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CONDITIONAL CHARACTERISTICS OF RISK-RETURN TRADE-OFF: A STOCHASTIC DISCOUNT FACTOR FRAMEWORK

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Abstract

Modeling of the stochastic discount factor using state variables to proxy for the marginal utility substituting for the failure of aggregate consumption data to account for the conditional characteristics of asset returns. The focus is on the conditional characteristics of historical stock returns specifically the trade-off between risk and return. The nonlinear least squares method inspired by the inference methodology of Britten-Jones (1999) is applied to estimate the parameters and estimate conditional volatility of the stochastic discount factor with GARCH and stochastic volatility models.

The results indicate that the estimated stochastic discount factor is conditionally heteroskedastic and its conditional volatility captures the conditional variation that coincides with the phases of the business cycle significantly better than consumption-based SDF. However, the proxy for the risk-return trade-off does not capture substantial conditional variation and magnitude relative to the empirically estimated conditional Sharpe ratios that prominent literature reports. The estimated SDF accounts for predictability but cannot account for the high equity premium particularly the long-run equity premium as the conditional volatility appears to be stationary.

Keywords

Stochastic discount factor – Conditional heteroskedasticity – Risk-return trade-off

Additional information

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1 INTRODUCTION

The conditional phenomena such as asset price volatility of Shiller (1981), equity premium puzzle of Mehra and Prescot (1985) and return predictability present fundamental challenges to describe in the historical asset data. The challenge for asset pricing models is to describe variable(s) which correlate with marginal utility to construct stochastic discount factor (SDF) which can define good and bad economic times to account for conditional asset pricing phenomena that we observe in the data. The obvious variable in constructing the SDF is aggregate consumption since the aggregate consumption correlates with the marginal utility of investors. However, the application of the aggregate consumption data in the standard consumption based asset pricing model produces low conditional volatility in the SDF that requires a time-varying or higher and implausible risk aversion coefficient to explain such conditional asset pricing phenomena. Mehra and Prescott (1985) and Hansen and Jagannathan (1991) address the problem in the equity premium puzzle and Hansen-Jagannathan bounds respectively. This motivates a search for alternative models that attempt to model SDF in response to these fundamental asset pricing problems.

The alternative models attempt to model SDF to describe bad economic times when marginal utility of investors is high to coincide with the predictability evidence, the fact that investors demand higher premium to hold stocks during bad economic times. In other words, stock prices are low in bad times because it takes a higher expected return or premium for investors to hold stocks and vice versa in good economic times when the marginal utility is low. For this reason, stocks are risky since stocks yield poor performance during bad times when investors face other economic uncertainties.

The works of Mehra and Prescott (1985) and Hansen and Jagannathan (1991) indicate that to capture the specific conditional phenomena, requires time varying risk aversion or conditional consumption volatility. Since consumption volatility does not vary as much, the purpose of the study is to seek an alternative SDF model which introduces variables other than the consumption data which correlate with marginal utility and describe different bad times in the economy. We select state variables which appear to forecast stock returns and some macroeconomic condition to signal time when the marginal utility and risk aversion is high. We assess the estimated SDF conditional volatility accountability to predictability and high equity premium. Specifically, the estimated SDF should be conditionally heteroskedastic to account for the high and volatile conditional risk-return tradeoff the Sharpe ratio that we observe in the historical data. Moreover, predictability evidence implies that the SDF conditional volatility should account for the countercyclical movement of the Sharpe ratio. The estimated SDF should match the Hansen and Jagganathan bounds conditionally to successfully explain predictability and equity premium puzzle.

We model the SDF from a multi-factor model with a nonlinear least squares (NLS) method applying inference methodology from Britten-Jones (1999) and estimate the conditional volatility from a GARCH-model and Stochastic Volatility model through Markov Chain Mote Carlo (MCMC) simulation. We apply monthly data from January 1959 to December 2011 which includes CRSP value-weighted return index and dividends from the Center for Research on Security Prices (CRSP) and the macroeconomic data from the Federal Reserve Economic Data (FRED).

The results indicate that the estimated SDF conditional expectation appears to be slightly less than one which is a plausible estimate and correlates with marginal utility. Moreover, the estimated SDF conditional volatility captures substantial conditional variation which coincides with phases of the business cycles. The estimated SDF model appears to perform better in capturing conditional characteristics relative to consumption-based SDF models despite smaller variation and magnitude in comparison with the empirically estimated Sharpe ratios. We conclude that the methodology that we apply in constructing the SDF can be applicable with alternative variables and estimation methodologies to account for the respective conditional characteristics of stock returns and pave the way towards explaining some of the anomalies in the empirical data.

The thesis proceeds as follows. In chapter 2, we review the asset pricing theory in relation to the stochastic discount factor. We also review predictability evidence and implication of the predictability evidence on the stochastic discount factor. Then, we explore different stochastic discount factor models and different forecasting variables in relation with predictability and marginal utility. Methodology that describes the estimation of the SDF and conditional volatility is discussed in chapter 3. Finally, we provide data definition and then report the empirical results and a conclusion.

2 LITERATURE REVIEW

This chapter reviews the background of asset pricing theory in relation to the stochastic discount factor and provide a theory to justify the variables that we apply in the estimation framework. Firstly, a brief review of the asset pricing theory with definition of key terms. Thereafter, the evidence and implication of excess stock returns predictability to the stylized facts on the conditional characteristics of asset returns and interest rates in relation to the stochastic discount factor. Then, we briefly explore consumption and factor based stochastic discount factors. Finally, the discussion and justification of forecasting variables in relation to marginal utility growth.

2.1 Asset Pricing Theory

The fundamental problem in asset pricing attempts to solve the problem of an investor who has a decision to make on saving, consumption and a portfolio of assets to hold. The basic asset pricing equation comes from the first order condition to the investors saving and consumption problem, that investors marginal utility should discount the expected payoffs to obtain the price today. The loss of marginal utility of investing on an asset and consume a little today, should equal the marginal utility gain of selling that asset and consume the proceeds on a future date. Unless this relationship satisfies, the investor should buy more of the specific asset.

The rate at which investors discount expected future cashflow to determine the price of an asset today is referred to as a stochastic discount factor. Therefore, the stochastic discount factor (SDF) should contain information and account for investors marginal utility. Harrison and Kreps (1979) shows the relationship between the SDF and asset prices stating that in equilibrium there exists a stochastic discount factor which prices all assets. Thus, any traded asset i with the stochastic discount factor must obey the following relations:

$$E_t(m_{t+1}x_{t+1}) = p_t \tag{1}$$

$$E_t(m_{t+1}R_{i,t+1}) = 1 \tag{2}$$

$$E_t(m_{t+1}Re_{i,t+1}) = 0 \tag{3}$$

where E_t is the conditional expectation, x_{t+1} and p_t is the payoff and price respectively, which expresses price as an expected discounted payoff. Equation (2) expresses the same equation in terms of returns by dividing by p_t on both sides, where m_{t+1} is the stochastic discount factor and $R_{i,t+1}$ is the asset gross return. This means that the expected value of the product of the stochastic discount factor and gross returns must equal unity. Equation (3) is a similar expression in terms of excess returns.

From the covariance definition, we can write the basic pricing equation for conditional expected returns as:

$$E_t(R_{i,t+1}) - R_{f,t+1} = -\frac{cov_t(m_{t+1}, R_{i,t+1})}{E_t(m_{t+1})} \tag{4}$$

where the left-hand side is the excess returns, the difference between conditional expected asset return $E_t(R_{i,t+1})$ and risk-free rate $R_{f,t+1}$, which indicates that the covariance between the payoff of an asset and the investors marginal utility should determine risk corrections in expected excess returns. Thus, we can determine the expected returns conditional on time t information, by the conditional covariance between returns and the stochastic discount factor. Applying the covariance decomposition, we can write the expected excess return as:

$$E_t(R_{i,t+1}) - R_{f,t+1} = -\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})})\sigma_t(R_{i,t+1})\rho_t(m_{t+1}, R_{i,t+1}) \tag{5}$$

With the assumption that consumption growth follows a lognormal distribution, we can write the approximation for the expected excess returns as

$$E_t(R_{i,t+1}) - R_{f,t+1} \approx \gamma_t\sigma_t(\Delta\, c_{t+1})\sigma_t(R_{i,t+1})\rho_t(\Delta\, c_{t+1}, R_{i,t+1}) \tag{6}$$

where γ_t is the relative risk aversion coefficient, $\sigma_t(\Delta c_{t+1})$ is conditional consumption volatility and $\rho_t(\Delta c_{t+1}, R_{i,t+1})$ is conditional correlation between consumption growth and returns. This means that variation in one or all the three components should account for the conditional variability in the conditional Sharpe ratio.

The conditional Sharpe ratio for a stochastic discount factor model is also equivalent to the following equation

$$\frac{E_t(R_{i,t+1})-R_{f,t+1}}{\sigma_t(R_{i,t+1})} = -R_{f,t+1}\sigma_t(m_{t+1})\rho_t(m_{t+1}, R_{i,t+1}) \tag{7}$$

which means that the reciprocal of the conditional mean of the SDF which is equivalent to the risk-free rate, conditional volatility of the SDF and conditional correlation between the SDF and returns determine the conditional Sharpe ratio.

Marginal utility is the feeling of an investor to an increase or decrease of consumption of goods or services from which in this case is an asset. Since the stochastic discount factor discounts expected future cashflow to obtain the price of an asset today in saving and consumption choice as relationship in equations (1-3) indicate, the stochastic discount factor should contain information which accounts for the marginal utility of investors.

