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TO SCAFFOLD OR NOT TO SCAFFOLD MATHEMATICS LEARNING; THAT IS THE QUESTION

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The purpose of this research is to study the effects of scaffolding on the conceptual and procedural knowledge of the students. It investigates how scaffolding affects the learning results and learning process in mathematical problem-solving in a teacher-to-classroom interaction. Mathematical competence development relies on conceptual and procedural knowledge. Learning of procedures must be connected to the learning of concepts and vice versa. Scaffolding assists the teacher to construct a learning trajectory which capitalises on the growth of student development. It also assists the students to develop both types of knowledge by focusing on the role of prior knowledge, self-efficacy, and sequencing of the study materials.

In this study, sixth-grade students \((n = 31)\) solved the questions related to algebraic problem-solving. This was done on a pre-structured worksheet which scaffolded their mathematical learning process and displayed the process to reach the learning results. The research questions were as follows: (1) What kind of difference is there in students’ knowledge level between the prior knowledge test and after the main activity worksheet? (2) How did scaffolds affect the learning results and learning process of the students? (3) How did self-efficacy develop across different thinking levels during the learning process?

The data was collected over the period of two days. On the first day, students completed the prior knowledge test in a ten-minute session and then the teacher instructed on an algebraic problem-solving in a forty-five-minute session. After the instruction, the students were provided with a problem-solving worksheet which had seven algebraic questions based on three thinking levels of revised Bloom’s Taxonomy; remembering, understanding and applying. It also had the elements of embedded and optional scaffolds namely self-assessment box and access to teacher’s and peer’s help during the learning process. On the second day, eight students were selected for an interview, based on their different performance in terms of the prior knowledge test and main activity? The findings show that there was a significant difference between the conceptual knowledge the prior knowledge test and the main activity. Self-efficacy beliefs (measured by self-assessment) of the students developed in a dynamic manner as the thinking levels in the worksheet progressed indicating that students’ zone of proximal development change according to the situation they face.

To conclude, the structured tasks can help teachers to know where students lie on a zone of proximal development during classroom interaction. Self-assessment embedded in the worksheet allows the students to evaluate their understanding. When students’ mathematical understanding progress is traceable, it is possible for teachers to design the activities and tasks which support student learning in mathematics as well as help the teachers to adjust and target their assistance precisely.

Keywords: Scaffolding, Conceptual Knowledge, Procedural Knowledge
“Unless someone like you cares a whole awful a lot, nothing’s going to get better. It’s not.”

Dr. Seuss
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1 Introduction

Our modern civilisation is based on the information we derive from the people’s ability to compute (Roman, 2004). The true meaning and importance of mathematics remains hidden from the students as well as unappreciated by the general population. But, mathematics is the bedrock of our modern society. We need to indulge students to find out the importance of mathematics in life. This is an information age and mathematics helps us to represent the information. Mathematics keeps our internet running, our airplanes in the sky and our rockets in the space. Similar to science and technology, mathematics is a persuasive tool which supports and provides enhancements to the other tools (Roman, 2004).

Unfortunately, our education still focuses on teaching the tools of life as separate subjects, and miss the importance of all the tools being inter-connected. This is one of the reasons why most of the students have a mistaken impression of mathematics where they don’t understand it completely (Roman, 2004). Even though mathematical knowledge is highly relevant in and to the society, many of the people believe mathematics to be irrelevant to them personally (Niss, 2003). Another reason is that the mathematics educators in western societies have not been successful to equip the majority of the population with knowledge, insights, and skills which are asked for (Niss, 2003).

In mathematics education, there have been numerous reforms over the past years. A new wave of documents and information surfaces about every ten years which offers methods to improve the teaching in mathematics (Myers, Sztajn, Wilson, & Edgington, 2015). But, they have “failed to change significantly the face of the mathematically successful students” (p. 11). Recently, mathematicians are developing on the concept of learning trajectories which includes the use of different instructional tasks. LT establishes a strong foundation of student learning with instruction in the school setting. It capitalises the central role that mathematics’ teachers play in the growth of student’s development and understanding of mathematics (Wilson, Sztajn, & Edgington, 2013). LT also traces the ways in which students transform their informal ideas into formal ones through appropriate instructional opportunities into refined mathematics learning. Early research into LT suggested that teachers when used LT, it subsequently improved their knowledge of mathematics (Mojica, 2010), guided their instructional decisions (Wilson P. H., 2009) and enhanced their abilities to use student thinking (Clements, Sarama, Spitler, Lange, & Wolfe, 2011).
Learning Trajectory is kind of a learning path in which a goal is decided, then the thinking levels and the activities and tasks are devised for the student’s learning. For the teacher, it helps to select appropriate activities and tasks for students and also it helps them to examine the learning paths of a student which eventually helps them to design activities and tasks which support student learning in mathematics (Wilson, Sztajn, & Edgington, 2013).

The type of instructional strategy to implement in the classroom which is a core part of a learning trajectory depends on what type of knowledge, a teacher wants to promote. In Mathematics, two types of knowledge are identified on which mathematical competence development relies; conceptual and procedural knowledge (Rittle-Johnson & Schneider, 2014). Knowledge of concepts is often termed as conceptual knowledge (Byrnes & Wasik, 1991). On the other hand, knowledge of procedures is known as procedural knowledge (Canobi, 2009).

The most well-accepted perspective is the iterative view (Rittle-Johnson et al., 2001). The iterative view promotes the relation between the two types, to be of bi-directional nature. With increase in conceptual knowledge, the procedural knowledge increases and vice versa (Baroody, 2003) (Rittle-Johnson & Siegler, 1998).

Over the past 15 years, researchers are gathering evidence of the positive correlation between the both types of knowledge. In several Mathematics’ domains including solving equations (Durkin, Rittle-Johnson, & Star, 2011), the strength of the relation has proven to be fairly high.

But, is this the case in the mathematics classrooms where both, conceptual and procedural knowledge have a bi-directional relationship? In the 21st century, students view school mathematics as a series of procedures to be memorized. Researchers have concluded that procedures which are memorized are prone to error and the learning of procedures must be connected to conceptual knowledge, to develop the understanding in mathematics (Hiebert & Carpenter, 1992). Muijs and Reynolds (2010) claim that overemphasis on learning procedures may lead to difficulty for students, in transferring knowledge in the other situations. On the other hand, development in only conceptual knowledge leads to less development in the procedural knowledge.

To equip teachers with strategies which helps the students to learn the knowledge, insights and skills used in mathematics and to change their mistaken impression of mathematics, that its irrelevant to them, mathematics’ educators need to choose a suitable approach or set of approaches which focus on the development of conceptual and procedural knowledge (Rittle-
Also, to prepare the teachers with appropriate instructional strategies would mean that they would be able to apply the learning trajectory results in the classroom which focuses on the growth of the students in the classroom instead of the results. The process of learning triumphs the product. (Myers, Sztajn, Wilson, & Edgington, 2015)

For this reason, the aim of this research is to study the effect of scaffolding on the conceptual and procedural knowledge in the context of mathematics in 6th-grade students. The instructional strategy and the tasks are based on the thinking levels of Revised Bloom’s Taxonomy (Krathwohl, 2002). The students not only solve the mathematical questions but they also explain their understanding level during the process. Students solved a prior knowledge test in a 10-minute session. Then, the mathematics teacher taught a lesson based on algebraic equations and students solve a worksheet with seven algebraic questions on it. Students had to solve the problems and explain their conceptual understanding in the worksheet. Afterward, interviews were conducted from the students.
2 Theoretical Framework

This research is based on three theoretical concepts; self-efficacy and duality of knowledge in mathematics, scaffolding and revised Bloom’s taxonomy. This chapter includes an overview of all the concepts. The focus is on how scaffolding has an effect on self-efficacy and conceptual and procedural knowledge in mathematics. The interconnection of all the concepts is also clarified.

2.1 Self-Efficacy and Duality of Knowledge in Mathematics

Learning in mathematics has been attributed to many factors and self-efficacy is one of the most important factors. It is not only important in learning mathematics but in different subjects as well. Self-efficacy beliefs are one of the factors affecting learning in general, and there is research showing that their role in mathematics learning is particularly important (Pajares & Kranzler, 1995). The concept of self-efficacy was introduced by Bandura, who states that,

“It is a belief in one’s own capabilities while organizing and executing courses of action in different situations. It is a belief of a person to produce designated levels of performance which influence over events which affect their lives. Self-efficacy belief determines how a person thinks, feels, motivate oneself and behave” (Bandura, 1977, p. 71).

Due to a strong sense of self-efficacy, the personal well-being of a person is enhanced. Because of the high self-efficacy, people approach difficult tasks as challenges instead of being afraid of the tasks and giving up eventually (Bandura, 1977). It also helps a person to set higher goals and their commitment to achieve those goals is higher as well. The interesting point is that a person with high self-efficacy persist in a type of a situation where other people would give up. Self-efficacy helps people to be consistent while they are working on the goals and the tasks (Bandura, 1994).

There are a number of factors which help self-efficacy to be developed and influences but performance accomplishments are one of the greatest contributors to a student’s confidence (Bandura, 1977). Success can increase the self-efficacy and failures can undermine it, especially if the failures take place at an early stage of the student’s learning (Bandura, 1994).
A research conducted by Stramel on (2010) “A naturalistic inquiry into the attitudes towards mathematics and mathematics self-efficacy beliefs of middle school students” used a naturalistic inquiry approach and used multiple sources to collect the data such as short-answer questions, classroom behaviours and one-on-one interviews with the students. The data was analysed for patterns, themes and relationships. This study found out that students attribute their high self-efficacy belief to the teacher or the higher grades. Conversely, they have low self-efficacy belief when they are unsuccessful and they attribute it to the low grades they receive on the daily assignments as well as not understanding mathematics.

