was to design an automatized measurement system for measuring phase and amplitude errors of a 5G phased array transceiver prototype.

The effect of phase and amplitude non-idealities on the performance of phased array transceivers was studied by using a Matlab model of a linear, uniformly spaced antenna array with uniform amplitude distribution. The code takes the essential parameters (number of antenna elements, operating frequency, spacing between elements and steering angle) as inputs and calculates the corresponding array factor. For analyzing the performance of arrays, separate functions were created to calculate the main lobe gain, the 3-dB beamwidth, the mainlobe direction and the side lobe level based on the calculated array factors. The effect of random phase and amplitude errors was modeled by performing Monte Carlo simulations. In order to do so, the Matlab program takes phase and amplitude errors standard deviations as additional inputs. The program calculates the array factor a defined number of times, each time giving each antenna element a random phase and amplitude error value from a Gaussian distribution that is defined by the given phase and amplitude errors standard deviations. The phase and amplitude error values between the elements are independent of each other. After all array factors have been calculated, the values of the defined array factors are calculated for each array factor. The output of the software is distributions of the performance parameters.

The Monte Carlo simulation was performed at 28 GHz operating frequency, half-wavelength inter-element spacing 10 000 times for various combinations of phase and amplitude error standard deviations and numbers of antenna elements. The results showed that increasing the phase and amplitude errors generally increase the deviations of the performance parameters. It was found that the phase errors can only decrease the mainlobe gain and only increase the sidelobe-level. On the other hand, the 3-dB beamwidth and the mainlobe direction can either increase or decrease because of the phase errors. The amplitude errors can either increase or decrease the values of all performance parameters, except the mainlobe direction; amplitude errors have no effect on it. It was also found that the effect of phase and amplitude errors combined on the array performance is more severe than the effects of only phase errors or only amplitude errors. Increasing the number of antenna elements has a positive effect on the array performance. Already with 16 antenna elements the phase and amplitude errors have only a little effect on each of the performance parameters, and with 64 elements the effect can be considered negligible. For the prototype phase and amplitude difference measurement, it was estimated that a phase measurement error of less than 5° and amplitude measurement error of less than 10 % will guarantee a sufficient accuracy.

From a more general point of view, the obtained performance parameter distributions fail to fully comply with the mathematical theory presented in Section 4.2. According to Equation 4.3, both phase and amplitude errors should decrease the mainlobe directivity but according to the simulations, the amplitude errors are as likely to increase as to decrease the mainlobe gain. The explanation for this may be
the ambiguity in terminology. In the simulations, the maximum value of the array factor was considered as the "gain", but directivity is generally defined as the ratio of maximum radiation intensity of the antenna to the radiation intensity of an isotropic source. In addition, Equation (4.6) does not explain why phase errors can only increase the sidelobe-level but amplitude errors are as likely to decrease as to increase the sidelobe-level. In order to fully understand the relation between the presented mathematical background and the simulation results, more work must be conducted. The software also has some minor bugs that occur in some rare cases. These must be fixed in case that the software is used in the future. The software could be used, for example, to analyze the effect of errors when amplitude tapering, such as Dolph-Chebyshev and Taylor, is used.

After the phase and amplitude error analysis, a measurement system was designed for measuring phase and amplitude errors of a 5G phased array transceiver prototype. Due to different measurement approaches between TX and RX modes, a separate measurement system layout was designed for both modes. In TX mode, a network analyzer is used to generate an IF signal to the transceiver. The phase and amplitude differences of two transceiver RF outputs are measured at a time by down-converting the signals to 100 MHz and feeding them to an oscilloscope. Two mixers and a signal generator are used to down-convert the RF signals. In RX mode, the network analyzer is used to create transceiver RF input signals. The amplitude response of each branch can be measured by feeding a signal to one RF input port at a time, down-converting the IF output signal and feeding it to the oscilloscope. The phase difference between two branches can be measured by feeding an RF signal to the corresponding RF input ports and measuring the IF output signal amplitude. The phase difference between the two branches can be calculated based on the single-branch amplitude response of both branches the combined amplitude response of the two branches. In addition to the measurement system layouts, separate measurement systems were designed for calibrating the measurement system itself. Detailed flow-charts describing the measurement processes were created. Finally, Matlab software for controlling the measurement instruments was created. The created instrument control software was tested and it seems to work as it should.

The disadvantage of the designed measurement system is that only two RF branches can be measured at a time. After measuring one branch relative to the reference branch, the cable must be detached from the current port to the next port. This is not only time-consuming, but it can also damage the SMP connectors and affect the measurement results. The reason for choosing this approach for the measurement is that the time was very limited and this approach was relatively easy to implement. In future measurement, a feedback loop of the antenna elements should be used in order to connect the outputs to the measurement instruments. This approach would enable a higher degree of automation of the measurements. The calibration of the feedback loop may cause some problems that must be solved. Finally, in order to fully automatize the measurement system, a software that can control both the transceiver and the measurement instrument must be created.
9. REFERENCES


10. APPENDICES

Appendix 1  Flowchart of the measurement procedure in TX mode
Appendix 2  Flowchart of the measurement procedure in RX mode
Appendix 1  Flowchart of the measurement procedure in TX mode
Appendix 2  Flowchart of the measurement procedure in RX mode

1. RF input amplitude and phase calibration of the measurement setup (separate setup and procedure)
2. Perform output amplitude calibration (separate setup and procedure)
3. Perform LO power calibration (separate setup and procedure)
4. Connect the first two array outputs to oscilloscope via mixers
5. Set the corresponding LO frequency (E8267D signal generator)
6. Set steering angle (phase shifter states)
7. Switch both RF branches ON and measure IF signal amplitude and save to file
8. Last RF frequency?
   - NO: Set the next RF frequency (N5247A network analyzer)
   - YES: Last steering angle?
     - NO: Set RF frequency back to the default value (N5247A network analyzer)
     - YES: Last RF branch?
       - NO: Switch only the nth RF branch ON and measure IF signal amplitude and save to file
       - YES: Switch only the first RF branch ON and measure IF signal amplitude and save to file
9. Set RF frequency and steering angle back to the default values
10. Turn all signals OFF and connect the cable to the next RF branch
11. Set RF frequency (N5247A network analyzer)
12. Instrument initialization: set signal generator power and VNA power and phase offsets based on calibration words
13. DONE