The implication of equations (1-3) is that we can determine the value of uncertain future cashflows by defining $m = f(\text{data})$. Investors fear stocks since they perform poorly in inconvenient times such as recessions when he/she also experiences losses in other sources such as labor income. As a result, investors demand a higher premium to hold stocks and thus prices are low during such times. The tendency of stocks to perform poorly during bad times makes stocks to be risky rather than a sense that the value of the stocks moves up and down. The inconvenient economic times are periods of high marginal utility when investors are in real need of money. Thus, the stochastic discount factor should determine changes in marginal utility implying the ability to proxy for bad and good economic conditions to capture the empirical facts that we observe in the historical asset data. We review the conditional characteristics of the asset returns historical data in the next section. The asset pricing models differ in the specification of the stochastic discount factor and different SDF specifications yield different results in explaining the conditional characteristics of asset returns. These different specifications all attempt to model marginal utility which describes the bad and good economic times to indicate risk bearing variation of investors. In return, we can understand price volatility and expected returns or predictability so as the equity premium. The models should indicate bad times as times when the marginal utility is high that risk aversion is higher to the extent that, investors avoid stocks despite having lower prices or high expected returns and at the same time interest rates remain low. However, the essence of the asset pricing models is consumption streams of investors.

Consumption is a useful indicator and an important link in describing the investors marginal utility, since consumption is low when marginal utility is high and vice versa. The standard consumption model (C-CAPM) with power utility reveals problems in capturing the specific asset pricing issues. For the C-CAPM requires high risk aversion coefficient since consumption data do not have enough variability to replicate the equity premium puzzle and predictability pattern. Conditionally, the model needs time varying consumption risk or time varying risk aversion. However, the equity premium puzzle and predictability require consumption volatility to be larger than the data indicates. Thus, the model requires high and constant risk aversion coefficient which is not plausible in consideration and perhaps conditionally a time varying risk aversion. Such challenges that the C-CAPM model presents motivate alternative models to describe such bad times in the economy to match the data.

There are various reasons which attribute to the problems of the consumption data. Different studies for example, Grossman et al. (1987), Wheatley (1988) and Breeden et al. (1989) account consumption data problems to measurement error and time aggregation. Mankiw and Zeldes (1991) indicate that the aggregate consumption may under proxy the consumption of asset-market participants. Kandel and Stambaugh (1990) report that consumption volatility does not correlate with the fitted mean and fitted volatility which both appear to be countercyclical.

The consumption data problems motivate a search for alternative forces other than consumption growth which may affect the marginal utility. There are alternative models which attempt to redefine the utility function and some to modify the consumption data. These models are such as the habit model of Campbell and Cochrane (1999), recursive utility and long-run risk models of Bansal and Yaron (2004), idiosyncratic risk model of Costantinides and Duffie (1996), heterogeneous preference model of Garleanu and Panageas (2015), rare disaster model of Barro (2006), intermediary asset pricing models and other models from behavioral finance perspective such as Barberis, Greenwood, Jin and Shleifer (2015). All these models which we briefly explore in section 2.4, have similar idea of introducing an extra recession-related state variable for expected returns to modify the SDF.

The focus of the thesis is on the multi-factor based SDF. We model the SDF with a combination of state variables which capture changes in marginal utility. The idea is that such variables which signal marginal utility change to indicate times when investors experience cashflow and consumption shocks and variation in risk aversion. However, to understand and reconcile with the conditional characteristics of asset returns we must concur with some degree of predictability of stock returns. In the next section, we review the predictability evidence and implication to the stylized facts about the conditional characteristics of asset returns and interest rates. We review the time varying characteristics of interest rates, equity risk premium and as a trade-off between risk and return.

2.2 Predictability Evidence and Stylized Facts about Equity Risk Premium, Interest Rates and Sharpe ratio

Excess return predictability implies that equity risk premium is time varying. The evidence on return predictability and time varying risk premia brings a new paradigm in explaining asset returns and implications on the stochastic discount factor. Permanent and transitory component of Fama and French (1988) and stock market volatility LeRoy and Porter (1981), Shiller (1981), Campbell (1991) and Cochrane (2005) are prominent studies which indicate evidence to suggest that excess returns may be predictable.

Fama and French (1988) provides an analysis on stock prices autocorrelation in different holding periods to test for predictability. The study finds negative autocorrelation in the stock prices which appears to be larger in holding periods longer than one year. This signifies mean reversion in stock returns and is consistent with the hypothesis that a transitory component in the stock prices account for variation in the stock returns.

Price to dividend ratio is also a good indicator to the predictability of equity risk premium. The fact that the ratio appears to be stationary and cointegrated implies that variation in dividend growth or in returns or both should account for variation in the price to dividend ratio. Literature on stock market volatility takes this route in providing evidence on return predictability. The prominent examples are LeRoy and Porter (1981) and Shiller (1981) where both studies argue that expected dividend growth discounted at a constant rate cannot account for high volatility of the price to dividend ratio, thus the expected return variation should account for the variation in price to dividend ratio.

Campbell and Shiller (1989) derive dynamic dividend growth model, from which Campbell (1991) and Cochrane (1991) decompose and quantify the variance of price to dividend ratio into two components. The components are the covariances with dividend growth and with stock returns. Cochrane (2005) quantify the covariance terms using annual data of value weighted NYSE and report that the return predictability accounts for more than 100 percent in the variation of price-dividend ratio. This means that changing forecasts of excess returns account for the variation in price-dividend ratio rather than expected future dividend growth. The changing forecast of excess returns is the variation in reward for bearing stock market risk which implies that risk premia moves over time and thus equity risk premium can be predictable. The fact that low prices today indicate high expected returns.

The predictability evidence has implications on characteristics and modelling of the stochastic discount factor. The best way to examine such a phenomenon is by reviewing the Hansen and Jagannathan bounds (HJ bounds) which we examine briefly in the next section. However, examining equation (6) and (7), the gross risk-free rate variation or the stochastic discount factor conditional volatility or risk aversion or the conditional consumption volatility respectively, or a conditional correlation between SDF and returns should account for the conditional Sharpe ratio characteristics.

Whitelaw (2000) states that in the United States (US) data the gross risk-free rate does not vary much as monthly data indicates variation between 1.00 and 1.02. And since correlation measures between -1 and 1, the stochastic discount factor conditional volatility should account for much of the variation in the conditional equity risk premium and Sharpe ratio which implies a conditional heteroskedastic SDF. Therefore, the conditional volatility of the SDF should account for much of the predictable countercyclical variation and high equity risk premium.

The next section explores the conditional characteristics of the mean and variance of the SDF given the predictability evidence and the asset return data. The relevance of the next section is to understand the stylized facts about the conditional characteristics of asset returns and interest rates for guidance in describing the SDF characteristics.

2.3 Predictability Implications on the Mean and Variance of the SDF

Asset return data restrict the moments of the SDF. Conditioning on the information available at the start of a period, a single period interest rate is equivalent to the conditional mean of the SDF. Consider that there is a short term real riskless asset with the payoff of 1, then equation (1) implies that the SDF is equivalent to the real price of the short term real riskless asset or reciprocal of its gross yield. Due to short term inflation shocks, there is no real riskless asset in the economy. However, in the United States of America (US) and other developed nations the short-term inflation shocks are modest thus treasury bill is a good proxy in practice.

Considering the standard practice, the conditional expectation of the SDF is equivalent to the expected real return on the treasury bills. Campbell (1999) using quarterly US data over the period between 1947 to 1996 reports a mean log return of 0.8 percent per year and a standard deviation of 1.76 percent. Arguably, half of the standard deviation is due to ex-post inflation shocks. Fama (1975) argues that in the period between 1950 and 1960 real interest rate is constant. There are lower frequency variations during early 1970s and very low or negative during late 1970s, followed by much higher real interest rate during early 1980s then drifts lower during later periods of 1980s. Therefore, a reasonable SDF model must have a conditional expectation which is slightly less than one that is relatively stable in the short run but with longer term variations.

The fact that interest rates are relatively stable presents a challenge to the asset pricing models. Since real interest rates should be equivalent to expected SDF, the conditional mean of the SDF should not vary much although the SDF should vary a lot. In the equity premium puzzle, for example the annual data which requires a risk aversion coefficient of 50 corresponds to a subjective discount factor of -0.5. The implication is that the consumption growth one percentage higher or lower requires real interest rate of 50 percentage point higher or lower than normal.

In a time-separable utility function, the large risk aversion coefficient implies that investors are unwilling to substitute expected consumption over time which is a large aversion to intertemporal substitution which should require large interest rate variation to induce the investors to make consumption growth variation. In other words, during bad times when marginal utility is high and the investor expects good times ahead should borrow against the future which would rise the real interest rate and cause large time variation. Cochrane (2005: 415-16) suggests an introduction of non-separability arguments to the marginal utility function which separates intertemporal substitution from risk aversion to explain the specific asset pricing puzzles.

Risk premia restricts the volatility of the SDF. Equation (4), indicate that risk determines expected excess returns which is the negative covariance between expected excess return and the SDF divided by the expected SDF which is equivalent to a risk-free rate or price of a risk less asset. During a period of high marginal utility when the SDF is high, the asset which has a large and negative covariance with the SDF appears to have low returns. In equilibrium, such an asset must have a higher return to compensate investors due to poor performance during the state of the world when wealth is particularly valuable to investors.

Since the correlation between the SDF and excess returns must be higher than minus one, the negative covariance in equation (4) must be less than the product of the standard deviation between the excess return and the SDF. Rearranging equation (4) we obtain

$$\frac{E_t(R_{i,t+1})-R_{f,t+1}}{\sigma_t(R_{i,t+1})} \leq \frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})} \approx \gamma_t \sigma_t(\Delta\, c_{t+1}) \tag{8}$$

The left-hand side of the equation (7) is the risk return trade-off or the Sharpe ratio. The Sharpe ratio puts the lower bound on the volatility of the SDF and the risky asset or portfolio of assets which has the highest Sharpe ratio gives the tightest lower bound. Shiller (1982) states the bound and Hansen and Jagannathan (1991) extends to indicate the construction of the frontier which relates the lower bound on the SDF volatility to the SDF mean with many risky assets without the riskless asset. This is like the mean-variance efficient frontier which relates the variance of a portfolio of the assets to the portfolio mean return. Hansen and Jagannathan (1997) indicate that standard deviation of the difference between a false SDF and a true SDF in a manner of equation (8), bound the largest possible pricing errors for a false economic SDF model.

Risk-return tradeoff is an important concept in the financial markets. The Sharpe ratio measures the risk-return tradeoff which is the expected mean excess returns in the numerator and standard deviation of the excess return in the denominator. A common practice is to model Sharpe ratio as two separate moments using different variables to forecast mean excess returns and standard deviation. Conditionally, Sharpe ratio appears to be highly volatile and countercyclical.

