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FACTOR INVESTING USING RISK PARITY OPTIMIZATION

Finance
September 2018
Investor’s dilemma is: "How to earn the highest possible return with the lowest possible risk." Yet, if we understood better what is driving the returns and risks, our portfolios could become better performing and diversified. We would also potentially encounter less unpleasant surprises during economic downturns. This seemingly easy question has become a challenging one since investors have failed to diversify portfolios well enough especially during bad times.

Factors are currently a popular topic in the financial industry. Yet, majority of the literature focuses on the significance of certain factors and less on how to apply factors in practice. Risk parity optimization serves an attractive alternative for optimizing portfolio weights. The aim of this study is to analyze whether our attention should be directed from asset classes to factors and what benefits such drift could possibly entail. Therefore, the research questions are organized as follows:

1. How chosen factors perform during different periods?
2. What factors seems to be the most persistent out of the six chosen factors?
3. How optimized portfolios perform with respect to the control portfolios?
4. How the chosen portfolio optimization methods work in terms of returns and risks?
5. What is the risk distribution of each constructed portfolio?
6. What do factors reveal from the MSCI World index?

We focus on the practical part of portfolio management and aim at outperforming a global equity fund. We use index performance data from the Morgan Stanley Capital International (MSCI) and our broad sample period is 30th Nov 1998 – 31st May 2018. The study is conducted by using long only factor exposures and static weights. As we do not need constant rebalancing and also because currently there are existing low cost ETFs to all used indices, trading costs are not included. This study shows how an attractive risk return based portfolio is constructed using Value, Momentum, Size, Quality, Low Volatility and Dividend factors. We optimize six static portfolios using different risk parity optimized methods and compare their performance to two benchmark portfolios; Equal Weighted (EW) or often called 1/N along with Restricted Minimum Variance.

First contribution of this study is that by combing the factors together investors ensure that they are exposing portfolio to the best performing factors. In addition to that, this study verifies that risk optimized portfolios lead win the horse race with EW. Yet, Restricted Minimum Variance portfolio is able to achieve the most attractive risk return tradeoff and wins the competition, but Beta Risk Parity offers better solution from the diversification point of view. Unlike than MVR that chooses only two factors; BRP uses all the six of them. Furthermore, this study confirms that it is generally advisable to shift attention from an asset class-based allocation towards the risk-based allocation. This is due to the fact that the performance of each sample portfolio seems to beat the performance of MSCI World. Such portfolio offers a more favorable return and risk reward relation that should be the simple goal of each rational investor.

Keywords
factors, style investing, factor investing, smart beta, alternative beta, value, momentum, size, quality, volatility, dividend, risk parity, 1/N, minimum variance, portfolio optimization, optimal allocation
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1 INTRODUCTION

1.1 Objectives and research questions of this study

Have we learned anything from the last financial crisis? Recently we broke the “all time high” of the legendary Standard and Poor’s 500 index record. The problems do not arise when stock market keeps on braking records. Instead they emerge when the times are bad for investors. Investor’s dilemma is: “How to earn the highest possible return with the lowest possible risk.” Yet, if we better understood, what is driving the returns and risk, our portfolios could be better performing and diversified. We would also potentially encounter less unpleasant surprises during economic downturns.

Traditionally, asset management, the skill and the art of choosing appropriate investments and asset classes, has focused on managing mean return and variance since Markowitz (1952). The objective of such action is to pursue an optimal portfolio with respect to return and risk. Diversification of assets to various asset classes is another conventional way of managing return and risk. In this approach, the impact of bad times can be balanced by means of the low correlation between different asset classes. This approach can be also linked to Peter Swensen (2000) who proposed asset class diversification to solve the problem in the spirit of the ‘Yale-Harvard’– asset class based diversification approach. Last two paradigms are dominating the toolbox of an active asset manager even today. Pursuant to this conventional approach, the nuances each manager can bring are mostly limited to 1) the selection of investment instruments and 2) allocation decisions regarding the percentages of each asset class for each investor.

Even today, a portfolio is often labeled as “balanced” when it contains 60% equity and 40% bond investments. Its riskiness derives mainly from the equity markets to the extent that more than 90% of its return variability links to the equity market (Kahra, 2015). Such approach is insufficient for detection of the actual drivers of return and risk of each investors’ portfolios, since it focuses purely on the low correlation between stocks and bonds. By questioning this approach, the way in which assets are currently managed can be implicitly challenged. There is an evident need for more adequate tools for tackling the yet unsolved problem.
Andrew Ang (2014) addresses the challenge of return and risk from a novel perspective. According to him we rely too heavily on the mean- variance framework and should shift our focus to factors. Looking at the current trend in the asset management industry and the boom of factor investing, his thoughts are starting to have many proponents. He resumes the earlier research suggesting a paradigm shift away from the traditional asset class thinking.

He defines every asset as a unique set of factors, and each unique combination of factors will eventually define bad times for the asset owner. He goes further stating that asset management (allocation) is after all factor management. Ang grasps an old topic and offers factors as a modern solution for gaining the best possible return with the lowest possible risk. In essence, his approach searches answers from the drivers of returns instead of blindly staring the labels of asset classes. He uses analogy of food and states that just like eating right means that we should look through food labels what nutrients we achieve, similarly should we focus our attention to factors underneath various asset classes (Ang, 2014.)

Despite the popularity of the topic, less attention is given to application of factors in practice. This study aims at analyzing whether risk parity optimization serves as a viable solution to this challenge. This study also pursues to make a proposal for optimization of factor-based portfolio by using risk parity optimization. Hence, the objective is to shed light on the asset class problem from the point of factor investing, but also from the risk parity perspective. In this context, we mean by risk parity optimization optimizing the amount of risk in a portfolio. Hence, we do not aim at optimizing the utility function with respect to the portfolio like in the case of the Markowitz’ model (Markowitz, 1952). Since, both topics are interesting already by themselves; the combination of these approaches creates even more fascinating outset for this research.

According to Ang (2014) the return of factors is reward for their possible underperformance during bad times. Ang addresses the problem, but his research leaves open a relevant question: “How one should use the different factors?” He only addresses this topic in a limited sense, generally by urging investors to define their individual setting of bad times and then picking factors that earn reward. Straight
answers or tools for selection of the actual factors and/or optimal combination of factors cannot be found from Ang’s work. Hannu Kahra (2015) continues developing Ang’s method and offers factors combined with risk parity as a modern solution for portfolio construction in asset management. This study experiments this approach and applies some of Kahra’s insights in different context than he initially does.

Ang does great when job listing the factors he considers relevant. However, his listing does not rule out the possibility that some other factors could earn premium as well. Therefore, in this study the set of chosen factor follows the criteria of Ang, but contributes by adding some factors Ang does not cover. We add factors to Kahra’s study and we also add the risk parity optimization methods to Ang’s research. We gain wider understanding on the possibilities and limitations of the respective approaches (Ang, 2014; Kahra, 2015.)

The aim of this study is to analyze whether our attention should be directed from asset classes to factors and what possible benefits such drift could entail. Therefore, the research questions are organized as follows:

1. How chosen factors perform during different periods?
2. What factors seems to be the most persistent out of the six chosen factors?
3. How optimized portfolios perform with respect to the control portfolios?
4. How the chosen portfolio optimization methods work in terms of returns and risks?
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6. What do factors reveal from the MSCI World index?

1.2 Scope of this study

This study compiles nearly two decades of market turmoil and uses data from November 1998 to April 2018 in the empirical part of this study. The data is provided by Morgan Stanley Capital International (MSCI) and covers a broad sample from 30th Nov 1998 to 30th April 2018. This particular period is challenging in many ways since it entails the tech bubble in addition to the financial crisis. Since our data comes entirely from a single commercial source, the scope of the research is limited in terms
of reference data. Yet, there are some benefits involved in choosing MSCI: There are existing products available. Hence, this study can be easily replicated, if needed. These benefits outweigh the limitations.

The scope of this study is also limited in a number of other considerations. First, it does not delve into the debate regarding active vs. passive portfolio management approaches. By active in this context we mean that we differ from passive stock market index. As a starting point it is assumed that both of these approaches entail benefits and some shortcomings. In contrast, the validity of such debate is challenged, as in this study, we use passive indices with the active allocation decisions. Perhaps, factor investing could be accepted as an intermediate solution for the two quite opposite approaches.

Second, this study is limited to a particular set of factors. The aim is not to list all available factors, but we have instead chosen to select a limited number of factors based on specific pre-determined criteria introduced in Chapter 3.

Another limitation to our study is that we do not question the significance of our factors. It is outside the scope of this study, but we provide a solid knowledge base of earlier research in Chapter 3 that reader can review for further details. Also the decision to choose MSCI World index as a horse race counterpart can be justified since it is widely used in the industry. MSCI World index serves as a viable benchmark of the old world since it can be used as a proxy for a global equity portfolio. Hence, should the methods applied in this study beat the MSCI World, there is a reason to reconsider the traditional assets class-based paradigm for portfolio optimization.

