OULU BUSINESS SCHOOL

UNIVERSITY OF OULU

Lauri Nikula

PORTFOLIO MANAGEMENT: ADJUSTING OPTIMAL PORTFOLIOS WITH MOMENTUM AND SENTIMENT FACTORS

Master's Thesis

Department of Finance

October 2022

UNIVERSITY OF OULU
Oulu Business School

ABSTRACT OF THE MASTER'S THESIS

Unit			
Department of Finance			
Author: Lauri Nikula		Supervisor: Andrew Conlin	
Title: Portfolio Management: Adjusting Optimal Portfolios with Momentum and Sentiment Factors			
Subject	Type of the degree	Time of publication	Number of pages
Finance	Master’s degree	October 2022	71

Abstract

This thesis aims to find whether a stock portfolio using Markowitz (1952) optimization method can achieve improved performance with addition of momentum and sentiment factors. The optimization method does by itself have its drawbacks, but the aim is to find whether the addition of momentum or sentiment factor can improve optimized portfolio’s performance while still keeping the simplified methods for optimization. The results suggest that momentum factor, which is based on time series momentum, provides consistent improvement on optimized portfolio with increased returns and increased Sharpe ratios over a long investment period. Sentiment factor was used as a contrarian indicator, and it provided more mixed results while having less significant effect on the portfolio performance than momentum factor. To measure sentiment a sentiment index was built out of indicators prone to capture investor sentiment.

Keywords
Optimization, momentum, sentiment, portfolio management

Additional information

CONTENTS

FIGURES

TABLES

1 INTRODUCTION

One part of modern portfolio theory is the investors desire to find a portfolio which effectively maximizes expected portfolio returns for a given level of risk, measured by variance of the portfolio. With the process of mean-variance analysis investor can find the weights for different assets to either minimize the risk of portfolio as low as possible, or to find weights for assets so that the portfolio maximizes its Sharpe ratio, which offers the highest ratio of returns for the level of risk.

This thesis will take a closer look at this type of portfolio optimization process as an investment strategy, and whether it is possible to enhance optimized portfolio's performance by using signal factors for weight adjustment. The investigated optimization method is based on Markowitz (1952) portfolio optimizing method. The optimization method includes some simplified assumptions which have been seen as causing drawbacks on the efficiency of the optimization method. This thesis investigates whether the optimized portfolio's performance can be enhanced by signal factors while keeping the simplified process for optimization.

The first added signal factor is a momentum factor. Momentum has been seen as one of the most persistent anomalies in explaining stock returns. Traditional views in finance argue that momentum should not explain future stock returns. For example, the efficient-market hypothesis states that asset prices already reflect all available information, meaning that past stock momentum should not affect future returns. However, previous research findings suggests that stock that have demonstrated positive return momentum during the previous 3–12-month period tend to keep outperforming either their peer stocks or the given benchmark (Jegadeesh and Titman, 1993; Moskowitz, Ooi and Pedersen, 2012).

The second factor used for stock weight adjustment is investor sentiment factor. Sentiment reflects the mood of the market and tells whether investors are bullish or bearish about the near future returns and this mood might drive stock prices in a way that is not reflected by their fundamental values. When sentiment is high, investors tend to be overoptimistic about near future returns. They may allocate bigger proportion of their funds towards risky assets, which drives up the valuations and lowers the expected future returns. On the contrast, lowering levels of sentiment may increase the expected future returns. Previous findings suggest that stocks that are more difficult to value tend to be sensitive to changes in sentiment. (Baker and Wurgler, 2007; Chau, Deesomsak and Koutmos, 2016; Chung, Hung and Yeh, 2012; Edelen, Marcus and Tehranian, 2010.)

This study investigates whether the addition of either momentum or sentiment factor, or both, improves the portfolio performance for the same set of selected stocks if they were equally weighted or if only optimized weights are used. Improved portfolio performance refers to increased returns, lowered standard deviation or increased Sharpe ratio. The total investment horizon is between years 1999-2021, which offer results for a long-term investor's perspective. Different rebalance times may affect the portfolio outcomes, which is why four different rebalance times are investigated. Relation between portfolio risk and return will be assessed with a selected regression model.

This study does not aim to find maximized returns for the set of stocks, as it would cause high risk in terms of standard deviation, and most likely it would be quite unreliable investment strategy. Instead, the goal is to find whether modest and consistent improvements can be achieved with the addition of selected factors. The rules for the selected factors will be based on previous financial literature. The motivation behind this thesis is to find whether such classic optimization method as Markowitz (1952) optimization could be considered as a valid investment strategy with the selected factors, and if the factors provide consistent improvements for portfolio performance.

This thesis will proceed with theoretical framework around the portfolio optimization process, followed by findings on momentum and sentiment factor with reviews of previous key literature. Following this will be section 3 which provides the research design. Section 4 describes the research data used and section 5 provides results for the study. Finally, section 6 provides conclusions.

2 THEORETICAL FRAMEWORK AND EARLIER RESEARCH

2.1 Portfolio optimization

The initial portfolio optimization will be done by using the optimization method introduced by Markowitz (1952). This method is based on the underlying theory of Modern Portfolio Theory, which states that in general, investors seek to maximize the rate of return for their portfolio for a given level of risk. Here, the risk refers to the standard deviation of the portfolio's return. A key part of Modern Portfolio Theory is diversification, and Markowitz (1952) argued, that the investor could achieve the best return to risk relationship by choosing an optimal mix between low risk and low return portfolio with high risk and high return portfolio. With this in mind, the investor could construct an optimal portfolio, which is a portfolio that aims to maximize return to risk relationship.

The first step in Markowitz (1952) portfolio optimization process is to identify the risk-return opportunities from a set of risky assets. These opportunities create portfolios with different levels of risk and returns, which are then presented by the minimum-variance frontier. This frontier is laying on a graph where x-axis corresponds to standard deviation, and y-axis corresponds to expected return, and it demonstrates the minimum variance that is attainable for a given level of portfolio expected return. The portfolio with the lowest attainable standard deviation lays on the leftmost part of the frontier, and it's called the global minimum-variance portfolio. The part of the frontier that lays on top of the global minimum-variance portfolio is called the efficient frontier. Since all the individual risky assets lie inside the efficient frontier, this demonstrates how risky portfolios containing only one asset are inefficient, since by diversifying into multiple assets the investor is able to attain higher expected return with lower standard deviation.

In the second part of Markowitz (1952) optimization method a risk-free asset is recognized and placed on the same graph with the efficient frontier. As the name states, the risk-free asset is considered to have zero risk with some level of expected return, so it sets somewhere on the y-axis. From the risk-free asset a straight line can be drawn to the point on the efficient frontier where the line is steepest and tangent to the efficient frontier. The slope of this line, the capital allocation line, presents the Sharpe ratio for the optimal portfolio, and the point where the capital allocation line is tangent to the efficient frontier presents the optimal risky portfolio, which offers the highest Sharpe ratio. The Modern Portfolio Theory continues by the investor choosing a mix of this optimal portfolio and risk-free asset according to their level of risk aversion, but this thesis focuses only on the optimal portfolio.

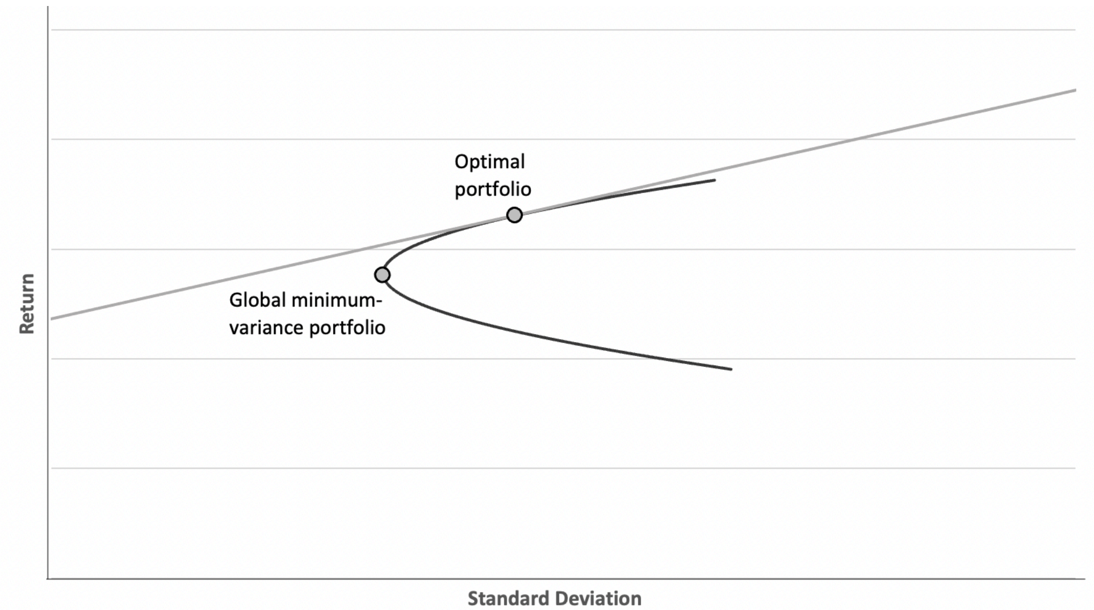

Figure 1. Efficient Frontier and Capital Allocation Line

The expected return of the portfolio is calculated as

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i), \tag{1}$$

where n is the number of stocks in the portfolio, w_i is the weight of each stock i and $E(r_i)$ is the expected return of each stock i. The variance of the portfolio is

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j), \tag{2}$$

where w_i is the weight of stock i and w_j is the weight of stock j, and $Cov(r_i, r_j)$ represents the covariance between stock i and j returns. Assuming that the portfolio is fully invested, then the sum of the weights adds up to 1.

$$\sum_{i=1}^{n} w_i = 1 \tag{3}$$

When moving past two assets it becomes necessary to use matrix multiplication to determine the optimal asset weights in the portfolio. Then the expected return for the portfolio is calculated as

$$E(r_p) = W^T R = \begin{bmatrix} w_1 & \cdots & w_j \end{bmatrix} \begin{bmatrix} E(r_1) \\ \vdots \\ E(r_j) \end{bmatrix} \tag{4}$$

where W^T is the transposed vector of weights of individual assets in the portfolio and R is the vector of expected returns of the individual assets in the portfolio. The standard deviation of the portfolio is calculated as

$$\sigma_p = \sqrt{W^T S(W)} = \left[\begin{bmatrix} w_1 & \cdots & w_j \end{bmatrix} \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1j} \\ \vdots & \ddots & \vdots \\ \sigma_{j1} & \cdots & \sigma_{jj} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_j \end{bmatrix} \right]^{\frac{1}{2}} \tag{5}$$

where W^T is the transposed vector of weights of individual assets in the portfolio and S is the variance-covariance matrix of the covariances between returns of each asset in the portfolio. The optimal weights in the portfolio are the ones that maximize the Sharpe ratio for the portfolio, which is calculated as

$$S_p = \frac{E(r_p) - r_f}{\sigma_p} \tag{6}$$

where $E(r_p)$ is the expected return of the portfolio, r_f is the return of a risk-free asset and σ_p is the standard deviation of the portfolio. The Sharpe ratio maximizing portfolio will be obtained by maximizing the return-risk trade-off of the portfolio, constraining that weights sum up to one, as

$$max \frac{W^T R}{\sqrt{W^T S W}} \tag{7}$$

$$s.t. 1^T W = 1 \tag{8}$$

$$W_d = \frac{S^{-1}R}{1^T S^{-1}R} \quad (9)$$

where R is the vector of expected returns, S is the variance-covariance matrix of the covariances between returns of each asset, 1 is a vector of ones ($1_n = 1$) and W_d is the vector of optimal portfolio weights.

When looking at the Markowitz (1952) optimal portfolio construction in more detail, estimates of covariance matrix of n x n stocks need to be defined for the set of stocks. The covariance matrix states the stocks' correlation between each other in the matrix. To capture the risk parameters for this matrix, daily returns of each stock will be used from a previous period of $t-1$ to find individual variance for each stock. The expected returns for each stock also need to be identified, and for this optimization method average annual returns of previous period will be used for each stock. As past annual returns provide an easy way of finding expected returns, they also have their limitations as potentially being quite inaccurate (Black, 1993). However, the aim of this study is to find whether different factors can improve the performance of this type of optimized portfolio which is based on a simple way of estimating expected returns.

The simplified nature of Markowitz portfolio optimization does have some benefits when implemented. The optimization method gives an easy way to include client objectives and constraints into the portfolio structure, such as the constraint of no short-selling. The optimization method can also be used to control portfolio's exposure to various risk components, which could be implemented from organization's investment philosophy. The optimization method also uses the investment information efficiently and is able to handle large amounts of data quickly. (Michaud, 1989.)

The simplicity of this optimization method does also have its drawbacks. According to Michaud (1989) the Markowitz optimization method's risk and return estimates are inevitably subject to estimation errors, and this optimization method tends to overweight stocks that have large estimated returns, negative correlations and small variances, so these stocks are the most likely to have large estimation errors. Jobson and Korkie (1980) found quantified evidence for the magnitude of estimation error with Markowitz optimal portfolio in their study. However, in their study they did not use a constraint of no short-selling, which is a common constraint for financial institutions. Applying this constraint could have reduced the magnitude of estimation error.

Another problem with Markowitz optimization method has to do with using past annualized stock returns for estimating the expected future returns. Stein (1956) has shown that the sample means are not a desirable estimator for expected returns under standard conditions. The optimization method also ignores the differentiation between levels of uncertainty associated with the inputs used, for example the levels of uncertainty with growth stocks and value stocks. The optimization method may also provide unstable results, meaning that small changes in the estimators may cause large changes in the overall portfolio. Also, the measure of diversifiable risk may be significantly underestimated. (Michaud, 1989.)