Lettau and Ludvigson (2010) using different variables to forecast mean and volatility indicate that the conditional Sharpe ratio is highly volatile and conditional excess returns is an important contributor to the Sharpe ratio volatility. The study also reports that the Sharpe ratio is countercyclical, that rises during recession when people are relatively more risk averse and demand higher premium to hold stocks so raises expected returns and vice versa during a period of expansion. Brandt and Kang (2004) and Ludvigson and Ng (2007) using latent VAR process and large number of predictors in a dynamic factor analysis framework respectively also indicate volatile and countercyclical Sharpe ratios.

Moreover, Tang and Whitelaw (2011) using predetermined set of financial variables to combine conditional mean and volatility of returns, indicate that the conditional Sharpe ratios have predictable time variation which coincides with business cycle patterns. Lustig and Verdelhan (2012) in studying the United States and other OECD countries also find Sharpe ratios that are higher in recession and lower in expansion, and changes in dividend growth rates cannot account to changes in expected returns.

The characteristics of conditional Sharpe ratio implies that the right-hand side of the equation (8), in the standard time separable model we require time varying consumption risk or time varying risk aversion. We see consumption risk does not vary as much so we need time varying risk aversion. Predictability requires that the SDF to be conditionally heteroskedastic but evidence does not indicate so for consumption growth data. Therefore, the need for alternative or extra variable which can define inconvenient times in the economy to drive asset prices and expected returns.

The brief review of the stylized facts about asset returns, the time variation of equity premium and as a trade-off between risk and return restricts and provides guidance on the characteristics of the SDF in explaining the respective facts. We learn that the SDF should be conditionally heteroskedastic that the conditional volatility should account for the variation and magnitude of the time varying equity premium and risk-return trade-off. The SDF conditional volatility should account for the countercyclical movement and high volatility of the conditional Sharpe ratio with relatively stable interest rates.

We also learn that the conditional expectation of the SDF should be slightly less than one showing relative stability in the short run with longer term variations. Moreover, the SDF should correlate with marginal utility in a sense that SDF is high during a period of high marginal utility and vice versa. In conclusion, we need the SDF model which captures the marginal utility in a way that asset prices correspond to a large equity risk premium which is time varying and correlates with business cycles.

The failure of consumption based models motivates a search for alternative asset pricing models. The alternative models are mainly different specifications of $m = f(\text{data})$ for modeling marginal utility of investors. Cochrane (2005: 49-50) suggests alternative ways to circumvent the problems with the consumption based models. These are such as applying different form of utility function, as there can be a problem in defining the utility functional form in the models.

Application of general equilibrium models can also be another alternative. The general equilibrium models attempt to link consumption to other variables in equilibrium decision rules. Such models link asset prices to other better measured macroeconomic aggregates such as income and investments to substitute for consumption data. Also arbitrage or near arbitrage pricing can be another alternative.

However, the focus of the thesis is on factor pricing model. We model the SDF from variables which correlate with marginal utility and appear to forecast economic state and stock returns. The theory is based upon the fact that these variables signal different economic times to correlate with business cycle variations. Such variables being recession state variables introduce shock to the consumption and cashflow of the investors and induce investors to become more risk averse. We estimate the stochastic discount factor from predicting variables and assess whether the conditional volatility of the SDF accounts for the conditional characteristics of the asset returns. We substitute bad consumption data with predicting variables that proxy for marginal utility to circumvent the problems of aggregate consumption data. We specify m_{t+1} as a linear combination of the predicting variables to form a stochastic discount factor representation that is multifactor beta-like.

There is an empirical evidence on the link between variables that forecast macroeconomic conditions and the conditional expected excess returns, which we examine in the next chapter. Such variables that appear to predict stock returns also seem to predict economic cycles which coincide with the equity risk premium and risk-return trade-off variation through time. Such an important link makes recession and financial distress factors to be useful indicators in constructing the stochastic discount factor because of correlation with and ability to signal changes in marginal utility of investors. Such factors are also better aggregate measures relative to smooth aggregate consumption data. Moreover, higher frequency data commonly generates heteroskedasticity thus we apply monthly data in this thesis.

The idea of factor models starts with the insight of Sharpe (1964), Lintner (1965) and Roll (1977) that if all investors are mean-variance optimizers in a single period then the market portfolio is mean-variance efficient. The implication is that there is a beta pricing relationship between all assets and the market portfolio. This is the case with the assumption of asymptotic no-arbitrage argument and that the market portfolio is the only source of undiversifiable risk. However, the multifactor model holds for more common factors which provide sources of undiversifiable risk which happens to be the case.

The SDF framework assumes that SDF is a linear combination of the common factors with the assumption that the common factors have mean zero and are orthogonal to each other. The approach resembles Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT) which often proxies marginal utility with broad-based portfolio such as market portfolio and a portfolio derived from factor analysis of covariance matrix of returns respectively. The approach also resembles Intertemporal Capital Asset Pricing Model (ICAPM) which applies macroeconomic variables and variables that forecast macroeconomic conditions and returns as factors to substitute for the consumption data.

In the next section, we review different asset pricing models with stochastic discount factor. The purpose is to gain an insight into different stochastic discount factors in terms of merits and shortcoming of the models. We also review the link between the predicting variables, stock returns and macroeconomy to theorize the selection and application of the predicting variables.

2.4 Asset Pricing Models with Stochastic Discount Factor

The consumption based theory is one of the major advances in financial economics during the last three decades. Lucas (1978), Breeden (1979), Grossman and Shiller (1981) and Hansen and Singleton (1982, 1983) indicate a simple relation between consumption and asset returns can capture the implications of inter temporal dynamics of asset returns. Inter temporal equilibrium models base on an economy of a representative agent with a definite utility function. Such models relate SDF to the marginal utility of aggregate consumption.

Lucas (1978) points out that we can derive the SDF with the exogenous consumption process and specification of the utility. The study derives a standard power utility model with the assumption that the economy has the representative investor who consumes aggregate consumption with a standard utility function. The first order condition of such investor determines the SDF. The study models aggregate stock market as paying a dividend equal to aggregate consumption which is equivalent to the assumption that stock market is the market portfolio of all wealth.

Thus, in the case of consumption based stochastic discount factor, consumption risk is investor's first order condition for optimal consumption choice which is a Euler equation for excess returns in relation to the marginal rate of substitution:

$$1 = \beta E_t\left(R_{t+1}\frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}}\right) \tag{8}$$

where β is the subjective rate of time preference, R_{t+1} is the gross stock return and $\beta\frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}}$ is the marginal rate of substitution in consumption. In this case, investors have utility function of the form $u(C_t)=\frac{C_t^{1-\gamma}}{1-\gamma}$, where $u(C_t)=\log(c_t)$ in the limit as $\gamma\rightarrow 1$ with constant relative risk aversion.

The standard power utility model has three problems with regards to the asset returns. The first problem is that the average stock returns are higher relative to the model display. With the assumption that consumption growth and asset returns are lognormal, the negative covariance between log return of an asset and log SDF is the product of risk aversion coefficient and the covariance of the asset with log consumption growth which should equal the risk premium of the asset.

The empirical data indicate that the consumption growth in the post war US data appears to be smooth with Campbell (1999) reporting a standard deviation of about one percent. Thus, regardless of high correlation between risky assets with the consumption growth, the model does not provide a large covariance with the consumption growth to explain the equity premium. To match the equity premium level, the risk aversion coefficient should be higher than a considered plausible estimate. This is the case we describe earlier as Mehra and Prescott (1985) and Hansen and Jagannathan (1991) equity premium puzzle.

The second problem is that the model cannot explain high volatility in stock returns. The innovations in consumption growth drive the stock returns which affect the expected future dividends and discount rates. Since consumption growth appears to be independent and identically distributed (IID) in post war data, then expected future consumption growth is constant and the unexpected stock return should equal the current unexpected consumption growth. This means that variation in expected future consumption growth causes an offset in variation in future expected dividend growth and stock returns when risk aversion is equal to one, whereby it cannot generate large variation in the current stock returns. Campbell (1999) describes the phenomenal as the stock market volatility puzzle.

Increasing the risk aversion has implication on the elasticity of inter-temporal substitution (EIS) within the power utility framework. The increasing risk aversion tends to lower EIS. Despite the evidence of a small EIS as we see in Campbell and Mankiw (1989) and Hall (1988), such small values for EIS imply that investors prefer a flat consumption. However, the historical upward drift in consumption implies that investors should borrow more from the future which should generate high real interest rates. This is a risk-free rate puzzle of Weil (1989).

The historical puzzles that the standard consumption models fail to reconcile motivate new ideas in the consumption based models. These are the models that attempt to redefine the marginal utility function to capture the dynamic characteristics of asset returns. The models attempt to describe bad economic times incorporating information in the marginal utility function with observables to explain the conditional characteristics and asset pricing puzzles.

Habit models define bad times as when the investors consumption falls below the habit. The habit here is defined as the minimum level of consumption which when the consumption falls below this level the marginal utility increases. The habit model can be external habit models in which the reference is the consumption of others or internal habit model where consumption is own personal past consumption. The habit captures time varying risk aversion since as the consumption falls towards the habit levels during bad times, investors become risk averse and do not want to hold stocks which result to decline in stock prices. Despite the high expected return that the stocks offer the investors is reluctant to hold because of the fear further losses in addition to consumption falling below the desired minimum levels. The prominent external habit model is Campbell and Cochrane (1999) model. The success of this model extends to not only explain equity premium but also low interest rates by arguing on precautionary savings. The borrowing motive of risk averse investors to even out consumption is offset by precautionary savings to reduce risk of losses in stocks and consumption falling below minimum level.

Long run risk models define bad times as when investors get bad news about future consumption growth. This high volatility about long run bad news generates high equity risk premium. The model identifies shocks coming from consumption and future stock dividends. Bansal and Yaron (2004) is the prominent long run risk model, however such models face challenge on identifying such news and a plausible mechanism in practice.