1.3 Structure of the study

This study is organized as follows: Subsequent to this introductory Chapter 1 (defining the objectives, research questions and the scope of the study), Chapter 2 provides a brief literature review shedding light on the theoretical background and explaining how have we reached this point. In Chapter 3, we delve into the factor theory itself and introduce the factors used in this study along with their selection criteria. Thereafter, the Chapter 4 explains the two benchmark portfolios and the risk parity methods are
used in this work. In the following Chapter 5 we explain the data and methodology that is used. The most interesting part of this study is the empirical part presented in the Chapter 6. Finally, findings of this work are concluded in Chapter 7. (*For an educated reader who is only interested in enhancing the allocation process, Chapter 6 is the most useful. Previous Chapters are not absolutely necessary for the interpretation of the results.*)
2 THEORETICAL FOUNDATION

This Chapter links the relatively new paradigm of factor-based investing into the historical framework of the financial theory. We explain briefly how the theoretical part of portfolio management has evolved, and what has led to a paradigm change within it. In order to understand properly the recent popularity of this topic, it is worth looking back at the evolution of the financial theory itself.

Hannu Kahra (2015) illustrates briefly financial theory background in his report to Finnish pension funds. His method is based on the original work by Mark Rubinstein (2006), who divides the history of financial theory into three different eras.

1. Era of antique: before 1950s
2. Era of classics: from 1950’s until 1980s and
3. Era of modern theory: after 1980s

Before 1950s financial theory and practice were merely a collection of anecdotal tales related to investing and Harry Markowitz’s doctoral thesis constituted the base for a systematic financial theory. Although the concept of diversification as such is nothing new. Already on the 4th century Middle-Eastern Rabbi Isac bar Aha’s advised in Jewish religious texts to divide one’s assets so that: “A third in Land, A third in merchandise, A third at Hand (Cash)”. Later on, William Shakespeare in England wrote Antoni to say in the Merchant of Venice: “I thank my fortune for it, My Ventures are not in one bottom trusted, nor to one place; nor is my whole estate…”. Another proponent of diversification was Daniel Bernoulli who already in 1738 France stated that “it is advisable to divide goods which are exposed to some danger into several portions rather than risk them all together” (Bernoulli, 1954). Before any formal theory, investors in various countries applied diversification in practice.

However, Markowitz’s work represents the first mathematical formalization of the diversification of investments (Markowitz, 1952; Rubinstein 2002). This formalization is known as the Modern Portfolio Theory (MPT). MPT divides investing into three different elements: return, risk and correlation. Return and risk are connected, and in order to earn higher returns, greater risk must be accepted.
MPT gives a normative rule how to optimize the portfolio using mean-and-variance. According to Kahra (2015) mean-variance-principle is the oldest and best known method of portfolio optimization. The essence MPT is that diversification reduces the risk, but it does not entirely eliminate it. Therefore, a rational investor should pursue maximizing the portfolio return, while minimizing the variance of the portfolio return. Important aspect of the MPT is that it is not the security’s own risk that matters to an investor, but the contribution to the variance of the total portfolio risk that is important. Individual securities have unique correlations with each other, and by adding more than one security into a portfolio total risk of the portfolio reduces “... the whole is greater than the sum of its parts” (Bernstein, 2002.)

Although, this model offered a solution for portfolio optimization, it was yet very sensitive to the input values of its expected returns. However, as explained later in the Chapter 4, one of the applied optimization methods is based on Markowitz’s work. Despite, nearly 70 years has lapsed since of the formalization of his theory, it certainly has not lost its relevance.

Sharpe (1964), Lintner (1965) and Mossin (1966) adapted the model of Markowitz, and developed it suitable for investing. The era of classical period further continued with the theory of Capital Asset Pricing Model (CAPM). Now risk was divided into two components: the avoidable risk, known as un-systematic risk, and the unavoidable risk called the systematic risk. CAPM states that the systematic risk is caused by economic developments in the market or the common characteristics of the asset itself.

For example, bond investors are exposed to different risk than equity investors. Clearly this theory is too restrictive and not any longer used in the industry. However, it was one of its kind upon its inception and showed the way for factor theory later on (Koedijk, Slager & Stork, 2013.) The systematic risk is the only risk to be compensated since it is the only risk that cannot be diversified away. On the contrary, un-systematic risk or “security specific –risk” can be reduced using diversification. Thus, no compensation should be paid for carrying it.
CAPM implies time-series regression estimating beta coefficient. In the equation (2.1) \( \alpha \) is the unexplainable return of the model, \( R_t^i \) can be the return of a security in this application of the model, but it could also be an asset class or market portfolio on at time \( t \) depending on the purpose.

\[
R_t^i - R_t^f = \alpha_{it} + \beta_{im} (R_t^m + R_t^f) + \epsilon_t^i
\]

\[
t = 1, 2, \ldots, T, \forall i
\]

\( \beta_{im} \) (2.2) stands for beta of the security with respect to the market portfolio as shown in the formula.

\[
\beta_{im} = \frac{\text{cov}(R_i, R_m)}{\sigma^2(R_m)}
\]

\( R_t^m \) and \( R_t^f \) are the return of market portfolio and risk free security, and \( \epsilon_t^i \) the error term that cannot be explained by the model (2.1). In CAPM world \( \alpha = 0 \) as there is no other risk to be rewarded as the market risk. This is presented in function (2.3).

\[
E(R_t^i - R_t^f) = \beta_{im} \lambda_m, \forall i.
\]

As the function (2.1) is applicable in calculating the return of the market portfolio, so can the risk-premia be estimated by using the expectation as in the following equation (2.2) is done (Kahra 2015.) \( \lambda_m \) is the market risk premium, and \( E(R_t^m - R_t^f) \) is the expectation value of the subtraction of market return minus the risk-free return.

\[
\lambda_m = E(R_t^m - R_t^f)
\]

Assets expected return is in linear relation with its exposure to the market portfolio risk and the market portfolio is the only risk that is compensated in the CAPM world. Thus, the return of the portfolio depends on the market beta, the only factor that is
compensated is the market risk-premium. This risk is measured by beta and which for
the market is always one. Therefore beta is not visible in function (2.4)

Thus, if an individual security has more risk than the market portfolio its beta is more
than one, likewise if it is exposed to less risk than the market portfolio it has beta less
than one. If this is not the case, then a security is mispriced until rational investors
balance the prices and an equilibrium between the security prices is reached at least
this is assumed to happen.

Stephen Ross (1976) further explained systematic risk in his Arbitrage Pricing Theory
(APT), and created an alternative theory to mean-variance investing. Ang (2014)
notices that the word “arbitrage” indicates that just like the single market factor in
CAPM, factors of the APT cannot be arbitraged or diversified away. In short, Ross
argues that as an investor bases expected risk on systematic risk, and systematic risk
changes due to various economic influences, is it then not more practical to relate
returns directly to those various economic influences? Changes in the expectations can
be explained by different economic factors. Unlike in CAPM, where there is only risk
factor that explains security return is market portfolio, in Ross’ paper there are various
risk factors that sum up to the total risk premium of a security.

Pricing models that have more than one risk-factor, add other risk-factors \( F^A, F^B, \ldots \),
to the model. In addition to the market portfolio now various other factors explain the
returns. Later on in Chapter 3 we suggest six factors to be in the place of “F” the model
2.5.

\[
R_t^i - R_t^f = \alpha_i + \beta_{im} (R_t^m + R_t^f) + \beta_{iA} F_t^A + \beta_{iB} F_t^B + \ldots + \varepsilon_t^i
\]

\( t = 1, 2, \ldots, T, \forall i \)

This implies that in equilibrium, investors must be compensated by bearing various
risk-factors. In this general presentation of the APT \( R_t^i \) stands for the expected return
of an asset \( i \), \( R_t^f \) for risk-free interest rate, \( \alpha \) for unexplainable return of the model
(residual), the $\beta_{im}$ for market beta, $R^m_t$ for the market return, $\beta_{iA} F^A_t$ for the sensitivity $\beta_{iA}$ for a factor $F^A_t$ and $\varepsilon^i_t$ for the error term. The subscript $t$ denotes that this pricing model explains a certain period of time. Indeed market beta is a factor in this context, even though it is written differently than other factors in this model.

Kahra (2015) points out in equation (2.6) that investors earn higher returns by accepting risks that implicitly relate to various factors.

$$E \left( R^i_t - R^f_t \right) = \beta_{im} \lambda_{im} + \beta_{iA} \lambda_{iA} + \beta_{ib} \lambda_{ib} + \cdots, \forall i$$

In this version of the APT, $\lambda$ stands for risk-premiums that are proportional to their betas $\beta$. This model states that investors are compensated by higher returns when they accept risks that are implicitly connected to factors.

Ross had an insight that various factors drive the asset returns. However, it was not clear that what particular factors drive the expected returns. Chen, Roll, and Ross (1986) discovered that macroeconomic factors: unexpected inflation, industrial production (economic growth), twist in the yield curve, investor confidence have a clear influence on equity returns.