When it comes to the weights of optimized portfolios, portfolio managers commonly impose constraints on the maximum weight of the portfolio that can be invested on a single stock. Having such constraints would make no sense in a situation where portfolio manager knows the actual risk and expected returns of selected stocks. However, the actual risk and expected returns are in practice unknown factors, so having such constraints may be justified. The absence of maximum weight constraints would lead to overinvestment in stocks with favorable estimation error and underinvestment in stocks with unfavorable estimation errors, causing estimation bias on the portfolio. Having an upper bound on portfolio weight may help improving portfolio performance by reducing this bias. (Frost & Savarino, 1988.)

Frost and Savarino (1988) found in their study that the absence of maximum weight constraints leads investors choosing less stocks in their portfolios than the minimum number of twenty, which was required by their weakest constraint of 5 percent maximum weight that can be invested in a single stock. This lead investors heavily favoring a small subset of stocks with favorable measurement errors, resulting in estimates of both portfolio expected return and risk being substantially biased. Frost and Savarino (1988) found that by having maximum weight constraint for individual stocks both reduces estimation bias and improves portfolio performance. They concluded these results being encouraging especially for stock analysis done only with historical data.

Another commonly used restriction on optimized portfolios is to restrict short-selling for each stock. Frost and Savarino (1988) demonstrated how allowing short-selling causes skyrocketing estimation risk. Unrestricted short-selling causes investors to heavily overinvest in stocks with favorable estimation error and short-sell stocks with unfavorable estimation error. In Frost and Savarino’s sample the unrestricted investor thinks they will receive up to 37.5 times the true expected portfolio returns at approximately 2.6 % of that portfolio’s true risk. This demonstrates the adverse effect of unrestricted short-selling on portfolio selection.

When concerning for different portfolio rebalance times the findings Tokat and Wicas (2007) suggests less frequent rebalancing being more optimal than frequent rebalancing. In a trending market where stock prices are rising each period, frequent rebalancing means that the investor would be selling the strong performing stocks and buying weak performing stocks. Same happens in downward-trending market where the investor would be continuously buying stocks as their prices decline, which leads to portfolio returns below a portfolio that is less frequently rebalanced. In a mean-reverting market theory suggests that frequent rebalancing could enhance portfolio returns, as falling stock prices follow a pattern of mean reversion, meaning that the investor could buy stocks with falling prices and sell them at after the price has appreciated. Still Tokat and Wicas (2007) found that on mean-reverting market conditions frequent rebalances leads to decrease in absolute average returns, as the additional returns from well-timed purchases and sales was less than the additional return produced by the higher equity allocation in a less frequently rebalanced portfolio.

2.2 Signal factors

In this study there are two selected factors which will affect the optimized stock weights of the portfolios. These factors are momentum factor and investor sentiment factor. Momentum is based on the observed phenomenon that stocks that have had high returns in the near past tend to keep outperforming stocks that have performed poorly in the near past. Sentiment refers to the overall investor mood in the financial markets. Sentiment aims to capture whether investors have optimistic, pessimistic, or neutral feelings for the near future market performance. Both factors have been widely studied and although traditional views in finance may find these factors more or less useless in portfolio construction process, the empirical results suggest that implementing these factors may benefit the investor.

2.2.1 Momentum factor

When looking at the momentum factor, we first need to define which type of momentum trading strategy the factor will be based on. Two commonly used strategies to capture stock momentum are cross-sectional momentum and time series momentum. The difference is that the cross-sectional momentum focuses on relative performance while the time series momentum focuses on absolute performance. This means that the cross-sectional momentum looks at how individual assets have performed compared to their peers, while the time series momentum only focuses on how the individual asset has performed in the past.

Momentum trading strategies can be challenged by theoretical frameworks, for example as Fama (1970) famously discussed the efficient-market hypothesis, which states that asset prices already reflect all available information. In his work Fama (1970) stated that the future prices cannot be predicted by analysing historical data. This assumption means that any momentum strategies would be useless for aims to enhance future returns, since they rely on historical price data, and the theory suggests that past returns cannot tell how the assets will perform in the future. However, later Fama and French (2008) admitted that the momentum seems to be one of the most persistent anomalies.

When it comes to stock momentum research, Jegadeesh and Titman (1993) demonstrated the effect of momentum with findings suggesting that previous “winners” tend to keep outperforming past “losers”. They found that the top performing decile of stocks from previous 3–12 months tends to keep outperforming the lowest performing decile of stocks for the next 3–12 months as well. This is a type of cross-sectional momentum trading strategy, where the momentum is based on the relative performance of the selected stocks. Another finding by Jegadeesh and Titman (1993) was that the momentum profits tend to reverse when the stocks are held for 24 months or longer.

When looking at time series momentum Moskowitz et al. (2012) found predictability for assets' future performance based on its own previous performance in futures markets. They found out that the asset's past 12-month positive excess return indicates positive returns for up to 12 months into future. However, this positive trend partially reverses when the holding period becomes longer than 12 months. Moskowitz et al. (2012) also demonstrate that this type of momentum trading strategy works for multiple different asset classes, such as equity indices, foreign currencies, commodity futures, and government bonds. For more conventional asset classes Georgopoulou and Wang (2017) document a significant time series momentum effect that has been consistent and robust across global equity and commodity markets from years 1969-2015.

Goyal and Jegadeesh (2018) criticize time series momentum trading strategies not being market-neutral, referring that the number of stocks outperforming the given benchmark is typically not the same as the number of stocks falling short from the benchmark. The basic idea is that in a bull market lots of stocks tend to perform well, and the standard alpha of the stock may not be meaningful to be considered as abnormal return. Goyal and Jegadeesh (2018) also found that when accounting for market performance the time series momentum profits are materially mitigated.

Although momentum trading strategies may seem like a profitable way of capturing some excess returns, there still may be some problems with such strategies. McLean and Pontiff (2016) examined more than ninety anomalies explaining cross-sectional stock returns and found that the average predictor's returns are more than halved after the anomaly has been published in academic journal. This set of anomalies included momentum factor. Chordia, Subrahmanyam and Tong (2014) find that the momentum profits have notably decreased after 2001, when stock quoting changed from quoting in fractions such as 1/8 and 1/16 to quoting in decimals. The decrease in the tick size resulted in improved liquidity, lower bid-ask spreads and decrease in trading costs, which caused a significant decrease in momentum profits as a result.

Although it seems that there has been a decrease in momentum strategies' profits, it will still be used as a signal factor in this study, since momentum has shown power in the past research. Since this thesis studies portfolio returns starting from 1999 and ending in 2021, it can be inspected whether momentum factor seems to show better relative strength during different market conditions.

2.2.2 Sentiment factor

Investor sentiment can be seen as a tool which measures the investors' overall mood or attitude towards individual assets or the markets as a whole. During a bull market when stock prices are going up investors are typically very optimistic about the near future, but during a bear market when stock prices go down, they tend to turn pessimistic. Traditional view in finance is that sentiment-driven investors are essentially trading on 'noise' and thus are irrational (De Long, Shleifer, Summers & Waldmann, 1990).

The investor mood might drive stock prices in a way that does not reflect their fundamental values. During a rising sentiment investors may allocate higher proportion of their funds towards risky assets, which drives up the valuations and lowers the expected future returns. Falling levels of sentiment on the other hand may indicate increase in the expected future returns. Changes in sentiment can spread quickly throughout the market, which may affect investors' level of risk aversion independently of intrinsic cash flow prospects of the investments. (Chau et al., 2016; Edelen et al., 2010.) Arguments against sentiment note that if sentiment-driven investors would drive prices away from their fundamental value, then rational investors could exploit the profit opportunities created by mispricing. However, if they cannot fully exploit such opportunities, the sentiment effect to stock prices becomes more likely. (Stambaugh, Yu & Yuan, 2012.)

The way of measuring sentiment can be done with either direct or indirect measures. Direct measures refer to surveys asking individuals directly how they feel about the current or future market conditions, while indirect measures use economic and financial variables that are prone to capture investors' state of mind. Survey measures may be seen as beneficial as they reflect the psychological dimension of individuals in accordance with their socioeconomic characteristics. One problem with survey measures is that the data is typically collected gradually during a week or month, so the survey results may not properly reflect the investor mood during a given point-in-time. The advantage of indirect measures is that they are quite easy to construct from simple market data and they are observed in real time. However, indirect measures may have problems concerning their validity, since the market indicators may not exclusively measure investor sentiment. (Beer & Zouaoui, 2013.)

Although traditional views in finance may see sentiment-driven investors as irrational, it may be that such investors make informed decisions based on the information extracted from the sentiment. Chau et al. (2016) examined whether and to what extent investor sentiment influences such investors' trading behavior. Their findings suggest that sentiment does indeed matter and that there is a group of sentiment-driven investors who can trade against the 'herd' by selling their positions as sentiment reaches overoptimistic levels, or by finding buying opportunities in times of low sentiment. Another finding was that these investors are more likely to trade based on a survey-based sentiment indicators rather than market-based indicators. Also, surveys based on individual investors are preferred over surveys based on institutional investors, because individual investors seem to be more prone to excessive optimism or pessimism, which are attitudes that may misguide asset prices from their fundamental values.

Fisher and Statman (2000) investigated the relationship between investor sentiment and stock returns. They recognized three different investor groups for measures of their sentiment. Small, referring to individual investors, medium, referring to writers of investment newsletters, and large, referring to Wall Street strategists. Fisher and Statman (2000) found negative and statistically significant relationship between S&P 500 returns and sentiment of individual investors and Wall Street strategists, but no statistically significant relationship for writers of investment newsletters. This means that the sentiment of individual investors and Wall Street strategists could be used as a reliable contrary indicator for future S&P 500 returns. Another thing to notice is that here was no statistically significant relationship between small-cap stocks' returns and sentiment of any investor group. For individual investors there was also positive and statistically significant relationship between short-term changes in S&P 500 returns and changes in sentiment, meaning that an increase of S&P 500 returns was associated with an increase in the bullish sentiment.

Although the findings of Fisher and Statman (2000) may seem quite straightforward, they should still be handled with caution. Fisher and Statman (2000) illustrated a multiple regression of S&P 500 returns in one month on the level of sentiment of each three group in the preceding month, and found R^2 of 0.08, indicating that sentiment would explain 8 percent of S&P 500 returns. This number may seem low but is still statistically significant. This also proposes an idea that even though the sentiment may not be the main driving force of stock returns, it could still be used as a good tool for forecasting future stock returns.

Sometimes sentiment may be captured from other sources than direct surveys and indirect economic indicators. Uhl (2014) measured sentiment from more than 3.6 million Reuters news articles to find either positive, neutral or negative sentiment by having an algorithm distinguish between negative and positive words. With gathered sentiment dataset Uhl (2014) aimed to find sentiment’s ability to forecast Dow Jones Industrial Average stock index’s returns. Their findings suggest that negative Reuters sentiment appears to have higher influence on stock returns than positive Reuters sentiment, while negative Reuters sentiment is more persistent than positive sentiment.

Chung et al. (2012) studied cross-sectional predictability patterns of investor sentiment in different states of the economy. They looked at economic expansion and contraction regimes, and only in expansion states of the economy does investor sentiment seem to have significant and robust predictive power in stock returns. During contraction state of economy sentiment generally had insignificant predictive power in stock returns. For the economic expansion state, the findings of the paper suggest that higher sentiment is associated with lower subsequent stock returns of firms with small size, young age, low book-to-market ratio, high return volatility, non-earnings, non-dividend-paying status, high intangible assets, and high growth opportunities.

There are similar findings regarding firm characteristics that may determine how sensitive a stock is to investor sentiment. Baker and Wurgler (2007) find that stocks that are difficult to arbitrage or to value are most affected by sentiment. Such companies are the ones that are younger, smaller, more volatile, unprofitable, non-dividend paying, distressed or with extreme growth potential. On the other hand, bond-like stocks, such as regulated utilities, are less affected by sentiment.

Baker and Wurgler (2006) also found that during times of low sentiment small stocks tend to earn higher subsequent returns, but during high sentiment they found no size effect at all. Although, they found lower subsequent returns during high sentiment for newly listed stocks, high volatility stocks, unprofitable stocks and non-dividend paying stocks. These types of findings suggest that it may not be desirable to treat each stock the same way with the sentiment factor, since firm characteristics may play a crucial role on how the sentiment will affect its performance.

When looking at different indirect sentiment indicators, one commonly used sentiment indicator is the put-call ratio. It is the ratio of the volume of put options traded to the volume of call options traded. In the short-term it is seen as one of the leading technical indicators to capture investors' sentiment concerning future moves of the market. Put-call ratio is often used as a contrarian indicator under the assumption that the investing public is generally wrong about the market, especially at significant turning points in the market. An increase in the volume of puts traded relative to the volume of calls provides a bullish indication for contrarian investors. Conversely a decrease in the volume of puts traded relative to the volume of calls traded suggest public confidence in the markets, which is a bearish signal for the contrarian investors. (Billingsley & Chance, 1988.)

Billingsley and Chance (1988) investigated a trading strategy based on put-call ratio, where a set of equities were bought when the put-call ratio gave bullish contrarian indication, and the position was held until the ratio turned back to neutral or bearish. Similarly, a short position of equities was opened if the put-call ratio gave a bearish contrarian indication and held until it turned back to neutral or bearish. The put-call ratios were calculated using both daily volume figures and five-day moving average of for the S&P 100 Index Option (OEX) and Chicago Board Options Exchange (CBOE) equity options.

The most profitable strategy for day-by-day measures of put-call ratios were achieved for CBOE 65/40 signal, receiving a mean daily return of 0,16 % for long positions and 0,15 % mean daily return for short positions. 65 was a threshold ratio for a buy signal and 40 a threshold for sell signal. Billingsley and Chance (1988) however found that by including transaction costs the received excess returns were wiped out. They still found put-call ratio being a good market forecasting tool which can be used to gauge the direction of the market. Similar findings of using put-call ratio as a contrarian indicator are presented by Simon and Wiggins (2001), as they found that an increase of the overall mean of put-call ratio was associated with increased subsequent returns of S&P 500 futures.