Idiosyncratic risk models are another kind of modification of the consumption based models. The idiosyncratic risk models define bad times as when investors face a lot of personal (idiosyncratic) risk to his consumption. Such models require that many investors face idiosyncratic risk at the same time so that cross-sectional consumption volatility is high. Costantinides and Duffie (1996) is a prominent study in this area. The cross-sectional consumption volatility is high in bad times which make the mechanism to be plausible, however the volatility is not high enough to account for the level and variation of equity premium in the data.

Lettau and Ludvigson (2010) estimate the empirical time varying risk-return tradeoff and compare the conditional characteristics with the leading consumption based models. The study compares the empirically estimated Sharpe ratio with the habit model of Campbell and Cochrane (1999), consumption volatility model and the standard power utility model which is considered in Bansal and Yaron (2004). The general findings are that the conditional risk-return tradeoff is countercyclical and highly volatile and the excess return appears to be the main contributor of the volatility in the Sharpe ratio. The prominent consumption models fail to significantly account for the Sharpe ratio conditional characteristics relative to the empirical Sharpe ratio. The empirically estimated Sharpe ratio ranges between -0.46 and 1.76 while the habit model ranges between 0 and 0.4 on quarterly basis.

The consumption based models produce a small countercyclical variation in the Sharpe ratio. The models that seek conditional heteroskedasticity in the consumption volatility fail to account for the conditional characteristics. Both models which directly apply consumption volatility have a -0.3 correlation with the empirical Sharpe ratio. This is consistent with the work of Kandel and Stambaugh (1990) which finds that consumption volatility does not capture the fitted mean and fitted volatility of returns countercyclical movements which appear to be higher in recessions relative to expansion periods. This could be another reason for the failure of the consumption based SDF apart from the smooth data problem and utility specifications as the previous section states.

Heterogeneous preference model by Garleanu and Panageas (2015) is another alternative model which defines investors having heterogeneous preferences in terms of risk tolerance. In this model investors do not face idiosyncratic risk but rather have different preferences as in different levels of risk aversion. The model describes bad times as times when investors who are risk tolerant and large stockholders become more risk averse after experiencing losses and thus the overall market becomes risk averse.

Intermediary asset pricing models are another alternative model for the standard consumption. Such models define bad times as when the intermediaries such as banks who are key investors face a large debt approaching bankruptcy and thus become more risk averse. There is a practical similarity with habit models and the difference being that the level of debt replaces the minimum level of consumption. That is as the intermediaries reach a certain amount of debt become more risk averse while the Habit model requires the minimum level of consumption. The argument against intermediary asset pricing models is the fundamental question on unlevered investors such as sovereign wealth funds and University endowments reluctance to buy stocks when the buying opportunity of high expected return and low prices emerges. The argument for is that the unlevered investors also become more risk averse and sell during bad time such as during the panic of 2008.

Other alternative models which generalize rational expectations consider probability assessments as the basis of the modification. These are models such as behavioral models which describe bad times as when investors irrationally think that the future expected returns are also low when prices are low. The models attempt to modify preferences and base on wrong probability assessments among investors. An example is a study of Barberis et al (2015) which suggests that investors over extrapolate from past returns that when the market falls investors irrationally think that it will continue to fall disregard that past returns have no relation with future returns. The models consider risk aversion and marginal utility from a probability distortion point of view.

Again, these alternative models attempt to describe a state variable for expected returns and build a mechanism to indicate bad economic times are times of high risk aversion and marginal utility and thus the risk bearing variation through different business cycles drives equity premium and prices. Surplus-consumption ratio, idiosyncratic risk, bankruptcy risk, cross-sectional consumption risk are the extra recession state variables in these modified models which describe the mechanism in different economic times. In this thesis, we want to link m to other data which account for aggregate marginal utility growth. Factor pricing model replaces consumption based expression in the following manner

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = a + b' f_{t+1}$$

where a and b are free parameters and f are the factors that proxy for marginal utility growth. The factors should be able to define bad state of the economy signaling changes in marginal utility of investors. The essence of asset pricing is that there are states of the economy when poor portfolio performance concerns investors. The investors are willing to tradeoff overall average performances for portfolios that do not perform bad in such states when the economy is weak such as during recessions. The factors can be variables that indicate or forecast such bad states of the economy. This relates to the consumption as when such states of the economy occur aggregate consumption responds in accordance.

Factor models start with the ideas of Sharpe and the CAPM (1964). For a long time, there seem to be that strategies with high average returns also had high market betas. Meaning that the co-variation between returns and the market portfolio accounts for the expected returns. However, since the work of Merton (1971, 1973) the asset pricing theory recognizes a need for more factors that affect time varying asset returns than the co-variation with the market portfolio alone.

Predictability implies that the equity risk premium varies in a way which coincides with the business cycles. The countercyclical variation of equity risk premium indicates that poor economic conditions associate with higher expected excess returns than good economic times. The implication is that there is a link between macroeconomy and conditional excess returns. The link is fundamental in explaining conditional characteristics of stock returns.

The empirical studies such as Fama and French (1989) and Ferson and Harvey (1991) indicate that expected returns increases during depressed business conditions and contracts during good times by plotting fitted values of the expected excess returns on the aggregate stock market. Harrison and Zhang (1999) applying non-parametric estimates of the conditional excess returns and correlate with business condition variables and find conditional excess returns to be countercyclical. Campbell and Diebold (2009) link the expected excess returns to the Livingstone expected business conditions survey data to indicate that the expected business conditions predict excess stock returns.

The link between equity risk premium and economic condition indicate the need for recession state variables. Investors depend more than their investments for their income that other aspects in the economy affect investment decisions. Bad economic times increases risk aversion and introduce cashflow shocks through other aspects such as labor income and borrowing constraints. The CAPM relaxes the assumption to indicate that investors only care about the performance of their investment portfolio that the market portfolio is the only source of undiversifiable risk. Stocks which are pro-cyclical should offer higher average returns than countercyclical stocks to convince investors to hold them regardless of the higher betas. Therefore, in a SDF notation m is linear to the market return.

With this fact in mind, it is inevitable to seek another dimension of priced risk which co-varies with the recession. Here emerges the idea of multifactor model where in the SDF notation m depends on multiple factors. This is a vector of factors where each factor defines different bad times. Unlike CAPM which restricts the SDF to be linear while the real world is nonlinear, the multifactor SDF can attempt to capture the nonlinearity. The asset pricing theory specifies that assets which underperform during bad economic times as in recessions need to have higher average returns to induce investors to hold them, that is investors are willing to sacrifice higher average returns for assets that do well in bad times. Therefore, the factor variables should measure or indicate such bad times.

The factors also should affect the average investor, that is the bidding up and selloff should affect the asset prices or expected returns of all investors on average. This implies that such factors should affect the aggregate marginal utility of investors, equilibrium prices or expected returns and not only the transfer of risk between transacting investors. Moreover, the factors should also indicate or signal business cycles and financial distress, appear to predict returns as well as to signal changing investment opportunity set.

The factor variables should not also be highly predictable as highly predictable factors predicts large interest rates variations. Therefore, the factors should somehow be independent over time since the real risk-free rate does not vary as much in comparison with the asset returns. The measured consumption growth also does not vary much and is the least predictable macroeconomic series especially accounting for temporal aggregation as the consumption series are quarterly averages. This implies that the factors should be stationary units such as growth rates rather than level, portfolio returns rather than prices or stationary series such as dividend to price ratio.

Consumption is a good indicator of marginal utility change and bad economic times. Consumption of investors decreases during and when there are prospects of bad times. Consumption is high during good economic times and low in bad times.

Consumption relates to variables such as broad-based return portfolio, interest rates, investment and other macroeconomic variables in a way that such variables measure wealth or the state of the economy. For example, when the market crashes or during recessions investors respond by reducing consumption.

Moreover, when variables change indicates a high future income the aggregate consumption tends to rise. Such variables are not measures of economic states rather tend to forecast such economic times. The implication is that consumption and marginal utility respond to news about the state of the economy. However, section 1.4 states the challenges to the efforts that link SDF to consumption data alone in empirical data. But consumption appears to be a good candidate as a forecasting variable in combination with other variables which define different bad times.

Change in labor income is also a good state variable as labor income can signal the state of the economy. Recessions affect investments which in turn affect productivity which can influence labor income. Such changes can influence investors decision making to withdraw funds from risky assets and thus any changes in the labor income might be priced in the stock market. Therefore, changes in labor income correlate with marginal utility and can signal the state of the economy and thus is a good proxy for bad economic times.

Price to dividend ratio appears to forecast stock returns as empirical studies such as Cochrane (2011) indicates. Prices are low when the expected returns rise since the expected cash flow is discounted at a higher rate. Low price to dividend ratio indicate that investors expect a higher return. This implies that the price to dividend ratio suggest a premium for holding risks related to recession or financial distress. The fact that this variable can predict stock returns and signal bad state of the economy is also a good candidate to be applied in our research. However, price to dividend ratio is a slow-moving variable which implies that the variable is good for long horizon return predictability.

Fama and French (1989) indicate that term spread which is the difference between long term and short-term bond yields can forecasts stock returns. The term spread is also a good indicator of bad times since it rises at the bottom of recession and inverted at the top of a boom period. The shape changes of the term spread signify that the variable carries information about marginal utility of investors. During bad times investors tend to shift their investments to less risky assets such as bonds. The shift tends to increase expected returns in risky assets such as stock and lower expected returns in bonds. In turn during such times we see stock prices plummeting while the bond prices rise.

Default spread which is the difference between high and low quality corporate bonds appears to have the forecasting power in the stock returns according to Fama and French (1988) study. During bad economic times such as the recession, the risk of holding corporate bonds tends to increase and cause the spread between different corporate bonds to increase. Such is the reverse case during good economic times such as expansion periods. Such a marginal utility correlation and signaling of state of the economy together with the predictive powers qualifies default spread to be applied in our framework.

Another state variable is short term interest rates which also appear to predict returns. Prominent studies such as Fama and Schwert (1977), Campbell (1991), Hodrick (1992) and Ang and Bekaert (2007) study and reveal the forecasting power of short term interest rate. Short term interest rate can also signal the expectation of the state of the economy. Central banks reactions in interest rates can signal changes in the expectations of the state of the economy.