In both models, CAPM and APT, the residual or unexplained excess return alpha is:

$$\alpha_i = E \left( R^i_t - R^f_t \right) - (\beta_{im} \lambda_{im} + \beta_{iA} \lambda_{iA} + \beta_{ib} \lambda_{ib} + \cdots).$$

Banz was the first to found out the outperformance of small market value companies (Banz, 1981). Later Fama and French explain asset returns with 3-factor model. (Fama & French, 1993). In addition to the CAPM market-factor they use two factors SMB and HML. SMB refers to the differential of the returns of a portfolio of Small stocks Minus Big stocks, it has a long position in small stocks and a short position in big stocks.

The second factor HML stands for the differential of the return of a portfolio of High book to market Minus Low book to market stocks (value). It has a long position in the
High book to market value stocks, and a short position in the low book to market stocks (growth).

Summing it up their model captures the effect of size and value/growth such that besides the CAPM market return, returns are explained by the size-factor (SMB) and the value-factor (HML).

\[
E(R_t^i) = R_t^f + \beta_{i,\text{MKT}}(R_t^m - R_t^f) + \beta_{i,\text{SMB}}E(SMB) + \beta_{i,\text{HML}}E(HML)
\]

Later on Jagadeesh and Titman (1993) introduced the momentum factor and Mark Carhart (1997) complimented with his four factor model. Momentum is simply a strategy where winners are bought and losers are sold. Momentum strategy is based on the idea that the stocks that have gone up will continue to win whereas the stocks that have gone done will continue to lose. We use the notation applied by Ang (2014) where buying winners minus losers stands for the forth factor WML. So now the factor model is:

\[
E(R_t^i) = R_t^f + \beta_{i,\text{MKT}}(R_t^m - R_t^f) + \beta_{i,\text{SMB}}E(SMB) \beta_{i,\text{HML}}E(HML) + \beta_{i,\text{WML}}E(WML)
\]

Towards the end of nineties more money started to flow to equities. Even though asset management techniques evolved they could not solve the puzzle of risk. A good example of this statement is the collapse of the Long Term Capital Management (LTCM) that collapsed in 1998. Two years later tech bubble bursted and left deep wounds in the idea of efficient diversification (Koedjik et al., 2013.)

David Swensen who was in charge of the Yale university endowment fund suggest that institutions like universities should invest in categories that are not so liquid, because they can afford to endure longer periods of market turmoil. This way they can earn premium from markets that would otherwise be unattainable for small investors. He further suggested that the key for efficient diversification was to divide investments between different asset classes (Swensen, 2000.) This led many advisers to copy this
model of Yale in their own work. However, even this model could not provide shelter that it was meant to give, since it performed badly during the financial/credit crisis in 2007 (Koedjik et al., 2013).

A key insight from credit crisis is that investments seem not to be as unique and as has been though. However, this created a fruitful discussion about the factors, both known and yet unknown, and their impact on risk and reward. Investments were no longer seen in isolation. Instead, they have all kinds of connections or better-phrased factors. So, it became then important to identify what are such factors and understand how they drive otherwise unrelated investments. Nor were portfolios so diversified as thought. Norwegian pension fund launched an investigation about its funds active management policy. Great results had been destroyed by the underperformance of the 2008 and now the returns of the last 10 years are lost. (Koedjik et al., 2013.)

Ang, Goetsmann and Schaefer (2009) reported that the key for an efficient diversification is to diversify between factors and not just asset classes or between different securities. They highlighted that 70% of the return could be linked to systematic factors and their tilts. This observation is also at the core of our work. In essence this leads to much lower correlations than those between investment categories. On the next Chapter 4 we examine basics of factor theory and introduce factors that we use in this work.
3 FACTOR THEORY

This Chapter explains basics of the factor theory and links factors to the previous research. Needless to say this is not any novel investment strategy. Instead, for nearly 40 years have been factors available, but of course not all of them. In this Chapter we first define a factor. We then suggest a method for qualifying a factor. Subsequently, we relate factors to the existing debate between the ongoing active vs. passive debate and show how factor investing solves the problem. Thereafter, we present the six factors that we later use in the empirical part of this work. Finally, we end this Chapter by discussion main limitations of the factor investing approach.

3.1 Definition of a factor

There are several ways to define factors. It is important to define that we use the term factor to describe a factor that is significant and earns a premium over long periods. In addition, such factors are attractive candidates for long term factor investing. As Andrew Ang (2014) states: “assets can be viewed as bundle of factors that reflect deeper risks and rewards, just as any food is a bundle of nutrients.”

For instance, Koedjik et al., (2013) state that terms “risk factor”, “factor” and “factor premium” are often used as synonyms although they mean different things. Also terms “smart beta” and “alternative beta” are often used interchangeably. Asness, Ilmanen, Israel and Moskowitz (2015) for instance use the term “style”. Ang (2014) adds “investment factor” and “dynamic factor” to the list. Factors can also be divided into static and systematic factors. Stocks and bonds are examples of static factors whose risk premiums are captured by just buying the assets. Other factors like value and momentum require trading and long/short positions and they are thus dynamic factors (Ang, 2014). This is merely a simplification since investors can invest into value and momentum factors using long only positions, but the caveat is that then they expose themselves to the market risk factor as well and factors loose then some of their diversification benefits.
We find Roncalli’s (2014) definition clear since he states that factors are a subset of *smart beta*. Even though many smart beta indices have strong factor tilts only the ones that are designed to capture specific risk premium such as *value, size, volatility, quality* or *momentum* should carry the name factor. Nomenclature is not what matters. We are interested in finding out whether factors perform well capturing the sources of returns that are not present in the passive market cap weighted index. If they do capture how well does that happen?

### 3.2 Factors solve the active vs. passive debate.

Supporters of factor theory argue that factor diversification offers a more effective way to diversify than the traditional asset class-based diversification. Investors should not pay to fund managers if they cannot produce true benefit for investments. The popularity of passive indexing changes the box of tools for investors and makes it easier to compare actively managed funds to passively managed ones. Investor can easily access various index Exchange Traded Funds (ETFs). The question whether active investing is better approach than passive investing arouses especially during the bad market times. Although Roncalli (2014) points out that distinction between active vs. passive can be artificial. He further continues that after all every portfolio that deviates from the market portfolio is indeed active. In this study the return/risk discussion will be discussed more on the following Chapter and on the empirical part of this work.

*Figure 1. Factor investing solves the problem between active vs. passive investing*
Roncalli (2014) summarizes that if a fund’s performance can mostly be linked to its factor tilts and combinations of them then why do not try to replicate factors in a systematic and low-cost way? His opinion is that active managers should be compensated only for taking idiosyncratic risks. We agree partially with this view. It is worthwhile to pay for active managers even if they use factor ETFs if their method of optimizing portfolios is superior to our own. In the following Chapter 4 we illustrate that it is not enough to invest in factors since they suffer from cyclicality and in addition to that better risk return trade-offs as whole is achieved when portfolios are optimized.

Ilmanen and Kizer (2012) argue that there is room for active investing and merely it can lead to substantially better results than passive investing. Focus is shifted from dollar allocation to risk allocation. Later on in the empirical part of this study we apply this idea and differentiate dollar vs. risk allocations. They present that instead of just diversifying assets to different asset classes more diversified portfolios can be made using factors. They use the portfolio Sharpe ratios as the main performance characteristic. They argue that main reason why factors can and will be used are lack of familiarity, distrust in sustainability of factors, no consensus which factors should be used and finally the aversion of shorting and leveraging.

There are two kinds of explanations for why factor premiums exist in the first place. Risk-based explanations claim that factor premiums are result of willingly accepting higher risk than market cap weighted investing has. For example, Burton Malkiel (2014) claims that factor strategies outperform their benchmarks, but only occasionally. His view on the topic is that it is because such strategies are tilted on more risk and particularly size and value factor related risks.

Behavioral explanations find argument for factors from the behavioral patterns of investors. An example about behavioristic view is the work of Arnott, Beck and Kalesnik (2016) where they claim factors being successful an able to create alpha return only because they are known to investors. Because such known factors become popular this further increases the returns. This statement appears arguable. Many factors like value have been there for several decades yet they earn premium. Fama and French (1997) study value stock around the world and find that they deliver
superior return vs. growth stocks. Their research during 1975-1995 found 7.6% difference between value and growth stocks in 12 markets out of 13.

Some of the factors are usable in other asset classes as well. For example, Assness, Moskowitz and Pedersen (2013) show that value and momentum are an efficient combination together and that they deliver abnormal returns within several countries, across country equity indices, government bonds, currencies and commodities.

However, in this Chapter we focus on six different factors: value, momentum, size, quality, low volatility and dividend. We first introduce each one of the factors. We then present the returns they provide. Thereafter, we examine at the possible explanations behind each factor. And finally, we look what kind of empirical evidence supports our selection of factors.