Another indirect sentiment measure is the advance-decline ratio, which refers to the number of stocks that closed higher in a given day divided by the number of stocks that closed lower in a given day. It is a variable which is based on recent market performance which simply measures how many stocks are moving up and how many are moving down, aiming to capture the market breath. It can be interpreted in a way that when more stocks are moving up than the number of stocks moving down, the sentiment is bullish, and vice versa. It can be measured on different timeframes, for example on a daily, weekly, or monthly basis. (Brown & Cliff, 2004.)

Brown and Cliff (2004) examined whether surveys measuring investor sentiment are related to other popular indirect measures of sentiment, such as advance-decline ratio. They found significant positive relation between institutional sentiment and the advance-decline ratio on monthly regression, however there was significant negative relation on a weekly regression. For all sentiment measures Brown and Cliff (2004) found strong co-movement with the market, however with little short-run predictability in returns.

Changes in the use of margin debt in investors’ accounts can also be seen as one indirect indicator for sentiment. If investors believe the stock market is going up, they may wish the leverage their stock market position by using margin debt. This means they become more optimistic about the near future stock market returns. Alternatively, investors may see strong upward movements in the stock market as a validation for bull market, which would encourage them to buy more stocks on margin if they are already fully invested. Conversely, when the stock market returns start to decline the investors may become more pessimistic resulting in decline on margin balances. Some investors may also need to sell their stocks to meet the margin requirements. (Domian & Racine, 2006.)

Using margin debt may also lead to so called "pyramiding–depyramiding process" of stock prices and margin debt. This happens from an increased demand from margin buying pushing stock prices higher, which in turn will encourage investors taking more margin debt and push stock prices higher. When this bubble eventually bursts, and stock prices start going down some investors need to sell their stocks to meet the margin requirements. These forced sales keep pushing stock prices to even lower levels. Margin debt has even been blamed for historic stock market crashes. (Fortune, 2001.)

2.3 Relation between risk and return

In the financial literature there has been different models aimed to understand the relation between risk and return. Fundamental contribution to this started from the development of capital asset pricing model (CAPM) by Sharpe (1964) and Lintner (1965), where the central prediction is that the expected asset returns co-moves with the expected return of the market portfolio. According to this model differences in betas are sufficient to explain cross-sectional differences in stock returns. The CAPM is presented as

$$E(r_i) - r_f = \beta_i(E(r_m) - r_f) \tag{10}$$

where $E(r_i) - r_f$ represents the excess expected return of stock i, β_i is the beta of stock i, which measures the stock's volatility in relation to expected market return $E(r_m)$.

Since the development of CAPM multifactor models have been developed in attempts to answer return patterns that cannot be explained by CAPM alone. One of the most famous multifactor models was developed by Fama and French (1993) with their three-factor model, which adds factors relating to size and book-to-market ratio to the standard CAPM. Subsequent studies have shown three-factor model performing substantially well in describing the cross-section of stock returns (Fama & French, 1996; Griffin & Lemmon, 2002; Liew & Vassalou, 2000). The three-factor model is presented as

$$E(r_i) - r_f = \alpha_i + \beta_1\big(E(r_m) - r_f\big) + \beta_2 SMB + \beta_3 HML + \epsilon_i \tag{11}$$

where SMB factor (Small Minus Big) is the return of a diversified portfolio of small stocks minus the return of diversified portfolio of big stocks, which is based on the observation that small firms tend to outperform large firms in the long term. HML factor (High Minus Low) is the difference in returns between returns of diversified portfolios of high and low book-to-market stocks, which refers to observation that firms with high book-to-market values tend to have higher returns than those with low book-to-market values. β_1, β_2 and β_3 represent factor coefficients. α_i is a risk-adjusted measure of active return on investment. Positive α_i indicates that the investment has earned excess return for its risk according to three-factor model. Negative α_i on the other hand indicates that the investment has earned too little for its level of risk according to model. ϵ_i is a zero-mean residual.

In a more recent study Fama and French (2015) introduced a five-factor model. Previous studies found three-factor model being incomplete in terms that it misses much of the variation in average returns related to profitability and investment (Novy-Marx, 2013; Titman, Wei, & Xie, 2004). Five-factor model adds these two risk factors to the previous three-factor model. Five-factor model is presented as

$$E(r_i) - r_f = \alpha_i + \beta_1\big(E(r_m) - r_f\big) + \beta_2 SMB + \beta_3 HML$$

$$+ \beta_4 RMW + \beta_5 CMA + \epsilon_i \tag{12}$$

where RMW factor (Robust Minus Weak) is the difference between returns on diversified portfolios of stocks with robust and weak profitability, and CMA factor

(Conservative Minus Aggressive) is the difference between returns on diversified portfolios of the stocks of low and high investment firms. Fama and French (2015) find the five-factor model outperforming the three-factor model in explaining the cross-section of stock returns. They also conclude book-to-market factor becoming redundant in describing average returns when factors for profitability and investment are added. In line with this finding Hou, Xue and Zhang (2015) find support for a four-factor model which includes market factor, as well as factors for size, profitability and investment.

Fama and French (2017) have also considered a six-factor model, saying that a momentum factor for example is a common addition to three-factor model. Momentum factor is used by Carhart (1997) who evaluated mutual fund performance with three-factor model and added a fourth factor, which was a momentum factor. Carhart (1997) found the addition of momentum factor reducing the pricing error of the portfolio when compared to CAPM and three-factor model. In a more recent study Fama and French (2020) examined models which added a momentum factor and found momentum factors being important for explaining returns on portfolio which are formed based on momentum but found that they do not otherwise contribute much to asset pricing models.

3 RESEARCH DESIGN

The optimization method introduced by Markowitz (1952) may seem to be somewhat unreliable method for portfolio construction, especially when the expected returns are estimated by looking at historical average returns of individual stocks. Research suggests that portfolios constructed by such optimization method are subject to estimation errors, which may vary depending on how much constraints are used. Estimating expected returns with past average returns provides a simple way of measuring expected returns, but the simplicity comes with a cost of potentially being quite unreliable.

This thesis will investigate a trading strategy based on portfolio optimization method which uses historical averages for expected return estimation and for covariance matrix estimates. The aim of the thesis is to find whether the optimized portfolio's performance can be enhanced by implementing momentum and sentiment factors for stock weight allocation. The enhanced performance refers to increase in returns, decrease in standard deviation, or increase in Sharpe ratio. Previous research has mainly focused on enhancing optimal portfolio by improving variance-covariance matrix estimates (Fan, Fan & Lv, 2008; Ledoit & Wolf, 2003, 2004). This study focuses on performance enhancement based on factor adjustments. Both factors of this study will be used with clear sets of rules, which will be carried out throughout the entire investment process. The optimization process will also have a set of constraints which together with factor rules aim to provide modest increase in portfolio performance over a long period of time.

To measure relation between risk and return Fama and French (2015) five-factor model regression will be used with addition of momentum factor. Fama and French (2015) found five-factor model outperforming the three-factor model in explaining the cross-section of stock returns. Also, the addition of momentum factor is justified since the portfolio of this study is based on momentum factor adjustments, and Fama and French (2020) found momentum factor being an important part of this type of portfolio.

3.1 Portfolio construction

This thesis investigates the performance of four portfolios using the same stocks and rules for their construction, but with different rebalance times with the Markowitz (1952) optimization and momentum factor adjustments. Each portfolio will be optimized with Markowitz optimization method to maximize portfolio Sharpe ratio. The rebalance times of different portfolios are 1-month, 3 months, 6 months, and 1-year. The overall investment horizon for each portfolio will be from beginning of January 1999 to end of December 2021. The Markowitz optimization method requires estimates for expected returns and for covariance matrix estimates. In this study annualized average stock returns will be used for estimating expected returns, and covariance matrix estimates will be calculated from historical stock variances.

The estimates for expected returns are calculated from average annual historic stock returns. In this thesis each portfolio will be using the same length of historic time for these stock returns as the time between each rebalance with the given portfolio. This means that the portfolio with one month rebalance time uses previous one month's worth of daily stock price data to create estimates for expected returns, three-month portfolio uses previous three months' worth of historic price data and so on. The same timeframe is used for covariance matrix estimates. As the timeframes of these historic samples differ for each portfolio, each average returns from the previous periods will be annualized to give congruent measures for forming expected returns.

At each rebalance time the portfolios' weights will be optimized with the same procedure as with each previous rebalance, by first optimizing the weights with Markowitz optimization method, after which the optimized weights will be adjusted with momentum factor. The portfolios will be always fully invested, meaning that the factors will not signal portfolios to liquidate some capital to cash at any point. Portfolios will also not be leveraged.

The stocks in the portfolio will be selected from two indexes, S&P 500 and S&P 400. S&P 500 provides exposure to the largest companies in the US while the S&P 400 serves as a gauge for mid-cap equities in the US. The top 15 weighted stocks from each index will be selected to the portfolios of this study, adding a total of 30 stocks in the portfolios. Since the stocks in the indexes change and receive new weights over time, the stocks in the portfolios will be reselected every two years based on the changes of top 15 weighted stocks in selected indexes.

The portfolios will be having constraints regarding their weights during the Markowitz optimization phase. Portfolios will be having a constraint of 6 percent for maximum weight which can be invested in a single stock. This is because of findings by Frost and Savarino (1988), which suggests that having such constraint both reduces estimation bias and improves portfolio performance. Absence of maximum weight constraint would likely result in portfolios investing in much less stocks than desirable, causing estimation bias and decreasing portfolio performance.

Addition to maximum weight constraint the portfolios will also have a minimum weight constraint of 1 percent. The minimum weight constraint is introduced as a risk management tool, since the absence of minimum weight constraint would potentially lead to a situation in which many stocks with non-desirable estimates would receive a weight of zero percent. This might lead to a situation of overinvestment to stocks with favorable estimation errors, while neglecting stocks with non-favorable estimates. With a 6 percent maximum weight, the absence of minimum weight constraint could lead to a situation where only 17 stocks out of 30 will receive capital allocation, 16 stocks receiving 6 percent weight and 1 receiving 4 percent weight, adding to fully invested portfolio. This would mean that almost half of the available stocks would be neglected. By introducing minimum weight constraint each stock will have at least some weight in the portfolio. The minimum weight constraint aims to mitigate the adverse effects of estimation errors. Selected maximum and minimum weight constraint are meant for the stock optimization process, but after adjusting weights with momentum or sentiment factor the final weights may go slightly beyond 6 percent or slightly under 1 percent, but never to negative.

As each stock has constraint to never have negative weights, this means there is a constraint of no short-selling. Not allowing short-selling can be justified by findings of Frost and Savarino (1988), who demonstrated how allowing short-selling causes skyrocketing estimation risk. This thesis only investigates fully invested portfolios, which are long on each stock, so there will be no leveraging allowed either. The study focuses only on the adjusted optimal portfolio and based on the results the investor could determine how much they would be willing to invest in this portfolio and a risk-free asset, or if they wish to leverage this portfolio in accordance with their level of risk aversion.

The results will be compared to a portfolio which invests in the same set of stocks as the investigated portfolios, but with even weights and rebalance to even weights at the same time as other portfolios have rebalances with Markowitz optimization. Momentum and sentiment factors will also be examined by themselves with optimized portfolios, as well as by using both factors together with optimized portfolio. Portfolio using only optimized weights will also be investigated for comparison.

3.2 Application of momentum factor

When looking at the two commonly used momentum trading strategies, cross-sectional momentum and time series momentum, the strategy which the momentum factor will be based on is the time series momentum. The main problem with cross-sectional momentum is that with 30 stock portfolio the adjustments with past winners and losers in the highest and lowest decile of stock returns only consist of 3 stocks per decile, so the adjustments for 6 stocks should be quite noticeable if we wanted to see any significant results. Having big adjustments for only a few stocks could provide too much risk, but with time series momentum we can make small adjustments to each stock, which potentially offers small but consistent return improvement to the optimized portfolio.

The way time series momentum factor will be implemented is by first optimizing the 30 stocks' weights with Markowitz optimization method, and then looking at how each individual stock has performed during the last 6 or 12-month period. Portfolio held for one year will be having 12-month investigated past returns for momentum and other portfolios with shorter rebalance times will be having 6 months of past returns as indicator for momentum. The past returns from 6 or 12-month time period will be compared to a benchmark, which in this case will be Russell 3000 index, which seeks to benchmark the entire US stock market.

Those stocks that have performed worse than the benchmark during the past intermediate time period will receive 30 percent reduction from the optimized weights, and this residual weight will be allocated evenly to the stocks that have outperformed benchmark. This way each stock will receive a small adjustment from momentum factor rather than having a few big adjustments if cross-sectional momentum factor would be used. If a stock would have performed exactly same as benchmark during the past 6 or 12-month time period, it will receive no adjustments from the momentum factor. These adjustments are only valid for the next investment period, after which the portfolio will be rebalanced again first with Markowitz optimization method, following adjustments with momentum factor.

There is a slight problem with the momentum factor that needs to be taken into consideration. As Moskowitz et al. (2012) found, time series momentum excess returns seem to reverse after 12-month holding period, meaning that the investigated portfolio which will have 12-month holding period may not receive excess momentum returns for the stocks that have received additional weight adjustment from momentum factor during the previous rebalance. However, momentum reversal may still have some open questions regarding its validity. Jegadeesh and Titman (2001) found that during time period of 1965–1981 momentum profits seemed to reverse after 12-month holding period, but not during time period of 1982–1998, although during this time period the momentum profits levelled off during the months 13–60 of holding period.