Inflation changes influence the nominal and real terms of cash flows and interest rates. This impacts asset valuation due to changes in average inflation rate which have systemic effect to average investors. High inflation is a characteristic of bad times as often happens during such times as in recessions. Inflation has a cashflow effect as rising inflation affects profitability of firms as the process to direct cost increase to a final consumer is gradual and thus reduces profit margin of firms. Inflation also has discount rate effect since high inflation happens during bad times which has a rising effect on the expected return which cuts the stock prices.

The variables that signal changes in the state of the economy and appear to have stock returns forecasting power are the variables that interest this research in testing and applying in the stochastic discount factor framework. The state variables define different state of economy and introduce cashflow shock and signal times when risk aversion changes. We hypothesize that a combination of such state variables can better account for the conditional characteristics of the stock returns as a substitute for bad consumption data in the stochastic discount factor framework.

The asset pricing models with stochastic discount factor that apply and attempt to modify the consumption data such as standard consumption models and habit model that we discuss briefly above encounter similar problems in capturing the respective conditional characteristics. There is no model yet that successfully describes all the fundamental conditional characteristics. The specific problems with the aggregate consumption data motivate alternative models that seek to substitute the consumption data with variables which are better measured aggregates and define changes in the marginal utility of investors in constructing the SDF. The variables which define different bad times in a way that do not only restrict bad times to changes in consumption.

In this study, we attempt to model a stochastic discount factor with state variables which signal the state of the economy and proxy for the marginal utility growth to account for the dynamic characteristics of returns specifically risk-return tradeoff. We model the non-linear SDF model from the state variables to substitute bad consumption data to proxy for the changes in the marginal utility growth. These variables define bad times in different aspects as in cashflow shock due to income shocks and borrowing constraints and increase in risk aversion. The expectation is that the SDF should be conditionally heteroskedastic accounting for the magnitude and conditional characteristics of the risk-return trade-off.

We estimate the stochastic discount factor from the forecasting variables and assess the SDF conditional volatility accountability to the conditional characteristics of risk-return trade-off. The conditional characteristics we refer to here are in terms of high volatility and countercyclical movements which coincide with business cycle variations that we observe in empirical data. Technically, the SDF should account for the high equity risk premium and predictability of the excess returns. We achieve our purpose through SDF parameters and conditional volatility estimation and perform the heteroskedasticity test.

The estimated SDF should have certain characteristics to match the conditional properties of historical asset returns. The conditional expectation of the SDF should be slightly less than one with longer term variations but relatively stable in the short run. The SDF should also coincide with the marginal utility that is high during the periods of high marginal utility and low during the periods of low marginal utility. The SDF should be extremely volatile but conditionally should have low volatility. Moreover, the estimated SDF conditional volatility should account for the variation and countercyclical movements that we observe in the empirically estimated Sharpe ratio and equity premium. In summary, the estimated SDF should account for the high and countercyclical movement of the Sharpe ratio to indicate predictability of stock excess returns and high equity premium while manifests low and relatively stable interest rates.

3 DATA AND RESEARCH METHODOLOGY

3.1 Data and Variables

The market index data is NYSE, Amex and Nasdaq monthly value-weighted index from January 1959 to December 2011, obtained from the Center for Research on Security Prices (CRSP). Macroeconomic data is obtained from the Federal Reserve Economic Data (FRED). We apply monthly macro-economic variables which are higher frequency data that commonly appears to generate heteroskedasticity.

3.1.1 Default spread

Default spread is the difference between BAA corporate bond and AAA corporate bond. The credit ratings are Moody's seasoned corporate bond yield from the board of governors of the federal reserve system. The rating is a monthly frequency and in percentage.

3.1.2 Short term interest rate (relative bill rate)

Short-term interest rate is the monthly Treasury bill rate minus its 12 months moving average. The 3-months treasury bill is secondary market rate in monthly frequency. The spread between is a proxy for short-term interest rate.

3.1.3 Term spread

Term spread is the difference between the long-term government bond yield and short-term bond yield. In our data, we have 10-year Treasury bond yield, constant maturity rate and 3-months Treasury yield. The yields are in percent and monthly frequency and the term spread is the difference of the two.

3.1.4 Consumption

We define population as 1 divided by real disposable income per capita times real disposable personal income. Consumption variable is the percentage change of consumption per capita which is personal consumption expenditure for non-durable goods and services divided by population.

3.1.5 Dividend-price ratio

Dividend price ratio is simply calculated by dividing the aggregate dividends by the CRSP value weighted index price. In our data, we calculate dividend yield from the difference between total return on index and capital gains since total return is the sum of the dividend yield and capital gains.

3.1.6 Inflation

We calculate change in inflation as change in consumer price index for all urban consumers and all items. The series is seasonally adjusted monthly frequency.

3.1.7 Labor income

We calculate percentage change in labor income by taking the percentage change in disposable personal income per capita. This is a change in disposable personal income divided by population. The series is seasonally adjusted annual rate in monthly frequency.

3.2 Methodology

The chapter introduces the methodology that we undertake in this paper. First, is parameter estimation of the stochastic discount factor and perform the heteroskedasticity test. Thereafter, is the estimation of conditional volatility of the estimated stochastic discount factor. We estimate conditional volatility of the stochastic discount factor applying two different methods.

3.2.1 Stochastic discount factor estimation.

The estimation of the stochastic discount factor starts with parameter estimates from a non-linear regression. Nonlinear least squares (NLS) method is useful in capturing the nature of the model and improves efficiency. The method differentiates by taking partial derivative of the function of the mean equation with respect to the parameters. The likelihood function is maximized when the residual square is minimized.

The parameter estimation methodology is inspired by the ordinary least square estimation and inference method from Britten-Jones (1999). In estimating the portfolio weights, Britten-Jones attempts to minimize the squared deviation between the constructed excess returns portfolio and the excess returns in $\mathbf{1}$.

$$\boldsymbol{X}\boldsymbol{b} + \boldsymbol{u} = \mathbf{1}$$

where $\boldsymbol{X}$ is a matrix of excess returns, $\mathbf{1}$ is a vector of ones and $\boldsymbol{u}$ is a residual vector indicating deviation of the portfolio of excess returns from $\mathbf{1}$. The regression has no intercept with a non-stochastic dependent variable and the residual vector which correlates with stochastic regressors. The dependent variable $\mathbf{1}$ is a sample counterpart arbitrage profits which is a positive excess returns with zero standard deviation. The estimated coefficient $\boldsymbol{b}$ weights produce a return vector which is the closest in terms of the least squares distance to the arbitrage return vector $\mathbf{1}$.

From the basic pricing equation (2), we define the stochastic discount factor as a nonlinear combination of the state variables. To estimate parameters $b_1 ... b_n$ on each variable, we replicate a vector of ones of the same length as the data to apply as a dependent variable in a non-linear least square regression of the following form

$$m_{t+1} = a + b_1 f_1 + b_2 f_2 + \cdots + b_n f_n$$

$$E_t\big((a + b_1 f_1 + b_2 f_2 + \cdots + b_n f_n \ ) * R_{i,t+1}\big) = 1$$

$b_{1\ldots.}b_n$ are the estimated parameters that we obtain from the fitted values of the regression. We then create a time series of our estimated SDF of parameter estimates along with the predicting variables. The estimated parameters are equivalent to the portfolio weights in Britten-Jones mean-variance efficient portfolio.

3.2.2 Conditional volatility estimation

In modeling conditional volatility, we consider the maximum likelihood and Bayesian inference methodologies. We first consider conditional volatility of the estimated stochastic discount factor as a GARCH process. We apply a standard GARCH (1,1) model to the fitted values of the expected conditional SDF. Secondly, we consider a stochastic volatility model to conduct Bayesian inference using Markov Chain Monte Carlo (MCMC) methodology. The latter is a non-deterministic method of modeling volatility which models the logarithm of squared volatilities through latent autoregressive process of order one.

4 EMPERICAL RESULTS

In chapter 3 we discuss and evaluate the methodology for the SDF estimation. In this chapter, we report the empirical results of the SDF estimation from the combination of the state variables. Working with factor variables, the factors should be orthogonal to each other with mean zero and no autocorrelation. The variables should also appear to price the stock return thus forecast stock returns and correlate with marginal utility to signal different states of the economy.

Table 1. Summary statistics of variables

	Mean	Std.dev	Autocorrelation
def	0.000	0.001	-0.096
rrel	0.001	0.427	-0.110
dp	0.003	0.002	0.931
cons	0.465	0.006	0.073
income	0.553	1.474	0.171

This table reports summary statistics of predicting variables. def represents the default spread, rrel is the short-term interest rate, dp is the dividend to price ratio, cons is the percentage change in consumption per capita and income is percentage change in income per capita. The sample runs from January 1959 to December 2011.

Table 1 reports the summary statistics of the predicting variables. The variables have sample means of zero as measured by Ljung-box test. The variables appear to have mild autocorrelation except for the dividend yield which is highly autocorrelated as expected. The high autocorrelated variables can yield errors-in-variables problem which can provide bias estimates on the loadings and downward estimates of the statistical significance as stated in Chen, Roll and Ross (1986).

However, dividend yield is an important variable in predicting stock returns especially in the longer horizons, but here in our sample dividend yield appears to price the return index. Since we use a portfolio index, the error-in-variable problem is mitigated unlike in the application of individual stock returns which could hinder the estimates. Labor income appears to have a higher standard deviation relative to other variables which do not appear to vary as much.

The variables which appear on the table 1 are the final variables that we apply in our framework. The term spread is redundant since it does not appear to price returns in our sample. The reason could be that the term spread is a long-term variable that does not price returns in higher frequency data since we apply the monthly frequency unlike the dividend yield in this sample. Another variable that is redundant is inflation which also does not price the monthly index returns in our sample and thus does not appear in our final results.

Table 2. Correlation matrix of variables

	Def	rrel	dp	cons
rrel	-0.204			
dp	0.051	-0.023		
cons	-0.162	0.062	-0.106	
income	-0.02	0.053	-0.046	0.258

This table reports a correlation matrix of predicting variables. def represents the default spread, rrel is the short-term interest rate, dp is the dividend to price ratio, cons is the percentage change in consumption per capita and income is percentage change in income per capita. The sample runs from January 1959 to December 2011.