3.3 Criteria for factor selection

We select a limited number of factors for this study. In a world of numerous factors it is essential to distinguish factors that are relevant from the ones that are not. What then makes a factor worth using? Ang, Goetzmann and Schaefer (2009) in the so called professors report to the Norwegian Ministry of Finance lists four criteria for determining which factors investors should choose. A factor should:

1. Be justified by academic research
2. Have exhibited significant premiums that are expected to persist in the future
3. Have return history available for bad times
4. Be implementable in liquid traded instruments

There is strong evidence that factor premia exists. Despite factors are used across the academia and among practitioners, there is no clear consensus on what factors should be used. However, there still seems to exist a consensus on what factors deliver the highest premiums. (Koedjik et al., 2013).
John Cochrane (2011) suggests that the current state of research to be a “factor zoo” and he summarizes the key questions of research as follows:

1. Which factors are independent?
2. Which factors are important or are some of them more relevant?
3. Why do factors move prices?

There are more than 300 factors according to Harvey, Liu and Zhu (2016) and more than 600 known factors according to Levi and Welch (2014). We need to find the ones that are actually usable and within the reach of an average investor. We shall also have a look at the possible explanations to particular factors, both risk and behavioral based ones.

Although some factors are applicable across different asset classes, we do not use them in this work in other asset classes than equities (Asness, Moskowitz & Pedersen, 2013). Assumed that factors really work, then we should direct our focus towards efficient diversifying from the typical asset-based way of diversification. However, if the results are promising, it raises an interesting question for the future research: Are these factors usable in other asset classes as well? Ang (2014; 456-457) brings up another concern that factors can come and disappear. He refers to Banz (1981) that found the small stocks factor premia. When this anomaly was found it suddenly disappeared. Arnott et al. (2016) mention the same concern.

3.4 Chosen factors

We choose six risk factors to be used: size, value, momentum, quality, low volatility, quality and dividend. First, all of these fulfill the four criteria presented by Ang in the previous Chapter. Second, all six of them can easily be accessed through existing products.
3.4.1 Size

Size factor captures the excess returns of smaller firms (Bender, Briand, Melas & Subramanian, 2013). Size factor is one of the Fama and French (1992) FF-models factors discussed in the Chapter two. Small-cap stocks have outperformed the Large-cap stocks and this factor refers going long in small stocks and short selling the large cap stocks. Although Fama and French’s study is probably the most well-known one, but Banz had presented similar results (Banz, 1981). Over the period of 1927-2010 the size premium has averaged 2.8% per year return in addition to stock market return (Ilmanen & Kizer, 2012). In this work we use Size to describe this factor. There are explanations why this factor exists. Fama and French (1992, 1993) present that small companies deliver high return because they have more systematic risk with them. One possible explanation to why this factor exist is that institutions cannot invest to small companies because of poor liquidity and low trading volume.

3.4.2 Value

Value factor aims to capture stocks that are cheap. In this context means High book value Minus Low book value. Bender et al. (2013) summarizes it by stating that value factor captures the positive link between stocks that have low prices relative to their fundamental value and returns in excess of the capitalization-weighted benchmark. If we speak about the value premium in its traditional sense it is a combination of various different criteria that Benjamin Graham used in his 1931 Book (Graham, 1934). It was later formalized by De Bondt and Thaler (1987).

This factor has performed very well both inside and outside the U.S. and has averaged a 3.9% return (Ilmanen & Kizer, 2015). It is still debatable why value factor exists. Fama and French (1993,1996) show that value assets include default risk that gets compensated by higher returns. Behavioristic explanation for value is given by Lakonsihok, Schleifer and Vischny (1994) who say that value factor exists because investors have the tendency to overreact to current news and extrapolate too far the past performance. Also they claim that the higher risk explanation is not justified.
3.4.3 Momentum

The third factor we include is momentum. Momentum factor captures the return that is based on buying stocks that have performed relatively well compared to the ones that relatively have not. Capturing this factor can be done using various periods of time e.g. one year. First documented by De Bondt and Thaler (1985) and later perhaps better known by Jegadeesh and Titman (1993) and later Carhart (1997), this factor has earned 7.7% on average in U.S. from 1927-2010 (Ilmanen & Kizer, 2015).

It is very close to short and long term reversal. Also in them one goes long in stocks that have performed relatively well while simultaneously shorting the stocks that have underperformed. Similarly, like in value investing the discussion circles around two possible explanations of why this premium exists. Risk-based explanations claim like with value factor that momentum factor return compensates from accepting higher risk. For example, Daniel and Moskowitz (2015) show how momentum strategies have crashed severely during poor market conditions. Behavioristic explanations, on the other hand, suggest the underreacting of investors to be the reason for this factor to exist (Barberis, Schleifer & Vischny 1998).

3.4.4 Quality

The fourth factor we apply is quality. Quality factor categorizes such that it captures companies that have good profitability and stability. It is able to capture those companies that are “high quality” vs. the market (Bender et al., 2013). Evidence shows that profitable firms earn higher returns even when higher valuation is considered (Novy & Marx, 2012). Also Asness et al. (2013) work document high risk-adjusted returns that can be captured using quality factor. Quality factor distinguishes companies that are profitable, high yielding and low in debt. Now what is a good quality is often subjective and can be questioned. For instance, Bender et al. (2013) list ROE, earnings stability, dividend growth stability, strength of balance sheet, financial leverage, accounting policies, strength of management, accruals, cash flows as common sources for capturing quality factor. Thus, it is not a surprise that quality factor gets support from active managers (Bender et al., 2013). Cambell and Vouleteenaho (2009) provide a fundamental evidence that both quality and value factor
have common sources of systematic risks. This is to say that if two stocks have
common accounting characteristics their return is driven by the same risk sources of
their fundamentals. According to Assness et al. (2013) quality factor links to no known
risk. Similarly, there is no consensus on the behavioristic explanations of this factor.

3.4.5 Volatility

The fifth factor we use is low volatility factor. Stocks that have low volatility earn
higher returns. From now on we use volatility to describe this factor. Volatility factor
dates back Black et al. (1972). It is very closely related to “Betting against the Beta
factor” (BAB) founded by Frazzini and Pedersen (2013). However, it is different from
BAB factor in a way that it considers a stocks volatility separately and not related to
the market volatility. According to Blitz et al. (2013) there are various explanations
for this factor, but short-selling constraints providing the strongest evidence.
Consequently, investors who do not have such constraints can earn this premium. Blitz
(2017) contradicts this view and states that low volatility stocks are the most
significant explainer of the hedge fund aggregate returns, but surprisingly they tend to
bet against the low volatility stocks. This finding is interesting and removes the
concern of low volatility factor being “overcrowded” and indeed presents a totally new
factor (high volatility) A behavioristic explanation for volatility factor is that most
institutional investors tend to underweight such low risk assets (Ang,2014; 345,.) Also
the fact that many institutional managers follow closely market cap indexes may itself
lead to this factor to exist (Baker et al., 2011).

3.4.6 Dividend

The sixth factor we use is high dividend yield factor. This factor uses excess return of
high dividend stocks with respect of low dividend stocks. Fama and French (1988)
state that dividend yields have the power to forecast future stock returns. They explain
this by high autocorrelation and with the growth of the variance of the regression
residuals. Shocks to expected returns are associated with the opposite shocks to current
prices. In addition, Zhou and Roland (2006) build a study that examines dividends and
earnings growth on a company specific level and find that higher dividend payout is
related to high future earnings’ growth. Ang and Bekaert (2007) contradict this and
find out that in the long-term dividend yield is not statistically significant return predictor.

### 3.5 Discussion

The key idea of factor investing is to tilt portfolios towards factors and this eventually leads to better performance and to the return that is not explained with the market factor. Factor investing is not a free lunch. Bender et al. (2013) remind that although factor indices have exhibited strong risk-adjusted excess returns during short periods of time they have also experienced major cyclicality. This means that if one were to invest in factors the ride can be volatile at times. When most of the people have relatively short investment periods this could partly explain why factors still earn significant excess returns despite the fact that they have been known for several years by academics and practitioners. Malkiel (2015) provides a simple explanation that by tilting portfolios towards factors higher returns are achievable, but this is due to the riskiness and not the superiority of factors.

Further Ilmanen and Kizer (2012) add that there is support for the idea that the various factor premiums are result for bearing some kind of systematic risk, capturing market inefficiencies or both. They search answers for the prevalence of factor premiums and suggest that 1) lack of familiarity 2) distrust in stability of premiums 3) no consensus on the factors that should be included and 4) unwillingness to shorting and leveraging are the reasons factor premiums still exists. Blitz (2017) argues that the concern that factors would rapidly be arbitraged away is vain. Cazalet and Roncalli (2014) warn that the explanatory power of risk factors other than the market risk factor has declined over the last few years, because market beta has been back since 2003.

Since chasing a factor premium is always an active move away from the market index it involves certain constructing risks. Another critical aspect to think about is the fact that most of the academic factors are not equal to their commercial peers meaning products that market to capture some kind of factor premia. To give an example, Blitz (2016) that at least in the case of MSCI High Dividend index there is a huge exposure (83%) low volatility stocks and that the outperformance with respect to the replicating portfolio is statistically insignificant. Thus, some of the factors are overlapping each
other and the distinction is not that straightforward. Bender et al. (2013) list estimate methods like principal component analysis, factor mimicking portfolios, cross-sectional regressions, panel regression, Bayesian models, latent factor models etc. It is clear that such variety of different techniques can affect to the profitability of factor investment strategies.