Another study conducted by Yu-Chang Chen (2010) on “Sources of self-efficacy beliefs in mathematics in seventh and eighth-grade Taiwanese students” found out that Taiwanese students have high self-efficacy when compared to the American students in mathematics. Taiwanese students have been outperforming American students in international mathematics tests due to the high self-efficacy belief recorded in them in the early stage.

Usta (2016) conducted a study on “Mathematics self-efficacy level based on PISA 2012 results in China-Shanghai, Turkey and Greece” found out that there was a positive relationship between self-confidence, teacher support and attitude towards school and mathematics self-efficacy belief in all the three countries.

2.1.1 Conceptual Knowledge

The research included conceptual and procedural knowledge because the evidence showed of the importance of both types of knowledges in mathematics (Rittle-Johnson & Schneider, 2014). On the other hand, the measure of both these knowledge types is important to find out the effect of scaffolding on the learning process and learning results in this research.

A difference has been drawn out between conceptual and procedural knowledge in mathematics by researchers and mathematicians (Rittle-Johnson, Siegler, & Alibali, 2011) Even though, both of the knowledge are connected but the interplay of conceptual and procedural knowledge is important to understand when it comes to the learning process in mathematics (Hiebert, 1986) (Schneider & Stern, 2010).

Conceptual knowledge is considered to be a deep and flexible knowledge of the underlying principles in mathematics (Star, 2005). Knowledge of concepts is considered to be conceptual knowledge (Rittle-Johnson, Siegler, & Alibali, 2011) (Canobi, 2009). On the other hand,
Hiebert (Conceptual and procedural knowledge: The case of mathematics, 1986) gives a deeper and richer definition of conceptual knowledge when it comes to mathematics, “Conceptual knowledge is characterized most clearly as the knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network” (pp. 8-9). Most of the scholars are of the view that conceptual knowledge increases and becomes richer by the increase of expertise in a particular area.

2.1.2 Procedural Knowledge

Procedural knowledge is considered to be the knowledge of series of steps and procedures taken to accomplish a goal. Knowledge of procedures is often termed as procedural knowledge (Rittle-Johnson, Siegler, & Alibali, 2011) (Canobi, 2009). Procedural knowledge is considered to be the ‘what’ and ‘how’ part of the process. It means that how a certain procedure is performed and what are the mandatory steps needed to be taken to complete the process.

The next question is, what do mathematicians mean by procedures? Procedures are known as constructs which include skills, strategies, productions and interiorized actions (Byrnes & Wasik, 1991). Procedures can either by algorithms; a prearranged sequence of action which will lead to the correct product when executed correctly or possible actions which must be sequenced properly to solve a given problem.

2.1.3 Relationship Between Conceptual Knowledge and Procedural Knowledge

Four theoretical causal relations have been drawn out when it comes to conceptual and procedural knowledge which are as follows (Haapsalo & Kadijevich, 2000) (Baroody, 2003) (Rittle-Johnson & Siegler, 1998);

1. Concepts-first Views; It means that a child first learns the conceptual knowledge through guidance and then build the procedural knowledge by continually practicing problem-solving (Halford, 1993).
2. Procedures-first Views; It means that a child first learns the procedural knowledge through exploration and then gradually develop the conceptual knowledge by abstraction process (Karmiloff-Smith, 1992).
3. Inactivation View; It means that conceptual and procedural knowledge develop independently (Haapsalo & Kadijevich, 2000).

4. Iterative View; It means that conceptual knowledge increases when procedural knowledge increases and procedural knowledge increases when conceptual knowledge increases. The relationship is said to be of a bi-directional nature (Baroody, 2003).

The fourth view, the iterative view is considered to be a well-accepted perspective. Iterative view considers gradual improvement in both types of knowledge important. It also advocates that the acquisition of conceptual or procedural knowledge first, depends upon the environmental input and prior knowledge of the topic. It also depends on which type of knowledge is being promoted first by the teacher in the lesson design, to assist the students in their learning. The positive relation between both has been found out in several domains of mathematics such as counting (Dowker, 2008) (LeFevre, et al., 2006) addition and subtraction (Canobi, Reeve, & Pattison, 1998), fractions and decimals (Hallett, Nunes, & Bryant, 2010) and equation solving (Durkin, Rittle-Johnson, & Star, 2011). The strength of the relation of these two knowledge types is considered fairly high but it also varies over time and the type of studies conducted. Not only a positive relation, but a bi-directional relation has also been found in different domains from fractions to equations. Schneider et al. (2011) explain a study conducted in a middle-school where student’s conceptual and procedural knowledge was measured in equation solving. A prior knowledge test was conducted and researchers established two measurement points. Then, students studied and explained mathematical examples with a partner. Those examples were assessed and scored. This study showed that conceptual and procedural knowledge had a bi-directional relation that wasn’t moderated by prior knowledge. The results showed that when complex knowledge structures change, it also assists in the development of mathematical competence.

2.2 Revised Bloom’s Taxonomy

Research in education has shown that the sequencing of learning material matters as much as the content itself which is being taught (McNeil, et al., 2012) (Murdock, 1962) (Saffran, Aslin, & Newport, 1996). This is, how the material is being introduced to the students is important for their acquisition of knowledge, no matter which type it is. For example, several research show that learners in mathematics benefit when different problem types are interconnected rather than
sequenced in a blocked order (Rohrer, 2012). Many experiments have shown that merely rearranging the order in which students encounter examples or questions without altering or changing the nature of these example or questions, can boost results (Rohrer, 2012).

Bloom’s Taxonomy (1956), “specifies a framework in which distinctive cognitive learning activities are identified for each of the six sequential stages through which the acquisition of knowledge and skills takes place” (Ugur, Constantinescu, & Stevens, 2015, p. 93). The six stages of Bloom’s taxonomy are as follows; Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation (Bloom, 1956). However, Krathwohl (2002) revised the taxonomy by labeling several of the stages again and transforming the taxonomy into a bi-dimensional structure which not only supports different types of knowledges but also a cognitive process. The stages for revised Bloom’s taxonomy are as follows; Remembering, Understanding, Applying, Analysing, Evaluating and Creating.

![Revised Bloom's Taxonomy](image)

Figure 1. Revised Bloom's Taxonomy

It has been more than 50 years now; Bloom’s Taxonomy has been used by several educators to facilitate the process of deciding learning objectives whether it comes to assessing students’ skills and knowledge. There are several studies on the effectiveness of Bloom’s Taxonomy in different fields of education. One such study was published in (2012) where researchers incorporated Bloom’s Taxonomy into their multiple-choice examination questions in a course and assessed its effectiveness in detecting areas of improvement in learning. They found out that
well-designed multiple-choice examination questions which incorporate learning domains of Bloom’s taxonomy lead to a potential method of assessing critical thinking skills (Kim, Patel, Uchizono, & Beck, 2012). With the help of these six sequential stages, teachers can establish the learning objectives according to the knowledge and skills they are trying to promote in the classroom. Then, these learning objectives translate into the learning activities or questions students solve during their learning.

2.3 Scaffolding

Scaffolding has been proposed as a mechanism to support learners in reaching new levels of understanding. It has been implemented as a method to learn mathematics. Scaffolding was introduced by Woods, Bruner, and Ross (1976), “adult controlling those elements of the task that are essentially beyond the learner’s capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence” (p. 89). Scaffolding term has been linked to the work of Vygotsky who was of the view that an adult or more knowledgeable person guides the child to complete a task which would have been otherwise difficult. Children then learn to bridge the gap between ‘actual’ and ‘potential’ with this help, depending on the resources provided by the adult (Vygotsky, 1978). Scaffolding as an instructional method is more than supporting the natural development; it is about providing such “scaffolds” for individuals that they are able to reach a level that they wouldn’t be able to accomplish without (Vygotsky, 1978).

According to Puntambekar & Hübscher (2005) Scaffolding can be broken down into following components;

- Inter-subjectivity; It can be defined as the same understanding of the task and can be achieved when the adult and child collaboratively reach a consensus on the goal which needs to be achieved.
- On-going Diagnosis; The adult should provide appropriate support based on the on-going diagnosis of the child’s current level of understanding. It would mean that adult should be aware of the child’s needs and capabilities which change as the task progress.
- Graduated Assistance; On-going diagnosis leads to graduated assistance. It means that whenever the child needs help and whatever type of help or resources are needed, the adult is responsible for providing it. It is based on the constantly changing of child’s knowledge and skills.
Fading: This is the last stage in Scaffolding and in this stage adult can withdraw the support and the transfer of responsibility is made sure from the adult to the child. After fading, a process known as ‘internalisation’ takes place which was described by Vygotsky (1978) as when child takes complete control of its learning and no longer needs support. It is important to mention over here that, the child not only completes the task successfully but he should be able to generalize his learning process for the upcoming task. So, the support is no longer needed.

Scaffolds can be defined as the way or method to achieve scaffolding. For example; ‘training walkers’ act as a scaffold when a child is learning to walk for the first time and an adult guiding the process. There are numerous types of scaffolds but their purpose is unified, which is to provide support. With the help of different scaffolds, students can complete more advanced tasks and engage in critical problem solving and thinking (McNeill, Lizotte, & Krajcik, 2004). One important point over here is that scaffolds need to lie in the Zone of Proximal Development of a child to promote and develop understanding. Zone of Proximal development can be defined as the area between a child’s own level of problem-solving skills and the level of problem-solving skills with the guidance of an adult or any other resource (Vygotsky, 1978). It is argued that scaffolds allow children to reach a higher level in their Zone of Proximal Development (Stone, 1993). Zone of Proximal Development not only discloses the actual development of a child but the course of development and maturity of the child in the future (Vygotsky, 1978).

For that purpose, most of the scaffolds are based on the prior knowledge of the child and they are challenging enough for the child to motivate him towards learning. Scaffolds should have enough information so that learners make the progress on their own (Hogan & Pressley, 1997).