The predicting variables should be orthogonal to each other to be applicable in the factor model framework. Table 2 depicts the correlation matrix of the predicting variables. The variables indicate the correlation between -0.162 and 0.258 which is close to no correlation between variables. The variables appear to be far from perfect correlation and therefore the variables cannot substitute each other.

The highest correlation between variables is between consumption growth and changes in labor income which is 0.258. This is a reasonable sign and magnitude since a change in labor income has a direct effect on the consumption of individuals. As income rises or when there is an expectation of income rise individual consumption tend to rise as well and so is the vice versa.

The least correlated variables are consumption growth and default spread which appears to be -0.162. This is also a reasonable estimate and sign as a decrease in consumption signals bad economic times during which the default spread increases. Also during good economic times when the default spread appears to decrease is when the consumption increases. The mild correlation that appears in our estimates qualifies the factors to be orthogonal and applicable in our framework since the correlation of below 0.5 is a reasonable estimate for factor model application.

Once we identify the orthogonality between the factors and summary statistics of the variables, we then check for the forecasting power of the state variables in our sample. For the variables to qualify in the application of our framework, variables should appear to forecast stock returns apart from being orthogonal to each other. As such predicting variables carry information about the marginal utility of investors and some of these variables also appear to forecast different states of the economy. Again, the state variables define different bad times in the economy when investors are risk averse and have cashflow shock.

Table 3. Regression estimates on predicting variables

						Estimate
Intercept	0.009	0.009	0.004	0.002	0.007	-0.004
	(4.911)	(4.688)	(1.170)	(0.760)	(3.219)	(-1.020)
def	-8.176					-8.577
	(-3.615)					(-4.200)
rrel		0.009				-0.015
		(-2.245)				(-3.443)
dp			1.884			2.600
			(1.725)			(-2.417)
Cons				0.013		0.011
				(2.637)		(-2.887)
Income					0.0030	0.002
					(2.253)	(2.199)
N						635

N = number of observations, *t*-values in parentheses
This table reports estimates from OLS regressions of stock returns on lagged variables. The dependent variable is the excess level of the return on the CRSP value-weighted stock market index. The regressors are def represents the default spread, rrel is the short-term interest rate, dp is the dividend to price ratio, cons is the percentage change in consumption per capita and income is percentage change in income per capita. Newey-West corrected *t*- statistics appear in the parentheses below coefficient estimates. The sample runs from January 1959 to December 2011.

Table 3 presents results of in-sample predictive regression of monthly excess returns on the value-weighted stock market index from the CRSP in excess of the monthly Treasury bill rate to the forecasting variables. Table 3 reports the regression coefficient and heteroskedasticity and autocorrelation consistent *t*-statistic for each regression. We estimate the monthly time series regression through the whole sample period from January 1959 to December 2011 to identify the state variables which appear to price excess returns.

Default spread appears to affect returns in higher magnitude while consumption growth, short term interest rate and changes in labor income seem to have marginal significant effect to changes in returns. Dividend yield does not appear to forecast excess returns univariately in this sample, however the dividend yield predictive ability enhances in a multivariate regression with other forecasting variables. The term spread and inflation do not appear to have the forecasting power for monthly excess returns in this sample hence both variables do not appear on table 3 as noted above.

Predictability check of the variables to excess returns permits us to select variables to be applicable to the estimation of the SDF. As the methodology chapter explains, we base our estimation of the SDF on the nonlinear least squares methodology and inference methodology of Britten-Jones (1999). The estimation provides the parameter estimates from which the fitted values provide us with the unconditional expectation of the SDF.

Table 4. SDF Parameter estimates

	Parameter
a	1.002
	(249.547)
b1	8.158
	(3.756)
b2	0.005
	(1.185)
b3	-2.579
	(-2.421)
b4	-0.011
	(-2.773)
b5	-0.0020
	(-2.141)

This table reports parameter estimates from NLS regressions of the stochastic discount factor estimation. The dependent variable is the vector of ones. The regressors are a product of gross returns and predicting variables. a is an intercept parameter, b1 is the default spread parameter, b2 is the short-term interest rate parameter, b3 is the dividend to price ratio parameter, b4 is the percentage change in consumption per capita parameter and b5 is percentage change in income per capita parameter. Newey-West corrected *t*-statistics appear in the parentheses below coefficient estimates.

Table 4 presents the parameter estimates of the estimated SDF from the nonlinear least squares. Table 4 reports the regression coefficient and heteroskedasticity and autocorrelation consistent *t*-statistic on each parameter. The estimated parameters are a result of the non-linear model of the form of equation (2).

The fitted values from the estimated parameters that table 4 presents form the estimated unconditional SDF values. Parameter b1 which is a default spread parameter has a high magnitude in this fit as well as b3 which is a dividend yield parameter. The intercept indicates a sign of spurious due to the high *t*-value which is for the bias that the NLS model and variables cannot account for.

Table 4 presents the unconditional SDF parameter estimates however, the focus of this study is on the conditional estimates. To obtain the conditional expected SDF we re-estimate by regressing the forecasting variables against the estimated parameters. Figure 1 depicts the conditional expected SDF. Conditionally, SDF appears to correlate with marginal utility as the SDF is high during depressed business conditions when marginal utility is high and vice versa.

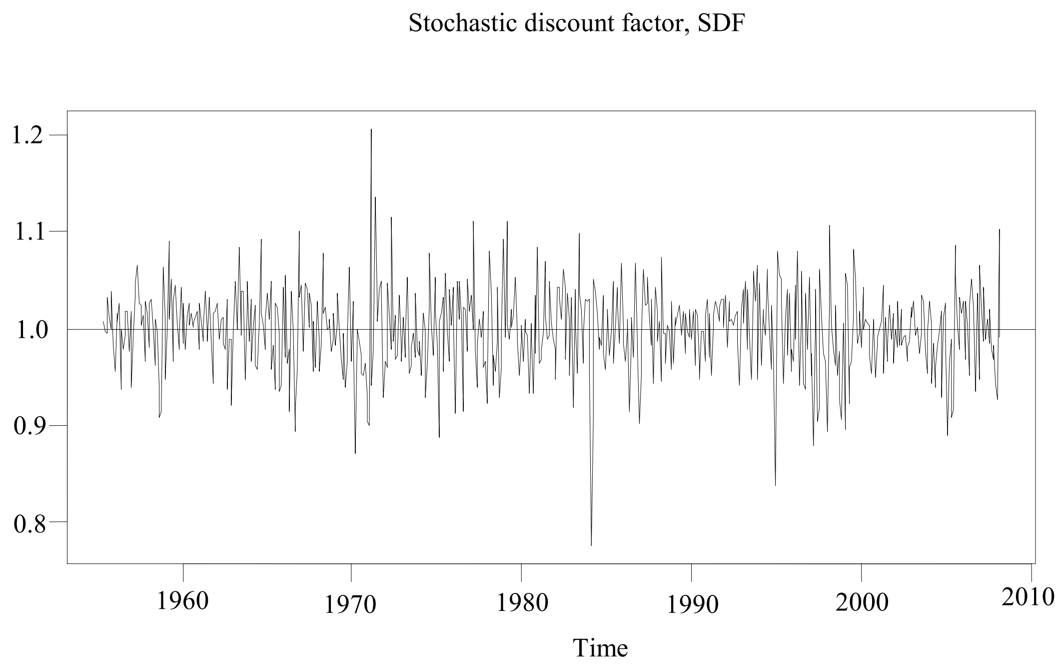

Figure 1. Stochastic discount factor

Figure 1 plots an estimate of the stochastic discount factor for the period between January 1959 and December 2011. The figure plots the fitted values from the re-estimated regression of parameters that table 4 depicts along the forecasting variables. The estimated conditional SDF is close to unity most of the sample period and is bound between the intervals of 0.7758 and 1.2061. The sample mean for the estimated SDF is 0.9982 which is a reasonable estimate, since the conditional expectation of the SDF should be slightly less than one.

The figure also indicates a positive correlation between the estimated SDF and marginal utility. The periods of high marginal utility as during the recession periods, the estimated SDF appears to be higher relative to expansion periods which characterize periods of low marginal utility. We characterize in detail the different business cycle phases in the next few paragraphs when we analyze the conditional volatility of the SDF.

The estimated SDF measures an unconditional volatility of 15% more than consumption growth. The consumption volatility appears to have a mean of 1% relative to 15% of the estimated SDF. This accounts for the high magnitude of the unconditional Sharpe ratio relative to the consumption volatility which requires a higher and implausible risk aversion coefficient to match the magnitude of the Sharpe ratio. This difference implies that our estimated SDF substantially accounts better for the high magnitude of Sharpe ratio unconditionally with 15 times more in terms of volatility relative to the consumption data.

Even though the estimated unconditional volatility appears to be slightly lower the sort of magnitude appears to match the monthly realized and conditional Sharpe ratio of Tang and Whitelaw (2011). The mean of the realized Sharpe ratio that Tang and Whitelaw (2011) report is about 0.15. The realized Sharpe ratio calculations are from the realized volatility monthly obtained from the sum of the squared daily returns within a month. The other estimated monthly conditional Sharpe ratio means appear to be in the region between 0.15 and 0.18. In these other estimates, Tang and Whitelaw (2011) estimate the Sharpe ratio from the two conditional moments separately and take the ratio. This is about the same magnitude that our estimated SDF unconditional volatility exhibits which is about 0.15.

The unconditional volatility from the estimated SDF implies that unconditionally, the estimated SDF does a better job to account for the magnitude of the Sharpe ratio relative to the consumption based SDF. Since the consumption growth appears to be I.I.D and with low volatility, consumption based models require implausible risk aversion coefficient in which is referred to in the equity premium puzzle. However, the focus of the study is on the conditional volatility of the SDF. The objective is to assess whether the SDF is conditional heteroskedastic to account for the conditional characteristics of the Sharpe ratio which accounts for predictability and high equity premium. The SDF conditional volatility should coincide with high volatility of the conditional Sharpe ratio and countercyclical movements through business cycle phases.