Some have suggested that factor investing is a result of data mining. However, if you look close enough you find a wide array of evidence supported by academic literature. Harvey et al. (2015) suggest that the hurdle for significance should be greater than three instead of roughly two and go further stating that most of the claimed research findings in finance are likely to be false.
4 RISK PARITY AND BENCHMARK PORTFOLIOS

The original concept of risk parity has evolved from 1990s Bridgewater research to the modern day and what was once optimizing the portfolio inversely to its volatility is much more nowadays. In this Chapter we add two benchmark portfolios and six different risk parity optimization methods to the horse-race contest.

Diversification is still important, but not as we know it. According to Asness, Ilmanen, Israel and Moskowitz (2015) most portfolios seem to have their risk concentrations around the equity risk. For example, a 60/40 stock/bond portfolio has its return correlated 0.99 to a portfolio that has 100% equity. This problem exists both among the institutional investors and mutual funds. They further state that the focus should be shifted towards factors instead of asset classes. As discussed in the previous Chapter, each factor has pro-and counter arguments for its usage. Since we cannot be certain about the optimal factor portfolio structure beforehand, we now introduce systematic methods that all lead us to different kinds of allocations later on in Chapter 5. In other words, the concept of risk parity entails calculating portfolio weights according to some specific objective function like for example inverse volatility. Various optimization methods help investors to decide appropriate weights for each factor in their portfolio. Risk parity offers a solution for those who are not happy with the traditional 60% equity 40% bond portfolio.

One way to find returns that are uncorrelated is to focus on seeking the alpha. But finding a true alpha seems challenging even for the successful investors like Warren Buffet. Frazzini et al (2013) study that when his returns are examined from a factor perspective most of the good performance comes down to exposures of BAB and Quality-Minus-Junk factors and his alpha return becomes eventually becomes insignificant.

Since the modern portfolio theory and ‘Yale-Harvard’-model have failed to offer enough diversification the focus needs to be directed to factors and risk budgeting. As we saw in Chapter 2 both ways have resulted portfolios that are not efficiently diversified. After all more action should lead to higher Sharpe ratios, otherwise it is not justified.
Risk-parity methods have the idea that expected returns are hard or even impossible to estimate in the short-term, but risk measured as a variance-covariance world is fairly easy to estimate. Thus, by using risk parity portfolios our goal is to further diversify the risk of our portfolio by optimizing the factor weights in our portfolio. We compare the risk parity-optimized portfolios to equal weight and minimum variance portfolio.

Ilmanen and Assness (2015) state that if portfolios are let to be dominated by market fluctuations this results a “roller-coaster” kind of ride and performance. Hsu (2006) complements this stating that portfolios should be constructed from weights that do not depend on prices. This leads to better performance than the traditional cap-weighted portfolios. This leads us to choose risk parity optimization in its various forms as our tool to create portfolios.

We choose also two portfolio optimization methods that do not fall under the name of risk parity. We compare the performance of equally weighted portfolio and minimum variance portfolio to six other methods and they serve as benchmark portfolios towards our six optimized portfolios. Hereafter we briefly introduce each calculation method and on the Chapter 5 we apply them into practice starting from our benchmark portfolios first.

4.1 Equal weight (1/N)

\[
W_i^{EW} = \frac{1}{N}
\]

First one is the equally weighted portfolio (EW). It’s also known as a naïve diversification method since it uses \textit{ad hoc} measure that has no economic intuition behind it nor does it rely on any historical information. In an EW portfolio each asset has an equal weight. Despite what kind of securities one has in a portfolio its weight is counted using 1/N method. Equal weighted portfolio is interesting benchmark first because its calculation is very simple: each asset of the portfolio gets equal weight. Secondly, because its calculation needs no estimation of expected returns. Thirdly, because it has low turnover. Fourth, DeMiguel, Garlappi and Uppal, (2009) suggest its usage as a first obvious benchmark, because the benefit diversification is often offset
by the estimation error using the mean variance methods. Although, simplistic this method needs not to be overlooked. Benartzi and Thaler (2011) state that even Harry Markowitz is using the same method saying that he prefers minimizing his regret and thus divides his wealth half in equity and the other half in bonds. If we are being careful we notice that this is the very “old method of Rabbi” mentioned in Chapter 2.

4.2 Minimum variance

\[
W_{MV} = \frac{\Sigma^{-1} 1}{1^T \Sigma^{-1} 1}
\]

Harry Markowitz (1952) was the first to formalize Minimum Variance portfolio. Minimum variance portfolio is a special case of mean variance optimized portfolio. It assumes all the means to be same and does not estimate them. In minimum variance portfolio weights are calculated so that weights are assigned to each asset according to each asset’s individual volatility and this leads to minimal variance of the total portfolio. The method we use here is based on the work of Kempf and Memmel (2003). Where \(W_{MV}\) stands for the weights of the minimum variance portfolio, 1 is a column vector of ones and \(\Sigma\) is the \(N \times N\) matrix including variance and the covariance. They present a one-step method to estimate weights and this simplified method is therefore preferred over Markowitz’s.

4.3 Inverse volatility

\[
W_i^{IV} = \frac{1/\sigma_i}{\sum_n 1/\sigma_i}
\]

The second method used is inverse volatility (IV) it is very simple method to use just like EW. Each stock has portfolio weight in accordance to its volatility. The smaller the volatility the greater the weight. Even though this method is simple it has the problem that it does not take into account correlations among the securities. The logic behind it is such that it gives penalty to the securities that have high volatility. (Kahra, 2015.)
4.4 Equal risk contribution

(4.4) \[ W_i^{\text{ERC}} = \arg \min \sum_{i=1}^{n} \sum_{i=j}^{n} (W_i \text{cov}(r_i r_p) - w_i \text{cov}(r_i r_p))^2 \]

The third method used is equal risk contribution (ERC). The idea of ERC is to balance the risk of each component to be equal in overall portfolio. It forces the same Marginal Contribution to Risk (MCRT) to each component of the portfolio. (Maillard, Roncalli & Teiletche 2010.)

4.5 Alpha risk parity

(4.5) \[ W_i^{\text{ARP}} = \arg \min \sum_{i=1}^{n} \sum_{i=j}^{n} \left( \frac{W_i \text{cov}(r_i r_p)}{\alpha_i} - \frac{W_i \text{cov}(r_i r_p)}{\alpha_j} \right)^2 \]

Another application of the ERC-method is alpha risk parity (ARP) it uses alpha as the main factor to determine how the total risk is allocated. Looking carefully the only thing that changes is the denominator that gets alpha to its place. In this particular extension, we are allocating risk-budget in proportion to alpha. However, alpha could be any signal that constructs portfolio (Kahra, 2015.)

4.6 Beta risk parity (systematic risk parity)

(4.6) \[ W_i^{\text{SRP}} = \arg \min \sum_{i=1}^{n} \sum_{i=j}^{n} \left( \frac{W_i \text{cov}(r_i r_p)}{\beta_i} - \frac{W_i \text{cov}(r_i r_p)}{\beta_j} \right)^2 \]

Kahra (2015) continues his own application of ERC-method called systematic risk parity, and allocates the risk proportionate to the systematic risk loadings of each individual asset. In this method Kahra substitutes alpha to the systematic equity market risk and it serves as restrictive budget to the weights. This method can be also called beta risk parity as well since beta in this context implies the systematic equity market risk. We later refer to this optimization method as BRP (Kahra, 2015.)
4.7 Maximum Diversification

\[
W_i^{MD} = \arg \max_w \frac{\sum_n w_i \sigma_i}{\sqrt{w^* \Sigma_w}}
\]

Another method we use is the maximum diversification MD. Choueifaty and Coignard (2008) introduce this method although they refer to it as “the Most diversified portfolio”. Its function is to allocate lots of weight to assets that maximize the difference of average volatility of the portfolio minus the total volatility of the portfolio. It favors assets that have a low correlation with each other. Their results show that higher Sharpe ratios are achievable than the market cap-weighted indices provide. They find also that it is able to beat both naïve (1/N) and minimum variance portfolios. That sets and interesting challenge to our empirical study in Chapter six.

4.8 Diversified risk parity

\[
W_i^{DRP} = \arg \max_w \exp(-\sum p_i \ln p_i)
\]

The final method we use is the diversified risk parity DRP. It focuses on the \textit{ex-ante} risk characteristics especially to the amount of uncorrelated bets. For a portfolio to be well-diversified its overall risk should therefore be evenly distributed across these principal portfolios. (Lohre, Opfer & Orzág, 2013.)

4.9 Some limitations of risk parity and benchmark portfolios

Risk parity offers attractive, but heuristic approach to portfolio diversification. However, there are few questions to be considered before rushing into conclusions. First, although we use in our empirical work short sell and leverage restrictions meaning that the weights need to be constantly between the 0 and 1, that is not always the case.