A more recent empirical paper by Conrad and Yavuz (2017) provides evidence that stocks that have displayed momentum in the first 6-month period do not display significant reversal in any time horizon up to 5 years. Here, the investigated time period was between 1965–2010. They also found that so called “contrarian” portfolio, including original winner and loser stocks that exhibit reversal in the first 6 months after portfolio formation, display significant reversal in the 12–24-month period, although Conrad and Yavuz (2017) found no statistical difference in returns between contrarian portfolio and stocks that had realized momentum profits after the first two years. Their findings suggest that the stocks realizing momentum profits are less likely to exhibit reversal than what would be expected by random chance.

When concerning for momentum reversal, the continuous portfolio rebalances in this study offer a chance for more dynamic approach rather than once looking at past intermediate timeframe stock returns and making momentum decisions for years to come. Conrad and Yavuz's (2017) findings suggest that trying to time a potential reversal for stocks that have realized momentum returns seems undesirable, as they found that these stocks do not tend to display significant reversal in any time frame for up to 5 years.

Based on these findings the momentum factor of this study works as described before, by looking at past intermediate time period's returns for each stock and comparing if it has exceeded the benchmark's returns or not and based on that either given additional weight or subtracted weight from the optimized weight. In other words, there will not be specific attempts trying to time momentum reversals. Previous research seems to have differing views on the momentum reversal after 12-month holding period, but the advantage of this study's portfolios is that none of them is held for more than 12 months before the next rebalance, which gives an opportunity to adjust weights for the next period if the momentum factor gives differing results from last period.

3.3 Application of sentiment factor

The aim of the sentiment factor is to recognize when the overall mood in the stock market becomes either overly optimistic or overly pessimistic and adjust portfolio's stocks that are most affected by sentiment. During overly optimistic times the expected future stock returns tend to be low, compared to times of overpessimism when the expected future stock returns tend to be high. With this recognition the sentiment works as a contrarian indicator, meaning that it suggests contrary actions when compared to prevailing market mood. This means being bullish during times of low sentiment and being bearish during high sentiment.

Since the aim of this thesis is trying to enhance performance of optimized and fully invested portfolio, the sentiment factor's signals will not be used for determining when to step entirely out of stock market by selling stocks for cash and later investing again. Instead, stocks' weights will be adjusted based on sentiment factor's signals. Unlike with momentum factor, adjustments based on sentiment factor's signals will only be done in situations when the sentiment factor provides signals of overly optimistic or overly pessimistic market sentiment. The factor aims to determine times of overoptimism or overpessimism in the market, which would give contrarian signal to the stocks most sensitive to sentiment.

The contrarian nature of the sentiment factor means that when sentiment reaches levels of extreme optimism in the market, weights of the stocks that are most sensitive to sentiment will be reduced by a fixed rate and this excess capital will be evenly moved to stocks that are not as sensitive to sentiment. When the sentiment reaches levels of extreme pessimism, adjustments will be done by adding more weight to sentiment sensitive stocks by reducing a fixed rate of weight from stocks not as sensitive for sentiment, and allocating this excess capital evenly to sentiment sensitive stocks. These adjustments can be done at any time when the sentiment factor gives a signal of overoptimism or overpessimism, even if the signals come before a time of rebalancing with Markowitz optimization method and momentum factor. For measuring sentiment, a sentiment index will be built based on different factors which have been seen as prone to capture investor sentiment.

By allowing sentiment factor adjustments to be done outside the times of rebalancing with Markowitz optimization and momentum factor it solves the potential problems that would arise if sentiment adjustments would always be done at the same time as momentum adjustments, meaning that there would be a problem if momentum factor tells to increase weight of one stock while sentiment factor tells to decrease its weight. However, there is a chance that sentiment factor gives a signal of extreme optimism or pessimism at the same day as rebalancing with Markowitz optimization and momentum factor would occur. If sentiment factor would give signal on such day, then the stock adjustments will only be done if momentum and sentiment factor both state that the weight should be added or decreased, but no sentiment adjustments will be done if the factors give mixed signals, meaning that other factor states weight adjustment while other suggests weight reduction.

As previous research suggests, stocks that are most difficult to value or arbitrage tend to be most affected by sentiment (Baker & Wurgler, 2007; Chung et al., 2012). Such stocks are typically firms with small size, young age, low book-to-market ratio, high return volatility, non-earnings, non-dividend-paying status, high intangible assets, or high growth opportunities. For this thesis the sentiment sensitive stocks will be determined by looking at the company size and whether is classified as a growth stock. The size will be measured by stock's market capitalization and growth stocks will be identified with book-to-market ratio, as stocks with low book-to-market ratio have been conventionally seen as a sign of growth stocks (Chen, 2017). As S&P 500 stocks represent the largest companies in the US, they will never be considered as smallest stocks in the selected portfolio which also has mid-cap stocks. Sentiment sensitive stocks based on size will always be from S&P 400 stocks, but sentiment sensitive stocks based on book-to-market ratios can be from either large- or mid-cap stocks.

Based on the size, the sentiment sensitive stocks will be determined from the two lowest quintiles of market values from the selected stocks. This means that 6 stocks out of 30 will be classified as sentiment sensitive stock based on their size. When looking at book-to-market values, the lowest quintile of book-to-market values will be classified as sentiment sensitive based on their book-to-market ratio. This includes 3 stocks from the portfolio with lowest book-to-market ratios, which represent the growth stocks of the portfolio. In total there can be a maximum of 9 stocks out of 30 which will be classified as sentiment sensitive. In case a same stock is on the lowest two quintiles of market value and on the lowest quintile of book-to-market value, it may bring down the total number of stocks which are sentiment sensitive, but the minimum amount will always be 6 stocks. The market capitalization and book-to-market ratios will be measured on a daily basis, and the determination of 6 to 9 sentiment sensitive stocks will be done on the date when the sentiment index gives a signal of overoptimism or pessimism, and the weight adjustments will be implemented from the next day forward.

When the sentiment index provides a signal of overoptimism, the portfolio adjustments will be done by reducing sentiment sensitive stocks' weight by 30 percent, and this excess capital will be allocated evenly to other stocks which are not classified as sentiment sensitive. When the sentiment index provides a signal of overpessimism, the adjustments will be done by reducing the weights of non-sentiment sensitive stocks by 13 percent and this excess capital will be allocated to sentiment sensitive stocks. With these rates the percentage changes of sentiment sensitive stocks remain close to each other during either overoptimistic or ovepessimistic sentiment, since there will always be more stocks in the portfolio which are considered as non-sensitive to sentiment, so decreasing their weight by 13 percent leads to additional weight of 30 percent to sentiment sensitive stocks. With this method the portfolio will be always fully invested, as it is with momentum adjustments. The adjusted weights based on sentiment factor will be held until the next rebalance with Markowitz optimization and momentum factor adjustments.

3.4 Sentiment index

The sentiment index of this study will be built out of five indicators, two direct indicators and three indirect indicators. The direct indicators will be based on the findings of Fisher and Statman (2000), where American Association of Individual Investors (AAII) survey data will be used to represent individual investors' sentiment and Investors Intelligence (II) survey data will be used to represent newsletter writers' sentiment. As Fisher and Statman (2000) found combination of survey indicators may provide a good tool to forecast future stock returns. According to their findings individual investors and newsletter writers form their sentiment as if they expect continuations of short-term returns.

When looking at how much the two direct indicators should be weighted in the entire sentiment index, the survey data of individual investors will be given more weight. This is due to findings suggesting that individual investors seem to be more prone to excessive optimism or pessimism, which may misguide asset prices from their fundamental values, creating potential portfolio adjustment opportunities as a contrary action (Chau et al. 2016). Survey data of AAII will be weighted by 35 percent of the entire sentiment index, while II survey data will be weighted by 15 percent of the entire sentiment index. As both surveys provide information on how many investors are either bullish or bearish, the sentiment of these factors are measured with a bull-bear spread, which is a ratio calculated by dividing bullish answers by bearish answers. A higher ratio indicates growing optimism while lower ratio indicates of more pessimistic sentiment.

The three indirect sentiment indicators will be selected based on Brown and Cliff's (2004) recognition of different categories of indirect sentiment indicators, which are indicators based on either market performance, type of trading activity, and derivatives variables. The three indirect sentiment indicators of this study will be each from one of these categories. The market performance indicator of this study will be advance-decline ratio, trading activity indicator will be the percentage change in margin borrowing and the derivatives indicator will be put-call ratio.

The advance-decline ratio refers to the number of stocks that closed higher in a given day divided by the number of stocks that closed lower in a given day, meaning that it offers an indicator based on resent market performance. Rather than looking at individual daily values of the ratio, it will be measured with a five-day simple moving average of daily ratios. This moving average aims to capture trends with the advance-decline ratio, meaning that increasing trend in ratio could reveal bullish market and decreasing trend in ratio could reveal bearish market. The advance-decline ratio ignores the magnitude of advance or decline in each stock, but by looking at its trend it provides information on how many stocks participate in the market movement. The advancing and declining stocks will be measured from S&P 500 index since it is one of the most followed indexes in the world and it is seen to represent US market performance. It also has enough stocks to provide reasonable results for the ratio. Advance-decline ratio will have a total of 15 percent weight in the index.

The change in margin borrowing will be measured from FINRA Margin Statistics, where they update monthly balances in customers' securities margin accounts. The monthly change in margin borrowing will be measured with 6-month moving average. This indicator has noticeably longer time period between new data entries when compared to other indirect indicators, but it aims to capture bigger trends in sentiment rather than looking at day-to-day changes. Significant increases on investors' use of margin borrowing tells that they are becoming optimistic about future stock returns as they are willing to take more risk. Noticeable decrease in margin borrowing on the other hand tells about increased pessimism in the markets. Change in margin borrowing indicator will have a total weight of 15 percent in the index.

The put-call ratio of sentiment index will be based on the put-call ratio of Chicago Board Options Exchange (CBOE) equity options. The put-call ratio will be followed similarly with other indirect indicators, by looking at the 5-day moving average of the ratio. The moving average aims to provide signals of optimistic or pessimistic trends in the markets. When the 5-day moving average ratio increases, meaning that investors are buying more put options than call options, this provides a signal of increased pessimism. When the 5-day moving average ratio decreases, it tell of increasing optimism. The put-call ratio will have a 20 percent weight in the index.

The construction of the sentiment index begins by calculating the mean and standard deviation of each sentiment indicator prior to year 1998. For advance-decline ratio as well as AAII and II surveys, data was collected starting from year 1993. Data for FINRA Margin Debt and CBOE equity options put-call ratio were collected starting from 1997 as data for them was not available before that. Advance-decline ratio, margin debt and put-call ratio were measured with moving averages. After this, standard scores, or z-scores, were calculated for each variable starting from beginning of 1998 as

$$z = \frac{x-\mu}{\sigma} \tag{13}$$

where x represents the raw score of the variable, μ is the sample mean for the variable and σ is the standard deviation of the variable.

The z-scores are measured for the first six months of 1998. From the z-scores are calculated scores for each indicator above which lies the data over 95th percentile. The z-scores above the 95th percentile will receive the same value as the score at 95th percentile. This process of winsorizing aims to limit extreme outliers' effect on the mean values. For margin debt values below 5th percentile are also winsorized to match score at 5th percentile, as it has outlier values on the lower end of the sample. The winsorized values are then multiplied by the weights assigned to them and the values are summed together to receive values for the sentiment index. The z-score values for put-call ratio need to be divided by -1 since otherwise increase in its value would indicate increased pessimism, but each indicator needs to be defined so that an increase in its value indicates increase in investor optimism.

The first sentiment index values are measured for the year 1998, from which the mean and standard deviation are calculated. For selected indicators first 6 months of 1998 are from initial z-score values, after which the winsorized values are used. For both z-scores and sentiment index, calculations of means and standard deviations are recalculated every 6 months by adding the last 6 month's data to the previous sample for calculating means and standard deviations. This way the means and standard deviations are always based on current historic data, and the sentiment index is not built by "looking into future" by calculating means and standard deviations from the entire sample going to year 2021. Also, the 95th and 5th percentile values for winsorizing are calculated every 6 months by adding the latest 6 month's data to the sample to represent current values.

As the portfolio observations start from the beginning of 1999, the sentiment index has had 1 year worth of data to collect its first mean and standard deviation values. The overoptimism and over pessimism will be determined by how many standard deviations away the sentiment index's value is from its mean value. Li (2020) used 2 standard deviations over mean value as threshold for overoptimism, and 2 standard deviations below mean as a threshold for overpessimism, and found these thresholds being statistically significant.

This thesis' sentiment index will have a slightly looser thresholds of $\pm$ 1.9 standard deviations, since many indicators use moving averages and threshold of $\pm$ 2.0 standard deviation may not capture extreme sentiment values if the movements of sentiment are quick and return as quickly from the extreme values. So, values of sentiment index over 1.9 standard deviations from the mean value are considered as overoptimistic sentiment, and values below 1.9 standard deviations from mean are considered overpessimistic sentiment. The means and standard deviations are calculated from the past values, and every 6 months the latest 6 months' worth of data is added to the sample where index mean and standard deviation are calculated.

3.5 Research questions and objective

The research questions of this study are "Is it possible to enhance optimized portfolio performance by using momentum and sentiment factors with available data?" and "How much do the selected factors contribute to the performance of adjusted portfolios?". The research objective is to find an enhanced optimal portfolio that is fully invested and long on each stock. The investor could then make decisions on how much to invest between this portfolio and a risk-free asset, or if they wish to leverage this portfolio in accordance with their level of risk aversion.

3.6 Research limitations

This study does have some limitations to it. First, the adjustments of the optimal portfolio are based on two factors, momentum and sentiment, but there could be other factors driving the stock market that is not captured by momentum or sentiment. Second, the factors could work better with different adjustments to their rules. This study has a set of rules for both momentum and sentiment factors which are carried out throughout the entire investment process of 23 years. This study investigates how momentum and sentiment factors that are based on the selected rules perform over the entire investment process. This study also ignores transaction costs, which could affect the results if implemented.