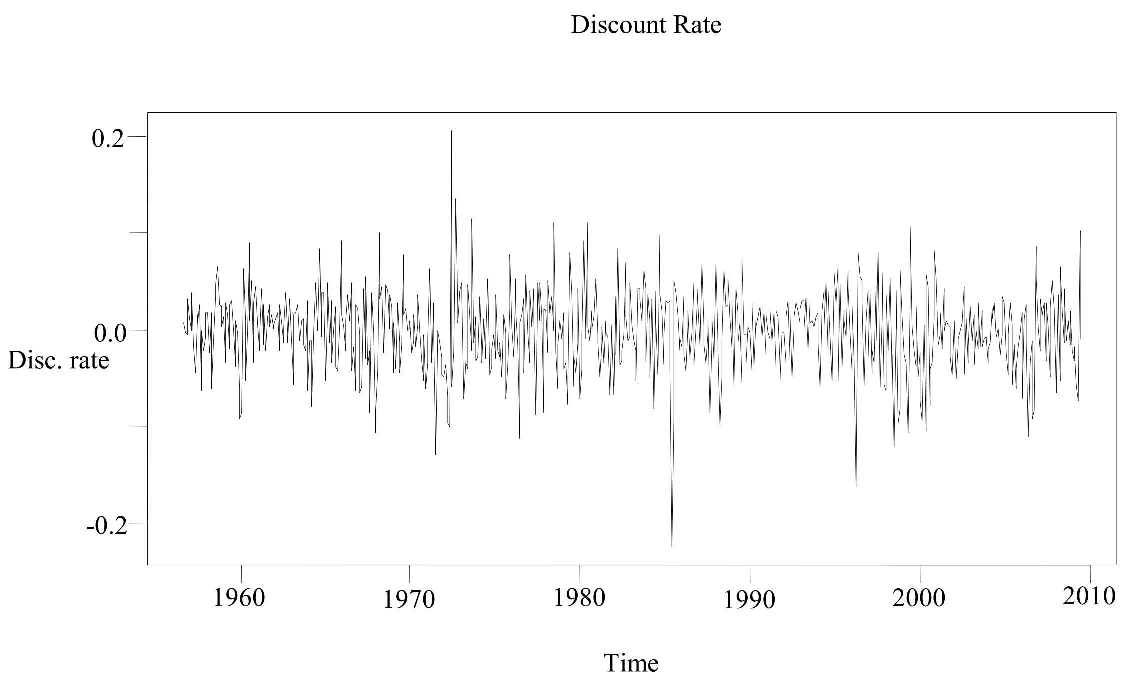

Figure 2. Conditional discount rate

Figure 2 plots the equivalent of the discount rate. The plot is a time series of the inverse of the estimated SDF which is equivalent to risk free rate or the discount rate. The empirical studies and historical data indicate that the risk-free rate is low and relatively stable. Our estimated risk-free rate equivalent appears to fluctuate around the zero mean even though has much variability relative to the observed historical data. The heteroskedasticity that the estimated SDF generates cause the short-term variations that we observe on figure 2.

The reason for the discount rate to remain low could be that the indication of bad times induce investors to transfer the funds from risky assets to less risky assets such as government bonds which tend to keep the interest rates lower. The demand for the less risky assets boosts the prices of safer assets which lowers the expected return. The transfer of funds from stocks and corporate bonds during bad times attribute for the default spread and equity risk premium to rise. This is a result of bad times which increases risk aversion and cashflow shocks to investors who hold risky assets during good economic times.

Assessing the accountability of the estimated SDF conditional volatility to conditional Sharpe ratio characteristics we refer to the HJ bounds. We concentrate on the ratio between the SDF conditional volatility and the expected conditional mean of the SDF, the right-hand side of equation (8). We do not focus on the conditional correlation and risk-free rate since as noted above, these two components are conditionally relatively stable to account for the conditional variability and variation of the risk-return trade-off. The focus is on the conditional heteroskedasticity of the SDF mainly the conditional volatility of the estimated SDF. The conditional expected SDF also appears to be relatively stable which implies that SDF conditional volatility accounts to most of the Sharpe ratio conditional variation.

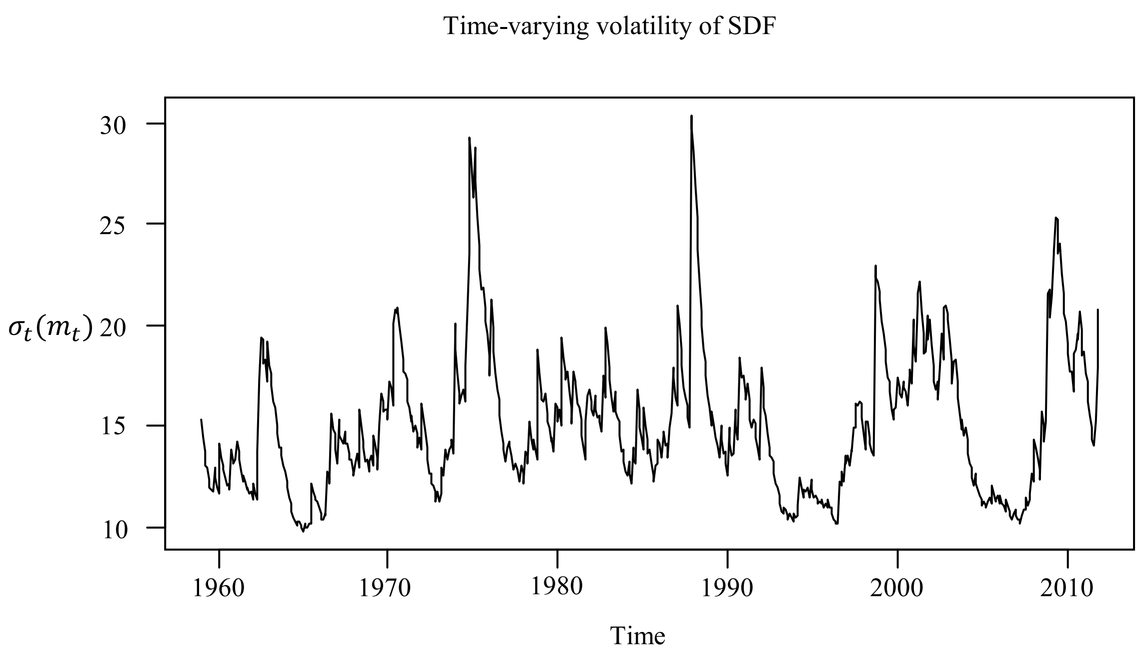

Figure 3. Stochastic discount factor conditional volatility

The predictability evidence implies that the Sharpe ratio is not constant that conditionally varies in ways which coincide with the business cycle. Therefore, the stochastic discount factor should be conditionally heteroskedastic to account for conditional variation of Sharpe ratio in the specific predictable countercyclical pattern. We account the SDF conditional volatility because the correlation of the discount factor on the volatility bound or the mimicking portfolios for discount factors should be 1 and the risk-free rate is relatively stable. In testing whether the estimated SDF is conditionally heteroskedastic we perform the Ljung-Box test on the square of the estimate.

According to the Ljung-Box test, we fail to reject the null hypothesis with p-value of 0.6954. The Ljung-Box test provides the null hypothesis of zero dependence, since the p-value is above 0.01 we fail to reject the null indicating that the estimated SDF is conditionally heteroskedastic. The implication of the Ljung-Box test is that the conditional volatility varies through time and vindicate that the Sharpe ratio is not constant but varies through time to support the predictability evidence. This conditional variation is also evident on figure 4 which depicts time variation that coincides with phases of the business cycles.

Figure 3 plots the conditional volatility of the estimated stochastic discount factor. This is a maximum likelihood estimation from a GARCH (1,1) model. The plot indicates the SDF conditional volatility that moves countercyclically which coincides with phases of the business cycles. The estimated SDF volatility appears to rise at the beginning of recessions and fall over a period of expansion. The recession years in the US such as during early 1960 the conditional volatility appears to slightly rise. This is a period of short monetary recession which occurs because of the Federal Reserve raising interest rates in 1959.

The other period of mild recession is during the year 1969 and 1970. This mild recession is a result of fiscal tightening because of budget closing of the Vietnam war and another Federal Reserve rate hike. The period between 1973 to 1975 is another period of recession because of 1973 oil shock and a stock market crash of 1973 to 1975. Between this period there is a significant rise to the conditional volatility as the figure 4 depicts.

During the 1990's US experiences the longest period of growth in history. The estimated SDF conditional volatility appears to fall during the respective period and appears to be lower relative to the recession periods. This period ends with the collapse of the dot com bubble and a fall of business investments and outlays between 2000 and 2001 coupled with the September 11 terror attacks. During this period of recession, the estimated conditional volatility indicates a significant rise.

Another notable recession is a great recession of between 2007 and 2009 from the subprime mortgage crisis. This is the collapse of housing related assets that causes the global financial crisis. There is a significant notable rise in the estimated SDF conditional volatility during the respective period. This period of stock market boom precedes the respective recession period and figure 4 depicts a significant fall in the conditional volatility. The countercyclical movement is consistent with the empirical estimate of the conditional Sharpe ratio that prominent studies such as Lettau and Ludvigson (2010) and Tang and Whitelaw (2011). The conditional variation is also consistent with the predictability evidence and conditional heteroskedasticity of the estimated SDF.

Three of the empirically estimated Sharpe ratios of Tang and Whitelaw (2011) appear to have substantial conditional variation relative to our estimated SDF conditional volatility. Tang and Whitelaw (2011) report in-sample time variation of three of the estimated Sharpe ratios which range between -0.20 and 0.90 monthly. This variation is more substantial than our estimated SDF conditional volatility which is between the range of 0.09 and 0.30.

However, the estimated SDF conditional volatility appears to perform better relative to the Tang and Whitelaw (2011) estimated Sharpe ratio in which the unconditional in-sample mean or realized volatility replaces one of the conditional moments which is equivalent of replacing a vector of conditioning variables in a regression with a constant. The Sharpe ratio exhibits low variation with an unconditional volatility of about 0.04 and ranges between 0.10 and 0.23.