Some of the empirical works that suggest the use of risk parity portfolios are done using both shorting and leveraging. In practical sense this increases the costs related to risk parity investing. Secondly, in this work we speak about risk parity in the context
of all equity factor portfolios, but instead some of the work that supports risk parity theories is done using different asset classes. In order to generalize risk parity methods, more evidence is needed across various asset classes.

Lee (2011) points out that so far no theory predicts *ex-ante* that any risk parity portfolios are more efficient than other portfolios. He finds that most studies eventually compare the performance of risk based portfolios *ex post* even though their objective function *ex ante* was to minimize risk somehow. This very aspect in his view is inconsistent. He further continues that portfolios that are constructed using risk-based optimization methods do not deserve to outplace the traditional mean-variance portfolio setting. He justifies this by saying that until each risk parity method justifies its objective beforehand mean variance methods should remain the base metrics for portfolio performance analysis. We agree on this especially considering how much chances there are to err in estimating the weights from the returns (except the EW). However, we do not share Lee’s (2011) view on that point, as every active allocation that differs from the market portfolio, would need an opposite position and that this would lead otherwise underperforming of either holder of the position. Investors may prefer other assets that have very little correlation with their income and that yield when the “bad times” hit. Ang (2014;198) uses an example of insolvency lawyer and a banker. When the economy booms the banker does well, and when the economy busts, the lawyer does well. Thus, they prefer different portfolios since they are vulnerable to different economical risks. However, we continue that this is an optimal allocation for both and not something that should be avoided.

Risk parity-optimized portfolios are formed by calculating weights from the previous returns and then using a pre-determined objective function resulting weights. This is to say that they also do predict the future returns and risk using this method and that is problematic since we have only one existing history to analyze. Another concern Lee brings up is that risk parity techniques are heuristic and lack sound economic link so far (Lee, 2011.) Instead of basing the idea of diversification to the volatility one might as well consider the correlations of the portfolios. Since we now have various techniques to calculate portfolio weights next we apply them and see what kinds of results each one them provides, but before that we name our data and define our methodology.
5 DATA AND METHODOLOGY

5.1 Data

For the empirical research we use MSCI equity indices to data for the six factors as plus world equity index performance. We use “end of day” data search. The motive for using MSCI is that it is widely known in the industry and lots of asset managers use MSCI indices as benchmarks. We use price data that is quoted in euro (EUR). All of our indices are tilt indices with high capacity. We choose to use high capacity over high exposure because investability is critical and narrow factor indexes may not have sufficient liquidity and capacity due to their concentrated nature.

One reason for using MSCI is that there are ETFs for each index and therefore these indices are readily available for the investor and on the other hand they are cost efficient alternatives. This implies also that the methods of this study are also available for practical purposes. It is important to notice that the data MSCI uses in factor tilt indexes is back-tested, as there are often differences between back-tested and actual results. This covers all the data prior to December 2014. This means that the performance is a description on how the index might have performed over that time-period had the index existed. Blitz (2016) notices that many factor indices exhibit similar performance with the factor they mimic. However, it seems that they are not fully able to capture the potential that can be offered by factor premiums.

MSCI ACWI World (World) stands for our benchmark index. It is a World equity index that is based on the market cap of the World’s biggest companies and includes large and mid-cap securities across 23 Developed Markets and 24 Emerging markets. (MSCI, 2018)

MSCI ACWI Dividend Tilt Index (Dividend) is based on the MSCI ACWI World Index, its parent index. This index is designed to reflect the performance of equities in the parent index with high investment capacity. The index is created by including all the dividend-paying constituents in the parent Index and tilting the market capitalization weights of these securities based on their Dividend Yield Score. (MSCI, 2018)
MSCI ACWI Momentum (Momentum) aims to reflect the performance of a Momentum strategy with relatively high investment capacity. The index is created by tilting the market capitalization weights of all the constituents in the parent index based on the Momentum scores and then re-weighting them. (MSCI, 2018)

MSCI ACWI Quality Tilt Index (Quality) aims to reflect the performance of a Quality strategy with relatively high investment capacity. The index is created by tilting the market capitalization weights of all the constituents in the parent index based on the Quality scores and then re-weighting them. (MSCI, 2018)

MSCI ACWI Size Tilt Index (Size) aims to reflect the performance of a low size strategy with relatively high investment capacity. The index is created by including all the constituents in the parent index and weighting the constituents using the square root of their market capitalization weight. (MSCI, 2018)

MSCI ACWI Value (Value) aims to reflect the performance of Value strategy with relatively high investment capacity. The is created by tilting the market capitalization weights of all the constituents in the parent index based on the stocks that have low prices relative to their fundamental value. The value investment style characteristics for index construction are defined using three variables: book value to price, 12-month forward earnings to price and dividend yield. (MSCI, 2018)

MSCI ACWI Volatility Tilt Index (Low Volatility) aims to reflect the performance of a low volatility strategy with relatively high investment capacity. The index is created by tilting the market capitalization weights of all the constituents in the parent index based on the inverse of security price variance and then re-weighting them. (MSCI, 2018)

5.2 Methodology

We use the raw end-of-day price data of the seven mentioned equity indices. We calculate logarithmic returns and use them in our research. First, we find out how factors and world index have performed. We examine their performance as individual factors and later on as constructed portfolios. The eight methods we introduce in
Chapter define the methods that we apply here. Thus, we then have six risk parity portfolios and two benchmark portfolios to controlling our results. We focus on analyzing their performance together with their risks. Throughout the study MSCI World index serves as an example of the old world and the time before factors and risk parity methods.

We decide our sample period to be from January 1999 to December 2007. During that sample period investors faced the tech bubble. However, the sample period end before the equally famous financial crisis. Thus, both sample and out of sample period face a major drawdown period with respect of the returns.

First, we calculate all of the weights that we are using in our out of sample study. We then form six different portfolios using the risk parity methods presented in Chapter 4 plus we include minimum variance and 1/N portfolio and use them as a comparison. In minimum variance portfolio we limit the weights between 0 and 1. This means that we do not allow short selling nor leveraging. Since we have these restrictions our results are later on more reliable for majority of investors.

Finally, we have a brief case study at the MSCI World index and its factor loadings. The Sharpe style analysis that we use serves as a valuable tool for graphical demonstration of the factor loadings and their style drifts. The reason we compare constantly our results to the MSCI World Index is its wide usage in the financial industry. Sharpe ratio is used as a main measure of performance since it combines the risk and return in a one figure.

Figure 2. Our investment process summarized
According to Ilmanen and Kizer (2012) factor-based perspective requires that focus shifts from dollar allocations to risk allocations. Our hypothesis is that we are able to construct better portfolios measured by Sharpe ratio than the MSCI World index. We shift our focus to the risk using dollar vs. risk budgets. Then we measure have they outperformed 1/N portfolio and Minimum Variance portfolios. We show which technique leads to the best performing portfolio. Sharpe ratio is used as a main measure of performance since it combines the risk and return in a one figure.

We measure the performance of all the portfolios in long-only strategies. Long/short means buying long the stocks that belong to each style and short selling the ones that have not the style criteria at that particular point of time. This matters since some of the academic factor studies suggest using the long/short portfolios. Cazalet and Roncalli (2014) state that long vs. long/short factors behave differently meaning that not always are the long only factors riskier. We consider that it is not necessary to use long/short strategy based on the paper of Israel and Moskowitz (2012). Also Hui et al. (2014) support our argument adding that long-only strategies are preferable since costs and decay may set off the benefits of long/short strategies. We agree with both and prefer the long-only version because of its practicality. This method has its drawbacks since it not only exposes to the different factors, but also to the market factor. Thus we cannot nullify the impact of market factor, the CAPM beta totally.
6 EMPIRICAL STUDY

6.1 Factor dominance of nearly two decades

Below we present broad sample returns from December 31, 1998 to May 31 2018. All of the factor indices out-perform the MSCI World index. Relatively small companies that get capture by size factor deliver the highest returns and out of all six factors the value and quality have the equally lowest. Based on the whole sample dividend factor provides the best annual Sharpe ratio (risk/return tradeoff). Surprisingly this is the factor we have least the evidence from and even state it being insignificant return predictor (Ang & Bekaert, 2007). Every factor on its own beats MSCI World when measured by return, risk, Sharpe-ratio and Maximum drawdown. This is in accordance with the previous studies; factors do earn premium.

Even though a period of nearly 20 years is relatively short, it still raises questions how come factors have had less severe drawdowns than the World index. This suggest that the explanation that factors earn premium because their exposure risks during bad times is not complete. The broad sample has two major drawdown periods in it, yet not a single factor has suffered equally much as the MSCI World index. Another observation we make is that factors do correlate a lot with the MSCI World and this is due to the fact use long only factors. Market factor becomes dominant and it is present in all six factors. However, it still pays off to use style factors instead of market factor.