Figure 2. Zone of Proximal Development presented by Vygotsky 1978
Fyfe, Rittle-Johnson & DeCaro (2012) studied the effect of feedback as a form of scaffold during exploratory mathematical problem-solving. Feedback is any kind of information which a child can use to accept, reject or change his prior knowledge (Mory, 2004). Prior knowledge of a child is an important characteristic when evaluating the effect of feedback as a scaffold. Scaffolds need to be aligned with the prior knowledge of a child so they can make progress during the process of problem solving (Fyfe, Rittle-Johnson, & DeCaro, 2012). Learners with higher prior knowledge do not need extensive instructional support whereas learners with low prior knowledge need the instructional support. For example; novices learn more with aided problem solving than studying independently (Tobias, 2009) (Kalyug, 2007) (Fyfe, Rittle-Johnson, & DeCaro, 2012). In the study, ‘Designing Knowledge Scaffolds to Support Mathematical Problem Solving’ (Rittle-Johnson & Koedinger, 2005) students’ prior knowledge was evaluated. Then, the students completed a computer-based classroom instruction which aimed to scaffold contextual, conceptual and procedural knowledge of the student. The results of this study suggested that scaffolding these three types of knowledge are promising tools for improving student learning. The study didn’t encompass the effect of combining different types of scaffolds on the problem solving in Mathematics.

The discussion conducted by Julia Anghileri (2006) on scaffolding practices which enhance Mathematics learning, compiled teaching approaches based on scaffolding. This discussion particularly looked at several mathematics instructions and how the interaction of the teacher and student took place inside the classroom. Anghileri (2006) concluded that teachers are most effective if they are able to use a range of teaching approaches in an environment that encourages student learning. The goal of teaching is to create self-regulated learners and flexible and dynamic scaffolding serves the purpose. In Mathematics teaching, a student actively constructs meaning as he participates in mathematical practices with the help of different ways and their re-enactment (Cobb, Yackel, & McClain, 2000). The role of the teacher changes from being the instructor to a responsive guide. The guidance during this process is crucial because it helps students to develop their mathematical reasoning. Anghileri (2006) recognizes 3 levels of scaffolds; environmental provisions, explaining, reviewing and restructuring and developing conceptual thinking. All of the three levels included different teaching practices which were used in different mathematical situations including geometry and algebraic problem solving. Teachers can use any number of scaffolds which deals with different needs of the students in a classroom setting.
When it comes to scaffolding, fading is an important part of the whole process in a learner’s development. There are numerous approaches to use fading. One of them is the use of self-assessment. By letting students decide when to use the support and when not to, based on their needs is beneficial for their self-regulated learning (Järvelä & Hadwin, 2013). Students can influence the fading process by using self-assessment according to their own competence level (Noroozi, Kirschner, Biemans, & Mulder, 2017). Noroozi et al. (2017) also found some studies which concluded that using self-assessment as fading can have opposite effects on the student learning where they cannot decide correctly what they need to learn and they only use the support when they want which is not always appropriate or even relating to their competence level.

As mentioned earlier, self-efficacy in mathematics plays an important role in learning (Bandura, 1994). Self-efficacy beliefs are also strengthened by the scaffolds provided during a learning process. When a student starts working on a task and teacher provides graduated assistance, there comes a time when less and less assistance is required by the student. Thus, resulting in more confidence which is related to self-efficacy (Larkin, 2001).

This research deals with the unanswered question suggested in the study conducted by (Rittle-Johnson & Koedinger, 2005) to find out the effect of combining different types of scaffolds on the problem solving in Mathematics. For this research different teaching practices were taken from all the three levels (Anghileri, 2006) to design a lesson and a written scaffold for the students while they solved algebraic problem solving questions.

Another reason why this research is important, there is a little research on teacher and student scaffolding in the whole classroom setting (Hogan & Pressley, 1997). One of the reasons discussed by Hogan and Pressley was that it's difficult for teachers to interact with students one to one in the whole classroom. Let alone, provide on-going diagnosis and graduated assistance to everyone with the limited amount of time.

Another issue with scaffolding when it comes to teacher and whole classroom interaction is that all the students have different zone of proximal developments and there cannot be one scaffold which deals with each zone of proximal development which exists in the classroom which leads to a concern that teacher and student scaffolding cannot be carried out effectively in the whole classroom setting (Stone, 1998). But, different strategies and tools can be applied to carry out scaffolding in the whole classroom setting as effectively as possible (Anghileri, 2006).
This research will look at the effects of scaffolding at the classroom level. It is important to remember over here that zone of proximal development doesn’t include only people but also tools and artifacts which can be used in different activities and lessons, provided by the teacher. Vygotsky (1978) termed it as ‘More Knowledgeable Other’. In the case of this research, one of these tools used is a written scaffold which will have embedded features such as self-assessment, different levels of thinking and teacher and student help. For the sake of this research, written scaffold is termed as an ‘embedded scaffold’.

This research used learning in mathematics as a context of student learning, revised Bloom’s taxonomy which lays out the thinking levels and then different scaffolds which are accompanied by instruction, activities, and tasks. The mathematical questions are challenging enough to drive the students to go a step further and be involved in them. These questions promote capacity to think, reason and problem solve which is a basis for selecting an instructional task included in a LT (Smith & Stein, 1998).
3 Methods

This chapter describes the research questions, participants, and context. Also, the procedure of this research has been accounted for. The aim of this research was to study the effect of scaffolding on the conceptual and procedural knowledge in the context of mathematics in 6th-grade students.

This research is set to answer the following questions;

1. What kind of a difference is there in the students’ knowledge level between the prior knowledge test and after the main activity worksheet?
2. How did scaffold affect the learning results and the learning process of the students?
3. How did self-efficacy develop across different thinking levels during the learning process?

3.1 Participants and Context

The participants were 31 pupils in grade six in the primary school in Oulu, Finland. One mathematics teacher was there to assist the process. The pupils consisted of 15 males (48 %) and 16 females (52 %). Their ages ranged from eleven to thirteen years. They were in mathematics lesson and the instructional language was Finnish. All the participating students and their guardians signed a consent form before the data was collected.

3.2 Procedure

Students completed their prior knowledge test in one 10-minute session. After the prior knowledge test (see Appendix 2), teacher carried out a lesson plan (see Appendix 1) provided by the researcher and students took part in the activity and started working with the worksheet with different questions provided to them (see Appendix 3). The activity took place in four parts; i) Concept and procedure instruction, ii) problem solving while applying their knowledge, iii) on-going diagnosis of the students by the teacher and then teacher provided assistance (the embedded scaffold in the worksheet), iv) teacher helped out students who opted to ask for help (action taken by the students first and then teacher or other student provides assistance (the optional scaffold). One day later, interviews were conducted from the selected students.
3.2.1 Instruction

The instruction took place in a classroom where teacher discussed the concept of Algebra and what does it mean to find an ‘x’ unknown value in an equation with the help of examples. It was important for the teacher to involve students as much as possible during this phase to reach the inter-subjectivity which is required for scaffolding to take its roots during the learning process (Puntambekar & Hübscher, 2005).

3.2.2 Problem Solving

Mathematics lesson was designed in such a way that it was a part of the students on-going curriculum. 6th-grade students in Finland study the concept of the unknown in algebra. They examine the algebraic equations and solve them by reasoning and experimentation (Opetushallitus, 2016). This worksheet was only based on simple algebraic expressions and single equation problem solving questions. After the prior knowledge test, students were given a worksheet which had seven algebraic questions. The difficulty level of the questions increased as the students progressed from question one to seven. (see Appendix 2).

Problem solving phase was assisted by a written scaffold, which had the embedded scaffold in it. The questions varied from understanding level to applying level which are the thinking orders taken from Revised Bloom’s Taxonomy (Krathwohl, 2002). Another thing which was added into the written scaffold was self-assessment criteria for the students. At the end of each question, they had to assess themselves with the help of three different criteria; i) I can teach others ii) I understand it iii) I don’t understand it. They marked the best assessment criteria which suited them.

3.2.3 Written Scaffold (Embedded Scaffold)

With the help of above-mentioned assessment criteria, the teacher moved onto the next phase which was on-going diagnosis and assistance. Teacher diagnosed the worksheets without disturbing the students and offered help where and when it was required. The teacher was not allowed to give out correct answers instead he posed questions such as what do you think can be done here? Have you tried thinking in another way?
3.2.4 Student Prompted Assistance (Optional Scaffold)

Before the work on written scaffold started, students were given, ‘help me’ exhibitors. There was a green cup and a red cup provided to each student individually. They used them in the following manner:

1. If they could teach the question to others, the green cup was on the top of the red cup.
2. If they only understood the question, both the cups were parallel to each other.
3. If they didn’t understand the question, the red cup was on the top of the green cup.

The students used these ‘help me’ exhibitors to portray their level of understanding. Number 3 assessment level assisted the students to receive help either from their peers or from the teachers. Number 1 assessment level assisted the students to give help to their peers where either students willingly taught others or gave help when asked. They chose to use these signs at any given point and recorded the use of it on their sheets for the researcher to know how many times it was used.

Figure 3. “Help Me” exhibitors showcasing assessment level 2
3.2.5 Interview

Interviews are conducted to seek the meaning of people’s actions, feelings, knowledge, and behaviors. Interviews are useful to know the thought process behind the experiences of people (Ary, Jacobs, Sorensen, & Walker, 2013). For this research, interviews were conducted with those students who had shown varied results in the prior knowledge test than the main activity worksheet. These interviews were conducted to follow up with the students after they solved the main activity worksheet. Interviews focused on to investigate the responses of the students in the worksheet. Whether students attributed scaffolds for their success or failure was targeted with the help of the interviews. Students were shown their worksheet and it helped the researcher to ask the questions and get validated responses from the students. The aim was to find out the student’s opinion and knowledge after they solved the questions on the main worksheet. There was a Finnish translator present during the interviews and eight students were interviewed based on the initial analysis.
4  Data Collection

This chapter describes the data collection sources such as prior knowledge test, main activity worksheet, and interviews. It also accounts for the explanation as to what type of data these sources collected. Both prior knowledge test and main activity worksheet was administered to 31 students and then interviews were conducted with 8 students in total as seen in Table 1.