The estimated SDF conditional volatility appears to be stationary which implies that the conditional volatility does not increase with horizon. The volatility of the SDF does not appear to grow with the horizon which does not add extra equity premium at the long horizon. For this reason, the model does not do well in capturing the long-run equity premium despite some magnitude in the short-run equity premium.

Apart from estimating the SDF conditional volatility through maximum likelihood methodology with a standard GARCH (1,1) model, we also estimate from Bayesian inference with the stochastic volatility model through Markov Chain Monte Carlo (MCMC) simulation. In estimating the stochastic volatility model, we set the prior parameters as default since the choice do not matter much when the data is informative and do not impact the posterior estimates. The prior mu is set at (0,100), prior phi is set at (5, 1.5) and prior sigma is set at 1. These are the default prior values in the R stochvol package for estimating stochastic volatility models.

Table 5. Posterior draws of parameters

Parameter	mean	sd
μ	0.001	0.060
Α	0.834	0.121
Σ	0.024	0.020
exp(μ/2)	1.001	0.030
σ^2	0.001	0.002

The table reports posterior draws of parameters from the summary of 10000 MCMC draws. μ represents mu, α represents phi and σ represents sigma.

Table 5 presents the posterior means of the four parameters. The posterior mean of the phi is 0.834 and a posterior mean of mu is 0.001 which indicate a moderate persistent autoregressive process. The moderate persistence can be depicted from figure 5 that indicate that the volatility path is smoother. The volatility of volatility is moderately low at 0.024. The overall level of volatility appears to be very low with the mean of mu at 0.001.

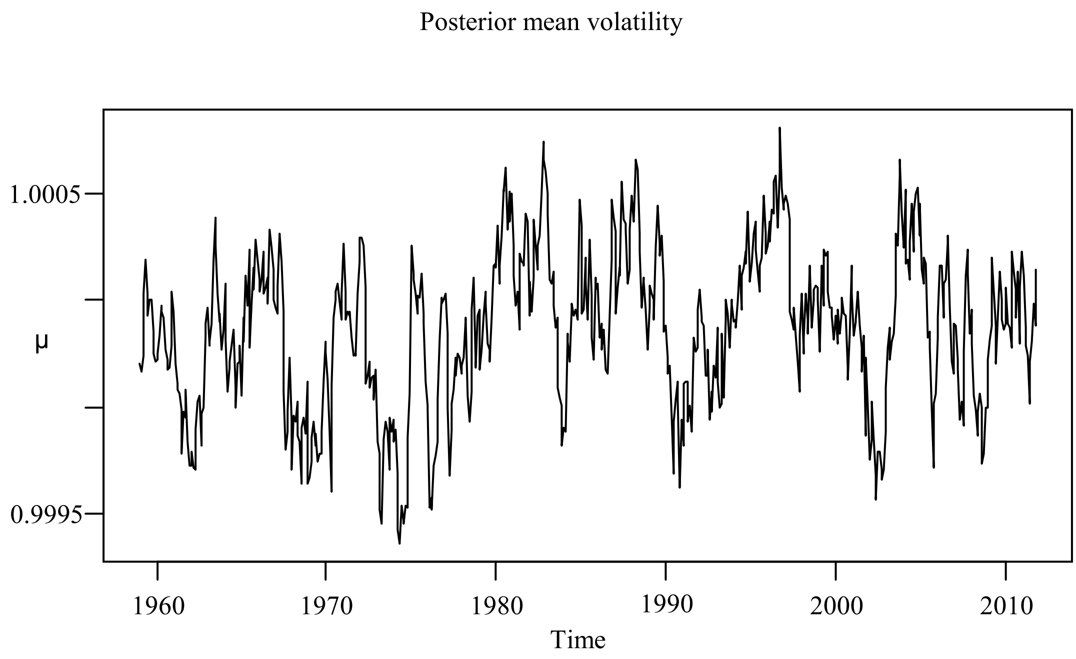

Figure 4. Posterior mean volatility

Figure 4 plots the posterior mean volatility from the stochastic volatility model. The volatility path appears to be smoother but not as smooth as the volatility estimate from the GARCH model. The moderate persistence as stated above that the posterior mean of phi indicates, generates somehow the smoother volatility path. Figure 4 also indicate the countercyclical movement of the estimated volatility. The countercyclical variation appears to coincide with the phases of the business cycle however, the GARCH estimate indicate substantial countercyclical variation relative to the stochastic volatility as can be compared between figure 3 and 4.

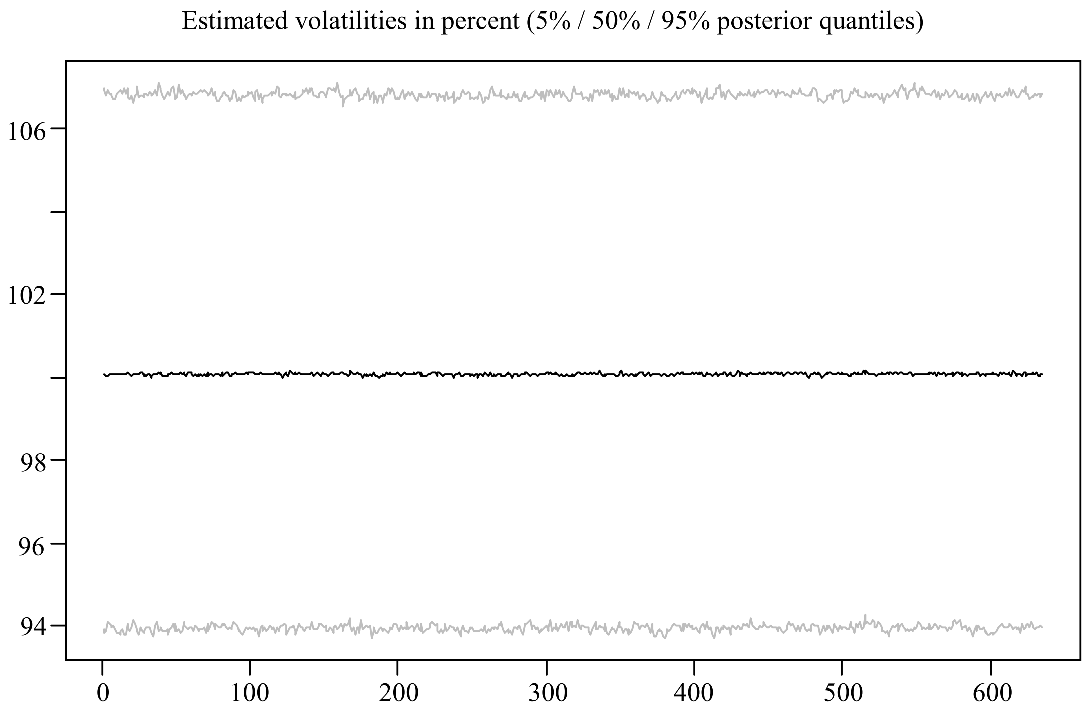

Figure 5. Estimated instantaneous volatilities

Figure 5 depicts the estimated volatilities in different quantiles. The range is between about 94% and 106% which is between standard deviation of 0.27 and 0.31. The stochastic volatility model appears to capture the high magnitude of the conditional volatility relative to the volatility estimated from the GARCH model. However, due to low persistent autoregressive process the variation appears to be insignificant and intermittent relative to the GARCH estimates. The dark line in the middle of the plot presents the median volatility estimation which is a 50% posterior quantile of the estimated volatility.

The estimated SDF model appears to capture predictability of excess returns and time variation of the risk-return tradeoff. The conditional volatility coincides with the Sharpe ratio conditional characteristics which appears to be countercyclical high in bad times and low in good times. The different state variables which define different bad times in the economy introduce different shocks to the cashflow of the investors and signal times when investors become more risk averse. The state variables in our framework define bad times as such times of borrowing constraints and consumption and labor income shocks and signal times of high risk aversion. For this reason, investors do not buy stocks in bad times despite being a good buying opportunity of low prices and higher expected returns.

5 CONCLUSION

The asset pricing models with consumption based stochastic discount factor failure to account for the conditional properties of returns in empirical data motivates a search for alternative models. The predictability evidence of excess returns indicate that the stochastic discount factor conditional volatility should account for the conditional properties of excess returns per unit risk. The SDF should be conditionally heteroskedastic to account for countercyclical variation and high volatility of the Sharpe ratio to explain predictability and high equity premium. The purpose of the thesis is to estimate the SDF from the multi-factor model framework with different state variables that define different bad times in the economy to assess the SDF conditional volatility accountability for the respective conditional properties.

The main results of the thesis are that the estimated SDF conditional volatility appears to capture substantial conditional variation of the Sharpe ratio relative to consumption based estimated SDF. The conditional volatility of the estimated SDF appears to be conditionally heteroskedastic with higher variability relative to the consumption growth based SDF. The estimated SDF conditional volatility appears to be countercyclical capturing significant business cycle patterns which explains predictability. Even though the estimated SDF captures the business cycle patterns, the conditional variation is smaller and fails in terms of the magnitude relative to empirically estimated conditional Sharpe ratio from the prominent studies. The results are consistent with the predictability evidence but differ in the magnitude of the estimates. The conditional volatility of the SDF also appears to be stationary and thus do not explain the long-run equity premium as the conditional volatility does not increase with the horizons.

The estimation of the SDF is based on the nonlinear least square (NLS) on orthogonal state variables. Other methods of parameter estimation such as GMM estimation can yield a better estimate of the SDF. Future research can attempt to apply different sort of variables such as the Fama and French factors in modeling the SDF, since some of the variables we apply in this thesis for example the dividend yield can be subject to structural change also with significant predictability effect in horizons longer than one month. The application of higher moments could also yield a better estimate of SDF since skewness and kurtosis risks appear to be priced. We could suggest future research to apply latent variables from the factor analysis since the factor model variables can still under proxy the marginal utility and misrepresent the real factors that can affect stock returns. But factor analysis lacks the economic theory to justify and define bad times in the economy. Conditional correlation is another angle for exploration and application but we do not think that conditional correlation can have significant improvement on the magnitude of the estimate since correlation ranges between one and minus one.

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