Table 1. Factors performance Nov 30th 1998 - May 31st 2018

<table>
<thead>
<tr>
<th></th>
<th>World</th>
<th>Dividend</th>
<th>Momentum</th>
<th>Quality</th>
<th>Size</th>
<th>Value</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return % p.a.</td>
<td>0.040</td>
<td>0.052</td>
<td>0.052</td>
<td>0.047</td>
<td>0.054</td>
<td>0.047</td>
<td>0.049</td>
</tr>
<tr>
<td>Volatility % p.a.</td>
<td>0.141</td>
<td>0.123</td>
<td>0.140</td>
<td>0.137</td>
<td>0.141</td>
<td>0.146</td>
<td>0.122</td>
</tr>
<tr>
<td>Sharpe p.a.</td>
<td>0.284</td>
<td>0.423</td>
<td>0.371</td>
<td>0.344</td>
<td>0.381</td>
<td>0.320</td>
<td>0.400</td>
</tr>
<tr>
<td>Maximum Annual Drawdown</td>
<td>-0.609</td>
<td>-0.470</td>
<td>-0.586</td>
<td>-0.574</td>
<td>-0.524</td>
<td>-0.561</td>
<td>-0.500</td>
</tr>
</tbody>
</table>
6.2 Calculating portfolio weights in sample

We now calculate weights for each of the eight portfolios. Rebalancing is not done in our study, so the weights remain unchanged during the out-of-sample period. Already now it is obvious that the content of each portfolio differs from other. It is also noticeable that in MVR portfolio the factor tilts are only in dividend and volatility factors. Another portfolio that uses only few factors is MD. Leverage is not allowed thus we have no negative weights in any of the portfolios. It is interesting that even though size and value factors deliver the best performing returns during our sample period, none of them is included in MVR. Hence, from a behavioristic point of view it would demand plenty of courage from a factor portfolio manager to leave four factors out of six out, especially when the remaining two are not among the best performing ones and taking into account the weak forecasting power of dividend factor.
Table 2. Portfolio weights calculated from sample period Jan 1999 - Dec 2007

<table>
<thead>
<tr>
<th></th>
<th>ARP</th>
<th>BRP</th>
<th>DRP</th>
<th>ERC</th>
<th>IV</th>
<th>MD</th>
<th>EW</th>
<th>MVR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend</td>
<td>19,92</td>
<td>21,57</td>
<td>1,08</td>
<td>18,27</td>
<td>18,28</td>
<td>22,93</td>
<td>16,66</td>
<td>37,18</td>
</tr>
<tr>
<td>Momentum</td>
<td>0,62</td>
<td>14,48</td>
<td>6,79</td>
<td>15,93</td>
<td>15,55</td>
<td>44,67</td>
<td>16,66</td>
<td>0</td>
</tr>
<tr>
<td>Quality</td>
<td>39,26</td>
<td>13,76</td>
<td>11,8</td>
<td>15,69</td>
<td>15,77</td>
<td>0</td>
<td>16,66</td>
<td>0</td>
</tr>
<tr>
<td>Size</td>
<td>21,95</td>
<td>14,66</td>
<td>18,51</td>
<td>16,02</td>
<td>16,12</td>
<td>0</td>
<td>16,66</td>
<td>0</td>
</tr>
<tr>
<td>Value</td>
<td>18,24</td>
<td>14,27</td>
<td>25,25</td>
<td>15,9</td>
<td>15,91</td>
<td>32,39</td>
<td>16,66</td>
<td>0</td>
</tr>
<tr>
<td>Volatility</td>
<td>0</td>
<td>21,23</td>
<td>36,55</td>
<td>18,16</td>
<td>18,35</td>
<td>0</td>
<td>16,66</td>
<td>62,82</td>
</tr>
</tbody>
</table>

Figure 3. Factor returns Jan 1999 - Dec 2007

6.3 Dollar budgets vs. risk budgets

Since we know have the weights that we are going to use out of our sample, we want to know what is their relation to risk. We present risk budgets using the Marginal Contribution to Risk (MCTR) method. This technique gives us an insight of our portfolio risk contribution and how it is distributed. Dollar weight stands for the
amount of currency that is allocated to each factor in each optimized portfolio whereas risk weight stands for amount of risk that truly is allocated. We notice that in every optimized portfolio these weights differ except the restricted MVR portfolio. Two clear examples are ERC and EW portfolios. By definition ERC allocates the risk weights equally among the factors measured by MCTR whereas EW divides dollar weights evenly despite of the MCTR. This is also the benchmark portfolio that is able to beat all of our optimized portfolios. However this portfolio is heavily concentrated to only two different factors. Instead BRP portfolio is more evenly distributed across the factors.
6.4 Out of sample results

We measure how well our portfolios handle the out of sample period. We do not change the weights during our sample period so our weights remain static and this naturally keeps the trading cost in minimum since no rebalancing is needed. All the portfolios are formed in the beginning of the sample and then hold steadily until the end of the sample. We are not able to beat the MVR portfolio.

Best performing risk parity portfolio is BRP-portfolio that is done by using Kahra’s method. Surprisingly, none of the portfolios is able to beat the performance nor the risk/return trade of the MVR portfolio. We conclude that MSCI world index can be...
beaten and gets easily outperformed by all of our portfolios performance. It also lose the comparison when portfolio risk/return tradeoff is taken account. Albeit the trading costs are not taken into account in our results they could not nullify our results since we do not trade our portfolio except the initial positioning. This particular period has a major drawdown period and as it shows from the graph it took from four to six years to recover from the credit crisis losses.

When we compare just single factor returns it shows that momentum delivers highest returns as well as the highest return/risk ratios. Even though we beat the performance of the MSCI World index we are not able to achieve such returns that momentum factor itself. Nor do we achieve as high return/risk ratios.

Table 4. Factor performance out of sample Dec 2007 - Apr 2018

<table>
<thead>
<tr>
<th></th>
<th>Dividend</th>
<th>Momentum</th>
<th>Quality</th>
<th>Size</th>
<th>Value</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>0.062</td>
<td>0.069</td>
<td>0.068</td>
<td>0.060</td>
<td>0.048</td>
<td>0.063</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.117</td>
<td>0.127</td>
<td>0.125</td>
<td>0.136</td>
<td>0.144</td>
<td>0.116</td>
</tr>
<tr>
<td>Sharpe</td>
<td>0.531</td>
<td>0.539</td>
<td>0.545</td>
<td>0.441</td>
<td>0.335</td>
<td>0.542</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-0.415</td>
<td>-0.447</td>
<td>-0.425</td>
<td>-0.473</td>
<td>-0.510</td>
<td>-0.414</td>
</tr>
</tbody>
</table>
It is highly interesting that MSCI World Index has higher volatility than every portfolio that we have constructed. Contrary to this, MSCI does not have higher volatility than each factor by itself. The exceptions are size and value that both have higher volatility than MSCI World. That is not dramatic since they both deliver higher return/risk ratio and are thus better choices even by themselves for constructing a portfolio.

These results are in accordance with previous studies especially Lakonishok (1994) of the value stocks overreacting and Fama and French (1992, 1993) that small companies earn premia because higher systematic risk involved. We can conclude that it is advisable to use factors however by themselves they have more risk than the traditional MSCI World. Therefore one needs to optimize portfolio in such a manner that optimal return/risk ratio is achieved and we keep consistent with the spirit of risk parity.

Also this study proves that optimization makes even better results than just picking randomly factors. From this part our results contradict those of Roncalli (2014). In our contest, the MVR portfolio was able to beat EW in this particular time period. Yet,
when it is taken into account that EW portfolio does not need any estimation its performance is quite impressive.

One can ask whether risk parity optimization is needed at all. As it shows that during our out of sample period pure factors offer higher Sharpe ratios than the optimized portfolios that have more than just one factor. The equal weighted EW portfolio gets higher Sharpe ratio than value and size factors by themselves. Now if we look back at the Figure 3 we see that size factor was the best performing factor at our broad sample. Thus, despite its return exceeds other factors its return is also poorly compensated risk wise. Therefore, it is not fair to compare single factors to more diversified portfolios.

<table>
<thead>
<tr>
<th></th>
<th>ARP</th>
<th>BRP</th>
<th>DRP</th>
<th>ERC</th>
<th>EW</th>
<th>MVR</th>
<th>IV</th>
<th>MD</th>
<th>WORLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>0.062</td>
<td>0.062</td>
<td>0.060</td>
<td>0.062</td>
<td>0.063</td>
<td>0.062</td>
<td>0.061</td>
<td>0.061</td>
<td>0.057</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.127</td>
<td>0.124</td>
<td>0.126</td>
<td>0.125</td>
<td>0.116</td>
<td>0.125</td>
<td>0.128</td>
<td>0.131</td>
<td></td>
</tr>
<tr>
<td>Sharpe</td>
<td>0.484</td>
<td>0.500</td>
<td>0.475</td>
<td>0.496</td>
<td>0.494</td>
<td>0.540</td>
<td>0.496</td>
<td>0.477</td>
<td>0.438</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-0.449</td>
<td>-0.443</td>
<td>-0.453</td>
<td>-0.446</td>
<td>-0.447</td>
<td>-0.414</td>
<td>-0.446</td>
<td>-0.460</td>
<td>-0.465</td>
</tr>
</tbody>
</table>
6.5 Factors reveal secrets of MSCI World Index

As a final part of our empirical study we have a brief look on the factor loadings of MSCI World. The reason for this is that although factors have been available for decades still many asset manager benchmark discretionary fund management portfolios with MSCI World index. This is an example that could easily be modified to analyzing fund’s factor tilts towards known factors. We show that style analyzing with factor leads us to understand what is driving returns and what factors are contributing the most to our portfolios wins and losses. We use both static and dynamic Sharpe ratio style analysis in this process. Our estimation window is 12 month.
During our broad sample the returns of MSCI World are explained well with quality factor. Also value explains well the returns, but its explanatory is not that frequent. These findings are interesting since one of the most well-known investor Warren Buffet is mainly a value investor. However, if we look at his style he is also tilted strongly towards quality and value (Frazzini et al., 2013.) Even the static weights are in accordance with this observation and from the Figure 6 we see that more than 80% of the MSCI World Index performance can be explained using quality and value factors.