There may be more limitations to sentiment factors than momentum factors. Momentum factor is mainly limited to the chosen way of capturing momentum and to the selected magnitude of adjustments based on momentum factor. For sentiment factor there are many ways to identify sentiment sensitive stocks, and this study focuses on only two of them, size and book-to-market ratio. Also, as the stocks in the portfolios are collected from large-cap and mid-cap indexes, there are no truly small stocks in the portfolio, which could be considered better choices as sentiment sensitive stocks. Data for small-cap indexes' constituents and their price performance were not as available as for large-cap and mid-cap indexes, which led to choosing stocks from these indexes. The signals of this study's sentiment index could be used for forecasting small-cap indexes' future performance, but since this study required data on individual stocks, top constituents from S&P 500 and S&P 400 were used.

There are also different ways how the sentiment index's signals could be used for stocks adjustments. This study only investigates the effect of sentiment adjustments that are done when the sentiment index reaches overoptimistic or overpessimistic levels, but adjustments could also be done for example based on rapid changes in sentiment index's values, even if the values do not reach extreme values.

Lastly, this study focuses on US stocks and factors based on US markets, so the results could vary if the factors and stocks would be based on different markets, for example Europe or emerging markets. The results still should give some idea how the factors might behave in different markets, as the financial markets are nowadays highly interconnected, where financial and economic changes in major economic areas are reflected on different markets.

4 RESEARCH DATA

The data for this thesis is collected from Refinitiv database. The daily data for stock performance in the portfolio has been calculated by the changes in Return Index (RI), which measures stock price performance and includes the reinvestment of dividends. The stocks in the portfolio are collected from S&P 500 and S&P 400 indexes by taking top 15 weighted stocks from each index, totaling to 30 stocks. As the stocks and their weights change in the indexes over time, the stocks are reselected every two years from the top 15 weighted spots. Both indexes use float-adjusted market cap weighting method, which measures the stocks' market cap based on their shares that are available for public trading. Top 15 stocks from both indexes were collected using this method every second year. For the stocks in the portfolio daily market values and book-to-market values were collected for each stock as they were needed to determine sentiment sensitive stocks. Daily yield for US 10 Year Treasury bond was collected to serve as risk-free asset.

Momentum factor required data for a benchmark, which in this case was Russell 3000 index. Return Index values were used to calculate benchmark returns. For the sentiment index some data was more available than others. AAII and II survey data were available, and data was collected from the start of 1993 to end of 2021. CBOE Equity put-call ratio data and FINRA Margin Debt data were available from start of 1997, and data for these indicators was also collected to the end of 2021. Data for the advance-decline ratio was not readily available for S&P 500 index, but it was calculated from start of 1993 to end of 2021 by using daily stock price data for each stock in the index, and by calculating the number of stocks advancing and declining during each day. Data for five-factor model plus momentum factor was collected from Kenneth R. French data library (2022).

5 EMPIRICAL RESULTS

Table 1 presents the descriptive statistics of the data sample, which consisted of top 15 weighted stocks of S&P 500 and S&P 400 indexes, or 15 large-cap stocks and 15 mid-cap stocks. Table shows the mean annual return of a single stock from both indexes during the investment period of this study, as well as average standard deviation of a large-cap and mid-cap stock. Also, the average market capitalization of a large-cap stock and mid-cap stocks are presented.

Table 1. Descriptive statistics

	Mean annual return	Average standard deviation	Average market cap
S&P 500 stocks	0,078	0,017	$290 billion
S&P 400 stocks	-0,037	0,026	$8,9 billion

From table 1 it can be noticed that over the entire investment horizon between 1999-2021 large-cap stocks have yielded on average 7,8 percent annual returns, which is quite close to average market returns over a long time horizon. What is interesting to see is that top 15 weighted stocks from S&P 400 index yielded on average negative 3,7 percent annual returns. The sample mid-cap stocks suffered quite heavy negative returns during dot-com bubble and financial crisis, but they also saw negative annual return during the latter half of 2010's, when S&P 500 stocks had quite consistent positive annual returns. S&P 400 stocks also had on average higher standard deviation than S&P 500 stocks. This study did not aim to beat the overall stock market, but the table 1 results show that investing in top 15 constituents of S&P 400 index may not be desirable by itself if one wishes to achieve returns close to market returns.

When looking at the average market capitalizations of large-cap and mid-cap stocks in table 1 it can be seen that large-cap stock are considerably bigger in their size when compared to mid-cap stocks. Since S&P 500 index represent the 500 largest companies in the United States, the top 15 constituents average weight grew over time, and at the end of the investment period average large-cap stock from the sample had already close to $800 billion market capitalization. The average size of mid-cap stocks did not change that much during the investment period, since the S&P 400 index has market-cap limits for companies to be considered part of the index, so if they grow over that limit they were taken away from the index.

The sectors of top 15 constituents of both indexes changed over time. During the first half of investment period top constituents of S&P 500 had quite big weight on financial sector, having stocks like Citigroup, American International Group and Bank of America at the top of S&P 500 constituents. However, these financial sector stocks had quite severe crashes during the financial crisis, which led to other sectors taking place in top constituents. During 2010's big tech stocks started to take place on the top constituents of S&P 500, having companies like Apple, Alphabet and Amazon on the top constituents of the index. S&P 400 index's top 15 constituents changed more frequently than S&P 500 and their sectors were quite diversified overall. During the beginning of investment period top constituents of S&P 400 included companies like Gap, Goodyear and MGIC Investment. At the end of the investment period top mid-cap constituents had companies like Aspen Technology, Interactive Brokers Group and HF Sinclair.

5.1 Summarized results

Table 2 presents the results for the portfolios with 1-year rebalance time with Markowitz optimization and momentum adjustments. Table shows mean annualized returns for investigated portfolios, as well as standard deviations and Sharpe ratios. Also, the total return for the entire investment horizon is presented. The investigated portfolios included a portfolio using only Markowitz optimization, a portfolio using optimization with momentum adjustments, a portfolio using optimization with sentiment adjustments, a portfolio using optimization with both sentiment and momentum adjustments, and a portfolio with equal weights rebalanced to equal weights each time other portfolios were rebalanced with Markowitz optimization.

Table 2. Results for 1-year portfolios

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
Mean Annual Return	0,056	0,058	0,057	0,062	0,040
Stand. Dev.	0,190	0,193	0,191	0,195	0,197
Sharpe Ratio	0,328	0,333	0,335	0,349	0,229
Total Return	1,781	1,889	1,811	2,098	0,895

The results of table 2 show that every other portfolio beats equal weight portfolio, both with higher returns and lower standard deviations with 1 year rebalance. The results also show that applying momentum, sentiment, or both factors to the optimized portfolio led to increased portfolio performance by increasing returns and Sharpe ratios. Standard deviation is lowest on the portfolio using only optimizing. For 1-year portfolios the most noticeable results are between equal weight portfolio and the portfolio using both momentum and sentiment factors. Equal weight portfolio has a mean annual return of 4.0 percent and Sharpe ratio of 22,9 percent, whereas portfolio with added momentum and sentiment factors to optimized weights has mean annual return of 6.2 percent and Sharpe ratio of 34.9 percent. Total return differences are also significant between these portfolios.

Table 3 presents results for same portfolios with 6-month rebalance times between Markowitz optimization and momentum adjustments. Table 3 has somewhat similar results as table 2. With 6-month rebalance times equal weight portfolio has noticeably lower Sharpe ratio than other portfolios. Although equal weight portfolio has similar mean returns and standard deviation as the portfolio with only optimized weights, the deviation of risk-free rate over investment time was unbeneficial for equal weight portfolio when concerning Sharpe ratio.

Table 3. Results for 6-month portfolios

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
Mean Annual Return	0,047	0,060	0,051	0,060	0,048
Stand. Dev.	0,180	0,182	0,181	0,182	0,183
Sharpe Ratio	0,325	0,383	0,342	0,380	0,251
Total Return	0,989	1,675	1,339	1,677	1,283

When comparing the table 3 results between portfolio using only optimized weights and portfolios with added sentiment or momentum factors, it is again noticeable that each portfolio with added momentum or sentiment adjustments performed better than the portfolio with only optimized weights, both in terms of returns as well as with Sharpe ratio. However, this time the sentiment factor seemed to add less significant benefit when compared to momentum factor. Sentiment factor by itself increased returns when compared to only optimized portfolio, but its addition to portfolio having momentum factor did not provide excess returns. The best performance in terms of Sharpe ratio was with optimized portfolio with momentum factor, which had Sharpe ratio of 38.3 percent. Standard deviations were quite close to each other.

Table 4 shows results for portfolios with 3-month rebalance times. Once again, the equal weight portfolio performs the worst out of other portfolios by having lower returns and lower Sharpe ratio. With 3-month rebalance times the portfolio with optimized weights and sentiment adjustments performed worst out of other portfolios except the equal weight portfolio, by having lower returns and lower Sharpe ratio. This means that the addition of sentiment factor to the optimized weights by itself was not desirable. The two portfolios with momentum factor added to optimized weights, with or without sentiment factor, performed the best out of five portfolios, both in terms of higher returns and higher Sharpe ratios.

Table 4. Results for 3-month portfolios

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
Mean Annual Return	0,048	0,054	0,044	0,052	0,039
Stand. Dev.	0,182	0,182	0,183	0,183	0,188
Sharpe Ratio	0,235	0,278	0,233	0,270	0,198
Total Return	1,339	1,601	1,023	1,439	0,840

Table 5 shows the results for 1-month rebalance times portfolios. The results are similar to results in table 4. This time equal weight portfolio performed quite similarly to the optimized only portfolio and optimized portfolio with sentiment adjustments in terms of returns, but the equal weight portfolio had higher Sharpe ratio. Portfolios with added momentum factor to the optimized weights, with or without sentiment, performed the best out of five portfolios, both in terms of returns and Sharpe ratio. One thing to notice is that the returns and Sharpe ratios noticeably decreased in each portfolio when compared to the 3-month results.

Table 5. Results for 1-month portfolios

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
Mean Annual Return	0,033	0,047	0,033	0,047	0,036
Stand. Dev.	0,175	0,174	0,176	0,175	0,183
Sharpe Ratio	0,143	0,226	0,141	0,224	0,194
Total Return	0,705	1,304	0,736	1,354	0,711

Tables 2-5 present somewhat similar results with portfolios using different rebalance times with optimization and momentum adjustments. Adding the momentum factor to optimized weights or optimized weights with sentiment factor seems to provide the best results with each rebalance time, both by mean and total returns, as well as Sharpe ratios. The best overall results are with portfolios using 1-year and 6-month rebalance times, as the mean returns and Sharpe ratios slightly decrease when the rebalance times shortens to 3 months or 1-month. Standard deviations are on somewhat similar levels with each rebalance times, highest standard deviation being with 1-year rebalance times portfolios, and lowest with 1-month rebalance times portfolios.

The sentiment factor did not seem to add as much difference to optimized weights as the momentum factor. One reason being that while momentum adjustments were done during each rebalance time with optimization process, sentiment adjustments were only done during times of overoptimistic or overpessimistic sentiment, which meant that during neutral sentiment there were no adjustments done using the sentiment factor, so it was used less than the momentum factor. Sentiment factor had the most impact on returns and Sharpe ratios in table 2 with 1-year rebalance times with optimization and momentum adjustments, and in table 3 with 6-month rebalance time when added to only optimized portfolio.

Another way of looking at portfolio performances is to see if the portfolios seem to perform differently during different time periods. To do this portfolio performances between years 1999 to 2010 and 2011 to 2021 have been presented for each rebalance time portfolios in tables 6-9. These two time periods represent different kinds of periods in the stock market in a sense that during the years 1999-2010 there were both dot-com bubble in the late 1990's and early 2000's, as well as global financial crisis between years 2007-2008, both of which resulted in huge stock market crashes. The period between years 2011-2021 on the other hand was mainly prime time for the stock market, as the markets have mainly gone up during that time without as major bear markets as global financial crisis or dot-com bubble.

Table 6. 1-year portfolios between years 1999-2010 and 2011-2021

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
	Panel A: Years 1999-2010				
Mean Annual Return	0,025	0,025	0,023	0,023	0,021
Stand. Dev.	0,214	0,218	0,216	0,220	0,221
Sharpe Ratio	0,115	0,119	0,121	0,110	0,075
Total Return	0,128	0,125	0,087	0,082	0,053
	Panel B: Years 2011-2021				
Mean Annual Return	0,089	0,093	0,094	0,104	0,061
Stand. Dev.	0,164	0,167	0,164	0,167	0,172
Sharpe Ratio	0,560	0,567	0,568	0,609	0,396
Total Return	1,464	1,567	1,587	1,863	0,800

Table 7. 6-month portfolios between years 1999-2010 and 2011-2021

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
	Panel A: Years 1999-2010				
Mean Annual Return	0,004	0,016	0,012	0,018	0,018
Stand. Dev.	0,209	0,210	0,210	0,211	0,212
Sharpe Ratio	0,011	0,065	0,041	0,066	0,034
Total Return	-0,181	-0,044	-0,039	-0,022	0,044
	Panel B: Years 2011-2021				
Mean Annual Return	0,093	0,108	0,094	0,106	0,082
Stand. Dev.	0,148	0,151	0,148	0,151	0,152
Sharpe Ratio	0,666	0,730	0,672	0,722	0,487
Total Return	1,428	1,797	1,434	1,738	1,188

Table 8. 3-month portfolios between years 1999-2010 and 2011-2021

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
	Panel A: Years 1999-2010				
Mean Annual Return	0,022	0,025	0,013	0,020	0,019
Stand. Dev.	0,202	0,202	0,203	0,203	0,210
Sharpe Ratio	0,019	0,043	-0,007	0,022	0,045
Total Return	0,127	0,142	-0,038	0,051	0,033
	Panel B: Years 2011-2021				
Mean Annual Return	0,076	0,084	0,078	0,087	0,061
Stand. Dev.	0,160	0,161	0,160	0,161	0,164
Sharpe Ratio	0,470	0,533	0,475	0,540	0,364
Total Return	1,074	1,278	1,103	1,320	0,781

Table 9. 1-month portfolios between years 1999-2010 and 2011-2021

	Optimized only	Optimized + Momentum	Optimized + Sentiment	Optimized + Sentiment + Momentum	Equal Weights
	Panel A: Years 1999-2010				
Mean Annual Return	0,005	0,012	0,006	0,013	0,017
Stand. Dev.	0,197	0,195	0,198	0,196	0,206
Sharpe Ratio	-0,112	-0,073	-0,115	-0,073	0,043
Total Return	-0,076	0,001	-0,058	0,027	0,004
	Panel B: Years 2011-2021				
Mean Annual Return	0,064	0,085	0,064	0,084	0,057
Stand. Dev.	0,152	0,150	0,152	0,153	0,157
Sharpe Ratio	0,421	0,552	0,421	0,547	0,359
Total Return	0,845	1,301	0,842	1,292	0,705

From the results in tables 6-9 it is clear to see how much better each portfolio has performed during the latter years of 2011-2021. Mean and total returns have increased significantly, standard deviations are lower, and Sharpe ratios are higher. In the first period between 1999-2010 Sharpe ratios were lower both because of lower portfolio returns, but also because of higher yield on the risk-free asset of US 10 Year Treasury bond during that time, which decreased the excess return of the portfolios. Also, higher standard deviations during years 1999-2010 lowered Sharpe ratios. During years 2011-2021 the US 10 Year Treasury bond had significantly lower yield, portfolios had lower standard deviations and the portfolio returns were higher, all resulting in higher Sharpe ratios during the latter years.