Table 1. Data Collection Instruments

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior Knowledge Test</td>
<td>31</td>
</tr>
<tr>
<td>Main Activity Worksheet</td>
<td>31</td>
</tr>
<tr>
<td>Interviews</td>
<td>8</td>
</tr>
</tbody>
</table>

4.1 Prior Knowledge Test

The prior knowledge test was designed by the researcher with the help of a mathematics teacher at the University of Oulu. This test was used to set a baseline of procedural and conceptual knowledge’s level of the individual students. The test was translated into the Finnish language prior to distribute them among the students. All 31 students completed the test and returned it back to the researcher during the time of the data collection. Prior knowledge test provided the researcher with the necessary data to gauge where the individual students stand in terms of their knowledge of algebraic problem solving.

Prior knowledge test had one question in it with a very specific set of instruction which is as follows;

Please solve this question below and explain why you think this is the correct answer. How did you reach this conclusion? You can either write in words or draw, whatever is easy for you to explain.

Veera has 50 euros, which is 8 euros more than twice what Eetu has. How many euros does Eetu have?

To have a clear distinction between the procedural and conceptual knowledge, the response section was divided into two parts, mathematical procedure and explanation (see Appendix 1).
4.2 Main Activity Worksheet

After the prior knowledge test was administered, students were given the main activity worksheet. At this time, students were already given a lesson by their teacher in which the teacher introduced the topic and then gave a few examples to make the concept tangible. Main activity worksheet was designed with the help of core idea of Revised Bloom’s Taxonomy (Bloom, 1956). This worksheet had seven mathematical questions. First two questions were based on the ‘Remembering’ level in Revised Bloom’s Taxonomy. Question three and four were based on the ‘Understanding’ level in the taxonomy and question five, six and seven were based on the ‘Applying’ level (see Appendix 3).

During the solving process, if students asked help from their peers or teachers, they had a ‘recording box’ at the end of each question. They used the ‘help me exhibitors’ to ask for help either from their peer or the teacher. After solving these questions, students had to self-assess themselves based on their understanding level after solving each of the seven questions. Self-assessment was based on the following criteria;

- I can teach others
- I understand
- I don’t understand

From the main worksheet four types of data were collected, conceptual knowledge, procedural knowledge, self-assessment level and help received.

4.3 Interviews

Interviews were conducted with those students who had shown variated results in the prior knowledge test than the main activity worksheet to investigate what they think about scaffolds whether it helped them in their mathematical learning or not. Variated result means students who fall under the following criteria;

- Students who performed high in prior knowledge test and high in the main worksheet
- Students who performed low in prior knowledge test and low in the main worksheet
- Students who performed high in prior knowledge test but low in the main worksheet
- Students who performed low in prior knowledge test but high in the main worksheet
Interviews were conducted with the help of the main activity worksheets students worked on during the problem solving phase. The interview was a semi-structured interview based on following questions focusing on the scaffolds provided:

1- Did you find optional scaffold useful? Why?
2- Did you find embedded scaffold useful? Why?

Through interviews students’ impression of scaffolds was recorded. Afterwards, the coded data was examined for different patterns, relations, and theme.
5 Data Analysis

Research method used in this research is a mixed-method. The data was collected on the paper and then it was exported into Microsoft Excel. NVivo and IBM SPSS was used as analysis softwares. The first step was calculating the grade of conceptual and procedural knowledge given the coding categories using NVivo. The grades for both types of knowledge were exported in IBM SPSS and then Wilcoxon’s Signed Rank Test and Spearman’s Rho correlation test were run. Self-Assessment level and Help frequency were determined by descriptive analysis. Interview data was analysed based on the given coding categories and placed in themes accordingly. Collected data was analysed based on the methods seen in table 2.

Table 2. Data Analysis Methods

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior Knowledge Test</td>
<td>Content, Descriptive and Statistics Analysis</td>
</tr>
<tr>
<td>Main Activity Worksheet</td>
<td>Content, Descriptive and Statistical Analysis</td>
</tr>
<tr>
<td>Interviews</td>
<td>Thematic Analysis</td>
</tr>
</tbody>
</table>

5.1 Content Analysis

As it was mentioned earlier that prior knowledge test and main activity worksheet was designed in such a way that they required students to explain their thought process in a separate section ‘explanation’ which gave the researcher an insight into their level of conceptual knowledge. To analyse the explanation, content analysis method was used.

Content analysis has roots in communication studies; it basically focuses on the characteristics present in the materials (Ary, Jacobs, Sorensen, & Walker, 2013). It’s an analysis method used in research to identify different underlying meanings or characteristics in a written, visual or audio material. Content analysis is used for a number of different purposes but in this research it is used to identify the existence of conceptual knowledge and then the relationship between scaffold and conceptual knowledge (Ary, Jacobs, Sorensen, & Walker, 2013). Content analysis has an emergent design framework. Following are the steps involved in content analysis;
– Specifying the topic to be investigated. In the case of this research, finding out the effects of scaffold on conceptual knowledge was the topic. This research aimed to find out whether scaffolds support the construction of conceptual knowledge or not.
– Selecting the instruments to collect the data. In the case of this research, prior knowledge test, main activity worksheet and interviews were devised.
– Formulating exhaustive and mutually exclusive coding categories.

In the case of this research, comprehensive coding categories were devised to gauge the level of conceptual knowledge of the individual students. These coding categories were inspired two researchers who have extensively defined and explained about conceptual knowledge (Rittle-Johnson, Siegler, & Alibali, 2011) (Canobi, 2009) and used them in another mathematics related research (Fyfe, Rittle-Johnson, & DeCaro, 2012). Mathematical concept was a code to measure the conceptual knowledge and it was divided into five sub-codes. As seen in table 3, description for each sub-code was laid out.

Table 3. Coding Categories

<table>
<thead>
<tr>
<th>Name of the code</th>
<th>Sub Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Concept</td>
<td>Interrelation</td>
<td>Student mentioned his use of prior knowledge while solving the word problem and connected the prior knowledge with the problem at hand</td>
</tr>
<tr>
<td>Mathematical Terminology</td>
<td></td>
<td>Student used mathematical terminologies e.g. equality, subtraction, addition, multiplication, division, unknown etc.</td>
</tr>
<tr>
<td>Reasoning</td>
<td></td>
<td>Student explained the equation by answering questions such as ‘how’ and ‘why’ himself.</td>
</tr>
<tr>
<td>Equalizing</td>
<td></td>
<td>Student explained his understanding of equalizing the equation</td>
</tr>
<tr>
<td>Evaluation</td>
<td></td>
<td>Student evaluated his decoding and problem solving skills in writing</td>
</tr>
<tr>
<td>Unsolved Problem</td>
<td>With a reason</td>
<td>Student didn’t solve the problem but wrote something in the explanation section</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Without a reason</td>
<td>Student didn’t solve the problem and didn’t write anything in the explanation section</td>
</tr>
</tbody>
</table>

5.2 Descriptive Analysis

The procedural part of the data where students had to solve mathematical problems was graded 0 - 4 based on the criteria which can be seen in table 4. Each number has a criteria and the mathematical problems were analysed and coded according to the criteria mentioned.

Table 4. Grading Criteria for Mathematical Problems

<table>
<thead>
<tr>
<th>Grade</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The steps and the product is incorrect</td>
</tr>
<tr>
<td>1</td>
<td>There is a slight evidence of the correct steps involved in the process</td>
</tr>
<tr>
<td>2</td>
<td>There is a half evidence of the correct steps involved in the process</td>
</tr>
<tr>
<td>3</td>
<td>The steps are correct but the product is incorrect</td>
</tr>
<tr>
<td>4</td>
<td>The steps and the product is correct</td>
</tr>
</tbody>
</table>

Both the content analysis results and the grading criteria were converted into percentages to combine both of the results and have a clear level of knowledge both conceptual and procedural.

The students self-assessed themselves based on the three options; “I don’t understand”, “I understand” and “I teach others”. These options were given a value of 0, 1 and 2 respectively. Since, there were seven questions in the main activity worksheet, so students self-assessed themselves seven times. After placing a value for each student’s worksheet questions individually, an average was calculated to get one self-assessment value for the whole worksheet. For example; Student 6 self-assessed herself as 1,2,2,1,2,2, and 2 on the seven questions in the worksheet. An average of 1.7 was calculated for her which was rounded
off to 2. Similarly, the value was rounded off for all those students who had a decimal point value of 0.5 or above. So, $0.5 \geq$ and $1.5 \geq$ were rounded off to 1 and 2 respectively.

On the other hand, to analyse the use of ‘teacher’s help’ and ‘student help’ during the main worksheet activity, a frequency distribution was allotted. A systematic arrangement of individual measures from the lowest to the highest is called a frequency distribution (Ary, Jacobs, Sorensen, & Walker, 2013). The number of times teacher and student help was taken was listed in an order of lowest to the highest and then it was tallied from the worksheets student’s used to mark these both types of help. The interview data was also first converted into frequencies and then converted into percentages to give readers more context.

### 5.3 Thematic Analysis

Thematic Analysis is about examining and recording the patterns which appear in the data set (Dawson, 2007) (Fraenkel, Wallen, & Hyun, 2012). The patterns help the researcher to analyse the data and answer the research question with the evidence provided from the data set.