Based on this short analysis dividend factor has very limited explanatory power in the MSCI World returns. The finding is interesting since some investors tend to believe that dividends are the main explanatory factor in stock returns. One possible explanation although behavioristic could be the preference to earn cash-flow from
dividends. Also the static weight graphics indicate null explanatory power for dividends.

Figure 7. Static portfolio weights of the MSCI World index

6.6 Discussion

In this Chapter we show that factors really earn better than the pure market return. We also demonstrate that optimizing portfolios is necessary and looking forwards it is hard to define which factor will earn the greatest performance. We show that when simplicity of EW portfolio is taken into account and the fact that it requires no estimation, it is a great solution for a not so mathematically-orientated investor. MVR portfolio wins this particular horse race, but it comprises of two factors only. Yet, we urge in the usage of BRP portfolio. Our argument for it is that it diversifies across all six factors. Together both MVR and BRP are better solutions for a diversified portfolio than the MSCI World Index.
We observe that during our broad sample period returns seem to stay under the long term equity return. Ilmanen (2011:7) reports a 5.4% annual return between 1900 – 2011 in the global equity market. Thus, our nominal returns are not that extraordinary. But this stems with the current forecast of 5.9% of nominal equity return for the years 2018 – 2047 (Botham, 2018.) Realized and expected returns may vary and are closely related to the particular sample period. For MSCI World index this particular period has not been that good either.

In this study all of the indices and their performance are provided by MSCI. That raises question about the generalization of our findings. Although we do not question the quality of the data by any means it yet leaves open, whether the results would be similar if another index provider was used.

Also our results do not take trading costs into account by any means. On the other hand since we urge to the usage of readily available ETFs provided by MSCI other than the initial portfolio construction costs need not taken into account. We keep our weights constant in all of the six risk parity portfolio plus our two control portfolios. Totally another kind of question is whether or not we would benefit from the constant rebalancing during our examination period. Our results are also restricted in a way that we focus solely to the equity asset class. An interesting question for the future research is what kind of results we would get if we utilized other asset classes as well. What if we applied the principles of factor investing together with risk parity optimization to other asset classes?

Our results are in contrast with Choueifaty and Coignards (2008) since we find that EW portfolio provides a better performing portfolio than MD they suggest without sophisticated mathematics nor a need to estimate future returns. Finally, we find compelling the fact that value, quality and size and especially the first two factors have such a major role explaining the return distribution of MSCI World. This result is in accordance with the Frazzini et al. findings that quality and value factors explain a major share of Warren Buffets success (Frazzini, 2013.) For example during the year 2003 in table Figure 7 all of the MSCI world return could be explained by value and quality factors.
It is arguable whether the risk budget method is a proper measure for the total risk of the portfolios. We mainly use it to illustrate the difference between dollar allocation and risk allocation, nothing more.

Our results do not however contribute to the reason why factors earn premia and how persistent is that premia. Nor do our results provide a link between the real economy and the factors. Summing it up, our empirical study shows evidence for the usage of factors and risk parity portfolios. Factors earn premia and risk factors lead to better results than just naïve diversification of assets.
7 CONCLUSIONS

Traditional diversification between asset classes fails to provide adequate protection. We have during the last 20 years witnessed two major financial crisis and currently we are in the middle of a period of low interests. This means that in the future environment it is even more important to concentrate on the attractive return and controlled risk. This has lead academics and finance professionals to seek for new methods and eventually this has caused factor investing to become one of the most talked topics in finance.

Our results confirm that the modern way of allocating funds based on factors together with risk parity methods is worthwhile using. We apply a clear method by first using factors as the tools and then optimizing the weight of each factor in the portfolio. To our surprise, every single optimized portfolio beats the MSCI World index with respect of return and risk. In this thesis we confirm that factor indices are really able to beat the MSCI World index performance. Their returns vary and are conditional to the sample period. It is surprising that they do so with more optimal risk/return ratio than the MSCI World index itself. Therefore, this study shows that factor performance is closely related to the chosen time period. However, this study leaves open what factors performance is the most persistent. Based on our results we also cannot tell with certainty what factors should an investor choose out of the six because their performance tends to vary and is time related and cannot be predicted.

The empirical part of the thesis is done using only factors that have gained a broad acceptance among the finance academia. However, there are hundreds of factors out there and the results of this thesis are not to be generalized because of uncertainty of possible data mining. In this thesis we examine the returns of MSCI factor indices compared to market weighted MSCI World. All of the indices have counterpart ETF’s thus the following results can be easily applied in real life.

We find evidence that risk parity optimization can help investor to beat equally weighted factor portfolios. BRP, ERC and IV - portfolio all beat the the Sharpe ratio of an EW portfolio. However, the best Sharpe is achieved using the MVR-portfolio. For us this finding is the most surprising and we were expecting to achieve better
portfolio than minimum variance optimized. Yet, we do not encourage use MVR since it leaves out 4 out of 6 factors we used, including the second best performing factor. Another point is that the differences are not that alarming when compared by Sharpe ratios. Also when we consider that during our sample period the best performing factors are totally different from the ones that performed best during the out of sample period relying solely on MVR seems too risky.

MVR bets two thirds to the low volatility factor and the remaining third to the dividend factor. Blitz (2017) presents evidence that Hedge Funds are betting against low volatility factor and this raises our concern for the future prevalence of this factor. Another concern we have is the heavy loading for the dividend factor since dividends face critics from Ang and Bekart (2007) who claim dividends to be insignificant factor of explaining returns. There for we argue that MVR should not be used a sole optimization method since it loads the most on these controversial factors.

As for the risk, we also conclude that risk budgeting should be a part of the toolbox when we look at the answers about the drivers of the returns since the factor indices do not provide enough information about the risk concentrations of the portfolio. In all eight portfolios we use dollar-based weights differ from the risk-based ones, except for the MVR portfolio. However, we admit that this method has its fallacies and it uses mainly volatility as a measure of risk. We prove enough evidence to show that using risk parity-optimized portfolios results are better than using EW (naïve diversification), but too little evidence that the paradigm shift should be transferred fully away from mean and variance environment.

We do not take into account trading costs that could possibly lower the real-life achievable returns. On the other hand, we keep our portfolio weights constant since we have calculated them. This means that we do not dynamically change our positions during the out of sample period. Another aspect to be considered is that we have only one existing history and these returns are achieved using back-tested data of MSCI. We are using all the time long only positions and we do not use any leverage. This means that limits of arbitrage do not prevent any institutional investor to benefit from the results of this work. Simultaneously, it provides an easy way to any individual investor to achieve better returns than MSCI World since equally weighted portfolio
demands no estimation process. On the other hand it leaves open the question whether similar results could be achieved using totally different data set.

More research is needed to verify the statistical significance of any factor returns. Another interesting topic would be testing whether similar out performance could be achievable using own constructed indices. MSCI World index is explained using Sharpe style analysis and majority of its return is due to a high loading in quality and value factor. Contrary to that dividend factor nearly has non explanatory power of the world index.

Our results contradict the previous studies from the part that we cannot confirm that portfolios that are built from factors would result in more volatile portfolios. This implies that we do not agree on the risk-based explanations of factors, but are more incline to behavioral explanations. Our results also contradict previous studies so, that we are not able to show that naïve diversification is necessarily outdated. ARP, DRP and MD all fail to have higher Sharpe ratio than the EW portfolio.

In terms of validity, our results cannot be generalized since we are using back-tested indices that are provided by the commercial sources. Data mining can exist and we cannot fully rule out the concern of it. This clearly does not nullify the insight that investor should look carefully to the added value a manager can create. If the return is explainable by the known factors fund manager lacks the true uncorrelated return that should be compensated.

As a key finding of this study, it is possible to form a portfolio that outperforms the MSCI World by using all factors selected six together with the presented risk parity optimization methods. This study confirms that, in general, it is advisable to shift attention from an asset class-based allocation towards the risk-based allocation since the performance of each sample portfolio beats the performance of MSCI World. Such portfolio offers a more favorable return and risk reward relation that should be the simple goal of each rational investor.
REFERENCES