When looking at the individual portfolios during years 1999-2010 with different rebalance times, it can be seen that adding a momentum factor to an optimized portfolio or to an optimized portfolio with sentiment adjustments seemed to have slightly positive effect on returns on other rebalance times other than 1-year rebalance time, but when looking at the total returns the differences are not that significant for a 12-year investment horizon. Adding a momentum factor increased Sharpe ratios on other rebalance times except for 1-year rebalance times, when it decreased Sharpe ratio when added to an optimized portfolio with sentiment adjustments.

During years 1999-2010 an addition of sentiment factor to an optimized portfolio or to an optimized portfolio with momentum factor adjustments slightly increased annual mean returns and total returns with 6-month and 1-month rebalance times. With 1-year or 3-month rebalance times adding a sentiment factor decreased annual mean returns and total returns for the same portfolios. Sharpe ratios were mainly increased with 6-month rebalance times and decreased or had mixed results with other rebalance times. During years 1999-2010 equal weighted portfolio performed better than other portfolios in terms of total returns with 6-month rebalance time. With 3-month rebalance time equal weighted portfolio had the highest Sharpe ratio.

For the years 2011-2021 adding a momentum factor to an optimized portfolio or to an optimized portfolio with sentiment factor increased both annual mean returns and total returns, as well as Sharpe ratios for all rebalance times. Most noticeable differences in total returns are with 1-month rebalance times during latter years. Adding a sentiment factor to an optimized portfolio or to an optimized portfolio with momentum factor increased annual mean returns and total returns, as well as Sharpe ratios with 1-year and 3-month rebalance times, but 6-month and 1-month rebalance times provided more mixed results for sentiment factor. With 6-month and 1-month rebalance times sentiment factor did not have any major effects on total returns during years 2011-2021. Equal weighted portfolio performed noticeably worse than other portfolios both in terms of returns and Sharpe ratios during years 2011-2021.

5.2 Momentum factor performance

Tables 10-13 present results of the momentum factor performance for different rebalance times. Tables present the total amount of momentum adjustments done and how many times the addition of momentum factor led to increased returns before next adjustment with optimization plus momentum factor adjustments. Panel A shows results of adding momentum factor to an optimized portfolio and panel B shows results of adding momentum factor to an optimized portfolio with sentiment adjustments.

Table 10. Momentum adjustment performance, 1-year portfolio

	Total	Times beat	Times lost
	Panel A: Momentum factor added to optimized portfolio		
Total adjustments	23	14 60,9 %	9 39,1 %
	Panel B: Momentum factor added to optimized portfolio with sentiment adjustments		
Total adjustments	23	12 52,2 %	11 47,8 %

Table 11. Momentum adjustment performance, 6-month portfolio

	Total	Times beat	Times lost
	Panel A: Momentum factor added to optimized portfolio		
Total adjustments	46	26 56,5 %	20 43,5 %
	Panel B: Momentum factor added to optimized portfolio with sentiment adjustments		
Total adjustments	46	26 56,5 %	20 43,5 %

Table 12. Momentum adjustment performance, 3-month portfolio

	Total	Times beat	Times lost
	Panel A: Momentum factor added to optimized portfolio		
Total adjustments	92	55 59,8 %	37 40,2 %
	Panel B: Momentum factor added to optimized portfolio with sentiment adjustments		
Total adjustments	92	55 59,8 %	37 40,2 %

Table 13. Momentum adjustment performance, 1-month portfolio

	Total	Times beat	Times lost
	Panel A: Momentum factor added to optimized portfolio		
Total adjustments	276	142 51,4 %	134 48,6 %
	Panel B: Momentum factor added to optimized portfolio with sentiment adjustments		
Total adjustments	276	141 51,1 %	135 48,9 %

The results on tables 10-13 show that the addition of momentum factor has always led to increased returns more times than it has led to decreased returns. The highest difference is with 1-year rebalance time when the momentum factor is added to optimized portfolio, having resulted in increased returns 60,9 % of the time. Although with 1-year rebalance time adding momentum factor to optimized portfolio with sentiment factor adjustments led to increased returns fewer times.

When concerning both panels A and B, the best results seem to be in table 12 with 3-month rebalance time. With 3-month rebalance time the addition of momentum factor led to increased returns 59,8 % of the time for both cases, meaning that the results were the same if momentum factor was added to only optimized portfolio or to portfolio with optimized weights plus sentiment adjustments. The worst results are with 1-month rebalance time in table 13, where the addition of momentum factor led to increased returns almost the same number of times as it led to decrease in returns. Although with 1-month rebalance time the number of increased returns is almost the same as number of decreased returns, this does not consider how much the returns were increased or decreased. From tables 5 and 9 we can see that with 1-month rebalance time the addition of momentum factor led to increased returns as well as increased Sharpe ratio.

5.3 Sentiment factor performance

Tables 14-17 present results on the sentiment factor performance for each rebalance times. Tables show how many times in total the addition of sentiment adjustments has led to an increase in returns before the next weight adjustments, both if the sentiment factor has been added to optimized only portfolio, and if the factor has been added to portfolio with optimized weights plus momentum factor adjustments. Also, the overoptimistic and overpessimistic signals have been examined separately.

Table 14 shows sentiment factor results for 1-year rebalance times. In total the addition of sentiment factor has led to an increase in returns slightly more times than it has led to a decrease in returns on both panels. With overpessimistic signal the results split evenly with addition of sentiment factor from overpessimistic signal resulting on increasing returns as many times as it has led to a decrease in returns. However, with overoptimistic signals the results show more times of increased returns from overoptimistic signal than times it has led to a decrease in returns on both panels, panel B having slightly more times sentiment factor favoring the upcoming returns.

Table 14. Sentiment adjustment performance, 1-year portfolio

	Total	Times beat	Times lost
	Panel A: Sentiment factor added to optimized portfolio		
Total adjustments	13	7 53,8 %	6 46,2 %
Total overoptimistic	7	4 57,1 %	3 42,9 %
Total overpessimistic	6	3 50,0 %	3 50,0 %
	Panel B: Sentiment factor added to optimized portfolio with momentum factor		
Total adjustments	13	8 61,5 %	5 38,5 %
Total overoptimistic	7	5 71,4 %	2 28,6 %
Total overpessimistic	6	3 50,0 %	3 50,0 %

Table 15 shows sentiment factor results for 6-month rebalance times. The results are somewhat similar as with table 14 results, in terms that in total the addition on sentiment factor led to an increase in returns before next adjustments slightly more times than it led to a decrease in returns on both panels. However, this time the overpessimistic signal led to an increase in returns more times than it led to a decrease in returns on both panels, and with overoptimistic signals the results split almost evenly on both panels. So, with 6-month rebalance times adjustments from the overpessimistic signal seemed to be more beneficial than adjustments from overoptimistic signal.

Table 15. Sentiment adjustment performance, 6-month portfolio

	Total	Times beat	Times lost
	Panel A: Sentiment factor added to optimized portfolio		
Total adjustments	17	9 52,9 %	8 47,1 %
Total overoptimistic	9	4 44,4 %	5 55,6 %
Total overpessimistic	8	5 62,5 %	3 37,5 %
	Panel B: Sentiment factor added to optimized portfolio with momentum factor		
Total adjustments	17	9 52,9 %	8 47,1 %
Total overoptimistic	9	4 44,4 %	5 55,6 %
Total overpessimistic	8	5 62,5 %	3 37,5 %

Table 16 shows sentiment factor results for 3-month rebalance times. This time the total amount of sentiment factor adjustments has led more times to a decrease in returns rather than an increase in returns on both panels. Adjustments from overoptimistic and overpessimistic signals have led to similar results in a way that for both signals there are more times the sentiment adjustments have led to a decrease in returns, so the times of decreased returns were not caused by the other signal alone. Results on table 16 show that the addition of sentiment factor was not beneficial on 3-month rebalance times.

Table 16. Sentiment adjustment performance, 3-month portfolio

	Total	Times beat	Times lost
	Panel A: Sentiment factor added to optimized portfolio		
Total adjustments	24	9 37,5 %	15 62,5 %
Total overoptimistic	12	5 41,7 %	7 58,3 %
Total overpessimistic	12	4 33,3 %	8 66,7 %
	Panel B: Sentiment factor added to optimized portfolio with momentum factor		
Total adjustments	24	8 33,3 %	16 66,7 %
Total overoptimistic	12	4 33,3 %	8 66,7 %
Total overpessimistic	12	4 33,3 %	8 66,7 %

Table 17 presents sentiment factor results for 1-month rebalance times. Out of all 42 total adjustments the results are quite even as adding the sentiment factor increased returns 22 times while it decreased returns 20 times on both panels. However, the results between overoptimistic and overpessimistic signals are quite interesting. The signals had equal number of adjustments based on their signal, but on both panels A and B the overpessimistic signal led to increased returns noticeably more times than it led to a decrease in returns. Overoptimistic signal on the other hand led to a decrease in returns more times than it led to an increase in returns on both panels A and B.

Table 17. Sentiment adjustment performance, 1-month portfolio

	Total	Times beat	Times lost
	Panel A: Sentiment factor added to optimized portfolio		
Total adjustments	42	22 52,4 %	20 47,6 %
Total overoptimistic	21	9 42,9 %	12 57,1 %
Total overpessimistic	21	13 61,9 %	8 38,1 %
	Panel B: Sentiment factor added to optimized portfolio with momentum factor		
Total adjustments	42	22 52,4 %	20 47,6 %
Total overoptimistic	21	8 38,1 %	13 61,9 %
Total overpessimistic	21	14 66,7 %	7 33,3 %

5.4 Evaluation of portfolio optimization and signal factors

The portfolio optimization process used optimization method developed by Markowitz (1952) to find optimal weights for portfolios including 30 stocks. Under investigation were portfolios with four different rebalance times, which were 1-month, 3 months, 6 months and 1-year. Each portfolio used the same length of historic daily stock price data as the time between each rebalance for given portfolio to construct covariance matrixes and average annual returns for return estimates in the optimization process. Each portfolio had minimum weight constraint of 1 percent for each stock and maximum weight constraint of 6 percent for each stock during optimization process.

The aim of the portfolio constraint was to mitigate the adverse effects of estimation errors, which would be caused by nonrestricted portfolios. The optimization constraints combined with the set of rules with momentum and sentiment factors aimed to offer modest increase in portfolio performance, without drastically increasing portfolio risk. When looking at the results in tables 2-5 it can be noticed that the standard deviations between each portfolio with same rebalance time were quite similar, but the standard deviation was always highest for the portfolio which was always rebalanced to equal weights. The equal weight portfolio always had the lowest Sharpe ratio, and typically lower mean returns. Optimized portfolios with both momentum and sentiment factor always beat the equal weight portfolio and only optimized portfolio in terms of higher Sharpe ratio and higher mean returns.

When concerning different rebalance times in tables 2-5 it can be noticed that the rebalance times of 1-year and 6-months seem to be performing best, and the performance drops with shorter rebalance times of 3 months and 1 month. Portfolio with 3-month and 1-month rebalance times have lower returns and lower Sharpe ratios than 6-month and 1year portfolios. The highest Sharpe ratio out of all portfolios comes from the 6-month rebalance time portfolio with both momentum and sentiment factor, reaching Sharpe ratio of 38.0 percent. 6-month portfolios also have lower standard deviations than portfolios with 1-year rebalance time. Similar results can be seen from tables 6-9 with split results between years 1999-2010 and 2011-2021, although years 1999-2010 seemed to be better in terms of returns and Sharpe ratios for 1-year rebalance time portfolios rather than 6-month portfolios. For years 2011-2021 portfolios with 1-year and 6-month rebalance times had similar performances.

These results suggest that with these methods for portfolio optimization and factor adjustments, selecting rebalance times of either 1-year or 6 months seems to be the best choice, as the returns and Sharpe ratios drop noticeably with shorter rebalance times. The results are beneficial when concerning real world trading with transaction costs, which would increase with more frequent portfolio adjustments in 1-month and 3-month portfolios, when compared to lesser adjustments with 6-month or 1-year portfolios. Higher transaction costs with 1-month and 3-month portfolios would have negative effect on portfolio returns, making these portfolios even less desirable when compared to 6-month and 1-year portfolios. The results are also supported by findings of Tokat and Wicas (2007), who found frequent rebalancing lowering average returns on trending markets, and even on mean-reverting markets they found frequent rebalancing leading to a decrease in absolute average returns.