Although thematic analysis is somewhat similar to content analysis, in the case of this research semi-structured interviews were conducted. Students were shown there work to provide reference for the questions and help them recall the process they went through. The purpose of the interviews was to find out the answer for the second research question, how scaffold affected the students learning. A coding scheme was created to analyse the interview data by adapting the theoretical guidelines of scaffolding as outlined by Puntambekar & Hübscher (2005). The coding scheme includes; student’s understanding of the task, teacher’s and student help and self-assessment as it can be seen in table 5 with description for each one of them. However, the coding scheme was not completely based on the previous studies but emerged from the data as well. So, this analysis was a mix of deductive and inductive techniques.
Table 5. Categories used to Analyse Effect of Scaffolds

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Understanding</td>
<td>When the student expresses that he/she has understood the topic and task</td>
</tr>
<tr>
<td>Teacher’s Help</td>
<td>When the student uses ‘optional scaffold’ to call for a teacher</td>
</tr>
<tr>
<td>Student’s Help</td>
<td>When the student either receives or gives help to another student</td>
</tr>
<tr>
<td>Importance of Self-Assessment</td>
<td>When the student expresses the importance of self-assessment while problem-solving</td>
</tr>
<tr>
<td>Feedback</td>
<td>When the student expresses a positive concern towards others who might be going through a different situation</td>
</tr>
</tbody>
</table>

5.4 Statistical Analysis

Wilcoxon Signed Rank Test is a non-parametric test which is used to compare two related samples to assess whether their population mean rank differs (Ary, Jacobs, Sorensen, & Walker, 2013). This test was administered to investigate whether there was a difference between the conceptual knowledge in the prior knowledge test and main activity worksheet. Also, whether there was a difference between the procedural knowledge in the prior knowledge test and the main activity worksheet. Wilcoxon Signed Rank Test can be interpreted given the p-value from the analysis. When p-value is equal or less than 0.05, it indicates a significant relation between the variables. When p-value is equal or greater than 0.05, it indicates an insignificant relation between the variables.

Spearman’s Rho correlation test is a non-parametric measure of rank correlation. It basically assesses how well the relationship between two variables can be described (Ary, Jacobs, Sorensen, & Walker, 2013). This test was administered to investigate whether scaffolds (i.e. self-assessment reporting) had an effect on the conceptual and/or procedural knowledge of the students in the main activity worksheet. Spearman’s Rho can be interpreted in a similar way to Pearson r. The coefficient of correlation ranges from -1.00 to +1.00. When each individual has the same rank on both variable, the correlation is +1.00. When their ranks on one variable are opposite to that of the other variable then the correlation is -1.00. When there is no relation at all between the ranking then the correlation is 0.
6 Results

The aim of the research was to find out the effect of scaffolding on the conceptual and procedural knowledge with the context of mathematics on 6th graders. To go deeper into the results, it was important to find out the level of conceptual and procedural knowledge in the prior knowledge test and the main worksheet. Also to find out the self-assessment relation with the knowledge which was collected from the main worksheet. The frequency of teacher’s help and student’s help taken during the main worksheet solving process. The findings below correspond to each research question;

6.1 What kind of difference is there in students’ knowledge level between the prior knowledge test and main activity worksheet?

To answer this question, students’ conceptual and procedural knowledge scores from prior knowledge test and the main activity worksheet were compared using percentages.

6.1.1 Prior Knowledge Test

*Conceptual Knowledge Level (CK<sub>p</sub>)*

Prior knowledge test was given an individual grade from 0 – 5 for conceptual knowledge and the test was scored individually for each student. As it can be seen in table 6, 13 students scored 0 which constitutes 41.9 % of the total and only 6 students scored 5 which constitutes 19.4 % of the total.

Table 6. Percentage of the Conceptual Knowledge in Prior Knowledge Test

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>41.90</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.22</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>9.67</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>12.90</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>12.90</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>19.35</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>100</td>
</tr>
</tbody>
</table>
Procedural Knowledge Level (PKp)

Prior knowledge test was given a grade from 0 – 4 for procedural knowledge and the test was scored individually for each student. As it can be seen, 5 students scored 0 which constitutes 16.12% of the total and 25 students scored 4 which constitutes 80.64% of the total.

Table 7. Percentage of the Procedural Knowledge in Prior Knowledge Test

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>16.12</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3.22</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>80.64</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>100</td>
</tr>
</tbody>
</table>

![Student 21](image)

Figure 4. Example for a high conceptual knowledge in Prior Knowledge Test
Two examples have been shown above. Student 21 has scored a grade 4 in procedural and 5 in conceptual knowledge as seen in figure 4. Student 23 has scored a grade 0 in both knowledge types as seen in figure 5.

6.1.2 Main Activity Worksheet

Conceptual Knowledge Level (CKₘ)

The main activity worksheet was given a grade from 0 – 5 for conceptual knowledge and the worksheet was scored individually for each student. As it can be seen, 1 student scored 0 which constitutes 3.22 % of the total and 16 students scored 5 which constitutes 51.61 % of the total.
Table 8. Percentage of the Conceptual Knowledge in Main Worksheet

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>3.22</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.22</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6.45</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>35.48</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>51.61</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>100</td>
</tr>
</tbody>
</table>

Procedural Knowledge Level \((PK_m)\)

The main activity worksheet was given a grade from 0 – 4 for procedural knowledge and the worksheet was scored individually for each student. As it can be seen, 0 students scored 0 which constitutes 0 % of the total and 26 students scored 4 which constitutes 83.87 % of the total.

Table 9. Percentage of the Procedural Knowledge in Main Worksheet

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.22</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>12.90</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>83.87</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>100</td>
</tr>
</tbody>
</table>

6.1.3 Comparison between Conceptual Knowledge of Prior Knowledge Test and Main Activity Worksheet

Wilcoxon Signed-Ranks Test was applied to the scores of conceptual knowledge in prior knowledge test and conceptual knowledge in main activity worksheet to check the difference between the conceptual knowledge level of students in prior knowledge test and main activity worksheet. The Wilcoxon Signed-Ranks test indicated that conceptual knowledge in the main activity worksheet \((M=4.26)\) was significantly higher than the conceptual knowledge in the prior knowledge test \((M=2.10), Z= -3.872, p < .001.\)
Table 10. Results of Wilcoxon Signed-Ranks Test to Compare CKp and CKm

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>CK_p</td>
<td>31</td>
<td>2.10</td>
<td>-3.872</td>
<td>.000</td>
</tr>
<tr>
<td>CK_m</td>
<td>31</td>
<td>4.26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05, **p < .01, ***p < .001

6.1.4 Comparison between Procedural Knowledge of Prior Knowledge Test and Main Activity Worksheet

Wilcoxon Signed-Ranks Test was applied on the scores of procedural knowledge in prior knowledge test and procedural knowledge in main activity worksheet to check the difference between the procedural knowledge level of students in prior knowledge test and main activity worksheet. The Wilcoxon Signed-Ranks test indicated that procedural knowledge in the main activity worksheet (M=3.81) didn’t change from the procedural knowledge in the prior knowledge test (M=3.32), Z= -1.549, p > .05.

Table 11. Results of Wilcoxon Signed-Ranks Test to Compare PKp and PKm

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>PK_p</td>
<td>31</td>
<td>3.32</td>
<td>-1.549</td>
<td>.121</td>
</tr>
<tr>
<td>PK_m</td>
<td>31</td>
<td>3.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05, **p < .01, ***p < .001

6.2 How did scaffold affect the learning results and the learning process?

To answer this question, the self-assessment score and teacher’s and student’s help score were used. Then to provide a more in-depth understanding of the effectiveness of scaffold, 8 student interview responses were then analysed in more details
6.2.1 Self-Assessment Effects on the Learning Results and Process

As it can be seen in table 12, 20 students out of 31 self-assessed themselves as ‘I understand’ which showed a better understanding on their part while solving the questions in the whole worksheet. 9 students self-assessed themselves as ‘I can teach others’ and 2 students self-assessed themselves as ‘I don’t understand’. These scores were recorded on the main activity worksheet when students were solving the mathematical questions.

Table 12. Self-Assessment Score

<table>
<thead>
<tr>
<th>Self-Assessment Criteria</th>
<th>Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I don’t understand</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>I understand</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>I can teach others</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>31</td>
</tr>
</tbody>
</table>

Self-Assessment Level and Conceptual Knowledge

To check whether there was a relationship between students’ self-assessment and the conceptual knowledge grade from their worksheet, the following table was created. As it can be seen, 10 students who marked themselves as ‘I understand’ received a grade 5 in their conceptual knowledge and 6 students who marked themselves as ‘I can teach’ received a grade 5 in their conceptual knowledge. Also, 10 students who marked themselves as ‘I understand’ received a grade 4 in their conceptual knowledge and 1 student who marked himself as ‘I can teach’ received a grade 4 in his conceptual knowledge. The following table shows that the students’ conceptual knowledge and their own self-assessment of their knowledge have a relationship.

Table 13. Cross-Tab for Self-Assessment and Conceptual Knowledge

<table>
<thead>
<tr>
<th>Grade</th>
<th>I don’t understand (0)</th>
<th>I understand (1)</th>
<th>I can teach others (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>20</td>
<td>9</td>
</tr>
</tbody>
</table>
Spearman’s rho correlation test was administered on the conceptual knowledge grades from the worksheet and self-assessment scores of the students to see whether conceptual knowledge grades and self-assessment scores were related. A two-tailed test of significance indicated that there was an insignificant weak positive relationship between the conceptual knowledge and self-assessment in the main activity worksheet $r_s (31) = .193$, $p > .05$.