When looking at the differences between equal weight portfolios to the portfolios with optimized weights with or without momentum and sentiment factor it can be noticed that the differences are somewhat modest during the first half of the investment horizon, but the equal weight portfolio starts to be left behind other portfolios' performances during latter years of investment horizon. Applying the optimization method and signal factors seem to improve the portfolio performance quite consistently from equal weights in terms of returns and Sharpe ratios over a long timeframe. As each portfolio essentially uses the same set of stocks and same rebalance times, the results suggest that the application of these optimization principles and rules for momentum and sentiment factors gets more out of the same stocks if compared to having equal weights across the board, but the investment time needs to be long enough. And on top of that having sentiment or momentum factor adjustments seems to generally improve the performance of only optimized portfolio, although momentum factor showed more consistent improvements. Portfolio weight constraints ensured that each stock has at least some weight in the portfolio and that the maximum weight cannot be too drastic, and the results suggest that this helped to mitigate adverse effects of estimation errors which would be caused from non-constricted portfolios.

When concerning the overall performance of the momentum factor it can be seen that it has been overall a well performing factor, which has led to increased returns over the entire investment process. The addition of the momentum factor has also led to increased Sharpe ratios. When looking at tables 6-9 the momentum factor seemed to lead in higher returns during latter years of investment horizon, but during first half of the investment horizon it did not have as noticeable effect on returns. This would suggest that the momentum factor performs better during an overall bullish market, as the first half of investment horizon saw two major stock market crashes resulting in decreasing stock prices across the market. Still the momentum factor performance supports the findings of Moskowitz et al. (2012) and Georgopoulou, and Wang (2017), who found time series momentum being able to predict assets' future performance based on its own historic performance. Momentum still seems to be quite persistent anomaly, even though studies like McLean and Pontiff (2016) suggest that momentum returns with other anomalies should be mitigated as new predictors are being published in academic journals.

When looking at the overall performance of sentiment factor it can be seen that it did not provide as consistent results as momentum factor. When concerning the overall returns in tables 2-9 when a sentiment factor is added to an optimized portfolio or to a portfolio with optimized weights with momentum adjustments, the sentiment factor seemed to have most positive impact on returns and Sharpe ratios with 1-year rebalance time, and during the latter half of the entire investment period. Positive impact from sentiment factor was also with 6-month rebalance times during the first half of the entire investment period. Otherwise, the sentiment factor did not seem to have as much impact on returns and Sharpe ratios, either by not resulting in much of a change or even by leading to decreased returns or Sharpe ratios.

Tables 14-17 may provide more insight to times when the addition of sentiment factor is beneficial. Tables 15 and 17 suggest that the sentiment adjustments may be more beneficial when implemented during overpessimistic sentiment, as the sentiment factor led to increased returns noticeably more times than it did when implemented during overoptimistic sentiment. In table 16 both signals led more times to decreased returns than to increased returns, and in table 14 the results were quite even but slightly tilting towards working better during overoptimistic signals. The overall tilt towards better performance during overpessimistic sentiment could be explained by the findings of Baker and Wurgler (2006) who found that during times of pessimistic sentiment small stocks tend to earn higher subsequent returns, but during optimistic sentiment they found no size effect at all. One part of recognizing sentiment sensitive stocks in this study was by their size, so the size effect may not have had as much difference during overoptimistic sentiment. Similarly, Uhl (2014) found negative sentiment having more influence on stock returns than positive sentiment.

One reason why the sentiment factor did not seem to add as much results as the momentum factor was due to the fact that adjustments based on sentiment factors signal were much less frequent than with momentum factor, which was used each time as optimization method was used. The sentiment adjustments were only done during times when sentiment reached overoptimistic or overpessimistic levels based on the sentiment index. From table 14 it can be seen that only 13 years out of 23 on the entire investment process were such years when sentiment reached these extreme levels, so there were 10 entire years when no sentiment adjustments were done at all. This led to a situation where portfolio performances with added sentiment factor were quite similar to ones without the sentiment factor.

Another reason why sentiment factor may not have performed as well as momentum factor has to do with the set of rules given to sentiment adjustments. As sentiment adjustments were only done during times when sentiment index reached overoptimistic or overpessimistic levels, the adjustments may have occasionally happened close to rebalance time with optimization and momentum adjustments, meaning that the weights provided by sentiment index's signals did not have a chance to last that long. This may explain why sentiment factor seemed to be most beneficial with 1-year and 6-month rebalance times, as the sentiment adjustments potentially had more time to effect portfolio returns. Sentiment could have also provided different or more noticeable results if different trading strategies were used, such as implementing sentiment adjustments based on rapid changes in sentiment even if sentiment does not reach extreme values.

The sentiment index on the other hand seemed to work as wanted. It did not provide unnecessary signals during neutral times in the markets, but for example during dot-com bubble, financial crisis and COVID-19 stock market crashes it provided signals of overpessimism, which suits the market mood during those times. But during the aftermath bull markets after these crashes the index provided signals of overoptimism. Each indicator in the sentiment index seemed to work quite well in terms of offering a balanced index for measuring sentiment. If one wishes to use looser or stricter rules for finding extreme values of sentiment it can be done easily by changing how many standard deviations away from mean value one concerns to be overoptimistic or overpessimistic.

5.5 Five-factor model + momentum regression

This section investigates the relation between risk and return of the portfolios in this study. A regression analysis is done based on the Fama and French (2015) five-factor model with addition of a momentum factor. The model is presented as

$$E(r_i) - r_f = \alpha_i + \beta_1\big(E(r_m) - r_f\big) + \beta_2 SMB + \beta_3 HML$$

$$+ \beta_4 RMW + \beta_5 CMA + \beta_6 MOM + \epsilon_i \tag{14}$$

where sixth factor MOM represent the average return of high prior return portfolio minus the average return of low prior return portfolio. Tables 18-21 present the regression coefficients and their significance for different rebalance times and the adjusted R Squares, which tell what proportion of variance in the dependent variable can be explained by the independent variable, when adjusted on the number of parameters in the model. T statistics is presented in parenthesis.

Table 18 presents the five-factor plus momentum factor model's regression results on the 1-year rebalance time portfolios. When first looking at the alphas of regression, which represents the intercepts, each portfolio has slightly negative alpha and the results are statistically significant, so it can be concluded that none of the 1-year rebalance time portfolios provided excess return to their benchmark when factoring all selected factors. Each portfolio also has $Mkt - RF$ coefficients close to 1 and statistically significant, meaning that the portfolios move quite closely with the overall markets. For none of the portfolios is SMB factor's coefficients statistically significant. HML factor's coefficient showed significance for optimized portfolio with sentiment factor and for optimized only portfolio.

When looking at the RMW factor on table 18 each portfolio showed positive and statistically significant coefficients for the factor. This suggests that 1-year rebalance time portfolios benefit during times when firms with robust profitability are outperforming firms with weak profitability. CMA factor's coefficients are positive but show significance at 10 percent significance level only on optimized portfolio with both momentum and sentiment factor as well as with optimized portfolio with sentiment factor. Lastly, MOM factor's coefficients are positive and statistically significant for each portfolio, meaning that the 1-year rebalance time portfolios benefit during times when high prior return stocks are outperforming low prior returning stocks. The adjusted R Squares for 1-year rebalance time portfolios are around 83-85 percent, meaning that 83-85 percent of the variability observed in the portfolios is explained by the regression model.

Table 18. Five-factor model + MOM regression factor coefficients, 1-year portfolios

Portfolios	Alpha	Mkt-RF	SMB	HML	RMW	CMA	MOM	Adj. R Square
Opt + mom + sent	-0,004***	1,032***	-0,015	0,051	0,134**	0,126*	0,124***	0,832
	(-3,204)	(33,235)	(-0,351)	(1,027)	(2,533)	(1,746)	(5,061)	
Opt + mom	-0,004***	1,014***	-0,032	0,051	0,142***	0,089	0,154***	0,833
	(-3,439)	(33,591)	(-0,764)	(1,051)	(2,767)	(1,272)	(6,444)	
Opt + sent	-0,004***	1,013***	-0,021	-0,08*	0,152***	0,124*	0,076***	0,853
	(-3,669)	(35,528)	(-0,541)	(1,76)	(3,132)	(1,875)	(3,37)	
Opt only	-0,004***	0,985***	-0,038	0,089**	0,157***	0,070	0,106***	0,853
	(3,780)	(35,872)	(-0,990)	(2,022)	(3,369)	(1,099)	(4,878)	

Alpha is the intercept, t statistics presented in parentheses, (***), (**), (*) coefficient is significant at 1%, 5%, 10% significant level

Table 19 presents the regression results for the 6-month portfolios. Alphas are once again slightly negative, but this time show significance only with optimized portfolio with sentiment factor and with optimized only portfolio. Coefficients for $Mkt - RF$ factor are again positive and statistically significant, but with slightly smaller values than in table 18 results. Portfolios in table 19 still move quite closely with the overall markets. For factors SMB, HML and RMW only the optimized only portfolio seems to show statistically significant coefficients for these factors, SMB being negative and others positive. This means that with 6-month rebalance time portfolio with only optimized weights benefits when large stocks are outperforming small stocks, when value stocks are outperforming growth stocks and when robust profitability firms are outperforming weak profitability firms. These results seem to disappear when either momentum or sentiment factor is added, except for optimized portfolio with sentiment factor, since it has a negative SMB coefficient at 10 percent significance level.

For table 19 results CMA factors coefficients did not show any significance for any portfolio. A noticeable difference to table 18 results is with MOM factors coefficients in table 19. None of the portfolios in table 19 show statistical significance for MOM factor's coefficient, when in table 18 each portfolio showed significance at 1 percent significance level. 6-month rebalance time portfolios were the first portfolios which used previous six months' worth of historical data on stock performance to evaluate if the stock had momentum when compared to benchmark, but with 1-year portfolio the momentum was evaluated from previous 1-year performance. This might be one factor explaining the difference. However, the adjusted R Square results in table 19 range between 50-65 percent, which is noticeably lower than in table 18 results. This shows that the selected model did not explain as much portfolio variability than in table 18 results.

Table 19. Five-factor model + MOM regression factor coefficients, 6-month portfolios

Portfolios	Alpha	Mkt-RF	SMB	HML	RMW	CMA	MOM	Adj. R Square
Opt + mom + sent	-0,003	0,839***	-0,106	0,12	0,096	0,111	0,045	0,503
	(-1,560)	(14,903)	(-1,360)	(1,330)	(1,005)	(0,853)	(1,016)	
Opt + mom	-0,004	0,836***	-0,118	0,115	0,092	0,121	0,072	0,497
	(-1,649)	(14,945)	(-1,515)	(1,282)	(0,969)	(0,935)	(1,62)	
Opt + sent	-0,004**	0,838***	-0,150*	0,130	0,158	0,173	-0,011	0,509
	(-2,034)	(14,855)	(-1,912)	(1,442)	(1,646)	(1,324)	(-0,242)	
Opt only	-0,004**	0,889***	-0,194***	0,132**	0,142**	0,168	-0,003	0,652
	(-2,455)	(19,952)	(-3,127)	(1,847)	(1,873)	(1,627)	(-0,083)	

Alpha is the intercept, t statistics presented in parentheses, (***), (**), (*) coefficient is significant at 1%, 5%, 10% significant level

Table 20 present regression model results for the 3-month rebalance time portfolios. Once again, alphas are slightly negative and statistically significant, meaning that the portfolios did not provide excess return to their benchmark when factoring all selected factors. $Mkt - RF$ factors are close to 1 and statistically significant, meaning that these portfolios also move quite closely with the market. None of the portfolios showed statistical significance for SMB factor.

Each 3-month rebalance time portfolio in table 20 shows statistical significance at 1 percent significance level for HML factor's coefficient, and each coefficient is positive. This means that the portfolios benefit during times when value stocks are outperforming growth stocks. RMW factor's coefficients are also positive and show statistical significance at 1 percent significance level for each portfolio except for the optimized portfolio with sentiment factor, as it shows no significance at any level. For other portfolios it can be concluded that they benefit during times when robust profitability firms are outperforming weak profitability firms.

When it comes to CMA factor's coefficients none of the portfolios in table 20 show statistical significance for this factor's coefficients. With MOM factor the two portfolios with added momentum factor show statistical significance at 5 percent significance level for MOM factor's coefficients, meaning that they benefit during times when high prior return stocks are outperforming low prior return stocks. Optimized only and optimized portfolio with sentiment adjustments do not show significance for MOM factor's coefficient. The adjusted R Squares in table 20 range between 83-84 percent, meaning that the regression model explained similar amount of variability observed in the portfolios as in table 18.

Table 20. Five-factor model + MOM regression factor coefficients, 3-month portfolios

Portfolios	Alpha	Mkt-RF	SMB	HML	RMW	CMA	MOM	Adj. R Square
Opt + mom + sent	-0,005***	1,000***	0,026	0,191***	0,178***	0,049	0,048**	0,839
	(-3,760)	(32,636)	(0,608)	(3,902)	(3,415)	(0,695)	(2,001)	
Opt + mom	-0,004***	0,980***	0,031	0,193***	0,184***	0,025	0,060**	0,838
	(-3,577)	(32,604)	(0,748)	(4,011)	(3,606)	(0,538)	(2,512)	
Opt + sent	-0,005***	0,985***	0,040	0,183***	0,200	0,087	-0,024	0,833
	(-3,821)	(-30,970)	(0,907)	(3,596)	(3,697)	(1,180)	(-0,944)	
Opt only	-0,004***	0,966***	0,045	0,185***	0,207***	0,066	-0,015	0,830
	(-3,639)	(30,700)	(1,024)	(3,669)	(3,866)	(0,904)	(-0,619)	

Alpha is the intercept, t statistics presented in parentheses, (***), (**), (*) coefficient is significant at 1%, 5%, 10% significant level

Finally, table 21 presents regression results for the 1-month rebalance time portfolios. Similar to table 20, the alphas on table 21 are slightly negative and statistically significant, meaning that the portfolios did not provide excess return to their benchmark when factoring all selected factors. $Mkt - RF$ coefficients are again close to 1 and statistically significant, so the portfolios move quite closely with the overall markets. None of the portfolios show statistical significance for SMB factor's coefficients. When looking at HML factor, each portfolio shows either 1 or 5 percent significance for HML factor's coefficients them being positive, meaning that the portfolios seem to benefit during times when value stocks are outperforming growth stocks.