Table 14. Correlation Test Results between Conceptual Knowledge and Self-Assessment

<table>
<thead>
<tr>
<th></th>
<th>Conceptual Knowledge</th>
<th>Self-Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s rho</td>
<td>Correlation Coefficient</td>
<td>1.000</td>
</tr>
<tr>
<td>Knowledge</td>
<td>Sig. (2 tailed)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>31</td>
</tr>
<tr>
<td>Self-Assessment</td>
<td>Correlation Coefficient</td>
<td>.193</td>
</tr>
<tr>
<td></td>
<td>Sig. (2 tailed)</td>
<td>.299</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>31</td>
</tr>
</tbody>
</table>

*Self-Assessment Level and Procedural Knowledge*

To check whether there was a relationship between students’ self-assessment and the procedural knowledge grade from their worksheet, the following table was devised. As it can be seen, 18 students who marked themselves as ‘I understand’ received a grade 5 in their procedural knowledge and 8 students who marked themselves as ‘I can teach’ received a grade 5 in their procedural knowledge. Also, 2 students who marked themselves as ‘I understand’ received a grade 4 in their procedural knowledge. The following table shows that the students’ procedural knowledge and their own self-assessment of their knowledge have a relationship.
Table 15. Cross-Tab for Self-Assessment and Procedural Knowledge

<table>
<thead>
<tr>
<th>Grade</th>
<th>I don’t understand (0)</th>
<th>I understand (1)</th>
<th>I can teach others (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>20</td>
<td>9</td>
</tr>
</tbody>
</table>

Spearman’s rho correlation test was administered on the procedural knowledge grades from the worksheet and self-assessment scores of the students to see whether procedural knowledge grades and self-assessment scores were related. A two-tailed test of significance indicated that there was an insignificant weak positive relationship between the procedural knowledge and self-assessment in the main activity worksheet $r_s (31) = .164, p > .05$.

Table 16. Correlation Test Results between Procedural Knowledge and Self-Assessment

<table>
<thead>
<tr>
<th></th>
<th>Self-Assessment</th>
<th>Procedural Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s rho</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Assessment</td>
<td>Correlation Coefficient 1.000</td>
<td>.164</td>
</tr>
<tr>
<td></td>
<td>Sig. (2 tailed)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>31</td>
</tr>
<tr>
<td>Procedural Knowledge</td>
<td>Correlation Coefficient .164</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Sig. (2 tailed)</td>
<td>.377</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>31</td>
</tr>
</tbody>
</table>

*Interview Data Results*

After the students solved the worksheet, interviews were conducted to find out the effect of scaffolds on their knowledge level. The researcher used already devised coding categories and the category of ‘feedback’ was devised while analysing the interviews. The purpose of interview analysis was to have a clear understanding from the students’ point of view that the scaffold’s which were provided to them, assisted them to develop their conceptual and procedural knowledge. All these categories were coded separately for each interviewee and then they were
combined as a frequency. For example; 7 students out of 8 mentioned the ‘subject understanding’ in their interviews. ‘Feedback’ as a coding category emerged as the interviews progressed with the individual students. 4 students out of 8 mentioned ‘feedback’ and explained it further. For example; Interviewee 5 said that, “The optional scaffold is useful for students who need more help than me”.

Table 17. Frequencies for the coded reasons for the effect of scaffolds

<table>
<thead>
<tr>
<th>Category</th>
<th>F</th>
<th>Percentage</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Understanding</td>
<td>7</td>
<td>87.50</td>
<td>‘The teacher’s instruction in the beginning was quite helpful’</td>
</tr>
<tr>
<td>Teacher’s Help</td>
<td>6</td>
<td>75</td>
<td>‘The teacher always know more’</td>
</tr>
<tr>
<td>Student’s Help</td>
<td>7</td>
<td>87.50</td>
<td>‘Other students would understand me more because they are going through the same problem’</td>
</tr>
<tr>
<td>Importance of Self-Assessment</td>
<td>6</td>
<td>75</td>
<td>‘It helps me to think deeply whether I actually understood or not’</td>
</tr>
<tr>
<td>Feedback</td>
<td>4</td>
<td>50</td>
<td>‘The optional scaffold is useful for students who need more help than me’</td>
</tr>
</tbody>
</table>

Eight students were selected for the interviews and as it can be seen in Table 17, 7 out of 8 students mentioned that subject understanding in the beginning of the lesson helped them to understand what the task was and how to go about it. Also, the prior knowledge test initiated thought process in the students before the lesson started.

*Example 1: Subject Understanding*

Interviewer: Do you think teacher’s help assisted you more or peer’s help?

Student 4: I didn’t ask for help during the solving part. The teacher explained very well at the beginning of the class how to do about the questions and it helped.

*Example 2: Activation of thoughts in Prior knowledge test*

Interviewer: What was going through your head when you were taking the prior knowledge test?

Student 3: It was a normal mathematics task where you had to think about the question.

Student 4: First, it was difficult but when I thought about it I could understand.
Student 2: It was quite easy but I had to think about it.

In all of the above statements by the students subject understanding and thinking is evident but if we look at the sentences with the context of self-efficacy, it can be seen that all the three students had different levels of self-efficacy when they began the prior knowledge test and it was continued when the scaffolds were introduced.

When students talked about using self-assessment, they were quite open to the idea. 6 out of 8 students mentioned that they thought that self-assessment helped them to know whether they actually understood the questions or not. Student 7 mentioned that it was easier if someone else initiates the help (Student 7 was pointing towards the self-assessment level 2 ‘I can teach others’). For student 7 it was difficult to evaluate whether it has the capability of helping others or not which reflects towards low efficacy belief.

Example 3: Importance of Self-Assessment

Interviewer: Did you think self-assessment was useful to you?

Student 3: Yes. It was useful to know whether I understood the concept or not. I was confident in knowing that I had full command over the questions.

Student 4: It was useful in a sense that you can teach others if you yourself understand it well. I thought deeply about it whether I actually understood or not.

Student 7: No. If someone asks me to help it is easy to help them but it is hard to assess by myself whether I am capable of helping them or not.

6 out of 8 students mentioned that teacher’s help was useful for them and 7 out of 8 students mentioned that peer’s help was also useful for them. It was evident from the interview transcripts that all of the 8 students had their own reasons for choosing both the help option during their learning process.

Example 4: Teacher’s Help and Peer’s Help

Interviewer: Did you find teacher’s and peer’s help useful? Why?

Student 8: I only asked for help a few times and I think peer’s help was more useful.

Student 7: Most of the times, I was doing the questions on my own but I asked for my friend’s help and she helped me.
Student 6: Peer help was useful to me because my friends were trying to solve the same questions as I was. So, they understood me.

Student 5: Both of them helped because they knew where and when I needed help. (This student also mentioned the use of ‘help me exhibitors’ for asking or receiving help)

Student 3: I solved my questions on my own. I checked my answers with my friend and then we moved on.

Student 2: Teacher’s help is better because he knows more than us.

The examples above clearly indicate that they wanted to rely on both types of help but some students like student 3, 7 and 8 didn’t even use that much of a help because they were already doing good.

While designing the interview coding categories, ‘feedback’ wasn’t part of them but it emerged as the interviews with the students progressed. 4 out of 8 students mentioned feedback towards others in this learning process where they talked about the use of ‘optional scaffold’. They mentioned that optional scaffold which was basically asking and receiving help while using ‘help me’ exhibitors (the use of red and green cup) could have been more useful for the students who needed it more than them.

*Example 5: Feedback*

Interviewer: Did you find optional scaffold useful? Why?

Student 4: I didn’t use it but I think it would have been more useful for the students who needed help.

Student 5: I don’t know. I used it a little but I think it would have been useful for the students who used it a lot.
6.3 How did self-efficacy develop across different thinking levels during the learning process?

Self-efficacy’s development was measured by the self-assessment criteria given to the students after each of the seven questions presented in the main activity worksheet. As mentioned earlier, the seven questions were based on the thinking levels; remembering, understanding and applying. As it can be seen in figure 6, question 1 (Q1) had a moderate start with the self-efficacy belief and then a rise can be seen towards question 2 (Q2) and 3 (Q3). In question 4 (Q4) there seems to be a fall of self-efficacy belief. Only 5 students marked themselves in level 2. From question 5 (Q5) to 7 (Q7), level 2 was more evident than before. Students marked their self-efficacy belief to be in level 1 which was ‘I understand’. From question 1 to 7, level 1 seemed to be evident and then it changed into the level 2. Five students marked their self-efficacy belief to be in level 0 in question 7.

![Self-Efficacy Development Across Different Thinking Levels](image)

**Figure 6. Self-Efficacy Development Across Different Thinking Levels**

To analyse the development of self-efficacy across the different thinking levels, level 2 of self-efficacy belief was isolated and it can be seen in figure 7. For the remembering level, half of the students had marked themselves at level 2 which is a stronger self-efficacy belief. And then strength of self-efficacy belief decreased when the students entered from remembering to understanding level at question 3. In question 4, level 2 was even less used by the students. In Graph 3, it can be seen that students received teacher’s and peer’s help mostly around question 7.
4 which indicated that students were having difficulty solving the question. In question 4 students had to express four sentences into algebraic expressions. One of the sentences was; Express the weight of Alphie the dog \((x)\) and Cyrus the cat \((y)\). For this question students either asked for their peer’s help or for their teacher’s help or both. This question gave the students a bit of a challenge. After this, in question 5, 6 and 7 students maintained their strong self-efficacy belief by marking themselves at level 2. These three questions were in the thinking level of applying.

![Self-Efficacy Level 2 Across Different Thinking Levels](image)

**Figure 7. Self-Efficacy Level 2 Across Different Thinking Levels**

It was important to observe the help received by the students per question, in order to draw connection with self-efficacy development and the support provided to the students. Question 1 and question 2 no one asked for help but in question 3 some students asked for help from both students and teachers. Question 4 received the maximum number of help both from students and teachers. And then question 7 received help from both.
6.4 Summary of the Results

The results show that there was a significant difference between the level of conceptual knowledge of the students in the prior knowledge test and the main activity worksheet. Results indicated an insignificant difference between the level of procedural knowledge of the students in the prior knowledge test and the main activity worksheet.

The effect of self-assessment as a scaffold could be seen on the grades of the students both in the conceptual knowledge and procedural knowledge in the main activity worksheet. But, statistically an insignificant weak positive relation was found out of both types of knowledge with the self-assessment. However, during the interviews the students reported that provided scaffolds related to subject understanding, teacher and peer help, self-assessment and feedback played a role in the development of their knowledge both procedural and conceptual.

The development of self-efficacy was dynamic which could be clearly seen from the results. The students had a moderate self-efficacy belief and then they had a high self-efficacy belief as the thinking levels in the mathematical questions progressed. Students had a difficulty at ‘understanding level’ in question 4 which is evident from the help they asked both from peers and
teachers. Consequently, their self-efficacy belief was quite low at this point. From question 5 to 7 their self-efficacy belief was again high and was persistent.
7 Discussion

The focus of this research was to explore different types of scaffolds as an effective instructional strategy which helps the students to learn the knowledge, insights and skills used in mathematics and to change their mistaken impression of mathematics, that it’s irrelevant to them, mathematics’ educators need to choose a suitable approach or set of approaches which focus on the development of conceptual and procedural knowledge (Rittle-Johnson & Schneider, 2014). Also, to prepare the teachers with appropriate instructional strategies which they would be able to apply in the classroom which focuses on the growth of the students in the classroom instead of the results. The process of learning triumphs the product (Myers, Sztajn, Wilson, & Edgington, 2015). The aim of this study was to find out the effects of scaffolding on the conceptual and procedural knowledge of the 6th-grade students in Algebra on classroom level.

The first important finding of this research was that the scaffolds had an effect on the conceptual knowledge but not on the procedural knowledge of the students in algebra. The reason behind an insignificant difference between the procedural knowledge in the prior knowledge test and the main activity worksheet can be speculated. The students’ procedural knowledge in the prior knowledge test was already high. This speculation is an indication of the study findings by Pål Lauritzen (2012) on ‘Conceptual and Procedural Knowledge of Mathematics Functions’ in which he states that, the school teaching has been focused on simple procedures without links to the abstract conceptual knowledge. The conceptual knowledge in the study didn’t surpass the procedural knowledge of the functions which was also confirmed by the student interviews at the end during his studies. These results are aligned with the study in which it was found out that learners with higher prior knowledge of a certain type do not need extensive instructional support whereas learners with low prior knowledge need the instructional support. For example; novices learn more with aided problem solving than studying independently (Tobias, 2009) (Kalyug, 2007) (Fyfe, Rittle-Johnson, & DeCaro, 2012).

In comparison to the previous study, ‘Designing Knowledge Scaffolds to Support Mathematical Problem Solving’ conducted by Rittle-Johnson and Koedinger (2005) designed three types of scaffolds; contextual, conceptual and procedural to support mathematical problem solving. They found out in the posttest results that students solved more problems correctly in the post-
test after the scaffolds were implemented compared to the pretest. When it came to the conceptual, contextual and procedural knowledge students’ accuracy significantly improved on seven out of eight items which were included in the problem solving process.

Roschelle, et al. (2009) conducted a study which scaffolded group explanation and feedback with handheld devices to see the effects on the conceptual knowledge. They found out that students who were conditioned to the scaffolds learned more than the control group. The data showed that the students participated more socially in questioning, explaining and discussing disagreements which indicated the improved conceptual knowledge of the students who were conditioned to the scaffolds. Also in the study by Kaur, Kohli and Devi (2008) on ‘the instructional strategies enhancing mathematical skills which included both procedural and conceptual knowledge in learning disabled children’ found out that the selected strategies produced significant enhancement in the mathematical skills in the posttest which included readiness, number concept, problem solving and total achievement scores in the students.

Similarly, in this research students showed a significant development of the conceptual knowledge after the scaffolds were introduced which is parallel to the previous study findings. However, as mentioned earlier procedural knowledge didn’t develop because students’ procedural knowledge was already high. It means that when it comes to students’ procedural knowledge they were not in their zone of proximal development but in their actual development level.

It is important to take a look back at this research’s methods. The procedures and conceptual explanation were accounted as a learning result. Self-assessment, teacher’s help, peer’s help and thinking levels of Revised Bloom’s Taxonomy were incorporated in the worksheet in such a way that these scaffolds worked together and were accounted as the learning process.

The second important finding of this research was that quantitatively, self-assessment as a scaffold didn’t affect the conceptual and procedural knowledge of the students. During the qualitative interviews conducted from the students, the results indicated that they understood the concept of algebra and their understanding is clear than before.

These study findings are similar to the study conducted by Chen and Bradshaw (2007) in which they scaffolded conceptual knowledge of the students in problem solving in a course of Psychology in a web-based learning environment. Their quantitative results didn’t find any significant difference between the students who received the scaffolds with regards to who did not
receive the scaffolds. But, in their qualitative results they found out that the students felt that they had a better understanding of the concepts after going through the scaffolds even though they had difficulties during the problem solving process. Students indicated that they fostered new and stronger connections in their conceptual knowledge while going through the instruction scaffolds in the web-based learning environment.

The third important finding was that self-efficacy (which was measured through ‘self-assessment’ levels) belief of the students developed in a dynamic manner as the thinking levels progressed in the main activity worksheet. This type of development is quite typical in the students with high self-efficacy belief where they don’t see difficult task (in this case question 4) as a problem but as a challenge. After the challenge, students self-efficacy beliefs are restored (Bandura, 1994). Also, this kind of varied judgment of self-efficacy has been observed by Bernacki, Nokes-Malach and Aleven (2015) when they examined self-efficacy during learning. In this study self-efficacy was frequently recorded to observe the variability during the learning process of the students. Students were using a software to log their self-efficacy after every fourth algebraic problem. The software logged the students’ problem solving performance and behaviour. In the results, it was found out that the self-efficacy belief varied over the learning task. It also predicted that students consider multiple factors to judge their self-efficacy. With the varied self-efficacy during the learning process students self-regulated learning processes also change.

Also, when it comes to self-assessment as a fading approach some patterns can be seen of the students where students chose whether to use teacher’s help or peer’s help during their learning process. It can be speculated over here that the students who were nearer to the zone 1 in which they were able to perform the questions ‘unaided’ didn’t use the ‘help scaffold’ and made a judgment about their understanding level while self-assessing themselves after each question. The students who were in zone 2 in which they were able to perform the questions with help were the ones who used the help scaffold. Around question 4 a lot of students found themselves in zone 2 from zone 1 and then by the end of the worksheet in question 5, 6 and 7 a lot fewer students were in zone 2 which was also confirmed by the marking of their self-assessment. Similar to self-efficacy it is safe to conclude that with the scaffolds which were provided had a very dynamic relationship with the learning process in this research. It also indicates that they were in the zone 1 of the zone of proximal development (Vygotsky, 1978) where they can solve the questions unaided or with a little help. It also shows one more thing that these students were self-aware of their competence and they were able to switch on and off the support whenever
they deemed necessary during the learning process (Noroozi, Kirschner, Biemans, & Mulder, 2017).
8 Evaluation

In this chapter, the evaluation of the research is described. The reliability, validity, ethical issues and limitations of the research are accounted for.

8.1 Reliability and Validity

To construct an objective, reliable and valid interpretations of this study results reliability and validity were taken care of. A study is reliable when it is consistent in its results. It means that if the study were to be replicated it would correspond to the original results (Ary, Jacobs, Sorensen, & Walker, 2013). In this study, there is detailed information and argumentation of how the data was collected, analysed and then presented. The transparency of the analysis process and designing the coding categories contributed to the complete understanding of the study. On the other hand, a study is valid when the research results are credible (Ary, Jacobs, Sorensen, & Walker, 2013). In this study, all the data collected from the participants wasn’t manipulated in any way by the researcher. All the information in the data sets was accurate. The research results have been carefully tied with the discussion and all the research questions have been answered with the collected data by the researcher and connections were drawn with the previous studies. The statistical tests were administered and mentioned in this research even though they were insignificant but they contributed towards the reliability of this research. The participants voluntarily took part in this research. This research was a part of their on-going mathematics class and the participants were in their usual environment.

8.2 Ethical issues

Consent from the parents/guardians, teachers and students was taken before the data was collected. The school’s name and all the students’ names have been kept anonymous under the confidentiality and privacy laws. Original authors of all the literature presented in this research have been referenced according to APA 6th edition referencing system. Data was collected, analysed and findings were reported accurately without any manipulation and subjectivity of the researcher.
8.3 Limitations

There are two limitations which are important to acknowledge for this study and can be built upon in further studies. Internalization of a learning process is extremely important in scaffolding. When learners receive enough support, the responsibility is transferred from more knowledgeable others to the learners (Vygotsky, 1978). It allows a learner to not only continue the learning process on his own but also apply the knowledge in different contexts and situations. Due to constraints of time and human resources, this study only included ‘self-assessment’ as a fading approach where researcher could observe the transfer of responsibility but there was no indicator to know whether students internalized the learning process or not. It can be speculated that if all the scaffolds were removed during that one session, students’ knowledge level and self-efficacy would incline. To eliminate this limitation further studies can design the lesson in such a way that it is spread across different lessons. The scaffolds are slowly faded away and then to measure whether internalization successfully took place or not, a retention test or post-test can be administered after a few weeks when the lessons end. Also, a larger sample size would be appropriate for this kind of study. A small sample size is not the representation of the population and not many statistical tests could be administered to generalize the results across the given population.