Each portfolio in table 21 also shows statistical significance at 1 percent significance level for RMW factor's coefficients, which are all positive. This suggests that the portfolios benefit during times when robust profitability firms are outperforming weak profitability firms. CMA factor's coefficients are also positive and statistically significant at either 5 or 10 percent significance levels for each portfolio. This suggests that the portfolios in table 21 benefit during times when low investment firms are outperforming high investment firms. MOM factor's coefficients show statistical significance only for optimized portfolio and for optimized portfolio with sentiment adjustments. The coefficients are negative, which suggests that these two portfolios benefit during times when low prior return stocks are outperforming high prior return stocks. The adjusted R Squares in table 21 range between 83-84 percent, meaning that the regression model explained similar amount of variability observed in the portfolios as in tables 18 and 20.

Table 21. Five-factor model + MOM regression factor coefficients, 1-month portfolios

Portfolios	Alpha	Mkt-RF	SMB	HML	RMW	CMA	MOM	Adj. R Square
Opt + mom + sent	-0,005***	0,992***	0,006	0,119**	0,193***	0,145**	-0,019	0,838
	(-3,812)	(32,217)	(0,138)	(2,412)	(3,686)	(2,035)	(-0,763)	
Opt + mom	-0,005***	0,987***	0,009	0,121**	0,186***	0,138*	-0,017	0,840
	(-3,881)	(32,420)	(0,212)	(2,481)	(3,594)	(1,955)	(-0,721)	
Opt + sent	-0,006***	0,997***	0,015	0,141***	0,228***	0,163**	-0,099***	0,838
	(-4,440)	(30,853)	(0,337)	(2,726)	(4,145)	(2,175)	(-3,886)	
Opt only	-0,006***	0,991***	0,020	0,144***	0,219***	0,159**	-0,098***	0,838
	(-4,472)	(30,815)	(0,440)	(2,803)	(3,994)	(2,133)	(-3,871)	

Alpha is the intercept, t statistics presented in parentheses, (***), (**), (*) coefficient is significant at 1%, 5%, 10% significant level

The regression results in tables 18-21 have some similar aspects to each other, but there are also some differences. When concerning for similarities, each portfolio with different rebalance times had slightly negative alpha and positive $Mkt - RF$ coefficient which was close to 1, suggesting that the portfolios move quite closely with the overall market. This can be explained by the fact that each portfolio used the same set of stocks, and half of the stocks were the biggest constituents of S&P 500 index, which is often used as a market performance benchmark index. The SMB factor did not seem to explain the performances for almost any of the portfolios. HML factor showed clear significance for portfolios with 1- or 3-month rebalance times, but not as clear results for 6-month or 1-year rebalance time portfolios.

The RMW factor seemed to show clear significance for other rebalance times other than with 6-month rebalance times in table 19, where the RMW factor's coefficients did not show any significance other than for only optimized portfolio. CMA factor's coefficients mainly showed significance for 1-month rebalance time portfolios in table 21. MOM factor also had some different results. For 1-year rebalance times in table 18 the factor's coefficients showed clear significance with positive values for each portfolio. In table 19 with 6-month rebalance time portfolios the coefficient was not significant for any portfolio, but with 3-month rebalance time results in table 20 the MOM factor's coefficient showed statistical significance for the two portfolios using momentum factor with positive coefficients, meaning that they benefitted during times when stocks exercising momentum returns outperformed stocks with low prior returns. With 1-month rebalance times MOM factor's coefficients showed negative values with statistical significance for the two portfolios that did not have momentum adjustments done to them.

Overall, the regression model seemed to explain quite well the amount of variability observed in the portfolios, as the adjusted R Squares were around 83-85 percent. The only exception was for the 6-month rebalance time portfolios in table 19, where the adjusted R Square values were noticeably lower than for other tables. For 6-month rebalance times the returns were mainly explained by the market factor $Mkt - RF$. None of the other factors' coefficients explained the portfolio returns as much as the market factor, which can be seen with lower values for other coefficients. This can be explained by the fact that the stocks in the portfolios were quite diversified into different types of stocks, so none of the portfolios were created to capture only certain risk factors by themselves, such as for example a portfolio consisting of only small growth stocks, which would have higher exposure to SMB and HML factors.

6 CONCLUSION

Mean-variance analysis with Markowitz (1952) optimization method has shown its drawbacks in financial literature. The optimization method's risk and return estimates have been seen as subject to estimation errors, which leads to overinvestments in stocks with favourable estimates and underinvestment in stocks with unfavourable estimates. Also, having no weight restrictions causes estimation errors to skyrocket. Previous literature has mainly focused on enhancing optimal portfolio by improving variance-covariance matrix estimates.

This study investigated whether Markowitz's optimal portfolio's performance could be improved by having the simplified estimates for the optimization process but including momentum and sentiment factors for weight adjustments. Momentum factor was based on time series momentum and sentiment factor worked as contrarian indicator, which provided signals of overoptimism and overpessimism based on constructed sentiment index. The optimization process had some constraints and signal factors had a set of rules, which both aimed to offer modest improvements to portfolio performance over the investment period.

The results suggest that the addition of momentum factor provides consistent improvements to portfolio performance, which can be seen as increased returns and increased Sharpe ratio. The momentum factor seems to provide better results during an overall bull market than during bear markets. The addition of sentiment factor provided mixed results, and it had less significant impact on portfolio performance than momentum factor. This may be due to the selected rules for sentiment factor, but mostly the lesser significance of sentiment factor may be due to the fact that sentiment adjustments were only done during extreme sentiment signals, which were less frequent than momentum adjustments. Sentiment factor's results also tilted more towards working better with overpessimistic signals with 6-month and 1-month rebalance times. Sentiment index worked as planned so different trading rules based on sentiment may lead to different results.

Under investigation were portfolios with four different rebalance times, which were 1-year, 6-month, 3-month and 1-month rebalance times. 1-year and 6-month rebalance times provided the best portfolio performances, which was seen as higher returns and higher Sharpe ratios. These results are supported by findings of Tokat and Wicas (2007), who found frequent rebalancing lowering average returns. Five-factor model plus momentum factor regression model showed that each portfolio moves quite closely with the overall markets, but different portfolios had differing factors showing statistical significance in explaining portfolio returns.

The overall results show that equal weight portfolio's performance can be enhanced by implementing optimization method and signal factors to the same set of stocks over a long time. Momentum factor seems to remain as one of the most persistent anomalies in explaining portfolio returns. Sentiment factor's mixed results suggest having different rules for sentiment adjustments being beneficial. Further research could investigate how different signal factors would affect optimal portfolio performance, if the covariance matrix estimates and expected return estimates were also enhanced for weight optimization.

REFERENCES

Baker, M., & Wurgler, J. (2006). Investor sentiment and the cross-section of stock returns. *The journal of Finance*, *61*(4), 1645-1680.

Baker, M., & Wurgler, J. (2007). Investor sentiment in the stock market. *Journal of economic perspectives, 21*(2), 129-152.

Beer, F., & Zouaoui, M. (2013). Measuring stock market investor sentiment. Journal of Applied Business Research (JABR), 29(1), 51-68.

Billingsley, R. S., & Chance, D. M. (1988). Put-call ratios and market timing effectiveness. *Journal of Portfolio Management, 15*(1), 25.

Black, F. (1993). Estimating expected return. *Financial Analysts Journal, 49*(5), 36.

Brown, G. W., & Cliff, M. T. (2004). Investor sentiment and the near-term stock market. *Journal of empirical finance*, *11*(1), 1-27.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57-82.

Chau, F., Deesomsak, R., & Koutmos, D. (2016). Does investor sentiment really matter?. *International Review of Financial Analysis, 48*, 221-232.

Chen, H. (2017). Do cash flows of growth stocks really grow faster?. *The Journal of Finance*, *72*(5), 2279-2330.

Chordia, T., Subrahmanyam, A., & Tong, Q. (2014). Have capital market anomalies attenuated in the recent era of high liquidity and trading activity?. *Journal of Accounting and Economics, 58*(1), 41-58.

Chung, S. L., Hung, C. H., & Yeh, C. Y. (2012). When does investor sentiment predict stock returns?. *Journal of Empirical Finance, 19*(2), 217-240.

Conrad, J., & Yavuz, M. D. (2017). Momentum and reversal: Does what goes up always come down?. *Review of Finance, 21*(2), 555-581.

De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. *Journal of political Economy*, *98*(4), 703-738.

Domian, D. L., & Racine, M. D. (2006). An empirical analysis of margin debt. *International Review of Economics & Finance*, *15*(2), 151-163.

Edelen, R. M., Marcus, A. J., & Tehranian, H. (2010). Relative sentiment and stock returns. *Financial Analysts Journal, 66(*4), 20-32.

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance, 25*(2), 383–417. https://doi.org/10.2307/2325486

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3-56.

Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The journal of finance*, *51*(1), 55-84.

Fama, E. F., & French, K. R. (2008). Dissecting anomalies. *The Journal of Finance*, *63*(4), 1653-1678.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of financial economics*, *116*(1), 1-22.

Fama, E. F., & French, K. R. (2017). International tests of a five-factor asset pricing model. *Journal of financial Economics*, *123*(3), 441-463.

Fama, E. F., & French, K. R. (2020). Comparing cross-section and time-series factor models. *The Review of Financial Studies*, *33*(5), 1891-1926.

Fan, J., Fan, Y., & Lv, J. (2008). High dimensional covariance matrix estimation using a factor model. *Journal of Econometrics*, *147*(1), 186-197.

Fisher, K. L., & Statman, M. (2000). Investor sentiment and stock returns. *Financial Analysts Journal*, *56*(2), 16-23.

Fortune, P. (2001). Margin lending and stock market volatility. *New England Economic Review*, 3-26.

Frost, P. A., & Savarino, J. E. (1988). For better performance: Constrain portfolio weights. *Journal of portfolio management*, *15*(1), 29.

Georgopoulou, A., & Wang, J. (2017). The trend is your friend: Time-series momentum strategies across equity and commodity markets. *Review of Finance*, *21*(4), 1557-1592.

Goyal, A., & Jegadeesh, N. (2018). Cross-sectional and time-series tests of return predictability: What is the difference?. *The Review of Financial Studies*, *31*(5), 1784-1824.

Griffin, J. M., & Lemmon, M. L. (2002). Book-to-market equity, distress risk, and stock returns. *The Journal of Finance*, *57*(5), 2317-2336.

Hou, K., Xue, C., & Zhang, L. (2015). Digesting anomalies: An investment approach. *The Review of Financial Studies*, *28*(3), 650-705.

Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance, 48*(1), 65.

Jegadeesh, N., & Titman, S. (2001). Profitability of momentum strategies: An evaluation of alternative explanations. *The Journal of finance*, *56*(2), 699-720.

Jobson, J. D., & Korkie, B. (1980). Estimation for Markowitz efficient portfolios. *Journal of the American Statistical Association*, *75*(371), 544-554.

Kenneth R. French (2022). Data library, home page. Retrieved from https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Ledoit, O., & Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of empirical finance*, *10*(5), 603-621.

Ledoit, O., & Wolf, M. (2004). A well-conditioned estimator for large-dimensional covariance matrices. *Journal of multivariate analysis*, *88*(2), 365-411.

Li, J. (2020). The momentum and reversal effects of investor sentiment on stock prices. *The North American Journal of Economics and Finance*, *54*, 101263.

Liew, J., & Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth?. *Journal of Financial Economics*, *57*(2), 221-245.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics & Statistics, 47*(1), 13.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*(1), 77–91. https://doi.org/10.2307/2975974

McLean, R. D., & Pontiff, J. (2016). Does academic research destroy stock return predictability?. *The Journal of Finance*, *71*(1), 5-32.

Michaud, R. O. (1989). The Markowitz optimization enigma: Is 'optimized'optimal?. *Financial analysts journal*, *45*(1), 31-42.

Moskowitz, T. J., Ooi, Y. H., & Pedersen, L. H. (2012). Time series momentum. *Journal of financial economics, 104*(2), 228-250. ISO 690

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of financial economics*, *108*(1), 1-28.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

Simon, D. P., & Wiggins III, R. A. (2001). S&P futures returns and contrary sentiment indicators. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *21*(5), 447-462.

Stambaugh, R. F., Yu, J., & Yuan, Y. (2012). The short of it: Investor sentiment and anomalies. *Journal of Financial Economics*, *104*(2), 288-302.

Stein, C. (1956). Inadmissibility of the usual estimator for the mean of a multivariate normal distribution. In *Contribution to the Theory of Statistics* (197-206). University of California Press.

Titman, S., Wei, K. J., & Xie, F. (2004). Capital investments and stock returns. *Journal of financial and Quantitative Analysis*, *39*(4), 677-700.

Tokat, Y., & Wicas, N. W. (2007). Portfolio rebalancing in theory and practice. *The Journal of Investing*, *16*(2), 52-59.

Uhl, M. W. (2014). Reuters sentiment and stock returns. *Journal of Behavioral Finance*, *15*(4), 287-298.