The second limitation is not only a limitation but also a recommendation for further research. The use of only three thinking levels of Revised Bloom’s Taxonomy. These three levels were chosen for this study because it was important to know the starting point in the learning process of the students. These three levels set the base for the development of the conceptual and procedural knowledge. The worksheet can trace student’s learning path up to application level. The next three levels help the students to practice their knowledge and even create new knowledge with the help of the previous ones. To eliminate this limitation, the mathematics worksheet can have seven or more questions which are based on analysing, evaluating and creating thinking levels.
9 Conclusion and Future Research

The purpose of this research was to explore the effects of different approaches in scaffolding on conceptual and procedural knowledge on 6th-grade students in Algebra. Other research can be built upon by taking the following recommendations into consideration. Revised Bloom’s Taxonomy has six thinking levels; remembering, understanding, applying, analysing, evaluating and creating (Krathwohl, 2002). This research studied first three levels. Further research can benefit from studying the conceptual knowledge and procedural knowledge in the context of analysing, evaluating and creating because these levels require high order thinking in the students which is an indication of stronger conceptual and procedural knowledge.

This research looked at the effects of different set of approaches based on scaffolding and the results which are presented can be used by the teachers in the field of mathematics. By using the worksheet which was embedded with different elements of scaffolding, teachers can follow the learning progress of the students during the learning process where they self-assess themselves letting the teacher know where they stand at the understanding level at a certain point in learning. For teachers it is important to know where a student lies in the zone of proximal development. The worksheet provides this information and teachers can adjust their assistance according to the information provided by the students. The design of the worksheet will allow the teacher to focus on those students who lie in the zone 2 instead of zone 1 of the zone of proximal development. The previous research suggests that novice students benefit more from the assistance of a more knowledgeable other when they are at the beginning of their learning process (Noroozi, Kirschner, Biemans, & Mulder, 2017). The paired nature of assessment (teachers assessing procedures and concepts and students self-assessing) in this worksheet gives teachers a tool to not only analyse the learning results but learning process, as mentioned above.

It is important in mathematics for the students to know where they stand when it comes to their understanding of the procedures and concepts. Teachers can use both the assessments to design future activities and tasks according to the needs of the students.

Each student is different and they have different zones of proximal development (Vygotsky, 1978). We often oversee the fact that these students in mathematics class are more connected not because of the differences they have but attitudes, feelings and reactions which make them similar to each other. In mathematics classroom, if these similarities are stressed upon and used to cater to all the zones of proximal development then teachers wouldn’t need to design a dif-
different activity or a task to cater all the students in one classroom. They can do so by implementing the worksheet which has embedded features of scaffold. This research has evidence that learning is dynamic process in mathematics and the zones of proximal development can change according to the activities or tasks which students face as suggested by previous studies (Vygotsky, 1978; Bernack, Nokes-Malach, & Aleven, 2015). If teachers know whom to help and at what time and the students know when to switch on or off the support system by showcasing their levels of understanding (self-assessment), then a difference can be seen in their understanding of the procedures and concepts. After all, the zone of proximal development not only allows to observe what has been developed but also to observe the child’s course of maturity. What a student can do with assistance today, can do it by himself tomorrow (Vygotsky, 1978).

All this information is important to build up a Learning Trajectory in the mathematics classrooms. LT, with this information, will allow the teachers to trace the ways in which students transform their informal ideas into formal ones through appropriate instructional opportunities into refined mathematics learning. Research in LT suggested that teachers when used LT, it improved their knowledge of mathematics (Mojica, 2010), guided their instructional decisions (Wilson P. H., 2009) and enhanced their abilities to use student thinking (Clements, Sarama, Spitler, Lange, & Wolfe, 2011). This will help the teachers to follow the learning paths of a student which eventually helps them to design activities and tasks which support active student learning in mathematics (Wilson, Sztajn, & Edgington, 2013).
References


Appendix 1

Title:
Decoding and solving the algebraic world problems

Grade:
6th

Keywords:
Addition, multiplication, Subtraction, Division, unknown, equality

Aim:
By the end of the lesson, students will be able to justify their course of action while decoding and solving algebraic word problems.

Objectives:
1. Recognize the mathematical keywords in the world problems
2. Form an equation after decoding the world problem
3. Rationalize their procedures and concepts related to algebraic word problems
4. Evaluate their learning process related to algebraic word problems
5. Build a relationship between unknown and equality

Driving Question:
Why do we create meaningful connections between our prior knowledge and new knowledge?

Work Form:
Mostly, individual but can receive and give peer help during the task time.

12:45 – 12:50 The teacher starts off with the question in the beginning of the lesson. “Why do we need to learn how to write numbers in words?” It’s a student lead discussion. Teacher can
encourage students to give answers to the questions. Teacher receives a couple of answers (correct or incorrect). Then, gives out the answer himself. We learn to write numbers in words because we would be writing a cheque at some point in life. To avoid confusion when dealing with banks we learn how to write numbers in words. We also use words to write numbers for pronunciation and while writing a description type answer in any language. Did you ever think about it before? Do you think about why do you learn what you learn? How is it important? Keep this thought in mind and let’s move on to our lesson today.

12:50 – 13:10 The teacher starts off with the definition of Algebra. “Algebra is an area of Mathematics where numbers and quantities are represented by letters in an equation. In short, Algebra is used for putting real life problems into equations. For example; \( x - 7 = 18 \). Here ‘x’ is the unknown. You find the value for ‘x’ by using four basic functions; division, multiplication, subtraction or addition. In this case you will solve this equation in the following manner:

\[
\begin{align*}
x - 7 & = 18 \\
x & = 18 + 7 \\
x & = 25
\end{align*}
\]

It’s important for the teacher to show them how to balance the equation as well like below. So, they understand that balancing is an important part of the process.

\[
\begin{align*}
x - 7 & = 18 \\
25 - 7 & = 18 \\
18 & = 18
\end{align*}
\]
Now, I know that we have already studied Algebra but this time we will look at word problems. Word problems in algebra, are statements given to us from which we have to make equations. Word Problems are like mysteries. They have little clues hidden them so a student can solve the mystery.

Teacher can put forth the question over here, if students like to solve mysteries? To make the lesson more engaging. Teacher gives an example of a word problem: If Riina has 3 times more money than Roope and she has a total of 33 Euros. How much money does Roope have?

Can someone find the clues in the word problem?

Clues: Times more, Total

Solution: \[3 \times x = 33\]
\[x = 33/3\]

More examples:

What is the sum of 8 and \(y\)?

What if the number (\(x\)) of pizza was reduced by 6?

It’s all about finding the clues in the words and then using the clues to decode the equation and then solve it.

13:10 – 13:45 Teacher asks everyone if they understand. Then, move forward. Students have their worksheets in front of them already and then they have their help signs as well. Teacher describes the rule for using help sign, if they think they don’t understand anything and also they have asked help from their peers. Then, they can use it to ask help from the teacher. After each time the help is taken either from the peer or teacher, students record it on the sheet. After each question, there is a self-assessment box. They get to self-assess themselves, on three different levels. Please take a look at the sheet for that. The reason for the self-assessment is so the teacher would know whom to give assistance. There are two parts of the worksheet, one tests procedural knowledge (Qs. 1 – 3) and the other tests both procedural and conceptual knowledge (Qs. 4-7). Students will also get a “clue sheet”, which you can observe below;
### Mathematical Operations and Key Words

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
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<tbody>
<tr>
<td>add(ed) to</td>
<td>decreased by</td>
</tr>
<tr>
<td>all together</td>
<td>difference</td>
</tr>
<tr>
<td>both</td>
<td>fewer than</td>
</tr>
<tr>
<td>combined</td>
<td>how many more</td>
</tr>
<tr>
<td>in all</td>
<td>left</td>
</tr>
<tr>
<td>increase by</td>
<td>less</td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
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<td>perimeter</td>
<td>minus</td>
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<td>plus</td>
<td>remaining</td>
</tr>
<tr>
<td>sum</td>
<td>take away</td>
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<tr>
<td>total</td>
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</table>

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Division</th>
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<tr>
<td>multiplied by</td>
<td>how many each</td>
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<tr>
<td>of</td>
<td>out of</td>
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<tr>
<td>per</td>
<td>percent</td>
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<tr>
<td>product of</td>
<td>quarter</td>
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<td>rate</td>
<td>quotient of</td>
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<tr>
<td>times</td>
<td>percent</td>
</tr>
<tr>
<td>triple</td>
<td></td>
</tr>
<tr>
<td>twice</td>
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</table>
Appendix 2

Prior Knowledge Test

Please solve this question below and explain why you think this is the correct answer. How did you reach this conclusion? You can either write in words or draw, whatever is easy for you to explain.

*Veera has 50 Euros, which is 8 euros more than twice what Eetu has. How many euros does Eetu have?*

<table>
<thead>
<tr>
<th>Maths Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>
Appendix 3

Worksheet

Name: ______________________  Concept: ______________________

1. Definition;

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

I can teach others  I understood it  I didn't understand it

2. Examples

(a)  It is an example because…

(b)  It is an example because…

________________________________________________________________________
________________________________________________________________________
3. Solve the following;
4. Now, it’s time to use your “clue sheet”. Quickly write an expression for the following sentences:

Express the total weight of Alphie the dog (x) and Cyrus the cat (y)

What is four less than y

I drive my car at 55 miles per hour. How far will I go in "x" hours?

What is the quotient of y and 3

---

I asked for help...
Yes ☐ No ☐
Teacher helped me....
Yes ☐ No ☐

I can teach others  I understood it  I didn't understand it
5. Niina collects pet rocks. She was given 23 for her 9th birthday, and then she bought more with her own money. Now, she has a total of 29 pet rocks. Can you find out how many more rocks did she buy?

Can you explain your answer here? How did you solve the above question?

I asked for help...
Yes □  No □
Teacher helped me....
Yes □  No □

I can teach others  I understood it  I didn't understand it
6. The class of x number of students was divided into groups of three students each. Total groups formed were 10. Can you find out how many students are there in the class?

Can you explain your answer here? How did you solve the above question?
7. Finnkino Cinema charges 14 Euros for a movie. But, recently they increased the movie ticket price by 3 Euros. If you have to buy 5 times more tickets, how much will you pay for it?

Can you explain your answer here? How did you solve the above question?